$$p(x) = \begin{cases} \frac{\theta-1}{x^\theta}, & x \geq 1 \\ 0 ; & x < 1 \end{cases} \qquad \theta > 1$$

$$p(x) = \frac{\theta-1}{x^\theta} \, I\{(1, +\infty)\}, \quad \theta > 1$$

$$F(x) = 1 - \frac{1}{x^{\theta-1}} \, I\{(1, +\infty)\}, \quad \theta > 1$$

a) ОМП: $L(\theta) = \prod_{i=1}^{n} \left(\frac{\theta-1}{x^\theta}\right) = (\theta-1)^n \cdot \prod_{i=1}^{n} \frac{1}{x_i^\theta} \to max$

$$\ln L(\theta) = n \ln(\theta-1) - \theta \sum_{i=1}^{n} \ln x_i \to max$$

$$\frac{\partial \ln L(\theta)}{\partial(\theta)} = \frac{n}{\theta-1} - \sum_{i=1}^{n} \ln x_i = 0 \to \tilde{\theta}_1 = \frac{n}{\sum_{i=1}^{n} \ln x_i} + 1$$

в) $\hat{x}$ - мед. $\int_{-\infty}^{\hat{x}} p(x)dx = \frac{1}{2}$

$f(\vec{x}_n, \theta) \sim g(t)$

$$\int_1^{\hat{x}} (\theta-1) \frac{1}{x^\theta} dx = \theta - 1 \int_1^{\hat{x}} x^{-\theta} dx = -x^{-\theta+1} \Big|_1^{\hat{x}} = -\hat{x}^{-\theta+1} + 1 = \frac{1}{2}$$

$$\hat{x}^{-\theta+1} = \frac{1}{2} \to \hat{x} = 2^{\frac{1}{\theta-1}}$$

Многомерная ЦПТ для ОМП. $\quad \Big| \; \frac{f(\tilde{\theta}) - f(\theta)}{\sigma(\tilde{\theta})} \sqrt{n} \sim N(0,1)$

$f(\theta) = 2^{\frac{1}{\theta-1}}; \; f(\tilde{\theta}) = 2^{\frac{1}{\tilde{\theta}-1}} \quad \Big| \; \sigma(\tilde{\theta}) = \sqrt{\nabla f^T(\tilde{\theta}) I^{-1} \nabla f(\tilde{\theta})}$

$$\nabla f = 2^{\frac{1}{\theta-1}} \ln 2 \frac{1}{(\theta-1)^2} \qquad \tilde{\theta} = \frac{n}{\sum_{i=1}^{n} \ln x_i} + 1$$

$\ln(\theta-1) - \theta \ln(x)$

$$I = M\left[\left(\frac{\partial \ln p}{\partial \theta}\right)^2\right] = \int_{-\infty}^{\infty} \left(\frac{1}{\theta-1} - \ln x\right)^2 \frac{\theta-1}{x^\theta} dx =$$

$$= \int_1^{\infty} \frac{1}{\theta-1} \frac{1}{x^\theta} dx + \int \ln^2 x \frac{\theta-1}{x^\theta} dx - \int \frac{2 \ln x}{x^\theta} dx = \frac{1}{(\theta-1)^2}$$

$\frac{1}{(\theta-1)^2} \qquad \frac{2}{(\theta-1)^2} \qquad 2(\theta-1)^2$

$$I^{-1}(\tilde{\theta}) = (\tilde{\theta}-1)^2$$

$$\sigma(\tilde{\theta}) = \sqrt{2^{\frac{1}{\theta-1}} \ln 2 \frac{1}{(\theta-1)^2} (\tilde{\theta}-1)^2 \cdot 2^{\frac{1}{\theta-1}} \ln 2 \frac{1}{(\theta-1)^2}} =$$

$$= 2^{\frac{1}{\theta-1}} \ln 2 \frac{1}{\theta-1}$$

$$\frac{f(\tilde{\theta}) - f(\theta)}{\sigma} \sqrt{n} \sim N(0,1)$$

$$-1{,}96 < \frac{2^{\frac{1}{\tilde\theta-1}} - 2^{\frac{1}{\theta-1}}}{2^{\frac{1}{\tilde\theta-1}}\ln 2 \,\frac{1}{\tilde\theta-1}}\sqrt{n} < 1{,}96$$

$$-1{,}96\frac{2^{\frac{1}{\tilde\theta-1}}\ln 2}{(\tilde\theta-1)\sqrt{n}} + 2^{\frac{1}{\tilde\theta-1}} < \hat{x} < 1{,}96\frac{2^{\frac{1}{\tilde\theta-1}}\ln 2}{(\tilde\theta-1)\sqrt{n}} + 2^{\frac{1}{\tilde\theta-1}}$$

c) Регул: $0 \stackrel{?}{=} \int_1^\infty \frac{\partial}{\partial\theta} p(\vec{x},\theta)dx = \int_1^\infty \frac{\partial}{\partial\theta}\left(\frac{\theta-1}{x^\theta}\right)dx =$

$= \int_1^\infty \frac{x^\theta - x^\theta \ln x(\theta-1)}{x^{2\theta}}dx = \int_1^\infty \frac{1-\ln x(\theta-1)}{x^\theta}dx = x^{1-\theta}\ln x\Big|_1^\infty = 0$ ✓

· ОМП: $\frac{f(\tilde\theta)-f(\theta)}{\sigma(\tilde\theta)}\sqrt{n} \sim N(0,1)$

$f(\theta) = \theta$ $\quad \nabla f = 1$ $\quad I^{-1} = (\tilde\theta-1)^2$ $\quad \tilde\theta = \frac{n}{\sum_{i=1}^n \ln x_i} + 1$

$f(\tilde\theta) = \tilde\theta$ $\quad \sigma = \tilde\theta - 1$

$$\frac{\tilde\theta-\theta}{\tilde\theta-1}\sqrt{n} \rightsquigarrow N(0,1)$$

$$-1{,}96 < \frac{\tilde\theta-\theta}{\tilde\theta-1}\sqrt{n} < 1{,}96$$

$$-1{,}96\frac{\tilde\theta-1}{\sqrt{n}} + \tilde\theta < \theta < 1{,}96\frac{\tilde\theta-1}{\sqrt{n}} + \tilde\theta$$

· ОММ: $\frac{f(\tilde\alpha)-f(\alpha)}{\sigma(\tilde\alpha)}\sqrt{n} \rightsquigarrow N(0,1)$

$\alpha_1 = M\xi = \int_1^\infty x\frac{\theta-1}{x^\theta}dx = \frac{\theta-1}{\theta-2} = 1 + \frac{1}{\theta-2}$

$\tilde\alpha_1 = \bar{x}$ $\quad 1 + \frac{1}{\theta-2} = \bar{x} \to \tilde\theta_2 = \frac{1}{\bar{x}-1} + 2, \ \theta > 2$

$f(\alpha) = \theta = \frac{1}{\alpha_1-1} + 2$ $\quad f(\tilde\alpha_1) = \tilde\theta = \frac{1}{\bar{x}-1} + 2$

$\nabla f(\tilde\alpha_1) = -\frac{1}{(\bar{x}-1)^2}$ $\quad k = K_{11} = \tilde\alpha_2 - \tilde\alpha_1^2 = \overline{x^2} - \bar{x}^2$

$$-1{,}96 < \frac{\frac{1}{\bar{x}-1} + 2 - \theta}{\frac{1}{(\bar{x}-1)^2}\sqrt{\overline{x^2}-\bar{x}^2}}\sqrt{n} < 1{,}96$$

$$-1{,}96\frac{\sqrt{\overline{x^2}-\bar{x}^2}}{(\bar{x}-1)^2\sqrt{n}} + 2 + \frac{1}{\bar{x}+1} < \theta < 1{,}96\frac{\sqrt{\overline{x^2}-\bar{x}^2}}{(\bar{x}-1)^2\sqrt{n}} + 2 + \frac{1}{\bar{x}+1}$$