Assignment 1

Applied Mathematics – I — BS-111

Lecturer: Dr. Anita Gupta (Class N)

Problem 1

If $\theta = t^n e^{\frac{-r^2}{4t}}$, find the value of n for which

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$$

Answer: $n = \frac{-3}{2}$

Problem 2

If $u = f(r)$ where $r^2 = x^2 + y^2 + z^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r}f'(r)$$

Problem 3

If $x + y = 2e^{\theta}cos\phi$ and $x - y = 2ie^{\theta}sin\phi$, show that

$$\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy\frac{\partial^2 u}{\partial x \partial y}$$

Problem 4

Show that the function $u = x + 2y + z$, $v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$ are functionally dependent. Find the relation between them.

Answer: $u^2 - v^2 = 4w$

Problem 5

If $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$, find $\frac{\partial(x_1,x_2,x_3)}{\partial(y_1,y_2,y_3)}$

Answer: $\frac{1}{4}$

Problem 6

If $u^3+v^3+w^3=x+y+z$, $u^2+v^2+w^2=x+y+z$, $u+v+w=x^2+y^2+z^2$, then prove that

$$\frac{\partial(u,v,w}{\partial(x,y,z)}=\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$$

Problem 7

If x increases at the rate of 2cm/sec at the instant when $x=3$ and $y=1$, at what rate must y be changing in order that the function $2xy-3x^2y$ shall be neither increasing nor decreasing?

Answer: y must be decreasing at the rate of $\frac{32}{21}$ cm/sec

Problem 8

Show that $dF=\frac{x}{(x^2+y^2}dy-\frac{y}{(x^2+y^2)dx}$ is an exact differential.

Problem 9

Show that

$$\int_0^x \frac{dx}{(x^2+a^2)^2}=\frac{x}{2a^2(x^2+a^2}+\frac{1}{2a^3}\,tan^{-1}\left(\frac{x}{a}\right)$$

by differentiating

$$\int_0^x \frac{dx}{(x^2+a^2)}=\frac{1}{a}\,tan^{-1}\left(\frac{x}{a}\right)$$

under the integral sign.

Problem 10

Prove that $\int_0^\infty \frac{e^{-ax}sin(\lambda x}{x}dx=tan^{-1}\frac{\lambda}{a}$, hence deduce that $\int_0^\infty \frac{sin\lambda x}{x}dx=\frac{\pi}{2}$

Problem 11

Discuss the maxima and minima of

$$f(x,y)=x^4+y^4-2x^2+4xy+2y^2$$

Answer: There is a minima at $(\sqrt{2},-\sqrt{2})$ and $(-\sqrt{2},\sqrt{2})$; Min $f=-8$ and the case is doubtful and further investigation is needed at $(0,0)$

Problem 12

Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2+y^2+z^2 = 1$

Answer: Maximum Distance = 14, Minimum Distance = 12