the step; similarly, the second derivative is nonzero at the onset *and* end of both the ramp and the step; therefore, property (2) is satisfied for both derivatives. Finally, we see that property (3) is satisfied also for both derivatives because the first derivative is nonzero and the second is zero along the ramp. Note that the sign of the second derivative changes at the onset and end of a step or ramp. In fact, we see in Fig. 3.36(c) that in a step transition a line joining these two values crosses the horizontal axis midway between the two extremes. This *zero crossing* property is quite useful for locating edges, as you will see in Chapter 10.

Edges in digital images often are ramp-like transitions in intensity, in which case the first derivative of the image would result in thick edges because the derivative is nonzero along a ramp. On the other hand, the second derivative would produce a double edge one pixel thick, separated by zeros. From this, we conclude that the second derivative enhances fine detail much better than the first derivative, a property that is ideally suited for sharpening images. Also, as you will learn later in this section, second derivatives are much easier to implement than first derivates, so we focus our attention initially on second derivatives.

3.6.2 Using the Second Derivative for Image Sharpening—The Laplacian

In this section we consider the implementation of 2-D, second-order derivatives and their use for image sharpening. We return to this derivative in Chapter 10, where we use it extensively for image segmentation. The approach basically consists of defining a discrete formulation of the second-order derivative and then constructing a filter mask based on that formulation. We are interested in *isotropic* filters, whose response is independent of the direction of the discontinuities in the image to which the filter is applied. In other words, isotropic filters are *rotation invariant*, in the sense that rotating the image and then applying the filter gives the same result as applying the filter to the image first and then rotating the result.

It can be shown (Rosenfeld and Kak [1982]) that the simplest isotropic derivative operator is the Laplacian, which, for a function (image) $f(x, y)$ of two variables, is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{3.6-3}$$

Because derivatives of any order are linear operations, the Laplacian is a linear operator. To express this equation in discrete form, we use the definition in Eq. (3.6-2), keeping in mind that we have to carry a second variable. In the x-direction, we have

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \tag{3.6-4}$$

and, similarly, in the y-direction we have

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \tag{3.6-5}$$

Therefore, it follows from the preceding three equations that the discrete Laplacian of two variables is

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) \tag{3.6-6}$$

This equation can be implemented using the filter mask in Fig. 3.37(a), which gives an isotropic result for rotations in increments of 90°. The mechanics of implementation are as in Section 3.5.1 for linear smoothing filters. We simply are using different coefficients here.

The diagonal directions can be incorporated in the definition of the digital Laplacian by adding two more terms to Eq. (3.6-6), one for each of the two diagonal directions. The form of each new term is the same as either Eq. (3.6-4) or (3.6-5), but the coordinates are along the diagonals. Because each diagonal term also contains a $-2f(x, y)$ term, the total subtracted from the difference terms now would be $-8f(x, y)$. Figure 3.37(b) shows the filter mask used to implement this new definition. This mask yields isotropic results in increments of 45°. You are likely to see in practice the Laplacian masks in Figs. 3.37(c) and (d). They are obtained from definitions of the second derivatives that are the negatives of the ones we used in Eqs. (3.6-4) and (3.6-5). As such, they yield equivalent results, but the difference in sign must be kept in mind when combining (by addition or subtraction) a Laplacian-filtered image with another image.

Because the Laplacian is a derivative operator, its use highlights intensity discontinuities in an image and deemphasizes regions with slowly varying intensity levels. This will tend to produce images that have grayish edge lines and other discontinuities, all superimposed on a dark, featureless background. Background features can be "recovered" while still preserving the sharpening

(a)

0	1	0
1	-4	1
0	1	0

(b)

1	1	1
1	-8	1
1	1	1

(c)

0	-1	0
-1	4	-1
0	-1	0

(d)

-1	-1	-1
-1	8	-1
-1	-1	-1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

effect of the Laplacian simply by adding the Laplacian image to the original. As noted in the previous paragraph, it is important to keep in mind which definition of the Laplacian is used. If the definition used has a negative center coefficient, then we *subtract*, rather than add, the Laplacian image to obtain a sharpened result. Thus, the basic way in which we use the Laplacian for image sharpening is

$$g(x, y) = f(x, y) + c\left[\nabla^2 f(x, y)\right] \tag{3.6-7}$$

where $f(x, y)$ and $g(x, y)$ are the input and sharpened images, respectively. The constant is $c = -1$ if the Laplacian filters in Fig. 3.37(a) or (b) are used, and $c = 1$ if either of the other two filters is used.

EXAMPLE 3.15: Image sharpening using the Laplacian.

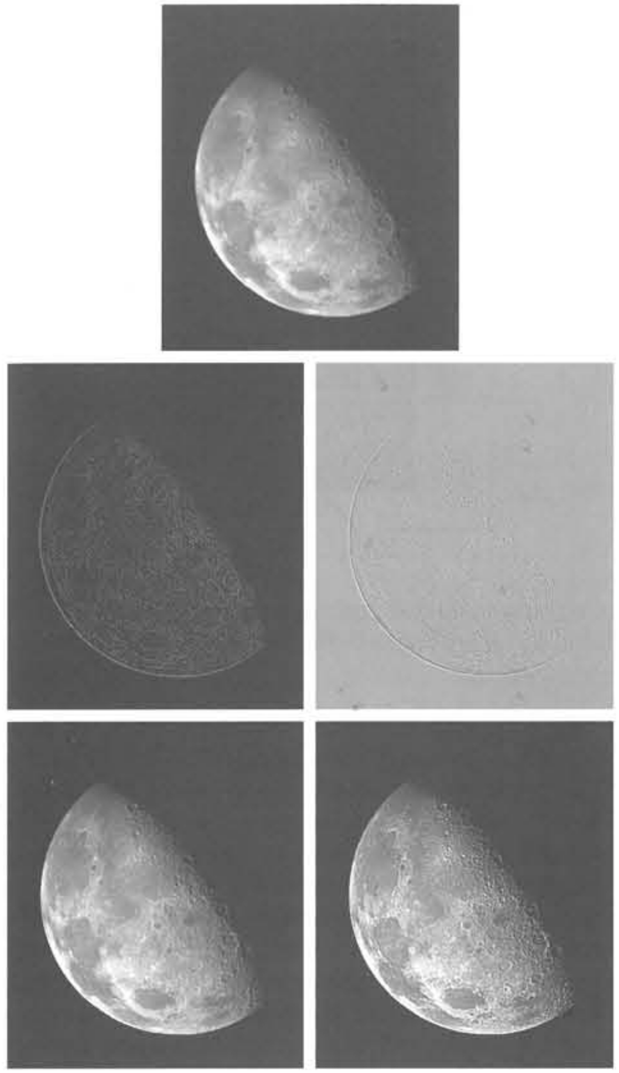
■ Figure 3.38(a) shows a slightly blurred image of the North Pole of the moon. Figure 3.38(b) shows the result of filtering this image with the Laplacian mask in Fig. 3.37(a). Large sections of this image are black because the Laplacian contains both positive and negative values, and all negative values are clipped at 0 by the display.

A typical way to scale a Laplacian image is to add to it its minimum value to bring the new minimum to zero and then scale the result to the full $[0, L - 1]$ intensity range, as explained in Eqs. (2.6-10) and (2.6-11). The image in Fig. 3.38(c) was scaled in this manner. Note that the dominant features of the image are edges and sharp intensity discontinuities. The background, previously black, is now gray due to scaling. This grayish appearance is typical of Laplacian images that have been scaled properly. Figure 3.38(d) shows the result obtained using Eq. (3.6-7) with $c = -1$. The detail in this image is unmistakably clearer and sharper than in the original image. Adding the original image to the Laplacian restored the overall intensity variations in the image, with the Laplacian increasing the contrast at the locations of intensity discontinuities. The net result is an image in which small details were enhanced and the background tonality was reasonably preserved. Finally, Fig. 3.38(e) shows the result of repeating the preceding procedure with the filter in Fig. 3.37(b). Here, we note a significant improvement in sharpness over Fig. 3.38(d). This is not unexpected because using the filter in Fig. 3.37(b) provides additional differentiation (sharpening) in the diagonal directions. Results such as those in Figs. 3.38(d) and (e) have made the Laplacian a tool of choice for sharpening digital images. ■

3.6.3 Unsharp Masking and Highboost Filtering

A process that has been used for many years by the printing and publishing industry to sharpen images consists of subtracting an unsharp (smoothed) version of an image from the original image. This process, called *unsharp masking*, consists of the following steps:

1. Blur the original image.
2. Subtract the blurred image from the original (the resulting difference is called the *mask*.)
3. Add the mask to the original.



a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Letting $\bar{f}(x, y)$ denote the blurred image, unsharp masking is expressed in equation form as follows. First we obtain the mask:

$$g_{\text{mask}}(x, y) = f(x, y) - \bar{f}(x, y) \tag{3.6-8}$$

Then we add a weighted portion of the mask back to the original image:

$$g(x, y) = f(x, y) + k * g_{\text{mask}}(x, y) \tag{3.6-9}$$

where we included a weight, k ($k \geq 0$), for generality. When $k = 1$, we have unsharp masking, as defined above. When $k > 1$, the process is referred to as