- **Adaptive step size for RK4:**
- Idea: in areas of rather straight flow the step size can be larger than in areas of highly curved stream lines
- do RK3 ➔ difference between RK4 and RK3 gives estimation for step size

$$\mathbf{f}_{\mathrm{RK3}}(\mathbf{x}_i) = \mathbf{x}_i + s \cdot \left(\frac{\mathbf{v}_1}{6} + \frac{\mathbf{v}_2}{3} + \frac{\mathbf{v}_3}{3} + \frac{\mathbf{v}(\mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i))}{6} \right)$$

gives $$\Delta = \mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i) - \mathbf{f}_{\mathrm{RK3}}(\mathbf{x}_i) = \frac{s}{6}(\mathbf{v}_4 - \mathbf{v}(\mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i)))$$

- **Adaptive step size for RK4:**

- Define error tolerance t. Then optimal step size is:

$$s^* = s \cdot \sqrt[5]{\rho \frac{t}{\Delta}}$$

$\rho > 1$: safety factor

- ask integrator to compute Δ with step size s
- If $\Delta > t$, then repeat current step with step size s^*
- otherwise proceed and take $s = \min(s^*, s_{\max})$ for next step

- This integration is called RK4(3).