№4

$\xi \sim R(\theta, 2\theta)$

$F(x,\theta) = \frac{x}{\theta} - 1 \; \{(\theta; 2\theta)\}$, $p(x,\theta) = \frac{1}{\theta}\{(\theta, 2\theta)\}$

- $M\xi = \frac{3}{2}\theta$
- $D\xi = \frac{\theta^2}{12}$

Выборка $\vec{X_n}$

ⓐ ОММ: $\alpha_1 = M\xi = \int\limits_{\theta}^{2\theta} \frac{x}{\theta} dx = \frac{x^2}{2\theta}\Big|_{\theta}^{2\theta} = \frac{3}{2}\theta$

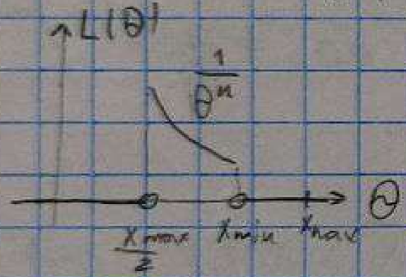
$\tilde{\alpha}_1 = \overline{X}$

$\frac{3}{2}\theta = \overline{X} \Rightarrow \tilde{\theta}_1 = \frac{2}{3}\overline{X} = \frac{2}{3n}\sum\limits_{i=1}^{n} x_i$

ОМП: $L(\theta) = \prod\limits_{i=1}^{n} p(x_i, \theta) = \left(\frac{1}{\theta}\right)^n = \theta^{-n} \{(\theta \le x_i \le 2\theta)\} \to \max$

$\left(\begin{array}{l} X_{max} \le 2\theta \\ X_{min} \ge \theta \end{array} \right)$

$\frac{1}{2} X_{max} < \theta < X_{min}$



$\Rightarrow \tilde{\theta}_2 = \frac{1}{2} X_{max}$

ⓑ • несмещ.

$- M[\tilde{\theta}_1] = M[\frac{2}{3}\overline{X}] = \frac{2n}{3n}\sum M\xi = \frac{2}{3}\cdot\frac{3}{2}\theta = \theta$ – несмещ $\tilde{\theta}_1$

$- M[\tilde{\theta}_2] = M[\frac{1}{2}X_{max}] = \frac{1}{2}\int\limits_{\theta}^{2\theta} x\, n\left(\frac{x}{\theta} - 1\right)^{n-1} \frac{1}{\theta} dx = \frac{2n+1}{2n+2}\theta$ – смещ $\tilde{\theta}_2$

$\{ \varphi_{max}(z) = n(F(z))^{n-1} \cdot F'(z) \}$

$\tilde{\theta}_2^* = \frac{n+1}{2n+1} X_{max}$

• сост.

$- D[\tilde{\theta}_1] = D[\frac{2}{3}\overline{X}] = \frac{4}{9n^2}\sum D\xi = \frac{4}{9n^2}\cdot\frac{n\theta^2}{12} = \frac{\theta^2}{27n} \xrightarrow[n\to\infty]{} 0$ – сост. (по дост усл)

$- D[\tilde{\theta}_2^*] = \left(\frac{n+1}{2n+1}\right)^2 D X_{max} = \left(\frac{n+1}{2n+1}\right)^2 [M X^2_{max} - M^2 X_{max}] =$

$$=\left(\frac{n+1}{2n+1}\right)^2\left(\int\limits_{\theta}^{2\theta} x^2 n\left(\frac{x}{\theta}-1\right)^{n-1}\frac{1}{\theta}dx-\left(\frac{2n+1}{n+1}\theta\right)^2\right)=$$

$$=\left(\frac{n+1}{2n+1}\right)^2\left(\frac{4n^2+8n+2}{n^2+3n+2}-\frac{4n^2+1+4n}{n^2+2n+1}\right)\theta^2=$$

$$=\frac{n\theta^2}{4n^3+12n^2+9n+2}=\frac{n\theta^2}{(2n+1)^2(n+2)}\xrightarrow{n\to\infty}0 \text{ — сост.}$$

c) $$D\tilde{\theta}_1=\frac{\theta^2}{27n}\overset{n\to\infty}{\sim}\frac{1}{27n}$$
$$D\tilde{\theta}_2^*=\frac{n\theta^2}{(2n+1)^2(n+2)}\overset{n\to\infty}{\sim}\frac{1}{4n^2}$$
$$\Rightarrow \tilde{\theta}_1 \text{ — наиб. эф.}$$

d) $f(\theta,\vec{x}_n)\sim \underset{\text{з-н распр.}}{g(t)}$ — изв

$$\frac{X_{max}}{\theta}=\eta\sim\underset{\text{ф распр.}}{\hat{F}(y)}$$

$$\hat{F}(y)=P(X_{max}<\theta y)=(F(\theta y))^n=(y-1)^n, \quad \in(1,2)$$

$$\hat{\varphi}(y)=n(y-1)^{n-1}$$

$t_1=\eta_{\frac{1-\beta}{2}}$ $\quad \int\limits_1^{t_1} n(y-1)^{n-1}dy=\frac{1-\beta}{2}\to(y-1)^n\Big|_1^{t_1}=\frac{1-\beta}{2}$

$$t_1=\sqrt[n]{\frac{1-\beta}{2}}+1$$

$t_2=\eta_{\frac{1+\beta}{2}}$ $\quad \hat{F}(t_2)=\frac{1+\beta}{2}$

$$t_2=\sqrt[n]{\frac{1+\beta}{2}}+1$$

$$\sqrt[n]{\frac{1-\beta}{2}}+1<\frac{X_{max}}{\theta}<\sqrt[n]{\frac{1+\beta}{2}}+1$$

$$\frac{X_{max}}{\sqrt[n]{\frac{1+\beta}{2}}+1}<\theta<\frac{X_{max}}{\sqrt[n]{\frac{1-\beta}{2}}+1}$$

e) ОММ: $\frac{f(\tilde{\alpha})-f(\alpha)}{\sigma(\alpha)}\sqrt{n}\rightsquigarrow N(0,1)$

$$\sigma(\alpha)=\sqrt{\nabla f^T K\nabla f}$$

$$\tilde{\theta}=f(\tilde{\alpha})=\frac{2}{3}\bar{x}\;;\quad \theta=f(\alpha)=\frac{2}{3}\alpha_1$$

$\bar{x} f = \frac{2}{3}$ $\qquad \alpha_1 = \bar{x}$ $\qquad \sigma(\tilde{\theta}) = \frac{2}{3}\sqrt{k} = \frac{2}{3}\sqrt{\bar{x}^2 - \bar{x}^2}$

$\alpha_2 = \overline{x^2}$

$\beta = 0{,}95$ ← делали

$$\frac{\tilde{\theta} - \theta}{\frac{2}{3}\sqrt{\overline{x^2} - \bar{x}^2}} \sim N(0,1) \qquad U_{\frac{1-\beta}{2}} \simeq -1{,}96 \qquad U_{\frac{1+\beta}{2}} = 1{,}96$$

$$-1{,}96 < \frac{\tilde{\theta} - \theta}{\frac{3}{3}\sqrt{\overline{x^2} - \bar{x}^2}}\sqrt{n} < 1{,}96$$

$$-2{,}94\sqrt{\frac{\overline{x^2} - \bar{x}^2}{n}} + \frac{2}{3}\bar{x} < \theta < 2{,}94\sqrt{\frac{\overline{x^2} - \bar{x}^2}{n}} + \frac{2}{3}\bar{x}$$