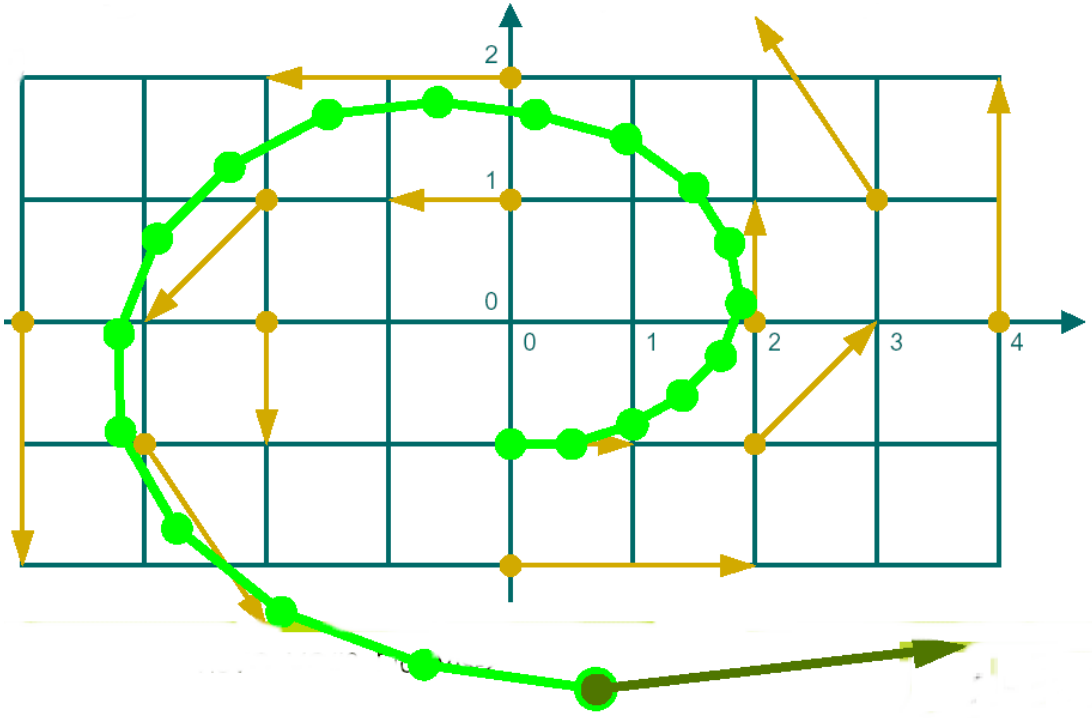
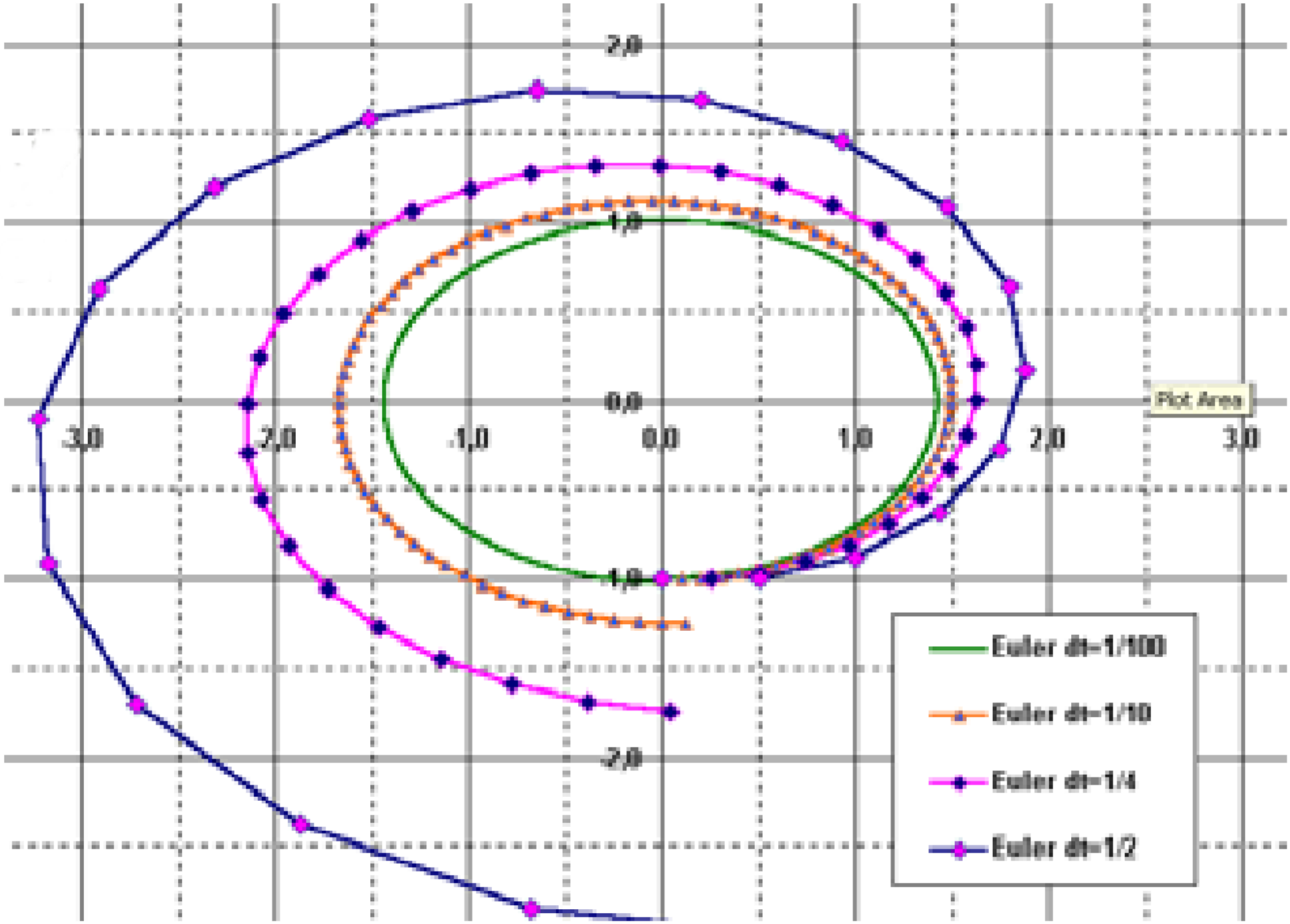
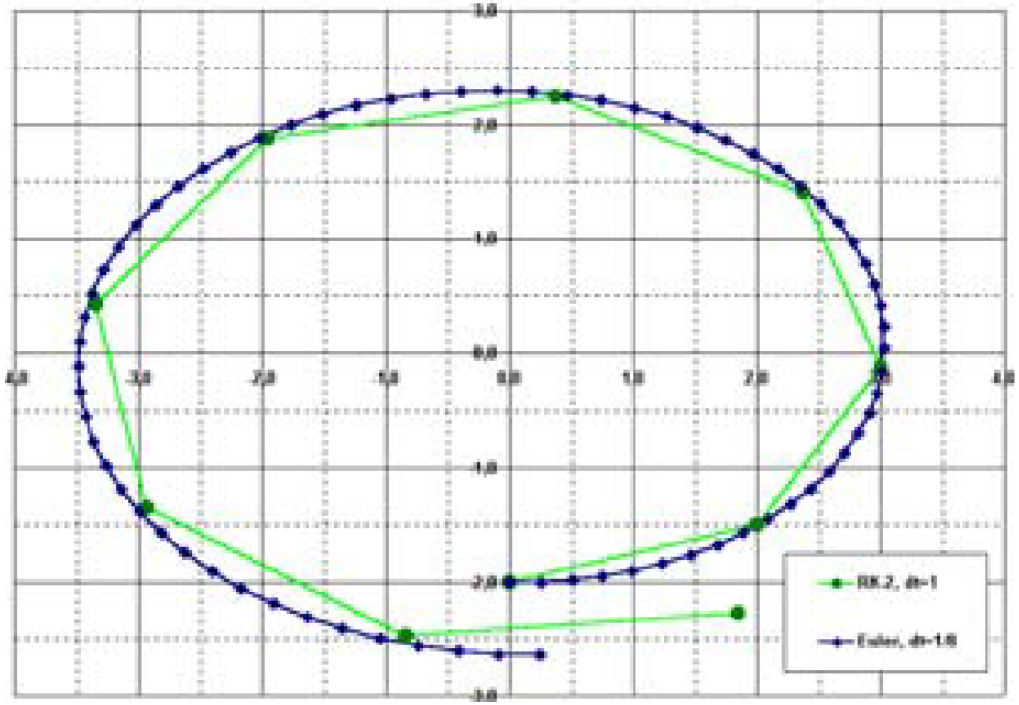
- **Euler integration**:
- Idea: $\mathbf{x}_{i+1} = \mathbf{x}_i + s\ \mathbf{v}(\mathbf{x}_i)$
- *s*: step size
- Large numerical error!
- Error proportional to s^2




2.0
1.0
0.0
-1.0
-2.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
Plot Area
Euler dt=1/100
Euler dt=1/10
Euler dt=1/4
Euler dt=1/2

- **Second order Runge Kutta integration**:
- Idea:
  - go half a step forward
  - evaluate vector there
  - use at starting point
- $\mathbf{x}_{i+1} = \mathbf{x}_i + s\ \mathbf{v}(\mathbf{x}_i + s/2\ \mathbf{v}(\mathbf{x}_i))$
- better than Euler



- **Fourth order Runge Kutta integration**:
- Standard; better than second order RK
  - Resembles the true solution up to s^4 (error proportional to s^5)
- Idea: step is convex
  - Combination of 4 vectors

$$\mathbf{x}_{i+1} = \mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i) = \mathbf{x}_i + s \cdot \left(\frac{\mathbf{v}_1}{6} + \frac{\mathbf{v}_2}{3} + \frac{\mathbf{v}_3}{3} + \frac{\mathbf{v}_4}{6} \right)$$

with

$$\mathbf{v}_1 = \mathbf{v}(\mathbf{x}_i) \quad , \quad \mathbf{v}_2 = \mathbf{v}\left(\mathbf{x}_i + \frac{s}{2} \mathbf{v}_1 \right)$$

$$\mathbf{v}_3 = \mathbf{v}\left(\mathbf{x}_i + \frac{s}{2} \mathbf{v}_2 \right) \quad , \quad \mathbf{v}_4 = \mathbf{v}\left(\mathbf{x}_i + s\, \mathbf{v}_3 \right)$$