International Reserve Management under Rollover Crises

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Motivation

To reduce the vulnerability to a debt crisis:

- Should the government reduce the debt or increase reserves?

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Answer unclear:

- Reserves provide liquidity, but reducing debt may be more effective

What we do

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  - Sunspot shocks, deterministic income
- How should the government exit crisis zone?

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- If heavily indebted, optimal to initially reduce debt and keep zero reserves
- Once debt is reduced sufficiently, optimal to increase debt and accumulate reserves
- Borrowing to accumulate reserves can reduce spreads

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Model

Environment

- Discrete time, infinite horizon. Constant endowment: $y_t = y$

- Government trades two assets...
  - short-term risk-free reserves, a
  - long-term defaultable debt, b
    - a bond issued in t promises to pay

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  - Discount future flows at rate r, assume $\beta(1+r) = 1$

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- Markov equilibrium w/ Cole-Kehoe (2000) timing:
  - Borrowing at the beginning of the period
  - Settlement (repay/default) at the end

Recursive Government Problem

- State is $s \equiv (a, b, \zeta)$

  ζ denotes an iid sunspot that coordinates the lenders

- The government chooses to repay or default

$$V(a, b, \zeta) = \max\left\{V_R(a, b, \zeta), V_D(a)\right\}$$

  If indifferent, assume repay

Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$

subject to

$$c \leq y + a - \frac{a'}{1+r}$$

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- Given $\beta(1+r) = 1$, this is

$$V_D(a) = \frac{u(y + (1-\beta)a) - \phi}{1-\beta}$$

Value of Repayment & Bond Price

$$V_R(a, b, \zeta) = \max_{a' \geq 0, b'} \left\{ u(c) + \beta \mathbb{E} V(a', b', \zeta') \right\}$$

subject to

$$c = y + a - \kappa b - \frac{a'}{1+r} + q(a', b', s) \left[b' - (1-\delta) b \right]$$

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Bond Price:

$$q(a', b', s) = \begin{cases} \frac{1}{1+r} \mathbb{E} \left[(1 - d(s')) \left(\kappa + (1-\delta) q(a'', b'', s') \right) \right] & \text{if } d(s) = 0 \\ 0 & \text{if } d(s) = 1 \end{cases}$$

where $a''(s')$ and $b''(s')$ are the future choice of reserves and debt

Multiplicity of Equilibria

- Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)
- If lenders expect...

  ... repayment, they lend, and the government repays

  ... default, they don't lend, and the government defaults

Repayment Value when the Government can rollover

$$V_R^{+}(a,b) = \max_{a' \geq 0, b'} \left\{ u(c) + \beta \mathbb{E} V(a', b', s') \right\}$$

subject to

$$c = y + a - \kappa b - \frac{a'}{1+r} + \tilde{q}(a', b') \left(b' - (1-\delta) b \right)$$

where $\tilde{q}(a', b')$ denotes fundamental bond price

Repayment Value in a Run

$$V_R^{-}(a, b) = \max_{a' \geq 0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$

subject to

$$c = y + a - \kappa b - \frac{a'}{1+r} + \cancelto{0}{\tilde{q}(a', b')\left(b' - (1-\delta)b\right)}$$

To pay debt, need to use reserves or cut consumption

Characterization

Safe, Default and Crisis Zones

- Immediate: $V_R^+(a,b) \geq V_R^-(a,b)$
- When $V_R^-(a,b) < V_D(a) \leq V_R^+(a,b)$, multiple equilibria

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$$\mathbf{S} = \left\{(a,b) : V_D(a) \leq V_R^-(a,b)\right\},$$

$$\mathbf{D} = \left\{(a,b) : V_D(a) > V_R^+(a,b)\right\},$$

$$\mathbf{C} = \left\{(a,b) : V_R^-(a,b) < V_D(a) \leq V_R^+(a,b)\right\}.$$

The Safe and Crisis Zones

- If $(a, b) \in \mathbf{S}$: we assume govt. stays in safe zone

$$V^S(a - b) = \frac{u(y + (1 - \beta)(a - b))}{1 - \beta}$$

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  - Speed of exit depends on curvature of $u(\cdot)$ and prob. of bad sunspot, λ

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Continuation value:

$$\mathbb{E}V(a', b', \zeta') = \begin{cases} V^S(a'-b') & \text{if } (a', b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a', b') + \lambda V_D(a') & \text{if } (a', b') \in \mathbf{C} \\ V_D(a') & \text{if } (a', b') \in \mathbf{D} \end{cases}$$

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How to exit: raise a or lower b?

Debt Thresholds

$V_R(a, b)$ decreasing in $b \Rightarrow$ for every a, there $\exists$ unique thresholds $b^-(a), b^+(a)$:

$$V_R^-\left(a, b^-(a)\right) = V_D(a)$$

$$V_R^+\left(a, b^+(a)\right) = V_D(a)$$

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Thresholds are such that:

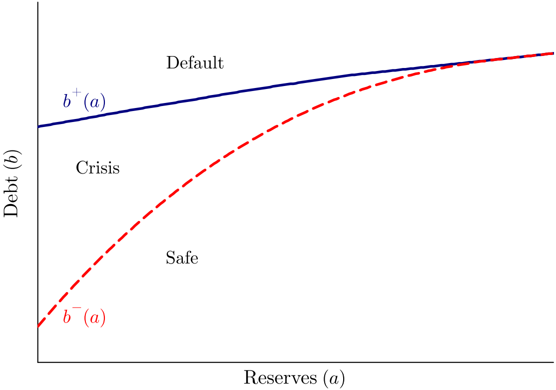
1. $(a,b) \in \mathbf{S}$ if and only if $b \leq b^-(a)$
2. $(a,b) \in \mathbf{C}$ if and only if $b^-(a) < b \leq b^+(a)$
3. $(a,b) \in \mathbf{D}$ if and only if $b > b^+(a)$

How does a affect these thresholds?

Proposition 2: $\frac{\partial b^-(a)}{\partial a} \geq \frac{\partial b^+(a)}{\partial a} > 0$

▸ Prelude

The Three Zones



Lowest-NFA safe portfolio

$$(a^\star, b^\star) = \operatorname*{argmin}_{a,b} a - b$$

$$\text{s.t.} \quad (a, b) \in \mathbf{S}$$

Lowest NFA position in the Safe Zone

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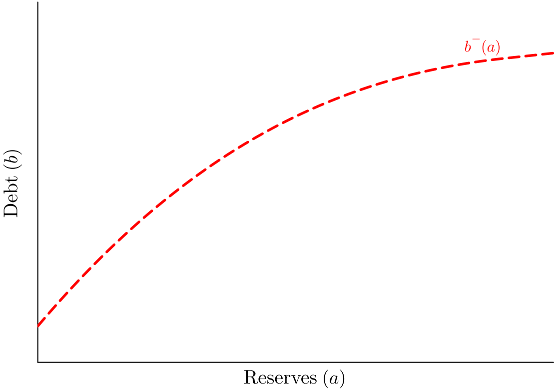
Lowest NFA position in the Safe Zone

Immediate: $b^\star = b^-(a^\star)$. Why? Since $(a, b) \in \mathbf{S}$ requires $b \leq b^-(a)$

If $a^\star > 0$, then we obtain:

$$\frac{\partial b^-(a^\star)}{\partial a} = 1$$

Lowest-NFA safe portfolio



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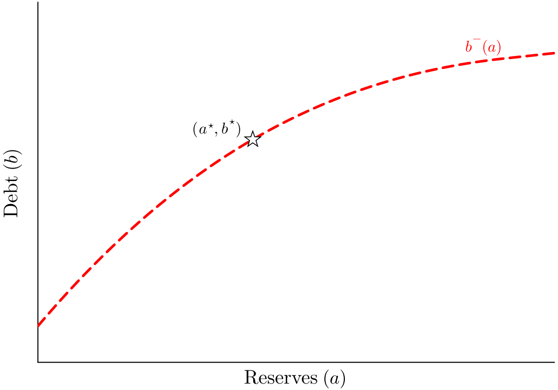

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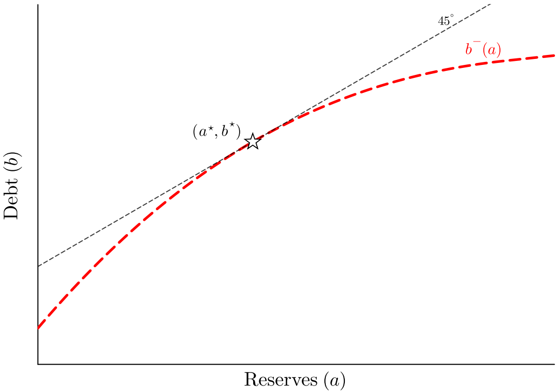

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Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^-(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^-(0)\right)\right] > u'(y).$$

Then, the lowest-NFA safe portfolio has strictly positive reserves, $a^\star > 0$

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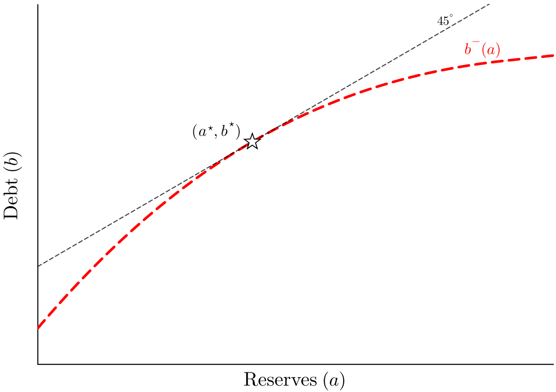
- Proposition implies $\left.\frac{\partial b^{-}(a)}{\partial a}\right|_{a=0} > 1$
- When does it fail? (i) low risk-aversion , (ii) one-period debt $(\delta = 1)$ [**Prop. 4**]

Lowest-NFA safe portfolio



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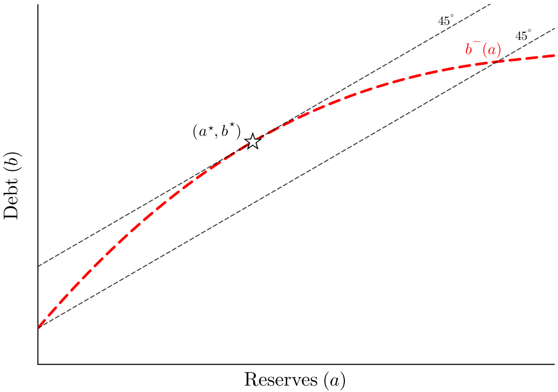

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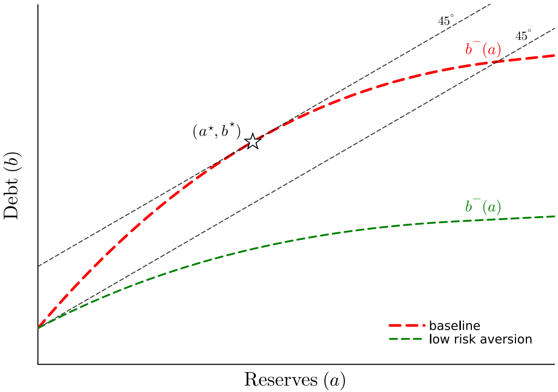


Simulations: Exiting the Crisis Zone

Optimal Exit Strategy

Q1: How many periods until exiting?

- Inside the Crisis Zone we can define Iso-T regions

Optimal Exit Strategy

Q1: How many periods until exiting?

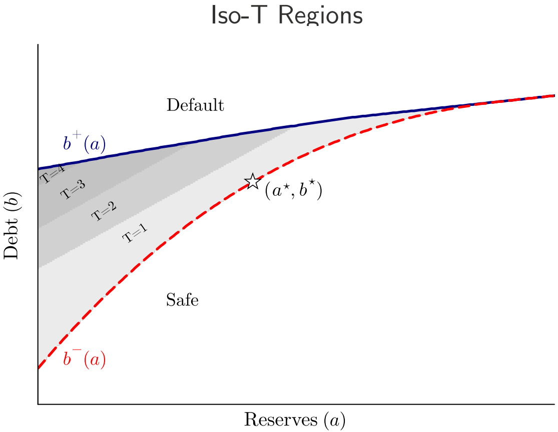
- Inside the Crisis Zone we can define Iso-T regions

Q2: What's the best strategy to exit the crisis zone?

- Should the government reduce its debt or increase reserves?
- If reserves are optimal, should govt. slowly build up its stock of reserves?

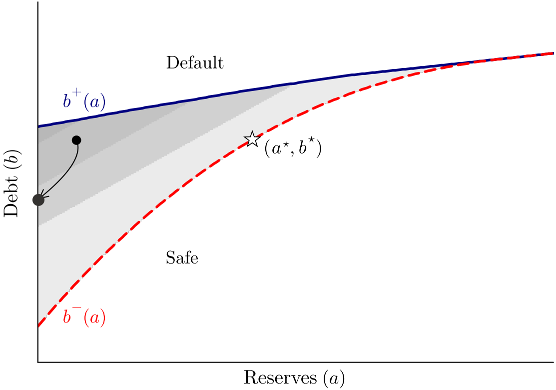
Calibration Policies

How many periods until exit



Deleveraging Path

Safety in Three Periods $\rightarrow (a^\star, b^\star)$



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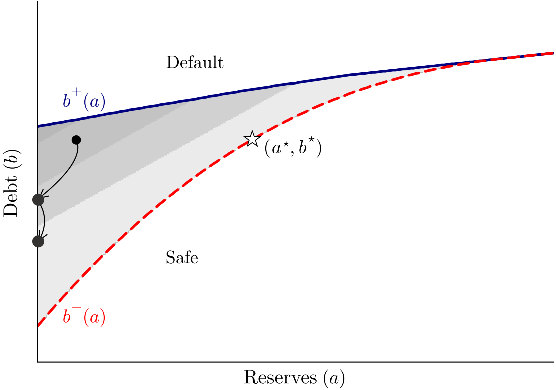

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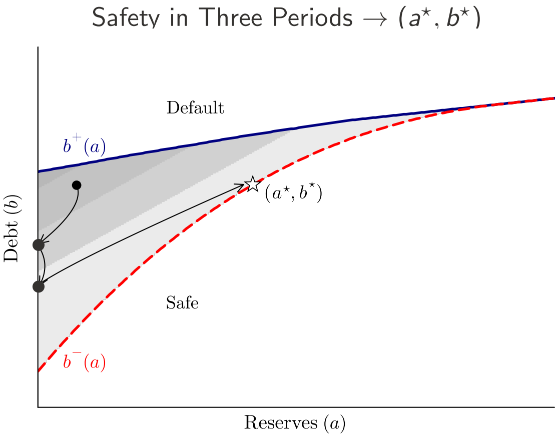
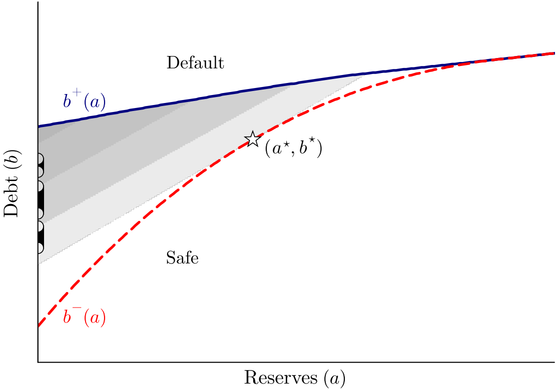


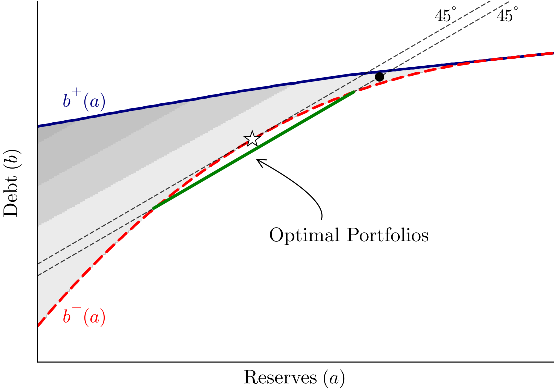
Deleveraging Path

Possible chosen portfolios for $a - b < a^\star - b^\star$

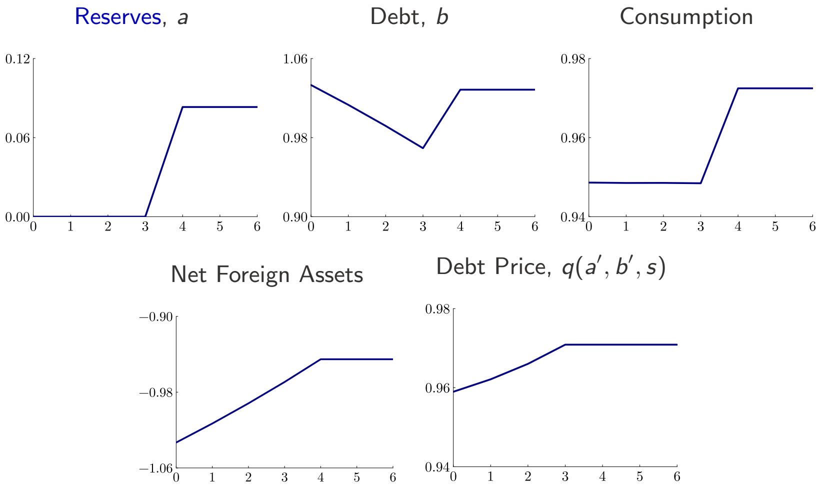


Deleveraging Path

Safety in One Period $\rightarrow (a - b > a^\star - b^\star)$



Deleveraging Dynamics



Conclusions

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- Optimal to accumulate reserves to reduce vulnerability

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Conclusions

- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safe zone
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
  - Following a debt crisis, IMF often prescribes increasing reserves
  - However, we find holding reserves <u>not optimal</u> at beginning of deleveraging process

Scan to find the paper!

THANKS!

Literature

back

- Alfaro and Kanczuk (2008): no reserves with one-period debt
  - Reserves make default attractive ⇒ worsen debt sustainability
- Bianchi, Hatchondo and Martinez (2018): positive reserves with long-term debt under *fundamental defaults*
  - Reserves help avoid rolling over debt at high spreads
  - Insurance within repayment states
- **Today:** reserve management under rollover crisis
  - Borrowing to accumulate reserves helps exiting the crisis zone
- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)

Parametrization

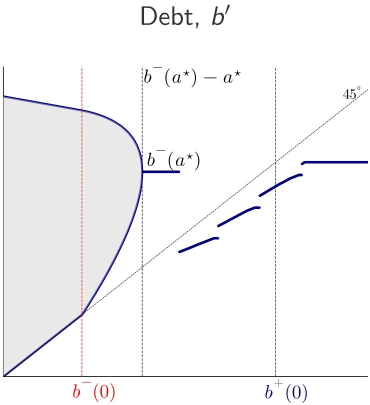
back

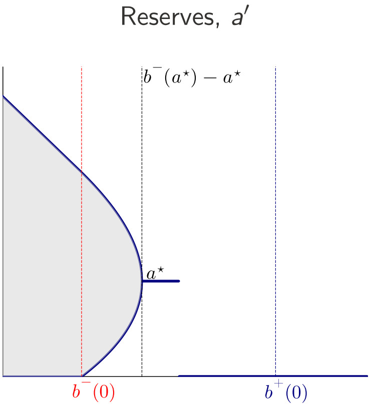
$$u(c) = \frac{(c - \underline{c})^{1-\sigma}}{1-\sigma}$$

Parameter	Value	Description	Source
y	1	Endowment	Normalization
σ	2	Risk-aversion	Standard
r	3%	Risk-free rate	Standard
$1/\delta$	6	Maturity of debt	Italian Debt
$\underline{c}$	0.68	Consumption floor	Bocola-Dovis (2019)
β	0.97	Discount factor	$\beta(1+r) = 1$
λ	0.5%	Sunspot probability	Baseline
ϕ	0.33	Default Cost	Debt-to-income =100%

Policies

▸ back





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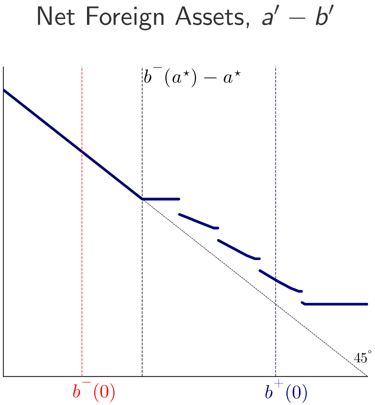

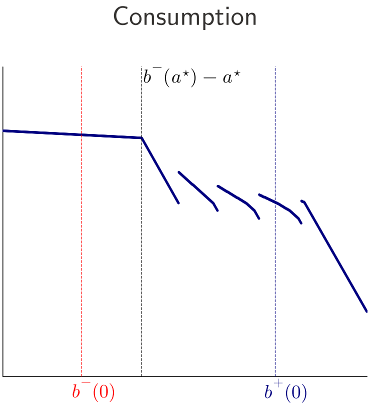

Prelude

▸ back

If government not vulnerable tomorrow after repaying in a run:

$$\max_{a'} u\left(y - \kappa b + a - \frac{a'}{1+r}\right) + \beta V^S(a' - (1-\delta)b))$$

- Solution: $a'(a,b) = \max[0, a - \delta b]$.
  - With low initial reserves, government constrained $\Rightarrow a' = 0$

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    - With low initial reserves, government constrained $\Rightarrow a' = 0$
- If $a \geq \delta b$ and $(a - \delta b, (1-\delta)b) \in \mathcal{S}$, then $V_R^-(a, b) = V_R^+(a, b)$.
    - If high reserves, govt. can achieve unconstrained consumption even in a run
    - Note reserves enough to pay all coupons not needed!

Taking Stock

To exit crisis zone, first deleverage, then raise debt and reserves

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- If initial portfolio (a, b) is such that $(a', b') \in \mathbf{C}$. Then, the optimal solution features $a' = 0$.

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Remark on maturity:

- With one-period debt, $\delta = 1$: ${V_R}^-$ and ${V_R}^+$ are unaffected by equal increases in debt and reserves $\Rightarrow$ issuing debt to accumulate reserves increases spreads
  - Zero reserves are optimal

Sensitivity

Experiment – How reserves help exit crisis zone

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- Exiting takes longer to exit <u>and</u> cuts more consumption

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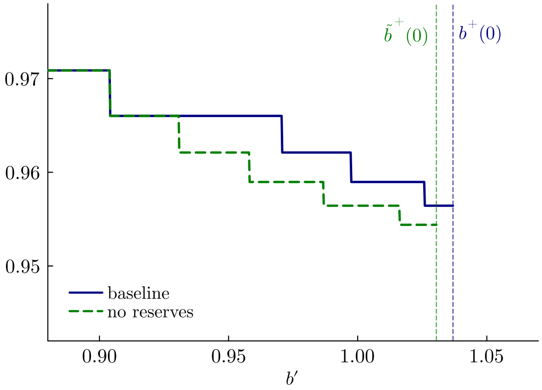
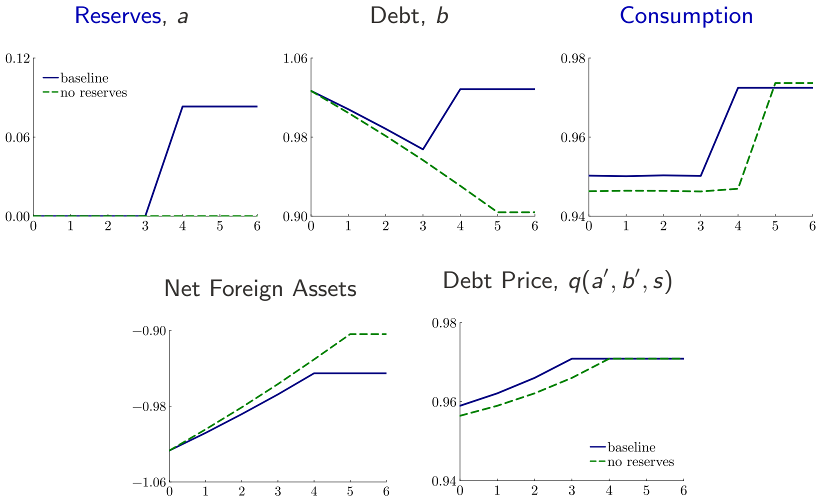
<u>Without reserves:</u> $\downarrow b^{+}$. More costly to deleverage $\Rightarrow$ lower debt-carrying capacity

Default zone expands

Price Schedule, $q(0, b')$

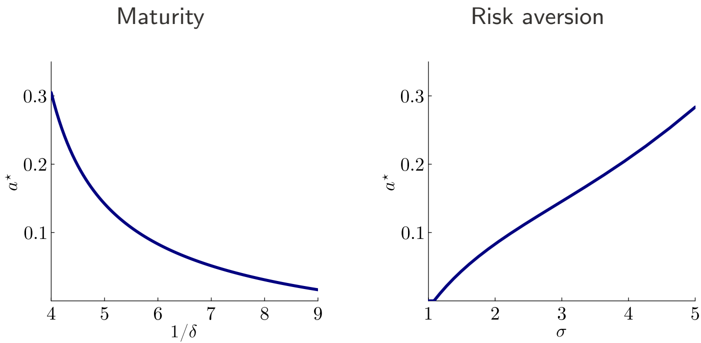


Lower consumption without reserves



Sensitivity: effect of maturity and risk-aversion on $a^\star$

back



Panels show the level of $a^\star$ for different values for δ and σ. The value of ϕ is recalibrated to match the same debt level $b^-(0)$ as in baseline.

Default zone expands

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