International Reserve Management under Rollover Crises

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Motivation

To reduce the vulnerability to a debt crisis:

• Should the government reduce the debt or increase reserves?

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Answer unclear:

- Reserves provide liquidity
 - ... but reducing debt may lower vulnerability more

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- Tractable model of rollover crises with long-duration bonds and reserves
 - Sunspot shocks, deterministic income
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- Once debt is reduced sufficiently, optimal to increase debt and accumulate reserves

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- Borrowing to accumulate reserves can reduce spreads

Model

Environment

- Discrete time, infinite horizon. Constant endowment: $y_t = y$
- Government trades two assets ...
 - short-term risk-free reserves: a
 - long-term defaultable debt: b
 a bond issued in t promises to pay

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- Risk-neutral deep pocket international investors:
 - Discount future flows at rate r, assume $\beta(1+r)=1$
- Markov equilibrium w/ Cole-Kehoe (2000) timing:
 - Borrowing at the beginning of the period
 - Settlement (repay/default) at the end

Preferences and resource constraint

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$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where $d_t = 0$ (1) denotes repayment (default)

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• If the government repays:

$$c_t = \underbrace{y + a_t - \kappa b_t}_{\text{resources avail.}} - \underbrace{\frac{a_{t+1}}{1+r}}_{\text{reserve purchases}} + \underbrace{q_t \left[b_{t+1} - (1-\delta)b_t\right]}_{\text{debt issuance}}$$

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If the government defaults:

$$c_t = y + \frac{a_t}{1 + r}$$
 Gov. saves on bond payments

and faces permanent exclusion and utility loss ϕ

Recursive Government Problem

• State is $s \equiv (a,b,\zeta)$ ζ denotes an iid sunspot that coordinates the lenders

The government chooses to repay or default

$$V(\mathbf{a}, b, \zeta) = \max\{V_R(\mathbf{a}, b, \zeta), V_D(\mathbf{a})\}\$$

If indifferent, assume repay

Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$

subject to $c \leq y + a - \frac{a'}{1+r}$

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 subject to
$$c \leq y + a - \frac{a'}{1+r}$$

• Given $\beta(1+r)=1$ and no uncertainty, we have constant consumption

$$V_D(a) = \frac{u(y + (1 - \beta)a) - \phi}{1 - \beta}$$

Two cases, depending on whether the investors want to rollover the debt

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Case 1. Investors want to rollover:

$$V_R^+(a,b) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + \tilde{q}(a', b') (b' - (1-\delta)b)$$

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Bond price depends on the portfolio and reflects default prob:

$$ilde{q}(a',b') = rac{1}{1+r} \mathbb{E}\left[\left(1-d(s')
ight)\left(\kappa+(1-\delta)q(a'',b'',s')
ight)
ight]$$

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Case 2. Investors don't want to rollover:

$$V_{R}(a,b) = \max_{a'>0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + \tilde{q}(a',b') \left(b' - (1-\delta)b\right)$$

To pay debt, need to use reserves or cut consumption

Multiplicity of Equilibria

 Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)

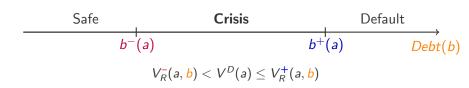
- If lenders expect...
 - ... repayment, then they rollover, and the govt repays
 - ... default, then they don't rollover, and the govt defaults

Characterization







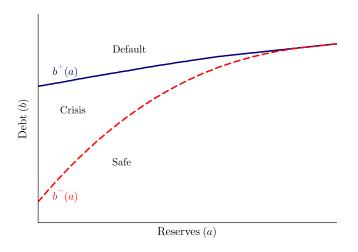


For a given level of reserves, two thresholds



Sunspot: assume government faces a run w/ prob λ when initial portfolio (a,b) is in the crisis zone

The Three Zones



Given debt: higher reserves lower vulnerability

Escaping the Crisis Zone

How to Exit the Crisis Zone?

Remaining in the crisis zone is risky:

• in case of a run, the gov't defaults

But exiting is also costly:

• requires cutting consumption and improving NFA

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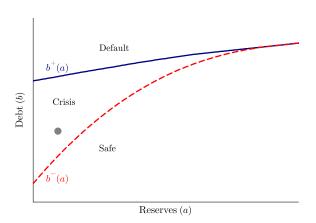
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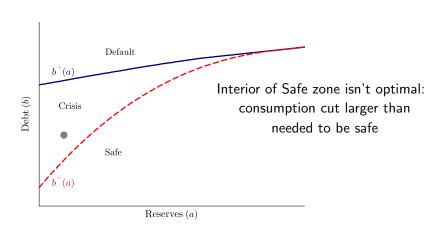
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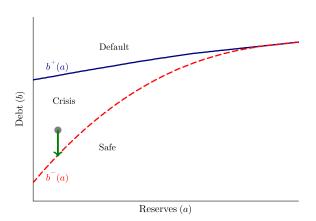
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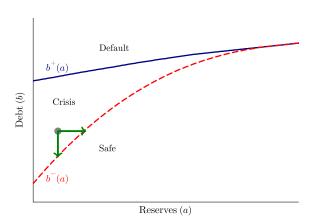
What's the best exit strategy for a country that is in the crisis zone (but didn't face a run today)?

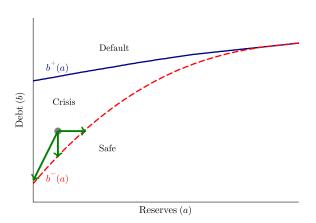
• Accumulate reserves $(a \uparrow)$ or reduce debt $(b \downarrow)$?



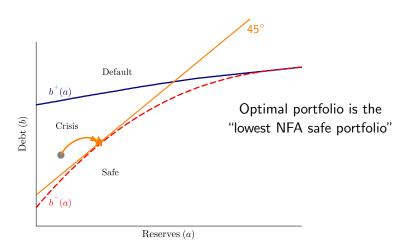




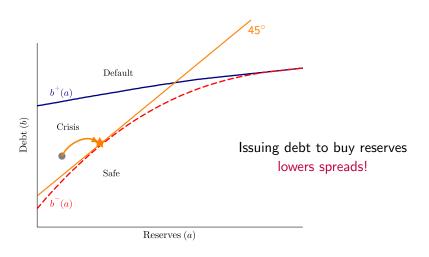




Possible Exit Paths



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Why do reserves help exit the crisis zone?

Getting to the safe zone requires $V_R^-(a,b) \ge V_D(a)$

More reserves help sustain higher gross debt & net debt
 ... even though reserves increase default value V_D(a).

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- Reserves are liquid and can be used in a run:

$$c = y + \underbrace{a - \kappa b}_{\text{more resources}} - \frac{a'}{1 + r}$$

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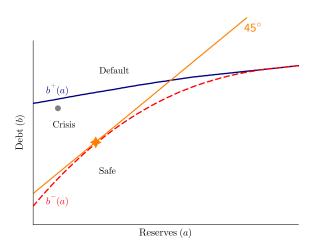
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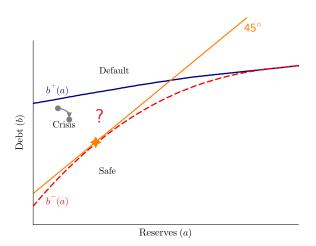
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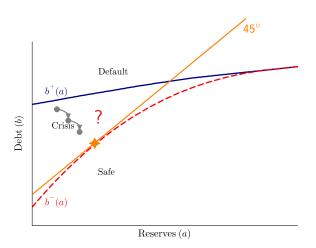
 Reserves also make default more attractive, but have lower marginal value:

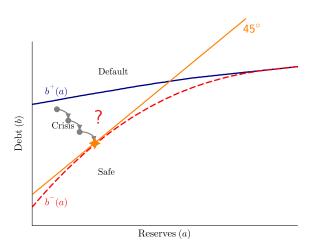
$$c_D = y + a - \frac{a'}{1+r}$$

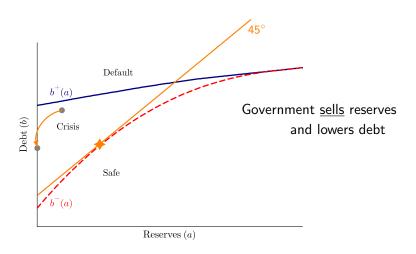
Country has higher initial debt level: what to do?

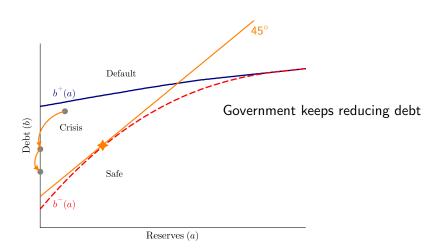


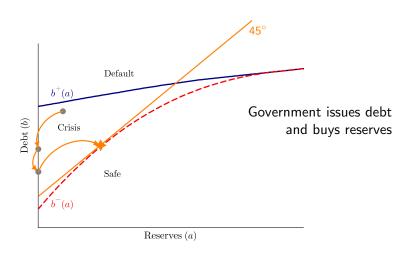










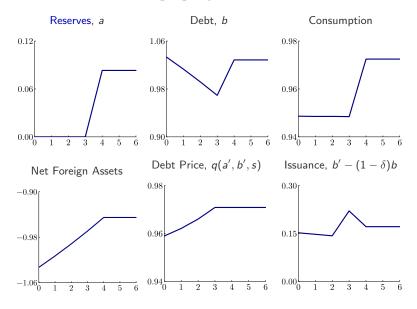


Why selling reserves (initially)?

- When the government is 'deep' in the Crisis Zone, on the margin reserves do not change the probability of a run
- Using the reserves to lower debt allows the govt to save on interest payments and helps deleveraging

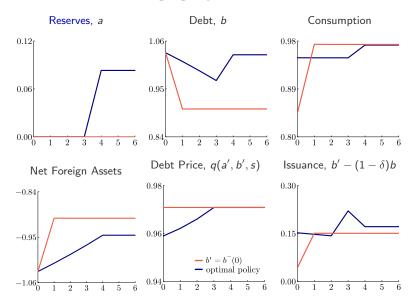
Deleveraging Dynamics





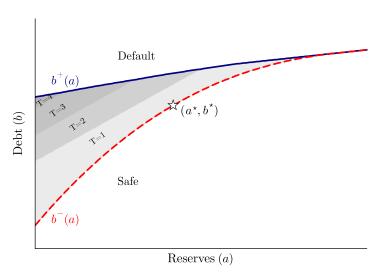
Deleveraging Dynamics





How many periods until exit?

Iso-T Regions



Formalizing the Results

Formalizing the Results: (a^*, b^*) portfolio

 (a^{\star},b^{\star}) is a focal point – we call it **Lowest-NFA safe portfolio**

Question: When do we have $a^* > 0$?

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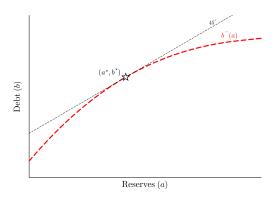
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Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^{-}(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^{-}(0)\right)\right]>u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves, $a^{\star}>0$

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- 1. low curvature in u(c),
- 2. one-period debt ($\delta = 1$) [**Prop. 4**]



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To exit crisis zone, first deleverage, then raise debt and reserves

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Proposition 5 (Optimal portfolio)

Consider an initial portfolio $(a, b) \in \mathbf{C}$. The optimal portfolio satisfies:

- If (a, b) is such that $a b < a^* b^*$ and $(a', b') \in \mathbf{S}$. Then we have T = 1 and $a' = a^*$, $b' = b^*$
- If (a, b) is such that $a b \ge a^* b^*$. Then, we have T = 1 and any portfolio $(a', b') \in \mathbf{S}$ and a b = a' b' is optimal. If $a = 0, b = b^* a^*$, then $a' = a^*, b = b^*$.
- If (a, b) is such that $(a', b') \in \mathbf{C}$. Then, the optimal solution features a' = 0.

We test two predictions of our theory:

- 1. Borrowing to save ($\uparrow b$ and $\uparrow a$ simultaneously) lowers spreads.
- 2. Once the government builds up its stock of reserves, it does not actually use it.

Theory predicts that borrowing to accum. reserves lowers spreads

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$$\Delta \log(\mathsf{Spread})_{it} = \beta_0 + \beta_{\mathsf{Res}} \, \Delta \mathsf{Reserves}_{it} + \beta_{\mathsf{Debt}} \, \Delta \mathsf{Debt}_{it} + \beta_{\mathsf{M}}' \, \mathsf{M}_{it} + \epsilon_{it}$$

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	Full Sample
Δ Reserves	-2.14***
	(0.74)
Δ Debt	0.46*
	(0.24)
Num.Obs.	4,468
R2 Adj.	0.352

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Theory also predicts stronger effect for low Debt or high NFA

Stronger effect in **low debt** periods ...

	Full Sample	Low Debt	High Debt
Δ Reserves	-2.14***	-3.72*	-1.23***
	(0.74)	(1.73)	(0.46)
Δ Debt	0.46*	1.24***	0.19
	(0.24)	(0.32)	(0.28)
Num.Obs.	4,468	2,559	1,909
R2 Adj.	0.352	0.424	0.263

All specs. include year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).

Robust standard errors in parentheses. p<0.1; p<0.05; p<0.05; p<0.01.



... also stronger effect in high NFA periods

	Full Sample	Low NFA	High NFA
Δ Reserves	-2.14***	-1.32***	-3.27**
	(0.74)	(0.51)	(1.56)
Δ Debt	0.46*	0.34	1.19**
	(0.24)	(0.25)	(0.49)
Num.Obs.	4,468	2, 226	2,242
R2 Adj.	0.352	0. 282	0.416

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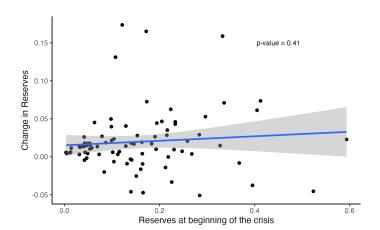
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- **Find:** no evidence that countries w/ higher reserves at beginning of crises used them more

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 - Not using them doesn't mean they're unnecessary
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
 - Following a debt crisis, IMF often prescribes increasing reserves
 - However, we find holding reserves <u>not optimal</u> at beginning of deleveraging process

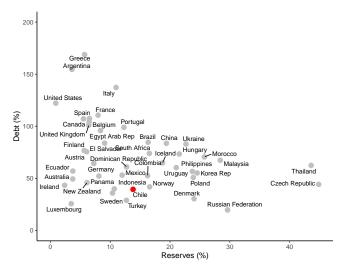


Scan to find the paper!



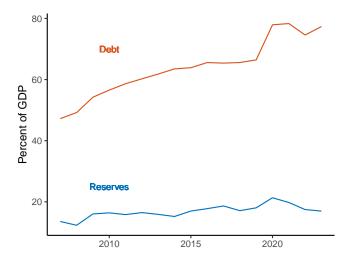
Data: Government Debt and International Reserves





Government debt and reserves (as % of GDP), 2023

Evolution of Debt and Reserves



Avg. Government debt and reserves (as % of GDP)



- Alfaro and Kanczuk (2008): no reserves with one-period debt
 - ullet Reserves make default attractive \Rightarrow worsen debt sustainability



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- Bianchi, Hatchondo and Martinez (2018): positive reserves with long-term debt under fundamental defaults
 - Reserves help avoid rolling over debt at high spreads
 - Insurance within repayment states



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- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

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Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)
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Characterization: Value in the Safe zone



• If $(a, b) \in S$: we assume gov. stays in safe zone

$$V^{S}(a-b) = \frac{u(y + (1-\beta)(a-b))}{1-\beta}$$

• **Note:** relevant state variable is the NFA, a - b

For a high enough δ : can establish that gov. finds it optimal to stay in ${\bf S}$

Characterization: Crisis zone



- If (a, b) ∈ C, govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
 - Staying in the crisis zone implies eventually costly default
 - Speed of exit depends on curvature of $u(\cdot)$ and probability of bad sunspot

Continuation value:

$$\mathbb{E}V(a',b',\zeta') = \begin{cases} V^{\mathcal{S}}(a'-b') & \text{if } (a',b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a',b') + \lambda V_D(a') & \text{if } (a',b') \in \mathbf{C} \\ V_D(a') & \text{if } (a',b') \in \mathbf{D} \end{cases}$$



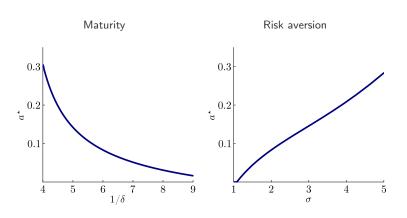
We have that for any T > 0 the bond price is given by

$$q(a',b') = \kappa \sum_{t=1}^{T-1} \left(\frac{1-\lambda}{1+r}\right)^t (1-\delta)^{t-1} + \left[\frac{(1-\lambda)(1-\delta)}{1+r}\right]^{T-1} \frac{1}{1+r}$$

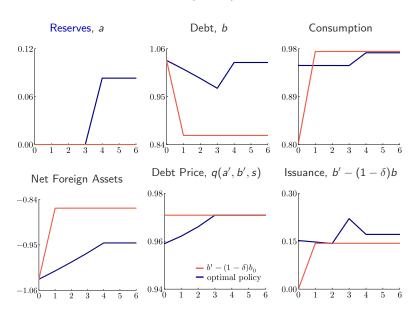
- First term: bond coupon payments investors expect to receive
- Second term: risk-free price of the bond once the government exits the crisis zone

Sensitivity: effect of maturity and risk-aversion on a^*

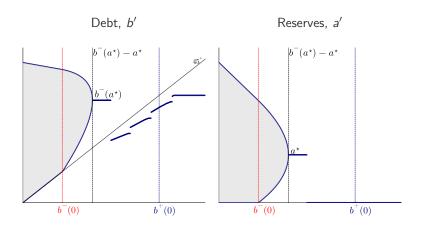




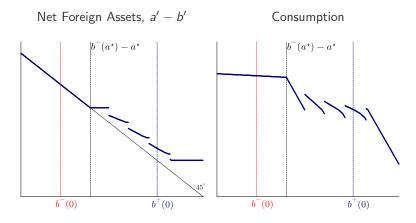






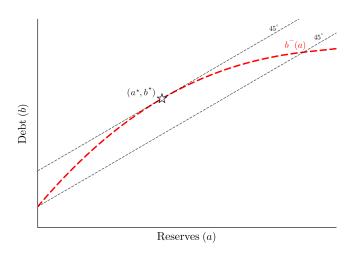






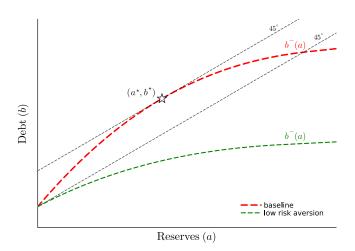
Lowest-NFA safe portfolio, (a^{\star}, b^{\star})





Lowest-NFA safe portfolio, (a^{\star}, b^{\star})





Parametrization



$$u(c) = \frac{(c - \underline{c})^{1 - \sigma}}{1 - \sigma}$$

Parameter	Value	Description	Source	
у	1	Endowment	Normalization	
σ	2	Risk-aversion	Standard	
r	3%	Risk-free rate	Standard	
$1/\delta$	6	Maturity of debt	Italian Debt	
<u>C</u>	0.68	Consumption floor	Bocola-Dovis (2019)	
β	0.97	Discount factor	$\beta(1+r)=1$	
λ	0.5%	Sunspot probability	Baseline	
ϕ	0.33	Default Cost	$Debt\text{-to\text{-}income} = \!\! 100\%$	
κ	$\frac{\delta+r}{1+r}$	Coupon	Normalization	

Experiment - How reserves help exit crisis zone

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- Exiting the crisis zone becomes more painful ⇒ (0, b⁻(0)) instead of (a*, b*)
- Exiting takes longer to exit <u>and</u> cuts more consumption

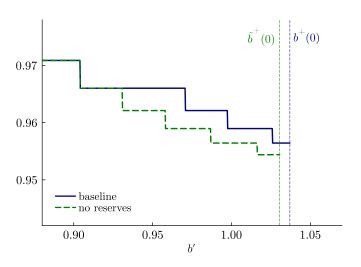
Experiment – How reserves help exit crisis zone

- Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0
- Exiting the crisis zone becomes more painful \Rightarrow $(0, b^-(0))$ instead of (a^*, b^*)
- Exiting takes longer to exit <u>and</u> cuts more consumption

<u>Without reserves:</u> $\downarrow b^+$. More costly to deleverage \Rightarrow lower debt-carrying capacity

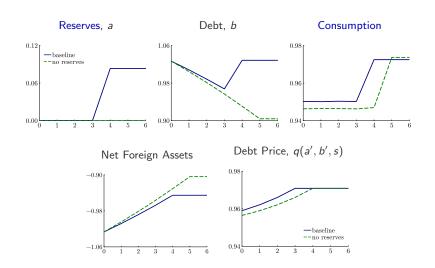
Price Schedule, q(0, b')



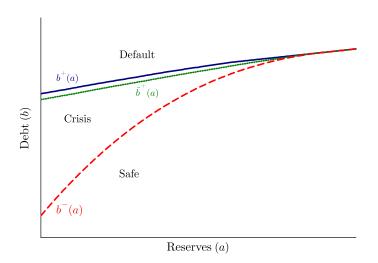


Lower consumption without reserves





Default zone expands



Increasing reserves and debt lowers spreads (levels)



Dep. Variable:	log(Spread)			
	(0)	(1)	(2)	
Reserves	-2.39***			
	(0.11)			
Sov.Debt	1.25***	-1.13***	1.58***	
	(0.10)	(0.14)	(0.20)	
NFA_public		-2.39***	-2.69 ***	
		(0.11)	(0.11)	
$(Sov.Debt)^2$			-5.48***	
			(0.31)	
Num.Obs.	4497	4497	4497	
R2	0.791	0.791	0.997	
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001				

All specs. include country FEs, year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).