Radiography

X-Ray Computed Tomography

Faezeh Moshfeghifar
Melanie Ganz-Benjaminsen

famo@di.ku.dk

UNIVERSITY OF COPENHAGEN

Objectives

- Describe the difference between gamma and x-rays
- Explain how an X-ray image is formed
- Write down the formula for the Radon transform
- Draw a schematic diagram describing back-projection
- Explain how filtered back-projection differs

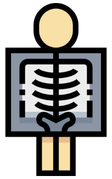
Outline

- Introduction to imaging modalities
- X-ray and CT basics
- Image reconstruction
- The radon transform
- Filtered back-projection

Medical imaging modalities

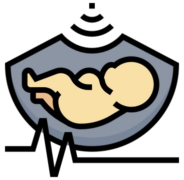
▽ Source of energy used for imaging

External Energies



Radiography (RX)
Computed Tomography (CT)

Dental Cone Beam CT



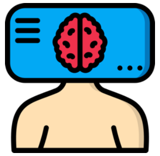
Ultrasound Imaging
& Tomography

Internal

Nuclear medicine:

- Single Photon Emission Tomography (SPECT)
- Positron Emission Tomography (PET)

Internal & External

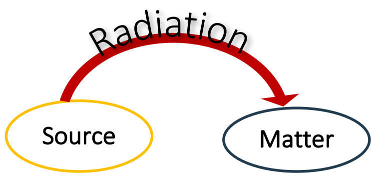


Magnetic Resonance Imaging:

- MRI
- FMRI
- PMRI

Figure 1. Different medical imaging modalities
Icons made by smashingstocks, Eucalyp, Eucalyp, Freepik, Smashicons from Flaticon

Radiation



- Emission of energy from a source through space or a material medium
- Described as ***wave*** and as particle-like units called ***photons***
- Produced in one of two ways
  - Interaction of a particle with matter
  - Radioactive decay of an unstable atom
- Transfer of energy from emitted radiation to matter
  - Ionization
  - Excitation

Table 1. The two types of radiation

Radiation	Energy	Mass	Charged	Examples
Electromagnetic	✓	×	×	- X-rays - gamma (γ) radiation
Particulate	✓	✓	✓×	- beta radiation (β) - alpha radiation (α) - neutrons

Electromagnetic (EM) wave

- Consisting of oscillating electric (E) and magnetic (B) waves
- E and B are perpendicular to each other and to the direction of propagation

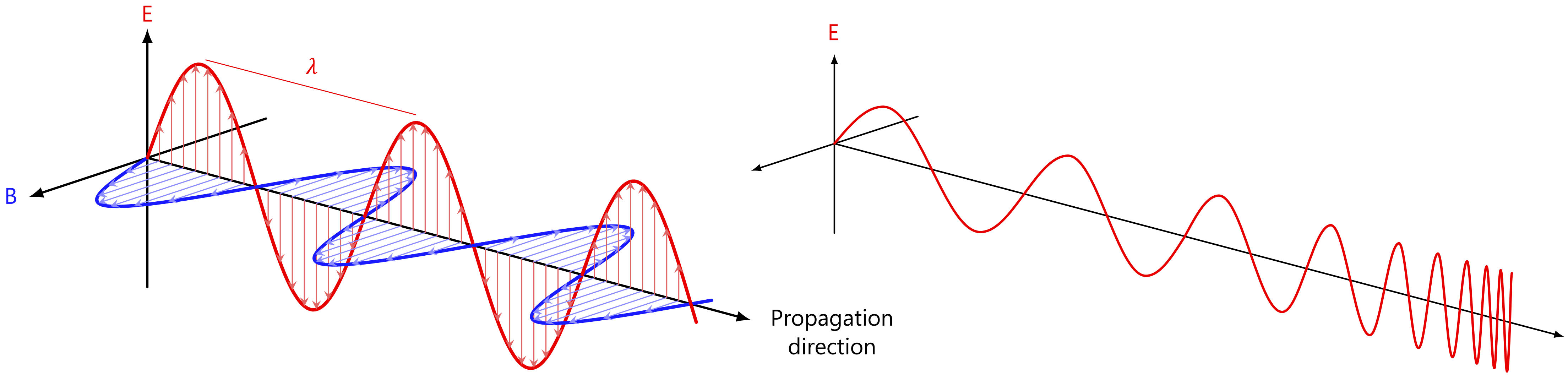


Figure2. The electromagnetic wave and wavelength
Figures made using TinZ.net

EM wave

- EM radiation is characterized by wavelength (λ), frequency (f), and energy per photon (E)
- The energy of a photon: $$E = hf = \frac{hc}{\lambda}$$

  - h : Planck's constant
  - c : *speed of the light*
  - hc = 1.2397 × 10−6 *eV*

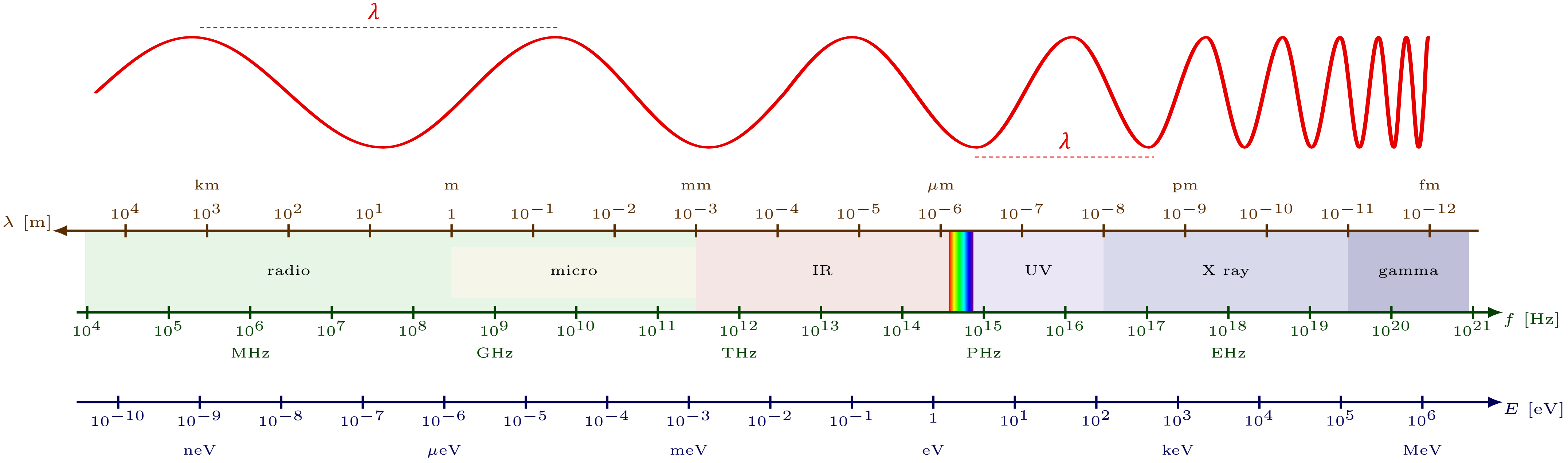
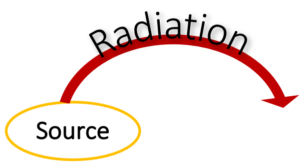
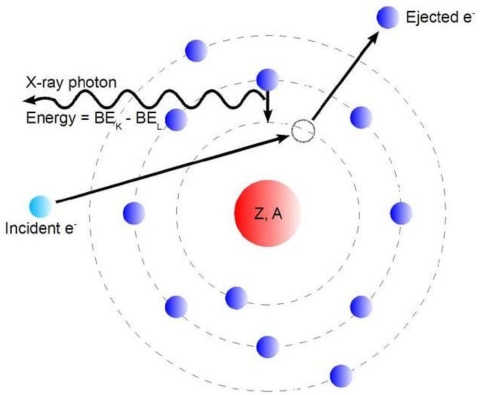


Figure 3. The electromagnetic spectrum
Figure made using TikZ.net and StackExchange



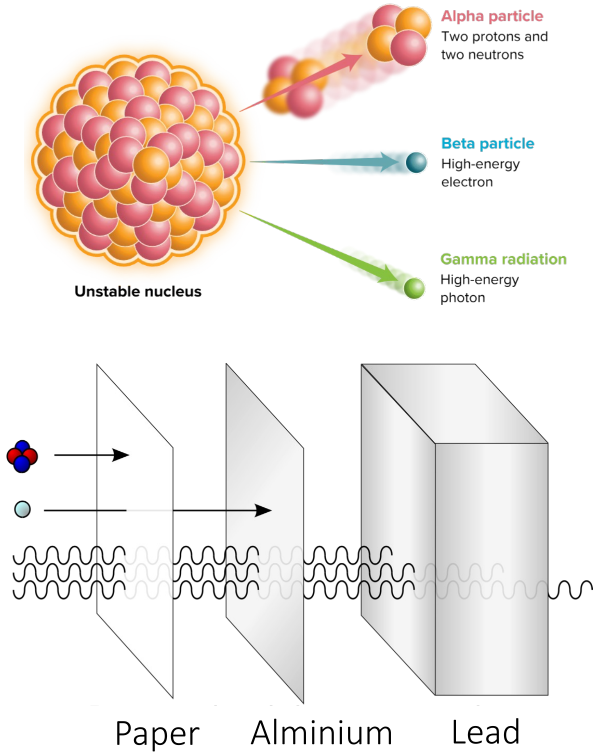
How is radiation produced?

- Interaction of a particle with matter



X-rays

- Radioactive decay of an unstable atom



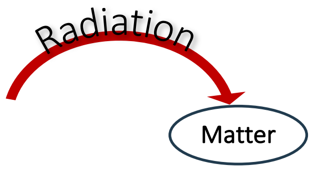
Helium nucleus
4-8 MeV

Few MeV
PET imaging modality

By-product of radioactive decay
100-200 keV
SPECT imaging modality

Figure 4. Radioactivity and the ability of ionizing radiation to penetrate matter
Figure from OMP, Knowablemagazine, and Wikipedia

Transfer of energy from emitted radiation to matter



Depending on its *energy* and ability to *penetrate matter* a radiation can be

- Ionizing radiation
  - High energy (>13.6 eV) radiation
  - Cause *ionization* in the medium through which it passes
  - Ionization: an atom or a molecule acquires a negative or positive charge by gaining or losing electrons to form ions
- Excitation (non-ionizing) radiation
  - Low energy (< 13.6 eV) radiation
  - Cannot form an ion in the medium through which it passes but change some configuration in the atom or molecule

- alpha particles
- beta particles
- neutrons

Figure 5. Transfer of energy from radiation to matter
Figure made using TikZ.net

X-rays vs. gamma rays

- Distinguish based on *wavelength*
  - X-rays: 10^{-9} - 10^{-11}
  - Gamma rays: 10^{-11} - 10^{-13}
  - ! Only possible if the wavelength is known

- Distinguish based on their *source*
  - X-rays are emitted by electrons
  - gamma rays are emitted by the atomic nucleus

Q: Compare the photon energy of x-ray and gamma-rays…

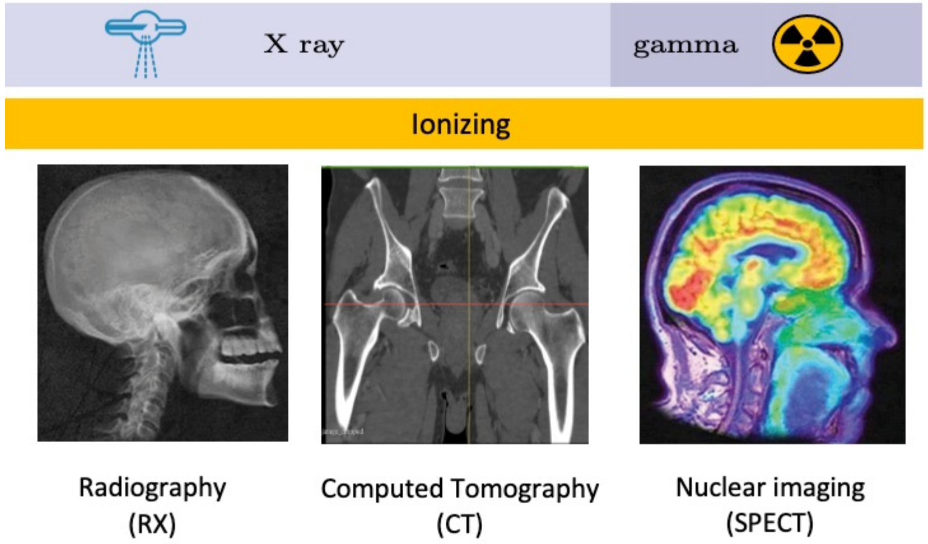
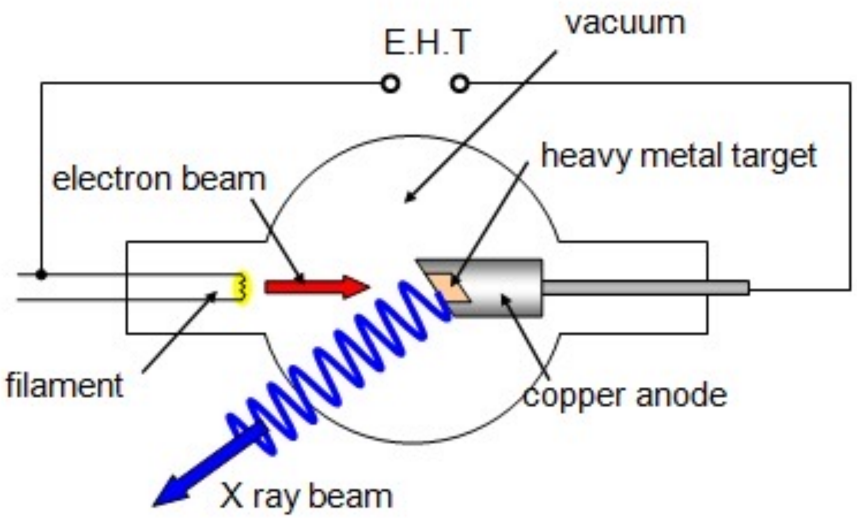


Figure 6. x-rays vs. gamma rays
Figures from Radnet

X-rays Source

X-ray tube diagram



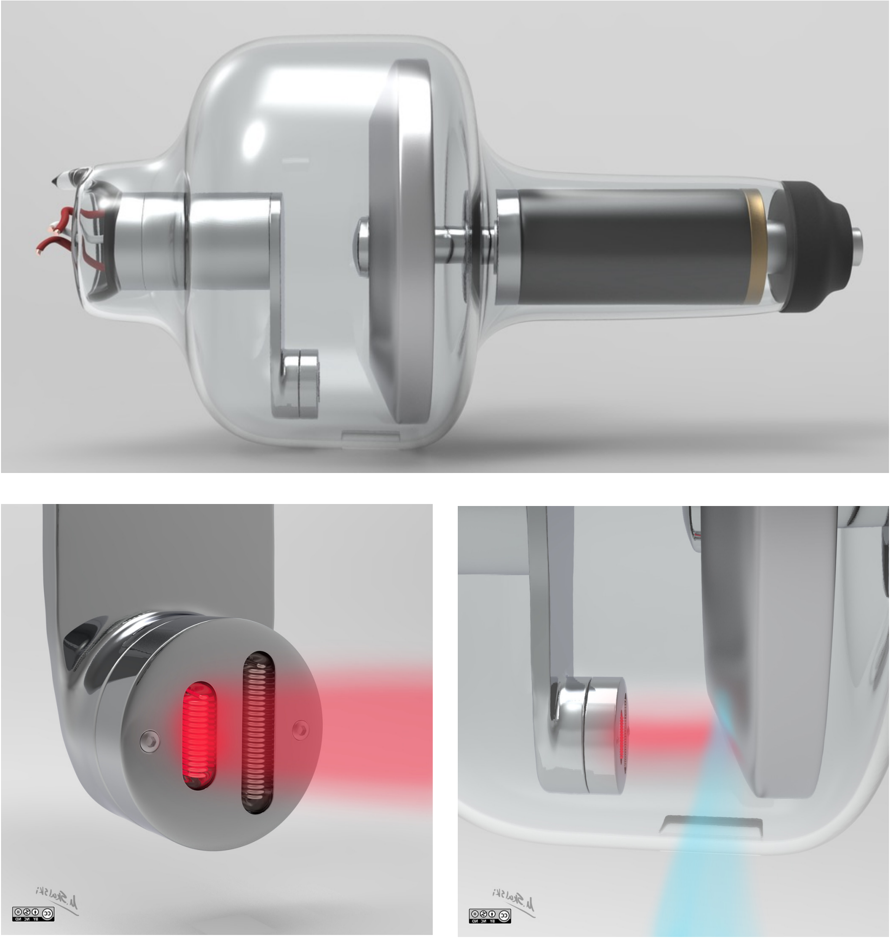


Figure 7. x-ray tube
Figures from E&C and TubeDiagrams

How are X-rays released?

Generated as the result of interactions of high-speed electrons with heavy target atoms

- ▷ Brehmsstrahlung : A continous X-ray spectrum
- ▽ Characteristic radiation
  - An accelerated electron loses energy in interaction with an atom and the loss of energy emits X-ray photons in a scattered direction

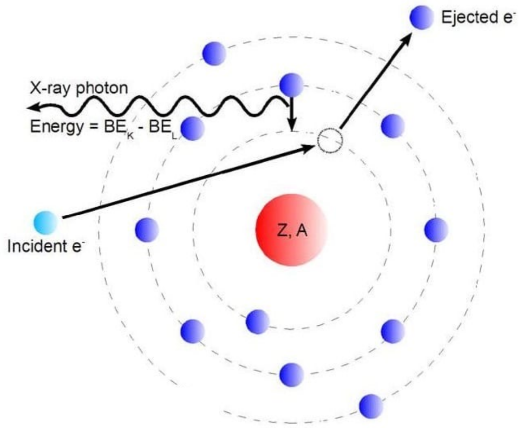
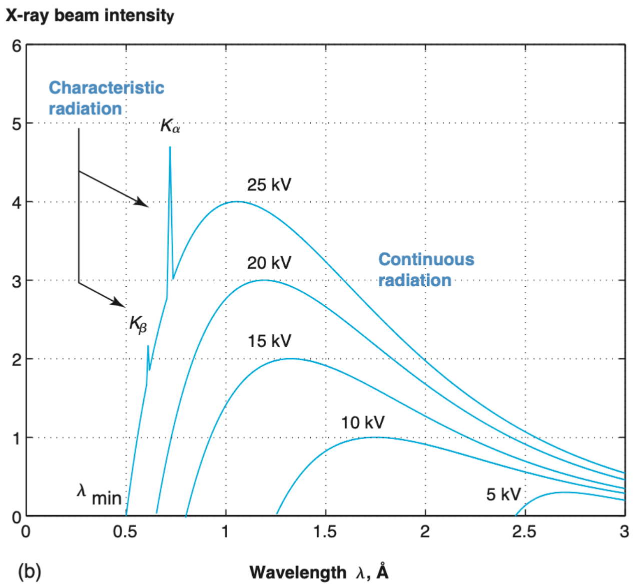


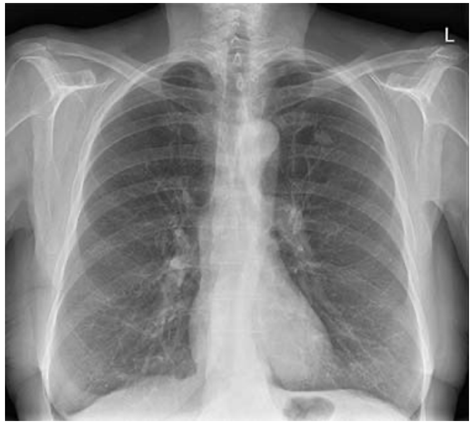
Figure 8. source of x-ray
Figure from OMP and FoMI



Radiography

Radiography images

- 2D images
- Diagnose of structural damage, abnormalities,…



Frontal chest radiograph
Figure from radiology cafe

Normal hip X-ray
Figure from CSS

Figure 9. Radiography examples

Interaction x-ray beam with tissue

- Material with thickness of $d = x_{out} - x_{in}$
- Homogeneous matter:

$$I_{out} = I_{in} e^{-\mu d} \qquad \mu \begin{cases} \text{linear attenuation coeff. (cm}^{-1}\text{)} \\ f(E, \text{material}) \end{cases}$$

μ (10keV, H_2O) = 5 cm^{-1}
μ (100keV, H_2O) = 0.17 cm^{-1}

- Non-homogeneous matter, single photon energy:

$$I_{out} = I_{in} e^{-\int_{x_{in}}^{x_{out}} \mu(x)dx}$$

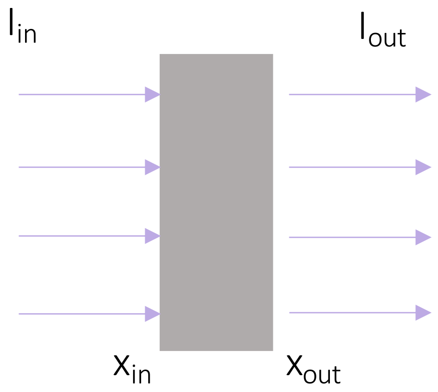
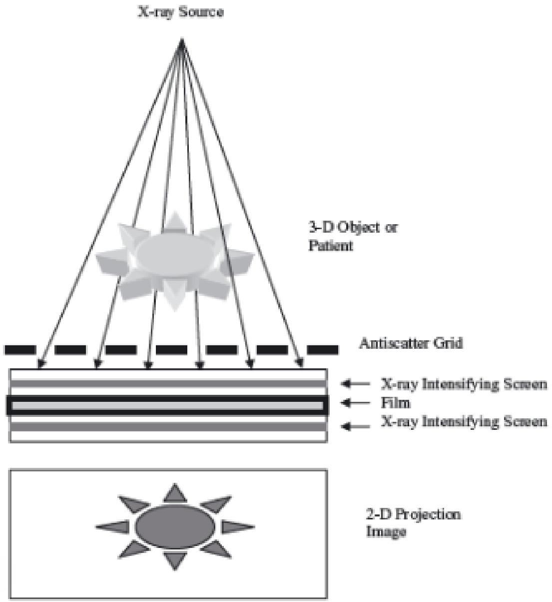


Figure 10. Attenuation

X-ray image reconstruction



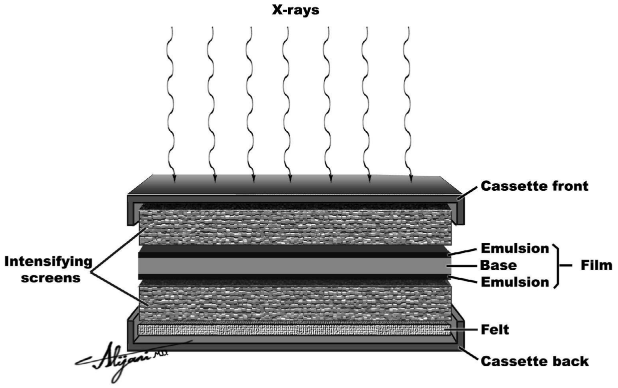


Figure 11. X-ray scanner machine and the screen-film detector
Figure from Siemens and MIA book,and the RK

X-ray image qualitty

- Resolution
  - The effective size of the X-ray tube
  - The distance between the X-ray source and the patient
  - The thickness of the intensifying screen
  - The speed of X-ray film
- Contrast: the intensity difference in adjacent regions of the image
  - The attenuation coefficient of what is imaged
  - Clinically one can inject/orally administer materials that increase the total attenuation coefficient
- Signal-to-noise ratio (SNR)
  - The X-ray exposure
  - Source and detector instrumentation
  - Thickness and heterogeneity of the tissue
  - Scattering
  - Contrast agents

BREAK
10 min

X-ray Computed Tomography (CT)

Computed Tomography (CT) images

- 3D detailed information of the structures inside the body
- Diagnose bone disorders, damage to internal organs, cancer , …

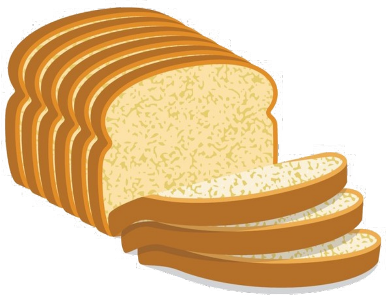
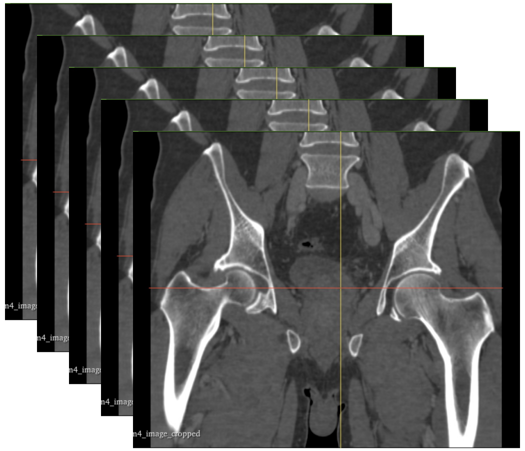
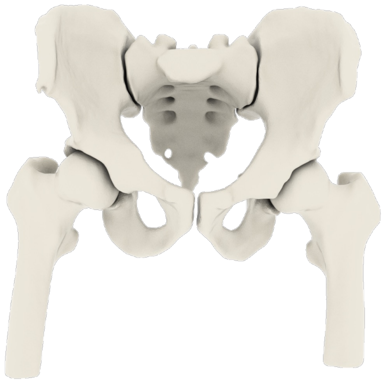




Figure 12. CT example of the hip joint area
Figures from LibHip github repository and ClipartMax

X-ray Computed Tomography (CT)

- Imaging modality that produces cross-sectional images of the body
- X-rays produced by an X-ray tube, attenuated by the patient, and measured by an X-ray detector

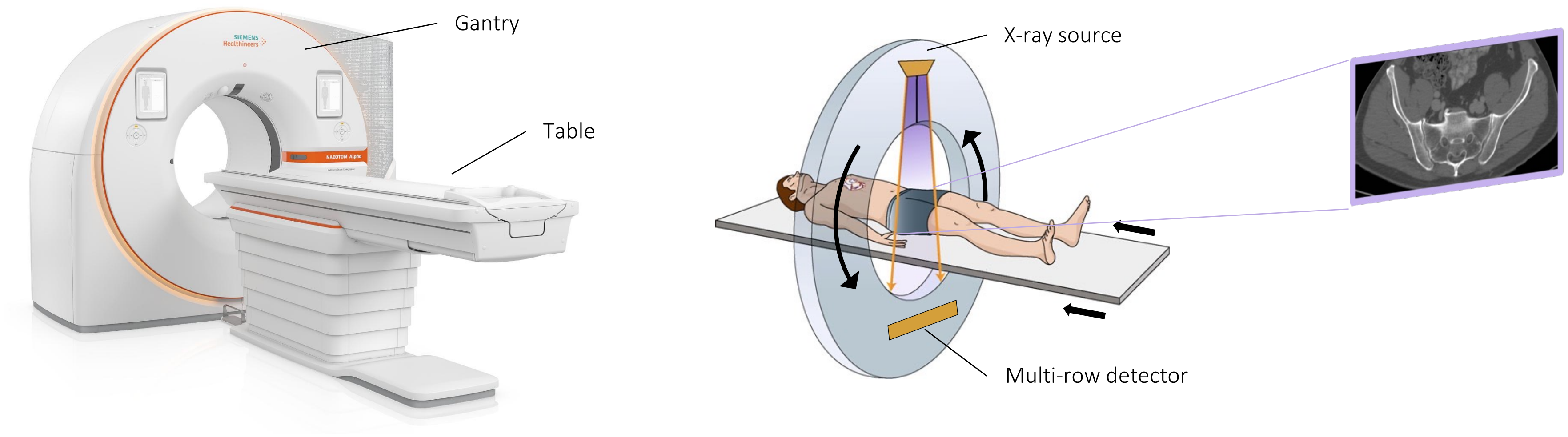
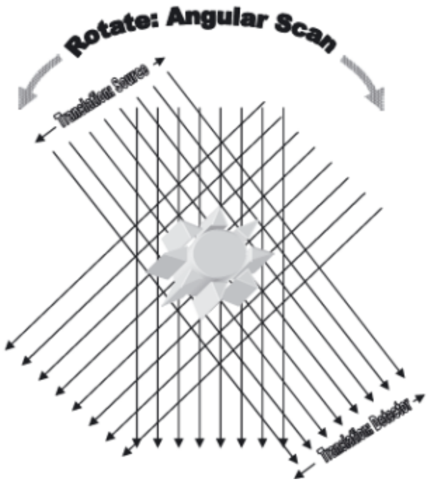
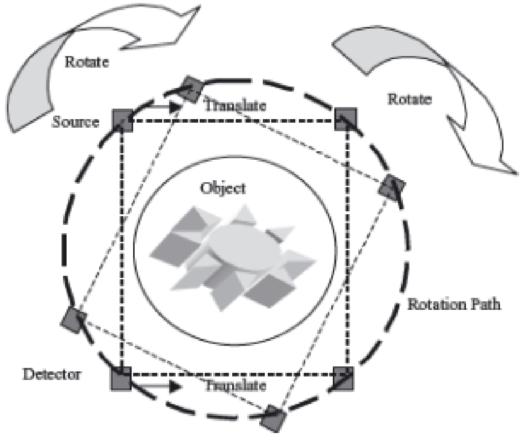


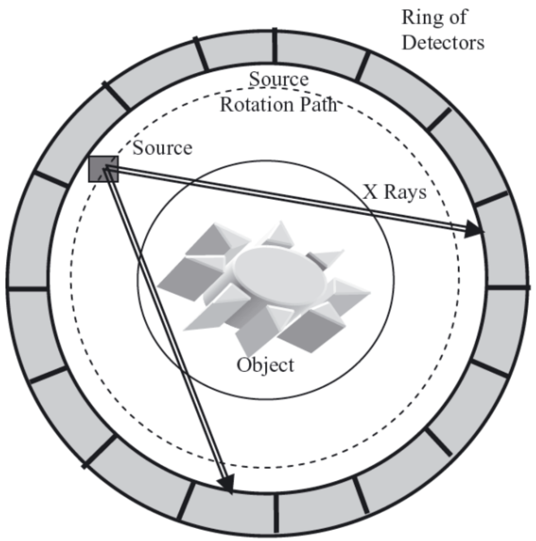
Figure 13. CT scanner machine and the main parts
Figure from Siemens, CardiacCT book, and LibHip

CT: many X-rays!



1st generation





4th generation

Figure 14. CT basic principles
Figure from the [MIA book]

Imaging in three dimensions

- Single-slice CT
  - Detector CT yields a single slice (2D) and via movement along the z direction one arrives at a 3D volume
- Multi-slice CT
  - Can acquire multiple 2D slices at the same time, hence creating a 3D volume
- Spiral CT (often already multi-slice):
  - Faster scan at lower radiation dose
  - Slightly sacrifices image resolution and sharpness
  - Higher temporal resolution allows gated imaging

CT image quality

- Same restrictions as for X-ray imaging
- Additional impact on resolution & SNR:
  - Correlated with the number of projections (angular sampling) and number of detectors (projection sampling); for higher spatial resolution, data must be acquired with higher sampling rate in the projection space
  - BUT the higher sampling rate is compromised by the limitations on scan time, method of data collection, and detector size and efficiency

Let’s do an experiment…

Spend 10 minutes in your study groups looking up and discussing what the Radon transform is and try to summarize what it does, by adding a post to my Radon transform thread in the discussion forum on Absalon!

CT image reconstruction

- Most common form is still (filtered) back projection
- Also exist iterative methods for reconstruction e.g. OSEM reconstruction
- Depends on type of imaging – transmission or emission
- Also depends on the scanner shape! E.g. Siemens HRRT is not circular and hence filtered back projection is not implemented, but only OSEM

Data acquisition

- Non-homogeneous matter, single photon energy:

$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

$$\begin{aligned} \mathrm{I}_\theta(r) &= \mathrm{I}_0 .\, \mathrm{e}^{-\int_{\mathrm{L}_{r,\theta}} \mu(\mathrm{x,y})\mathrm{ds}} \\ &= \mathrm{I}_0 .\, \mathrm{e}^{-\int_{\mathrm{L}_{r,\theta}} \mu(rcos\theta - ssin\theta, rsin\theta + scos\theta)\mathrm{ds}} \end{aligned}$$

$$\begin{aligned} P_\theta(\mathrm{r}) &= -\ln \frac{I_\theta(r)}{I_0} \\ &= \int_{L_{r,\theta}} \mu(rcos\theta\ - ssin\theta,\ rsin\theta + scos\theta)ds \end{aligned}$$

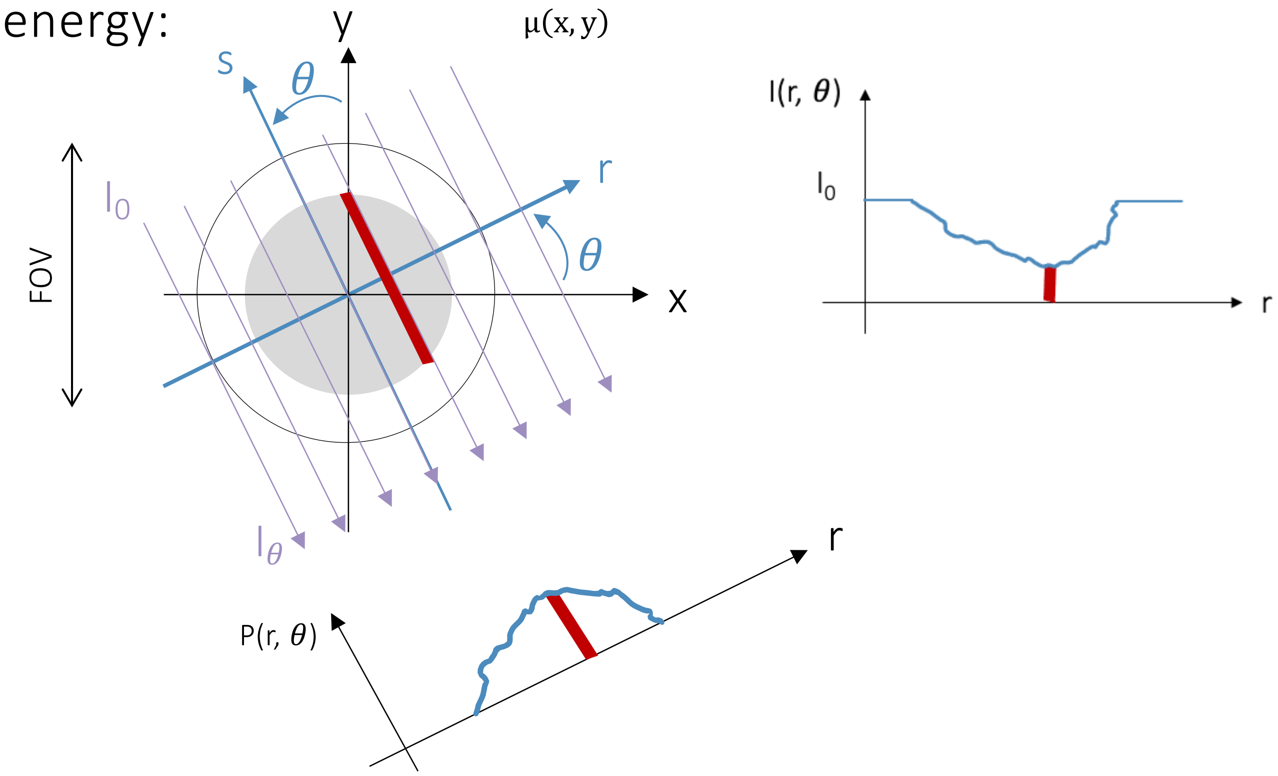


Figure 15. Data acquisition in CT

The Radon transform

Objective function → Projection domain

- Objective function *f(x,y)*

$$x\cos\theta + y\sin\theta = r$$
$$-x\sin\theta + y\cos\theta = s$$

$$P(r,\theta) = R\{f(x,y)\} = \int_L f(x,y)dl$$
$$= \int_{-\infty}^{\infty} f(r\cos\theta - s\sin\theta,\ r\sin\theta + s\cos\theta)d$$

where the **line integral** $\int_L$ is computed along the path L,

which are parallel arrow lines defined by angle θ

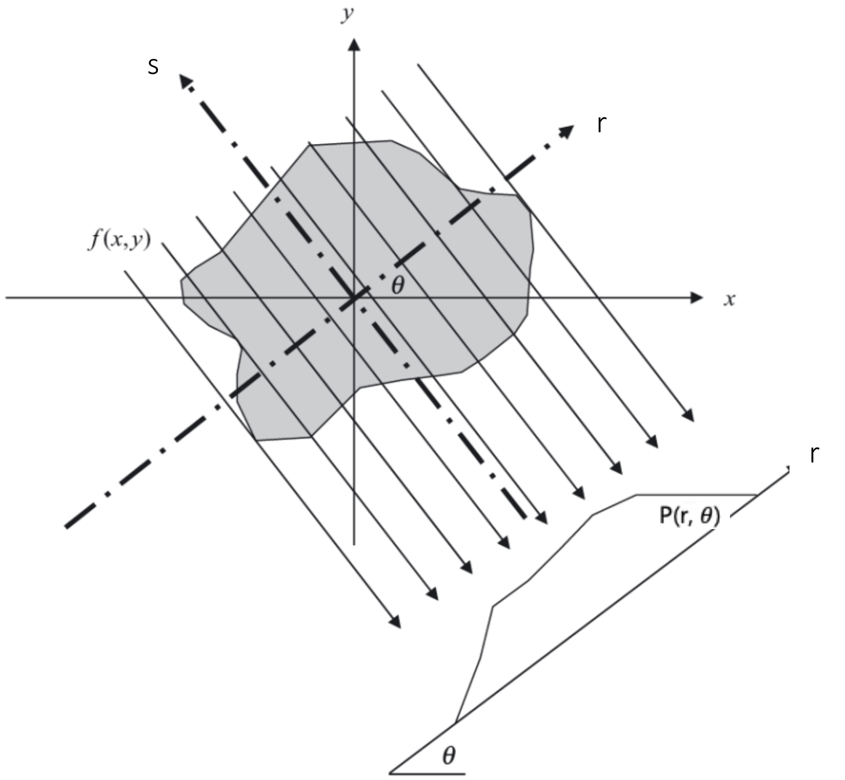


Figure 16. Radon transform from the MIA book

2D image reconstruction

- ▷ Back-projection of line integrals
- ▽ How to reconstruct the distribution $\mu(x, y)$ from the projection domain?
  - For a particular line (r, θ) assign the projection value P(r, θ) to all points (x,y) along that line
  - Repeat this (I.e., integrate) for θ ranging from zero to 180

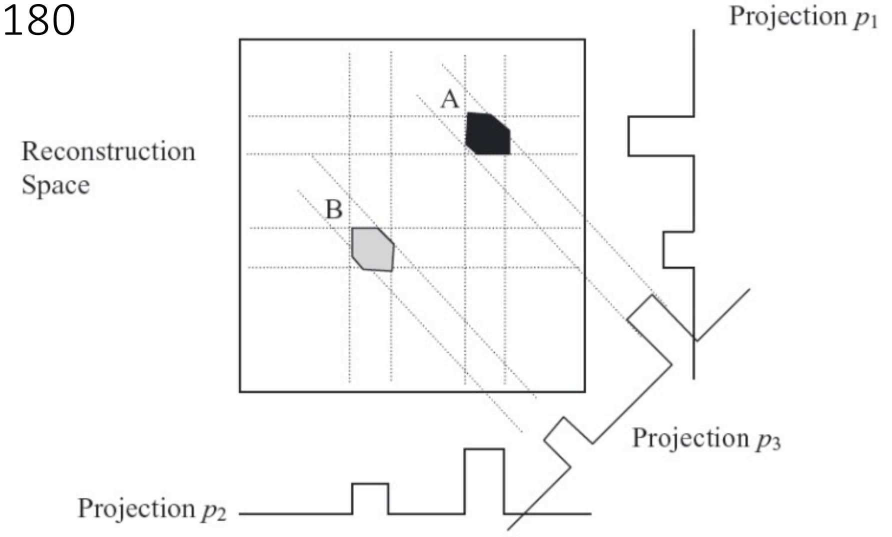


Figure 17. Back-projection

Quick check – Image vs. Fourier domain

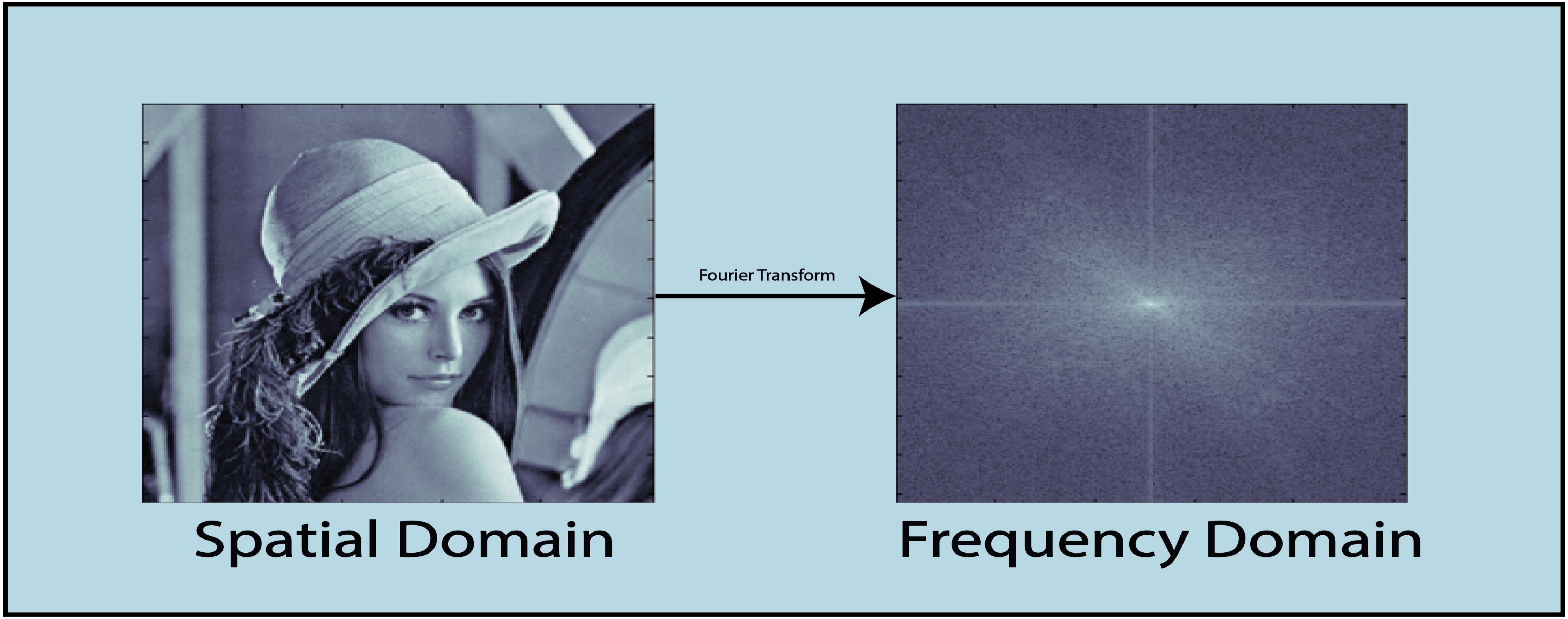
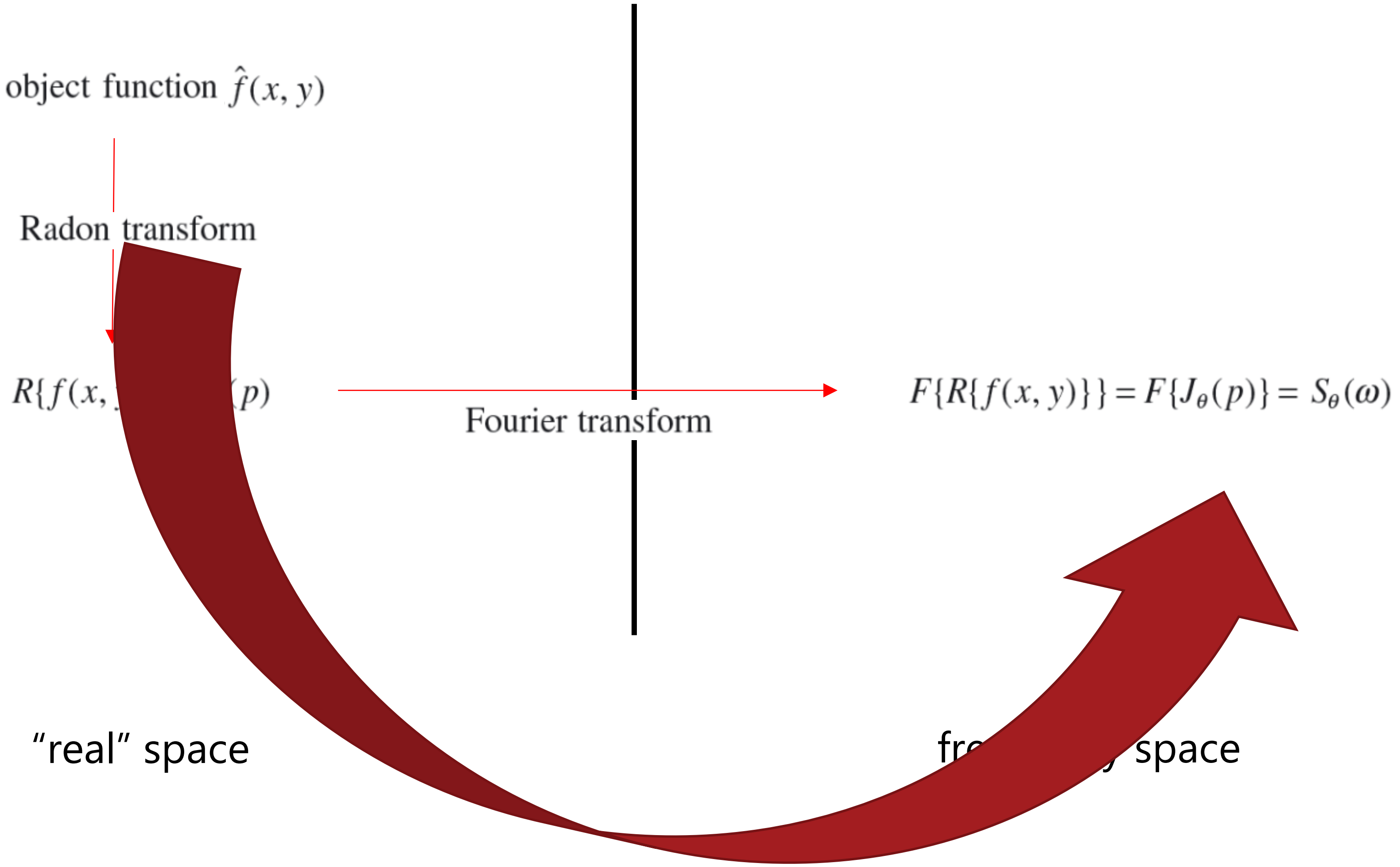
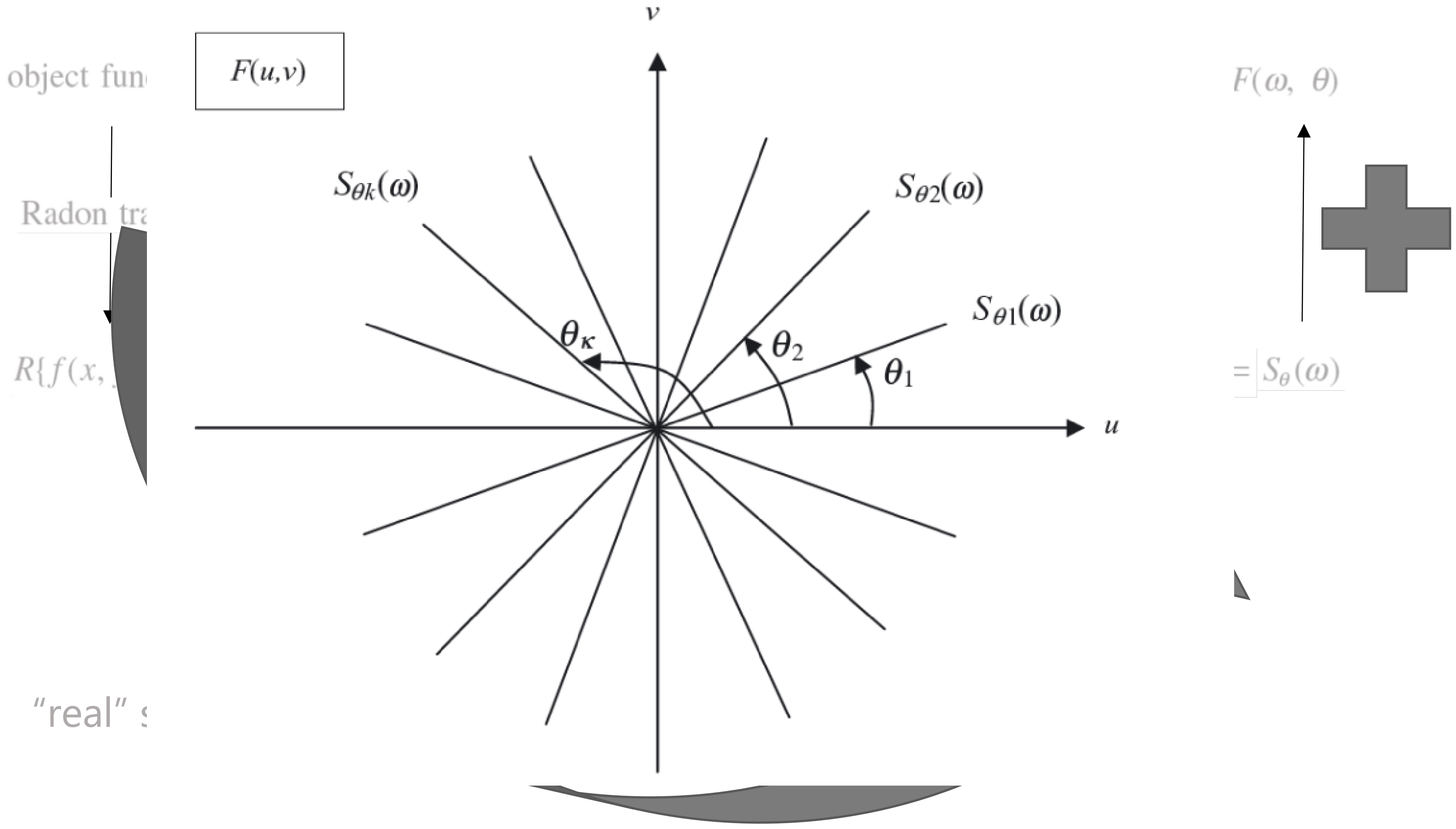


Figure 18. Figure from [Medium]

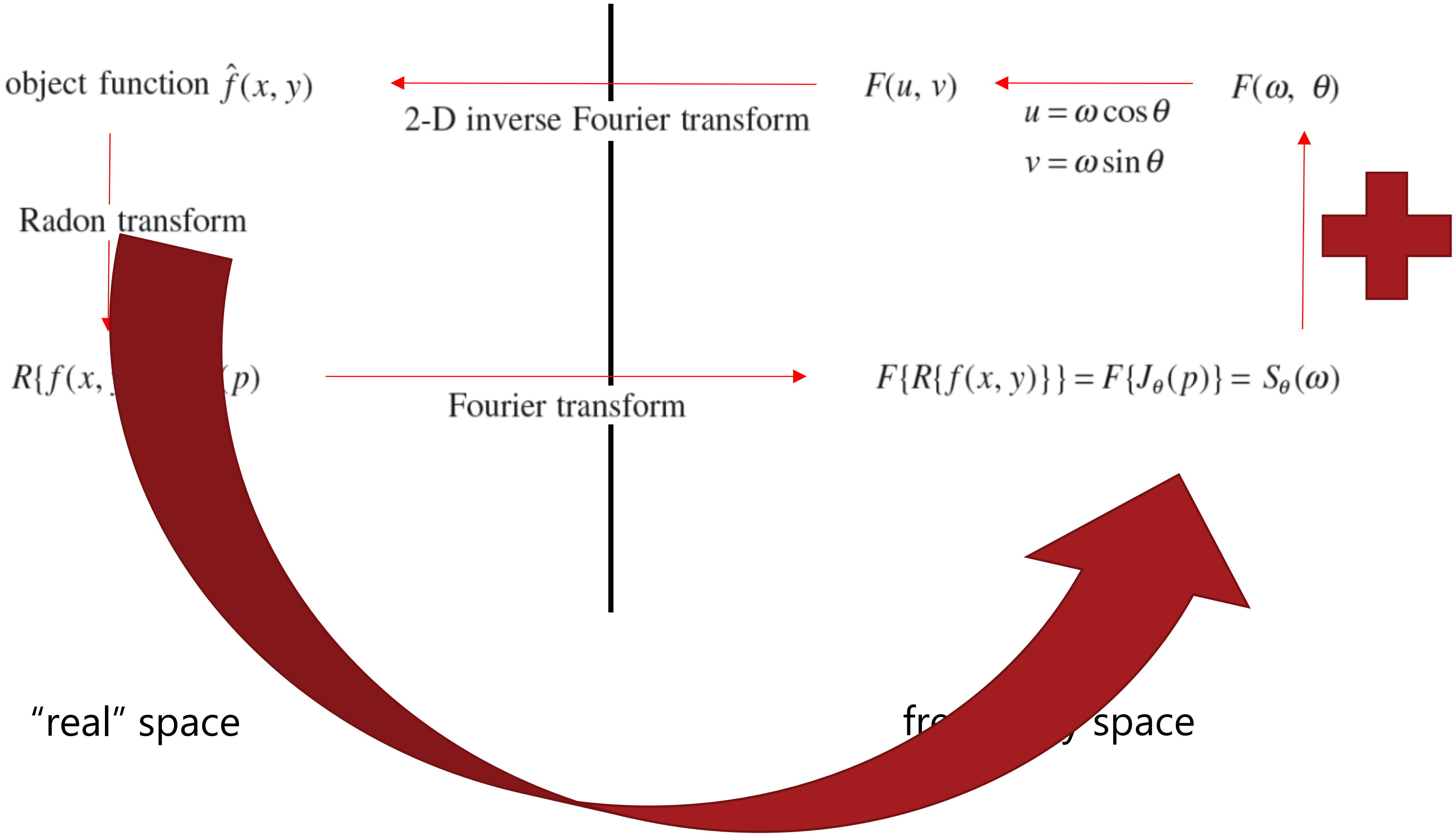
Central limit theorem & backprojection



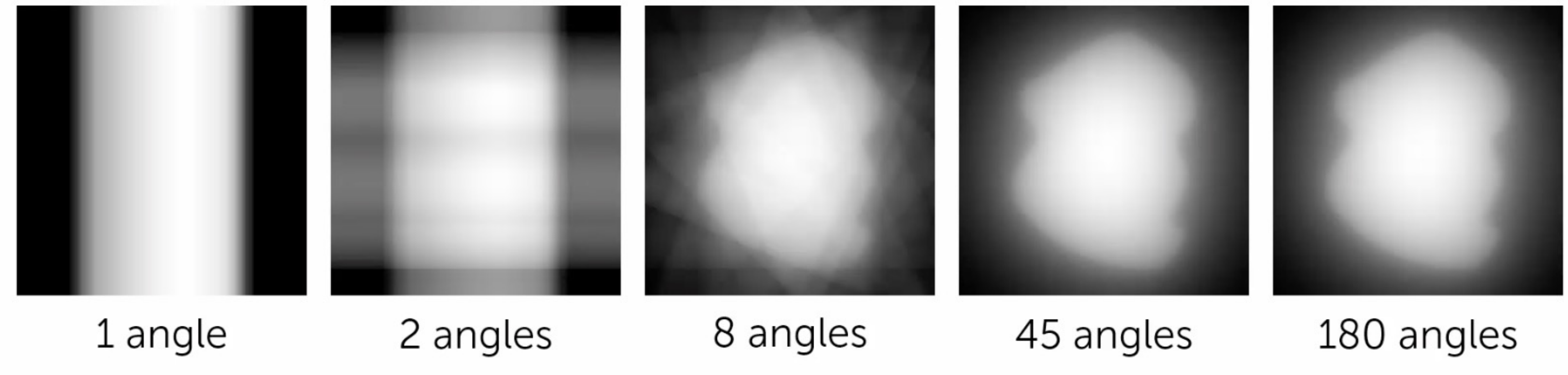
Central limit theorem & back-projection

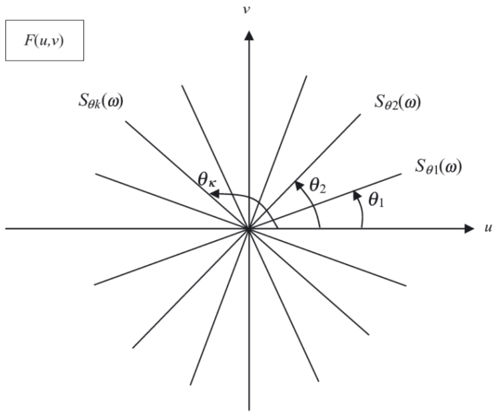


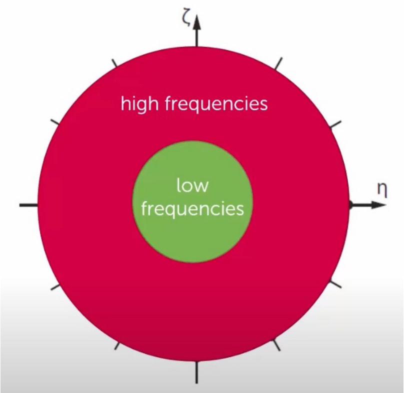
Central limit theorem & backprojection



Blurring in back-projection



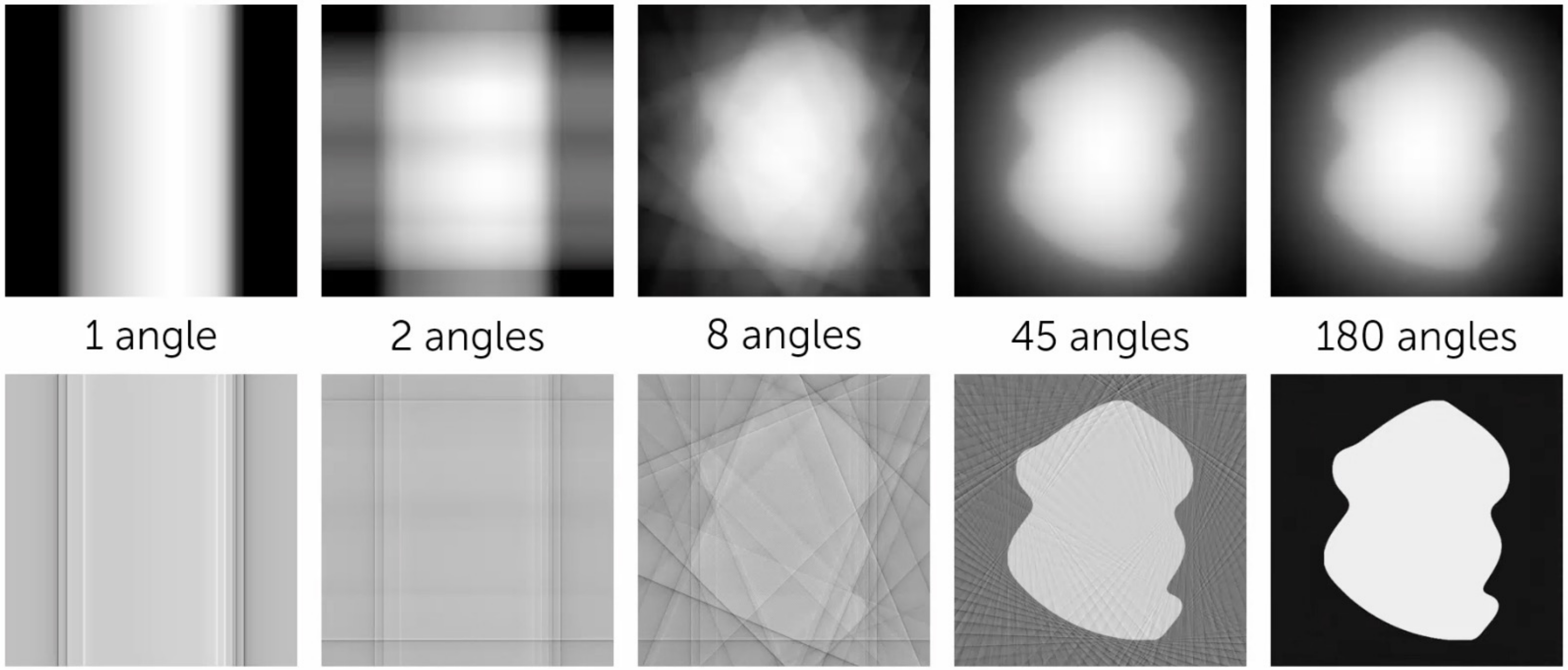




Figures are screenshot from ASTRA Toolbox video . Refer to the video for more information.

Filtered back-projection

- Filter out low frequencies to avoid blurring.



Figures are screenshot from [ASTRA Toolbox]() video . Refer to the video for more information.

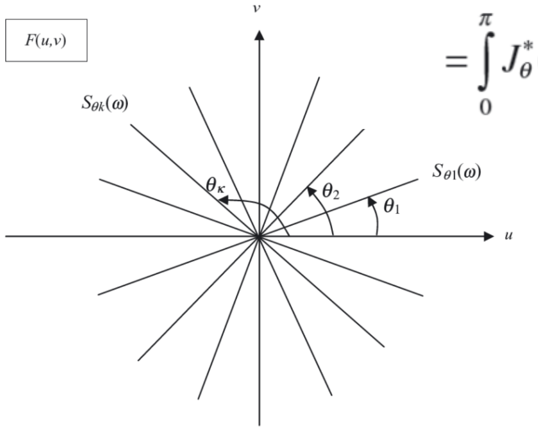
Filtered back-projection

object function $\hat{f}(\cancel{x, y})$ $\hat{f}(r, \theta)$

$$\hat{f}(r,\theta) = \int_0^{\pi}\int_{-\infty}^{\infty} F(\omega,\theta)e^{j2\pi w(x\cos\theta + y\sin\theta)}|\omega|\,d\omega\,d\theta.$$

$$\hat{f}(r,\theta) = \int_0^{\pi}\int_{-\infty}^{\infty} |\omega|\,S_\theta(\omega)e^{j2\pi\omega\,(x\cos\theta + y\sin\theta)}\,d\omega\,d\theta$$

$$= \int_0^{\pi} J_\theta^*(p')d\theta$$



$$J_\theta^*(p') = \int_{-\infty}^{\infty} |\omega|\,S_\theta(\omega)e^{j2\pi\omega\,(x\cos\theta + y\sin\theta)}\,d\omega$$

$$= \int_{-\infty}^{\infty} |\omega|\,S_\theta(\omega)e^{j2\pi\omega\,p}\,d\omega$$

$$= F^{-1}\{|\omega|\,S_\theta(\omega)\}$$

$$= F^{-1}\{|\omega|\}\otimes J_\theta(p)$$

$$= \int_{-\infty}^{\infty} J_\theta(p')h(p-p')\,dp'$$

By limiting spatial frequencies

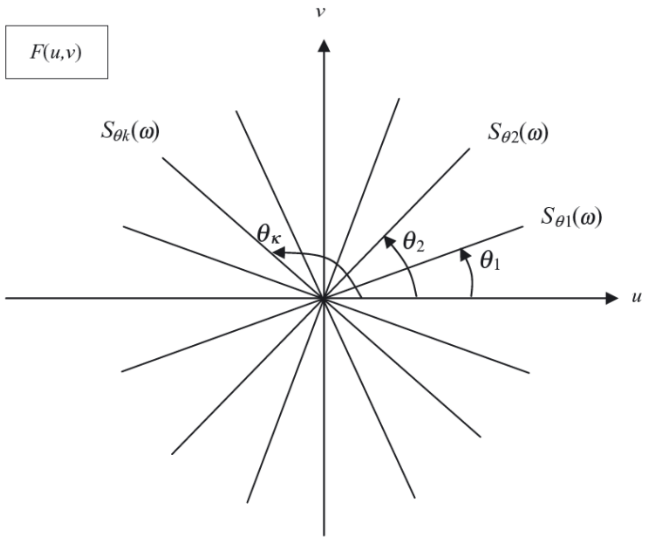
Trick 1 $\rightarrow$ $$\hat{f}(x, y) = \frac{\pi}{L}\sum_{i=1}^{L} J_{\theta_i}(p')$$

Limit number of necessary projections

Filtered back-projection

$$\hat{f}(r,\theta) = \int_0^{\pi} \int_{-\infty}^{\infty} |\omega| \, S_\theta(\omega) e^{j2\pi\omega\,(x\cos\theta + y\sin\theta)} \, d\omega \, d\theta$$

$$= \int_0^{\pi} J_\theta^*(p') d\theta$$



Trick 2?

Watch the video I shared about filtered back projection under the Modalities page (access. via modules) and try to figure out what the second trick of filtered back projection is.

Conclusion

- Describe the difference between gamma and x-rays
- Explain how an X-ray image is formed
- Write down the formula for the Radon transform
- Draw a schematic diagram describing back-projection
- Explain how filtered back-projection differs

Questions?

Next week

- Lectures on Tuesday 9-12 and Thursday 9-12
  - MRI
  - SPECT/PET
  - External lecture on Thursday by Cyril Pernet, Senior researcher, Rigshospital
- Exercises on Thursday from 13-15
  - 1st assignment is due Monday the 19th of September
  - 2nd assignment will be accessible from the same day on