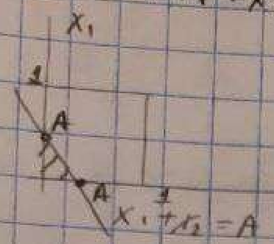
$\mathbb{P}(x_1+x_2 \le A \mid H_0) = \alpha$




$$\iint\limits_{x_1+x_2\le A} 1\cdot 1\, dx_1\, dx_2 = \alpha$$

$$\frac{1}{2}A^2 = \alpha \to A = \sqrt{2\alpha} \qquad G_{кр}: x_1+x_2 \le \sqrt{2\alpha}$$

к повш

$$W = \mathbb{P}(x_1+x_2 \le A \mid H_1) = \iint\limits_{x_1+x_2\le A} \frac{(e^{1-x_1})(e^{1-x_2})}{(e-1)\cdot(e-1)} dx_1 dx_2 =$$

$$= \frac{e^2}{(e-1)^2}\int_0^A dx_1 \int_0^{A-x_1} e^{-x_1} e^{-x_2} dx_2 = \frac{e^2}{(e-1)^2}\int_0^A e^{-x_1}(1-e^{-A+x_1})dx_1 =$$

$$= \frac{e^2}{(e-1)^2}\left(\int_0^A e^{-x_1}dx_1 - \int_0^A e^{-A}dx_1\right) = \frac{e^2}{(e-1)^2}[1-e^{-A}-e^{-A}A]$$

$$\alpha_2 = 1 - W$$

в) асимпт. крит.

$$l = \frac{L_1}{L_0} = \frac{\prod p_1(x_i)}{\prod p_0(x_i)} = \prod \frac{p_1(x_i)}{p_0(x_i)} \ge C$$

$$\text{(ЦПТ)} \quad \sum_{i=1}^{n} \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i} \ge \ln C$$

$$\text{ЦПТ}: \frac{\sum \eta_i - n M\eta_i}{\sqrt{n D\eta_i}} \rightsquigarrow N(0,1) \qquad \eta_i = \ln\frac{e^{1-x_i}}{(e-1)^1} = \ln\frac{e}{e-1} - x_i$$

$$H_0: \quad M\eta_i = M\left[\ln\frac{e}{e-1} - x_i\right] = \ln\frac{e}{e-1} - \frac{1}{2}$$

$$D\eta_i = D\left[\ln\frac{e}{e-1} - x_i\right] = Dx_i = \frac{1}{12}$$

$$\mathbb{P}(\ln l \ge \ln C \mid H_0) = \alpha$$

$$æ \rightsquigarrow N(0,1) \qquad A$$

$$\mathbb{P}(\sum\eta_i \ge \ln C \mid H_0) = \underset{\text{квант}}{\mathbb{P}}\left(\frac{\sum\eta_i - nM\eta_i}{\sqrt{n Dx_i}} \ge \frac{\ln C - n(\ln\frac{e}{e-1} - \frac{1}{2})}{\sqrt{n\frac{1}{12}}} \,\Big|\, \cancel{H_0}\right)$$

$$\int_A^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}dx = \alpha \qquad A = u_{1-\alpha}$$

$$\ln l \ge \ln C \qquad \sum\eta_i = n\ln\frac{e}{e-1} - \sum x_i$$

$$\frac{\ln C - n(\ln\frac{e}{e-1} - \frac{1}{2})}{\sqrt{n/12}} = u_{1-\alpha} \qquad \ln C = n\ln\frac{e}{e-1} - \frac{n}{2} + u_{1-\alpha}\sqrt{\frac{n}{12}}$$

$$\cancel{n\ln\frac{e}{e-1}} - \sum x_i \ge \cancel{n\ln\frac{e}{e-1}} - \frac{n}{2} + u_{1-\alpha}\sqrt{\frac{n}{12}} \qquad G_{кр}: \bar{x} \le \frac{1}{2} - u_{1-\alpha}\sqrt{\frac{1}{12}}$$

№10 $H_0: \xi \sim p_0(x) = 1 \{(0,1)\}$
$H_1: \xi \sim p_1(x) = \frac{e^{1-x}}{e-1} \{(0,1)\}$

а) $n = 1 \quad \alpha$

$$\ell = \frac{L_1}{L_0} = \frac{p_1(x)}{p_0(x)} = \frac{e^{1-x}}{(e-1)1} \geq C$$ разр. отн-но $x$

$$1 - x \geq \ln C(e-1) \to x \leq 1 - \ln C(e-1) = A$$

$G_{кр}: x \leq A \qquad \underline{P}(x \leq A \mid H_0) = \alpha$

$$\int_0^A p_0(x)dx = \alpha \qquad A = \alpha \qquad G_{кр}: x \leq \alpha$$

$\alpha_1 = \alpha$

$$W = P(x \leq A \mid H_1) = \int_0^\alpha p_1(x)dx = \int_0^\alpha \frac{e^{1-x}}{e-1}dx = -\frac{e^{1-x}}{e-1}\Big|_0^\alpha =$$

$$= \frac{e}{e-1}(1 - e^{-\alpha}) = \cancel{\frac{e}{e-1}} \quad \alpha_2 = 1 - W$$

б) $n = 2 \quad \alpha$

$$\ell = \frac{L_1}{L_0} = \frac{p_1(x_1)\,p_1(x_2)}{p_0(x_1)\,p_0(x_2)} = \frac{e^{1-x_1}e^{1-x_2}}{(e-1)^2 \cdot 1 \cdot 1} \geq C$$

$$-x_1 - x_2 \geq \ln \ldots \qquad G_{кр}: x_1 + x_2 \leq A$$

$$W = P\left(\overline{X} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} \,\Big|\, H_1\right) \qquad W = P(\ln C \cancel{\leq} \geq \ln C \mid H_1)$$

$\sum \eta_i$"

$$P\Bigg(\underbrace{\frac{\sum \eta_i - nM\eta_i}{\sqrt{nD\eta_i}}}_{\sim N(0,1)} \geq \overbrace{\frac{\ln C - nM\eta_i}{\sqrt{nD\eta_i}}}^{B}\Bigg) \ominus$$

та же

$$H_1: \quad M\eta_i = \ln\frac{e}{e-1} - M[X_i] = \ln\frac{e}{e-1} - \int_0^1 x\,\frac{e^{1-x}}{e-1}\,dx = \ln\frac{e}{e-1} - \frac{e-2}{e-1}$$

$$\frac{e-2}{e-1}$$

$$\ominus \int_B^{+\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\,dx$$

$$D\eta_i = \frac{e^2-3e+1}{(e-1)^2}$$

$$\alpha_2 = 1 - W \qquad \alpha_1 = \alpha$$

$$B = \frac{n\Big(\overbrace{\frac{e-2}{e-1} - \frac{1}{2}}^{-0{,}082}\Big) + u_{1-\alpha}\sqrt{\frac{n}{12}}}{\sqrt{n\,\frac{e^2-3e+1}{(e-1)^2}}} = E + \overset{<0}{F}\sqrt{n} \xrightarrow{n\to+\infty} -\infty \Rightarrow W \xrightarrow{n\to+\infty} \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\,dx = 1$$

О. Кр. проверки статист. гипотез наз. сост., если $W \xrightarrow{n\to\infty} 1$

d) $G_{kp}: X_{min} < C$ $\quad \mathbb{P}(X_{min} < C \mid H_0) = \alpha \Leftrightarrow$

$H_0: \quad F_0(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases}$ $\quad \Leftrightarrow F_{min}(C) = 1 - (1 - F_0(C))^n$

$F_0(C) = -\sqrt[n]{1-\alpha} + 1$

$G_{kp}: X_{min} < 1 - \sqrt[n]{1-\alpha}$ $\quad \alpha \in (0,1): \quad F_0(C) = 1 - \sqrt[n]{1-\alpha} = C$

$$W = P(X_{min} < c \mid H_1) \ominus \nearrow \int \frac{e^{1-x}}{e-1}\,dx = -\frac{e^{1-x}}{e-1} + C \qquad C = 1 + \frac{1}{e-1}$$

$$H_1 : F_0(x) = \begin{cases} 0 & ; \ x < 0 \\ -\frac{e^{1-x} + (-e)}{e-1} & ; \ x \in [0,1] \\ 1 & ; \ x > 1 \end{cases}$$

$$\ominus \ F_{min}(c) = 1 - (1 - F_0(c))^n = 1 - \left(1 + \frac{e^{1-x} - e}{e-1}\right)^n$$

$$\alpha_1 = \alpha \ ; \ \alpha_2 = 1 - W$$