International Reserve Management under Rollover Crises

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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To reduce the vulnerability to a debt crisis:

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Answer unclear:

• Reserves provide liquidity, but reducing debt may be more effective

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 - Sunspot shocks, deterministic income
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 - · Borrowing to accumulate reserves helps exiting the crisis zone
- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

Model

Environment

- Discrete time, infinite horizon. Constant endowment: $y_t = y$
- Government trades two assets ...
 - short-term risk-free reserves, a
 - long-term defaultable debt, b
 - a bond issued in t promises to pay $\left(\frac{\delta+r}{1+r}\right)\left[1,\,(1-\delta),\,(1-\delta)^2,\,....\right]$
- Risk-neutral deep pocket international investors:
 - Discount future flows at 1+r, assume $\beta(1+r)=1$

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- Risk-neutral deep pocket international investors:
 - Discount future flows at 1 + r, assume $\beta(1 + r) = 1$
- Markov equilibrium w/ Cole-Kehoe (2000) timing:
 - Borrowing at the beginning of the period
 - Repay/default at the end

Government

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where $d_t = 0$ (1) denotes repayment (default)

• If the government repays:

$$c_t = y + \frac{\delta + r}{1 + r}b_t + q_t(\frac{\delta}{\delta + 1}, b_{t+1})[b_{t+1} - (1 - \delta)b_t] - \frac{\frac{\delta}{\delta + 1}}{1 + r}$$

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• If the government defaults:

$$c_t = y + a_t - \frac{a_{t+1}}{1+r}$$

and faces permanent exclusion and utility loss ϕ

Recursive Government Problem

• State is $s \equiv (a,b,\zeta)$ $\zeta \mbox{ denotes an iid sunspot that coordinates the lenders}$

• The government chooses to repay or default

$$V(\mathbf{a},b,\zeta) = \max\{V_R(\mathbf{a},b,\zeta),V_D(\mathbf{a})\}$$

If indifferent, assume repay

Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$
 subject to $c \leq y + a - rac{a'}{1+r}$

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• Given $\beta(1+r)=1$, this is

$$V_D(a) = \frac{u(y + (1 - \beta)a) - \phi}{1 - \beta}$$

Value of Repayment

$$V_R(a,b,\zeta) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',\zeta') \right\}$$

subject to

$$c = y + a - \left(\frac{\delta + r}{1 + r}\right)b - \frac{a'}{1 + r} + q(a', b', s)\left[b' - (1 - \delta)b\right]$$

Equilibrium Bond Price

$$q(a',b',s) = egin{cases} rac{1}{1+r}\mathbb{E}\left[\left(1-d(s')
ight)\left(rac{\delta+r}{1+r}+(1-\delta)q(a'',b'',s')
ight)
ight] & ext{if } d(s)=0 \ 0 & ext{if } d(s)=1 \end{cases}$$

where a''(s') and b''(s') are the future choice of reserves and debt

Multiplicity of Equilibria

- Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)
- If lenders expect...
 - ... repayment, they lend, and the government repays
 - ... default, they don't lend, and the government defaults

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Next: incentives to default depending on initial portfolio and whether investors are willing to roll over or not

Repayment value when government can rollover

$$V_R^+(a,b) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',s') \right\}$$

subject to

$$c = y + a - \left(\frac{\delta + r}{1 + r}\right)b - \frac{a'}{1 + r} + \tilde{q}(a', b')\left(b' - (1 - \delta)b\right)$$

where $\tilde{q}(a',b')$ denotes fundamental bond price

Repayment Value in a Run

$$V_R^-(a,b) = \max_{a'>0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \left(\frac{\delta + r}{1+r}\right)b$$

To pay debt, need to use reserves or cut consumption

Characterization

Safe zone, crisis zone and default zone

- Immediate: $V_R^+(a,b) \ge V_R^-(a,b)$
- When $V_R^-(a,b) < V_D(a) \le V_R^+(a,b)$, multiple equilibria

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$$\mathbf{S} = \{(a,b) : V_D(a) \le V_R^-(a,b)\},$$

$$\mathbf{D} = \{(a,b) : V_D(a) > V_R^+(a,b)\},$$

$$\mathbf{C} = \{(a,b) : V_R^-(a,b) < V_D(a) \le V_R^+(a,b)\}.$$

The Value in the Safe zone

• If $(a, b) \in S$: we assume gov. stays in safe zone

$$V^{S}(a-b) = \frac{u(y+(1-\beta)(a-b))}{1-\beta}$$

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For a high enough δ : can establish that gov. finds it optimal to stay in **S**

- If $(a, b) \in \mathbf{C}$, govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
 - Staying in the crisis zone implies eventually costly default
 - Speed of exit depends on curvature of $u(\cdot)$ and probability of bad sunspot

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Continuation value:

$$\mathbb{E}V(a',b',\zeta') = \begin{cases} V^S(a'-b') & \text{if } (a',b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a',b') + \lambda V_D(a') & \text{if } (a',b') \in \mathbf{C} \\ V_D(a') & \text{if } (a',b') \in \mathbf{D} \end{cases}$$

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How to exit: raise a or lower b?

The Crisis Zone (ctd)

Consider portfolio $(a, b) \in \mathbf{C}$. If government exits in T(a, b) as long as $\{\zeta_t\}_{t=0}^{T-1}$:

$$q(a',b') = \frac{\delta+r}{1+r} \sum_{t=1}^{T-1} \left(\frac{1-\lambda}{1+r}\right)^t (1-\delta)^{t-1} + \left[\frac{(1-\lambda)(1-\delta)}{1+r}\right]^{T-1} \frac{1}{1+r}$$

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Proposition 1 (Monotonically increasing consumption path)

Consider an initial portfolio $(a_0, b_0) \in \mathbf{C}$ such that the government exit time is T. Then, if $\zeta_t = 0$ for all $t \leq T - 1$, we have $c_{t+1} \geq c_t$ for all $t \leq T$.

Debt Thresholds

 $V^R(a,b)$ decreasing in $b \Rightarrow$ for every a, there \exists unique thresholds $b^-(a), b^+(a)$:

$$V_R^-(a, b^-(a)) = V_D(a)$$

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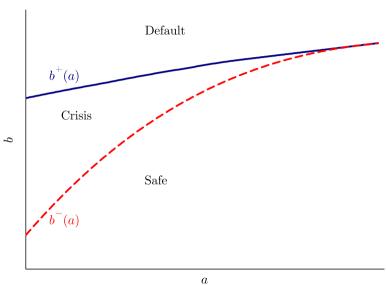
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Thresholds are such that:

- 1. $(a, b) \in \mathbf{S}$ if and only if $b \leq b^{-}(a)$
- 2. $(a, b) \in \mathbb{C}$ if and only if $b^{-}(a) < b \le b^{+}(a)$
- 3. $(a, b) \in \mathbf{D}$ if and only if $b > b^+(a)$

The Three Zones



The slopes of the two boundaries

Recall:
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 and $V_R^+(a, b^+(a)) = V_D(a)$

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Differentiating with respect to a

$$\frac{\partial b^{-}(a)}{\partial a} = \frac{\frac{\partial V_D(a)}{\partial a} - \frac{\partial V_R^{-}(a,b^{-}(a))}{\partial a}}{\frac{\partial V_R^{-}(a,b^{-}(a))}{\partial b}}$$

$$\frac{\partial b^{+}(a)}{\partial a} = \frac{\frac{\partial V_D(a)}{\partial a} - \frac{\partial V_R^{+}(a,b^{+}(a))}{\partial a}}{\frac{\partial V_R^{+}(a,b^{+}(a))}{\partial b}}$$

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Proposition 2 establishes: $\frac{\partial b^{-}(a)}{\partial a} \ge \frac{\partial b^{+}(a)}{\partial a} > 0$

$$(a^*, b^*) = \underset{a,b}{\operatorname{argmin}} a - b$$

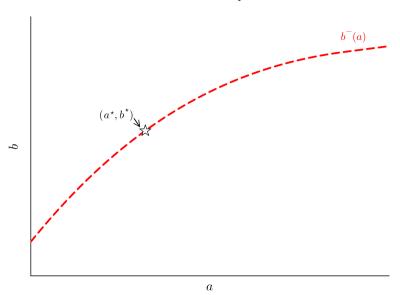
s.t. $(a, b) \in \mathbf{S}$

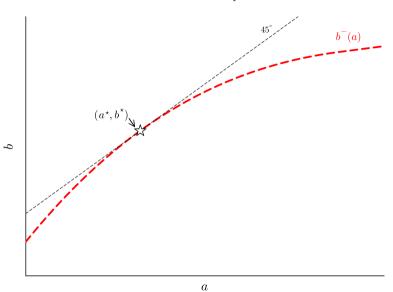
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Using that $(a, b) \in S$ if $b \le b^-(a)$ and assuming a *strictly interior solution for* a^* , we obtain:

$$\frac{\partial b^{-}(a^{\star})}{\partial a} = 1$$





 (a^{\star}, b^{\star}) is a focal point. When do we have $a^{\star} > 0$?

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Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

$$\beta(1-\delta)\left[u'\left(y-\left(\frac{\delta+r}{1+r}\right)b^{-}(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^{-}(0)\right)\right]>u'(y).$$

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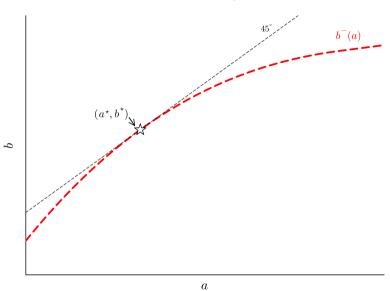
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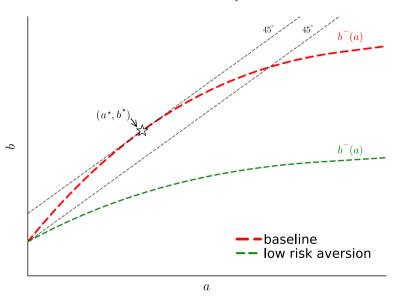
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- Proposition implies $\frac{\partial b^{-}(a)}{\partial a}\Big|_{a=0} > 1$
- ullet When does it fail? (i) low risk-aversion , (ii) one-period debt $(\delta=1)$ [Prop. 4]





Simulations: Exiting the Crisis Zone

Parametrization

$$u(c) = \frac{(c - \underline{c})^{1 - \sigma}}{1 - \sigma}$$

Parameter	Value	Description	Source
У	1	Endowment	Normalization
σ	2	Risk-aversion	Standard
r	3%	Risk-free rate	Standard
$1/\delta$	7	Maturity of debt	Italian Debt
<u>c</u>	0.70	Consumption floor	Bocola-Dovis (2019)
β	$(1+r)^{-1}$	Discount factor	$\beta(1+r)=1$
λ	0.5%	Sunspot probability	Baseline
ϕ	0.34	Default Cost	Debt-to-income =90%

Optimal Exit Strategy

Q1: How many periods until exiting?

• Inside the Crisis Zone we can define Iso-T regions

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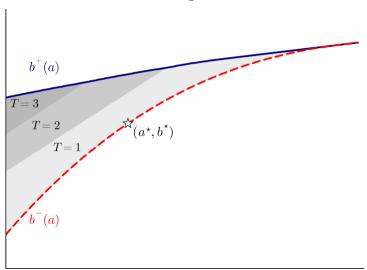
• Inside the Crisis Zone we can define Iso-T regions

Q2: What's the best strategy to exit the crisis zone?

- Should the government reduce its debt or increase reserves?
- If reserves are optimal, should gov. slowly build up its stock of reserves?

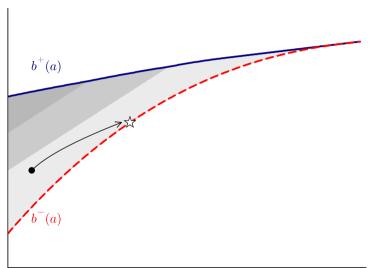
How many periods until exit





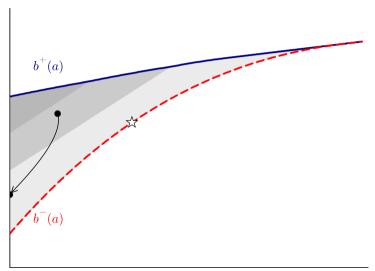
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Safety in One Period \rightarrow (a^{\star}, b^{\star})



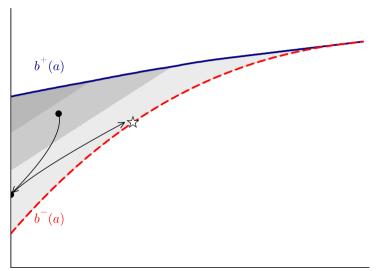
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Safety in Two Periods $o (a^\star, b^\star)$



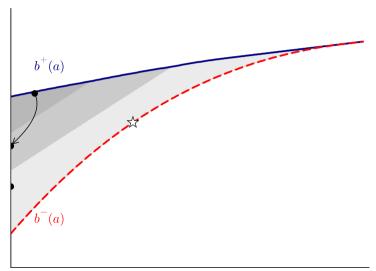
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Safety in Two Periods $\rightarrow (a^{\star}, b^{\star})$



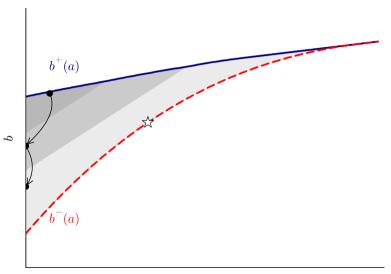
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Safety in Three Periods \rightarrow (a^*, b^*)

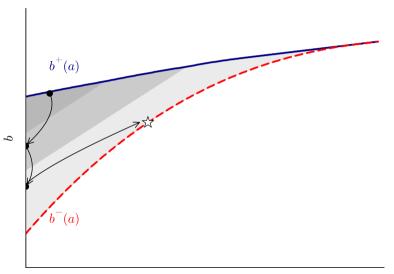


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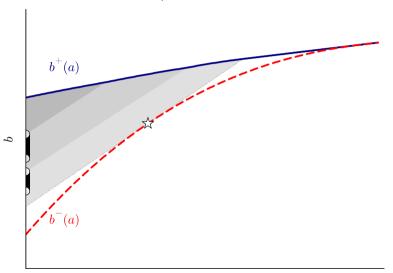
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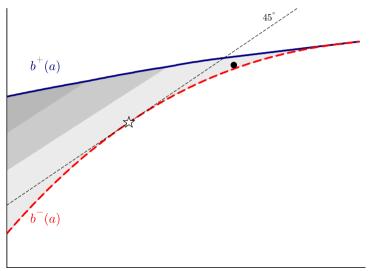
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Possible chosen portfolios for $a - b < a^* - b^*$

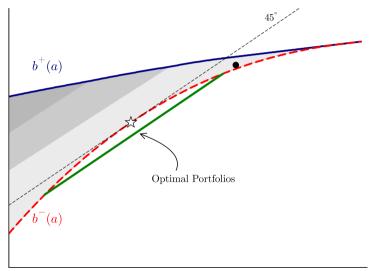


Safety in One Period \rightarrow $(a - b > a^* - b^*)$



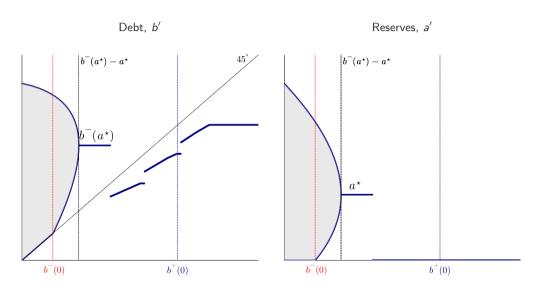
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Safety in One Period ightarrow $(a-b>a^\star-b^\star)$

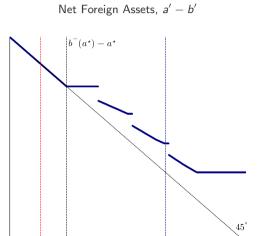


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Policies



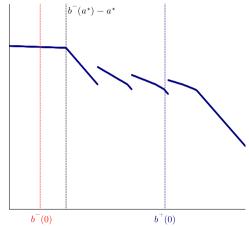
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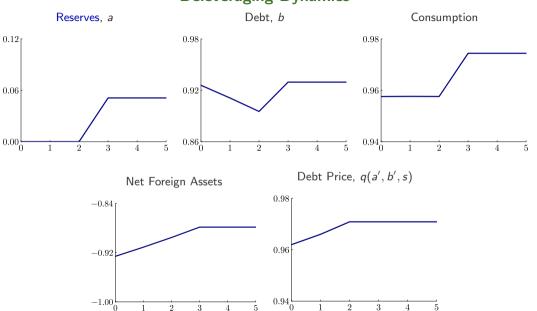
 $b^{+}(0)$

 $b^{-}(0)$

Consumption



Deleveraging Dynamics



28/34

Taking Stock

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- If initial portfolio (a, b) is such that $(a', b') \in \mathbb{C}$. Then, the optimal solution features a' = 0.

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Remark on maturity:

- With one-period debt, $\delta=1$: V_R^- and V_R^+ are unaffected by equal increases in debt and reserves \Rightarrow issuing debt to accumulate reserves increases spreads
 - Zero reserves are optimal

Experiment - How reserves help exit crisis zone

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- Either take longer to exit or cut more consumption

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<u>Without reserves:</u> $\downarrow b^+$. More costly to deleverage \Rightarrow lower debt-carrying capacity

▶ Default zone expands

Price Schedule, q(0, b') ${ ilde b}^+(0) \hspace{0.1cm} \Bigg| \hspace{0.1cm} b^+(0)$ baseline no reserves

0.95

0.90

b'

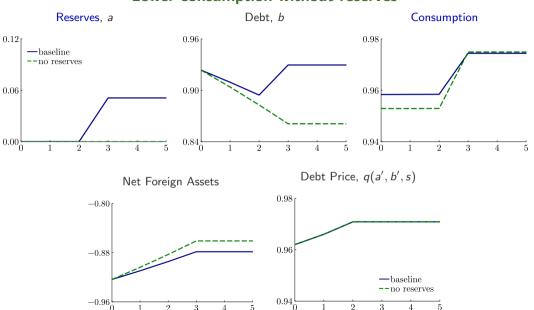
0.97

0.96

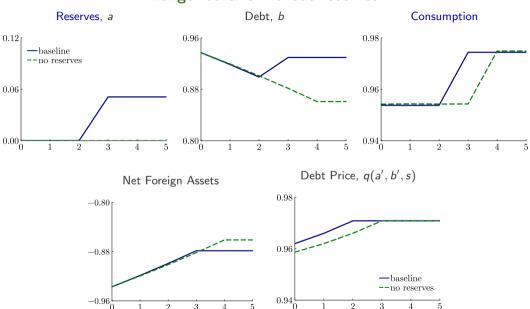
0.95

0.85

Lower consumption without reserves



Longer to exit without reserves



Conclusions

- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
 - However, only after debt has been reduced towards safe zone
- Issuing debt to accumulate reserves can reduce spreads

Conclusions

- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
 - However, only after debt has been reduced towards safe zone
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
 - Following a debt crisis, IMF often prescribes increasing reserves
 - However, we find holding reserves not optimal at beginning of deleveraging process



Scan to find the paper!





If government not vulnerable tomorrow after repaying in a run:

$$\max_{a'} u \left(y - \frac{\delta + r}{1 + r} b + a - \frac{a'}{1 + r} \right) + \beta V^{S} (a' - (1 - \delta)b))$$

- Solution: $a'(a,b) = \max[0, a \delta b]$.
 - With low initial reserves, government constrained $\Rightarrow a' = 0$

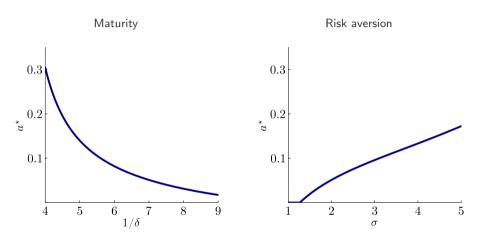


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 - With low initial reserves, government constrained $\Rightarrow a' = 0$
- If $a \ge \delta b$ and $(a \delta b, (1 \delta)b) \in \mathcal{S}$, then $V_R^-(a, b) = V_R^+(a, b)$.
 - If high reserves, govt. can achieve unconstrained consumption even in a run
 - Note reserves enough to pay all coupons not needed!
 - Just enough to repay the fraction of the debt that would allow the keep the same NFA.





Panels show the level of a^* for different values for δ and σ . The value of ϕ is recalibrated to match the same debt level $b^-(0)$ as in baseline.

Default zone expands



