MATH I ASSIGINMENT 1

Q

Qu1 If $\theta = t^n e^{-r^2/4t}$, find the value of n for which

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t} \qquad [\text{A } n = -3/2]$$

$$\theta = t^n e^{-r^2/4t}$$

$$\frac{\partial \theta}{\partial t} = (t^n)'(e^{-r^2/4t}) + (t^n)(e^{-r^2/4t})'$$

$$\Rightarrow (n t^{n-1})(e^{-r^2/4t}) + (t^n)(e^{-\frac{r^2}{4t}})\left(-\left(-\frac{r^2}{4t^2}\right)\right)$$

$$\Rightarrow e^{-r^2/4t}\left\{ n t^{n-1} + \frac{t^{n-1}(+r^2)}{4t} \right\}$$

$$\Rightarrow (e^{-r^2/4t})(t^{n-1})\left\{ n + \frac{r^2}{4t} \right\} \text{ --- } 1$$

$$\frac{\partial \theta}{\partial r} = t^n e^{-r^2/4t} \cdot \left(-\frac{2r}{4t}\right) \Rightarrow t^n e^{-\frac{r^2}{4t}}\left(-\frac{r}{2t}\right)$$

$$\Rightarrow -\left(\frac{t^{n-1}}{2}\right)(r)\left(e^{-\frac{r^2}{4t}}\right)$$

$$r^2\frac{\partial \theta}{\partial r} = -\left(\frac{t^{n-1}}{2}\right)(r^3)\left(e^{-\frac{r^2}{4t}}\right)$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = -\left(\frac{t^{n-1}}{2}\right)\left\{ 3r^2\left(e^{-\frac{r^2}{4t}}\right) + r^3\left(e^{-\frac{r^2}{4t}}\right)\left(\frac{-\cancel{2}r}{\cancel{4}_2 t}\right) \right\}$$

$$\Rightarrow -\left(\frac{t^{n-1}}{2}\right)\left(e^{-\frac{r^2}{4t}}\right)\left\{ 3r^2 - \frac{r^4}{2t} \right\}$$

$$\Rightarrow -\left(\frac{t^{n-1}}{2}\right)\left(e^{-\frac{r^2}{4t}}\right)(r^2)\left\{ 3\cancel{r} - \frac{r^2}{2t} \right\}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = -\left(\frac{t^{n-1}}{2}\right)\left(e^{-\frac{r^2}{4t}}\right)\left(\frac{\cancel{r^2}}{\cancel{r^2}}\right)1\left\{ 3\cancel{r} - \frac{r^2}{2t} \right\}$$

Q2

Que 2 If $u = f(r)$ where $r^2 = x^2+y^2+z^2$, prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$$

$$r^2 = x^2+y^2+z^2$$

$$\hookrightarrow 2r\frac{\partial r}{\partial x} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r} \quad \frac{\partial r}{\partial y} = y/r \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} \Rightarrow f'(r)\left(\frac{x}{r}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(f'(r)\left(\frac{x}{r}\right)\right) \Rightarrow \left\{f''(r)\left(\frac{x}{r}\right)\right\}\left(\frac{x}{r}\right) + f'(r)\left(\frac{r - r'x}{r^2}\right)$$

$$\Rightarrow f''(r)\left(\frac{x}{r}\right)^2 + f'(r)\left(\frac{r - \frac{x}{r}\times x}{r^2}\right)$$

$$\text{"} \quad + \quad \text{"} \left(\frac{r^2 - x^2}{r^3}\right)$$

$$\Rightarrow f''(r)\left(\frac{x^2+y^2+z^2}{r^2}\right) + f'(r)\left\{\frac{3r^2 - x^2 - y^2 - z^2}{r^3}\right)$$

$$\Rightarrow f''(r)\left(\frac{r^2}{r^2}\right) + f'(r)\left\{\frac{3r^2 - r^2}{r^3}\right)$$

$$\Rightarrow f''(r)(1) + f'(r)\left\{\frac{2\cancel{r^2}}{\cancel{r^3}}\right)$$

$$\Rightarrow f''(r) + \frac{2}{r}f'(r)$$

Q3

Que3 If $x+y = 2e^{\theta}\cos\phi$ and $x-y = 2ie^{\theta}\sin\phi$, show that

$$\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy\frac{\partial^2 u}{\partial x \partial y}$$

$$x+y = 2e^{\theta}\cos\phi \qquad x-y = 2ie^{\theta}\sin\phi$$

$$x+y = 2e^{\theta}\cos\phi$$
$$x-y = 2ie^{\theta}\sin\phi$$

$$[x = e^{\theta}\{\cos\phi + i\sin\phi\}] \rightarrow e^{\theta}\cdot e^{i\phi}$$

$$y = 2e^{\theta} - e^{\theta}\{\cos\phi + i\sin\phi\}$$
$$\Rightarrow [e^{\theta}\{2-(\cos\phi + i\sin\phi)\}] \quad e^{\theta}\{2-e^{i\phi}\}$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$\Rightarrow \frac{\partial u}{\partial x}\left[e^{\theta}\{\right.$$

$2^2\}$

Que 4 Show that the function $u = x+2y+z$, $V = x-2y+3z$ and $w = 2xy - xz + 4yz - 2z^2$ are functionally dependent. Find the relation between them. [A $u^2 - v^2 = 4w$]

$U = x + 2y + z$, $V = x - 2y + 3z$

$w = 2xy - xz + 4yz - 2z^2$

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Equating 1 & 2

$$\cancel{e^{-\frac{r^2}{4t}}}\cancel{(t^{n-1})}\left\{n+\frac{r^2}{4t}\right\} = -\left(\frac{\cancel{t^{n-1}}}{2}\right)\cancel{\left(e^{-\frac{r^2}{4t}}\right)}\left\{3 - \frac{r^2}{2t}\right\}$$

$$\Rightarrow \quad n + \frac{r^2}{4t} = \frac{r^2}{2t} - \frac{3}{2}$$

$$\Rightarrow \quad n + \cancel{\frac{r^2}{4t}} = -\frac{3}{2} + \cancel{\frac{r^2}{4t}}$$

$$\boxed{n = \frac{-3}{2}}$$

$\therefore$ Statement