# International Reserve Management under Rollover Crises

Mauricio Barbosa-Alves <sup>a</sup> Javier Bianchi <sup>b</sup> César Sosa-Padilla <sup>c</sup>

<sup>a</sup>University of Minnesota <sup>b</sup>Minneapolis Fed <sup>c</sup>University of Notre Dame & NBER

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

## Motivation

To reduce the vulnerability to a debt crisis:

• Should the government reduce the debt or increase reserves?

## **Motivation**

To reduce the vulnerability to a debt crisis:

• Should the government reduce the debt or increase reserves?

#### Answer unclear:

- Reserves provide liquidity
  - ... but reducing debt may lower vulnerability more

## What we do

- Tractable model of rollover crises with long-duration bonds and reserves
  - Sunspot shocks, deterministic income
- How should the government exit the 'crisis zone'?

## What we do

- Tractable model of rollover crises with long-duration bonds and reserves
  - Sunspot shocks, deterministic income
- How should the government exit the 'crisis zone'?

#### Preview:

- If heavily indebted, optimal to initially reduce debt and keep zero reserves
- Once debt is reduced sufficiently, optimal to increase debt and accumulate reserves

## What we do

- Tractable model of rollover crises with long-duration bonds and reserves
  - Sunspot shocks, deterministic income
- How should the government exit the 'crisis zone'?

#### Preview:

- If heavily indebted, optimal to initially reduce debt and keep zero reserves
- Once debt is reduced sufficiently, optimal to increase debt and accumulate reserves
- Borrowing to accumulate reserves can reduce spreads

# Model

#### **Environment**

- Discrete time, infinite horizon. Constant endowment:  $y_t = y$
- Government trades two assets ...
  - short-term risk-free reserves, a
  - long-term defaultable debt, b
     a bond issued in t promises to pay

$$\kappa [1, (1-\delta), (1-\delta)^2, ....]$$

## **Environment**

- Discrete time, infinite horizon. Constant endowment:  $y_t = y$
- Government trades two assets ...
  - short-term risk-free reserves, a
  - long-term defaultable debt, b
     a bond issued in t promises to pay

$$\kappa [1, (1-\delta), (1-\delta)^2, ....]$$

- Risk-neutral deep pocket international investors:
  - Discount future flows at rate r, assume  $\beta(1+r)=1$

## **Environment**

- Discrete time, infinite horizon. Constant endowment:  $y_t = y$
- Government trades two assets ...
  - short-term risk-free reserves, a
  - long-term defaultable debt, b
     a bond issued in t promises to pay

$$\kappa [1, (1-\delta), (1-\delta)^2, ....]$$

- Risk-neutral deep pocket international investors:
  - Discount future flows at rate r, assume  $\beta(1+r)=1$
- Markov equilibrium w/ Cole-Kehoe (2000) timing:
  - Borrowing at the beginning of the period
  - Settlement (repay/default) at the end

## Preferences and resource constraint

• Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where  $d_t = 0$  (1) denotes repayment (default)

#### Preferences and resource constraint

• Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where  $d_t = 0$  (1) denotes repayment (default)

• If the government repays:

$$c_t = \underbrace{y + a_t - \kappa b_t}_{\text{resources avail.}} - \underbrace{\frac{a_{t+1}}{1+r}}_{\text{reserve purchases}} + \underbrace{q_t \left[b_{t+1} - (1-\delta)b_t\right]}_{\text{debt issuance}}$$

4

## Preferences and resource constraint

Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi d_t]$$

where  $d_t = 0$  (1) denotes repayment (default)

• If the government repays:

$$c_t = \underbrace{y + a_t - \kappa b_t}_{\text{resources avail.}} - \underbrace{\frac{a_{t+1}}{1+r}}_{\text{reserve purchases}} + \underbrace{q_t \left[b_{t+1} - (1-\delta)b_t\right]}_{\text{debt issuance}}$$

If the government defaults:

$$c_t = y + \frac{a_t}{1 + r}$$
 Gov. saves on bond payments

and faces permanent exclusion and utility loss  $\phi$ 

## **Recursive Government Problem**

• State is  $s \equiv (a,b,\zeta)$   $\zeta$  denotes an iid sunspot that coordinates the lenders

The government chooses to repay or default

$$V(\mathbf{a}, b, \zeta) = \max\{V_R(\mathbf{a}, b, \zeta), V_D(\mathbf{a})\}\$$

If indifferent, assume repay

## Value of Default

$$V_D(a) = \max_{a' \geq 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$
 subject to  $c \leq y + a - rac{a'}{1+r}$ 

#### Value of Default

$$V_D(a) = \max_{a' \ge 0} \left\{ u(c) - \phi + \beta V_D(a') \right\}$$
subject to
$$c \le y + a - \frac{a'}{1+r}$$

• Given  $\beta(1+r)=1$ , we have constant consumption

$$V_D(a) = \frac{u(y + (1 - \beta)a) - \phi}{1 - \beta}$$

## Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors want to rollover:

$$V_R^+(a,b) = \max_{a' \geq 0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b') (b' - (1-\delta)b)$$

## Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors want to rollover:

$$V_R^+(a,b) = \max_{a'>0,b'} \left\{ u(c) + \beta \mathbb{E} V(a',b',s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b') (b' - (1-\delta)b)$$

Bond price depends on the portfolio and reflects default prob:

$$q(a',b') = rac{1}{1+r} \mathbb{E}\left[\left(1-d(s')
ight)\left(\kappa+(1-\delta)q(a'',b'',s')
ight)
ight]$$

## Value of Repayment

Two cases, depending on whether the investors want to rollover the debt

If investors don't want to rollover:

$$V_R^-(a,b) = \max_{a' \geq 0} \left\{ u(c) + \beta \mathbb{E} V(a', (1-\delta)b, s') \right\}$$

subject to

$$c = y + a - \frac{a'}{1+r} - \kappa b + q(a', b') (b' - (1-\delta)b)$$

To pay debt, need to use reserves or cut consumption

## Multiplicity of Equilibria

 Coordination failure may lead to self-fulfilling crises (Cole-Kehoe)

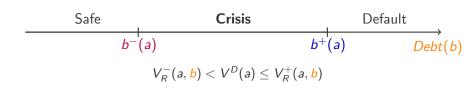
- If lenders expect...
  - ... repayment, then they rollover, and the govt repays
  - ... default, then they don't rollover, and the govt defaults

# Characterization







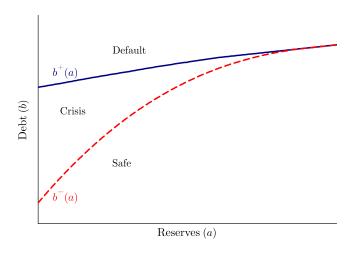


For a given level of reserves, two thresholds



Sunspot: assume government faces a run w/ prob  $\pi$  when initial portfolio (a,b) is in the crisis zone

## The Three Zones



Given debt: higher reserves lower vulnerability

# **Escaping the Crisis Zone**

## How to Exit the Crisis Zone?

Remaining in the crisis zone is risky:

• in case of a run, the gov't defaults

But exiting is also costly:

• requires cutting consumption and improving NFA

## How to Exit the Crisis Zone?

Remaining in the crisis zone is risky:

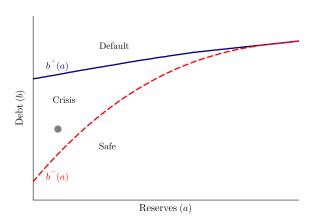
• in case of a run, the gov't defaults

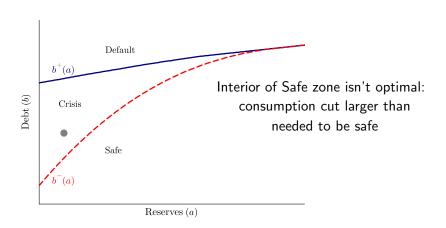
But exiting is also costly:

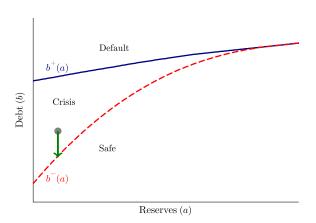
• requires cutting consumption and improving NFA

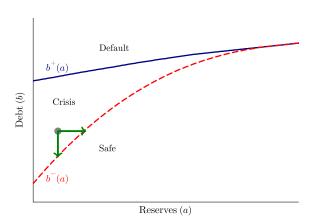
What's the best exit strategy for a country that is in the crisis zone (but didn't face a run today)?

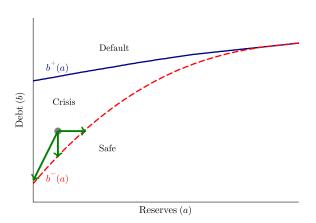
• Accumulate reserves  $(a \uparrow)$  or reduce debt  $(b \downarrow)$ ?

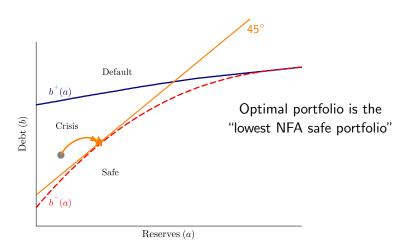




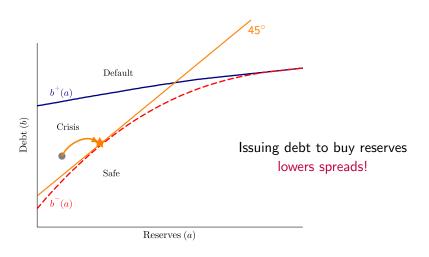








#### Possible Exit Paths



### Why do reserves help exit the crisis zone?

Getting to the safe zone requires  $V_R^-(a,b) \ge V_D(a)$ 

More reserves help sustain higher gross debt & net debt
 ... even though reserves increase default value V<sub>D</sub>(a).

### Why do reserves help exit the crisis zone?

Getting to the safe zone requires  $V_R^-(a,b) \ge V_D(a)$ 

- More reserves help sustain higher gross debt & net debt Intuition:
- ullet Only a fraction  $\kappa$  of debt is due every period
- Reserves are liquid and can be used in a run:

$$c = y + \underbrace{a - \kappa b}_{\text{more resources}} - \frac{a'}{1 + r}$$

### Why do reserves help exit the crisis zone?

## Getting to the safe zone requires $V_R^-(a,b) \ge V_D(a)$

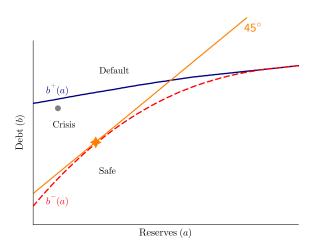
- More reserves help sustain higher gross debt & net debt Intuition:
- ullet Only a fraction  $\kappa$  of debt is due every period
- Reserves are liquid and can be used in a run:

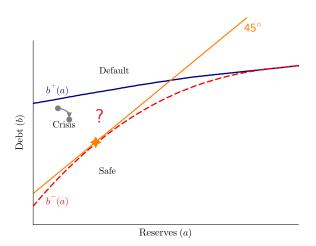
$$c = y + \underbrace{a - \kappa b}_{\text{more resources}} - \frac{a'}{1 + r}$$

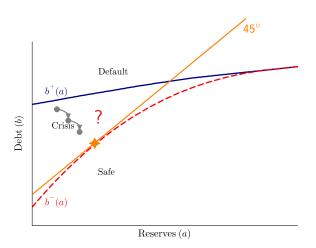
 Reserves also make default more attractive, but have lower marginal value:

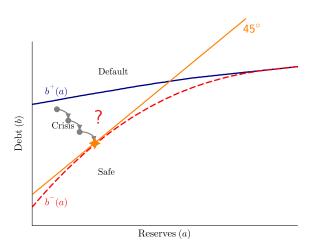
$$c_D = y + a - \frac{a'}{1+r}$$

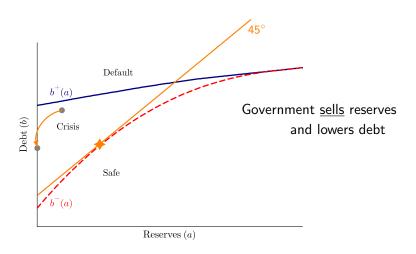
Country has higher initial debt level: what to do?

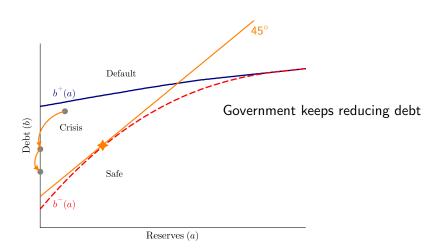


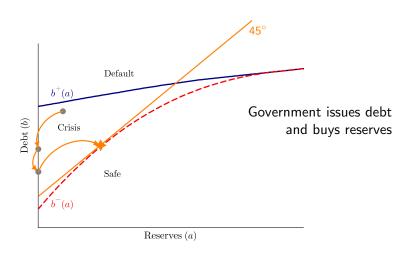










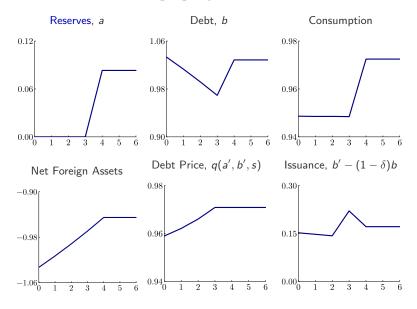


# Why selling reserves (initially)?

- When the government is 'deep' in the Crisis Zone, on the margin reserves do not change the probability of a run
- Using the reserves to lower debt allows the govt to save on interest payments and helps deleveraging

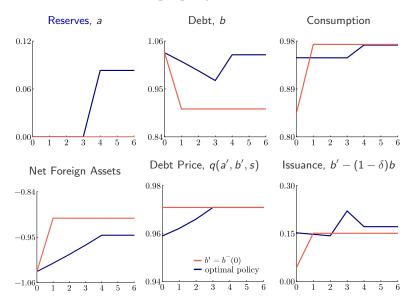
## **Deleveraging Dynamics**





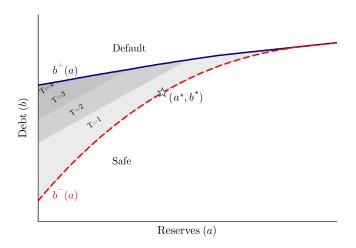
### **Deleveraging Dynamics**





# How many periods until exit?

Iso-T Regions



# Formalizing the Results

## Formalizing the Results: $(a^*, b^*)$ portfolio

 $(a^*,b^*)$  is a focal point – we call it **Lowest-NFA safe portfolio** When do we have  $a^*>0$ ?

# Formalizing the Results: $(a^*, b^*)$ portfolio

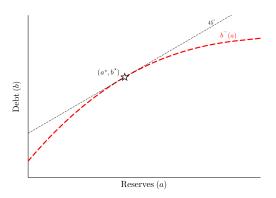
 $(a^*, b^*)$  is a focal point – we call it **Lowest-NFA safe portfolio** When do we have  $a^* > 0$ ?

**Answer:** when 
$$\left. \frac{\partial b^-(a)}{\partial a} \right|_{a=0} > 1$$

## Formalizing the Results: $(a^*, b^*)$ portfolio

 $(a^*, b^*)$  is a focal point – we call it **Lowest-NFA safe portfolio** When do we have  $a^* > 0$ ?

**Answer:** when  $\left. \frac{\partial b^-(a)}{\partial a} \right|_{a=0} > 1$ 



## Lowest-NFA safe portfolio, $(a^*, b^*)$

 $(a^*, b^*)$  is a focal point. When do we have  $a^* > 0$ ?

**Answer:** when  $\left. \frac{\partial b^-(a)}{\partial a} \right|_{a=0} > 1$ 

#### Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves  $b^-(0)$  satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^{-}(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^{-}(0)\right)\right]>u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves,  $\mathbf{a}^{\star}>0$ 

# Lowest-NFA safe portfolio, $(a^*, b^*)$

 $(a^*, b^*)$  is a focal point. When do we have  $a^* > 0$ ?

**Answer:** when  $\left. \frac{\partial b^-(a)}{\partial a} \right|_{a=0} > 1$ 

#### Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves  $b^-(0)$  satisfies

$$\beta(1-\delta)\left[u'\left(y-\kappa b^{-}(0)\right)-u'\left(y-(1-\beta)(1-\delta)b^{-}(0)\right)\right]>u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves,  $a^{\star}>0$ 

#### When does it fail?

1. low curvature in u(c),

## Lowest-NFA safe portfolio, $(a^*, b^*)$

 $(a^*, b^*)$  is a focal point. When do we have  $a^* > 0$ ?

**Answer:** when  $\frac{\partial b^{-}(a)}{\partial a}\Big|_{a=0} > 1$ 

### Proposition 3 (Positive reserves)

Suppose that the boundary of the crisis region at zero reserves  $b^-(0)$  satisfies

$$eta(1-\delta)\left[u'\left(y-\kappa b^{-}(0)
ight)-u'\left(y-(1-eta)(1-\delta)b^{-}(0)
ight)
ight]>u'(y)$$

Then, the lowest-NFA safe portfolio has strictly positive reserves,  $a^{\star}>0$ 

When does it fail?

- 1. low curvature in u(c),
- 2. one-period debt ( $\delta = 1$ ) [**Prop. 4**]



# Formalizing the Results: Optimal portfolio

To exit crisis zone, first deleverage, then raise debt and reserves

### Formalizing the Results: Optimal portfolio

To exit crisis zone, first deleverage, then raise debt and reserves

#### Proposition 5 (Optimal portfolio)

Consider an initial portfolio  $(a, b) \in \mathbf{C}$ . The optimal portfolio satisfies:

- If (a, b) is such that  $a b < a^* b^*$  and  $(a', b') \in \mathbf{S}$ . Then we have T = 1 and  $a' = a^*$ ,  $b' = b^*$
- If (a, b) is such that  $a b \ge a^* b^*$ . Then, we have T = 1 and any portfolio  $(a', b') \in \mathbf{S}$  and a b = a' b' is optimal. If  $a = 0, b = b^* a^*$ , then  $a' = a^*, b = b^*$ .
- If (a, b) is such that  $(a', b') \in \mathbf{C}$ . Then, the optimal solution features a' = 0.

Theory predicts that borrowing to accum. reserves lowers spreads

Theory predicts that borrowing to accum. reserves lowers spreads

$$\Delta \log(\mathsf{Spread})_{it} = \Delta \mathsf{Reserves}_{it} + \Delta \mathsf{Debt}_{it} + \mathsf{Controls}_{it} + \varepsilon_{it}$$

Theory predicts that borrowing to accum. reserves lowers spreads

$$\Delta \log(\mathsf{Spread})_{it} = \Delta \mathsf{Reserves}_{it} + \Delta \mathsf{Debt}_{it} + \mathsf{Controls}_{it} + \varepsilon_{it}$$

	Full Sample
Δ Reserves	-2.14***
	(0.74)
$\Delta$ Debt	0.46*
	(0.24)
Num.Obs.	4,468
R2 Adj.	0.352

Theory predicts that borrowing to accum. reserves lowers spreads

$$\Delta \log(\mathsf{Spread})_{it} = \Delta \mathsf{Reserves}_{it} + \Delta \mathsf{Debt}_{it} + \mathsf{Controls}_{it} + \varepsilon_{it}$$

	Full Sample
Δ Reserves	-2.14***
	(0.74)
$\Delta$ Debt	0.46*
	(0.24)
Num.Obs.	4,468
R2 Adj.	0.352

Theory also predicts stronger effect for low Debt or high NFA

Stronger effect in **low debt** periods ...

	Full Sample	Low Debt	High Debt
Δ Reserves	-2.14***	-3.72*	-1.23***
	(0.74)	(1.73)	(0.46)
$\Delta$ Debt	0.46*	1.24***	0.19
	(0.24)	(0.32)	(0.28)
Num.Obs.	4,468	2,559	1,909
R2 Adj.	0.352	0.424	0.263

All specs. include year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).

Robust standard errors in parentheses.  $^*p<0.1;$   $^{**}p<0.05;$   $^{***}p<0.01.$ 



#### ... also stronger effect in high NFA periods

	Full Sample	Low NFA	High NFA
Δ Reserves	-2.14***	-1.32***	-3.27**
	(0.74)	(0.51)	(1.56)
$\Delta$ Debt	0.46*	0.34	1.19**
	(0.24)	(0.25)	(0.49)
Num.Obs.	4,468	2, 226	2,242
R2 Adj.	0.352	0. 282	0.416

All specs. include year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).

Robust standard errors in parentheses. p<0.1; p<0.05; p<0.05; p<0.01.

- Simple theory of optimal res. management w/ rollover crises
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safety

- Simple theory of optimal res. management w/ rollover crises
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safety
- Reserves as 'buffer': after buildup, no use of reserves in eqm.
  - Not using them doesn't mean they're unnecessary

- Simple theory of optimal res. management w/ rollover crises
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safety
- Reserves as 'buffer': after buildup, no use of reserves in eqm.
  - Not using them doesn't mean they're unnecessary
- Issuing debt to accumulate reserves can reduce spreads

- Simple theory of optimal res. management w/ rollover crises
- Optimal to accumulate reserves to reduce vulnerability
  - However, only after debt has been reduced towards safety
- Reserves as 'buffer': after buildup, no use of reserves in eqm.
  - Not using them doesn't mean they're unnecessary
- Issuing debt to accumulate reserves can reduce spreads
- Findings speak to policy discussions on appropriate level of FX reserves (e.g. IMF)
  - Following a debt crisis, IMF often prescribes increasing reserves
  - However, we find holding reserves <u>not optimal</u> at beginning of deleveraging process

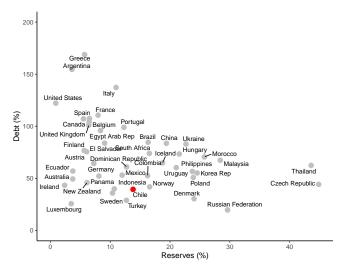


Scan to find the paper!



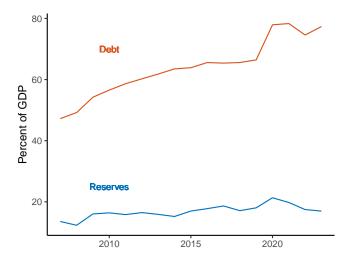
### **Data: Government Debt and International Reserves**





Government debt and reserves (as % of GDP), 2023

### **Evolution of Debt and Reserves**



Avg. Government debt and reserves (as % of GDP)



- Alfaro and Kanczuk (2008): no reserves with one-period debt
  - ullet Reserves make default attractive  $\Rightarrow$  worsen debt sustainability



- Alfaro and Kanczuk (2008): no reserves with one-period debt
  - Reserves make default attractive ⇒ worsen debt sustainability
- Bianchi, Hatchondo and Martinez (2018): positive reserves with long-term debt under fundamental defaults
  - Reserves help avoid rolling over debt at high spreads
  - Insurance within repayment states



- Alfaro and Kanczuk (2008): no reserves with one-period debt
  - Reserves make default attractive ⇒ worsen debt sustainability
- Bianchi, Hatchondo and Martinez (2018): positive reserves with long-term debt under fundamental defaults
  - Reserves help avoid rolling over debt at high spreads
  - Insurance within repayment states
- Today: reserve management under rollover crisis
  - Borrowing to accumulate reserves helps exiting the crisis zone



- Alfaro and Kanczuk (2008): no reserves with one-period debt
  - Reserves make default attractive ⇒ worsen debt sustainability
- Bianchi, Hatchondo and Martinez (2018): positive reserves with long-term debt under fundamental defaults
  - Reserves help avoid rolling over debt at high spreads
  - Insurance within repayment states
- Today: reserve management under rollover crisis
  - Borrowing to accumulate reserves helps exiting the crisis zone
- Hernandez (2019): numerical simulations w/ fundamental and sunspot shocks

```
Cole-Kehoe (2001); Corsetti-Dedola (2016); Aguiar-Amador (2020); Bianchi-Mondragon (2022); Bianchi and Sosa-Padilla (2023); Corsetti-Maeng (2023ab)
```

### Characterization: Value in the Safe zone



• If  $(a, b) \in S$ : we assume gov. stays in safe zone

$$V^{S}(a-b) = \frac{u(y + (1-\beta)(a-b))}{1-\beta}$$

• **Note:** relevant state variable is the NFA, a - b

For a high enough  $\delta$ : can establish that gov. finds it optimal to stay in  ${\bf S}$ 

### Characterization: Crisis zone



- If  $(a, b) \in \mathbf{C}$ , govt. seeks to exit in finite time (may default along the way if bad sunspot hits)
  - Staying in the crisis zone implies eventually costly default
  - Speed of exit depends on curvature of  $u(\cdot)$  and probability of bad sunspot

#### Continuation value:

$$\mathbb{E}V(a',b',\zeta') = \begin{cases} V^{\mathcal{S}}(a'-b') & \text{if } (a',b') \in \mathbf{S} \\ (1-\lambda)V_R^+(a',b') + \lambda V_D(a') & \text{if } (a',b') \in \mathbf{C} \\ V_D(a') & \text{if } (a',b') \in \mathbf{D} \end{cases}$$



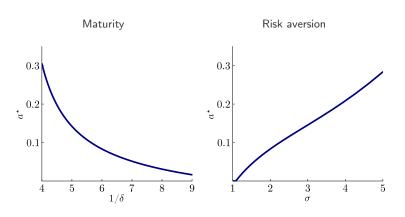
We have that for any T > 0 the bond price is given by

$$q(a',b') = \frac{\delta+r}{1+r} \sum_{t=1}^{T-1} \left(\frac{1-\lambda}{1+r}\right)^t (1-\delta)^{t-1} + \left[\frac{(1-\lambda)(1-\delta)}{1+r}\right]^{T-1} \frac{1}{1+r}$$

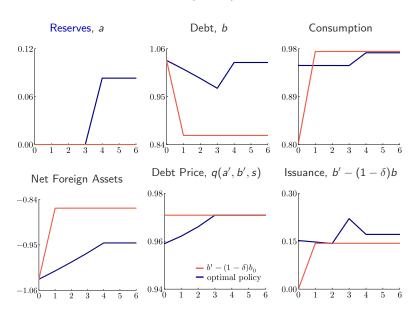
- First term: bond coupon payments investors expect to receive
- Second term: risk-free price of the bond once the government exits the crisis zone

## Sensitivity: effect of maturity and risk-aversion on $a^*$

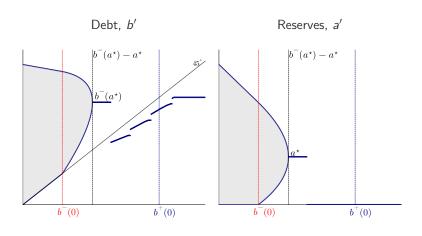




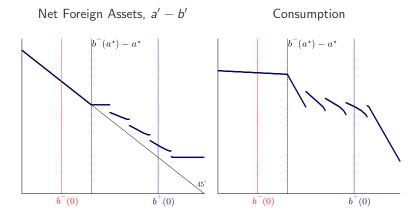






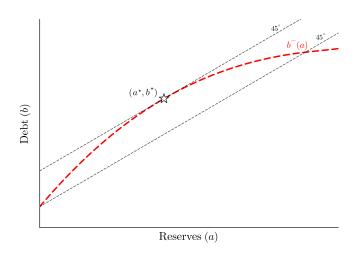






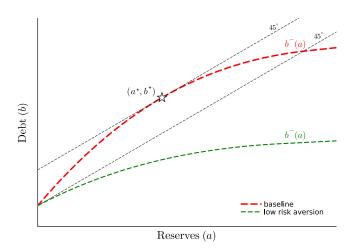
# Lowest-NFA safe portfolio, $(a^*, b^*)$





# Lowest-NFA safe portfolio, $(a^{\star}, b^{\star})$





### **Parametrization**



$$u(c) = \frac{(c - \underline{c})^{1 - \sigma}}{1 - \sigma}$$

Parameter	Value	Description	Source	
у	1	Endowment	Normalization	
$\sigma$	2	Risk-aversion	Standard	
r	3%	Risk-free rate	Standard	
$1/\delta$	6	Maturity of debt	Italian Debt	
<u>C</u>	0.68	Consumption floor	Bocola-Dovis (2019)	
$\beta$	0.97	Discount factor	$\beta(1+r)=1$	
$\lambda$	0.5%	Sunspot probability	Baseline	
$\phi$	0.33	Default Cost	$Debt\text{-to\text{-}income} = \!\! 100\%$	
$\kappa$	$\frac{\delta+r}{1+r}$	Coupon	Normalization	

### Experiment - How reserves help exit crisis zone

• Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0

## Experiment – How reserves help exit crisis zone

- Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0
- Exiting the crisis zone becomes more painful ⇒ (0, b<sup>-</sup>(0)) instead of (a\*, b\*)
- Exiting takes longer to exit <u>and</u> cuts more consumption

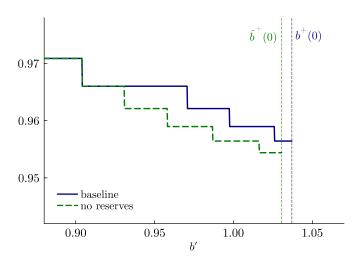
## Experiment – How reserves help exit crisis zone

- Assume gov. starts w/ portfolio (a, b), **but** from t+1 onward, a' = 0
- Exiting the crisis zone becomes more painful ⇒ (0, b<sup>-</sup>(0)) instead of (a\*, b\*)
- Exiting takes longer to exit <u>and</u> cuts more consumption

<u>Without reserves:</u>  $\downarrow b^+$ . More costly to deleverage  $\Rightarrow$  lower debt-carrying capacity

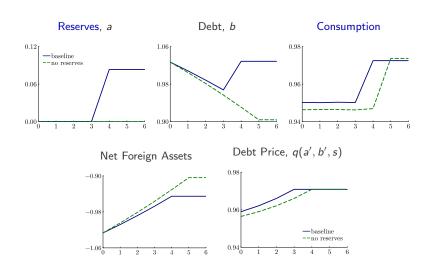
# Price Schedule, q(0, b')



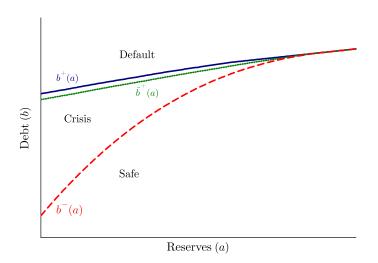


## Lower consumption without reserves





## **Default zone expands**



## Increasing reserves and debt lowers spreads (levels)



Dep. Variable:	log(Spread)			
	(0)	(1)	(2)	
Reserves	-2.39***			
	(0.11)			
Sov.Debt	1.25***	-1.13***	1.58***	
	(0.10)	(0.14)	(0.20)	
$NFA_public$		-2.39***	-2.69 ***	
		(0.11)	(0.11)	
$(Sov.Debt)^2$			-5.48***	
			(0.31)	
Num.Obs.	4497	4497	4497	
R2	0.791	0.791	0.997	
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001				

All specs. include country FEs, year dummies and additional macro controls (as in Sosa-Padilla and Sturzenegger, 2023).