# P S Solution

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## 1 Problem

Pullela Gopichand (PG) and Saina Nehwal (SN) are thinking of playing a series of badminton matches. However, as Pullela Gopichand is an old man now, he believes he has only a 40% chance of winning one game. To make the game equally challenging for both, he comes up with a twist. He says that PG & SN will keep on playing until either SN wins 4 consecutive games or PG wins 3 consecutive games.
With this scheme, what is the chance of PG winning?

## 2 Solution

Define the following events-

1. $E_{i,w}$ = PG wins the $i^{th}$ game
2. $E_{i,l}$ = PG loses the $i^{th}$ game
3. $E_w$ = PG wins finally

We need to find the $\mathrm{P}(E_w) = p$
Let $p_w$ denote $\mathrm{P}(E_w|E_{1,w})$
Let $p_l$ denote $\mathrm{P}(E_w|E_{1,l})$

By law of total Probability, `https://en.wikipedia.org/wiki/Law_of_total_Probability`

$\mathrm{P}(E_w) = \mathrm{P}(E_w \cap E_{1,w}) + \mathrm{P}(E_w \cap E_{1,l})$
$\Rightarrow \mathrm{P}(E_w) = \mathrm{P}(E_w|E_{1,w})\mathrm{P}(E_{1,w}) + \mathrm{P}(E_w|E_{1,l})\mathrm{P}(E_{1,l})$
$\Rightarrow p = p_w 0.4 + p_l(0.6) = 0.4p_w + 0.6p_l$

Now, we will attempt to find $p_w$ and $p_l$. Interestingly, this will be a solution to two equations in two variables which can be constructed as below:

For the calculations of $p_w$, we realize that PG can win in the following mutually exclusive ways:

1. PG wins the second game and PG wins the third game
2. PG wins the second game, loses the third game and then goes on to win somehow
3. PG loses the second game and then goes on to win somehow

Thus, again using the law of total probability

$p_w = \mathrm{P}(E_w|E_{1,w})= \mathrm{P}((E_w \cap E_{2,w})|E_{1,w}) + \mathrm{P}((E_w \cap E_{2,l})|E_{1,w})$

$\Rightarrow p_w = \mathrm{P}((E_w \cap E_{2,w})|E_{1,w}) + \mathrm{P}(E_w|(E_{2,l} \cap E_{1,w}))\ \mathrm{P}(E_{2,l}|E_{1,w})$

We now note that $\mathrm{P}(E_w|(E_{2,l} \cap E_{1,w})) = p_l$ and $\mathrm{P}(E_{2,l}|E_{1,w}) = 0.6$

Thus,
$\Rightarrow p_w = \mathrm{P}((E_w \cap E_{2,w})|E_{1,w}) + p_l(0.6)$

$\Rightarrow p_w = \mathrm{P}((E_w \cap E_{2,w} \cap E_{3,w})|E_{1,w}\ ) + \mathrm{P}((E_w \cap E_{2,w} \cap E_{3,l})|E_{1,w}\ ) + p_l(0.6)$

$\Rightarrow p_w = \mathrm{P}((E_w \cap E_{2,w} \cap E_{3,w})|E_{1,w}\ ) + \mathrm{P}((E_w \cap E_{2,w} \cap E_{3,l})|E_{1,w}\ ) + p_l(0.6)$

$\Rightarrow p_w = \mathrm{P}(E_w|(E_{2,w} \cap E_{3,w} \cap E_{1,w}))\ \mathrm{P}((E_{2,w} \cap E_{3,w})|E_{1,w}\ ) + \mathrm{P}((E_w \cap E_{2,w} \cap E_{3,l})|E_{1,w}\ ) + p_l(0.6)$

$\Rightarrow p_w = (1)(0.4)(0.4) + \mathrm{P}((E_w \cap E_{2,w} \cap E_{3,l})|E_{1,w}\ ) + p_l(0.6)$

$\Rightarrow p_w = (1)(0.4)(0.4) + \mathrm{P}(E_w|(E_{2,w} \cap E_{3,l} \cap E_{1,w}))\ \mathrm{P}((E_{2,w} \cap E_{3,l})|E_{1,w}) + p_l(0.6)$

$\Rightarrow p_w = (1)(0.4)(0.4) + p_l(0.4)(0.6) + p_l(0.6)$

A similar exercise for $p_l$ will give the following expression -

$\Rightarrow p_l = (0)(0.6)(0.6)(0.6) + p_w(0.6)(0.6)(0.4) + p_w(0.6)(0.4) + p_w(0.4)$

Solving these two equations will give -

$p_w = \frac{500}{1067}$

$p_l = \frac{392}{1067}$

Thus, from the equation - $p = p_w 0.4 + p_l(0.6)$, we get

p = 0.40787253983 approximately or 2176/5335

The results agree with the simulation results.