Elipsa: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $b=\sqrt{a^2-c^2}$; $\varepsilon=\frac{c}{a}=\sqrt{1-\frac{b^2}{a^2}}$; raze focale: $\begin{cases} r_1 = a+\varepsilon x \\ r_2 = a-\varepsilon x \end{cases}$, tangenta prin M_0 ($M_0\in$ elipsei): $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$

normala: $a^2y_0x-b^2x_0y-(a^2-b^2)x_0y_0=0$; tangenta pantă k: $y=kx\pm\sqrt{a^2k^2+b^2}$, panta tang. $k_{1,2}=\frac{-x_1y_1\pm\sqrt{b^2x_1^2+a^2y_1^2-a^2b^2}}{x_1^2-a^2}$ ($M_1\notin$ elipsei)

dacă $M_1\in x=\pm a \Rightarrow k=\pm\frac{y_1^2-b^2}{2ay_1}$ **Hiperbola:** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $b=\sqrt{c^2-a^2}$; $\varepsilon=\frac{c}{a}=\sqrt{1+\frac{b^2}{a^2}}>1$; raze focale: $\begin{cases} r_1 = a+\varepsilon x \\ r_2 = a-\varepsilon x \end{cases}$

tangenta ($M_0\in$ hip): $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$, normala: $a^2y_0x+b^2x_0y-(a^2+b^2)x_0y_0=0$; panta tangentei: $y=kx\pm\sqrt{a^2k^2-b^2}$, $k^2>\frac{b^2}{a^2}$!

panta tangentei ($M_1\notin$ hip): $k_{1,2}=\frac{y_1x_1\pm\sqrt{a^2y_1^2-b^2x_1^2+a^2b^2}}{x_1^2-a^2}$ dacă $M_1\in x=\pm a$: $k=\pm\frac{y_1^2+b^2}{2ay_1}$ **Parabola:** $y^2=2px$; tangenta ($M_0\in$ par.): $yy_0=p(x+x_0)$

tangenta prin panta k: $y=kx+\frac{p}{2k}$ ($k\neq0$), panta tangentei punct exterior: $y-y_1=k(x-x_1)$, unde $k: 2x_1k^2-2y_1k+p=0$ sau $y-y_1=\frac{p}{2y_1}$ ($M_1\in Oy$)

Cuadrica: coord. verifică $a_{11}x^2+2a_{12}xy+2a_{13}xz+a_{22}y^2+2a_{23}yz+a_{33}z^2+a_{14}x+a_{24}y+a_{34}z+a_{44}=0$; **Elipsoid** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

vârfurile sale $(\pm a,0,0)$; $(0,\pm b,0)$, $(0,0,\pm c)$; Elipsoid de rotație: $\frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=1$; cc plan tangențial $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}+\frac{zz_0}{c^2}=1$

Conul de gradul al doilea $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$; intersecția cu xOy $\left\{\frac{x^2}{a^2h^2/c^2}+\frac{y^2}{b^2h^2/c^2}=1, z=h\right\}$, intersecția cu xOz $\left\{\frac{x}{a}\pm\frac{z}{c}=0, y=0\right\}$ sau $\left\{\frac{z^2}{c^2h^2/b^2}-\frac{x^2}{a^2h^2/b^2}=1, y=h\right\}$; intersecția cu yOz $\left\{\frac{y}{b}\pm\frac{z}{c}=0, x=0\right\}$ sau $\left\{\frac{y^2}{b^2h^2/a^2}-\frac{z^2}{c^2h^2/a^2}=1, x=h\right\}$

plan tangent la con: $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}-\frac{zz_0}{c^2}=0$; con de rotație: $\frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=0$ **Hiperboloid cu o pânză** $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

$\cap$ cu $xOy\Rightarrow$ elipsa $\left\{z=h, \frac{x^2}{\left(a\sqrt{\frac{h^2}{c^2}+1}\right)^2}+\frac{y^2}{\left(b\sqrt{\frac{h^2}{c^2}+1}\right)^2}=1\right\}$; $\cap$ cu yOz $\left\{\begin{matrix} x=h \\ |h|<|a| \end{matrix}, \frac{y^2}{\left(b\sqrt{1-\frac{h^2}{a^2}}\right)^2}-\frac{z^2}{\left(c\sqrt{1-\frac{h^2}{a^2}}\right)^2}=1\right\}$; $\left\{\begin{matrix} x=h \\ |h|=|a| \end{matrix}, \frac{y}{b}\pm\frac{z}{c}=0\right\}$,

$\left\{\begin{matrix} x=h \\ |h|>|a| \end{matrix}, \frac{z^2}{\left(c\sqrt{\frac{h^2}{c^2}-1}\right)^2}-\frac{y^2}{\left(b\sqrt{\frac{h^2}{a^2}-1}\right)^2}=1\right\}$; plan tangent $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}-\frac{zz_0}{c^2}=1$; generatoare $\begin{cases} \lambda\left(\frac{x}{a}+\frac{z}{c}\right)=\mu\left(1-\frac{y}{b}\right) \\ \mu\left(\frac{x}{a}-\frac{z}{c}\right)=\lambda\left(1-\frac{y}{b}\right) \end{cases}$ $\begin{cases} \alpha\left(\frac{x}{a}+\frac{z}{c}\right)=\beta\left(1-\frac{y}{b}\right) \\ \beta\left(\frac{x}{a}-\frac{z}{c}\right)=\alpha\left(1+\frac{y}{b}\right) \end{cases}$

hip de rotație: $\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=1$; plan tangent la hip cu 2 pânze $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}-\frac{zz_0}{c^2}=-1$ **Paraboloid eliptic** $\frac{x^2}{p}+\frac{y^2}{q}=2z$:

$\cap$ cu xOy, $z=h$: $\{\emptyset, h<0\}$, $\{$originea, $h=0\}$ $\left\{\frac{x^2}{2ph}+\frac{y^2}{2qh}=1, h>0\right\}$ (elipsa); $\cap$ yOz $\left\{x=h, y^2=2qz-\frac{qh^2}{p}\right\}$; paraboloid de rotație $x^2+y^2=2pz$

planul tang: $\frac{xx_0}{p}+\frac{yy_0}{q}=p(z+z_0)$ **Paraboloidul hiperbolic** $\frac{x^2}{p}-\frac{y^2}{q}=2z$; $\cap$ cu xOy $\left\{\begin{matrix} \text{o hiperbolă} \\ h<0 \end{matrix}, z=h, \frac{y^2}{-2qh}-\frac{x^2}{-2ph}=1\right.$

$\left\{\begin{matrix} \text{drepte} \\ \text{pereche} \\ h=0 \end{matrix}; z=0; \frac{x}{\sqrt{p}}\pm\frac{y}{\sqrt{q}}=0\right\}$; $\left\{\begin{matrix} \text{hiperbola} \\ h>0 \end{matrix}; z=h, \frac{x^2}{2ph}-\frac{y^2}{2qh}=1\right\}$; plan tang $\frac{xx_0}{p}-\frac{yy_0}{q}=z+z_0$; generatoare

$\begin{cases} \lambda\left(\frac{x}{\sqrt{p}}-\frac{y}{\sqrt{q}}\right)=2\mu z \\ \mu\left(\frac{x}{\sqrt{p}}+\frac{y}{\sqrt{q}}\right)=\lambda \end{cases}$ și $\begin{cases} \alpha\left(\frac{x}{\sqrt{p}}+\frac{y}{\sqrt{q}}\right)=2\beta z \\ \beta\left(\frac{x}{\sqrt{p}}-\frac{y}{\sqrt{q}}\right)=\alpha \end{cases}$ **Cilindru eliptic** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $\cap$ cu xOy $\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \\ z=h \end{cases}$; $\cap$ cu yOz $\begin{cases} \frac{y^2}{b^2}=1-\frac{h^2}{a^2} \\ x=h \end{cases}\Rightarrow$

- $|h|<|a|$ 2 dr. $\parallel$ cu Oz: $\begin{cases} y=\pm b\sqrt{1-\frac{h^2}{a^2}} \\ x=h \end{cases}$
- $|h|=|a|$ o dreaptă
- $\emptyset$ $|h|>|a|$

plan tangențial $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$; cilindru eliptic de rotație ($a=b$): $x^2+y^2=a^2$ **Cilindru hiperbolic** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; $\cap$ cu xOy $\left\{z=h; \frac{x^2}{a^2}-\frac{y^2}{b^2}=1\right\}$; $\cap$ cu yOz $\left\{\frac{y^2}{b^2}=\frac{h^2}{a^2}-1, x=h\right\}\Rightarrow\emptyset$ $|h|<a$, o dreaptă $\parallel Oz$ $|h|=a$; o pereche de drept. $\parallel Oz$ $\begin{cases} y=\pm b\sqrt{\frac{h^2}{a^2}-1} \\ x=h \end{cases}$

$\cap$ cu zOx: $\begin{cases} x=\pm a\sqrt{\frac{h^2}{b^2}+1} \\ y=h \end{cases}$; planul tangent: $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$ **Cilindru parabolic** $y^2=2px$; $\cap$ xOy $\begin{cases} y^2=2px \\ z=h \end{cases}$; $\cap$ cu yOz

$\begin{cases} y^2=2px \\ x=h \end{cases}\Rightarrow\emptyset$ $h<0$, axa Ox $h=0$, o pereche de drepte ($h>0$) $\begin{cases} y=\pm2\sqrt{ph} \\ x=h \end{cases}$; $\cap$ cu xOz dreapta $\begin{cases} x=\frac{h^2}{2p} \\ y=h \end{cases}$

Generatoare de suprafețe; Suprafețe cilindrice: generatoare care se mișcă $\parallel$ cu o dreaptă directoare + $\cap$ cu curba dată.

$\Delta\begin{cases} P_1(x,y,z)=0 \\ P_2(x,y,z)=0 \end{cases}$ $(C)\begin{cases} F(x,y,z)=0 \\ G(x,y,z)=0 \end{cases}$ $G_{\lambda,\mu}\begin{cases} P_1(x,y,z)=\lambda \\ P_2(x,y,z)=\mu \end{cases}$ dependenți prin $\varphi(\lambda,\mu)=0$ $\Rightarrow$ ecuația suprafeței cilindrice $\varphi(P_1(x,y,z), P_2(x,y,z))=0$

suprafața conică: generatoare + trec prin punctul V + $\cap$ cu curba C.

vârf $(V)\begin{cases} P_1(x,y,z)=0 \\ P_2(x,y,z)=0 \\ P_3(x,y,z)=0 \end{cases}$ $G_{\lambda,\mu}\begin{cases} P_1(x,y,z)=\lambda P_3(x,y,z) \\ P_2(x,y,z)=\mu P_3(x,y,z) \end{cases}$ $\varphi(\lambda,\mu)\Rightarrow$ ecuația suprafeței conice: $\varphi\left(\frac{P_1(x,y,z)}{P_3(x,y,z)}, \frac{P_2(x,y,z)}{P_3(x,y,z)}\right)$

suprafața conoidă: generatoare + $\cap$ cu dreapta directoare + $\parallel$ cu un plan + $\cap$ cu curba C

$(\Delta)\begin{cases} P_1(x,y,z)=0 \\ P_2(x,y,z)=0 \end{cases}$ (π) $P(x,y,z)$ $G_{\lambda,\mu}\begin{cases} P_1(x,y,z)=\lambda P_2(x,y,z) \\ P(x,y,z)=\mu \end{cases}$ (analog φ) $\Rightarrow\varphi\left(\frac{P_1(x,y,z)}{P_2(x,y,z)}, P(x,y,z)\right)=0$

Suprafața de rotație: curba care se rotește în jurul unei drepte (axă de rotație)

$\Delta: \frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$ $G_{\lambda,\mu}\begin{cases} (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=\lambda^2 \\ lx+my+nz=\mu \end{cases}$

$\varphi\left(\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}, lx+my+nz\right)=0$

Vectori $\vec{a}\cdot\vec{b}=\|\vec{a}\|\cdot\|\vec{b}\|\cdot\cos(\angle\vec{a},\vec{b})$, $\|\vec{a}\|=\sqrt{a\cdot a}=\sqrt{a^2}$, $(\lambda\vec{a}+\beta\vec{b})\cdot\vec{c}=\lambda(\vec{a}\vec{b})+\beta(\vec{b}\cdot\vec{c})$, $\vec{a}\perp\vec{b}\Leftrightarrow\vec{a}\cdot\vec{b}=0$

$(\vec{a}(a_1,a_2,a_3),\vec{b}(b_1,b_2,b_3))$: $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$ $\|\vec{a}\|=\sqrt{a_1^2+a_2^2+a_3^2}$ $\|\vec{a}\times\vec{b}\|=\|\vec{a}\|\times\|\vec{b}\|\cdot\sin\alpha$

$$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}=\begin{vmatrix} a_2 & a_3\\ b_2 & b_3\end{vmatrix}i-\begin{vmatrix} a_1 & a_3\\ b_1 & b_3\end{vmatrix}j+\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}k$$; produs mixt: $(a,b,c)=(\vec{a}\times\vec{b})\cdot\vec{c}$

Dreapta în plan $M_0(x_0,y_0)$ $\vec{a}(l,m)$ – v. director $Ax+By+C=0$ (ec. gen) $p_1(-B,A)$ – v. director

$\begin{cases} x=x_0+l\cdot t\\ y=y_0+m\cdot t\end{cases}$; $\frac{x-x_0}{l}=\frac{y-y_0}{m}$; $(x-x_0)(y_1-y_0)=(y-y_0)(x_1-x_0)$; $y-y_0=\frac{m}{l}(x-x_0)$; $\vec{n}(A,B)$ – v. normal

$r-r_0=ta$; $\frac{x}{a}+\frac{y}{b}-1=0$; $d_1\perp d_2\Leftrightarrow A_1A_2+B_1B_2=0$; $d_1\parallel d_2$: $A_1B_2-A_2B_1=0$; $d(M_0,\Delta)=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

Dreapta în spațiu

$r=r_0+t\cdot\vec{a}$ r – v de poz

$\begin{cases} x=x_0+lt\\ y=y_0+mt\\ z=z_0+nt\end{cases}$; $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$; $\frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0}$

$\begin{cases} A_1x+B_1y+C_1z+D_1=0\\ A_2x+B_2y+C_2z+D_2=0\end{cases}$ (dreapta ∩ a 2 plane) $d(M_1,\Delta)=\frac{\|(r_1-r_0)\times\vec{a}\|}{\|\vec{a}\|}$

$\vec{a}=\vec{n_1}(A_1,B_1,C_1)\times\vec{n_2}(A_2,B_2,C_2)$

$d_1\perp d_2\Leftrightarrow l_1l_2+m_1m_2+n_1n_2=0$ $d_1\parallel d_2\Leftrightarrow\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$

Planul în spațiu π – plan $M_0(x_0,y_0,z_0)$ $a_1(l_1,m_1,n_1)$ $a_2(l_2,m_2,n_2)$ vectori necoliniari $\parallel$ cu π

$\begin{cases} x=x_0+l_1u+l_2v\\ y=y_0+m_1u+m_2v\\ z=z_0+n_1u+n_2v\end{cases}$ $\begin{vmatrix} x-x_0 & y-y_0 & z-z_0\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0$ (ec. det de 1 punct, 2 vectori) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$ (det de 3 puncte)

$\overrightarrow{M_1M_2}(x_2-x_1, y_2-y_1, z_2-z_1)$

$Ax+By+Cz+D=0$

$\vec{n}(A,B,C)$ – v. normal

$\pi_1\perp\pi_2\Leftrightarrow A_1A_2+B_1B_2+C_1C_2=0$ $\pi_1\parallel\pi_2\Leftrightarrow\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$

Dreapta și planul: π_1: $A_1x+B_1y+C_1z+D_1=0$ π_2: $A_2x+B_2y+C_2z+D_2=0$

• taie după 1 dreaptă ⇒ rang$\begin{pmatrix} A_1 & B_1 & C_1\\ A_2 & B_2 & C_2\end{pmatrix}=0$ • sunt $\parallel$: $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}\neq\frac{D_1}{D_2}$ • coincid: $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}$

• Δ și π au un punct comun $Al+Bm+Cn\neq0$ (ec param p Δ) $\Delta\parallel\pi$: $Al+Bm+Cn=0$ și $Ax_0+By_0+Cz_0+D\neq0$

$\Delta\in\pi$: $Al+Bm+Cn=0$ și $Ax_0+By_0+Cz_0+D=0$

$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0$ (ec plan det de 2 drepte)

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ l & m & n\end{vmatrix}=0$ (ec det de 1 dreaptă și un punct)

$A=\frac{1}{2}|\Delta|$ $\Delta=\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$