# Registration. Lecture 2

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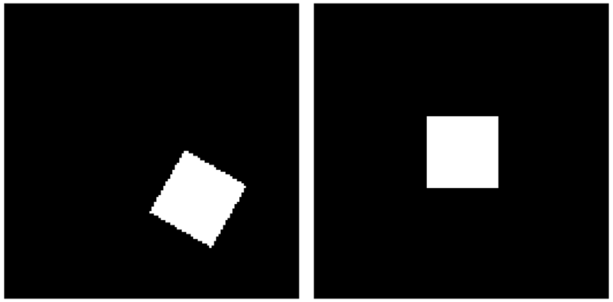
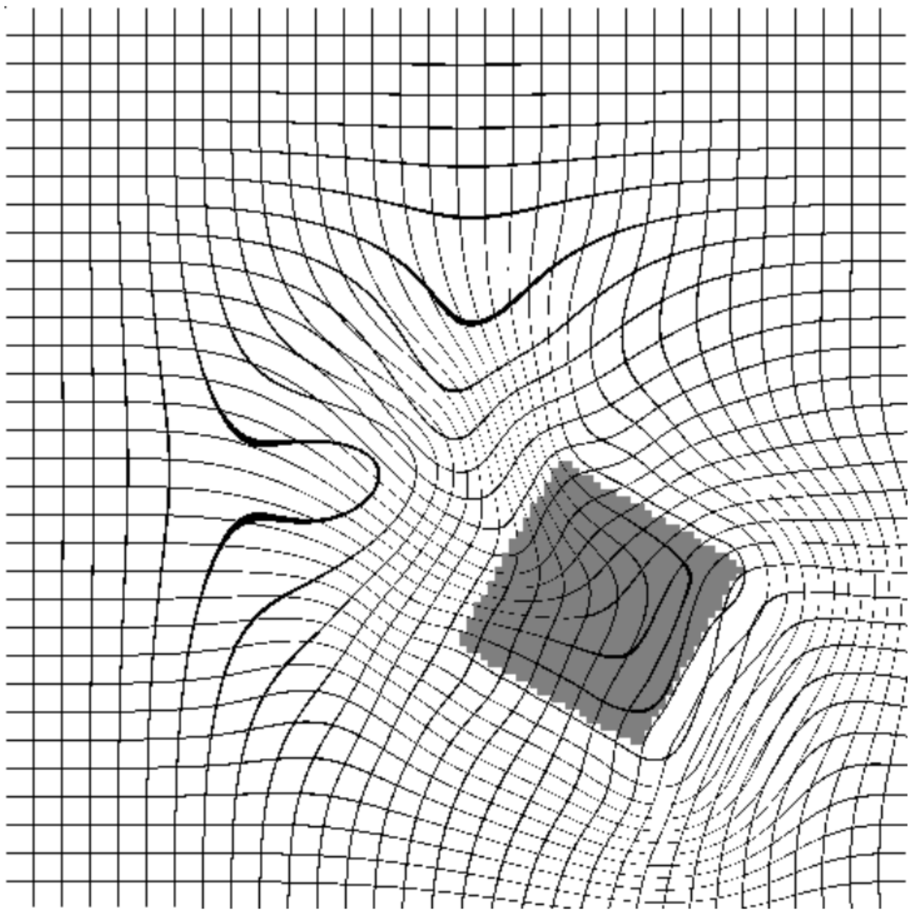
# Image registration: regularization

**Is this a good transformation field?**

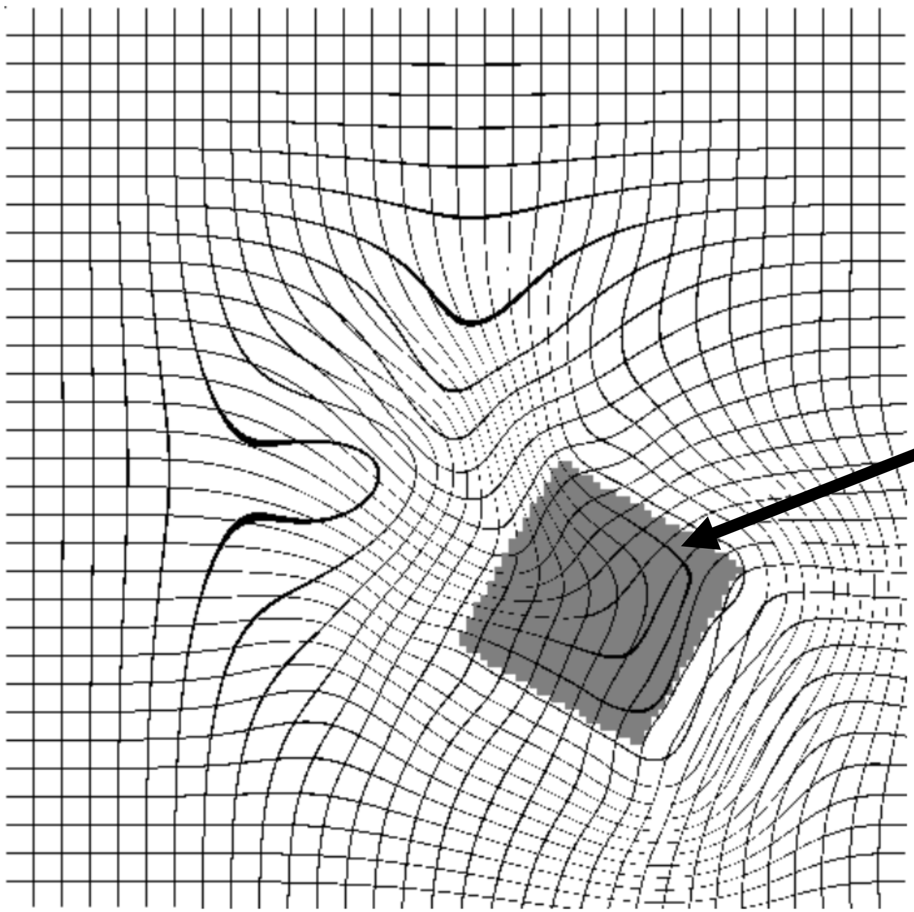


fixed moving



deformed moving

# Image registration: regularization



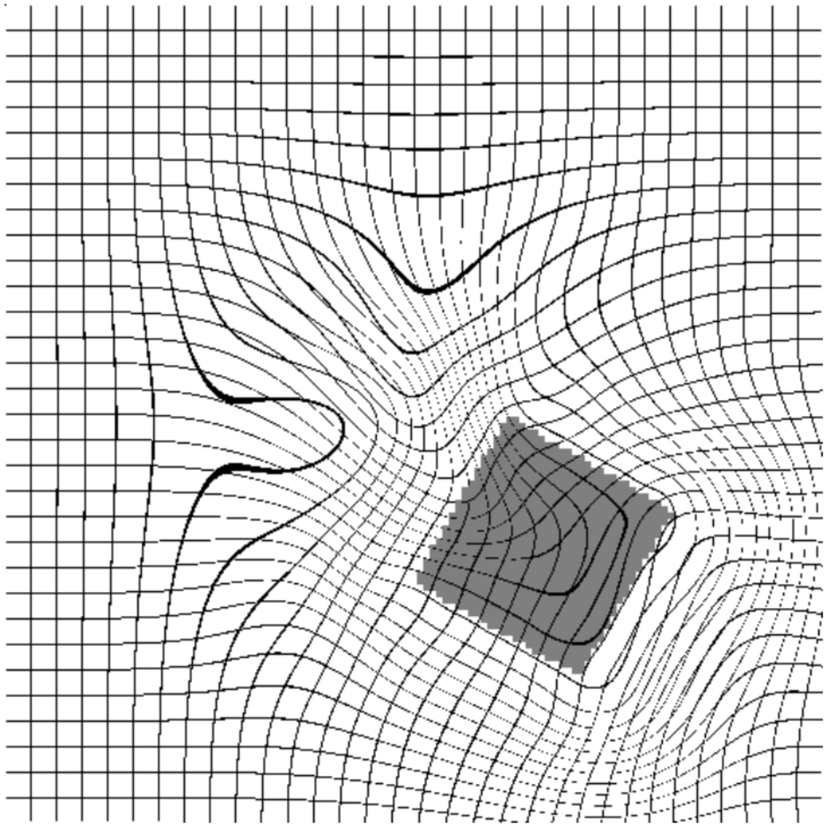
Strange internal distortions

# Image registration: regularization

**Regularization**:

- We make the algorithm pay the price for too much deformations

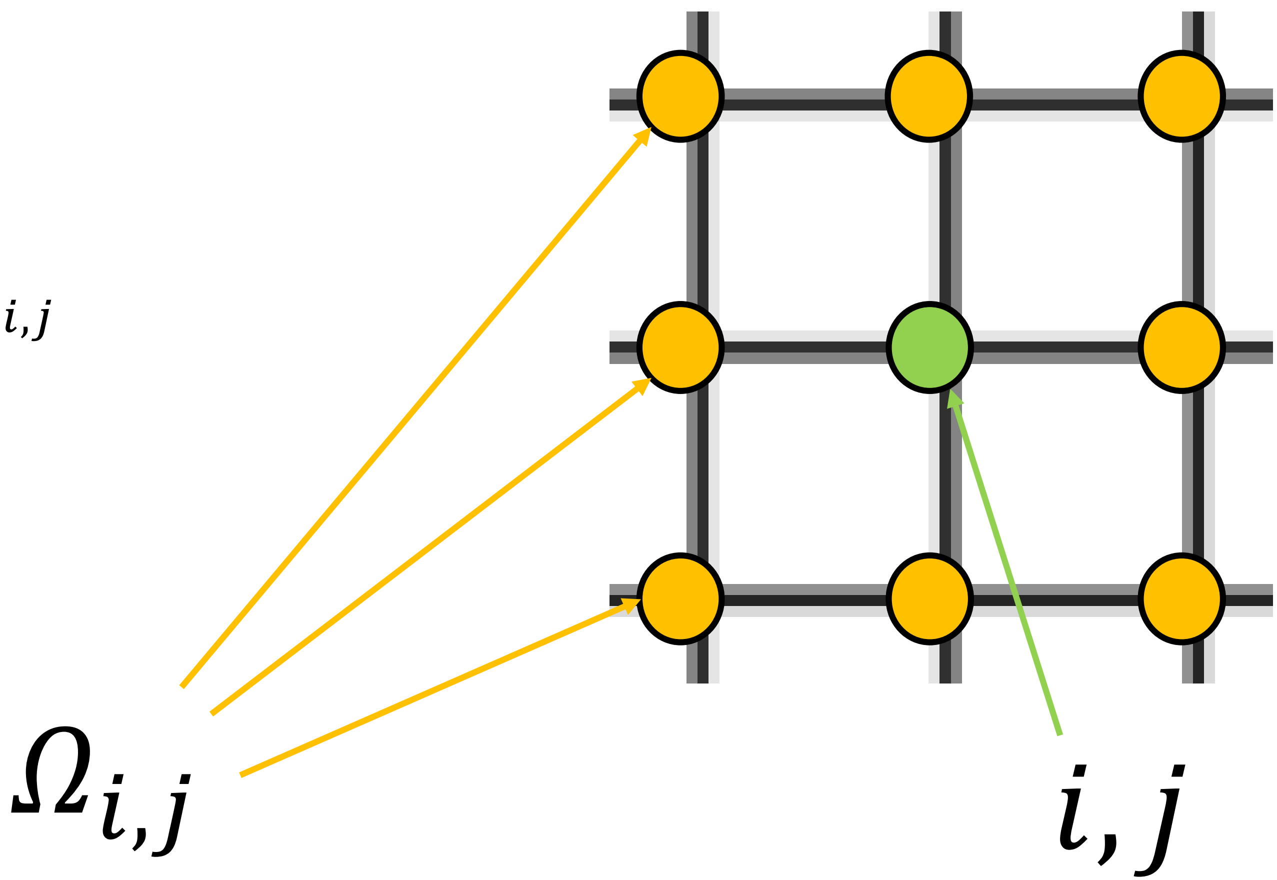
- Price should grow nonlinearly (better move many a little bit than one a lot)



# Image registration: regularization

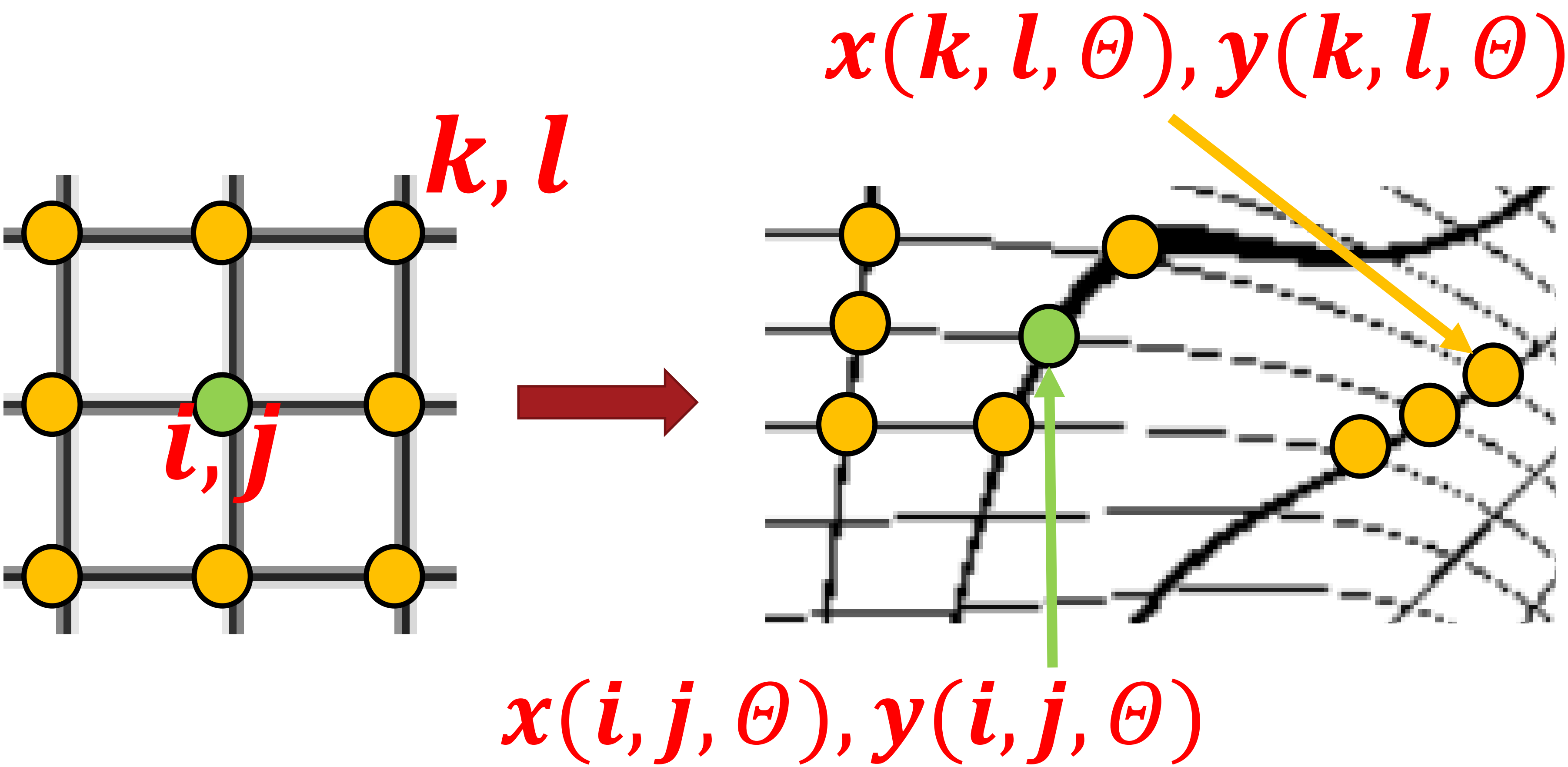
**We would like to preserve distances between**:

- Point $i, j$
- Point from $\Omega_{i,j}$



# Image registration: regularization

**We would like to preserve distances between points in original and deformed grid**



# Image registration: regularization

**Original optimization formulation**:

$$F(I,J,\Theta) = \min \sum_{i,j} d(I(i,j) - J(x(i,j,\Theta), y(i,j,\Theta)))$$

**Formulation with regularization**:

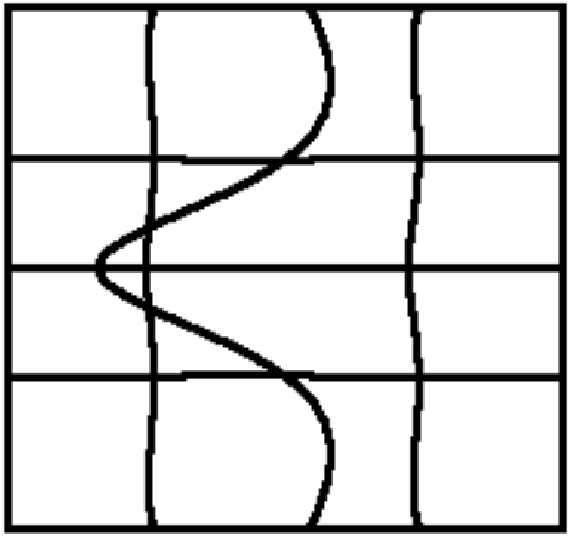
$$F(I,J,\Theta) = \min \left( \sum_{i,j} d(I(i,j) - J(x(i,j,\Theta), y(i,j,\Theta))) \right.$$
$$+ \lambda \sum_{i,j} \sum_{k,l \in \Omega_{i,j}} \left( ((i-k) - (x(i,j,\Theta) - x(k,l,\Theta)))^2 \right.$$
$$\left. \left. + ((j-l) - (y(i,j,\Theta) - y(k,l,\Theta)))^2 \right) \right)$$

# Image registration: regularization

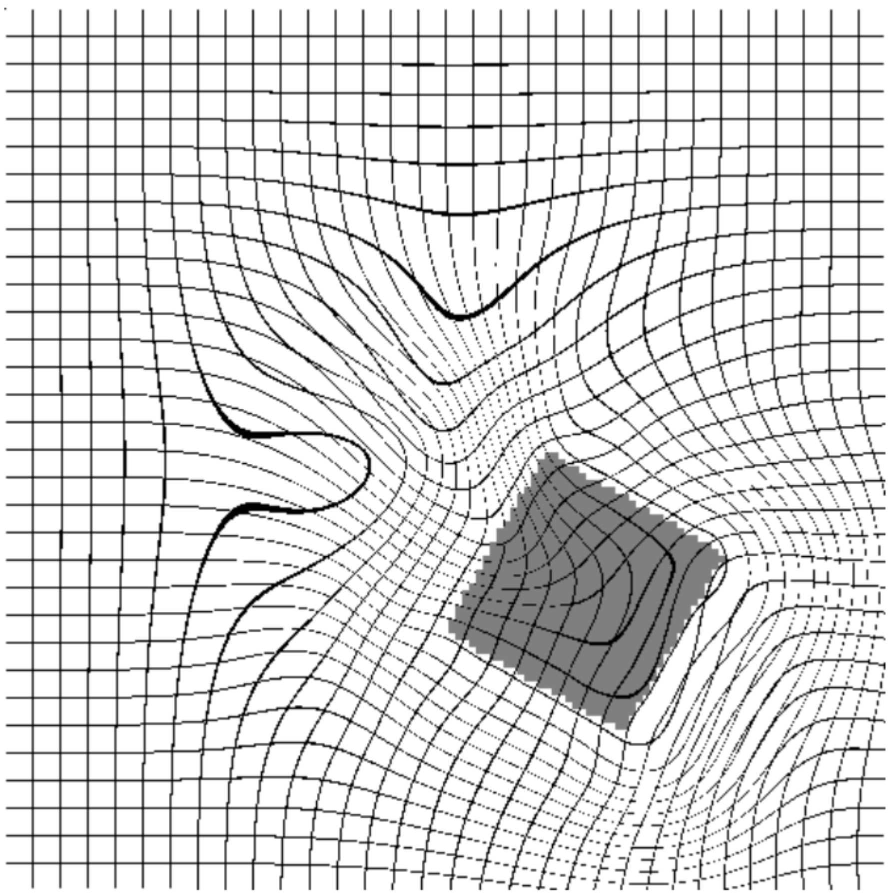
**What will this regularization do?**

$$\sum_{i,j} \sum_{k,l \in \Omega_{i,j}} \Big( \Big(1 - \mathrm{sign}\big((i-k) \cdot (x(i,j,\Theta) - x(k,l,\Theta))\big)\Big) + \Big(1 - \mathrm{sign}\big((j-l) \cdot (y(i,j,\Theta) - y(k,l,\Theta))\big)\Big) \Big)$$

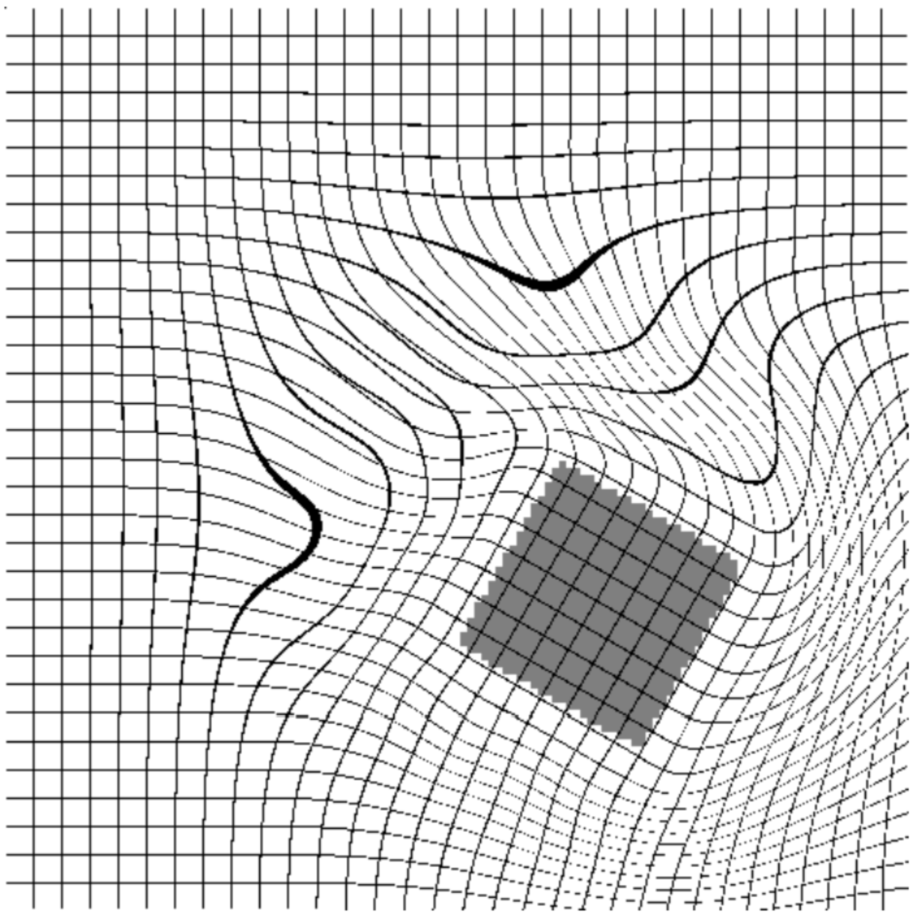
folding



# Image registration: regularization



**Unregularized**

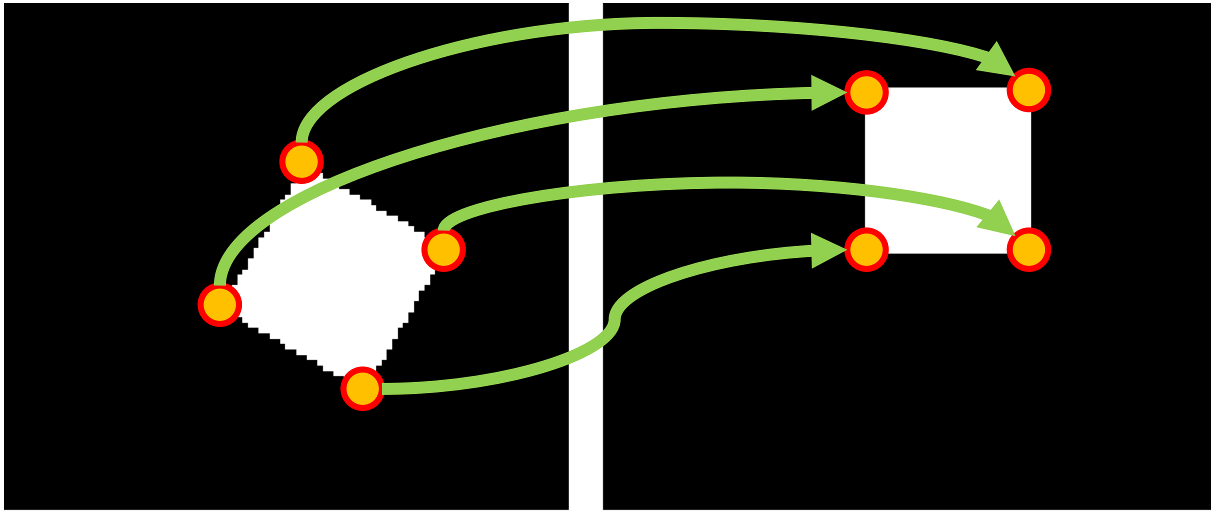


**Regularized**

# Image registration: landmarks

**Can we help registration of two squares?**

- Automatically detect square corners
- Force registration to align corners



fixed moving

# Image registration: landmarks

**New optimization function**:

$$F(I,J,\Theta) = \min\left(\sum_{i,j} d(I(i,j) - J(x(i,j,\Theta), y(i,j,\Theta))) + \lambda \cdot Reg + \gamma \cdot \sum_{t\in T} \left|p_t^I - p_t^J(\Theta)\right|^2\right)$$

Minimizing distance between landmarks in two image

**How to ensure the landmarks match?**

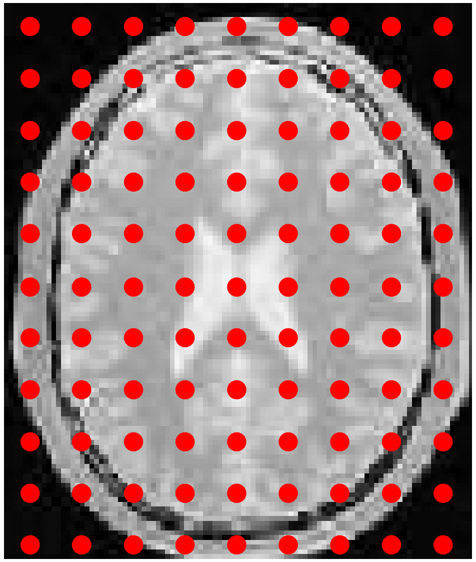
# Image registration: Summary algorithm

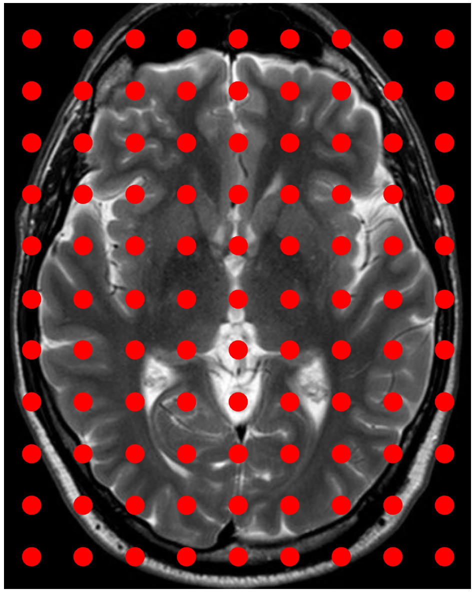
- **Define algorithm settings**:
  - Acceptable transformations (Rigid, Similarity, Non-rigid)
  - Similarity measure (Mean Squares, Correlation, Mutual Information)
  - Multi-resolution pyramid, grid schedule
  - Regularization

- **Prepare input images**:
  - Normalization of intensities
  - Rescaling

# Image registration: Summary algorithm

- **Optimization**:
  - Compute similarity measure over grids on both images
  - Compute gradients on grid on moving image
  - Move grid according to the gradients
  - Deform moving image according to the grid
  - Use splines to interpolate pixels outside the grid
- **Repeat until convergence**

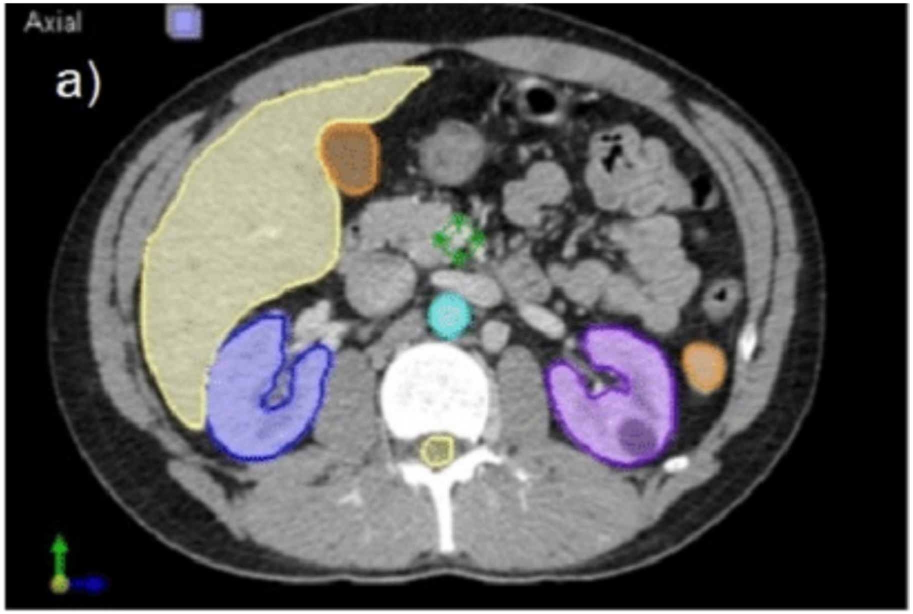




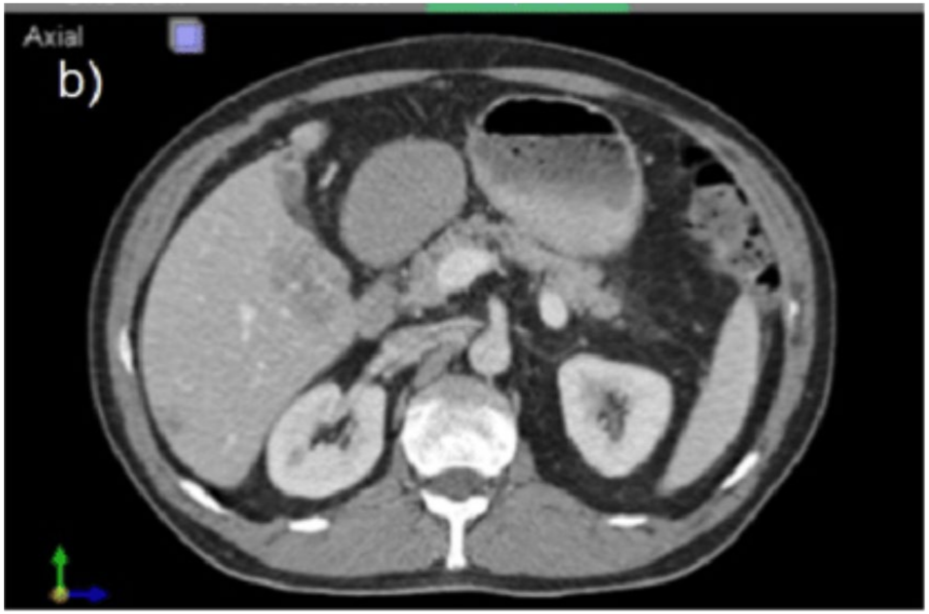
# Image registration: applications

## Atlas-based segmentation

Training images



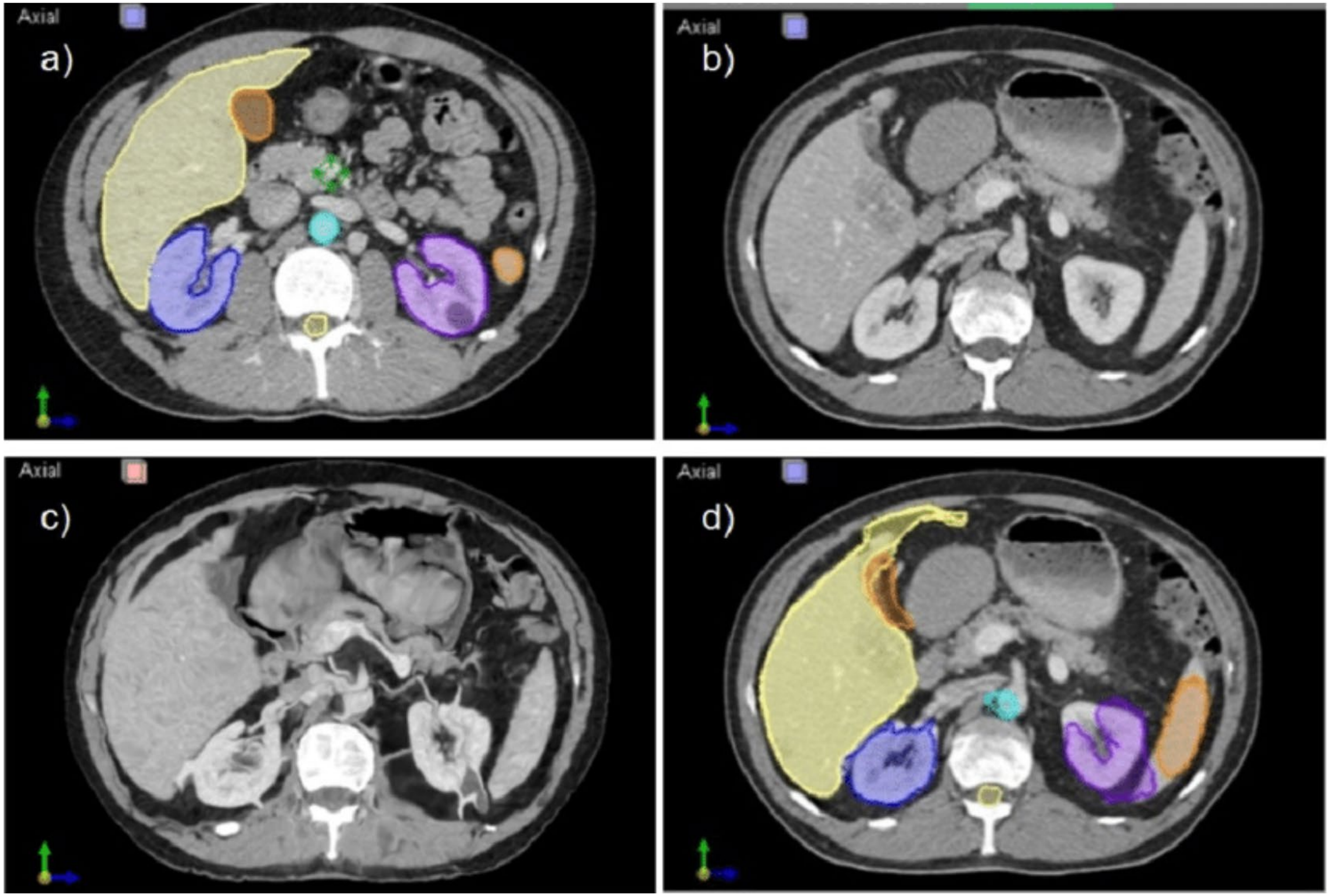

New image




Collection of segmented images

# Image registration: applications

## Atlas-based segmentation



# Image registration: applications

**Atlas-based segmentation:**

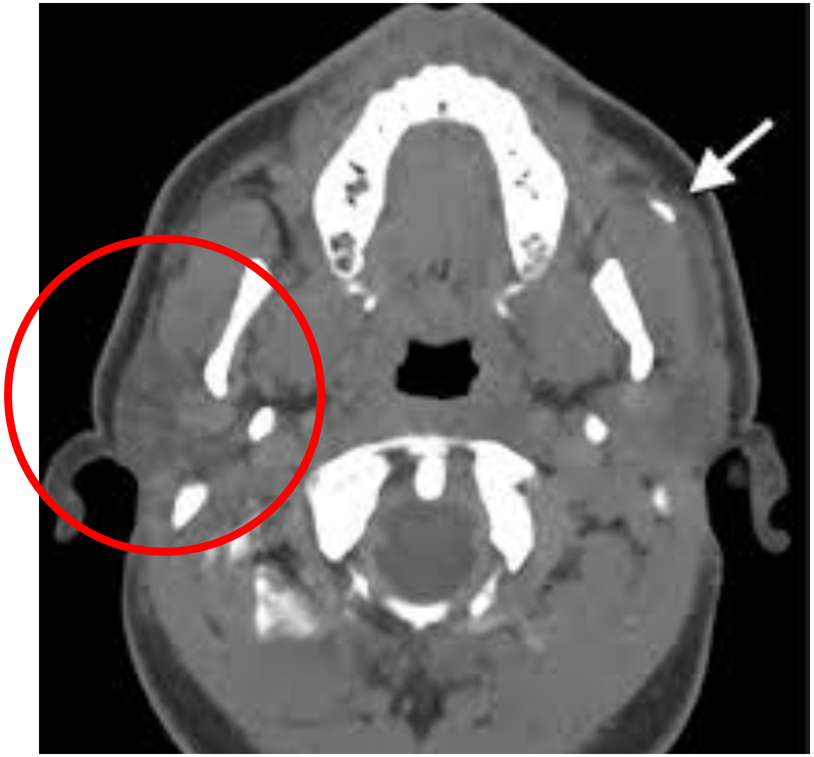
- Advantages:
  - Anatomically correct segmentation
  - Light training image requirements
  - Invariant to the number of target objects

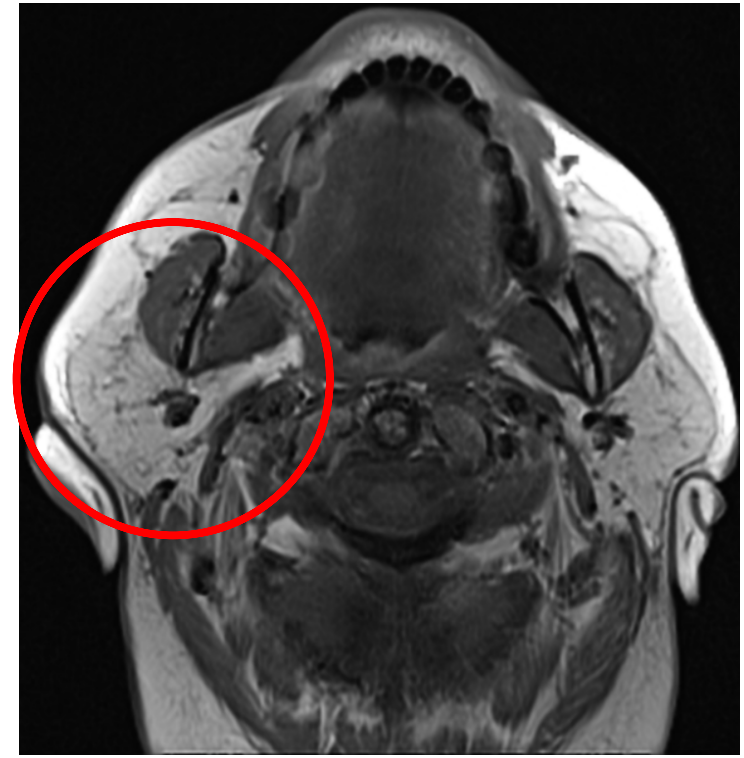
- Disadvantages:
  - Slow

# Image registration: applications

**Multi-modal image registration**:

- Improves visibility of structures (bones – CT, soft tissues – MR, tumors – PET and MR)

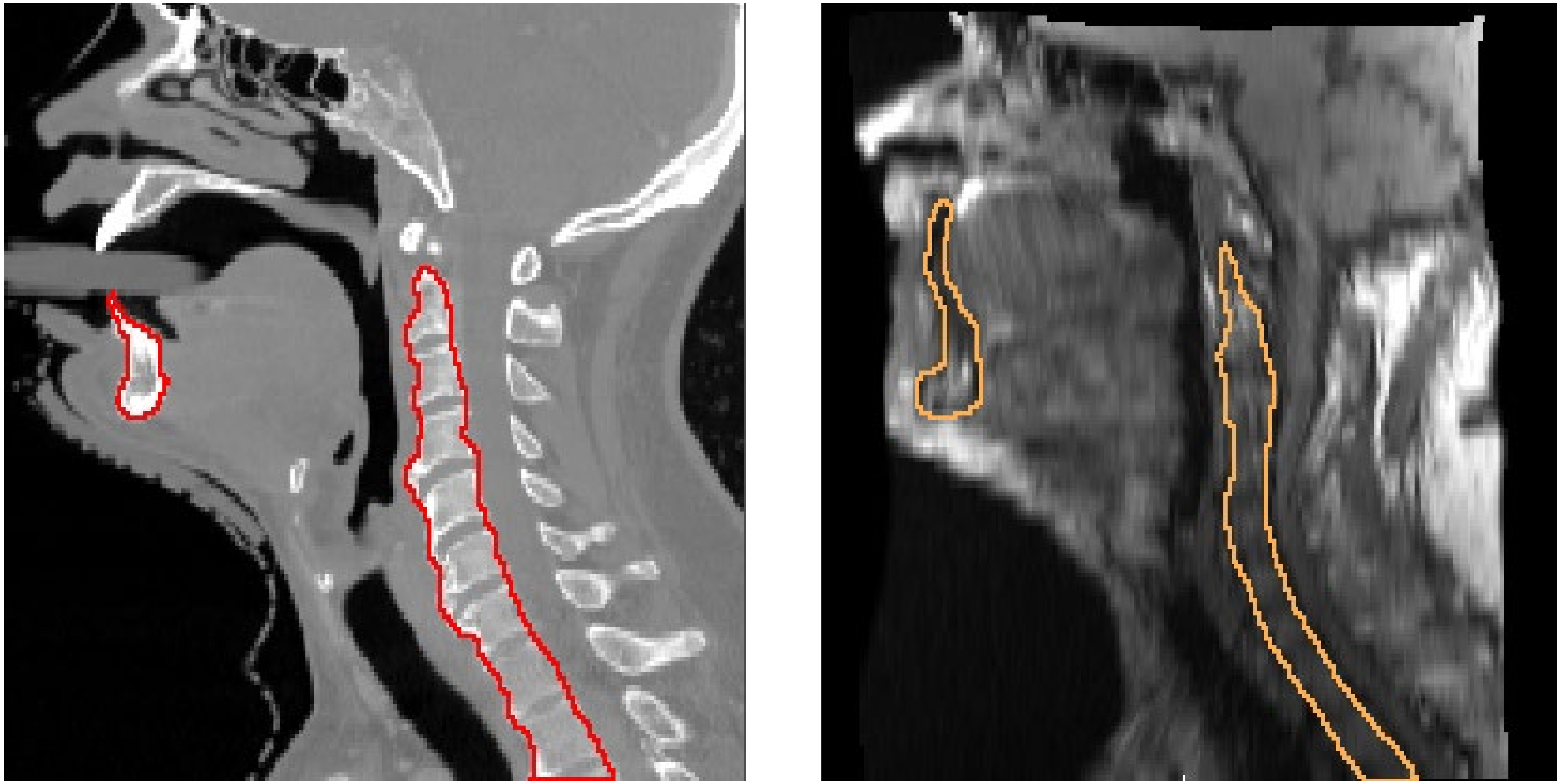




# Image registration: applications

**Multi-modal image registration**:

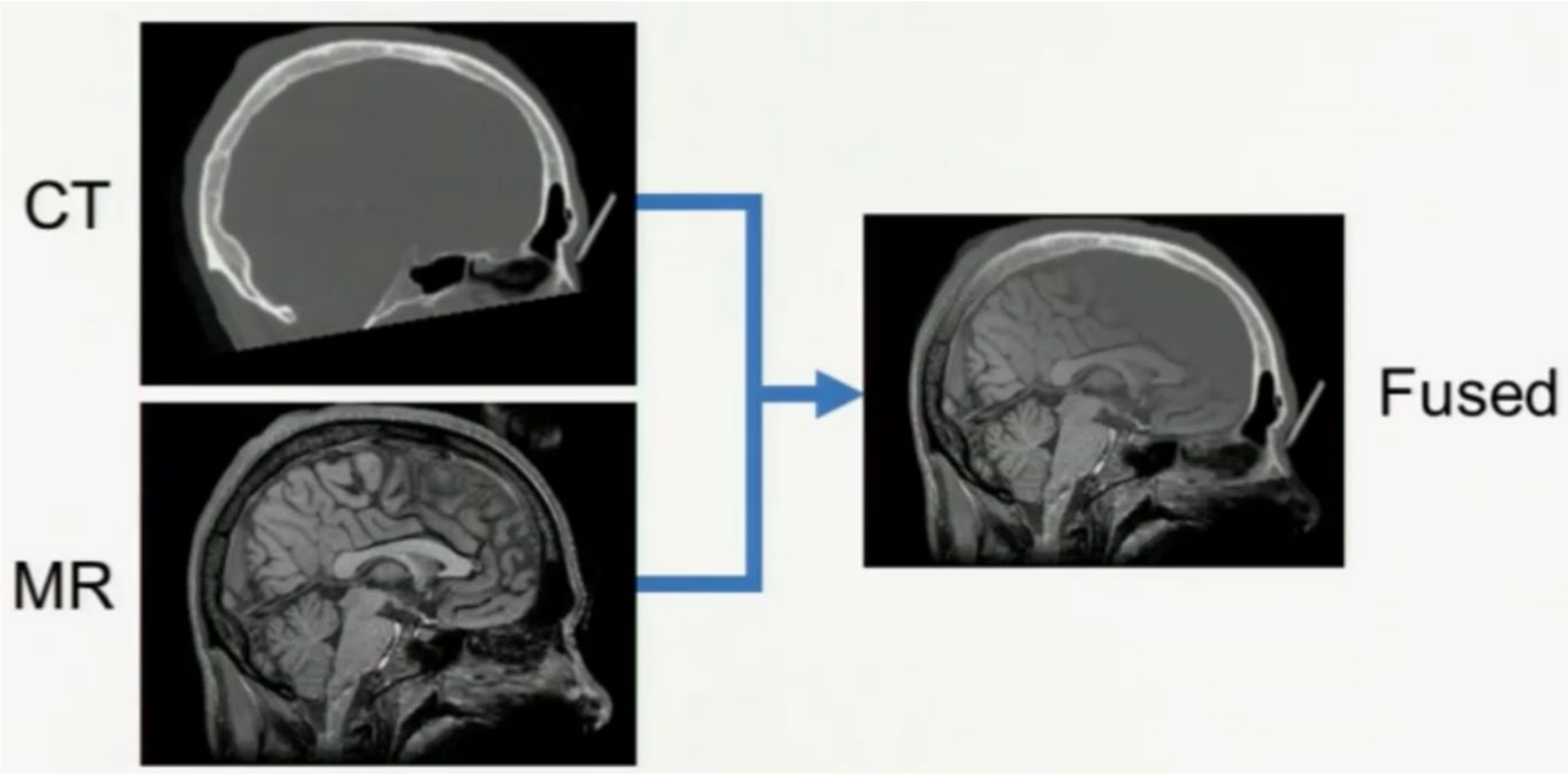
- Improves visibility of structures (bones – CT, soft tissues – MR, tumors – PET and MR)



# Image registration: applications

**Multi-modal image registration**:

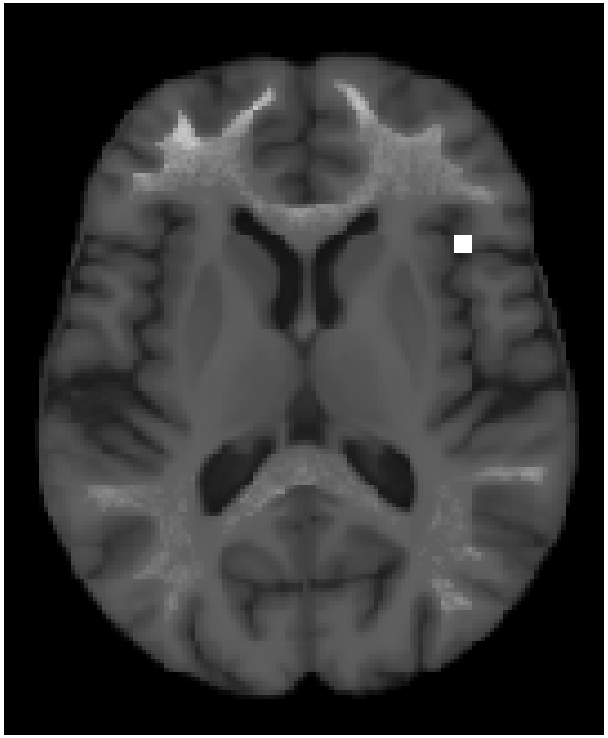
- Improves visibility of structures (bones – CT, soft tissues – MR, tumors – PET and MR)



# Image registration: applications

## Atlas-based abnormality detection

Healthy training images

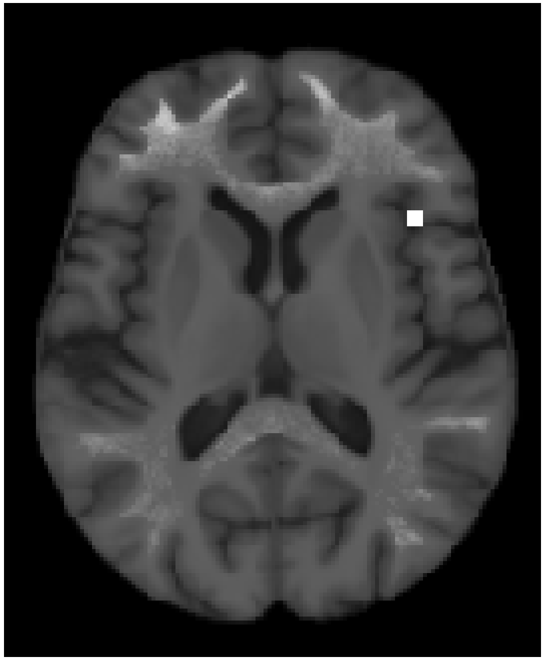


Testing image with tumor

# Image registration: applications

**Atlas-based abnormality detection**:

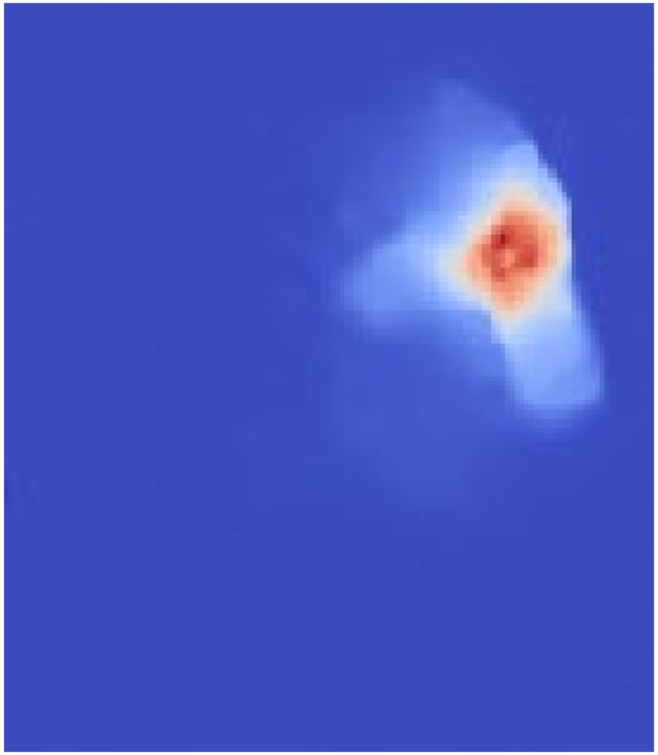
- Register healthy image to potentially abnormal
- Study deformation field to find abnormalities



# Image registration: applications

**Atlas-based abnormality detection**:

- Problem
  - Difficult to detect small abnormalities

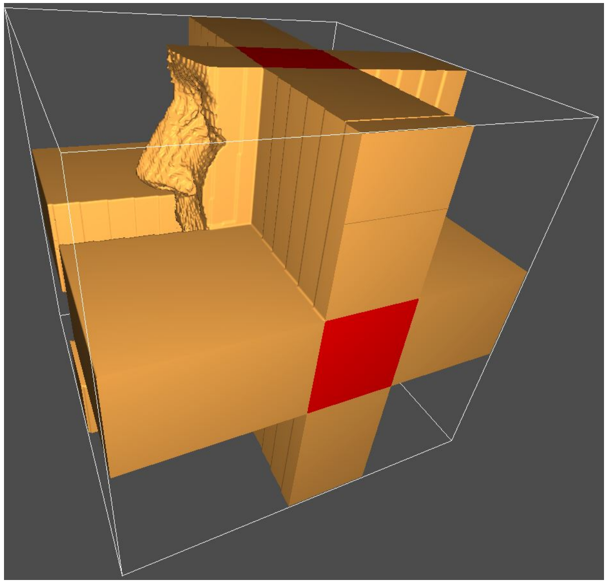


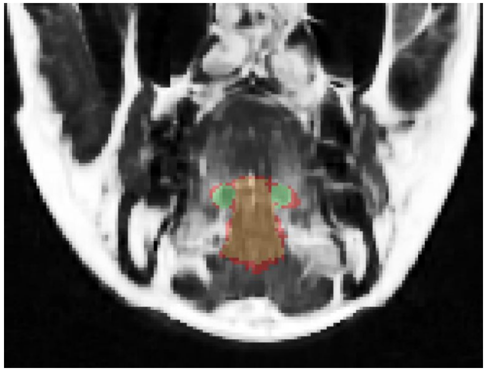
# Break

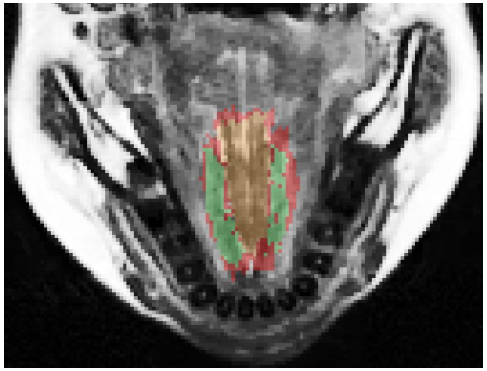
# Image registration: applications

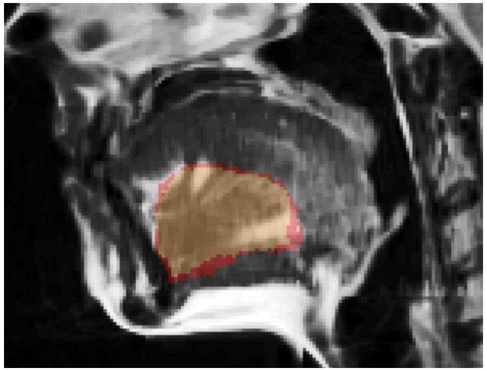
**Super-resolution images**:

- We have three orthogonal 3D MR images of high in-plane resolution but high slice thickness
- How to create a super-resolution 3D MR?





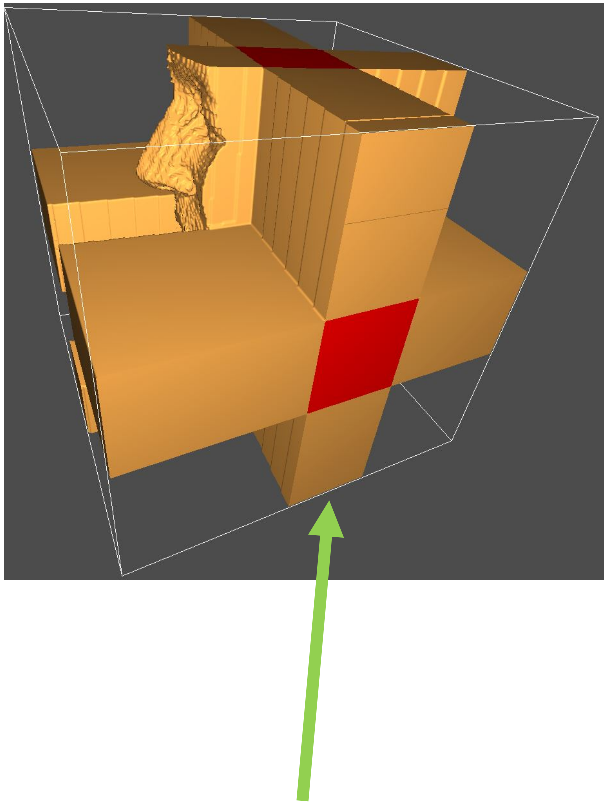




# Image registration: applications

**Super-resolution images**:

- We convert orthogonal images to full volume but adding empty space
- This empty space should not be taken into account during registration
- Idea of masked registration



The orthogonal images are "thin", they do not cover the whole volume

# Image registration: applications

**Super-resolution images**:

$$\sum_{i,j} M_I(i,j) \cdot M_J(x(i,j,\Theta), y(i,j,\Theta)) \cdot d(I(i,j) - J(x(i,j,\Theta), y(i,j,\Theta)))$$

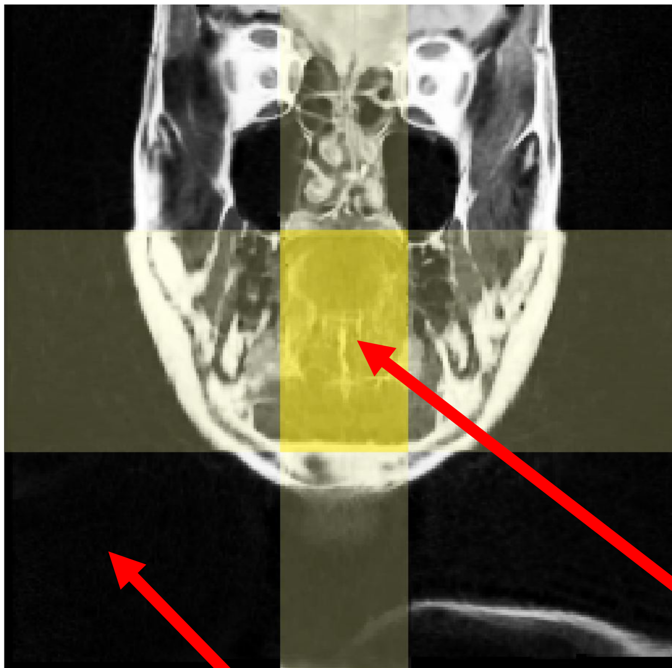
Binary mask for fixed image $I$

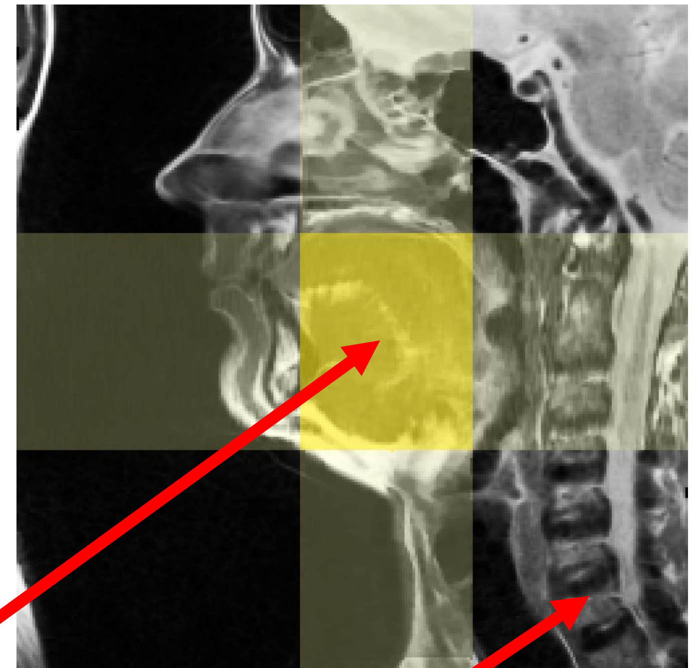
Binary mask for moving image $J$

We only take pixels that are not empty in both images

# Image registration: applications

## Super-resolution images:





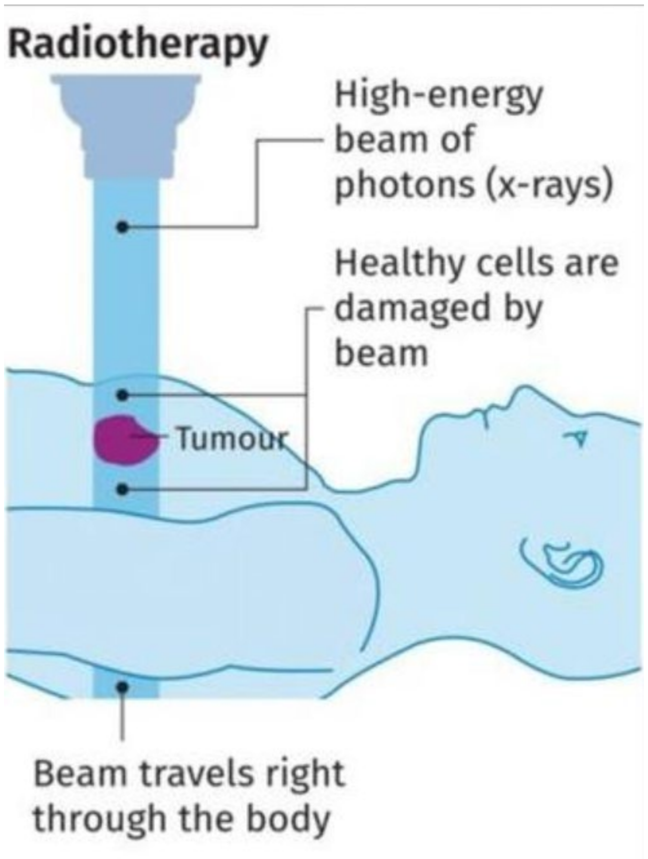
All three images are used here

These pixels are moved only under regularization forces

# Image registration: applications

**Adaptive radiotherapy planning**:

- Radiation kills both cancer and non-cancer cells
- We need to target tumor very precisely

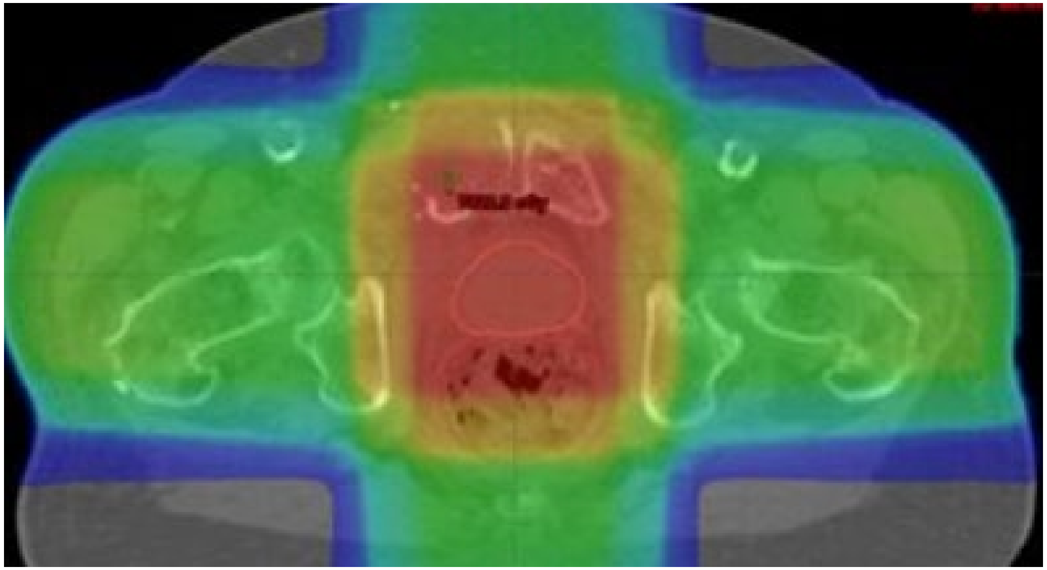


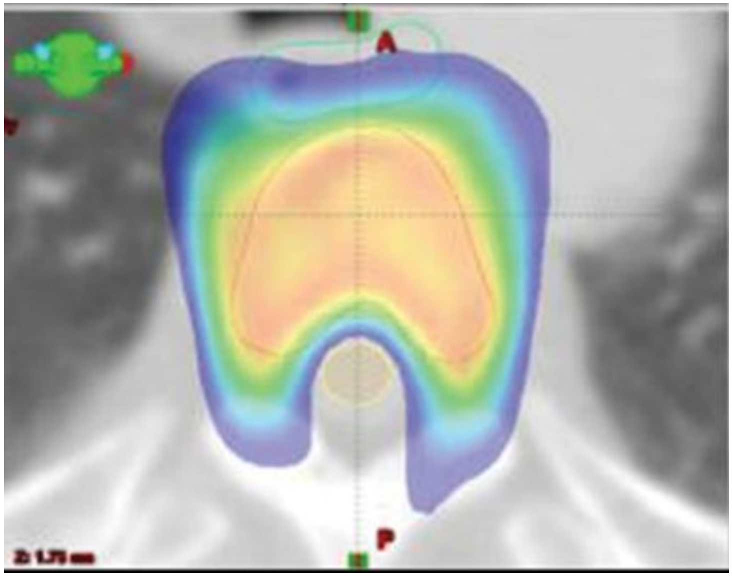

Source: Cancer Research UK

# Image registration: applications

**Adaptive radiotherapy planning**:

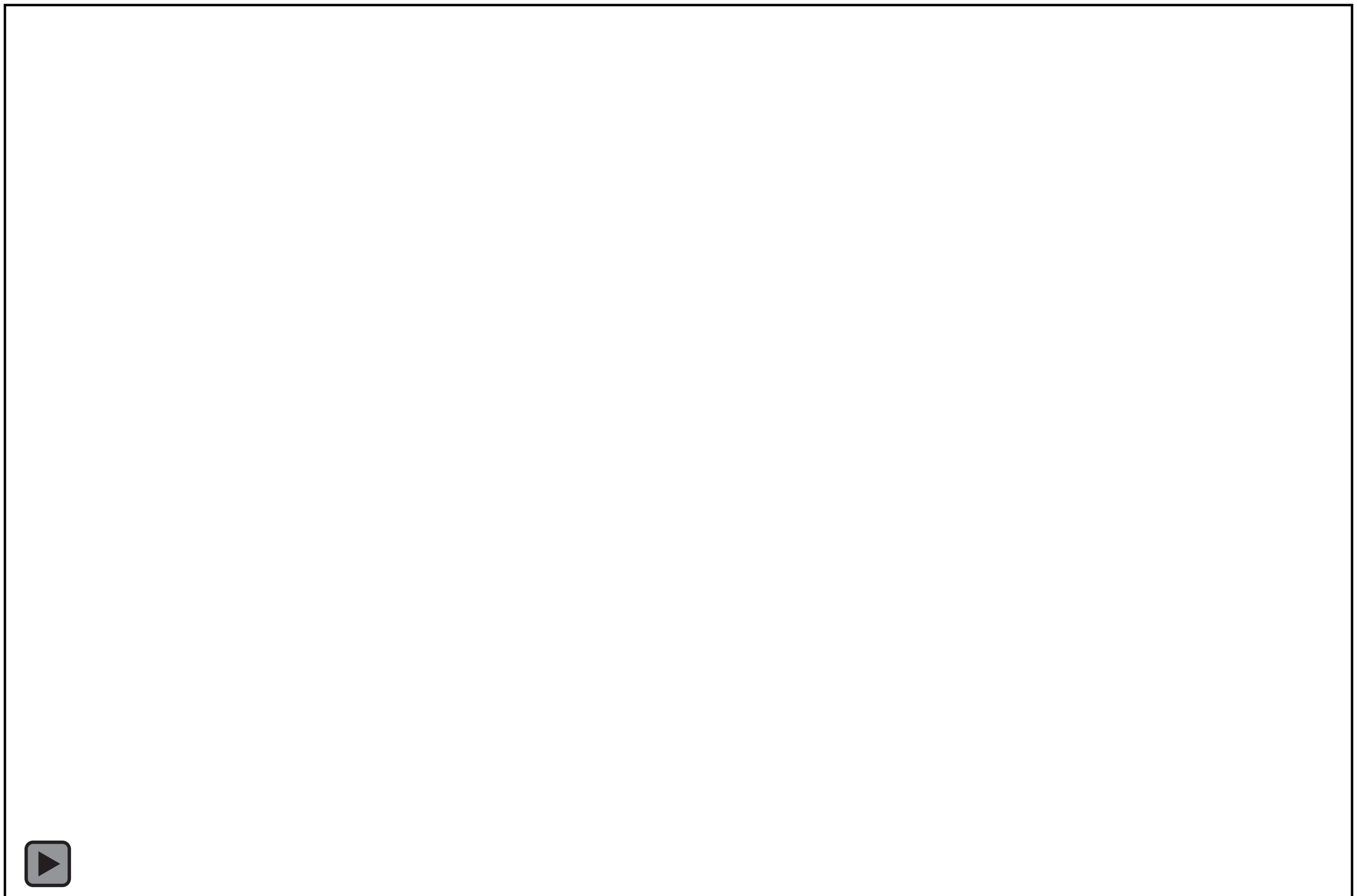
- Modern radiotherapy allows very high level of personalization





# Image registration: applications

**Adaptive radiotherapy planning**:



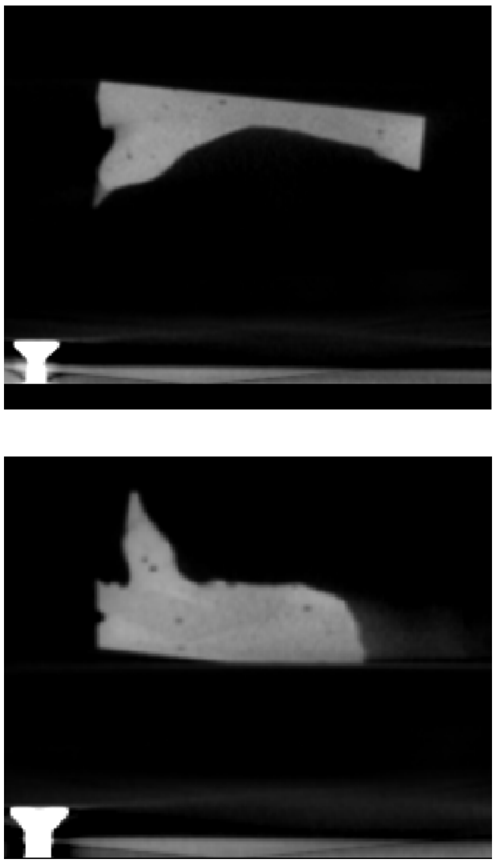
# Image registration: applications

**Adaptive radiotherapy planning**:

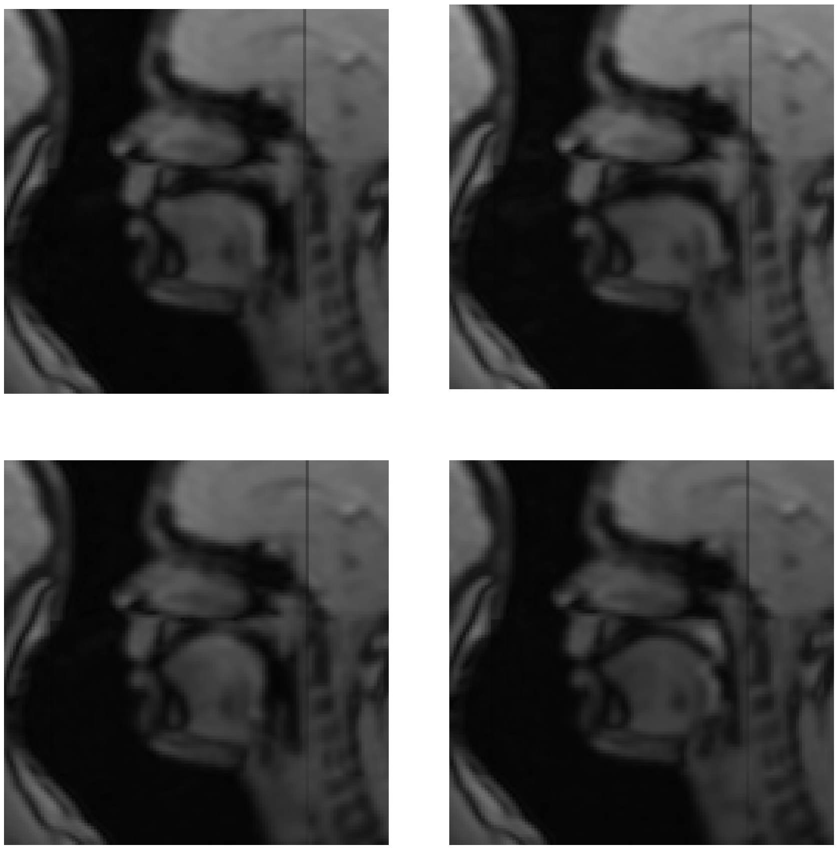
# Image registration: applications

**Landmark-based registration**:

Dental casts



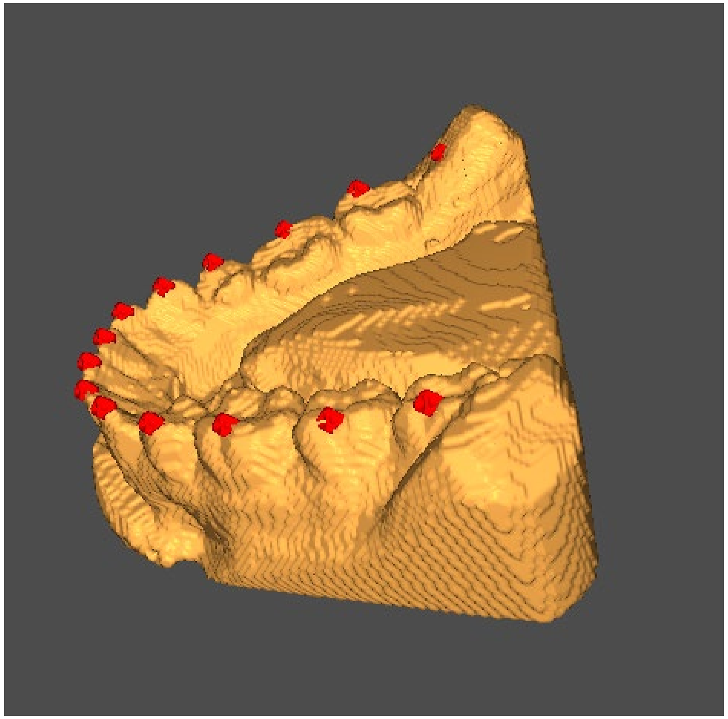
MR dynamic images



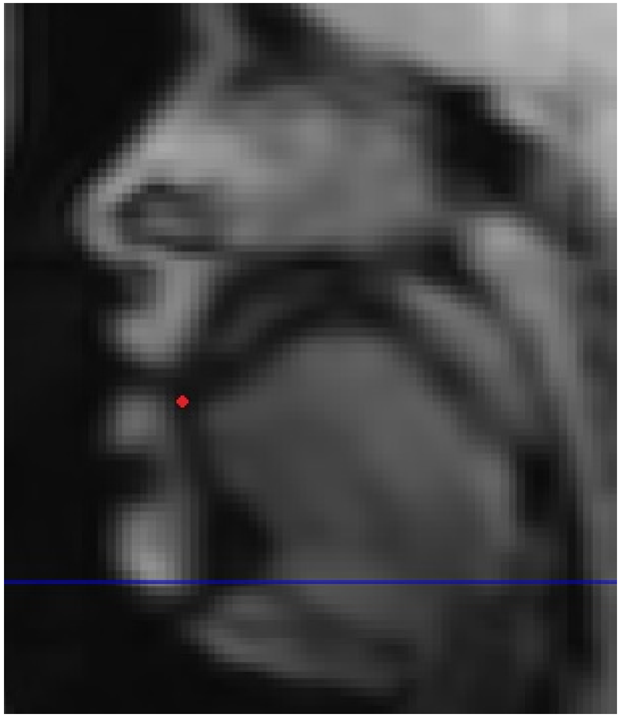
# Image registration: applications

**Landmark-based registration**:

Dental cast landmarking



MR image landmarking

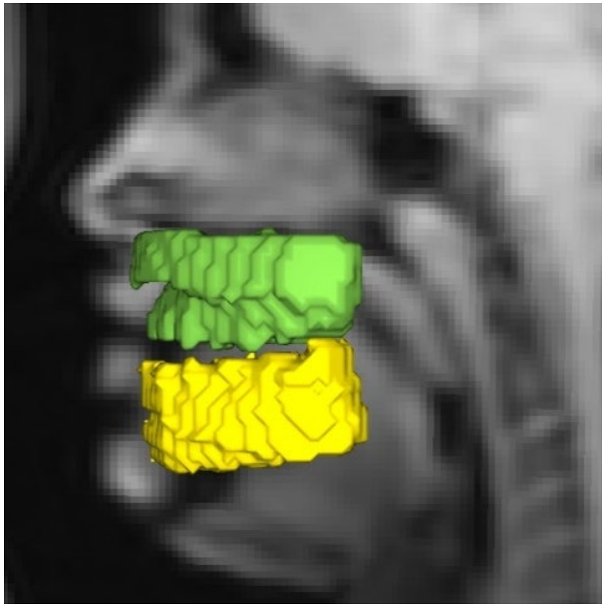


# Image registration: applications

**Landmark-based registration**:

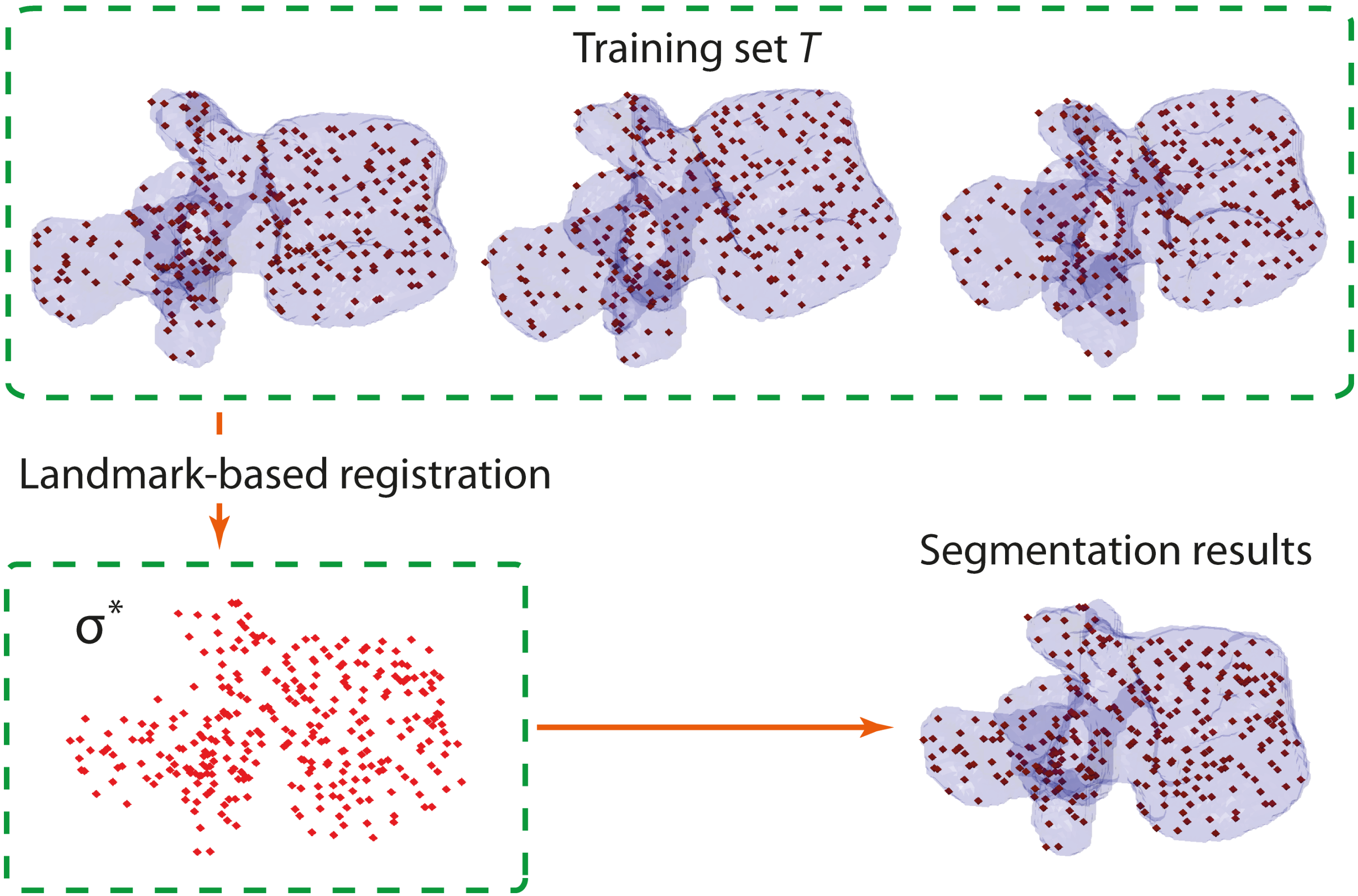
$$F(I,J,\Theta) = \min\left(\lambda \cdot Reg + \gamma \cdot \sum_{t \in T} \left|p_t^I - p_t^J(\Theta)\right|^2\right)$$

- What kind of Θ should be for dental cast registration?
- Θ is rigid (translation and scaling)
- $Reg$ is not needed



# Image registration: applications

## **Landmark-based registration**:



# Image registration: applications

**Landmark-based registration**:

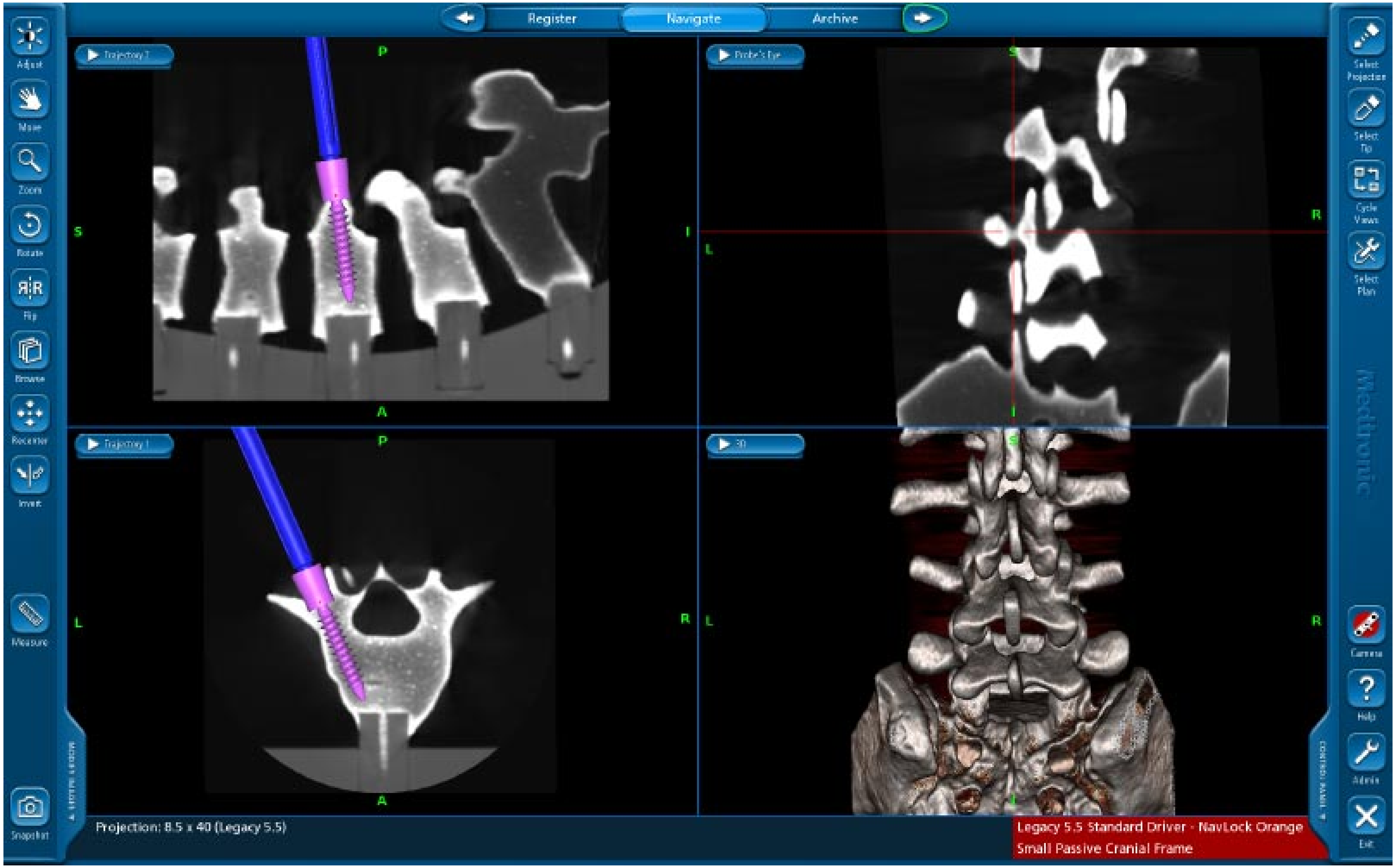
$$F(I,J,\Theta) = \min\left(\lambda \cdot Reg + \gamma \cdot \sum_{t \in T} \left|p_t^I - p_t^J(\Theta)\right|^2\right)$$

- $\Theta$ is non-rigid
- $Reg$ is needed
- Various algorithms exist. For example coherent point drift

# Image registration: Coherent Point Drift

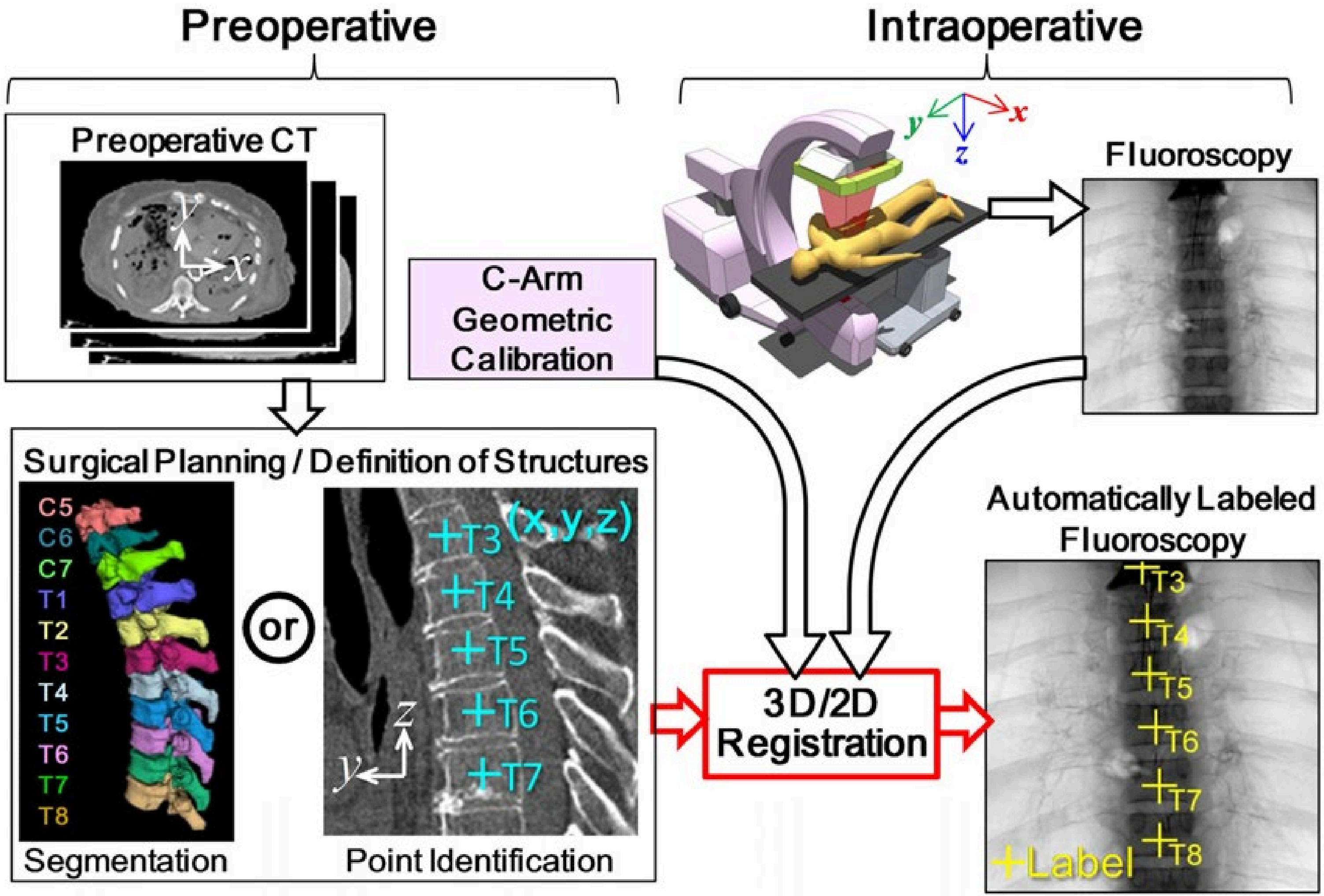
# Image registration: applications

## **2D-3D registration**:



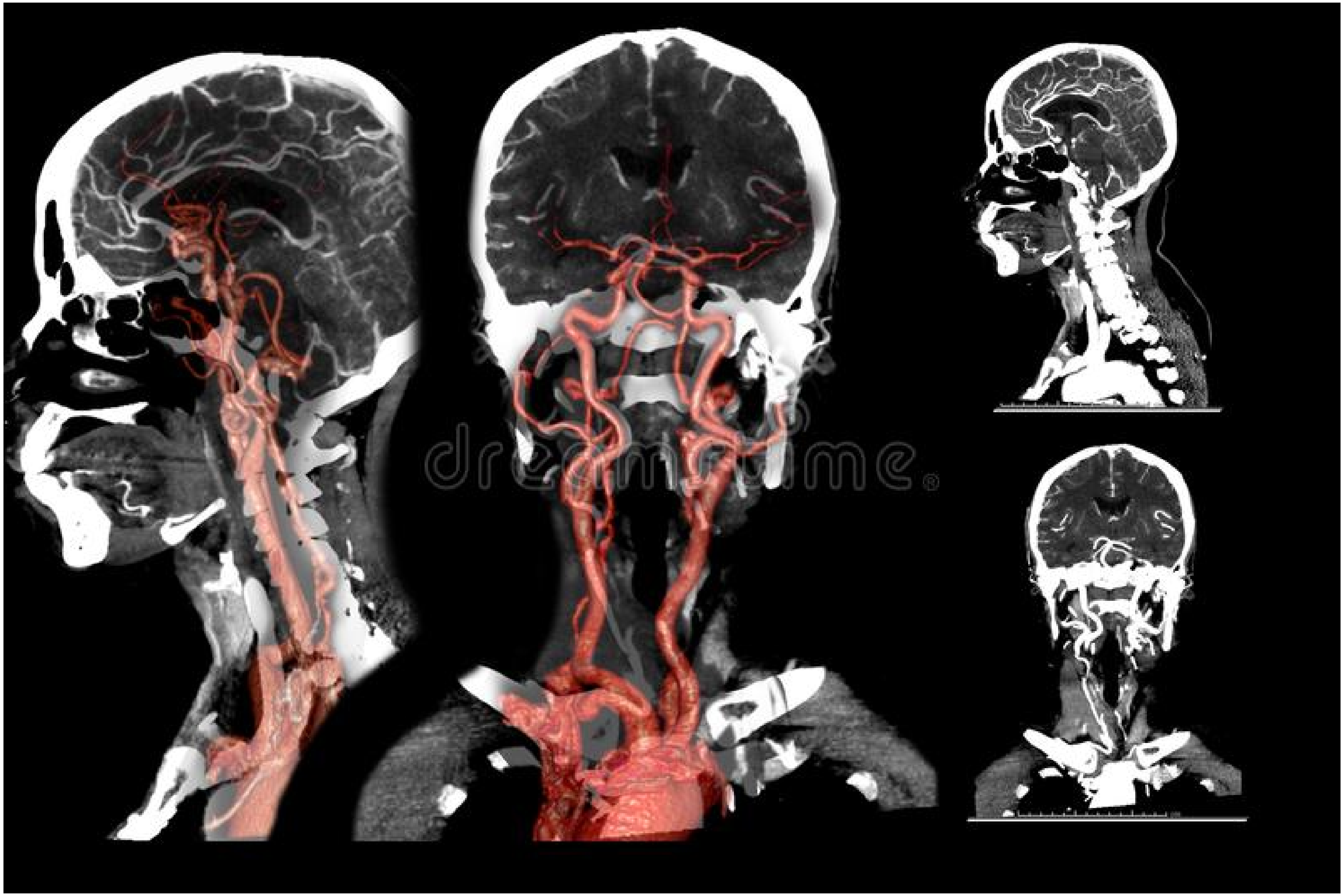
# Image registration: applications

## 2D-3D registration:



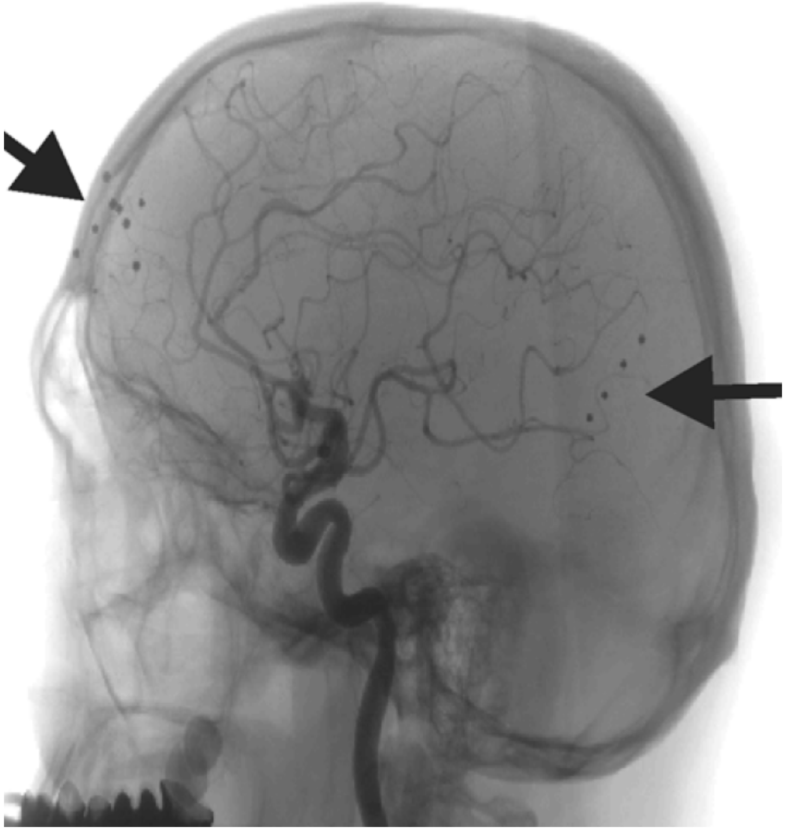
# Image registration: applications

**2D-3D registration**:

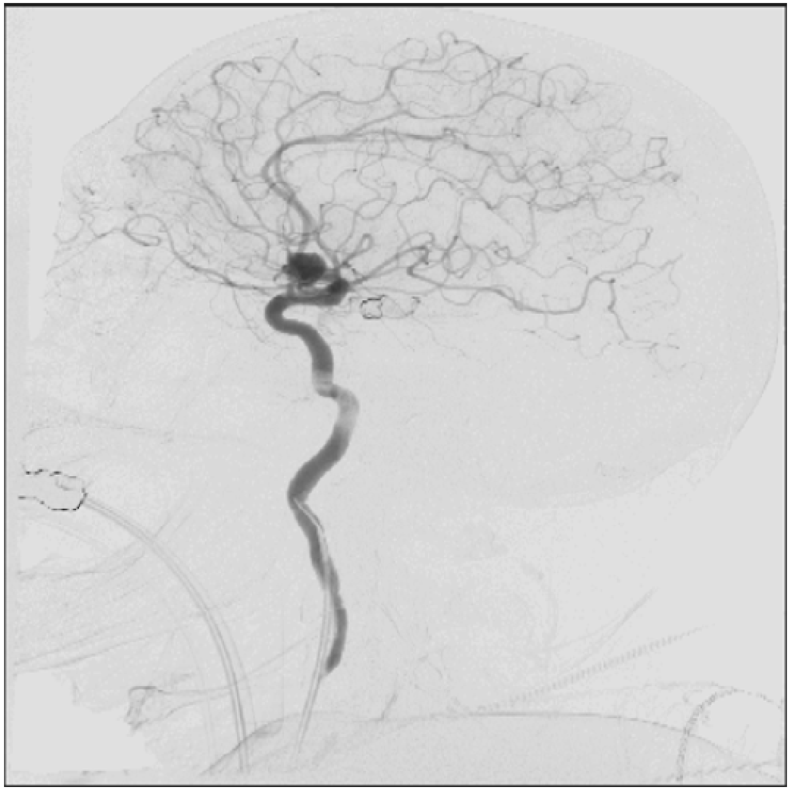


# Image registration: applications

**2D-3D registration**:



Digitally-reconstructed image

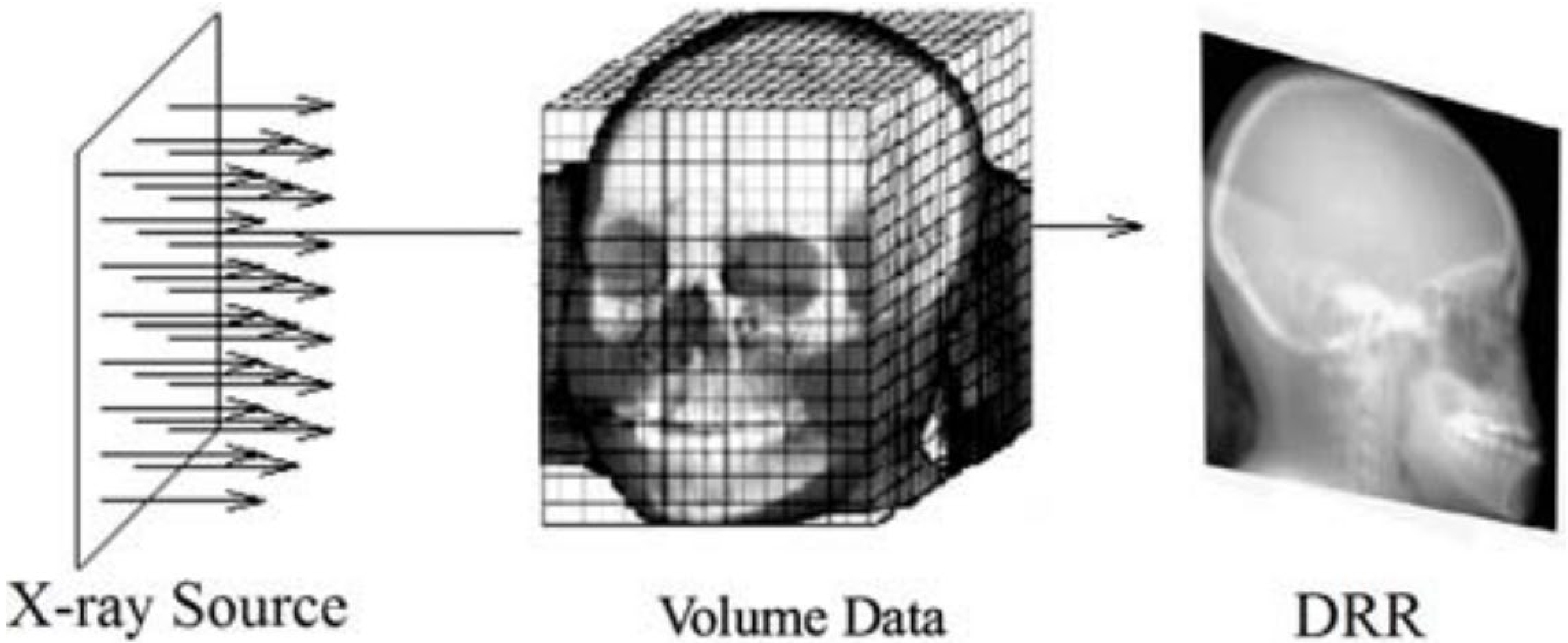


Intraoperative image

# 3D-2D registration

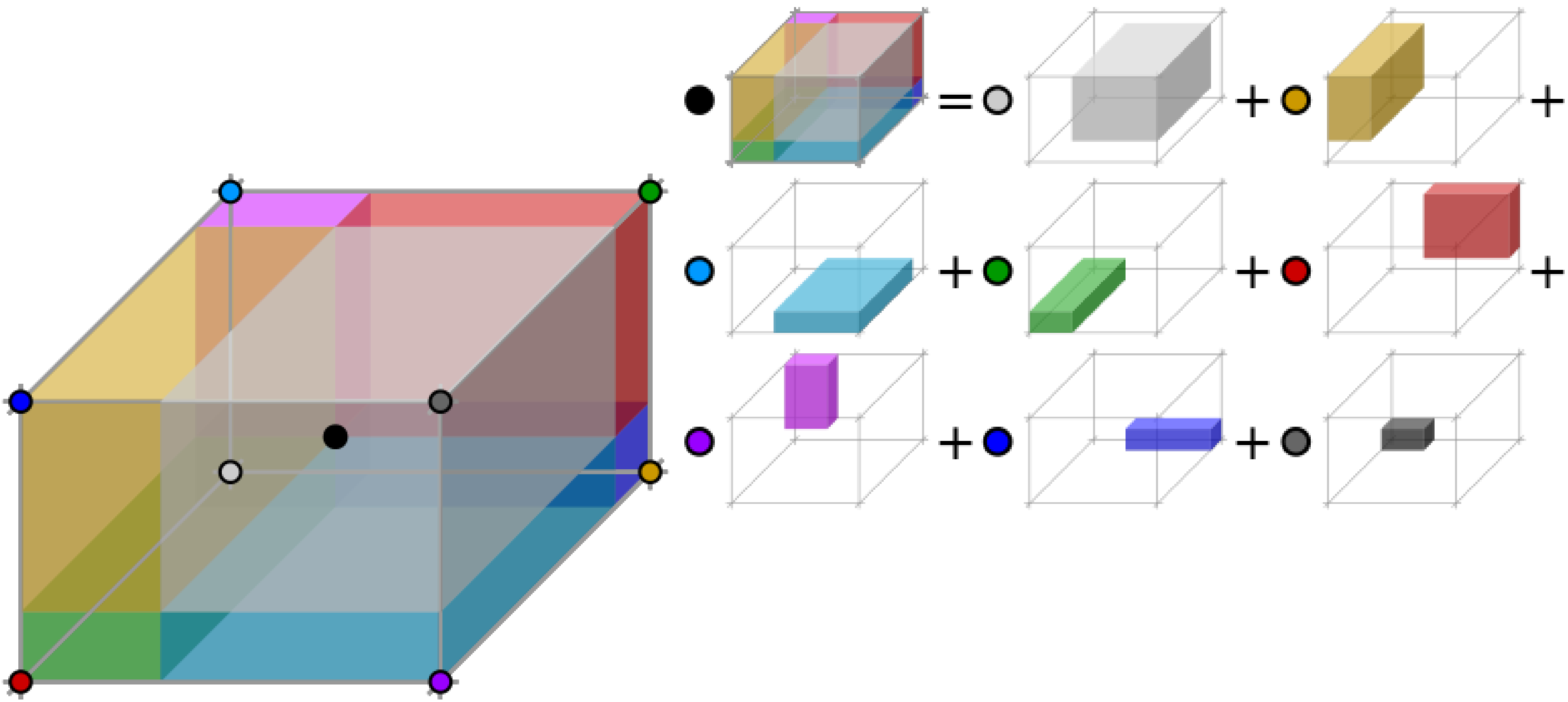
**Digitally reconstructed radiograph**:

- Location to the X-ray source
- Location to the detector



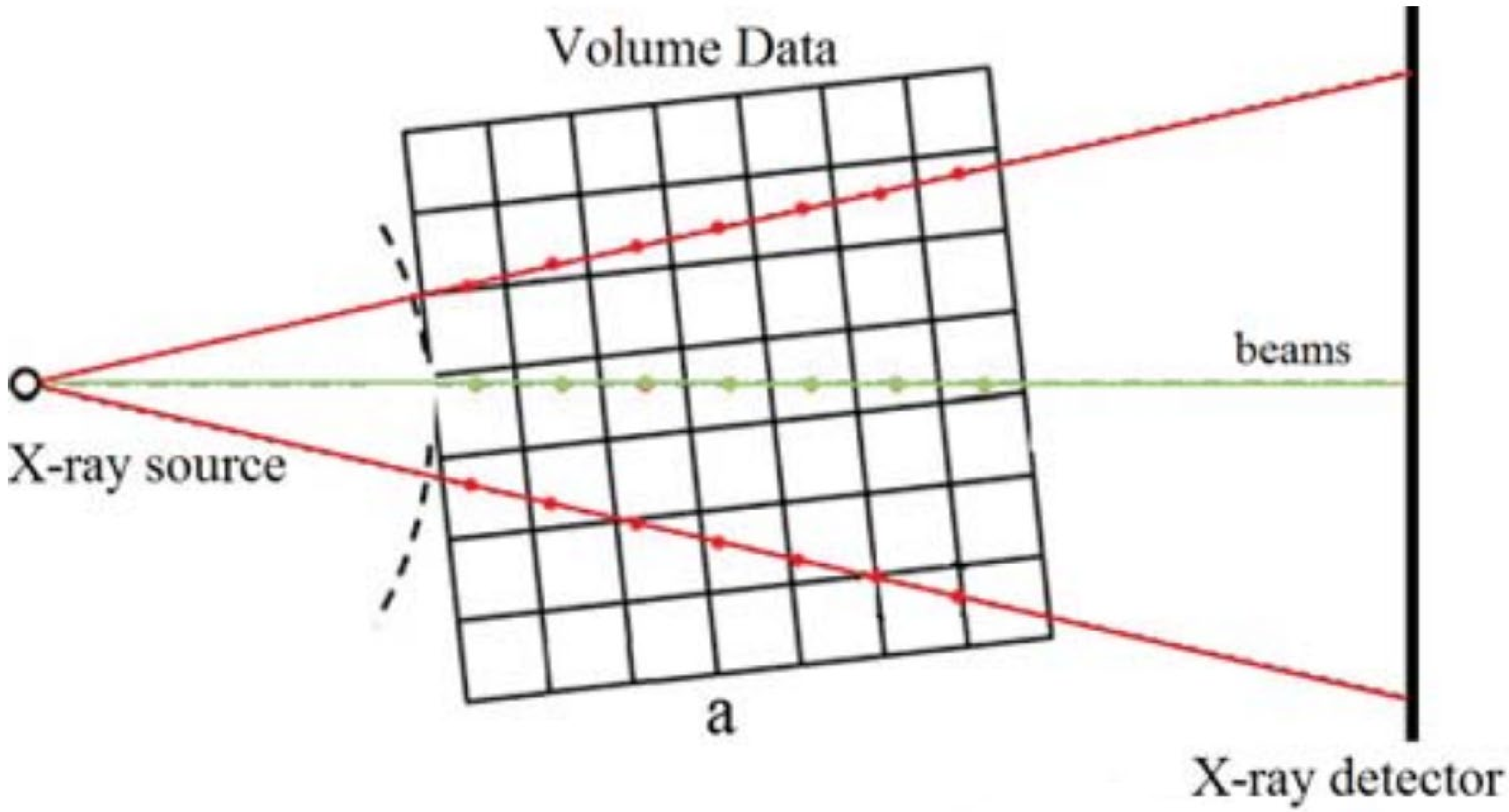
# 3D-2D registration

## We need 3D interpolations



# 3D-2D registration

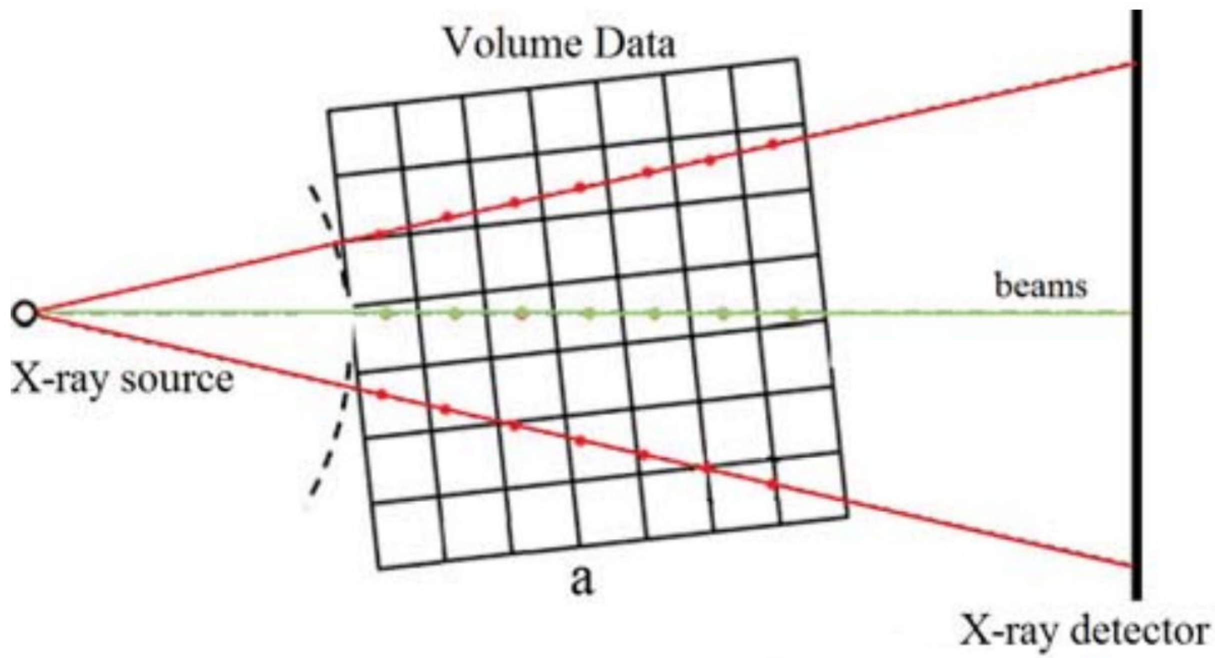
## Digitally reconstructed radiograph



# 3D-2D registration

**Digitally reconstructed radiograph**:

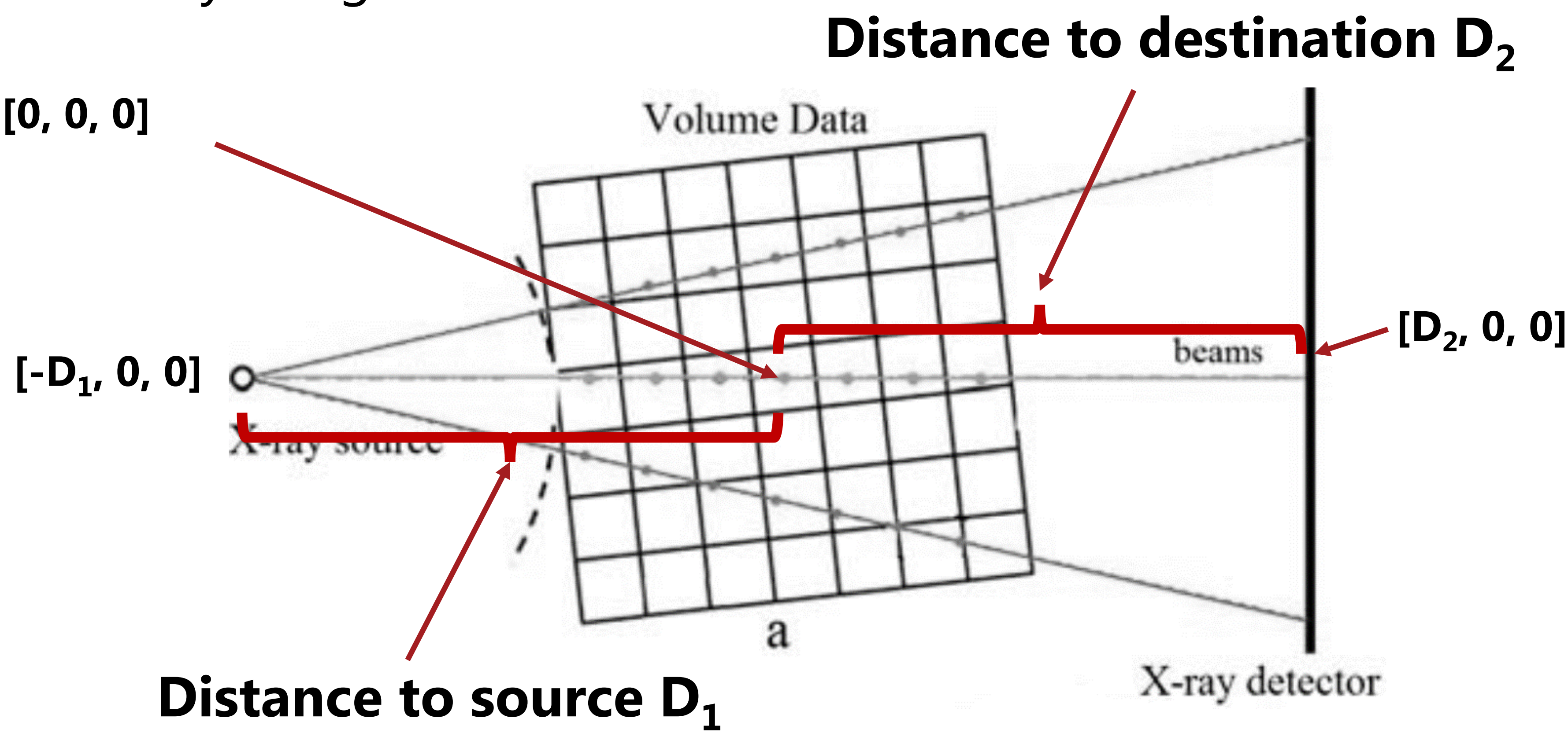
- Define the size of the DRR
- Go through every pixel of DRR
- Compute the maximum of the ray that goes from the source and detector



# 3D-2D registration

**Digitally reconstructed radiograph**:

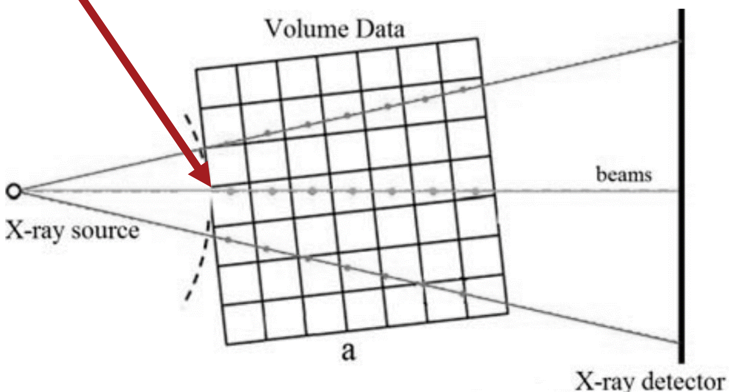
- CT image size $X \times Y \times Z$
- X-ray image size $A \times B$



# 3D-2D registration

**Let's say you want to find the value at pixel [0, 0]**:

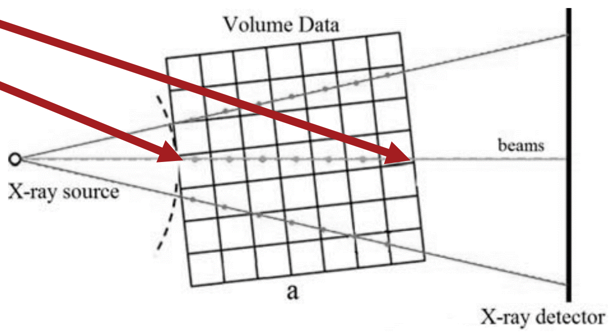
- This point has 3D coordinate [$D_2$, -A/2, -B/2]
- For every 1 mm passed in x-direction, we pass:
  - $-\frac{0.5A}{D_1+D_2}$ in y direction
  - $-\frac{0.5B}{D_1+D_2}$ in z direction
- We wait until cross the level –X/2, and compute:
  - y and z coordinates
  - Convert them into the 3D image space:
    - Add [X/2, Y/2, Z/2]



# 3D-2D registration

**Let's say you want to find the value at pixel [0, 0]**:

- Points with x in interval [–X/2; X/2]
  - $y = (D_1 + x) \cdot \frac{-0.5A}{D_1+D_2}$
  - $z = (D_1 + x) \cdot \frac{-0.5B}{D_1+D_2}$

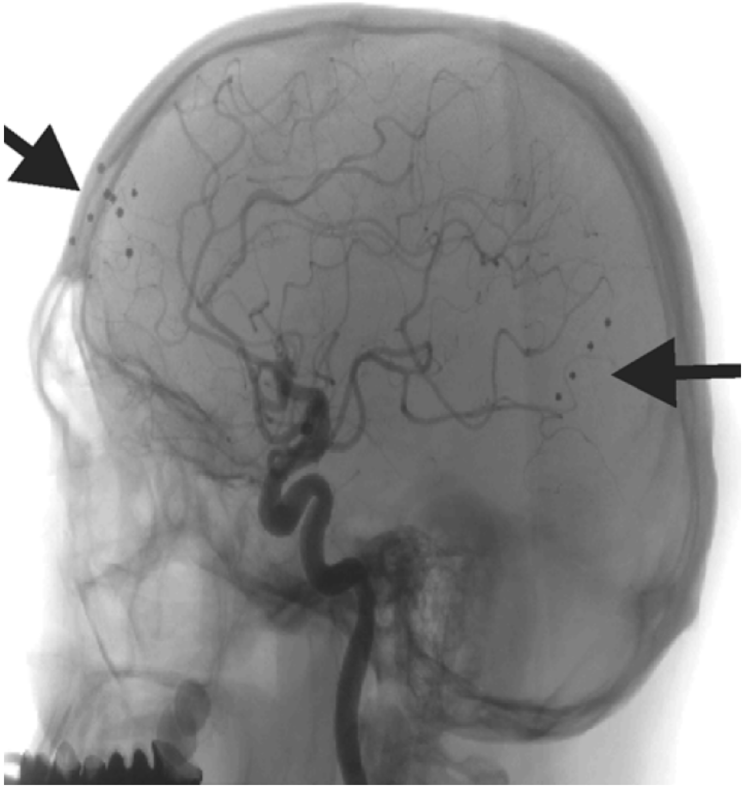



- Compute the interpolated values for voxels inside 3D image

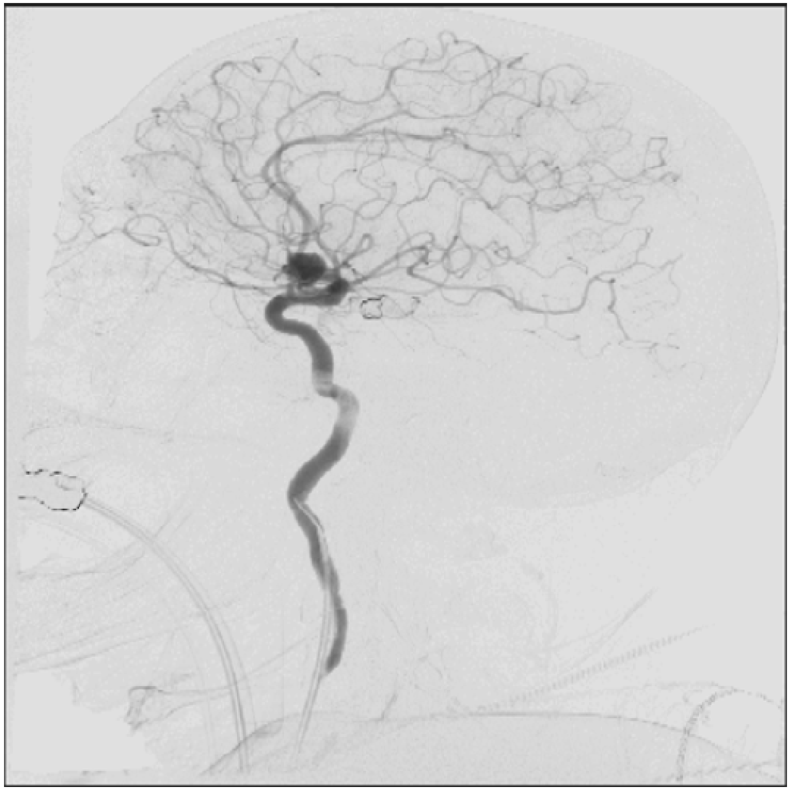
- Find the maximum value

# 3D-2D registration

**Matching DDR with true 2D image:**



Digitally-reconstructed image

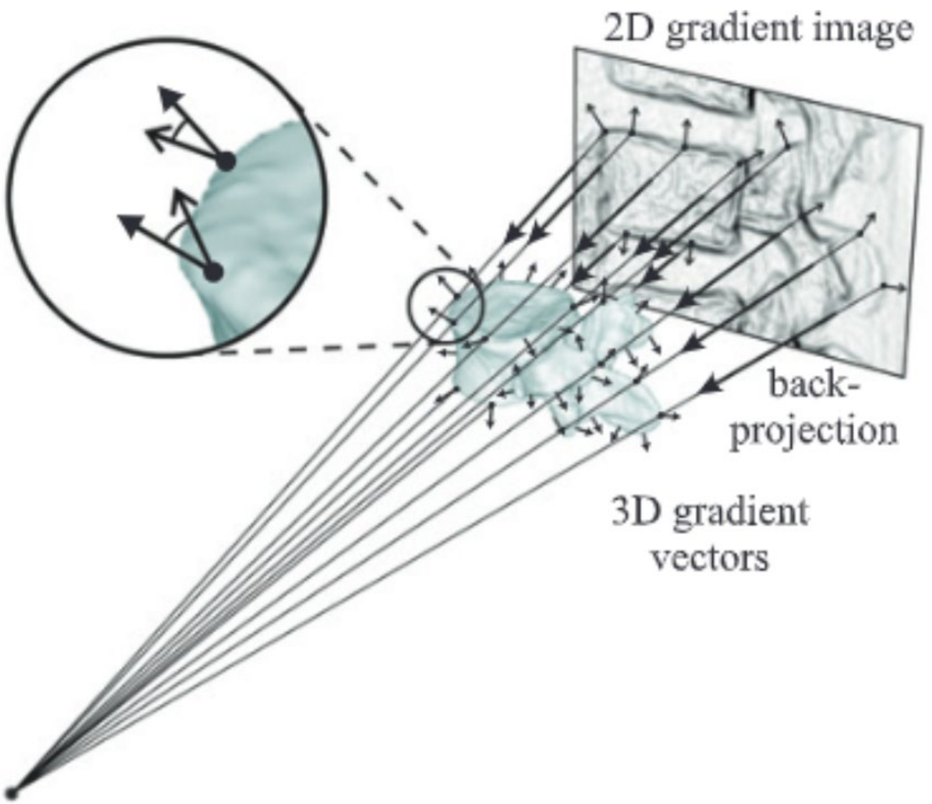


Intraoperative image

# 3D-2D registration

**Gradients**:

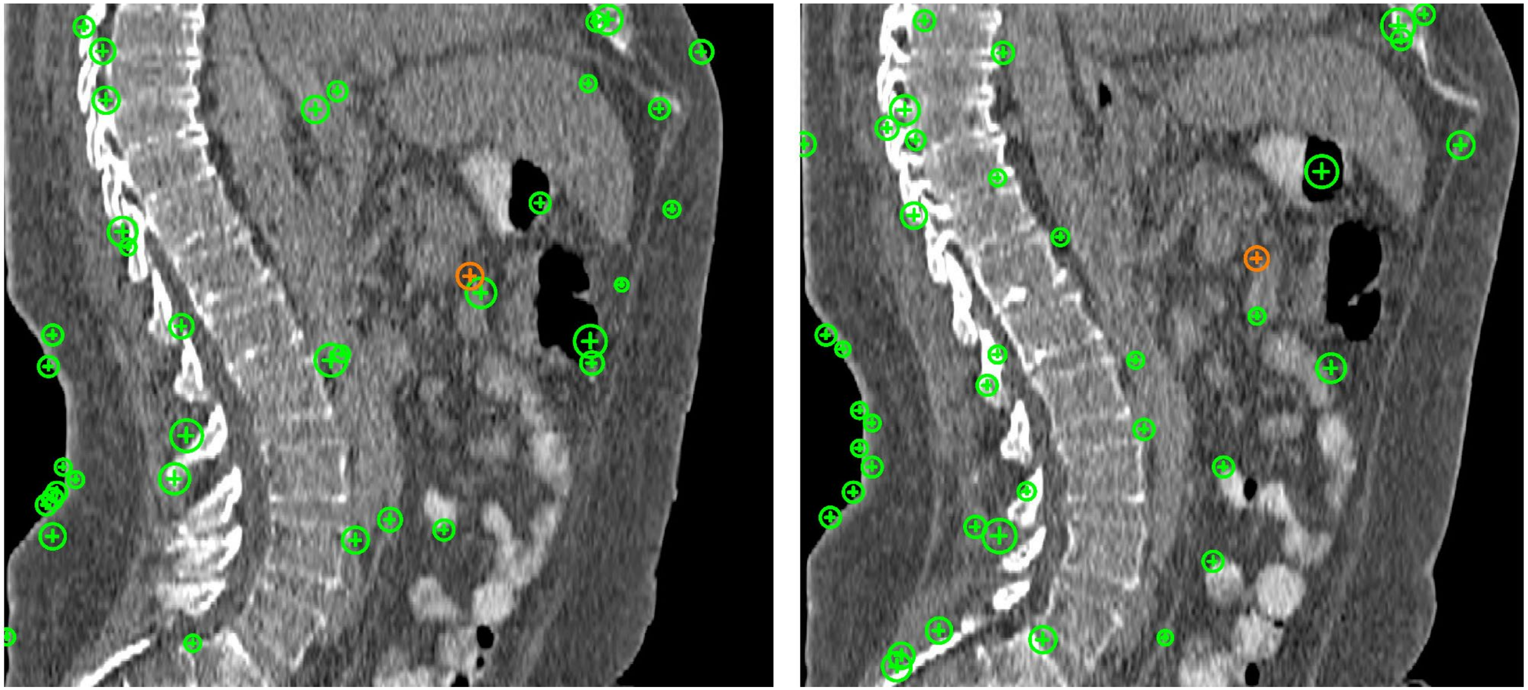
- Compute dot products between gradients in DDR and X-ray
- Find optimal 2D transformation between gradient images
- Use this transformation as a rough idea where to move source and detectors



# 3D-2D registration

**Descriptors**:

- Find pixels of interest in both images (corners, etc. Surf, Sift)
- Find optimal 2D transformation between gradient images
- Use this transformation as a rough idea where to move source and detectors



# Questions?