International Reserve Management under Rollover Crises

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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To reduce the vulnerability to a debt crisis:

• Should the government reduce the debt or increase reserves?

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Answer unclear:

 Reserves provide liquidity, but their return is lower than borrowing costs

What we do

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 - Sunspot shocks, deterministic income
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- Once debt is reduced sufficiently, optimal to increase debt and accumulate reserves
- Borrowing to accumulate reserves can reduce spreads

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 - Borrowing to accumulate reserves helps exiting the crisis zone
- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

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Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)
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Model

Environment

- Discrete time, infinite horizon. Constant endowment: $y_t = y$
- Government trades two assets ...
 - short-term risk-free reserves, a
 - long-term defaultable debt, b
 a bond issued in t promises to pay

$$\kappa [1, (1 - \delta), (1 - \delta)^2,]$$

- Risk-neutral deep pocket international investors:
 - Discount future flows at rate r, assume $\beta(1+r)=1$

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- Risk-neutral deep pocket international investors:
 - Discount future flows at rate r, assume $\beta(1+r)=1$
- Markov equilibrium w/ Cole-Kehoe (2000) timing:
 - Borrowing at the beginning of the period
 - Settlement (repay/default) at the end

Preferences and resource constraint

• Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where $d_t = 0$ (1) denotes repayment (default)

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• If the government repays:

$$c_t = \underbrace{y + a_t - \kappa b_t}_{\text{resources avail.}} - \underbrace{\frac{a_{t+1}}{1+r}}_{\text{reserve purchases}} + \underbrace{q_t \left[b_{t+1} - (1-\delta)b_t\right]}_{\text{debt issuance}}$$

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If the government defaults:

$$c_t = y + \frac{a_t}{1 + r}$$
 Gov. saves on bond payments

and faces permanent exclusion and utility loss ϕ

Recursive Government Problem

• State is $s \equiv (a,b,\zeta)$ ζ denotes an iid sunspot that coordinates the lenders

The government chooses to repay or default

$$V(\mathbf{a}, b, \zeta) = \max\{V_R(\mathbf{a}, b, \zeta), V_D(\mathbf{a})\}\$$

If indifferent, assume repay

Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$
 subject to $c \leq y + a - rac{a'}{1+r}$

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subject to
$$c \le y + a - \frac{a'}{1+r}$$

• Given $\beta(1+r)=1$, we have constant consumption

$$V_D(a) = \frac{u(y + (1 - \beta)a) - \phi}{1 - \beta}$$

Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors want to rollover:

$$V_R^+(a,b) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b') (b' - (1-\delta)b)$$

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Bond price depends on the portfolio and reflects default prob:

$$q(a',b') = rac{1}{1+r} \mathbb{E}\left[\left(1-d(s')
ight)\left(\kappa+(1-\delta)q(a'',b'',s')
ight)
ight]$$

Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors don't want to rollover:

$$V_R^-(a,b) = \max_{a' \geq 0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b') (b' - (1-\delta)b)$$

To pay debt, need to use reserves or cut consumption

Multiplicity of Equilibria

 Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)

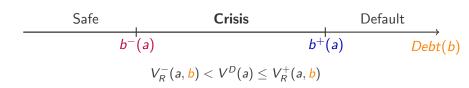
- If lenders expect...
 - ... repayment, then they rollover, and the govt repays
 - ... default, then they don't rollover, and the govt defaults

Characterization

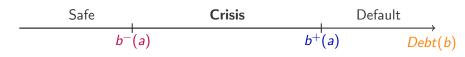






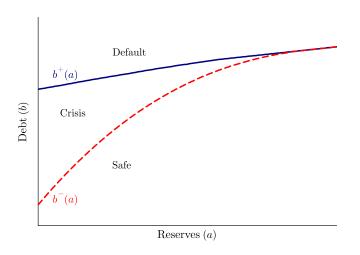


For a given level of reserves, two thresholds



Sunspot: government faces a run w/ prob π when initial portfolio (a,b) is in the crisis zone

The Three Zones



Proposition 2 establishes: $\frac{\partial b^-(a)}{\partial a} \geq \frac{\partial b^+(a)}{\partial a} > 0$

Escaping the Crisis Zone

How to Exit the Crisis Zone?

Remaining in the crisis zone is risky:

• in case of a run, the gov't defaults

But exiting is also costly:

• requires cutting consumption and improving NFA

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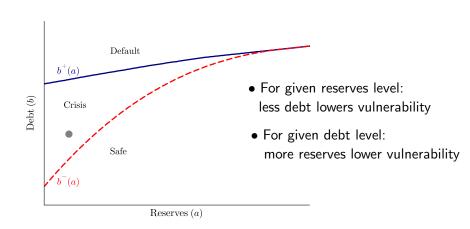
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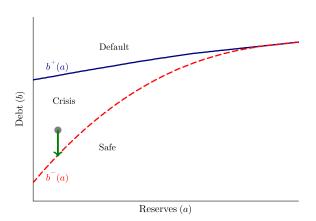
What's the best exit strategy for a country that is in the crisis zone (but didn't face a run today)?

• Accumulate reserves $(a \uparrow)$ or reduce debt $(b \downarrow)$?

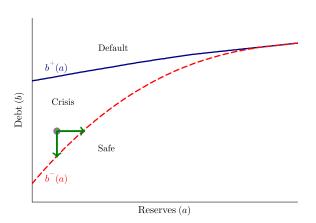
Possible Exit Paths

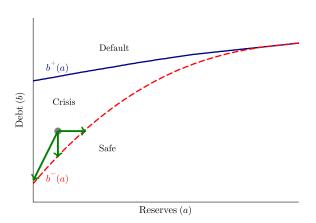


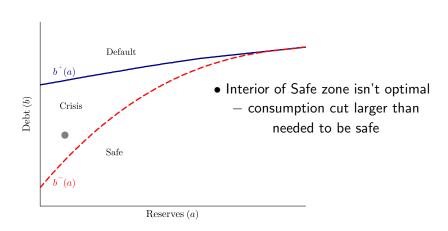
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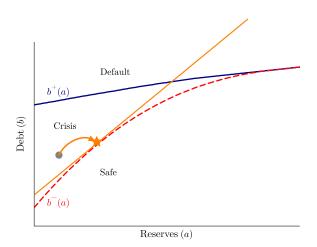


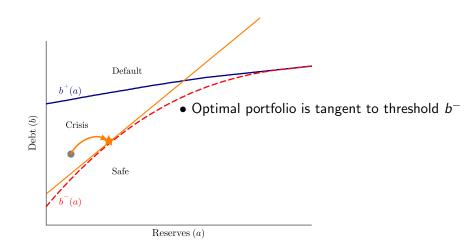
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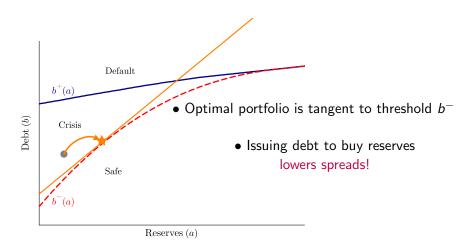












Getting to the safe zone requires $V_R^-(a,b) \ge V_D(a)$

• Accumulating reserves helps sustain higher <u>net debt</u> ... even though reserves increase default value $V_D(a)$.

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$$c = y + \underbrace{a - \kappa b}_{\text{more resources}} - \frac{a'}{1 + r}$$

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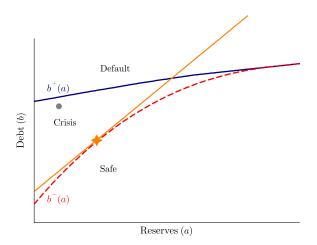
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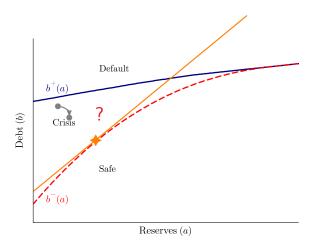
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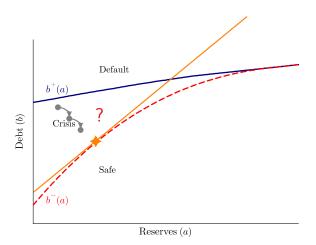
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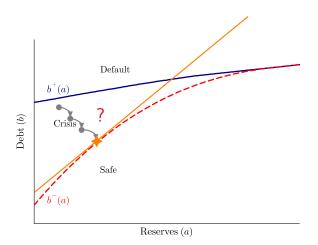
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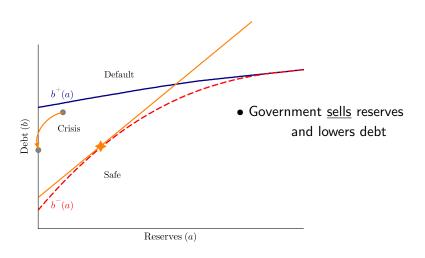
Country has higher initial debt level: what to do?

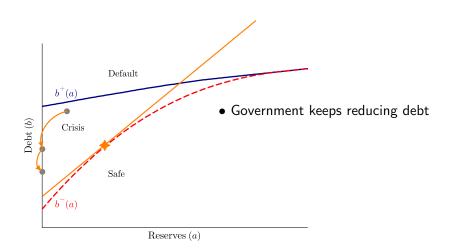


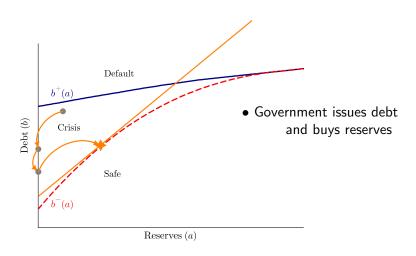










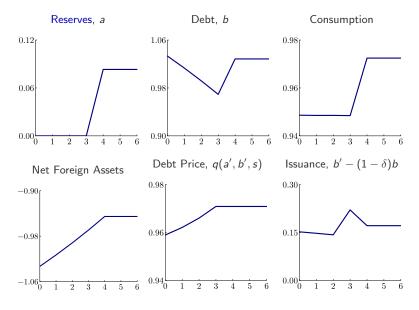


Why selling reserves (initially)?

- When the government is 'deep' in the Crisis Zone, on the margin reserves do not change the probability of a run
- Using the reserves to lower debt allows the govt to save on interest payments and helps deleveraging

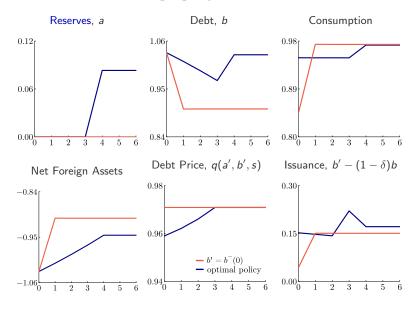
Deleveraging Dynamics





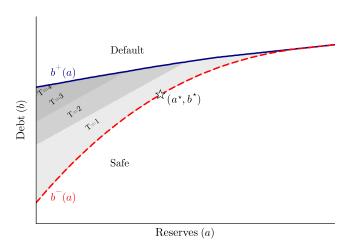
Deleveraging Dynamics





How many periods until exit?

Iso-T Regions



Formalizing the Results: (a^*, b^*) portfolio

 (a^{\star},b^{\star}) is a focal point – we call it **Lowest-NFA safe portfolio** When do we have $a^{\star}>0$?

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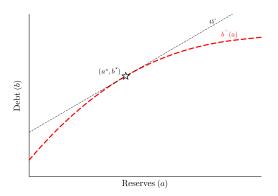
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Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^{-}(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^{-}(0)\right)\right]>u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves, $a^{\star}>0$

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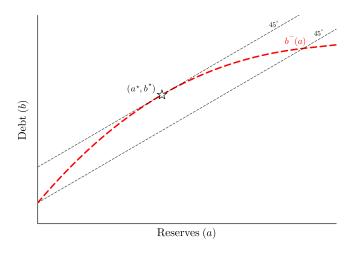
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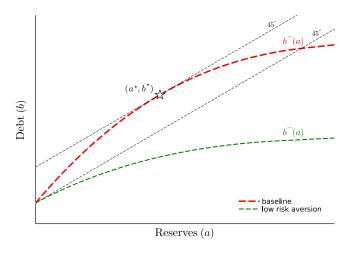
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- 1. low risk-aversion,
- 2. one-period debt ($\delta = 1$) [**Prop. 4**]





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Proposition 5 (Optimal portfolio)

Consider an initial portfolio $(a, b) \in \mathbf{C}$. The optimal portfolio satisfies:

- If initial portfolio (a, b) is such that $a b < a^* b^*$ and $(a', b') \in \mathbf{S}$. Then we have $a' = a^*, b' = b^*$
- If initial portfolio (a, b) is such that $(a', b') \in \mathbf{C}$. Then, the optimal solution features a' = 0.

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Remark on maturity:

- With one-period debt, $\delta=1$: V_R^- and V_R^+ are unaffected by equal increases in debt and reserves \Rightarrow issuing debt to accumulate reserves increases spreads
 - Zero reserves are optimal



Conclusions

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- Optimal to accumulate reserves to reduce vulnerability
 - However, only after debt has been reduced towards safety
- Reserves as 'buffer': after buildup, no use of reserves
 - Not using them doesn't mean they're unnecessary
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
 - Following a debt crisis, IMF often prescribes increasing reserves
 - However, we find holding reserves <u>not optimal</u> at beginning of deleveraging process

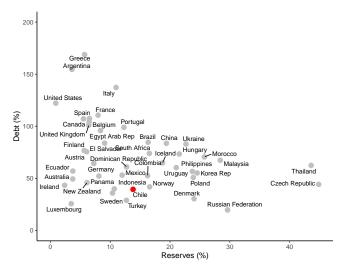


Scan to find the paper!



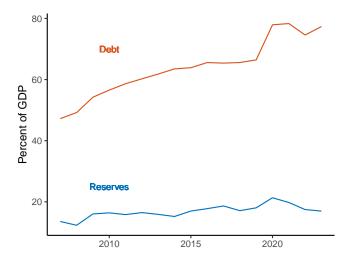
Data: Government Debt and International Reserves





Government debt and reserves (as % of GDP), 2023

Evolution of Debt and Reserves



Avg. Government debt and reserves (as % of GDP)

Characterization: Value in the Safe zone



• If $(a, b) \in S$: we assume gov. stays in safe zone

$$V^{S}(a-b) = \frac{u(y + (1-\beta)(a-b))}{1-\beta}$$

• **Note:** relevant state variable is the NFA, a - b

For a high enough δ : can establish that gov. finds it optimal to stay in ${\bf S}$

Characterization: Crisis zone



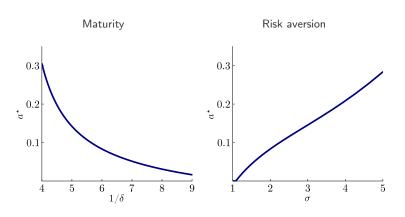
- If $(a, b) \in \mathbf{C}$, govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
 - Staying in the crisis zone implies eventually costly default
 - Speed of exit depends on curvature of $u(\cdot)$ and probability of bad sunspot

Continuation value:

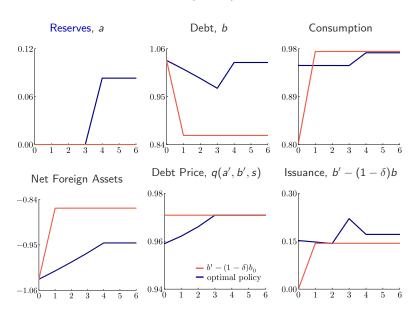
$$\mathbb{E}V(a',b',\zeta') = \begin{cases} V^{\mathcal{S}}(a'-b') & \text{if } (a',b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a',b') + \lambda V_D(a') & \text{if } (a',b') \in \mathbf{C} \\ V_D(a') & \text{if } (a',b') \in \mathbf{D} \end{cases}$$

Sensitivity: effect of maturity and risk-aversion on a^*

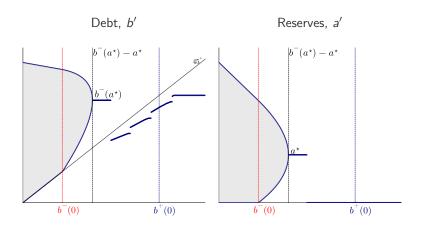




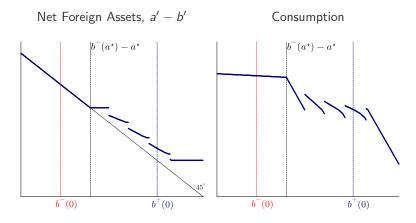












Parametrization



$$u(c) = \frac{(c - \underline{c})^{1 - \sigma}}{1 - \sigma}$$

Parameter	Value	Description	Source	
у	1	Endowment	Normalization	
σ	2	Risk-aversion	Standard	
r	3%	Risk-free rate	Standard	
$1/\delta$	6	Maturity of debt	Italian Debt	
<u>c</u>	0.68	Consumption floor	Bocola-Dovis (2019)	
β	0.97	Discount factor	$\beta(1+r)=1$	
λ	0.5%	Sunspot probability	Baseline	
ϕ	0.33	Default Cost	Debt-to-income =100%	
κ	$\frac{\delta+r}{1+r}$	Coupon	Normalization	

Experiment - How reserves help exit crisis zone

• Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0

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- Exiting the crisis zone becomes more painful ⇒ (0, b⁻(0)) instead of (a*, b*)
- Exiting takes longer to exit <u>and</u> cuts more consumption

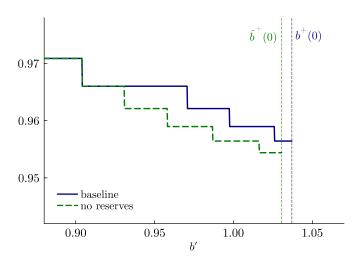
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<u>Without reserves:</u> $\downarrow b^+$. More costly to deleverage \Rightarrow lower debt-carrying capacity

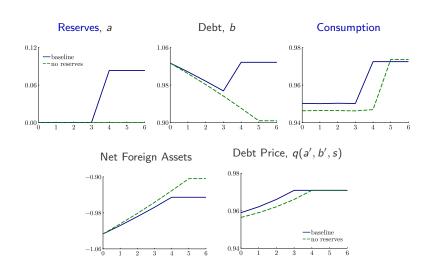
Price Schedule, q(0, b')



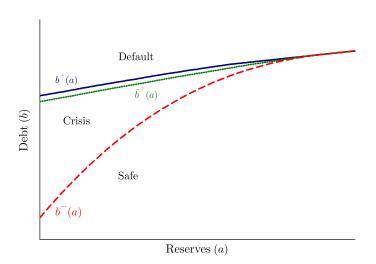


Lower consumption without reserves





Default zone expands



Data: increasing reserves and debt lowers spreads (preliminary)

▶ back

Dep. Variable:		log(Spread)		
	(0)	(1)	(2)	
Reserves	-2.39***			
	(0.11)			
Sov.Debt	1.25***	-1.13***	1.58***	
	(0.10)	(0.14)	(0.20)	
NFA_public		-2.39***	-2.69***	
		(0.11)	(0.11)	
$(Sov.Debt)^2$			-5.48***	
			(0.31)	
Num.Obs.	4497	4497	4497	
R2	0.791	0.791	0.997	
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001				

All specs. include country FEs, year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).