2-

1.

The likelihood function for the dataset X consists of the joint probability density of all observations, given that they are independent and identically distributed from a normal distribution.

$$L(\mu) = \prod_{i=1}^{n} f(x_i \mid \mu)$$

$$L(\mu) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \cdot 4}} \exp\left(-\frac{(x_i - \mu)^2}{2 \cdot 4}\right)$$

$$L(\mu) = \left(\frac{1}{\sqrt{8\pi}}\right)^n \exp\left(-\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{8}\right)$$

$$L(\mu) = \left(\frac{1}{\sqrt{8\pi}}\right)^n \exp\left(-\frac{1}{8}\sum_{i=1}^{n}(x_i - \mu)^2\right)$$

2.

We have the likelihood function from last part. The log-likelihood function ℓ(μ) is given by:

$$\ell(\mu) = \log L(\mu) = \log\left(\left(\frac{1}{\sqrt{8\pi}}\right)^n \exp\left(-\frac{1}{8}\sum_{i=1}^{n}(x_i - \mu)^2\right)\right)$$

Now we take the derivative of ℓ(μ) with respect to μ and set it to zero:

$$\frac{d\ell(\mu)}{d\mu} = -\frac{1}{8} \cdot 2\sum_{i=1}^{n}(x_i - \mu) = 0$$

Rearrange the equation to solve for μ:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sum_{i=1}^{n} x_i - n\mu = 0 \qquad \sum_{i=1}^{n}(x_i - \mu) = 0$$

3.

We have the log-likelihood function and first derivative from last part.

Now, we differentiate $d\ell(\mu) \backslash d(\mu)$ with respect to μ to get the second derivative:

$$\frac{d^2\ell(\mu)}{d\mu^2} = -\frac{1}{4}\sum_{i=1}^{n}\frac{d}{d\mu}(x_i - \mu)$$

Since $d/d\mu\ (x_i-\mu) = -1$, the second derivative becomes:

$$\frac{d^2\ell(\mu)}{d\mu^2} = -\frac{1}{4}\sum_{i=1}^{n}(-1) = -\frac{n}{4}$$

4.

where n is the number of data points. For the dataset X = {3,5,4,6,7}, we have: n=5

sum of data points = 25

sample mean = 5

Therefore, the maximum likelihood estimate (MLE) of μ for the given dataset is: 5