

# Reserve Accumulation, Macroeconomic Stabilization, and Sovereign Risk <sup>\*</sup>

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## Abstract

In the past three decades, governments in emerging markets have accumulated large amounts of international reserves, especially those with fixed exchange rates. We propose a theory of reserve accumulation that can account for these facts. Using a model of endogenous sovereign default with nominal rigidities, we show that the interaction between sovereign risk and aggregate demand amplification generates a macroeconomic-stabilization hedging role for international reserves. Reserves increase debt sustainability to the extent that financing reserves with debt accumulation may not lead to increases in spreads. We also study simple and implementable rules for reserve accumulation. Our findings suggest that a simple linear rule linked to spreads can achieve significant welfare gains, while those rules currently used in policy studies of reserve adequacy can be counterproductive.

JEL classification: F32, F34, F41.

*Keywords:* International reserves, sovereign default, macroeconomic stabilization, fixed exchange rates, inflation targeting.

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# 1 Introduction

The accumulation of international reserves—official public assets that are readily available for use—is one of the most salient features of the international monetary system over the past 30 years. While prevalent across emerging markets, the increase in reserves has been led by countries with fixed exchange rates, which as we document, increased their reserves-to-GDP ratios from about 10% in the 90s to 30% in recent years. What accounts for the increase in reserves, and what is the relationship between exchange rate regimes and the accumulation of international reserves?

In this paper, we argue that the interaction between sovereign risk and aggregate demand amplification generates a macroeconomic-stabilization role for international reserves. Our starting point is a model of endogenous sovereign default and nominal rigidities, in which the government follows a fixed exchange rate and can accumulate risk-free assets (reserves). In this context, we show how issuing debt to accumulate reserves allows the government to face less severe recessions in the future. Moreover, reserves help to stabilize macroeconomic fluctuations, so much so that financing reserves with debt accumulation may not necessarily lead to increases in spreads.

Our theory of how reserves have a special role under a fixed exchange rate contrasts with the traditional argument, articulated by [Krugman \(1979\)](#) and [Flood and Garber \(1984\)](#). In those studies, a fixed exchange rate regime precludes the use of seigniorage as a source of fiscal revenue: having a stock of international reserves allows the central bank to sustain a fixed exchange rate, even with persistent primary deficits. Rather than being based on a fiscal *need* for reserves to sustain a fixed exchange rate regime, our theory is based on a desire to hold reserves to manage macroeconomic stability under sovereign risk concerns. Quantitatively, we show that this motive for reserve accumulation can account for the high observed levels of reserves, a feature of the data that has proven difficult to reconcile with existing models.

To understand our argument, consider a negative shock that worsens the borrowing terms faced by a government. Such a shock could come from a decline in income for the government or from foreign lenders' risk premia. The optimal response for the economy is, naturally, a reduction in borrowing and consumption. In the presence of a fixed exchange rate and downward nominal wage rigidity, the reduction in consumption leads to a recession, which further deepens the contraction in consumption. Having reserves in these states allows the government to smooth the decline in consumption and mitigate the severity of the recession *ex post*. From an *ex ante* point of view, however, the government may also choose to reduce the sovereign debt as opposed to accumulating reserves. What generates an incentive to accumulate both reserves and debt as a macro-stabilization policy is that it is precisely in the states in which reserves allow for the reduction of unemployment that debt becomes more costly to roll over. In a nutshell, having reserves allows the government to avoid rolling over the fraction of debt maturing at high interest

rates and frees up resources to mitigate the recessionary effects. We label these ex-ante effects “macro-stabilization hedging”.

Perhaps surprisingly, we find that when the government issues debt to accumulate reserves, this does not necessarily lead to increases in spreads. In fact, while debt reduces the value from repayment, reserves increase both the value of repayment and default. We show, however, that when borrowing terms are particularly adverse, a larger stock of reserves is especially valuable under repayment because it is in precisely this situation that the use of reserves allows the government to mitigate a contraction in output. As a result, issuing debt may not increase sovereign spreads if the proceeds are used to accumulate reserves.

Our quantitative results show that the model features an equilibrium level of reserves that is roughly 16% of GDP, a value that is close to the observed levels in the data. Under the same calibration that matches debt levels and spreads, the amount of reserves falls to 7% of GDP when there are no nominal rigidities or when the government follows a flexible exchange rate. In line with these results, we document that in the data, countries with a lower degree of exchange rate flexibility hold on average more reserves. Moreover, while following a flexible exchange rate reduces significantly the need for reserves, we also find that governments should hold a significant amount of reserves under an inflation targeting regime.

Finally, motivated by practical policy considerations, we also explore the performance of rules for reserve accumulation. Our results show that it is possible to construct a simple, implementable policy rule that delivers significant gains. In fact, we find that a rule that is linear in spreads and debt delivers about one-third of the welfare gains achieved by following the optimal state contingent policy and significantly mitigates the costs from fixing the exchange rate. On the other hand, we argue that the Greenspan-Guidotti rule, which is often used in policy circles as a metric for reserve adequacy, performs quite poorly and that following this rule generates significant welfare losses.

**Related literature.** The paper contributes to several strands of the literature. In the first place, it relates to a vast literature on international reserves, in particular the one emphasizing the precautionary role of reserves.<sup>1</sup> The decision problem faced by the government in our model is similar to the one in [Bianchi, Hatchondo and Martinez \(2018\)](#), which in turn builds on the canonical sovereign default model in the tradition of [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#), and [Arellano \(2008\)](#). We depart from the existing work by incorporating

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<sup>1</sup>See, for example, [Aizenman and Lee \(2007\)](#); [Alfaro and Kanczuk \(2009\)](#); [Caballero and Panageas \(2008\)](#); [Jeanne and Ranciere \(2011\)](#); [Hur and Kondo \(2016\)](#); and [Arce, Bengui and Bianchi \(2019\)](#). A different strand of the literature analyzes the role of reserves in implementing exchange rate policies in the context of limits to international arbitrage ([Amador et al., 2017, 2018](#); [Fanelli and Straub, 2018](#)). A related line of research studies reserve accumulation and the lender-of-last-resort role of central banks ([Bocola and Lorenzoni, 2017](#), [Céspedes and Chang, 2019](#)).

nominal rigidities and show that the interaction with sovereign risk gives rise to a macroeconomic stabilization role for reserves. Furthermore, a contribution of our paper is to develop a theory that is quantitatively consistent with the observed levels of reserves in emerging markets and to link, both theoretically and empirically, the accumulation of international reserves to the exchange rate regime.

Our paper also belongs to a nascent literature analyzing fiscal and monetary policy in the context of sovereign default models with nominal rigidities. [Na et al. \(2018\)](#) first introduced nominal rigidities and showed that the optimal time-consistent exchange rate policy delivers full employment. Moreover, they show that the model can deliver the so-called “twin D”, that is, episodes in which large devaluations coincide with a sovereign default. [Bianchi, Ottonello and Presno \(2019\)](#) consider a fixed exchange rate regime and analyze the dilemma of choosing between austerity and stimulus, finding that the optimal fiscal policy is consistent with the observed procyclicality. [Arellano, Bai and Mihalache \(2020\)](#) study the interaction between monetary policy (conducted through interest rules) and sovereign risk. [Bianchi and Mondragon \(2018\)](#) show that an economy under a fixed exchange rate regime is more vulnerable to rollover crises. In contrast to these studies, we allow for the accumulation of international reserves and study how the optimal holdings of reserves differ depending on the monetary policy regime.

Our paper is also related to the literature on aggregate demand externalities under nominal rigidities and constraints on monetary policy (e.g., [Schmitt-Grohé and Uribe 2016](#); [Farhi and Werning 2016, 2017](#)). Our analysis of portfolio management shares elements with [Farhi and Werning \(2017\)](#), who show that, in general, the government can improve welfare by controlling households’ portfolios of Arrow-Debreu securities and redirecting demand across states.<sup>2</sup> However, we focus theoretically and quantitatively on the specific role of international reserves and provide a distinct mechanism linked to endogenous default risk.

Finally, our paper is also related to a closed economy literature that studies portfolio management in which the government faces distortionary taxation and is endowed with commitment. As shown by [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#), trading at different maturities allows the government to alter households’ marginal utility and bond prices and, through this channel, improve spanning and complete markets. Our model differs in that the government cannot commit to repay and fluctuations in bond prices arise because of changes in default probabilities. To our knowledge, our paper is the first to analyze how the presence of nominal rigidities shapes the optimal government portfolio and to uncover a macro-stabilization benefit from carrying larger

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<sup>2</sup>See also [Fanelli \(2017\)](#) for an example in which the government may find it optimal to distort savings but not the composition of the risky foreign asset portfolio.

gross positions.<sup>3</sup>

**Layout.** The rest of the article proceeds as follows. Section 2 presents the model of optimal reserve accumulation. Section 3 presents empirical evidence (in line with the model predictions) on the relationship between reserves, sovereign spreads, and exchange rate flexibility. Section 4 examines the calibration, and Section 5 presents the results of the quantitative analysis. Section 6 concludes.

## 2 Model

We consider a two-sector small open economy model in which the government issues long-term defaultable bonds and invests in short-term risk-free assets (i.e., reserves). We assume there is a stochastic endowment for tradable goods, while non-tradable goods are produced using labor. Labor markets are subject to downward nominal wage rigidity, as in Schmitt-Grohé and Uribe (2016), creating the possibility of involuntary unemployment. As the baseline case, we study a fixed exchange rate regime, and we will contrast the results with a flexible exchange rate regime and an inflation targeting regime.

### 2.1 Households

The small open economy is populated by a measure one of households. Households' preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where  $c_t$  denotes private consumption in period  $t$ ,  $\beta \in (0, 1)$  is the subjective discount factor, and  $\mathbb{E}_t$  denotes the expectation operator conditional on the information set available at time  $t$ . We assume that  $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ , where  $\gamma$  is the coefficient of relative risk aversion.

The consumption good is assumed to be a composite of tradable ( $c^T$ ) and non-tradable goods ( $c^N$ ), with a constant elasticity of substitution (CES) aggregation technology:

$$c = \mathcal{C}(c^T, c^N) = [\omega(c^T)^{-\mu} + (1 - \omega)(c^N)^{-\mu}]^{-1/\mu}, \quad (2)$$

where  $\omega \in (0, 1)$  and  $\mu > -1$ . The elasticity of substitution between tradable and non-tradable consumption is given by  $1/(1 + \mu)$ .

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<sup>3</sup>Other relevant papers in different strands of this literature include Arellano and Ramanarayanan (2012); Faraglia, Marcet and Scott (2010); Ottonello and Perez (2019); Debortoli, Nunes and Yared (2016); Bocola and Dovis (2019); and Lustig, Sleet and Yeltekin (2008). The last of these is particularly relevant in that it considers a fiscal hedging benefit of long-term nominal debt in an environment in which the government cannot accumulate assets and inflation is costly due to nominal rigidities.

In each period, households receive an endowment of tradable goods,  $y_t^T$ , which is assumed to follow a stationary first-order Markov process given by

$$\log(y_t^T) = (1 - \rho)\mu_y + \rho \log(y_{t-1}^T) + \varepsilon_t, \quad (3)$$

with  $|\rho| < 1$  and  $\varepsilon \stackrel{iid}{\sim} N(0, \sigma_\varepsilon^2)$ .

Households inelastically supply  $\bar{h}$  hours in the labor market. When the downward wage rigidity constraint (to be discussed below) binds, households will be able to work only the number of hours demanded by firms, and we will have  $h_t < \bar{h}$ . Households also receive profits from the ownership of firms producing non-tradable goods,  $\phi_t^N$ , and lump-sum transfers from the government,  $T_t$ , both expressed in terms of tradable goods, which will serve as the numeraire. As is common in the sovereign debt literature, we assume that households do not have access to external capital markets.

The households' budget constraint, expressed in terms of tradables, is therefore given by

$$c_t^T + p_t^N c_t^N = y_t^T + \phi_t^N + w_t h_t + T_t, \quad (4)$$

where  $p_t^N$  denotes the price of non-tradables and  $w_t$  denotes the wage rate. Assuming that the law of one price holds and that the price of tradable goods in foreign currency is constant and normalized to one, these prices can also be interpreted in terms of foreign currency.

The households' problem consists of choosing sequences of  $c_t^T$  and  $c_t^N$  to maximize (1) given the sequence of prices  $\{p_t^N, w_t\}_{t=0}^\infty$ , profits  $\{\phi_t^N\}_{t=0}^\infty$ , and transfers  $\{T_t\}_{t=0}^\infty$ . The optimality condition of this problem yields that the marginal rate of substitution is equated to the relative price of non-tradables:

$$p_t^N = \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{\mu+1}. \quad (5)$$

Because of the CES aggregation, the marginal rate of substitution depends only on the ratio of tradable to non-tradable consumption. As a result, in equilibrium, this ratio is increasing in the price of non-tradables.

## 2.2 Firms

There is a continuum of firms of measure one. They produce non-tradable goods with a decreasing returns production function such that  $y^N = F(h)$ , where  $F(h) = h^\alpha$  and  $\alpha \in (0, 1]$ . Firms' maximum profits in each period are then given by

$$\phi_t^N = \max_h p_t^N F(h) - w_t h_t. \quad (6)$$

The optimal choice of labor  $h_t$  equates the value of the marginal product of labor and the wage rate, all expressed in tradable units,

$$p_t^N F'(h_t) = w_t. \quad (7)$$

## 2.3 Government

The government operates a fixed exchange rate regime, chooses issuances of long-term bonds and holdings of risk-free assets (i.e., reserves), and provides lump-sum transfers to households. The government has no commitment and can default on its debt.

Long-term bonds are introduced following Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012), and are denominated in tradable goods. A bond promises a deterministic infinite stream of coupons that decreases at an exogenous constant rate. Namely, a bond issued in period  $t$  promises to pay  $\delta(1 - \delta)^{j-1}$  units of the tradable good in period  $t + j$ , for all  $j \geq 1$ . Thanks to the recursive structure, the evolution of the face value of the debt,  $b_t$ , can be represented by the following law of motion:

$$b_{t+1} = (1 - \delta)b_t + i_t, \quad (8)$$

where  $i_t$  is the amount of new bonds issued in period  $t$ . The government issues these bonds at a price  $q_t$ , which in equilibrium depends on the government's portfolio decisions and the exogenous shocks.

Reserves pay one unit of tradable consumption goods. We let  $a_t \geq 0$  denote the government's reserve holdings at the beginning of period  $t$  and let  $q_a$  denote the price of reserves. Letting  $d_t = 1(0)$  if the government repays (defaults), we have that the budget constraint of the government can be written as

$$T_t = \begin{cases} a_t + q_t i_t - q_a a_{t+1} - \delta b_t & \text{if } d_t = 0 \\ a_t - q_a a_{t+1} & \text{if } d_t = 1. \end{cases} \quad (9)$$

Notice that upon default, the government retains control of its reserves and access to savings but cannot borrow in the default period. A default entails a utility loss  $\psi^d(y^T)$ , which depends on the realization of the tradable endowment. We think of this utility loss as capturing various default costs related to reputation, sanctions, or misallocation of resources; we do not model these explicitly.<sup>4</sup>

We abstract from financial exclusion as an additional source of default penalty. That is, the

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<sup>4</sup>An alternative assumption in the literature specifies an exogenous cost of default in terms of output. Assuming log utility over the composite consumption and that output losses from default are proportional to the composite consumption in default, the losses from default are identical for the output and utility cost specifications.

government can once again borrow from international markets in the period following a default.

## 2.4 Foreign Lenders

Bonds are priced in a competitive market inhabited by a large number of identical lenders. To capture global factors that are exogenous to domestic fundamentals, we introduce risk premium shocks. These shocks are not critical for the mechanism but enrich the analysis and are in line with a large empirical literature on global shocks' role in driving spreads and credit flows.<sup>5</sup>

Foreign lenders price the payoffs of bonds using the following stochastic discount factor, following [Vasicek \(1977\)](#):

$$m_{t,t+1} = e^{-r - \kappa_t(\varepsilon_{t+1} + 0.5\kappa_t\sigma_\varepsilon^2)}, \quad \text{with } \kappa_t \geq 0. \quad (10)$$

Here,  $r$  is the international risk-free rate, and  $\kappa_t \geq 0$  is a stochastic parameter governing the risk premium shock. Notice that (10) implies that bond payoffs are more valuable for investors when the small open economy faces a negative shock, capturing the positive degree of correlation between the small open economy and the lenders' income process. To the extent that the government is more likely to default when there are negative shocks to the tradable endowment  $\varepsilon$ , this implies that lenders demand a positive risk premium to be willing to invest in government bonds.

The risk premium shock  $\kappa$  follows a two-state Markov switching regime with values  $\kappa_L = 0$  and  $\kappa_H > 0$  and transition probabilities  $\pi_{LH}$ ,  $\pi_{HL}$ . In the "risk-neutral regime," we assume that  $\kappa = \kappa_L = 0$  so that the stochastic discount factor reduces to  $m_{t,t+1} = e^{-r}$ , eliminating any risk premia. In the "risk premia regime,"  $\kappa = \kappa_H > 0$ , and lenders require a risk premium to invest in government bonds. The value of  $\kappa$  can be seen as capturing how correlated the small open economy is with the lenders' income process or, alternatively, the degree of diversification in foreign lenders' portfolios. Therefore, a higher  $\kappa$  is associated with stronger risk premium shocks.

The standard asset pricing condition for bonds is therefore

$$q_t = \mathbb{E}_t \left\{ m_{t,t+1} (1 - \hat{d}_{t+1}) [\delta + (1 - \delta)q_{t+1}] \right\}, \quad (11)$$

where  $\hat{d}_{t+1}$  is the equilibrium default decision in  $t+1$ . Notice that assuming that these investors also price the risk-free asset gives us that the price of reserves is  $q_a = 1/(1+r)$ , a result that follows from the log-normal structure. Moreover, this implies that if investors are in a risk-neutral state, the expected return on bonds equals the return on reserves.

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<sup>5</sup>See for example [Longstaff et al. \(2011\)](#); [Forbes and Warnock \(2012\)](#); [Uribe and Yue \(2006\)](#); [Rey \(2013\)](#); [Johri, Khan and Sosa-Padilla \(2019\)](#).



## 2.5 Equilibrium

In equilibrium, market clearing for non-tradable goods requires that output is consumed domestically:

$$c_t^N = F(h_t). \quad (12)$$

For the labor market, we follow [Schmitt-Grohé and Uribe \(2016\)](#) and assume that nominal wages in domestic currency are downwardly rigid such that  $W \geq \bar{W}$ .<sup>6</sup> Using  $e$  to denote the exchange rate expressed in units of domestic currency per foreign currency and setting  $\bar{w} = \bar{W}/e$ , we arrive at the constraint

$$w_t \geq \bar{w}. \quad (13)$$

Hence, in a fixed exchange rate, the rigidity in domestic currency becomes equivalent to a wage rigidity in foreign currency. The lack of a flexible exchange rate therefore prevents the government from using [Friedman's \(1953\)](#) traditional stabilization role of exchange rates to affect real wages and employment.

Labor market equilibrium implies that the following slackness condition must hold for all dates and states:

$$(w_t - \bar{w})(\bar{h} - h_t) = 0. \quad (14)$$

This condition implies that when the wage rigidity is slack (i.e.,  $w > \bar{w}$ ), the economy must be at full employment. On the other hand, when there is unemployment (i.e.,  $h_t < \bar{h}$ ), the market wage must be equal to the floor  $\bar{w}$ . In the latter case, note that employment is such that firms are always on their labor demand, given the prevailing wages and prices for non-tradables.

We then have that a competitive equilibrium for given government policies can be defined as follows:

**Definition 1** (Competitive equilibrium given policy). Given initial values  $\{a_0, b_0\}$ ; exogenous processes  $\{y_t^T, \kappa_t\}_{t=0}^\infty$ ; government policies for transfers, debt and reserves,  $\{T_t, b_{t+1}, a_{t+1}\}_{t=0}^\infty$ ; and a default decision  $\{d_t\}_{t=0}^\infty$ , a *competitive equilibrium* is a sequence of allocations  $\{c_t^T, c_t^N, h_t\}_{t=0}^\infty$  and prices  $\{p_t^N, w_t, q_t\}_{t=0}^\infty$  such that

1. allocations solve households' and firms' problems at given prices;
2. government policies satisfy the government budget constraint (9);
3. the bond pricing equation (11) holds;
4. the market for non-tradable goods clears; and
5. the labor market satisfies conditions (13), (14), and  $h \leq \bar{h}$ .

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<sup>6</sup>In [Schmitt-Grohé and Uribe \(2016\)](#), the wage floor depends on the previous period's wage. For numerical tractability, we follow [Bianchi et al. \(2019\)](#) and set the wage floor as an exogenous constant value.

Notice that using the households' budget constraint (4), the definition of the firms' profits, the market clearing condition (12), and the government budget constraint, we can write the economy's tradable aggregate resource constraint as

$$c_t^T + q_a a_{t+1} = a_t + y_t^T + [q_t i_t - \delta b_t] \quad (15)$$

in periods in which the government repays and

$$c_t^T + q_a a_{t+1} = a_t + y_t^T \quad (16)$$

when the government defaults.

Toward a characterization of the the optimal government problem, notice that we can combine (5), (7), and (12) and obtain

$$h_t \leq \mathcal{H}(c^T; \bar{w}) = \left[ \frac{1 - \omega}{\omega} \left( \frac{\alpha z}{\bar{w}} \right) \right]^{\frac{1}{1+\alpha\mu}} (c_t^T)^{\frac{1+\mu}{1+\alpha\mu}}. \quad (17)$$

This expression shows that employment is an increasing function of  $c^T$  as long as the economy is not at full employment. The underlying mechanism is that a higher level of tradable consumption must be associated in equilibrium with a larger price of non-tradables, which, given a rigid wage, leads to a higher demand for labor and more employment in equilibrium. In essence, this is an aggregate demand effect originating from a nominal rigidity. Condition (17) will constitute a key implementability constraint in the government's problem below.

## 2.6 Optimal Government Problem

The government is benevolent and is unable to commit to repayment or any other future policies. Thus, one may interpret this environment as a game in which the government choosing policies in period  $t$  is a player that takes as given the policies of other players (governments) that will decide after  $t$ . We consider a Markov equilibrium, in which all policies depend on the payoff-relevant states  $(b, a, s)$  where  $s \equiv \{y^T, \kappa\}$ . Every period, the government directly chooses the repayment/default decision, bond issuances, reserves, and transfers, as well as labor, wages, and consumption of tradables and non-tradables subject to the competitive equilibrium conditions.

The government evaluates whether to repay or default, by comparing the value function of repayment  $V^R(b, a, s)$  with the value function of defaulting  $V^D(a, s)$ . These value functions represent the lifetime utility of households under the two government decisions. We therefore have that the value is given by

$$V(b, a, s) = \max_{d \in \{0,1\}} \left\{ (1 - d) V^R(b, a, s) + d V^D(a, s) \right\}. \quad (18)$$

The government faces a bond price schedule  $q(b', a', s)$  that determines the bond price at which it can raise debt depending on the choice over reserves and debt. Using the implementability constraint (17) as well as resource constraints (15) and (16), we have that the value of repayment is given by

$$\begin{aligned} V^R(b, a, s) &= \max_{b', a', h \leq \bar{h}, c^T} \left\{ u(c^T, F(h)) + \beta \mathbb{E}_{s'|s} [V(b', a', s')] \right\} \\ &\text{subject to} \\ c^T + q_a a' + \delta b &= a + y^T + q(b', a', y^T) [b' - (1 - \delta)b], \text{ and} \\ h &\leq \mathcal{H}(c^T; \bar{w}). \end{aligned} \quad (19)$$

Notice that as long as  $\delta < 1$ , the value function upon repayment depends on the composition of the portfolio  $(b, a)$ , not just the net position. On the other hand, with one-period debt,  $\delta = 1$ , the state variable under repayment can be summarized entirely by the net foreign asset position,  $a - b$ .

The value of default is given by

$$\begin{aligned} V^D(a, s) &= \max_{c^T, h \leq \bar{h}, a'} \left\{ u(c^T, F(h)) - \psi_d(y^T) + \beta \mathbb{E}_{s'|s} [V(0, a', s')] \right\} \\ &\text{subject to} \\ c^T + q_a a' &= y^T + a, \text{ and} \\ h &\leq \mathcal{H}(c^T; \bar{w}). \end{aligned} \quad (20)$$

It is important to notice that while we focus on a fixed exchange rate regime, one can see from problems (19) and (20) that under a flexible exchange rate regime, the government could choose a depreciation large enough to ensure full employment, which would be the optimal policy.<sup>7</sup>

A Markov perfect equilibrium is then defined as follows.

**Definition 2** (Markov perfect equilibrium). A Markov perfect equilibrium is defined by value functions  $\{V(b, a, s), V^R(b, a, s), V^D(a, s)\}$ ; associated policy functions  $\{\hat{d}(b, a, s), \hat{a}(b, a, s), \hat{b}(b, a, s), \hat{c}^T(b, a, s), \hat{h}(b, a, s), \hat{T}(b, a, s)\}$ ; a bond price schedule  $q(b', a', s)$  such that

1. given the bond price schedule, policy functions solve problems (18), (19), and (20);
2. the bond price schedule satisfies the bond pricing equation

$$q(b', a', s) = \mathbb{E}_{s'|s} \left\{ m(s', s) \left[ 1 - \hat{d}(b', a', s') \right] [\delta + (1 - \delta)q(b'', a'', s')] \right\}, \quad (21)$$

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<sup>7</sup>This can be seen by noting that  $\mathcal{H}$  is decreasing in  $\bar{w}$  and that, by definition,  $\bar{w} = \bar{W}/e$ . It thus follows that a depreciation (i.e., an increase in  $e$ ) reduces  $\bar{w}$ , and since nominal variables do not appear anywhere else in the restrictions of the government, raising the exchange rate until full employment is reached is optimal. See Na et al. (2018) for an analysis of optimal exchange rate policy in a sovereign default model.

where

$$b'' = \hat{b}(b', a', s') \text{ and } a'' = \hat{a}(b', a', s').$$

## 2.7 The Macro-Stabilization Role of Reserves

In this section, we examine how aggregate demand and sovereign risk considerations shape the optimal government portfolio. To do so, we present the first-order necessary conditions from the government problem (18)-(20).

We let  $\xi_t$  denote the Lagrange multiplier on (17), the implementability constraint associated with the labor market equilibrium  $h \leq \mathcal{H}(c^T; \bar{w})$ . We have that when the economy is at full employment,  $\xi_t = 0$ , and when there is unemployment,  $\xi_t$  turns strictly positive. Specifically,

$$\xi_t = \begin{cases} 0 & \text{if } h = \bar{h} \\ u_N(c_t^T, h_t^\alpha) \alpha h_t^{\alpha-1} & \text{if } h < \bar{h}. \end{cases} \quad (22)$$

That is, when  $h < \bar{h}$ , we have that the marginal value from relaxing (17) is given by the increase in utility that results from the increase in output of non-tradables. This multiplier will play a key role in the analysis

Let  $u_T$  denote the marginal utility from tradable consumption. Using the first-order conditions and the envelope conditions, we can obtain the following necessary conditions for optimality:

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$$\left( u_T + \xi \frac{\partial \mathcal{H}}{\partial c^T} \right) \left[ q + \frac{\partial q(b', a', s)}{\partial b'} i \right] = \beta \mathbb{E}_{s'|s} \left[ \left( u'_T + \xi' \frac{\partial \mathcal{H}'}{\partial c^T} \right) [\delta + (1 - \delta)q'](1 - d') \right], \quad (23)$$

$$\left( u_T + \xi \frac{\partial \mathcal{H}}{\partial c^T} \right) \left[ q_a - \frac{\partial q(b', a', s)}{\partial a'} i \right] \geq \beta \mathbb{E}_{s'|s} \left( u'_T + \xi' \frac{\partial \mathcal{H}'}{\partial c^T} \right) \text{ with equality if } a' > 0. \quad (24)$$

Equation (23) represents the Euler equation with respect to debt. The left-hand side represents the marginal benefits of issuing an additional unit of debt in the current period. When the government borrows one more unit, it raises  $q$  units of consumption, but it also lowers the revenue from the inframarginal issuances by  $\frac{\partial q(b', a', s)}{\partial b'} i$ . Each unit of consumption has a direct marginal utility benefit of  $u_T$  and an indirect marginal utility benefit of  $\xi \frac{\partial \mathcal{H}}{\partial c^T}$ . The direct effect is simply the benefit from borrowing that an individual household would face. On the other hand, the indirect effects are a manifestation of an aggregate demand amplification at work. When there is currently unemployment, the government internalizes that additional borrowing would

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<sup>8</sup> For illustration purposes, we assume that the bond price and the value function are differentiable. Notice that Clausen and Strub (2017), however, show that in the canonical default model, the objective function is continuously differentiable at the optimal choices, and a version of the envelope theorem applies. Aguiar et al. (2016) show similar results with shocks to outside options to default. Our computational method does not rely on differentiability.

raise consumption and induce a positive effect on employment in equilibrium.

The right-hand side represents the marginal utility cost of carrying one more unit of debt. When the government borrows one more unit and repays in the next period, the cost is given by the coupon payments  $\delta$  plus the cost of carrying  $(1 - \delta)$  units of debt at the market price  $q'$ . Importantly, the marginal utility costs of repaying those resources have analogous direct and indirect effects. Namely, if the economy is in a situation with unemployment tomorrow, repaying those resources has an additional cost given by the negative effects on employment.

Notice that the possibility of default generates an extra cost from borrowing to finance consumption that goes beyond the expected repayment to the creditors. When the government borrows more, it raises the probability of default, and hence the costs associated with it. The default costs do not explicitly appear in (23), because in effect, they cancel out with the resources that the government “saves” by defaulting.<sup>9</sup> The fact that investors price the default risk, however, implies that the government obtains fewer revenues from bond issuances. In fact, the fall in revenue is given by the term  $\frac{\partial q(b', a', s)}{\partial b'} i$  discussed above.

Equation (24) represents the Euler equation with respect to reserves. The left-hand side represents the marginal costs of purchasing one unit of reserves in the current period. Purchasing one unit of reserves requires  $q_a$  units of consumption, but because reserves affect the price at which the government is able to issue debt, the overall cost is given by  $q_a$  net of the effect that the increase in reserves has on the revenue from the debt issuances,  $\frac{\partial q(b', a', s)}{\partial a'} i$ . The right-hand side shows the marginal utility benefits of consuming the proceeds from reserves in the next period. Again, importantly, the value of those resources today and tomorrow depends on the slack in the labor market.

It is worth highlighting three key elements behind the Euler equations. First, the presence of default risk alters the marginal costs of borrowing and acquiring reserves, as they both affect future incentives to default and current spreads. Second, the presence of long-term bonds implies that repaying debt is less costly when future bond prices are lower. Third, the value associated with resources in each state depends on the slack in the labor market. This last effect arises because the government internalizes the aggregate demand amplification at work, and it would not be present in the portfolio problem of an individual household.

**Debt-financed reserves.** To further inspect the trade-offs behind the optimal portfolio, let us analyze a financial operation by which the government purchases an additional unit of reserves and finances it by issuing debt. Specifically, starting from a set of initial states  $(b, a, s)$ , assume that the government sets a certain level of transfers to households,  $\bar{T}$  (which in turn determines real allocations for  $c^T$ ,  $c^N$ , and  $h$ ). We denote by “candidate portfolios” the pairs of  $(a', b')$  that

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<sup>9</sup>Technically, when taking first-order condition, the derivative of the default threshold with respect to  $b'$  multiplies the difference between  $V^R$  and  $V^D$  that equals zero at the indifference point.

are consistent with

$$\bar{T} = a + q(b', a', y^T)[b' - (1 - \delta)b] - q_a a' - \delta b. \quad (25)$$

Assuming that the government follows the optimal policies from tomorrow onward, we can consider trade-offs behind the range of all candidate portfolios.<sup>10</sup> In particular, we must have that at the optimum candidate portfolio, the government equates the net marginal benefits of debt-financed reserves to zero. That is,

$$\underbrace{\left( \frac{q + \frac{\partial q(b', a', s)}{\partial b'} i}{q_a - \frac{\partial q(b', a', s)}{\partial a'} i} \right)}_{\text{Reserves bought}} \mathbb{E}_{s'|s} \left[ u'_T + \xi' \frac{\partial \mathcal{H}'}{\partial c^T} \right] = \mathbb{E}_{s'|s} [1 - d'] \left\{ \mathbb{E}_{s'|s, d'=0} [\delta + (1 - \delta)q'] \mathbb{E}_{s'|s, d'=0} \left[ \left( u'_T + \xi' \frac{\partial \mathcal{H}'}{\partial c^T} \right) \right] \right. \\ \left. + \underbrace{\text{COV}_{s'|s, d'=0} \left( \delta + (1 - \delta)q', u'_T + \xi' \frac{\partial \mathcal{H}'}{\partial c^T} \right)}_{\text{Macro-stabilization hedging}} \right\}. \quad (26)$$

The left-hand side represents the marginal benefits of debt-financed reserves. This term is given by multiplying the reserves that can be bought by issuing one more unit of debt by the marginal utility benefits of having the reserves available in the next period. The amount of reserves that can be bought is essentially given by the relative prices of bonds and reserves, both adjusted by how the change in positions affects the inframarginal units of debt issued.

The right-hand side represents the marginal costs of debt-financed reserves. The first term is given by the expected marginal utility costs that the government faces for the average amount of resources repaid. A key term is the one we label “macro-stabilization hedging,” which emerges from the interaction between nominal rigidities and sovereign risk. In particular, because sovereign risk is countercyclical, states in which there is slack in the labor market will coincide with states with low  $q'$ , making it less costly for the government to repay the debt. In effect, by borrowing and accumulating reserves, the government is transferring resources to states of nature in which resources are valued especially for aggregate demand management (i.e., states with high  $\xi' \frac{\partial \mathcal{H}'}{\partial c^T}$ ). Importantly, the macro-stabilization hedging benefits arise only if bonds have long maturity as in [Bianchi et al. \(2018\)](#), but in contrast with the mechanism they highlight, the hedging benefits arise even if the government were risk neutral.<sup>11</sup>

<sup>10</sup>Notice that because of the presence of a Laffer curve, there is an upper bound on the value of reserves that a candidate portfolio may have.

<sup>11</sup>This can be seen by noting that if marginal utility were constant, the covariance would not vanish as long as  $\delta < 1$ .

**Illustration of main trade-off.** Equation (26) describes the condition that the government satisfies at the optimum candidate portfolio. While it is not possible to derive explicit analytical solutions for the optimum portfolio, we can numerically illustrate how the key terms in that condition vary for all possible candidate portfolios. In Figure 1, we plot three terms as a function of the amount of reserves purchased for given initial conditions and the target transfer presented in the note of the figure.<sup>12</sup> The first term, depicted with a solid blue line, is “Reserves bought,” which represents how many reserves the government can purchase by issuing one additional unit of debt while keeping transfers at a constant target. This term starts below one, reflecting that this is a costly operation, and it is downward sloping. That is, as the government increases the amount of the gross positions, spreads increase and the government is able to obtain fewer reserves for every additional unit of debt issuances.

The second term, depicted with a dashed red line, represents the normalized marginal costs, which we construct as the right-hand side of equation (26) divided by the expected marginal utility benefit of an additional unit of reserve. The crossing of this line with the blue straight line described above denotes the optimal portfolio, given the initial states. A key component of the marginal costs is given by the macro-stabilization hedging term that is represented with a dotted black line. This term is negative, indicating that the macro-stabilization hedging effect makes debt-financed reserves less costly. Moreover, it is also downward sloping, indicating that as the government increases the stock of reserves, the macro-stabilization hedging benefit becomes smaller at the margin.

To summarize, the key implication of the theory is that a government that follows a fixed exchange rate has incentives to accumulate reserves over and beyond those of a government that has a flexible exchange rate. Namely, there is a macro-stabilization hedging benefit from holding reserves when a government is unable to stabilize the economy by using monetary policy.

In the next section, we present empirical evidence suggesting that emerging economies under a fixed exchange rate do in fact accumulate more reserves than similar economies with flexible exchange rates. After that, we will calibrate the model and analyze its quantitative predictions.

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<sup>12</sup>The associated level of debt emerges from solving (25). Notice that the figure is constructed for the calibrated version of the model that we discuss in Section 4, and throughout the paper, debt and reserves are expressed in terms of GDP.

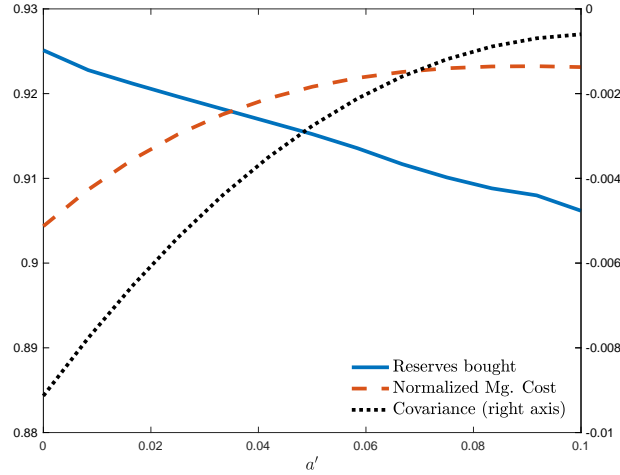


Figure 1: Macro-stabilization benefits of reserves

*Note:* The figure is computed ensuring that candidate portfolios keep current tradable consumption fixed at .8 (eighty percent of mean tradable income) with the associated transfer, and it assumes initial reserves equal to 6% of GDP, initial debt of 36% of GDP, tradable income at its mean, and no risk premium shock ( $\kappa = 0$ ).

### 3 Empirical Evidence

In this section, we present empirical evidence on international reserves, sovereign spreads, and exchange rate flexibility that illustrates the relationship between these variables. We document one key fact: consistent with the model, countries with less exchange rate flexibility tend to accumulate more reserves.

**Data.** Unless specified otherwise, we use data from 1990 to 2015 for a set of 22 emerging economies, which are commonly used in the literature.<sup>13</sup> Table 1 presents summary statistics of debt, sovereign spreads, reserves, and unemployment in our sample of countries. On average, the countries in our panel have a debt-to-GDP ratio of 45% and a reserves-to-GDP ratio of 16%. In addition, they face significant default risk. The average mean spread is 2.9%, and the standard deviation is 1.6%.

<sup>13</sup>The 22 countries in our panel are Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Egypt, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Poland, Romania, Russia, South Africa, Thailand, and Turkey. As is common in studies of emerging economies, we exclude crisis years. Crisis years are defined following Catão and Mano (2017). Appendix A has more details about the dataset and variable definitions.



Table 1: Summary statistics

	Mean	Median
Debt/GDP	.45	.41
Reserves/GDP	.16	.16
Spread (in %)	2.9	2.4
SD(Spread) (in %)	1.6	1.5
Corr(Reserves/GDP, Spread)	-.40	-.50

**Exchange rate flexibility and reserve holdings.** In Figure 2, we use the IMF’s classification to sort countries into two categories, “Fixed” and “Flex,” representing countries with rigid and flexible exchange rate regimes, respectively. This figure shows that while both sets of countries have experienced an increase in reserve holdings, the surge is most notable for countries with fixed exchange rates.

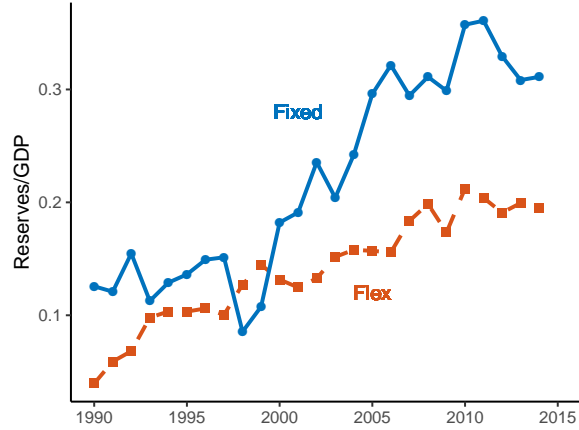


Figure 2: Reserve accumulation and exchange rate regime

In order to further document this fact, we follow the empirical literature (e.g., [Tenreyro, 2007](#)) and measure the exchange rate variability (ERV) for country  $i$  as the standard deviation of the log first-difference of the exchange rate of country  $i$ ’s currency (against the US Dollar). The standard deviation is computed using (centered) rolling windows of three years.

The simple correlation between ERV and reserve accumulation is mildly negative in our sample (-.13). Since the association between exchange rate variability and reserve accumulation might be driven by other confounding factors, we use a regression framework and control for time-invariant country fixed effect, the level of the world interest rate, and other country-specific explanatory variables. Specifically, we estimate

$$Res_{it} = \beta_{ERV} ERV_{it-1} + \phi' \mathbf{X}_{it-1} + \xi_i + \epsilon_{it}, \quad (27)$$

where  $Res_{it}$  is the (logarithm of) of the reserves-to-GDP ratio for country  $i$  in year  $t$ ;  $ERV_{it-1}$  is our measure of exchange rate variability for country  $i$  in year  $t$ ; and  $\mathbf{X}_{it-1}$  is a vector of commonly used control variables in reserves regressions: the debt-to-GDP ratio, the country spread, the cyclical component of GDP, and the world interest rate. Finally,  $\xi_i$  is a country fixed effect and  $\epsilon_{it}$  is a random error term.<sup>14</sup>

Table 2 shows that, other things equal, countries with less exchange rate variability tend to accumulate more reserves, and this finding is robust to various controls and specifications. The magnitudes are also economically significant. In specification (5), for example, a decrease of 1 standard deviation in a country's  $i$  exchange rate variability is associated with a 21% increase in reserve holdings.

Table 2: Panel regressions

	<i>Dependent variable: <math>\log(Reserves/y)</math></i>				
	(1)	(2)	(3)	(4)	(5)
ERV	−0.647* (0.367)	−0.656** (0.332)	−0.662** (0.334)	−0.281* (0.152)	−0.206* (0.121)
$\log(Debt/y)$		0.245 (0.214)	0.250 (0.244)	0.349 (0.240)	0.324 (0.203)
$\hat{y}$			−0.069 (1.227)	1.158 (1.326)	1.389 (1.007)
$\log(Spread)$				−0.155 (0.095)	−0.063 (0.093)
$r^{world}$					−0.119*** (0.038)
Number of countries	22	22	22	22	22
Observations	459	459	458	314	314
R <sup>2</sup>	0.02	0.04	0.04	0.12	0.24
F Statistic	7.28***	8.97***	6.53***	9.43***	18.24***

Note: All explanatory variables are lagged one period to control for endogeneity.  $\hat{y}$  is the cyclical component of GDP ( $y$ ). All specifications include country fixed effects. Robust standard errors (clustered at the country level) are reported in parentheses. \*p<0.1; \*\*p<0.05; \*\*\*p<0.01.

<sup>14</sup>All the specifications of equation (27) that we estimate lag the explanatory variables one period, to control for endogeneity.

## 4 Quantitative Analysis

### 4.1 Numerical Solution

The recursive problem is solved using value function iteration. As in [Hatchondo, Martinez and Saprizo \(2010\)](#), we solve for the equilibrium by computing the limit of the finite-horizon version of our economy. For each state, we solve the optimal portfolio allocation by searching over a grid of debt and reserve levels and then using the best portfolio on that grid as an initial guess in a nonlinear optimization routine. The value functions  $V^R$  and  $V^D$  and the function that indicates the equilibrium bond price  $q(\hat{b}(\cdot), \hat{a}(\cdot), s)$  are approximated using linear interpolation over  $y^T$  and cubic spline interpolation over debt and reserves positions.

### 4.2 Calibration

A period in the model refers to a year. We split the parameters of the model into two groups. The first group of parameters (those in the top part of [Table 3](#)) take values that can be set either directly from the data or using typical values from the literature. The second group of parameter values (those in the bottom part of [Table 3](#)) are set by simultaneously matching key moments from the data. As a data reference, we use a panel of emerging economies, which are described in the empirical section.

Following [Bianchi et al. \(2018\)](#), we assume the following functional form for the utility cost of default,

$$\psi_d(y^T) = \psi_0 + \psi_1 \log(y^T).$$

As in [Chatterjee and Eyigungor \(2012\)](#), having two parameters in the cost of default gives us enough flexibility to match the spread dynamics observed in the data.

The parameter values that govern the tradable endowment process are chosen to mimic the average behavior of logged and linearly detrended tradable GDP. This yields  $\sigma_\varepsilon = 0.045$  and  $\rho = 0.84$ . We set  $\mu_y = -\frac{1}{2}\sigma_\varepsilon^2$  so that mean tradable income is normalized to 1.

The values of the risk-free interest rate and the domestic discount factor are set to  $r = 0.04$  and  $\beta = 0.90$ , which are standard in quantitative sovereign default studies. We set  $\delta = 0.2845$ . With this value and the targeted level of sovereign spread, sovereign debt in the simulations has an average duration of three years, which is roughly the average duration of public debt in our panel of emerging economies.<sup>15</sup>

Following [Bianchi et al. \(2018\)](#), we use the average EMBI+ spread to parameterize the shock process to lenders' risk aversion. We assume that a period with high lenders' risk aversion is

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<sup>15</sup>We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by  $D = (1 + i_b)/(\delta + i_b)$ , where  $i_b$  denotes the constant per-period yield delivered by the bond.

Table 3: Parameter values.

Parameter	Description	Value
$r$	Risk-free rate	0.04
$\alpha$	Labor share in the non-tradable sector	0.75
$\beta$	Domestic discount factor	0.90
$\pi_{LH}$	Prob. of transitioning to high risk premium	0.15
$\pi_{HL}$	Prob. of transitioning to low risk premium	0.8
$\sigma_\varepsilon$	Std. dev. of innovation to $\log(y^T)$	0.045
$\rho$	Autocorrelation of $\log(y^T)$	0.84
$\mu_y$	Mean of $\log(y^T)$	$-\frac{1}{2}\sigma_\varepsilon^2$
$\delta$	Coupon decaying rate	0.2845
$1/(1+\mu)$	Intratemporal elast. of subs.	.44
$\gamma$	Coefficient of relative risk aversion	2.273
$\bar{h}$	Time endowment	1
Parameters set by simulation		
$\omega$	Share of tradables	0.4
$\psi_0$	Default cost parameter	3.6
$\psi_1$	Default cost parameter	22
$\kappa_H$	Pricing kernel parameter	15
$\bar{w}$	Lower bound on wages	0.98

one in which the global EMBI+ without countries in default is one standard deviation above the median over the sample period. With this procedure, we obtain three episodes of a high risk premium every 20 years with an average duration equal to 1.25 years for each episode, which implies  $\pi_{LH} = 0.15$  and  $\pi_{HL} = 0.8$ . On average, the global EMBI+ was 2 percentage points higher in those episodes than in normal periods.

The households' endowment of hours to work ( $\bar{h}$ ) is normalized to 1. The labor share in the production of non-tradable goods ( $\alpha$ ) is set to 0.75, the estimate found by Uribe (1997) for Argentina. We set  $1/(1+\mu) = 0.44$ , which is the elasticity of substitution between tradables and non-tradables estimated by Gonzalez-Rozada et al. (2003) and Akinci (2011). The coefficient of relative risk aversion is set to  $\gamma = 1 + \mu$ . We do this for two reasons: first, the implied value (2.273) is close to 2, a value commonly used in the literature; and second, this is a convenient parameterization because it implies that the dynamics of the “tradable block” (debt, reserves, and consumption of tradables) are independent of the “non-tradable block,” absent nominal rigidities.<sup>16</sup>

**Targeted moments.** The calibration strategy described so far leaves us with five parameters to assign values to: the weight of tradables in the utility function ( $\omega$ ), the default cost parameters

<sup>16</sup>This feature follows from the fact that the CRRA utility function and CES aggregator imply that the cross-partial derivatives  $u_{TN}$  equal zero when the intertemporal elasticity of substitution equals the intratemporal elasticity across goods.

( $\psi_0$  and  $\psi_1$ ), the risk premium parameter ( $\kappa_H$ ), and the lower bound on wages ( $\bar{w}$ ). We target the following five moments from the data: (i) a share of tradable output to total output of 41%, (ii) a mean debt-to-GDP ratio of 45%, (iii) a mean sovereign spread of 2.9%, (iv) an increase of 200 basis points in the spread during high-risk premium periods, and (v) an increase of 2 percentage points in the cyclical unemployment rate in a one year window around a default event.

To compute the sovereign spread that is implicit in a bond price, we first compute the yield  $i_b$ , defined as the return an investor would earn if he holds the bond to maturity (forever) and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \delta(1 - \delta)^{j-1} e^{-ji_b}.$$

The sovereign spread is then computed as the difference between the yield  $i_b$  and the risk-free rate  $r$ . Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate—that is,  $\frac{\delta}{1-(1-\delta)e^{-r}} b_t$ .

## 5 Results of the Quantitative Analysis

In this section, we present the results of the quantitative analysis. First, we show the ability of the benchmark calibration of the model to account for salient features of business cycles in our panel of emerging economies. Second, we study the macro-stabilization benefits of reserve accumulation. Third, we analyze the effects of reserve accumulation and nominal rigidities on equilibrium spreads and default policies and argue that reserves are not necessarily costly. Fourth, we examine the welfare implications of reserve accumulation. Fifth, we study the extent to which simple and implementable reserve accumulation rules can effectively approximate the optimal policy. Finally, we show that under inflation targeting, a strong macroeconomic stabilization role for reserves remains.

### 5.1 Simulation Moments

Table 4 reports moments from the data and from the model simulations. We present results for both the baseline model under a fixed exchange rate and an economy under a flexible exchange rate. Because the optimal exchange rate policy prescribes full employment across all states, the latter can also be interpreted as an economy with flexible wages.<sup>17</sup>

The top panel of Table 4 shows that our model closely matches the targeted moments. The middle panel shows that the model also performs well in accounting for several non-targeted

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<sup>17</sup>We compute the flexible-wage economy by setting  $\bar{w} = 0$  and recalibrating the other four parameters (set by simulation) in order to produce the same moments as before, except for the change in the unemployment rate (which, by construction, will be zero in the flexible-wage version).

Table 4: Key statistics – model and data.

	Data	Model	
		Benchmark	Flexible
<b>Targeted</b>			
Mean debt ( $b/y$ )	45	44	46
Mean $r_s$	2.9	2.9	3.0
$\Delta r_s$ w/ risk-prem. shock	2.0	2.0	1.9
$\Delta$ UR around crises	2.0	2.0	0.0
Mean $y^T/y$	41	41	41
<b>Non-Targeted</b>			
$\sigma(c)/\sigma(y)$	1.1	1.0	1.1
$\sigma(r_s)$ (in %)	1.6	3.1	2.9
$\rho(r_s, y)$	-0.3	-0.6	-0.8
$\rho(c, y)$	0.6	1.0	1.0
Mean Reserves ( $a/y$ )	16	16	7
Mean Reserves/Debt ( $a/b$ )	35	35	15
$\rho(a/y, r_s)$	-0.4	-0.4	-0.6

*Note:* Moments in the model are computed for the average of pre-default simulation samples (except for the change in the unemployment rate around default crises). We simulate the model for 1,000 samples of 300 periods each. We then take the last 35 observations of each sample in which the last default was observed at least 25 periods before the beginning of the sample.

moments of the data. In particular, we see that our benchmark model features consumption that is procyclical and also more volatile than output and sovereign spreads that are volatile and countercyclical.

The bottom panel of Table 4 provides the model predictions for reserves. We present the average level of reserves, expressed both in terms of output and as a fraction of debt, and the correlation of reserves with spreads. All these moments are non-targeted. Remarkably, one can see that the benchmark model produces a mean reserves-to-output ratio of 16%, in line with the average of our panel of emerging markets. In contrast, the flexible economy generates holdings of international reserves that are much lower, averaging 7% of GDP. This large difference underscores that the macroeconomic stabilization role for reserves is critical in accounting for the level of reserves observed in the data and the differences across economies with different degrees of exchange rate flexibility.

It is important to point out that while the economy under nominal rigidities features higher volatility and lower employment, this is *not* the reason why the government accumulates more reserves. In fact, assuming separable preferences for tradables and non-tradables (as we did in the calibration), a higher volatility of non-tradable output is completely irrelevant to the decision

to accumulate reserves under flexible wages. Key for our mechanism is that the government *internalizes* that through its portfolio choice, it can alter the level and the volatility of employment. In the next section, we further inspect this mechanism.

## 5.2 Macro-Stabilization Hedging Benefits

In this section, we inspect the key channel at work in the model and show how reserves contribute to improve macroeconomic stabilization. Namely, we show that a portfolio with larger gross asset and debt positions helps reduce the severity of future recessions.

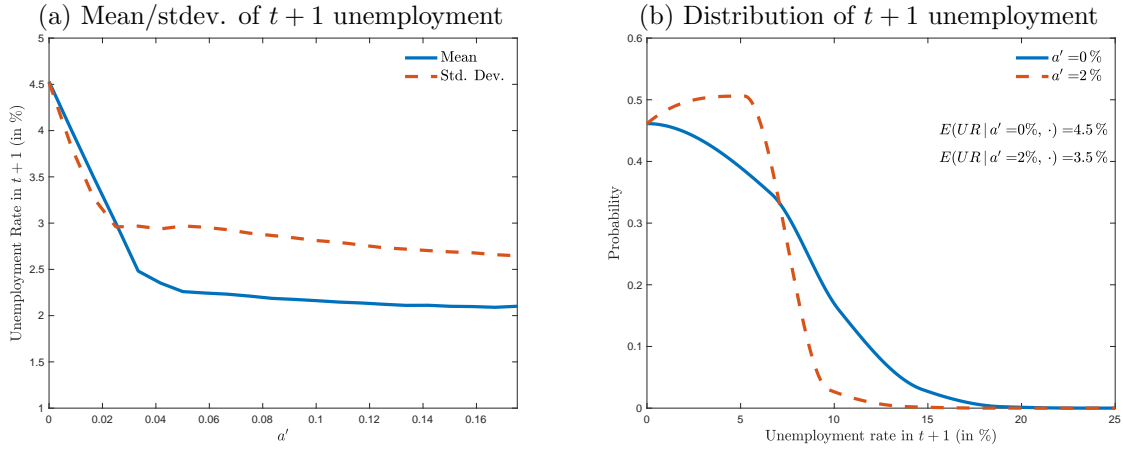


Figure 3: Unemployment tomorrow as a function of reserves

*Note:* The figure assumes an initial portfolio of  $a = 9\%$  and  $b = 31\%$  of mean GDP, an initial tradable income that is 1s std. dev. below mean, and  $\kappa = \kappa_L = 0$ .

We now examine how the entire set of candidate portfolios, as constructed in (25), affects the distribution of unemployment rates in the future. That is, we fix an initial state and consider all the possible portfolios from which the government could pick for a given amount of transfers to households. The results of this exercise are illustrated in Figure 3.<sup>18</sup> The left panel shows the mean and the volatility of next-period unemployment for a range of values of reserves. The right panel shows the entire distributions of  $t + 1$  unemployment rate for two possible values of reserve accumulation (0 and 2 percent of GDP).

The key result from Figure 3 is that portfolios with higher higher reserves (and debt) help reduce future unemployment. Panel (a) shows a lower mean and volatility of unemployment with higher reserves; Panel (b) shows how with higher reserves the distribution of unemployment places a lower mass of probability in states with higher unemployment levels. Notice that states with high unemployment in the distribution are associated with low tradable income shocks or

<sup>18</sup>It is convenient to note here that all figures in the paper express debt and reserves as percentage of mean GDP.

adverse risk-premium shocks. The figure therefore shows that having more reserves available in those states allows for a significant reduction of the slack in the labor market.

### 5.3 Portfolios and Spreads

A central element in the government’s optimal portfolio choice is how spreads respond to the portfolio composition. As highlighted in equation (23), the more the spread increases in response to a debt-financed reserves operation, the smaller the amount of reserves that can be purchased is, and hence the lower the net marginal benefits from accumulating reserves are. As we will show below, the exchange rate regime will play a key role in affecting spreads and therefore in the portfolio decision.

**Default sets.** To understand how portfolios affect spreads, it is useful to start by considering for which states the government finds it optimal to default or repay. In Figure 4, we fix the initial value of income  $y^T$  at one standard deviation below its mean and  $\kappa = 0$  (no risk premia case), then analyze the default decision for a range of  $(b, a)$ . We label “default set” the combinations of  $(b, a)$  such that  $V^D(a, b, y^T, \kappa) > V^R(a, b, y^T, \kappa)$ . For the benchmark economy, this set is the area with vertical stripes in Figure 4. For comparison, we also show the default set under a flexible exchange rate (for the same initial values of the exogenous state variables), which is illustrated by the shaded area in Figure 4.

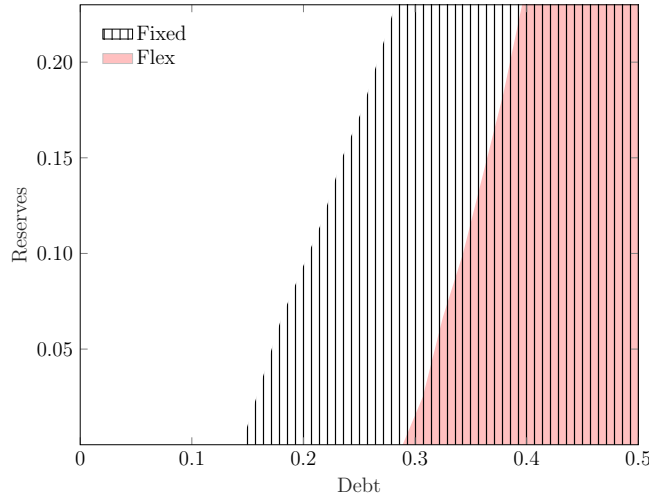


Figure 4: Default sets.

*Note:* The area with dark vertical stripes is the default set for the economy under a fixed exchange rate regime, and the area with a pink shading is the default set for the economy under a flexible exchange rate regime.

A first result that emerges from Figure 4 is that a fixed exchange rate worsens the incentives to repay. This result is illustrated by the fact that the default set under a flexible exchange



rate is contained by the default set under a fixed exchange rate. That is, for the initial states considered, there is no pair of  $(b, a)$  such that the government would repay under a fixed exchange rate and default under a flexible exchange rate. Moreover, if for a given  $(b, a)$  the government defaults under a flexible exchange rate, it also defaults under a fixed one.

To understand how the exchange rate regime affect the incentives for repayment, it is important to note that nominal rigidities affect both  $V^R$  and  $V^D$ . The key is therefore whether whether nominal rigidities are relatively more binding under repayment or under default. As noted by [Bianchi and Mondragon \(2018\)](#), when the government is in a net repayer position (i.e.,  $q(b', s)i < \delta b$ ), the value of repayment falls more than the value of default when nominal rigidities get tighter, causing the default set to expand as observed in the figure. This occurs because when the government is facing a recession, a net payment to creditors deepens the recession: it reduces households' disposable income and induces a reduction in the relative price of non-tradables, which leads to further under-utilization of labor absent the ability to depreciate the exchange rate and reduce real wages. Because the government has a negative net foreign asset position, it is indeed more common that the government is a net repayer position. Moreover, the key is evaluating how the values of repayment and default are affected at states in which the government is indifferent between defaulting and repaying, and in line with [Arellano \(2008\)](#), these are states in which the government is indeed in a net repayer position.<sup>19</sup> Furthermore, the exchange rate regime affect the decision to default even in states in which the economy is not facing a recession. If incentives to default are higher under a fixed exchange rate regime, this translates into worse borrowing terms and again a wider default set.

A second result that emerges from Figure 4 is that default sets are increasing in debt, as it is to be expected, but more importantly, they are decreasing in reserves. That is, for a given level of debt, higher reserves increases the incentives to repay. This result reflects that while both the value of repayment and the value of default are increasing in reserves, the former is even more so.

The result that the default set contracts with larger reserve holdings is further illustrated in Figure 5. This figure presents the value functions of repayment and default for a range of values of reserves, starting from a certain level of debt (in this case 20% of mean GDP) and considering the same shocks as in Figure 4. Starting from zero reserves, we have that  $V^R < V^D$  and hence the government prefers to default. When reserves reach around 10 percent of GDP, the inequality reverses and the government prefers to repay. Naturally, this result is a consequence of the fact that, as we can see in the figure, the slope of  $V^R$  is larger than the slope of  $V^D$ . To understand why this is the case, we can use the envelope condition to obtain the marginal effects of reserves

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<sup>19</sup> [Arellano \(2008\)](#), in fact, formally establishes this result when output follows an iid process and there is permanent autarky (see Proposition 2 in her paper). The idea is that if the government could obtain net debt inflows, it could simply consume more today and default tomorrow on a higher debt. While our model departs from hers in many dimensions, the result still carries.

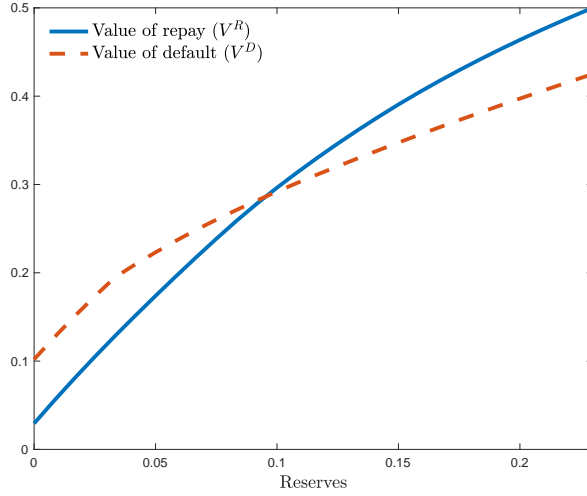


Figure 5: Value function for different levels of reserves.

*Note:* The initial states are such that debt equals 20% of GDP, tradable income is at one-standard deviation below its mean and  $\kappa = 0$  (no risk premium shock).

on the value functions under repayment and default

$$\frac{\partial V^i}{\partial a} = u_T + \xi \frac{\partial \mathcal{H}}{\partial c^T} \quad \text{for } i = R, D. \quad (28)$$

An additional unit of reserves has again both direct and indirect effects. As it turns out, what determines whether the value of repayment or the value of default increases more with reserves is the amount of tradable consumption in each of the two cases. Because the government is a net debtor, it tends to run trade surplus under repayment, and hence has a lower tradable consumption and a higher direct marginal benefit from reserves under repayment. Moreover, when there is slack in the labor market, additional reserves have an amplifying effect on the value by alleviating the contraction in output. Overall, this implies that reserves tend to be more valuable under repayment than under default, the default set shrinks with reserves, as shown in Figure 4.

**Spread-debt menus.** The previous results on the default set translate naturally into an equilibrium spread schedule that the government faces. In fact, for any candidate portfolio, the government will face a spread that must be consistent with the incentives to default in future periods, as illustrated by the default sets presented.

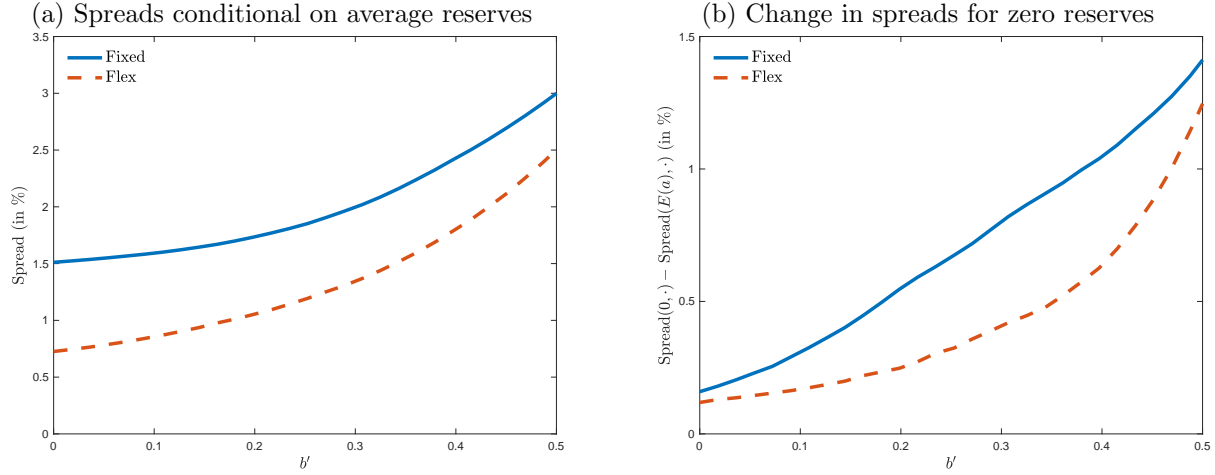


Figure 6: Spreads schedules, reserves and nominal rigidities.

*Note:* The left panel shows the equilibrium spread-debt menu under ‘Fixed’ and ‘Flex’, conditional on reserves being at the mean observed in the benchmark simulations. The right panel shows how much the spread will increase if the government were to choose zero reserves instead of the average value. Initial states correspond to average income, debt, reserves and no risk premium shocks.

The left panel in Figure 6 shows the menu of spreads and next-period debt level combinations from which the sovereign can choose, keeping fixed a level of  $a'$  at the mean value observed in the simulations and initial values for all the other state variables set at their means. In line with the results on the default sets, the spread is increasing in the end-of-period debt and the spread is higher under fixed than under flex for any given level of debt.<sup>20</sup>

The right panel of Figure 6 shows how much the spreads-debt menu worsens if the government were to accumulate zero reserves. Specifically, the right panel of Figure 6 compares the spread if the government accumulates zero reserves with the spread if the government were to accumulate the mean level of reserves. The figure shows that spreads worsen significantly when  $a' = 0$ , and especially so under a fixed exchange rate. In line with the theoretical results developed above, having more reserves helps stabilize the economy under a fixed exchange rate and makes repayment more likely.

**Debt-financed reserves and spreads.** The previous results highlight that when the government chooses to hold higher reserves, this reduces incentives to default in the future and lowers spreads. On the other hand, repaying debt rather than accumulating reserves could, presumably, lead to even lower spreads. We show, however, that this may not be the case.

In order to show that issuing debt to buy reserves need not raise sovereign spreads, we proceed to evaluate all the candidate portfolios starting from an arbitrary target level of consumption and initial conditions. As explained above, all these candidate portfolios are consistent with the

<sup>20</sup>Notice that because debt is long-term and investors price all future probabilities of default, spreads can reach 100 bps even for debt close to zero.

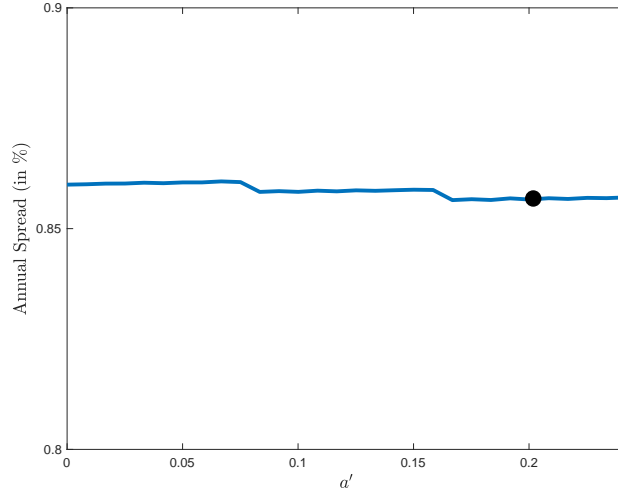


Figure 7: Spreads as a function of debt financed reserves

*Note:* The figure shows the spreads for different candidate levels of debt-financed reserves. The black dot denotes the optimal level of  $a'$ . The figure is computed ensuring that candidate portfolios keep current tradable consumption fixed at .8 (eighty percent of mean tradable income) and it assumes initial reserves equal to 13% of GDP, initial debt of 49% of GDP, tradable income at one-standard deviation above its mean and no risk premium shock.

budget constraint and the same level of consumption. The question we then ask is: Are portfolios with higher debt and reserves necessarily associated with higher spreads? Figure 7 shows that this may not be the case. In this figure, we can see that for the initial conditions considered, there is a negative relationship between  $a'$  and the spread paid in equilibrium.

Two features of the model are essential to obtain this surprisingly negative relationship between debt-financed reserves and spreads. A first feature is that bonds need to have long-term maturity. In a model with one-period debt, an increase in reserves and debt by one unit leaves unchanged the value of repayment while raising the value of default. As a result, with one period debt, spreads are always increasing in debt-financed reserves. On the other hand, with long-term maturity, debt is less costly to repay when income is low, and hence it is in principle possible that an increase in both debt and reserves lowers spreads. The second feature is the presence of a fixed exchange rate regime. Only when the macroeconomic stabilization motive is strong enough, we find that spreads may actually fall in response to an increase in debt-financed reserves.<sup>21</sup>

It is also important to highlight that this results emerges for a reduced portion of the state space. In fact, in Figure 1, we present other conditions under which spreads were decreasing in the amount of reserves. From a policy perspective, this result is important because it highlights that a government could, in some instances, judiciously issue debt and purchase reserves, at effectively, no cost. There is of course a limit to this. At some point, once debt-financed reserves become large enough, the spreads would move against the government.

<sup>21</sup>This is in line with the results of Bianchi et al. (2018) who found that spreads are always increasing in gross positions in an endowment economy.

## 5.4 Welfare Implications

In this section we ask what are the welfare gains of accumulating reserves and the extent to which they mitigate the costs of running fixed exchange rate.

We compute these welfare costs as follows:

$$\text{welfare costs} = 100 \times \left[ \left( \frac{(1-\gamma)(1-\beta)V_{\text{baseline}} + 1}{(1-\gamma)(1-\beta)V_{\text{alternative}} + 1} \right)^{1/(1-\gamma)} - 1 \right]. \quad (29)$$

where  $V_{\text{baseline}}$  and  $V_{\text{alternative}}$  denote the value function for some baseline or alternative economies. For all the exercises in this subsection we keep parameters at their values in the benchmark calibration and evaluate the welfare.<sup>22</sup>

In Figure 8 we present the main results of the welfare analysis. On the left panel, we present the welfare costs from nominal rigidities as functions of the tradable income level. These welfare costs are positive and sizable for both the benchmark and the no-reserves economy. They are also decreasing functions of the tradable income level: nominal rigidities are costlier at low levels of tradable income since these are states of nature in which aggregate demand is low and nominal rigidities are more binding. The average welfare costs of nominal rigidities are 0.89% of permanent consumption in the benchmark model and 1.03% of permanent consumption in the “no-reserves” economy. Interestingly, nominal rigidities are costlier if the economy cannot accumulate safe assets.<sup>23</sup>

The right panel of Figure 8 presents the welfare gains from accumulating reserves. That is, we compare the value function in an economy in which the government can accumulate reserves and an economy in which it cannot. We see that welfare gains of reserves are substantially larger in the economy under a fixed exchange rate, in line with the macroeconomic stabilization role we highlighted.<sup>24</sup>

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<sup>22</sup>The “no-reserves” economy is essentially the same model studied in Section 2 except that the government does not have access to short-term risk-free assets.

<sup>23</sup>These welfare costs are computed for an initial level of debt equal to zero. When we compute the welfare costs for a level of initial debt equal to the average observed in the simulations of the benchmark model, we obtain welfare costs of nominal rigidities that are higher: 2.83% of permanent consumption in the benchmark model and 2.95% of permanent consumption in the “no-reserves” economy.

<sup>24</sup>In fact, under some parameterizations, we find that in a flexible exchange rate, reserve accumulation may lead to negative welfare gains. The reason is that having the ability to accumulate reserves may end up leading to worse borrowing terms for the government because accumulating reserves exacerbates the time inconsistency problem associated with debt dilution. See Hatchondo, Martinez and Sosa-Padilla (2016) for a study on debt dilution and sovereign default risk.

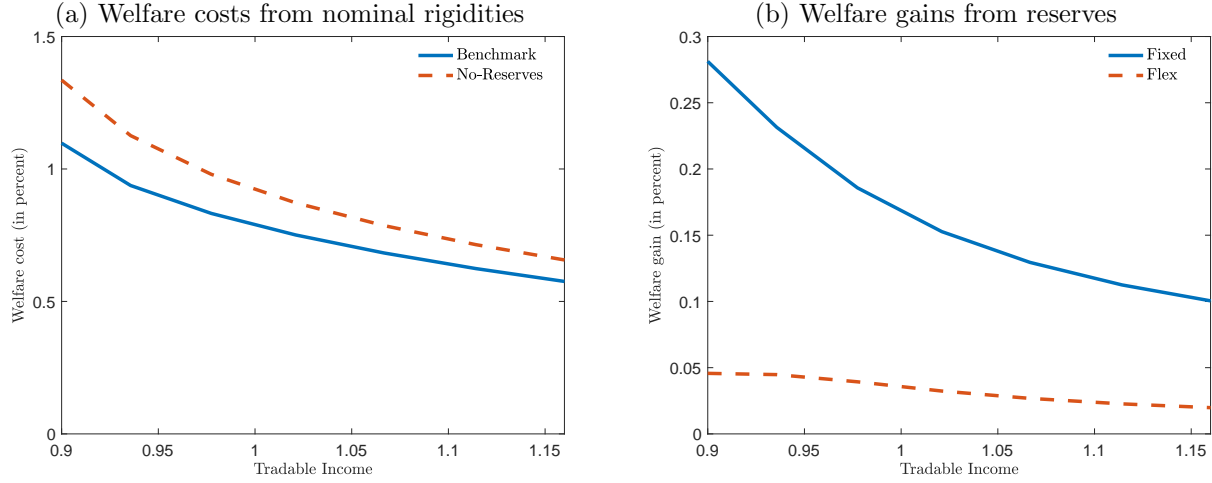


Figure 8: Welfare costs.

*Note:* The left (right) panel shows the costs of nominal rigidities (not having access to reserves). Both panels assume zero initial debt and reserves.

## 5.5 A simple reserve accumulation rule

An important discussion in policy circles is what constitutes an “adequate” amount of reserves. In fact, it is common that the IMF prescribes countries to hold a certain amount of reserve as a fraction of imports or some debt measure. Perhaps the most well-known example is the Greenspan-Guidotti rule, which prescribes that a country must hold reserves equal to 100% of its short term liabilities. In our analysis, so far, we have considered the optimal state contingent government policy. To help guide these discussions, we now explore the design of a simple and implementable rule and compare it with an implementation of the Greenspan-Guidotti rule.

Towards an operational policy, we explore the performance of reserve accumulation rules that are linear on the variables of interest in the model. We assume throughout that the government continues to optimize over debt, consumption and the default/repayment decision, subject to the reserve accumulation rule. Specifically, we explore a reserve accumulation that, under repayment, follows

$$a_{t+1} = \beta_0 + \beta_{debt} b_t + \beta_{spr} spread_t + \beta_{res} a_t + \beta_y y_t^T \quad (30)$$

Using different values for the coefficients of the rule, we can solve the model and evaluate the performance.<sup>25</sup> To compute the best rule, we first compute the welfare gains of moving from the economy without reserves to the economy in which this rule is followed for every possible initial state. Then, we maximize the increase in welfare starting from the ergodic distribution under no reserve accumulation.<sup>26</sup>

Notice that the optimization is a computationally intensive one, as there are many combina-

<sup>25</sup>We also explore rules for reserve accumulation upon default but these play a less important role.

<sup>26</sup>Appendix B has the detailed description of the equilibrium in the “rule” economy.

tions of coefficients to be explored. We proceed by first using simulated data from the model to estimate a regression using (30), and then we perform a grid search over the coefficients, centered around the estimated values. This exercise results in the following coefficients:

$$\beta_0 = 0.336, \beta_{debt} = 2.535, \beta_{spread} = -1.69, \beta_{res} = 0.422, \beta_y = 0.418. \quad (31)$$

These estimated rules imply a reserve accumulation policy that is increasing in the initial level of reserves and in tradable output, and decreasing in spreads and in initial level of debt. For example, under repayment, our estimated rules prescribe that a 1 percentage point increase in spreads, controlling for all other factors, should lead to reserves declining 1.69% of mean (tradable) output (which is roughly 0.70% of GDP).

Table 5 shows the simulated moments for the “rules” and the benchmark economies. It is quite remarkable to see that using the best rule produces simulated moments that are very similar to the ones in the benchmark economy (even though there is no re-calibration involved in this exercise). From a normative perspective, the average welfare gains amount to 0.07% of permanent consumption, starting from zero initial debt and reserves. On the other hand, the Greenspan-Guidotti rule, which prescribes that a country must hold reserves equal to 100% of its short term liabilities, performs quite poorly, featuring significant welfare losses.<sup>27</sup> Our interpretation is that the Greenspan-Guidotti rule may be too much of a straight jacket. A useful reserve accumulation rule should have some degree of state contingency.

It is also interesting to note that the preferred simple rule from our analysis yields a level of reserves that is higher than the one prescribed by Greenspan-Guidotti. That is, the precautionary motive in our theory suggest that the government should have more reserves than the ones needed to fulfill the short-term debt obligations. In other words, when borrowing conditions become adverse, the government must have (on average) liquidity not only to repay existing debt but also to have additional resources for macro-stabilization.

To explore which is the more important variable for the success of the simple rule, especially compared to the poor performance of the Greenspan-Guidotti rule, we restrict the simple rule to be invariant to some of the components. To do this we set (one at the time) the coefficients for spreads, debt, tradable income and reserves to zero and compute welfare gains of following these alternative rules versus not accumulating reserves at all. Spreads represent the most important coefficient of the rule, followed by debt. Following the rule without reacting to spreads implies an average welfare loss of  $-0.91\%$  of permanent consumption, while following a rule that does not react to debt implies a loss of  $-0.31\%$  of permanent consumption.

The main message, overall, is that a simple implementable rule for reserve accumulation may

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<sup>27</sup>Even though our model has only long-duration debt it is straightforward to compute short term liabilities as the debt payments that are due within 1 year. Since the calibration of the model is annual, these short term payments amount to  $\delta b'$ .

Table 5: Key statistics – benchmark and “rule” models.

	Benchmark	Rules	
		Best Rule	Greenspan-Guidotti
<b>Targeted</b>			
Mean debt ( $b/y$ )	44	42	19
Mean $r_s$	2.9	2.8	2.4
$\Delta r_s$ w/ risk-prem. shock	2.0	1.9	1.7
$\Delta$ UR around crises	2.0	2.0	1.8
Mean $y^T/y$	41	41	40
<b>Non-Targeted</b>			
$\sigma(c)/\sigma(y)$	1.0	1.0	1.0
$\sigma(r_s)$ (in %)	3.1	3.0	2.7
$\rho(r_s, y)$	-0.6	-0.6	-0.7
$\rho(c, y)$	1.0	1.0	1.0
Mean Reserves ( $a/y$ )	16	15	6
Mean Reserves/Debt ( $a/b$ )	35	38	31
$\rho(a/y, r_s)$	-0.4	-0.7	0.5
Reserves/S.T. liabilities	112	139	100
Welfare gain (vs. No-Reserves)	0.18	0.07	-0.22

go a long way in replicating the welfare gains of the fully optimal policy. However, rules that are too rigid, for example by not reacting to spreads, may end up being counterproductive.

## 5.6 Inflation Targeting

In this section we explore a variant to our model in which we study a different monetary policy regime. The goal is to study a regime under which the government has some degree of exchange rate flexibility, but it is unable to fully implement the equilibrium with full employment.

The government follows an inflation targeting regime, and in particular, targets a long-run aggregate price level of  $\bar{P} > 0$ , which is given by the price of the composite consumption good. Denoting by  $P^T$  and  $P^N$  the price of tradable and non-tradable goods expressed in domestic currency, we have that the price of the composite consumption good is given by

$$P(P^T, P^N) = \left( \omega^{\frac{1}{1+\mu}} (P^T)^{\frac{\mu}{1+\mu}} + (1 - \omega)^{\frac{1}{1+\mu}} (P^N)^{\frac{\mu}{1+\mu}} \right)^{\frac{1+\mu}{\mu}}.$$

Let us denote by  $\mathcal{P}^N(c^T; e)$  the equilibrium price for non-tradables when consumption is given by  $c^T$  and the exchange rate is  $e$ . Using (5), we have that

$$\mathcal{P}^N(c^T; e) = e^{\frac{1-\omega}{\omega}} \left( \frac{c^T}{\mathcal{C}(c^T, \mathcal{H}(c^T; \frac{W}{e}))} \right)^{1+\mu}.$$



Table 6: Model comparisons with inflation targeting

	Data	Model	
		Fixed Exchange Rate	Inflation Targeting
<b>Targeted</b>			
Mean debt ( $b/y$ )	45	44	51
Mean $r_s$	2.9	2.9	2.8
$\Delta r_s$ w/ risk-prem. shock	2.0	2.0	2.1
$\Delta$ UR around crises	2.0	2.0	0.5
Mean $y^T/y$	41	41	42
<b>Non-Targeted</b>			
$\sigma(c)/\sigma(y)$	1.1	1.0	1.1
$\sigma(r_s)$ (in %)	1.6	3.1	3.0
$\rho(r_s, y)$	-0.3	-0.6	-0.7
$\rho(c, y)$	0.6	1.0	1.0
Mean Reserves ( $a/y$ )	16	16	12
Mean Reserves/Debt ( $a/b$ )	35	35	23
$\rho(a/y, r_s)$	-0.4	-0.4	-0.3

Since we have normalized the price of tradables in foreign currency to one, we have, using the law of one price, that the price of tradables in domestic currency is given by  $P^T = e$ . Using this, and combining the previous two equations, we therefore have that under the inflation targeting regime the government must set  $e$  such that

$$\bar{P} = \left( \omega^{\frac{1}{1+\mu}} (e)^{\frac{\mu}{1+\mu}} + (1 - \omega)^{\frac{1}{1+\mu}} (\mathcal{P}^N(c^T; e))^{\frac{\mu}{1+\mu}} \right)^{\frac{1+\mu}{\mu}}. \quad (32)$$

We can now solve the government problem in a way analogous to the case with a fixed exchange rate (18)-(20) but allowing the government to choose  $e$  subject to (32). In this regime, the government has the ability to use monetary policy to stabilize macroeconomic fluctuations. Consider for example a shock worsening borrowing conditions. Because this shock is deflationary, the government can at least partially depreciate the exchange rate to reduce unemployment while still delivering the targeted inflation.

Table 6 presents the simulation moments for the model under inflation targeting and compares them with the moments from the baseline version.<sup>28</sup> As the third column shows, the government still accumulates about 12% of GDP in reserves under an inflation targeting regime. The key takeaway is that the macroeconomic stabilization role for international reserves is also very strong under an inflation targeting regime.

<sup>28</sup> We parameterize  $\bar{P}$  so that it is equal to the mean value observed in the simulations of the flexible-wage economy. All other parameters are kept at their benchmark calibration values (i.e. there is no recalibration involved).

## 6 Conclusions

We provide a theory of reserve accumulation based on the interaction between macroeconomic stabilization and sovereign risk. We show that governments with limited exchange rate flexibility find it optimal to accumulate a large amount of reserves to reduce the frequency and the severity of recessions. In contrast to the traditional argument based on the fiscal sustainability of a currency peg, the key channel we provide is based on a macroeconomic stabilization motive.

Our findings have important implications for international reserve management. From a positive perspective, we show that a precautionary theory of reserves coupled with macroeconomic stabilization can account for the levels of reserves observed in the data for emerging markets. On the normative side, our model can help provide guidance on how governments should manage reserves. In particular, we find that a simple linear reserve accumulation rule can go a long way towards replicating the fully optimal policy. We think that an interesting avenue for future research is to extend our analysis to study the accumulation of reserves when default can be triggered by self-fulfilling rollover crises.

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## A Data sources and variable definitions

Whenever possible we take the data from the online appendix of [Catão and Mano \(2017\)](#). We also follow them in terms of variable definitions for: debt, reserves, spreads, world interest rate, crisis years, and exchange rate regime classification. Here we provide a brief description of these variables:<sup>29</sup>

- **Debt:** we focus on “Total Government Debt.” The sources are IMF’s World Economic Outlook and World Bank’s World Development Finance databases.
- **Foreign Exchange Reserves:** as reported in IMF’s International Financial Statistics.
- **Spreads:** the main source for emerging market spreads is JP Morgan’s EMBI spreads.
- **World interest rate:** the real world interest rate is computed as a GDP weighted average of the (short-term) money market interest in all G7 countries plus Australia, deflated by CPI inflation. Money market rates, CPI and inflation and US dollar GDP data for all countries are from the IMF’s International Financial Statistics.
- **Exchange rate regime:** this is a dummy variable taking a value of 1 for countries deemed to be under a “Fix” regime and 0 otherwise. This dummy was constructed based on the IMF classification (categories “1” and “2”).
- **Crisis years:** are defined as years in which a given country experienced a “credit event.” These events are defined as all the years in between the initial default and full (or near full) settlement of arrears as per the Standard and Poor’s definition.

The variables that were not readily available from [Catão and Mano \(2017\)](#) were obtained (or constructed) as follows:

- **Exchange rate:** we use the official nominal exchange rate between a country  $i$ ’s currency and the USD from the World Bank’s World Development Indicators (WDI).
- **Tradable GDP:** we construct tradable GDP as the sum of Agriculture value added and Industry value added. Both series are expressed in constant local currency units and taken from the WDI.

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<sup>29</sup>See [Catão and Mano \(2017\)](#)’s data appendix for further details.

## B Recursive Problem in the “Rule” Economy

The government problem with access to financial markets and following the reserve accumulation rule can be formulated in recursive form as follows:

$$V_{rule}(b, a, s) = \max_{d \in \{0, 1\}} \{ (1 - d)V_{rule}^R(b, a, s) + dV_{rule}^D(a, s) \}, \quad (33)$$

where  $s \equiv \{y^T, \kappa\}$  summarizes the exogenous states and  $d \in \{0, 1\}$  is the discrete default decision, with 1 (0) representing default (repayment).  $V^R(b, a, s)$  and  $V^D(a, s)$  denote, respectively, the value of repayment and the value of default. The former is given by the Bellman equation

$$V_{rule}^R(b, a, s) = \max_{b', h \leq \bar{h}, c^T} \{ u(c^T, F(h)) + \beta \mathbb{E}_{s'|s} [V_{rule}(b', a', s')] \} \quad (34)$$

subject to

$$c^T + q_a a' + \delta b = a + y^T + q(b', a', y^T)(b' - (1 - \delta)b),$$

$$h \leq \mathcal{H}(c^T),$$

$$a' = \beta_0^R + \beta_{debt}^R b + \beta_{spr}^R spread + \beta_{res}^R a + \beta_y^R y^T$$

where  $q(b', a', s)$  denotes the bond price schedule and  $\beta^R s$  are the estimated rule coefficients under repay. The value of default is given by

$$V_{rule}^D(a, s) = \max_{c^T, h \leq \bar{h}} \{ u(c^T, F(h)) - \psi_d(y^T) + \beta \mathbb{E}_{s'|s} [V_{rule}(0, a', s')] \} \quad (35)$$

subject to

$$c^T + q_a a' = y^T + a,$$

$$h \leq \mathcal{H}(c^T),$$

$$a' = \beta_0^D + \beta_{res}^D a + \beta_y^D y^T$$

where  $\beta^D s$  are the estimated rule coefficients under default.

**Definition 3** (Markov perfect equilibrium). A Markov perfect equilibrium is defined by value functions  $\{V(b, a, s), V^R(b, a, s), V^D(a, s)\}$ , associated policy functions  $\{\hat{d}(b, a, s), \hat{b}(b, a, s), \hat{c}^T(b, a, s), \hat{h}(b, a, s), \hat{T}(b, a, s)\}$ , a bond price schedule  $q(b', a', s)$  and  $p^N(b, a, s)$  such that

1. Given the bond price schedule, policy functions solve problems (33), (34), and (35),
2. The bond price schedule satisfies the bond pricing equation,

$$q(b', a', s) = \mathbb{E}_{s'|s} \left\{ m(s', s) \left[ 1 - \hat{d}(b', a', s') \right] \left[ \delta + (1 - \delta)q(b'', a'', s') \right] \right\}, \quad (36)$$

where

$$b'' = \hat{b}(b', a', s') \quad \text{and} \quad a'' \text{ follows the rule.}$$