International Reserve Management under Rollover Crises

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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To reduce the vulnerability to a debt crisis:

• Should the government reduce the debt or increase reserves?

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Answer unclear:

Reserves provide liquidity, but reducing debt may be more effective

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 - Sunspot shocks, deterministic income
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- Today: reserve management under rollover crisis
 - Borrowing to accumulate reserves helps exiting the crisis zone
- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

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Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)
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Model

Environment

- Discrete time, infinite horizon. Constant endowment: $y_t = y$
- Government trades two assets ...
 - short-term risk-free reserves, a
 - long-term defaultable debt, b
 a bond issued in t promises to pay

$$\kappa [1, (1 - \delta), (1 - \delta)^2,]$$

- Risk-neutral deep pocket international investors:
 - Discount future flows at rate r, assume $\beta(1+r)=1$

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- Risk-neutral deep pocket international investors:
 - Discount future flows at rate r, assume $\beta(1+r)=1$
- Markov equilibrium w/ Cole-Kehoe (2000) timing:
 - Borrowing at the beginning of the period
 - Settlement (repay/default) at the end

Government

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where $d_t = 0$ (1) denotes repayment (default)

• If the government repays:

$$c_t = y + \frac{a_t}{1 + r} - \kappa b_t + q_t [b_{t+1} - (1 - \delta)b_t]$$

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• If the government defaults:

$$c_t = y + a_t - \frac{a_{t+1}}{1+r}$$

and faces permanent exclusion and utility loss ϕ

Recursive Government Problem

• State is $s \equiv (a,b,\zeta)$ ζ denotes an iid sunspot that coordinates the lenders

The government chooses to repay or default

$$V(\mathbf{a}, b, \zeta) = \max\{V_R(\mathbf{a}, b, \zeta), V_D(\mathbf{a})\}\$$

If indifferent, assume repay

Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$
 subject to $c \leq y + a - \frac{a'}{1+r}$

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• Given $\beta(1+r)=1$, this is

$$V_D(a) = \frac{u(y + (1 - \beta)a) - \phi}{1 - \beta}$$

Value of Repayment

$$V_R(a,b,\zeta) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',\zeta') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b', s) [b' - (1-\delta)b]$$

Equilibrium Bond Price

$$q(a',b',s) = egin{cases} rac{1}{1+r} \mathbb{E}\left[(1-d(s')) \left(\kappa + (1-\delta) q(a'',b'',s')
ight)
ight] & ext{if } d(s) = 0 \ 0 & ext{if } d(s) = 1 \end{cases}$$

where $a^{\prime\prime}(s^\prime)$ and $b^{\prime\prime}(s^\prime)$ are the future choice of reserves and debt

Multiplicity of Equilibria

- Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)
- If lenders expect...
 - ... repayment, they lend, and the government repays
 - ... default, they don't lend, and the government defaults

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Next: incentives to default depending on initial portfolio and whether investors are willing to roll over or not

Repayment value when government can rollover

$$V_R^+(a,b) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + \tilde{q}(a', b') (b' - (1-\delta)b)$$

where $\tilde{q}(a',b')$ denotes fundamental bond price

Repayment Value in a Run

$$V_R^-(a,b) = \max_{a' \geq 0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$
 subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + \tilde{q}(a',b') (b' - (1-\delta)b)$$

To pay debt, need to use reserves or cut consumption

Characterization

Safe zone, crisis zone and default zone

• Immediate: $V_R^+(a,b) \ge V_R^-(a,b)$

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$$\mathbf{S} = \{(a, b) : V_D(a) \le V_R^-(a, b)\},$$

$$\mathbf{D} = \{(a, b) : V_D(a) > V_R^+(a, b)\},$$

$$\mathbf{C} = \{(a, b) : V_R^-(a, b) < V_D(a) \le V_R^+(a, b)\}.$$

The Value in the Safe zone

• If $(a, b) \in S$: we assume gov. stays in safe zone

$$V^{S}(a-b) = \frac{u(y + (1-\beta)(a-b))}{1-\beta}$$

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• **Note:** relevant state variable is the NFA, a - b

For a high enough δ : can establish that gov. finds it optimal to stay in ${\bf S}$

- If (a, b) ∈ C, govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
 - Staying in the crisis zone implies eventually costly default
 - Speed of exit depends on curvature of $u(\cdot)$ and probability of bad sunspot

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Continuation value:

$$\mathbb{E}V(a',b',\zeta') = \begin{cases} V^{\mathcal{S}}(a'-b') & \text{if } (a',b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a',b') + \lambda V_D(a') & \text{if } (a',b') \in \mathbf{C} \\ V_D(a') & \text{if } (a',b') \in \mathbf{D} \end{cases}$$

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How to exit: raise a or lower b?

The Crisis Zone (ctd)

Consider portfolio $(a, b) \in \mathbf{C}$. If government exits in T(a, b) as long as $\{\zeta_t\}_{t=0}^{T-1} = 0$:

$$q(a',b') = \kappa \sum_{t=1}^{T-1} \left(\frac{1-\lambda}{1+r}\right)^t (1-\delta)^{t-1} + \left[\frac{(1-\lambda)(1-\delta)}{1+r}\right]^{T-1} \frac{1}{1+r}$$

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Proposition 1 (Monotonically increasing consumption path)

Consider an initial portfolio $(a_0,b_0) \in \mathbf{C}$ such that the government exit time is T. Then, if $\zeta_t = 0$ for all $t \leq T - 1$, we have $c_{t+1} \geq c_t$ for all $t \leq T$.

Debt Thresholds

 $V_R(a, b)$ decreasing in $b \Rightarrow$ for every a, there \exists unique thresholds $b^-(a), b^+(a)$:

$$V_R^-(a, b^-(a)) = V_D(a)$$

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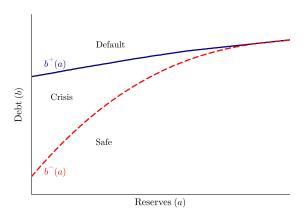
 $V_P^+(a, b^+(a)) = V_D(a)$

Thresholds are such that:

- 1. $(a, b) \in \mathbf{S}$ if and only if $b \leq b^{-}(a)$
- 2. $(a, b) \in \mathbb{C}$ if and only if $b^{-}(a) < b \le b^{+}(a)$
- 3. $(a, b) \in \mathbf{D}$ if and only if $b > b^+(a)$

▶ Prelude

The Three Zones



The slopes of the two boundaries

Recall:
$$V_R^-(a, b^-(a)) = V_D(a)$$
 and $V_R^+(a, b^+(a)) = V_D(a)$

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Differentiating with respect to a

$$\frac{\partial b^{-}(a)}{\partial a} = \frac{\frac{\partial V_D(a)}{\partial a} - \frac{\partial V_R^{-}(a,b^{-}(a))}{\partial a}}{\frac{\partial V_R^{-}(a,b^{-}(a))}{\partial b}}$$

$$\frac{\partial b^{+}(a)}{\partial a} = \frac{\frac{\partial V_D(a)}{\partial a} - \frac{\partial V_R^{+}(a,b^{+}(a))}{\partial a}}{\frac{\partial V_R^{+}(a,b^{+}(a))}{\partial b}}$$

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Proposition 2 establishes: $\frac{\partial b^{-}(a)}{\partial a} \ge \frac{\partial b^{+}(a)}{\partial a} > 0$

$$(a^*, b^*) = \underset{a,b}{\operatorname{argmin}} a - b$$

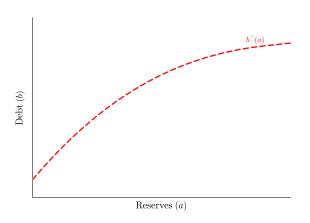
s.t. $(a, b) \in \mathbf{S}$

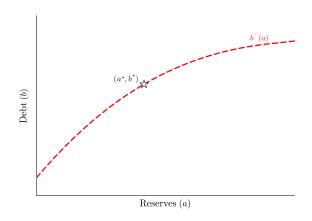
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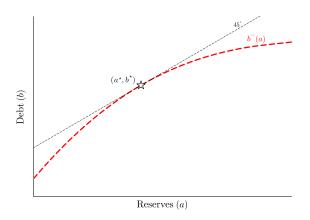
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Using that $(a, b) \in \mathbf{S}$ if $b \le b^-(a)$ and assuming a *strictly interior* solution for a^* , we obtain:

$$\frac{\partial b^{-}(a^{\star})}{\partial a} = 1$$







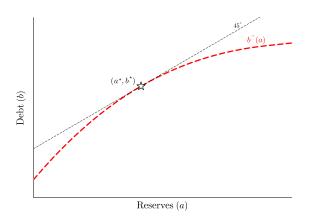
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Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves $b^-(0)$ satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^{-}(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^{-}(0)\right)\right]>u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves, $\mathbf{a}^{\star}>0$

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When does it fail?

1. low risk-aversion,

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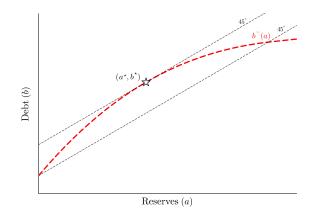
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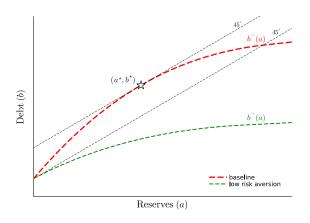
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When does it fail?

- 1. low risk-aversion,
- 2. one-period debt ($\delta=1$) [**Prop. 4**]





Simulations: Exiting the Crisis Zone

Parametrization

$$u(c) = \frac{(c - \underline{c})^{1 - \sigma}}{1 - \sigma}$$

Parameter	Value	Description	Source
у	1	Endowment	Normalization
σ	2	Risk-aversion	Standard
r	3%	Risk-free rate	Standard
$1/\delta$	6	Maturity of debt	Italian Debt
<u>C</u>	0.68	Consumption floor	Bocola-Dovis (2019)
β	0.97	Discount factor	$\beta(1+r)=1$
λ	0.5%	Sunspot probability	Baseline
ϕ	0.33	Default Cost	$Debt\text{-to\text{-}income} = \!\! 100\%$
κ	$\frac{\delta+r}{1+r}$	Coupon	Normalization

Optimal Exit Strategy

Q1: How many periods until exiting?

• Inside the Crisis Zone we can define Iso-T regions

Optimal Exit Strategy

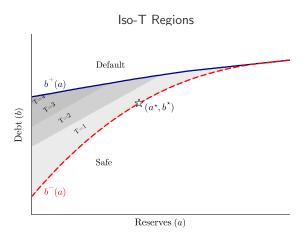
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Inside the Crisis Zone we can define Iso-T regions

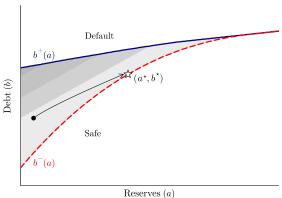
Q2: How to manage the portfolio in the transition?

- Should the government reduce its debt or increase reserves?
- If reserves are optimal, should gov. slowly build up its stock of reserves?

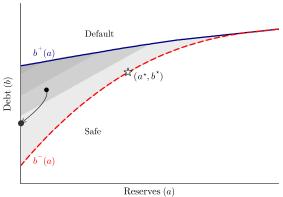
How many periods until exit



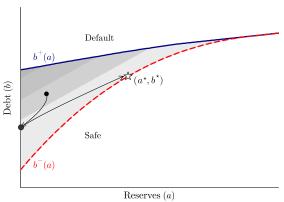




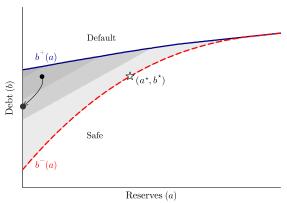




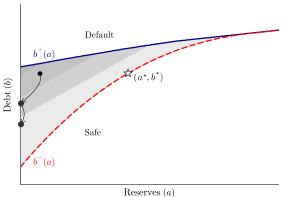




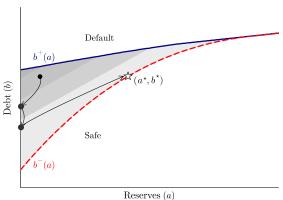




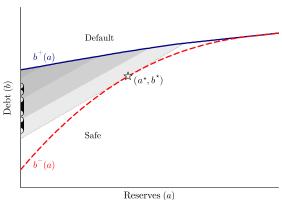


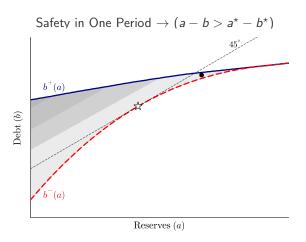


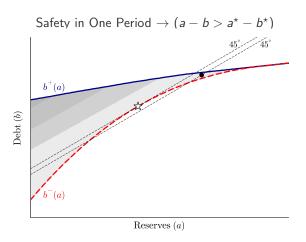


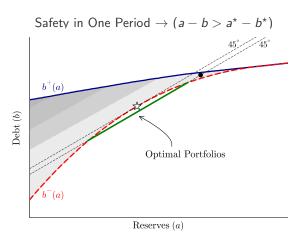


Possible chosen portfolios for $a - b < a^* - b^*$

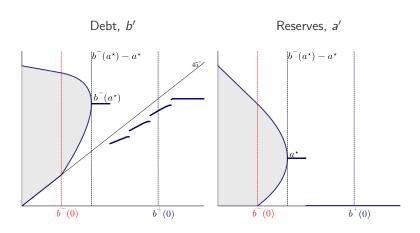




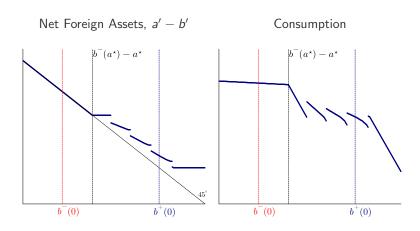


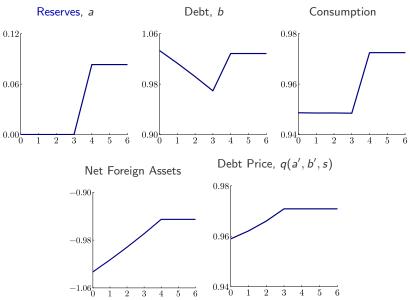


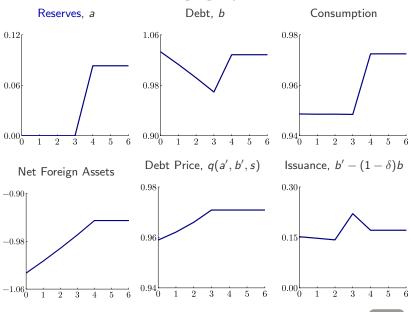
Policies



Policies







Taking Stock

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- If initial portfolio (a, b) is such that $a b < a^* b^*$ and $(a', b') \in \mathbf{S}$. Then we have $a' = a^*$, $b' = b^*$
- If initial portfolio (a, b) is such that $(a', b') \in \mathbf{C}$. Then, the optimal solution features a' = 0.

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Remark on maturity:

- With one-period debt, $\delta=1$: V_R^- and V_R^+ are unaffected by equal increases in debt and reserves \Rightarrow issuing debt to accumulate reserves increases spreads
 - Zero reserves are optimal



Experiment - How reserves help exit crisis zone

• Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0

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- Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0
- Exiting the crisis zone becomes more painful \Rightarrow $(0, b^-(0))$ instead of (a^*, b^*)
- Exiting takes longer to exit <u>and</u> cuts more consumption

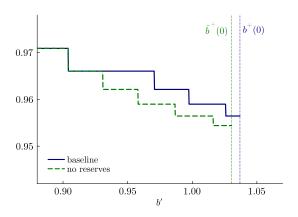
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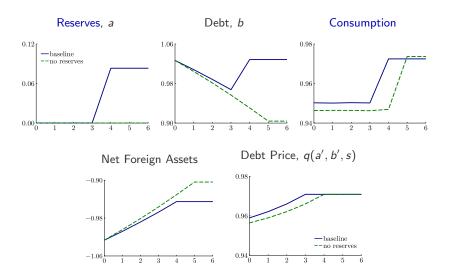
<u>Without reserves:</u> $\downarrow b^+$. More costly to deleverage \Rightarrow lower debt-carrying capacity

▶ Default zone expands

Price Schedule, q(0, b')



Lower consumption without reserves



Conclusions

- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
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- Simple theory of optimal foreign reserve management under rollover risk
- Optimal to accumulate reserves to reduce vulnerability
 - However, only after debt has been reduced towards safe zone
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
 - Following a debt crisis, IMF often prescribes increasing reserves
 - However, we find holding reserves <u>not optimal</u> at beginning of deleveraging process



Scan to find the paper!



Prelude



If government not vulnerable tomorrow after repaying in a run:

$$\max_{a'} u \left(y - +a - \frac{a'}{1+r} \right) + \beta V^{S} (a' - (1-\delta)b))$$

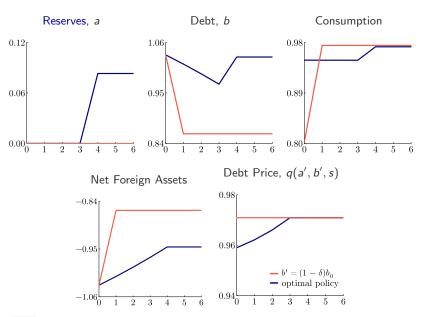
- Solution: $a'(a,b) = \max[0, a \delta b]$.
 - With low initial reserves, government constrained
 ⇒ a' = 0

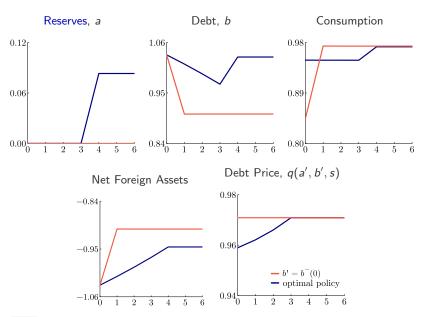


If government not vulnerable tomorrow after repaying in a run:

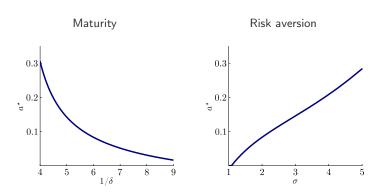
$$\max_{a'} u \left(y - +a - \frac{a'}{1+r} \right) + \beta V^{S} (a' - (1-\delta)b))$$

- Solution: $a'(a,b) = \max[0, a \delta b]$.
 - With low initial reserves, government constrained
 ⇒ a' = 0
- If $a \ge \delta b$ and $(a \delta b, (1 \delta)b) \in \mathcal{S}$, then $V_R^-(a,b) = V_R^+(a,b)$.
 - If high reserves, govt. can achieve unconstrained consumption even in a run
 - Note reserves enough to pay all coupons not needed!









Panels show the level of a^* for different values for δ and σ . The value of ϕ is recalibrated to match the same debt level $b^-(0)$ as in baseline.

Default zone expands



