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### Problem 1

On average 66 % of Finite Mathematics students spend some time in the Mathematics Department's resource room. Half of these students spend more than 90 minutes per week in the resource room. At the end of the semester the students in the class were asked how many minutes per week they spent in the resource room and whether they passed or failed. The passing rates are summarized in the following table:

Time spent in resource room	Pass %
None	23
Between 1 and 90 minutes	52
More than 90 minutes	67

If a randomly chosen student did not pass the course, what is the probability that he or she did not study in the resource room?

Answer: \_\_\_\_\_

*Correct Answers:*

- 0.494802494802495

+6pc-1pc

### Problem 2

One of two urns is chosen at random with one just as likely to be chosen as the other. Then a ball is withdrawn from the chosen urn. Urn 1 contains 2 white and 4 red balls, and urn 2 has 1 white and 2 red balls.

If a white ball is drawn, what is the probability that it came from urn 1?

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(Hint: Draw a tree diagram first)

*Correct Answers:*

- 0.5

+6pc-1pc

### Problem 3

A biomedical research company produces 40% of its insulin at a plant in Kansas City, and the remainder is produced at a plant in Jefferson City. Quality control has shown that 0.55% of the insulin produced at the plant in Kansas City is defective, while 1.2% of the insulin produced at the plant in Jefferson City is defective. What is the probability that a randomly chosen unit of insulin came from the plant in Jefferson City given that it is defective?

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(Hint: Draw a tree diagram first)

*Correct Answers:*

- 0.765957

+6pc-1pc

#### Problem 4

It is estimated that approximately 8.47% Americans are afflicted with diabetes. Suppose that a certain diagnostic evaluation for diabetes will correctly diagnose 98% of all adults over 40 with diabetes as having the disease and incorrectly diagnoses 2% of all adults over 40 without diabetes as having the disease.

- a) Find the probability that a randomly selected adult over 40 does not have diabetes, and is diagnosed as having diabetes (such diagnoses are called "false positives").
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- b) Find the probability that a randomly selected adult of 40 is diagnosed as not having diabetes.
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- c) Find the probability that a randomly selected adult over 40 actually has diabetes, given that he/she is diagnosed as not having diabetes (such diagnoses are called "false negatives").
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(**Note:** it will be helpful to first draw an appropriate tree diagram modeling the situation)

*Correct Answers:*

- 0.018306
- 0.898688
- 0.00188497

+6pc-1pc

#### Problem 5

A breathalyser test is used by police in an area to determine whether a driver has an excess of alcohol in their blood. The device is not totally reliable: 8 % of drivers who have not consumed an excess of alcohol give a reading from the breathalyser as being above the legal limit, while 10 % of drivers who are above the legal limit will give a reading below that level. Suppose that in fact 18 % of drivers are above the legal alcohol limit, and the police stop a driver at random. Give answers to the following to four decimal places.

##### Part a)

What is the probability that the driver is incorrectly classified as being over the limit?

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**Part b)**

What is the probability that the driver is correctly classified as being over the limit?

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**Part c)**

Find the probability that the driver gives a breathalyser test reading that is over the limit.

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**Part d)**

Find the probability that the driver is under the legal limit, given the breathalyser reading is also below the limit.

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**Solution:**

Part a)

$$\begin{aligned}P(\text{positive test and driver below limit}) &= P(\text{positive test}|\text{driver below limit})P(\text{driver below}) \\&= 0.08 \times (1 - 0.18) \\&= 0.0656.\end{aligned}$$

Part b)

Since

$$\begin{aligned}P(\text{positive test and driver above limit}) &= P(\text{positive test}|\text{driver above limit})P(\text{driver above}) \\&= 0.9 \times 0.18 \\&= 0.1620.\end{aligned}$$

Part c)

Events (a) and (b) are mutually exclusive and exhaustive for a driver testing positive so

$$\begin{aligned}P(\text{positive test}) &= 0.08 \times (1 - 0.18) + 0.9 \times 0.18 \\&= 0.2276.\end{aligned}$$

Part d)

Using Bayes' Theorem,

$$\begin{aligned}P(\text{driver below}|\text{reading below}) &= \frac{P(\text{driver below})P(\text{reading below}|\text{driver below})}{P(\text{driver below})P(\text{reading below}|\text{driver below})+P(\text{driver above})P(\text{reading below}|\text{driver above})} \\&= \frac{(1-0.18)(1-0.08)}{(1-0.18)(1-0.08)+0.1 \times 0.18} \\&= 0.9767.\end{aligned}$$

*Correct Answers:*

- 0.0656
- 0.162
- 0.2276
- 0.9767

+6pc-1pc

### Problem 6

Items in your inventory are produced at three different plants: 50% from plant A1, 30% from plant A2 and 20% from plant A3. You are aware that your plants produce at different levels of quality: A1 produces 5 percent defectives, A2 produces 7 percent defectives and A3 yields 8 percent defectives. You randomly select an item from your inventory and it turns out to be defective. Which plant is the item most likely to have come from?

- Plant A1
- Plant A2
- Plant A3

### Solution:

**SOLUTION:**

The correct answer is Plant A1

Draw a decision tree. If a sample is chosen at random, it could have come from A1 (probability 50%), A2 (30%), or A3 (20%). Now consider if the sample is defective. If it came from A1, the probability that it is defective is 5%. So the probability that the sample came from A1 AND is defective is 50% times 5%, or 2.5%. If you calculate the probability for the sample being defective in the other two cases, you will see that the first case has the highest probability.

*Correct Answers:*

- Plant A1

+6pc-1pc

**Problem 7**

Three airlines serve a small town in Ohio. Airline A has 47% of all scheduled flights, airline B has 28% and airline C has the remaining 25%. Their on-time rates are 84%, 63%, and 38%, respectively. A flight just left on-time. What is the probability that it was a flight of airline A?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.592614830381267

+6pc-1pc

**Problem 8**

A foreman for an injection-molding firm admits that on 14% of his shifts, he forgets to shut off the injection machine on his line. This causes the machine to overheat, increasing the probability that a defective molding will be produced during the early morning run from 4% to 21%. The plant manager randomly selects a molding from the early morning run and discovers it is defective. What is the probability that the foreman forgot to shut off the machine the previous night?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.460815047021944

+6pc-1pc

**Problem 9**

Your favorite team is in the World Series. You have assigned a probability of 60% that they will win the championship. Past records indicate that when teams win the championship, they win the first game of the series 72% of the time. When they lose the championship, they win the first game

26% of the time. The first game is over and your team has lost. What is the probability that they will win the World Series?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.362068965517241

+6pc-1pc

### **Problem 10**

Transplant operations have become routine and one common transplant operation is for kidneys. The most dangerous aspect of the procedure is the possibility that the body may reject the new organ. There are several new drugs available for such circumstances and the earlier the drug is administered, the higher the probability of averting rejection.

The New England Journal of Medicine recently reported the development of a new urine test to detect early warning signs that the body is rejecting a transplanted kidney. However, like most other tests, the new test is not perfect. In fact, 20% of people who do reject the transplant test negative, and 6% of people who do not reject the transplant test positive. Physicians know that in about 25% of kidney transplants the body tries to reject the organ. If the new test has a positive result (indicating early warning of rejection), what is the probability that the body is attempting to reject the kidney?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.816326530612245

+6pc-1pc

### **Problem 11**

The chartered financial analyst (CFA) is a designation earned after taking three annual exams (CFA I, II, and III). The exams are taken in early June. Candidates who pass an exam are eligible to take the exam for the next level in the following year. The pass rates for levels I, II, and III are 0.51, 0.78, and 0.8, respectively. Suppose that 3,000 candidates take the level I exam, 2,500 take the level II exam and 2,000 take the level III exam. A randomly selected candidate who took a CFA exam tells you that he has passed the exam. What is the probability that he took the CFA I exam?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.301181102362205

+6pc-1pc

### **Problem 12**

Bad gums may mean a bad heart. Researchers discovered that 79% of people who have suffered a heart attack had periodontal disease, an inflammation of the gums. Only 33% of healthy people have this disease. Suppose that in a certain community heart attacks are quite rare, occurring with only 12% probability.

A. If someone has periodontal disease, what is the probability that he or she will have a heart attack?

Probability = \_\_\_\_\_

B. If 43% of the people in a community will have a heart attack, what is the probability that a person with periodontal disease will have a heart attack?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.246105919003115
- 0.643615005683971

+6pc-1pc

### **Problem 13**

Suppose that at UVA, 77% of all undergraduates are in the College, 10% are in Engineering, 6% are in Commerce, 3% are in Nursing, and 4% are in Architecture. In each school, the percentage of females is as follows: 58% in the College, 24% in Engineering, 45% in Commerce, 85% in Nursing, and 33% in Architecture. If a randomly selected student is male, what is the probability that he's from the College?

Probability = \_\_\_\_\_

*Correct Answers:*

- 0.697433685572568

+6pc-1pc

### **Problem 14**

If  $P(F) = 0.4$  and  $P(E|F) = 0.8$ , then

$P(E \cap F) =$  \_\_\_\_\_

*Correct Answers:*

- 0.32

+6pc-1pc

**Problem 15**

If  $P(A) = 0.6$ ,  $P(B) = 0.45$  and  $P(A \text{ and } B) = 0.1$ , find the following probabilities:

- a)  $P(A \text{ or } B) = \underline{\hspace{2cm}}$
- b)  $P(\text{not } A) = \underline{\hspace{2cm}}$
- c)  $P(\text{not } B) = \underline{\hspace{2cm}}$
- d)  $P(A \text{ and } (\text{not } B)) = \underline{\hspace{2cm}}$
- e)  $P(\text{not } (A \text{ and } B)) = \underline{\hspace{2cm}}$

*Correct Answers:*

- 0.95
- 0.4
- 0.55
- 0.5
- 0.9

+6pc-1pc

**Problem 16**

If  $A$  and  $B$  are two mutually exclusive events with  $P(A) = 0.15$  and  $P(B) = 0.75$ , find the following probabilities:

- a)  $P(A \text{ and } B) = \underline{\hspace{2cm}}$
- b)  $P(A \text{ or } B) = \underline{\hspace{2cm}}$
- c)  $P(\text{not } A) = \underline{\hspace{2cm}}$
- d)  $P(\text{not } B) = \underline{\hspace{2cm}}$
- e)  $P(\text{not } (A \text{ or } B)) = \underline{\hspace{2cm}}$
- f)  $P(A \text{ and } (\text{not } B)) = \underline{\hspace{2cm}}$

*Correct Answers:*

- 0
- 0.9
- 0.85
- 0.25
- 0.1
- 0.15

+6pc-1pc

**Problem 17**

Urn A has 3 white and 7 red balls. Urn B has 12 white and 15 red balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. What is the probability that the coin landed heads?

*Correct Answers:*

- 0.402985074626866

+6pc-1pc

**Problem 18**

All that is left in a packet of candy are 10 reds, 5 greens, and 2 blues.

(a) What is the probability that a random drawing yields a red followed by a red assuming that the first candy drawn is put back into the packet?

Answer: \_\_\_\_\_

(b) Are the events 'red' and 'red' independent?

Answer: \_\_\_\_\_

*Correct Answers:*

- 0.346020761245675
- yes

+6pc-1pc

**Problem 19**

If  $P(E \cap F) = 0.032$ ,  $P(E|F) = 0.32$ , and  $P(F|E) = 0.8$ , then

(a)  $P(E) =$  \_\_\_\_\_

(b)  $P(F) =$  \_\_\_\_\_

(c)  $P(E \cup F) =$  \_\_\_\_\_

(d) Are the events  $E$  and  $F$  independent? \_\_\_\_\_ Enter *yes* or *no*.

*Correct Answers:*

- 0.04
- 0.1
- 0.108
- no

+6pc-1pc

**Problem 20**

Mutually exclusive events must be independent.

- A. False
- B. True

**Solution:**

The statement is false, since events A and B to be mutually exclusive implies that  $P(A \cap B) = 0$ , whereas for the two events to be independent we must have  $P(A \cap B) = P(A)P(B)$ , so the two conditions could only hold if A and B are null events.

*Correct Answers:*

- A

+6pc-1pc

**Problem 21**

A useful graphical method of constructing the sample space for an experiment is:

- A. a histogram
- B. a tree diagram
- C. a pie chart
- D. an ogive

If A and B are mutually exclusive events with  $P(A) = 0.70$ , then  $P(B)$ :

- A. cannot be smaller than 0.30
- B. can be any value between 0 and 0.70
- C. cannot be larger than 0.30
- D. can be any value between 0 and 1

*Correct Answers:*

- B
- C

+6pc-1pc

**Problem 22**

For two events  $A$  and  $B$ ,  $P(A) = 0.7$  and  $P(B) = 0.3$ .

(a) If  $A$  and  $B$  are independent, then

$$P(A \cup B) = \underline{\hspace{2cm}}$$

$$P(A|B) = \underline{\hspace{2cm}}$$

$$P(A \cap B) = \underline{\hspace{2cm}}$$

(b) If  $A$  and  $B$  are dependent and  $P(A|B) = 0.3$ , then

$$P(B|A) = \underline{\hspace{2cm}}$$

$$P(A \cap B) = \underline{\hspace{2cm}}$$

*Correct Answers:*

- 0.79
- 0.7
- 0.21
- 0.128571428571429
- 0.09

+6pc-1pc

### Problem 23

If  $P(A) = 0.3$ ,  $P(B) = 0.1$ , and  $P(A \cup B) = 0.4$ , then

$$P(A \cap B) = \underline{\hspace{2cm}}.$$

(a) Are events  $A$  and  $B$  independent? (enter YES or NO)   

(b) Are  $A$  and  $B$  mutually exclusive? (enter YES or NO)   

*Correct Answers:*

- 0
- no
- yes

+6pc-1pc

### Problem 24

Two fair dice, one blue and one red, are tossed, and the up face on each die is recorded. Define the following events:

$$E : \{ \text{The difference of the numbers is 3 or more} \}$$

$$F : \{ \text{A 1 on the blue die} \}$$

Find the following probabilities:

(a)  $P(E) = \underline{\hspace{2cm}}$

(b)  $P(F) = \underline{\hspace{2cm}}$

(c)  $P(E \cap F) = \underline{\hspace{2cm}}$

Are events E and F independent?

- A. no
- B. yes

*Correct Answers:*

- 0.333333333333333
- 0.1666666666666667
- 0.083333333333333
- A