Assignment-1

Que1 If $\theta = t^n e^{-r^2/4t}$, find the value of n for which

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

[A $n = -3/2$]

Que2 If $u = f(r)$ where $r^2 = x^2+y^2+z^2$, prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = f''(r)+\frac{2}{r}f'(r)$$

Que3 If $x+y = 2e^{\theta}\cos\phi$ and $x-y = 2ie^{\theta}\sin\phi$, show that

$$\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial \phi^2} = 4xy\frac{\partial^2 u}{\partial x\,\partial y}$$

Que4 Show that the function $u = x+2y+z$, $v = x-2y+3z$ and $w = 2xy-xz+4yz-2z^2$ are functionally dependent. Find the relation between them. [A $u^2-v^2 = 4w$]

Que5 If $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$, find $\frac{\partial(x_1,x_2,x_3)}{\partial(y_1,y_2,y_3)}$

[A 1/4]

Que6 If $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$, $u+v+w = x^2+y^2+z^2$, then prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$$

Que7 If x increases at the rate of 2 cm/sec at the instant when $x = 3$ cm and $y = 1$ cm, at what rate must y be changing in order that the function $2xy-3x^2y$ shall be neither increasing nor decreasing?

[A y must be decreasing at the rate of 32/21 cm/sec]

Que 8 Show that $df = \frac{x}{(x^2+y^2)}dy - \frac{y}{(x^2+y^2)}dx$ is an exact differential.

Que 9 Show that $\int_0^x \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right)$ by differentiating $\int_0^x \frac{dx}{(x^2+a^2)} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ under the integral sign.

Que 10 Prove that $\int_0^\infty \frac{e^{-ax}\sin\lambda x}{x}dx = \tan^{-1}\frac{\lambda}{a}$, hence deduce that $\int_0^\infty \frac{\sin\lambda x}{x}dx = \frac{\pi}{2}$

Que 11 Discuss the maxima and minima of

$f(x,y) = x^4+y^4-2x^2+4xy-2y^2$

[**Ans** There is minima at $(\sqrt{2},-\sqrt{2})$ and $(-\sqrt{2},\sqrt{2})$; Min $f=-8$ and the case is doubtful and further investigation is needed at $(0,0)$]

Que 12 Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$

[**Ans** Max. distance $= 14$, Min distance $= 12$]