№10 H_0: $\xi \sim p_0(x) = 1\{(0,1)\}$

H_1: $\xi \sim p_1(x) = \frac{e^{1-x}}{e-1}\{(0,1)\}$

а) $n = 1$ α

$$\ell = \frac{L_1}{L_0} = \frac{p_1(x)}{p_0(x)} = \frac{e^{1-x}}{(e-1)1} \geq C$$ разр. от-но x

$$1 - x \geq \ln C(e-1) \rightarrow x \leq 1 - \ln C(e-1) = A$$

$G_{кр}$: $x \leq A$ $\quad P(X \leq A \mid H_0) = \alpha$

$$\int_0^A p_0(x)dx = \alpha \qquad A = \alpha \qquad G_{кр}: x \leq \alpha$$

$$W = P(X \leq A \mid H_1) = \int_0^{\alpha} p_1(x)dx = \int_0^{\alpha} \frac{e^{e-x}}{e-1}dx = -\frac{e^{1-x}}{e-1}\Big|_0^{\alpha_1} = \alpha$$

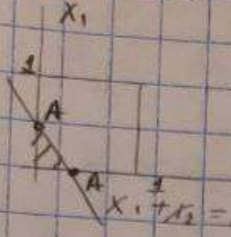
$$= \frac{e}{e-1}(1 - e^{-\alpha}) \rightarrow \alpha_2 = 1 - W$$

б) $n = 2$ α

$$\ell = \frac{L_1}{L_0} = \frac{p_1(x_1)\,p_1(x_2)}{p_0(x_1)\,p_0(x_2)} = \frac{e^{1-x_1}e^{1-x_2}}{(e-1)^2 \cdot 1 \cdot 1} \geq C$$

$$-x_1 - x_2 \geq \ln \ldots \qquad G_{кр}: x_1 + x_2 \leq A$$

$$\underline{P}(x_1+x_2 \le A \mid H_0) = \alpha$$



$$\iint_{x_1+x_2\le A} 1 \cdot 1 \, dx_1 dx_2 = \alpha$$

$$\frac{1}{2}A^2 = \alpha \to A = \sqrt{2\alpha} \qquad \text{Кр. обл.: } x_1+x_2 \le \sqrt{2\alpha}$$

к новым

$$W = \underline{P}(x_1+x_2 \le A \mid H_1) = \iint_{x_1+x_2\le A} \frac{(e^{1-x_1})(e^{1-x_2})}{(e-1)\cdot(e^A-1)} dx_1 dx_2 =$$

$$= \frac{e^2}{(e-1)^2} \int_0^A dx_1 \int_0^{A-x_1} e^{-x_1} e^{-x_2} dx_2 = \frac{e^2}{(e-1)^2} \int_0^A e^{-x_1}\left(1 - e^{-A+x_1}\right) dx_1 =$$

$$= \frac{e^2}{(e-1)^2}\left(\int_0^A e^{-x_1} dx_1 - \int_0^A e^{-A} dx_1\right) = \frac{e^2}{(e-1)^2}\left[1 - e^{-A} - e^{-A} A\right]$$

$$\alpha_2 = 1 - W$$

в) асимпт. крит.

$$\ell = \frac{L_1}{L_0} = \frac{\prod p_1(x_i)}{\prod p_0(x_i)} = \prod \frac{p_1(x_i)}{p_0(x_i)} \ge C$$

(лог)

$$\sum_{i=1}^{n} \ln \frac{p_1(x_i)}{p_0(x_i)} \eta_i \ge \ln C$$

ЦПТ: $\frac{\sum \eta_i - n M\eta_i}{\sqrt{n D\eta_i}} \sim N(0,1)$ $\qquad \eta_i = \ln \frac{e^{1-x_i}}{(e-1)\cdot 1} = \ln \frac{e}{e-1} - x_i$

H_0: $M\eta_i = M\left[\ln \frac{e}{e-1} - x_i\right] = \ln \frac{e}{e-1} - \frac{1}{2}$

$D\eta_i = D\left[\ln \frac{e}{e-1} - x_i\right] = Dx_i = \frac{1}{12}$

$$\underline{P}(\ln \ell \ge \ln C \mid H_0) = \alpha \qquad x \sim N(0,1)$$

$$\underline{P}\left(\sum \eta_i \ge \ln C \mid H_0\right) = P\left(\frac{\sum \eta_i - n M\eta_i}{\sqrt{n D x_i}} \ge \frac{\ln C - n\left(\ln \frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n \frac{1}{12}}} \mid H_0\right)$$

$$\int_A^\infty \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx = \alpha \qquad A = U_1 - \alpha \quad \text{(квант.)}$$

$$\ln \ell \ge \ln C \qquad \sum \eta_i = n \ln \frac{e}{e-1} - \sum x_i$$

$$\frac{\ln C - n\left(\ln \frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n/12}} = U_1 - \alpha \qquad \ln C = n \ln \frac{e}{e-1} + \frac{n}{2} + U_1 - 2\sqrt{\frac{n}{12}}$$

$$n \ln \frac{e}{e-1} - \sum x_i \ge n \ln \frac{e}{e-1} - \frac{n}{2} + U_1 - 2\sqrt{\frac{n}{12}} \qquad \text{Кр. обл.: } \bar{x} \le \frac{1}{2} - U_1 - 2\sqrt{\frac{1}{12}}$$

$W = P\left(\bar{X} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} \mid H_1\right)$ $\quad W = P\left(\ln \ell \ge \ln C \mid H_1\right)$

$\sum \eta_i''$

$\xrightarrow{n \to \infty} N(0,1)$ $\quad \beta$

$P\left(\frac{\sum \eta_i - n M\eta_i}{\sqrt{n D\eta_i}} \le \frac{\ln C - n M\eta_i}{\sqrt{n D\eta_i}}\right) \Leftrightarrow$ $\quad \frac{e-2}{e-1}$

$H_1: \quad M\eta_i = \ln \frac{e}{e-1} - M[x_i] = \ln \frac{e}{e-1} - \int_0^1 x \frac{e^{1-x}}{e-1} dx =$

$= \ln e - \frac{e-2}{e-1} \Leftrightarrow \int_\beta^{+\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$

$D\eta_i = \frac{e^2 - 3e + 1}{(e-1)^2}$ $\quad \alpha_2 = 1 - W \quad \alpha_1 = \alpha$

$\beta = \frac{n\left(\frac{e-2}{e-1} - \frac{1}{2}\right) + u_{1-\alpha}\sqrt{\frac{n}{12}}}{\sqrt{n \frac{e^2-3e+1}{(e-1)^2}}} = E + F\sqrt{n} \xrightarrow[n \to +\infty]{F < 0} -\infty \Rightarrow W = \int_{-\infty}^{n \to +\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx = 1$

0. Кр. проверки статист. гипотез наз. сост., если $W \xrightarrow{n \to \infty} 1$

d) $G_{kp}: x_{min} < C$ $\quad P(x_{min} < C \mid H_0) = \alpha \Leftrightarrow$

$H_0: \quad F_0(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases} \Leftrightarrow F_{min}(C) = 1 - (1 - F_0(C))^n$

$F_0(C) = 1 - \sqrt[n]{1-\alpha}$

$G_m: x_{min} < 1 - \sqrt[n]{1-\alpha}$ $\quad \alpha \in (0,1): \quad F_0(C) = 1 - \sqrt[n]{1-\alpha} = C$

$W = P(X_{min} < c \mid H_1) \Leftrightarrow \int \frac{e^{1-x}}{e-1} dx = \frac{e^{1-x}}{e-1} + C$

$H_1: F_0(x) = \begin{cases} 0 & ; x < 0 \\ \frac{e^{1-x} + (-e)}{e-1} & ; x \in [0,1] \\ 1 & ; x > 1 \end{cases}$

$\Leftrightarrow F_{min}(c) = 1 - (1 - F_0(c))^4 = 1 - \left(1 + \frac{e^{1-x} - e}{e-1}\right)^4$

$\alpha_1 = \alpha \; ; \; \alpha_2 = 1 - W$