All the shapes of spaces: a census of small 3-manifolds *

Sóstenes L. Lins and Lauro D. Lins

May 16, 2013

Abstract

In this work we present a complete (no misses, no duplicates) catalogue for closed, orientable and prime 3-manifolds induced by plane graphs with a bipartition of its edge set (blinks) up to 9 edges. Blinks form a universal encoding for such manifolds. We hope that this census becomes as useful for the study of concrete examples of 3-manifolds as the tables of knots are in the study of knots and links. Along the years we have made an issue in our computational work that it must be reproducible and independently checked by other researchers. Our software BLINK is available, but currently it lacks yet a good documentation and help is welcome to change this. An Wiki open source project is starting.

1 Introduction

After presenting some instances of closed 3-manifolds, P. Alexandroff says in the English translation (1961) of his joint work with D. Hilbert [1], first published (1932) in German, [2]: "These few examples will suffice. Let it be remarked here that, at present, in contrast with the two-dimensional case, the problem of enumerating the topological types of manifolds of three and more dimensions is in an apparently hopeless state. We are not only far removed from the solution, but even from the first step toward a solution, a plausible conjecture".

John Hempel in his book (1976) *3-Manifolds* [6] writes at the opening of Section 15, titled *Open Problems:*
"The ultimate goal of the theory would be in providing solutions to: *The homeomorphism problem*: provide an *effective* procedure for determining whether two *given* 3-manifolds are homeomorphic, together with *The classification problem*: *effectively* generate a list containing exactly one 3-manifold from each homeomorphism class."

It is amazing how much the picture has changed in the 80 years since Alexandroff's-Hilbert book. The progress was due to the deep breakthroughs in the 1950's and 1960's, starting in the early 1980's of W. Thurston, in the early 2000's, G. Perelman, and more recently 2010's, I. Agol and many others. Currently there is a great amount of important reserch issues going on and these are exciting times for 3-manifold theory. See the recent essay of E. Klarreich in the Simons Foundation, [11].

We present here our modest contribution to the topic. It is placed in the confluency of two deep research passions of the authors, apparently very far apart: The study of closed orientable

*2010 Mathematics Subject Classification: 57M25 and 57Q15 (primary), 57M27 and 57M15 (secondary)

3-manifolds and the study of plane graphs. We mean to provide a road for a segmented answer for Hempel's questions. The closed oriented 3-manifolds are partitioned by the number of edges in a minimum encoding of them by a certain class of plane graphs, shortly to be defined. Lexicography is used to define a representative unique plane graph (a canonical form) for each closed oriented 3-manifold. We explicitly solve the segmented problem up to 9 edges, see Theorem 4.1. An important fact is that there is an efficient algorithm to make available the canonical form of any manifold induced by plane graphs at the current level of the catalogue (currently 9 edges).

2 Encoding closed orientable 3-manifolds by plane graphs

Unexplored simplicity. This was the reason for birth of this work. A *blink* is a plane graph with an arbitrary bipartition of its edges in gray and black. Even though this object is around since 1994 when it was introduced in the joint research monography of L. Kauffman and S. Lins, [9], the fact that they encode oriented closed 3-manifolds remains basically unkown.

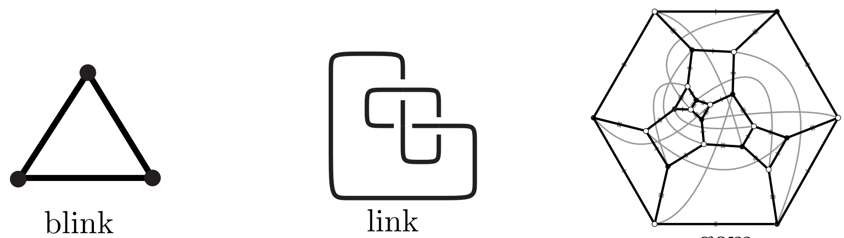


Figure 1: The minimum blink, minimum link and minimum gem inducing the binary tetrahedral space

Blinks are in 1-1 correspondence with blackboard framed links which in turn encodes in a very simple way an specific 3-gem inducing the 3-manifold. If we take the smallest of these three encodings of a 3-manifold, the blink is the one that has the smallest "perceptual complexity", see Fig. 1. Also, blinks are very easy to generate recursively. They have a rich simpifying theory which permits the generation of 3-manifold catalogues. Yet, their isomorphism problem is computationally simple.

We make here explicit for the first time that each class of homeomorphic closed oriented 3-manifolds is a subtle class of blinks where two membres of each class is linked by means of a small number of local simple moves. Our moves (see Fig. 5) are a slight reformulation for blinks of the one for framed links on Kirby's calculus ([10]) using Fenn and Rourke moves for the calculus, [5] recently published by Martelli, [16]. No doubt that blinks are simple mathematical objects. Nevertheless, as a consequence of Martelli's reformulation of Kirby's calculus, we see that they hold, in their gist, the mystery of 3-manifolds.

Reformulation of the original Kirby's moves and of the Fenn-Rourke move for blackboard framed links are worked by Kauffman in [8]. In a grounding breaking work, Lickorish in 1962, [12], proved that each closed orientable 3-manifold $\mathbb{M}^3$ can be encoded by a link in $\mathbb{S}^3$ where each one of its k components is endowed with an irreducible fraction (the framing) $\frac{\pm p}{q}$ where q could be 0, and in the case the fraction becomes $\pm\infty$. To construct the 3-manifold $\mathbb{M}^3$ from the framed link we act as follows: after removing an ϵ-neighborhood of each link component we are left with $\mathbb{M}^3\backslash(\mathbb{S}^1 \times \mathbb{D}^2)_k = \mathbb{S}^3\backslash(\mathbb{S}^1 \times \mathbb{D}^2)_k$. The fraction specifies, in the toroidal boundary inside

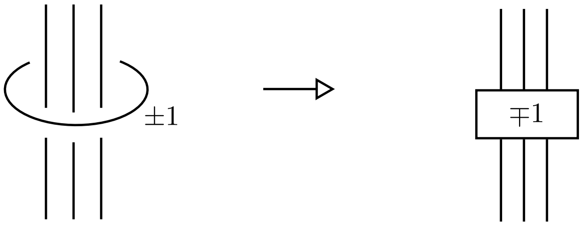


FIGURE 2. A topological blow-down. The number n 0 of vertical strands crossing the unknot is arbitrary (here n = 3). The box marked with +1 (−1) indicates one full counterclockwise (clockwise) twist.

Figure 2: Case n=3 of the Fenn-Rourke Move

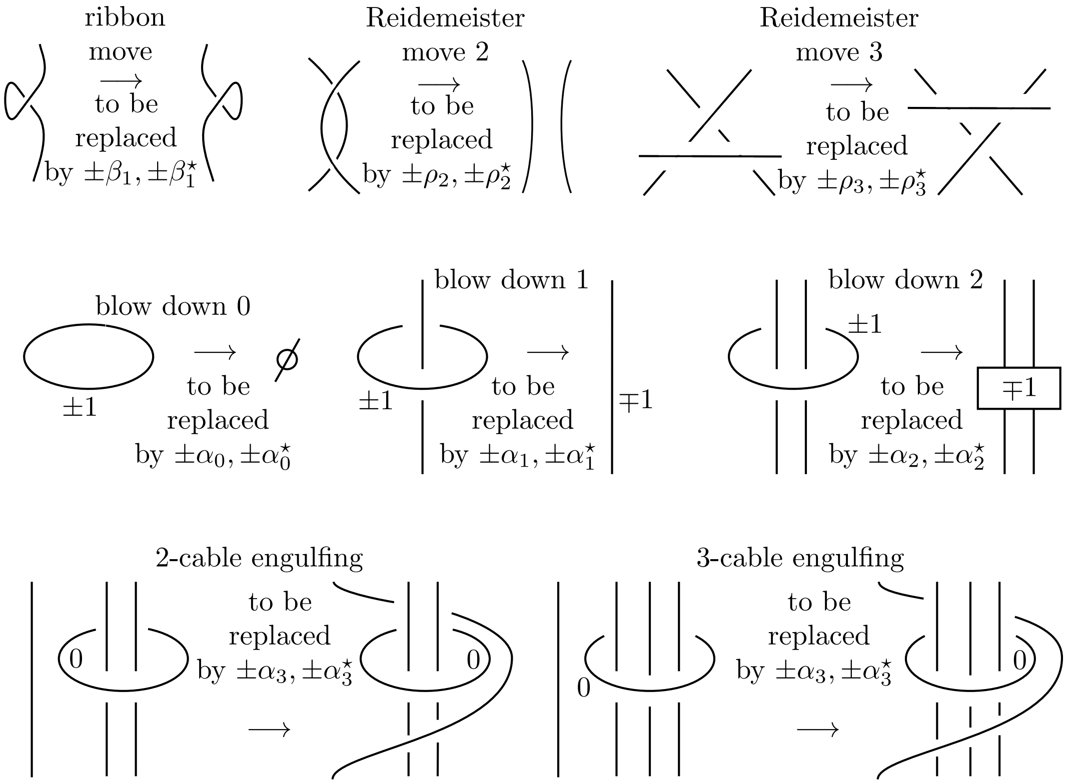


Figure 3: Martelli's finite set of 14 local moves for framed links. The ribbon move is redundant in the calculus, since the configurations are in $\mathbb{R}^3$. It is included to help the translation to blinks, which is a strictly planar calculus.

$\mathbb{S}^3$, the homology type $(\pm p, q)$ of the curve that is contractible in the solid torus inside $\mathbb{M}^3$. For each component, we then identify the simple curve given by the homological base pair with the meridian of a canonical copy of a solid torus in $\mathbb{R}^3$ so as to completely specify the pasting of the solid torus closing the toroidal hole. Lickorish's breakthrough was to prove that any $\mathbb{M}^3$ has inside it a finite number k of disjoint solid tori so that $\mathbb{M}^3 \backslash (\mathbb{S}^1 \times \mathbb{D}^2)_k = \mathbb{S}^3 \backslash (\mathbb{S}^1 \times \mathbb{D}^2)_k$.

(2.1) Proposition. *Given any fractional framed link it is possible to obtain, by an effective algo-*

rithm, an integer framed link inducing the same 3-manifold.

Proof. See [17] for an algorithmic proof and a simple complete example in page 79 of [14], which illustrate the general algorithm. This example relates to the closed hyperbolic orientable 3-manifold with smallest volume known, [7]. □

A *blackboard framed link* is an integer framed link, given as a fixed projection in the plane (with upper and lower strands recorded) where the integer framing of a component is the algebraic sum of the signs of its self-crossings.

(2.2) Proposition. *Given any integer framed link it is possible to obtain a blackboard framed link inducing the same 3-manifold.*

Proof. Just introduce an adequate set of positive or negative curls to the components to have their self-writhes agreeing with the required integer framing. □

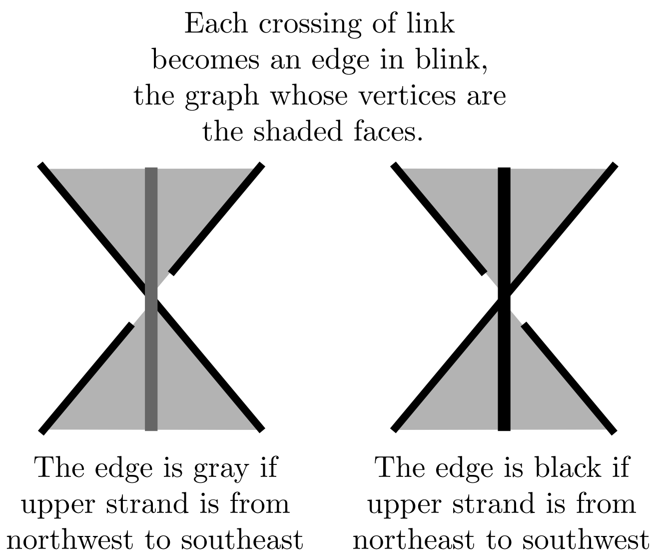


Figure 4: Obtaining the blink from the link. The inverse procedure is straighforward.

3 A formal calculus on blinks

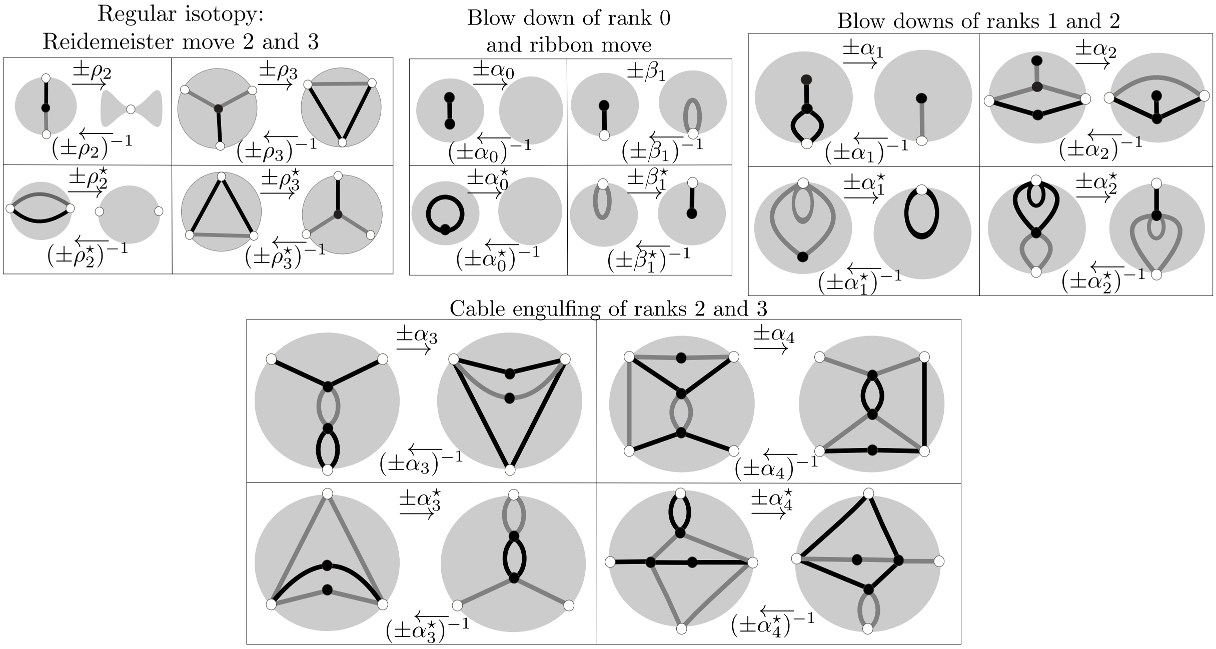


Figure 5: **Blink-coin reformulation of Martelli's calculus**: there are 8 types of moves, each having four moves, a total of 32 local inverse pairs of moves. The four moves of the same type are obtained either by changing sign which means to interchange the gray and black colors of the edges (changing the parity) or by plane duality (represented by the starred moves) followed by the interchanging of gray and black (which produces the same associated blackboard framed link). There are a grand total of 64 moves, because each move goes from left to right (which names the moves) or from right to left (its inverse move, which must have a -1 exponente) in each one of the 32 pairs. For each move a local disk (named *coin*) with an internal specific configuration is replaced by another coin with another internal specific configuration. The complement of the disk contains a completely arbitrary blink whose intersection with the the internal blink is formed by some isolated attachment vertices represented by small white circle in the boundary of the coin. The number of such attachment vertices is the first index of the move. In all but moves $\pm\rho_2$ corresponding to Reidemeister move 2 in the blackboard framed link, the replacing coin is a disk; in the four exceptional moves one disk becomes a pinched disk (or vice-versa), but this causes no harm. We made no effort in minimizing the number of moves. For instance moves $\pm\rho_3$, $\pm\rho_3^\star$ and $\pm\alpha_3$, $\pm\alpha_3^\star$ are equivalent because they are self-dual. As a matter of fact, beyound the self-dual moves we can show that every $\alpha_{ij}^\star$ move is redundant by global considerations. The last 4 types of moves, on the contrary of the previous ones, are not direct translations from Martelli' s calculus restricted to blackboard framed links. However it is easy to show that our whole set of moves imply and are implied by the direct translation of Martelli' s. Our last four types of moves seem more convenient as compared with those of Martelli's, because they maintain the number of edges and are either self-dual, $\{\pm\alpha_3, \pm\alpha_3^\star\}$ involving 6 edges, or clearly involutions, the biggest ones involving 9 edges, $\{\pm\alpha_4, \pm\alpha_4^\star\}$.

(3.1) Theorem. *Two blinks represent the same oriented closed 3-manifold if and only if they are linked by a finite sequence where each term is in the subset*

$$\{\pm\rho_2, \pm\rho_2^\star, \pm\rho_3, \pm\alpha_0, \pm\beta_1, \pm\alpha_1, \pm\alpha_2, \pm\alpha_3, \pm\alpha_4\}$$

of Fig. 5 or their inverses.

Proof. The moves are basically a reformulation for blinks of the calculus in [16] restricted to blackboard framed links. The only difference are in moves $\pm\alpha_3, \pm\alpha_3^\star$ and $\pm\alpha_4, \pm\alpha_4^\star$ which can be easily checked to produce an equivalent set of moves for blackboard framed links as those of Martelli's. □

Any plane drawing whatsoever of a graph with an arbitrary bipartition of its edge set, that is, a blink, corresponds to a unique closed oriented 3-manifold via the associated blackboard framed links. An important aspect about blinks is that each one possesses an easily obtainable *canonical form* inducing the same 3-manifold: it is named the *representative of the blink* and is obtained by lexicography from a small number of conventions, fixed in advance. This is explained, with a great amount of details, in L. Lins' thesis, [13].

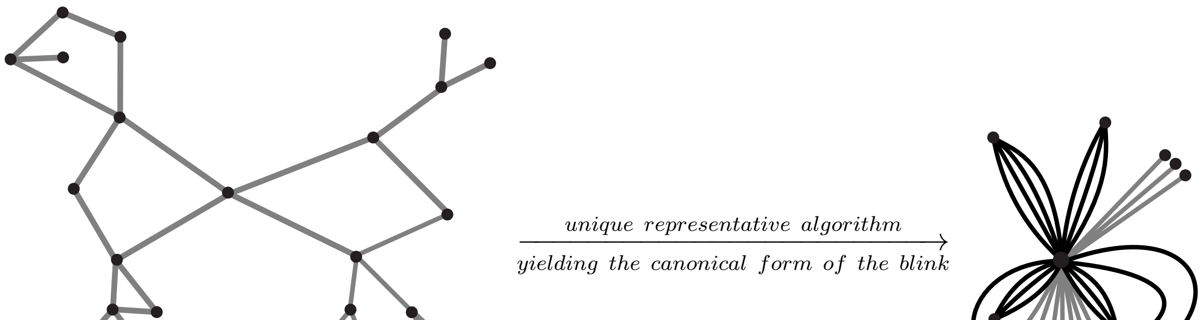


Figure 6: Blink representative algorithm: doglike blink with 27 edges and its representative with 25 edges.

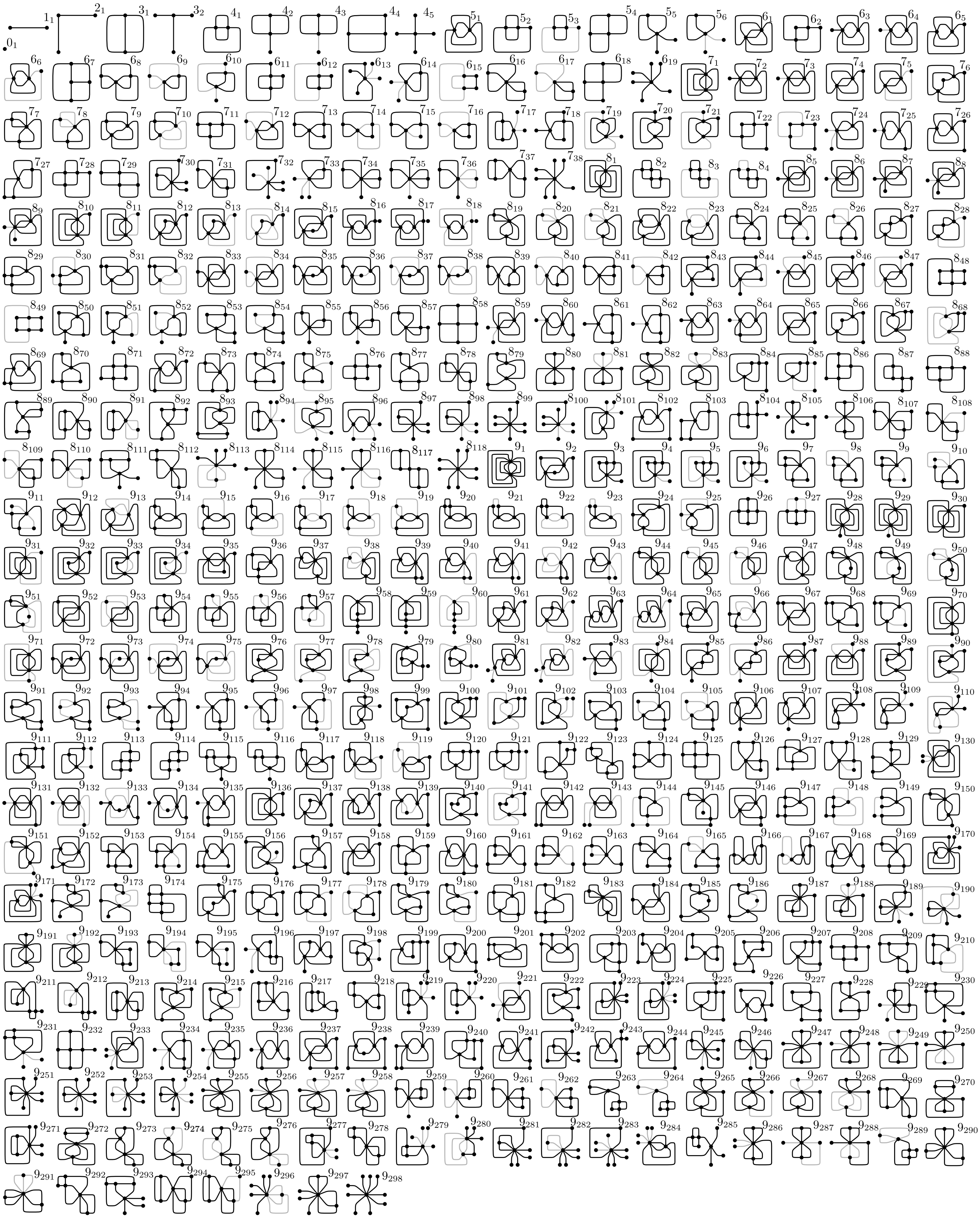
4 A complete duplicate free census of 9-small 3-manifolds

In references [9], [13] and [14] we have defined and show how a *blink*, that is, a plane graph with an arbitrary partition of its edges (here presented as colors black and gray) induces a well defined closed oriented 3-manifold. Moreover each such a manifold is induced by a blink (in fact, by infinite blinks). An *n-small* is a closed, orientable, and prime 3-manifold is a manifold induced by a blink with at most n edges. Relative to [13] the blinks of next theorem have receive two additions, the representative blinks $U[1563]$ and $U[2165]$. Also the previous HG12QI-class 6_5 became the homemorphism class 0_1 corresponding to $\mathbb{S}^2 \times \mathbb{S}^1$. We have decreased by 1 the numbering of the $HG12QI$-classes $6_6, 6_7, \ldots, 6_{20}$ become the homeomorphisms classes $6_5, 6_7, \ldots, 6_{19}$. This is because the HG12QI-classes 9_{126} and 9_{199} of [13] split into two topological classes. An objective of the present work is to prove that the splitings indeed take place.

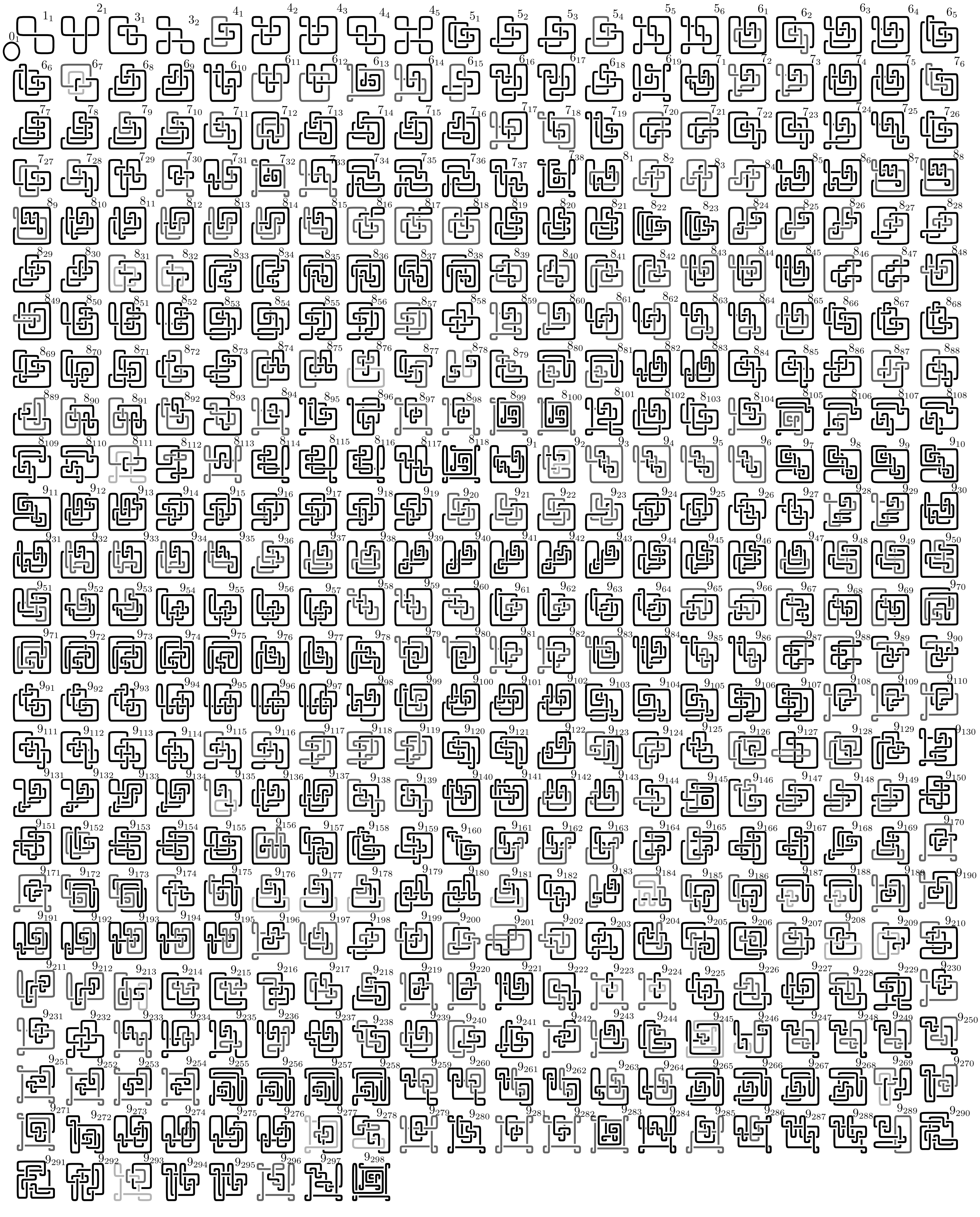
We observe that the blinks are enlarged in the appendix, showing them together with the corresponding blackboard framed links. The notation n_i attached to each blink below, is the name of its homeomorphism class, not merely its hgqi-class, as in [13].

This paper concludes the proof of the following theorem:

(4.1) Theorem. *Let M^3 be a closed, oriented and prime 3-manifold induced by a blink with at most 9 edges. Then M^3 is homeomorphic to exactly one of the 3-manifolds induced by the 489 blinks below. Morever all of these are pairwise non-homeomorphic.*



0_1 1_1 2_1 3_1 3_2 4_1 4_2 4_3 4_4 4_5 5_1 5_2 5_3 5_4 5_5 5_6 6_1 6_2 6_3 6_4 6_5
6_6 6_7 6_8 6_9 6_{10} 6_{11} 6_{12} 6_{13} 6_{14} 6_{15} 6_{16} 6_{17} 6_{18} 6_{19} 7_1 7_2 7_3 7_4 7_5 7_6
7_7 7_8 7_9 7_{10} 7_{11} 7_{12} 7_{13} 7_{14} 7_{15} 7_{16} 7_{17} 7_{18} 7_{19} 7_{20} 7_{21} 7_{22} 7_{23} 7_{24} 7_{25} 7_{26}
7_{27} 7_{28} 7_{29} 7_{30} 7_{31} 7_{32} 7_{33} 7_{34} 7_{35} 7_{36} 7_{37} 7_{38} 8_1 8_2 8_3 8_4 8_5 8_6 8_7 8_8
8_9 8_{10} 8_{11} 8_{12} 8_{13} 8_{14} 8_{15} 8_{16} 8_{17} 8_{18} 8_{19} 8_{20} 8_{21} 8_{22} 8_{23} 8_{24} 8_{25} 8_{26} 8_{27} 8_{28}
8_{29} 8_{30} 8_{31} 8_{32} 8_{33} 8_{34} 8_{35} 8_{36} 8_{37} 8_{38} 8_{39} 8_{40} 8_{41} 8_{42} 8_{43} 8_{44} 8_{45} 8_{46} 8_{47} 8_{48}
8_{49} 8_{50} 8_{51} 8_{52} 8_{53} 8_{54} 8_{55} 8_{56} 8_{57} 8_{58} 8_{59} 8_{60} 8_{61} 8_{62} 8_{63} 8_{64} 8_{65} 8_{66} 8_{67} 8_{68}
8_{69} 8_{70} 8_{71} 8_{72} 8_{73} 8_{74} 8_{75} 8_{76} 8_{77} 8_{78} 8_{79} 8_{80} 8_{81} 8_{82} 8_{83} 8_{84} 8_{85} 8_{86} 8_{87} 8_{88}
8_{89} 8_{90} 8_{91} 8_{92} 8_{93} 8_{94} 8_{95} 8_{96} 8_{97} 8_{98} 8_{99} 8_{100} 8_{101} 8_{102} 8_{103} 8_{104} 8_{105} 8_{106} 8_{107} 8_{108}
8_{109} 8_{110} 8_{111} 8_{112} 8_{113} 8_{114} 8_{115} 8_{116} 8_{117} 8_{118} 9_1 9_2 9_3 9_4 9_5 9_6 9_7 9_8 9_9 9_{10}
9_{11} 9_{12} 9_{13} 9_{14} 9_{15} 9_{16} 9_{17} 9_{18} 9_{19} 9_{20} 9_{21} 9_{22} 9_{23} 9_{24} 9_{25} 9_{26} 9_{27} 9_{28} 9_{29} 9_{30}
9_{31} 9_{32} 9_{33} 9_{34} 9_{35} 9_{36} 9_{37} 9_{38} 9_{39} 9_{40} 9_{41} 9_{42} 9_{43} 9_{44} 9_{45} 9_{46} 9_{47} 9_{48} 9_{49} 9_{50}
9_{51} 9_{52} 9_{53} 9_{54} 9_{55} 9_{56} 9_{57} 9_{58} 9_{59} 9_{60} 9_{61} 9_{62} 9_{63} 9_{64} 9_{65} 9_{66} 9_{67} 9_{68} 9_{69} 9_{70}
9_{71} 9_{72} 9_{73} 9_{74} 9_{75} 9_{76} 9_{77} 9_{78} 9_{79} 9_{80} 9_{81} 9_{82} 9_{83} 9_{84} 9_{85} 9_{86} 9_{87} 9_{88} 9_{89} 9_{90}
9_{91} 9_{92} 9_{93} 9_{94} 9_{95} 9_{96} 9_{97} 9_{98} 9_{99} 9_{100} 9_{101} 9_{102} 9_{103} 9_{104} 9_{105} 9_{106} 9_{107} 9_{108} 9_{109} 9_{110}
9_{111} 9_{112} 9_{113} 9_{114} 9_{115} 9_{116} 9_{117} 9_{118} 9_{119} 9_{120} 9_{121} 9_{122} 9_{123} 9_{124} 9_{125} 9_{126} 9_{127} 9_{128} 9_{129} 9_{130}
9_{131} 9_{132} 9_{133} 9_{134} 9_{135} 9_{136} 9_{137} 9_{138} 9_{139} 9_{140} 9_{141} 9_{142} 9_{143} 9_{144} 9_{145} 9_{146} 9_{147} 9_{148} 9_{149} 9_{150}
9_{151} 9_{152} 9_{153} 9_{154} 9_{155} 9_{156} 9_{157} 9_{158} 9_{159} 9_{160} 9_{161} 9_{162} 9_{163} 9_{164} 9_{165} 9_{166} 9_{167} 9_{168} 9_{169} 9_{170}
9_{171} 9_{172} 9_{173} 9_{174} 9_{175} 9_{176} 9_{177} 9_{178} 9_{179} 9_{180} 9_{181} 9_{182} 9_{183} 9_{184} 9_{185} 9_{186} 9_{187} 9_{188} 9_{189} 9_{190}
9_{191} 9_{192} 9_{193} 9_{194} 9_{195} 9_{196} 9_{197} 9_{198} 9_{199} 9_{200} 9_{201} 9_{202} 9_{203} 9_{204} 9_{205} 9_{206} 9_{207} 9_{208} 9_{209} 9_{210}
9_{211} 9_{212} 9_{213} 9_{214} 9_{215} 9_{216} 9_{217} 9_{218} 9_{219} 9_{220} 9_{221} 9_{222} 9_{223} 9_{224} 9_{225} 9_{226} 9_{227} 9_{228} 9_{229} 9_{230}
9_{231} 9_{232} 9_{233} 9_{234} 9_{235} 9_{236} 9_{237} 9_{238} 9_{239} 9_{240} 9_{241} 9_{242} 9_{243} 9_{244} 9_{245} 9_{246} 9_{247} 9_{248} 9_{249} 9_{250}
9_{251} 9_{252} 9_{253} 9_{254} 9_{255} 9_{256} 9_{257} 9_{258} 9_{259} 9_{260} 9_{261} 9_{262} 9_{263} 9_{264} 9_{265} 9_{266} 9_{267} 9_{268} 9_{269} 9_{270}
9_{271} 9_{272} 9_{273} 9_{274} 9_{275} 9_{276} 9_{277} 9_{278} 9_{279} 9_{280} 9_{281} 9_{282} 9_{283} 9_{284} 9_{285} 9_{286} 9_{287} 9_{288} 9_{289} 9_{290}
9_{291} 9_{292} 9_{293} 9_{294} 9_{295} 9_{296} 9_{297} 9_{298}



Proof. The proof follows from L.Lins' thesis and from the discussion about the lengths of the smallest geodesics of the classes 9_{126} and 9_{199} $\square$

5 Definition of Gem

For completeness we briefly recall the basic definitions of gem theory, leading to its definition, [14]. A *4-graph* G is a finite bipartite 4-regular graph whose edges are partitioned into 4 colors, 0,1,2, and 3, so that at each vertex there is an edge of each color, a proper edge-coloration, [3]. For each $i \in \{0, 1, 2, 3\}$, let E_i denote the set of i-colored edges of G. A $\{j, k\}$-residue in a 4-graph G is a connected component of the subgraph induced by $E_j \cup E_k$. A 2-residue is a $\{j, k\}$-residue, for some distinct colors j and k. A *gem* is a 4-graph G such that for each color i, $G\backslash E_i$ can be embedded in the plane such that the boundary of each face is a 2-residue. From a gem there exists a straightforward algorithm to obtain a closed orientable 3-manifold, in two different, dual ways. Every such a manifold is obtainable in this way. An unecessary big gem is obtained from a triangulation T for a manifold by taking the dual of the barycentric subdivision of T. Here the colors corresponds to the dimensions. Doing simplifications in the gem completely destroys this correspondence.

6 The resolution of the doubts left in L. Lins' thesis

The topological classification of the 9-small spaces was nearly completed in [13]. This work develops a theory for generating a distinguished set of blinks named U_n and indexed lexicographically, $U_n[i]$ is the i-th such blink. The relevance of U_n is that it misses no closed, orientable, prime and irreducible 3-manifold which is induced by a blink with n-edges.

The 3-manifolds of [13] are classified by homology and the quantum WRT$_r$-invariants $r = 3, \ldots, u$, up to d decimal digits forming $hgqi_u^d$-classes. Our algorithm for computing the WRT_r^d-invariants are based on the theory developed in [9].

After 6 years we have put our doubts as a Challenge to topologists and group algebraists, [15]. [18]

The basic tool to differentiate the manifold is the software SnapPy, [4].

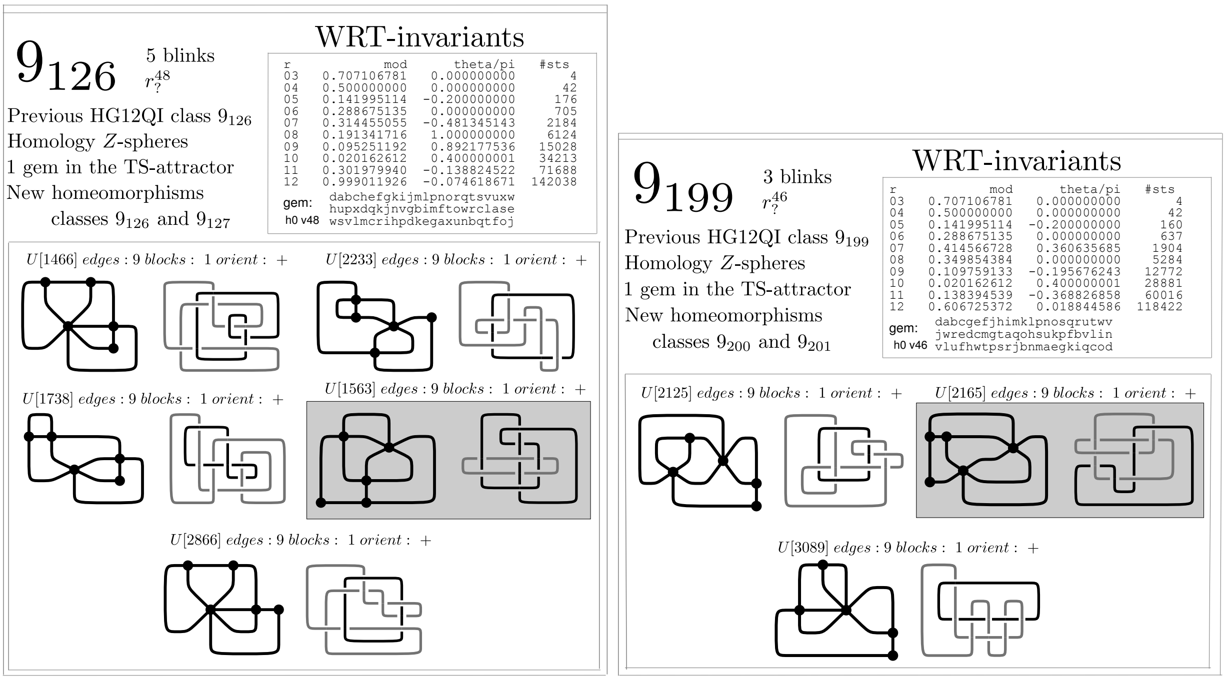
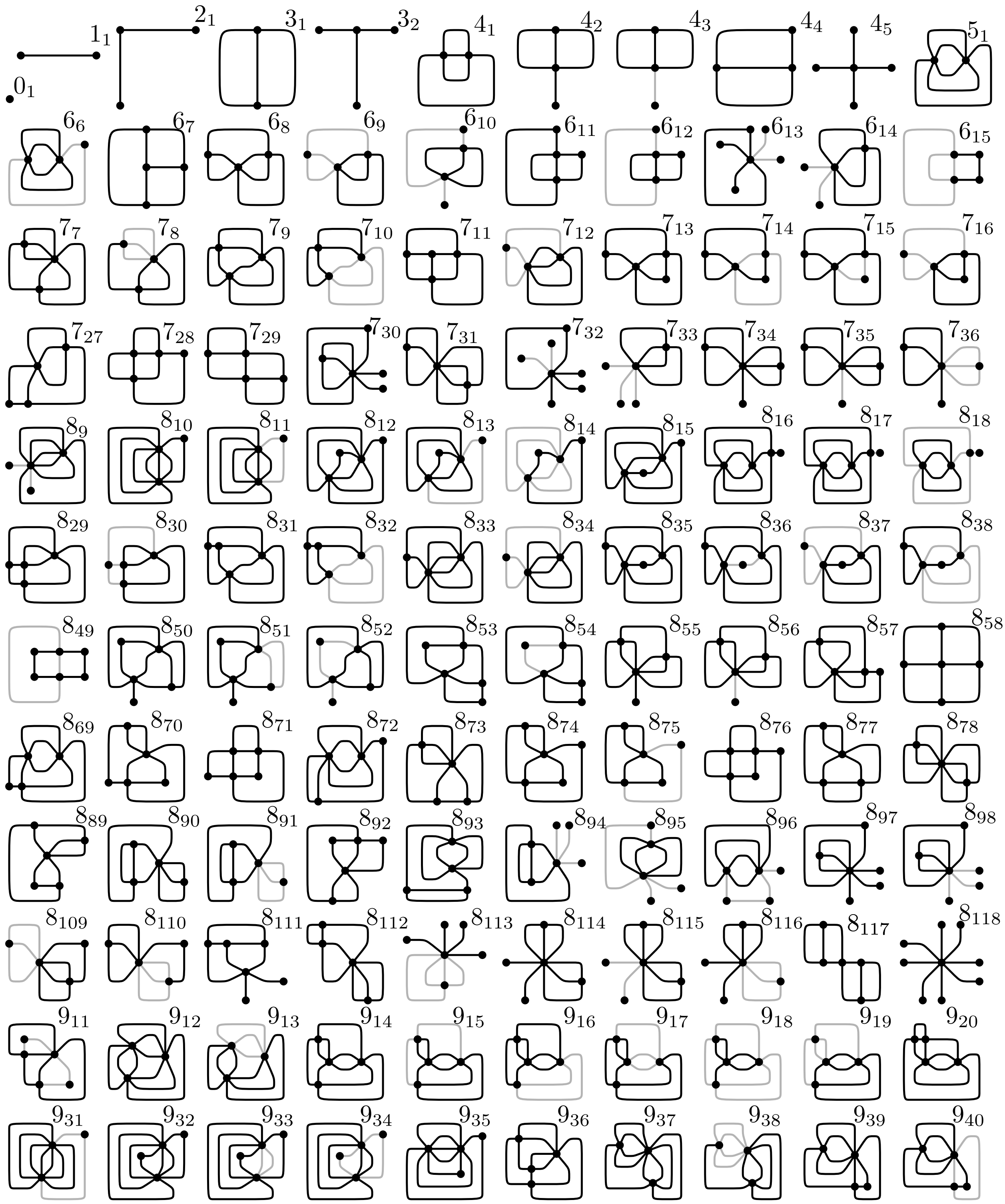


Figure 7: Solving the last two doubts in L. Lins's thesis. The latter are homeomorphic.

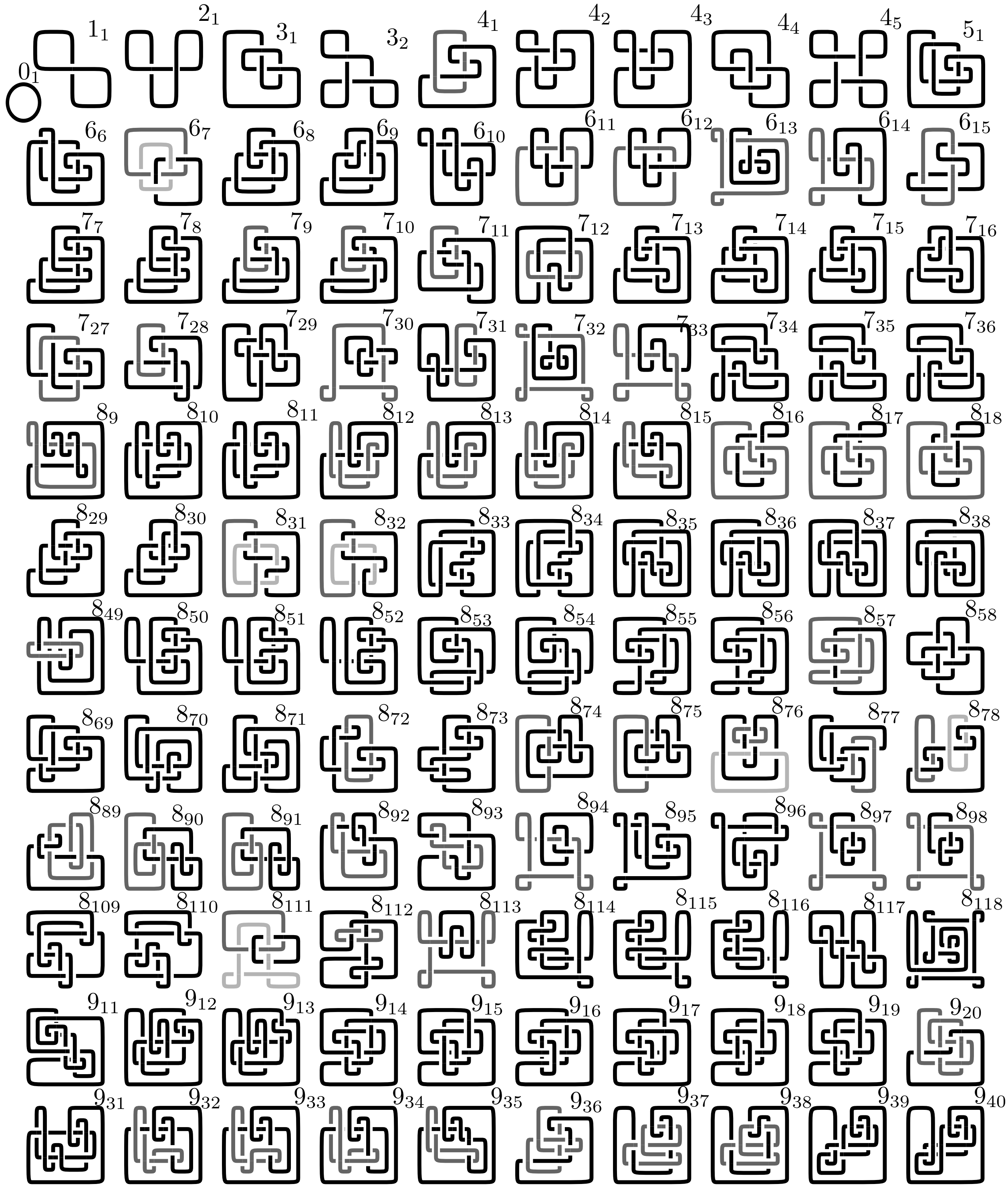
.

7 Appendix: census (no misses, no duplicates) of 9-small 3-manifolds

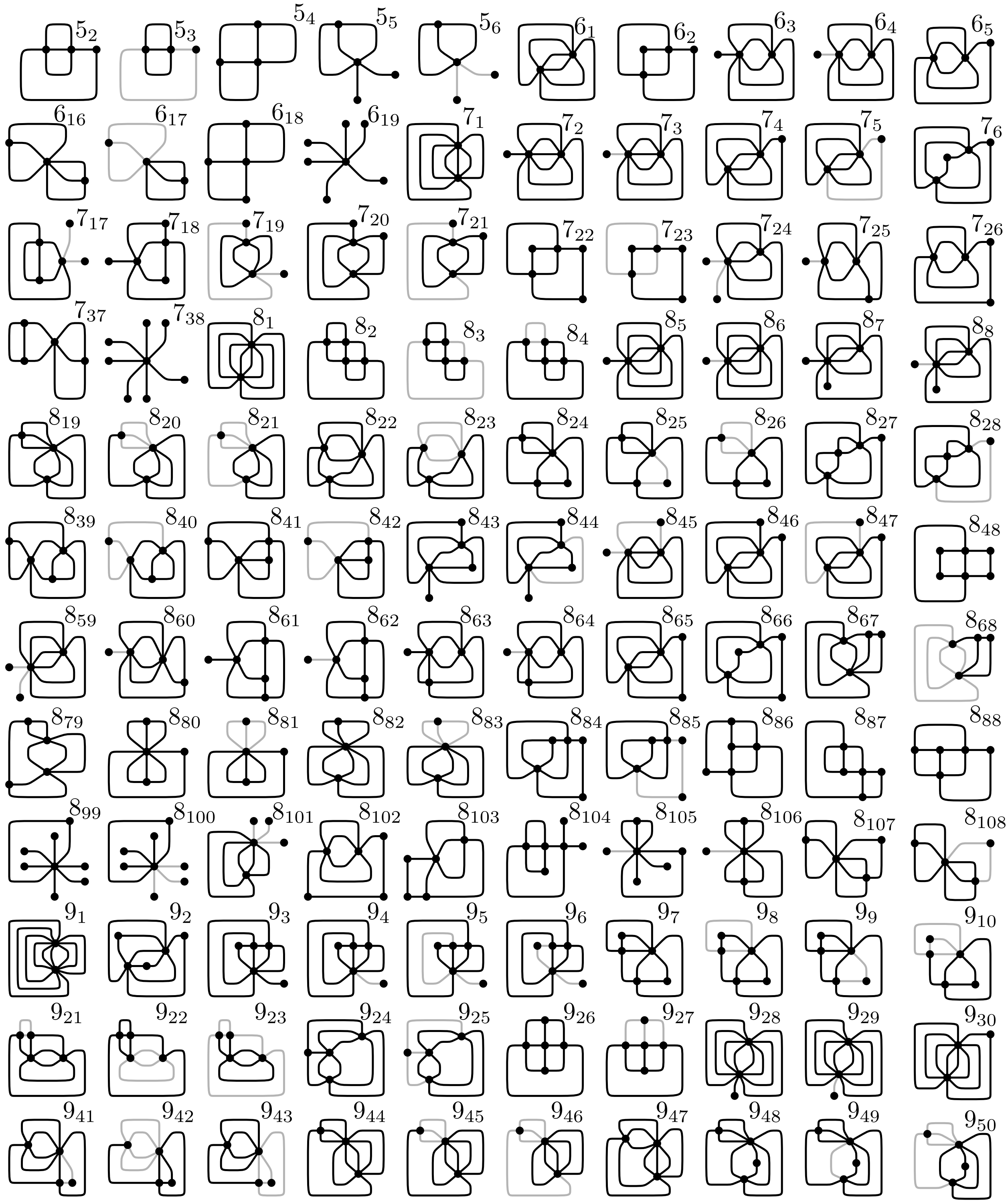
Part 1/4 in terms of blinks:



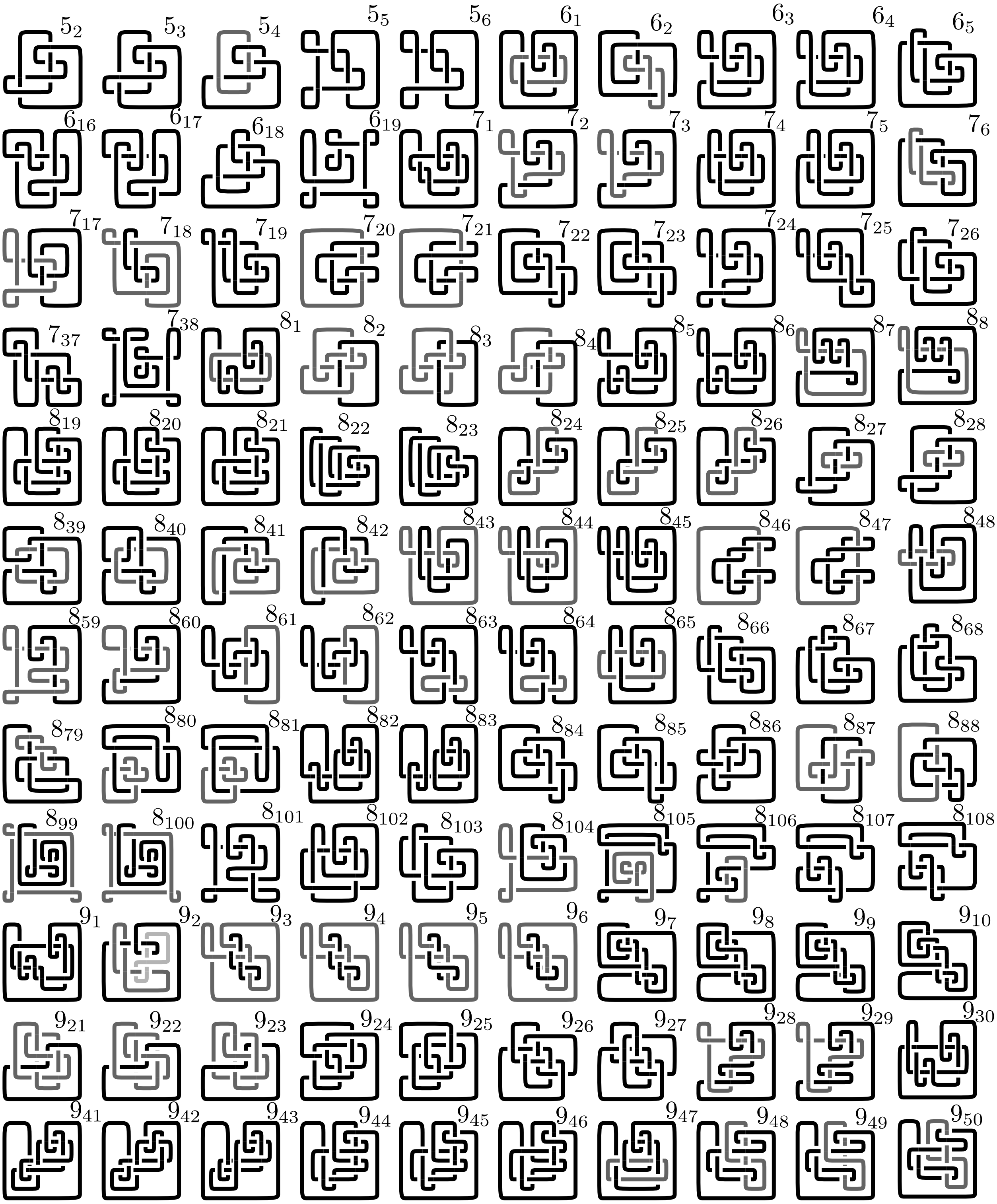
Part 1/4 in terms of blackboard framed links:



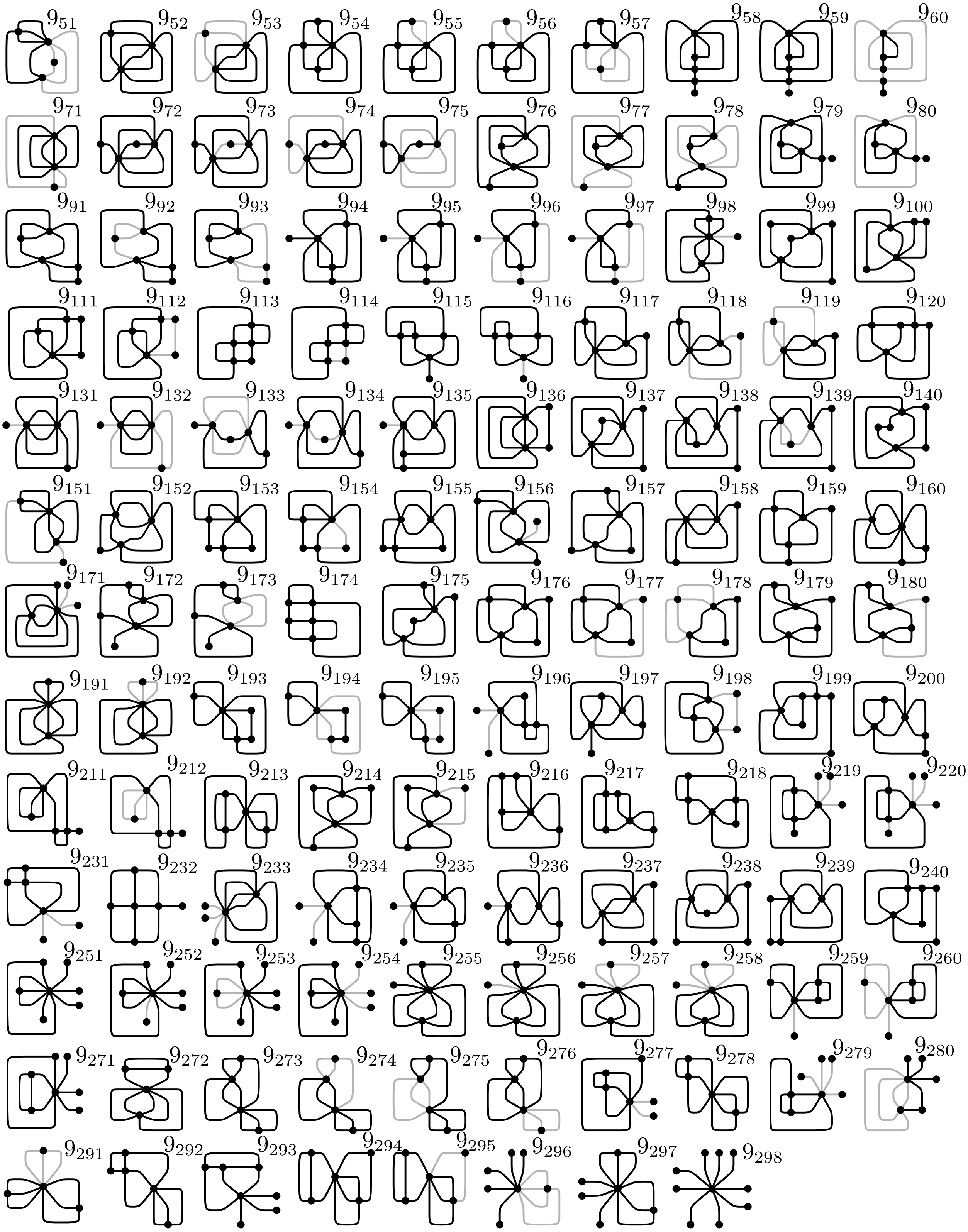
Part 2/4 in terms of blinks:



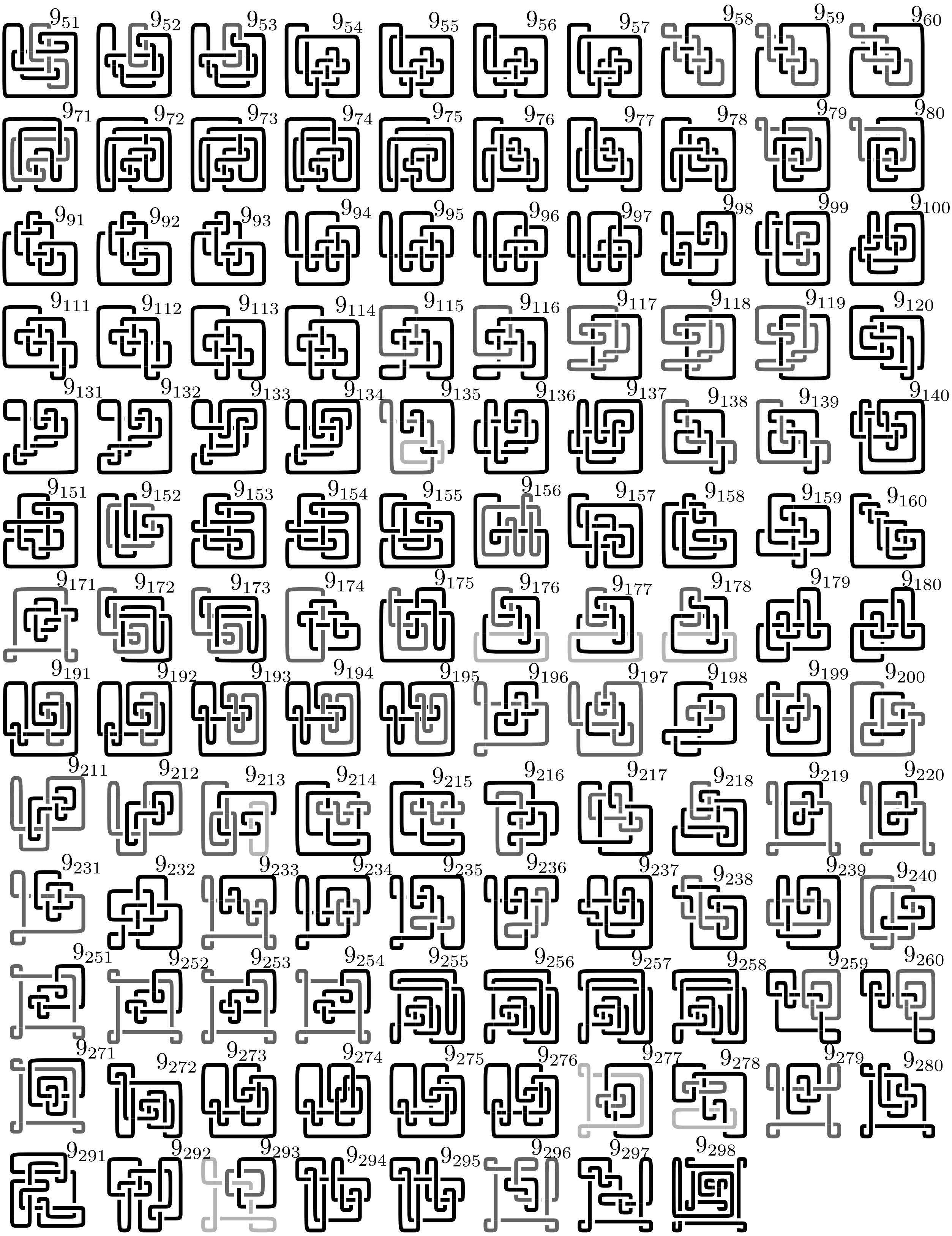
Part 2/4 in terms of blackboard framed links:



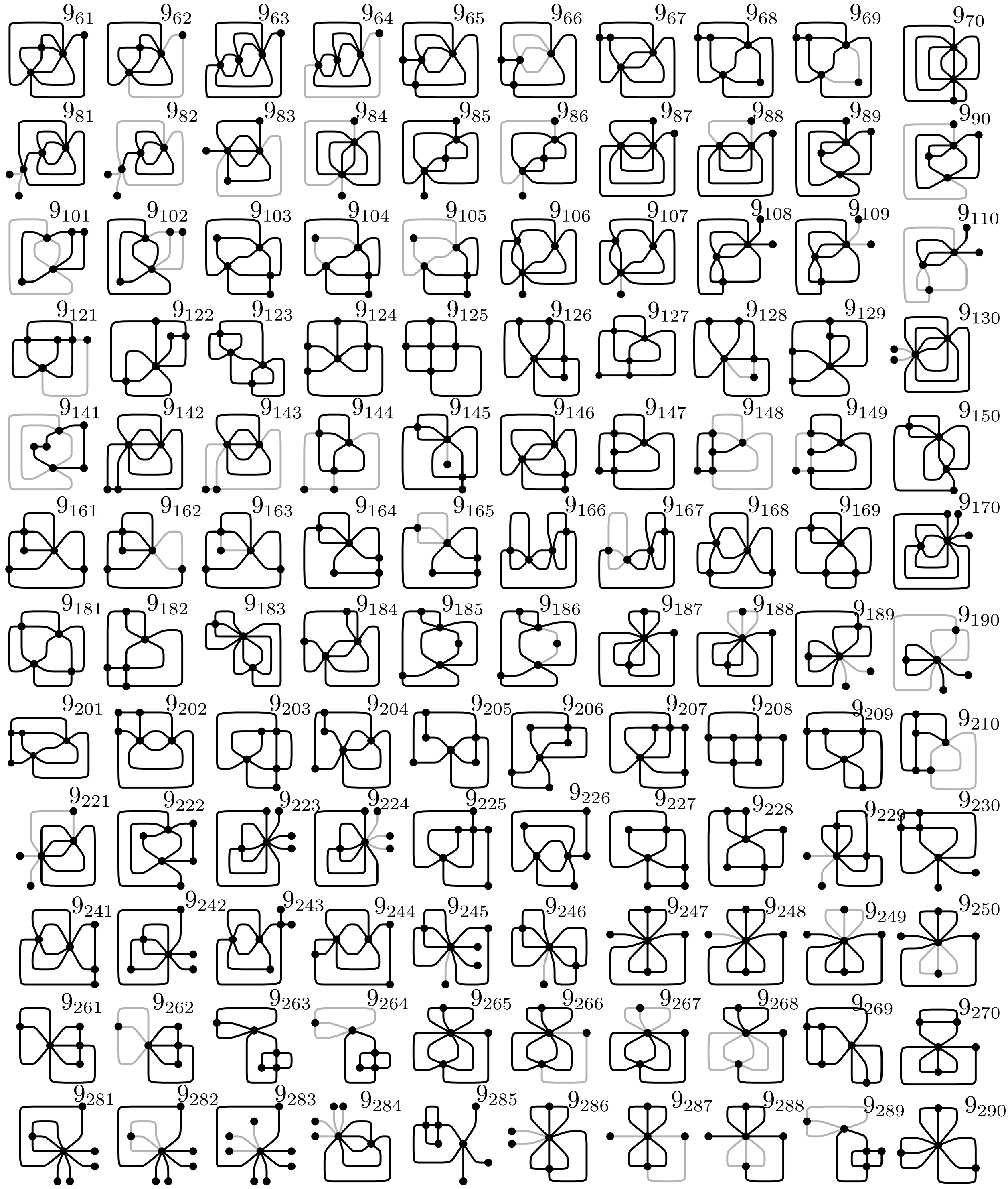
Part 3/4 in terms of blinks:



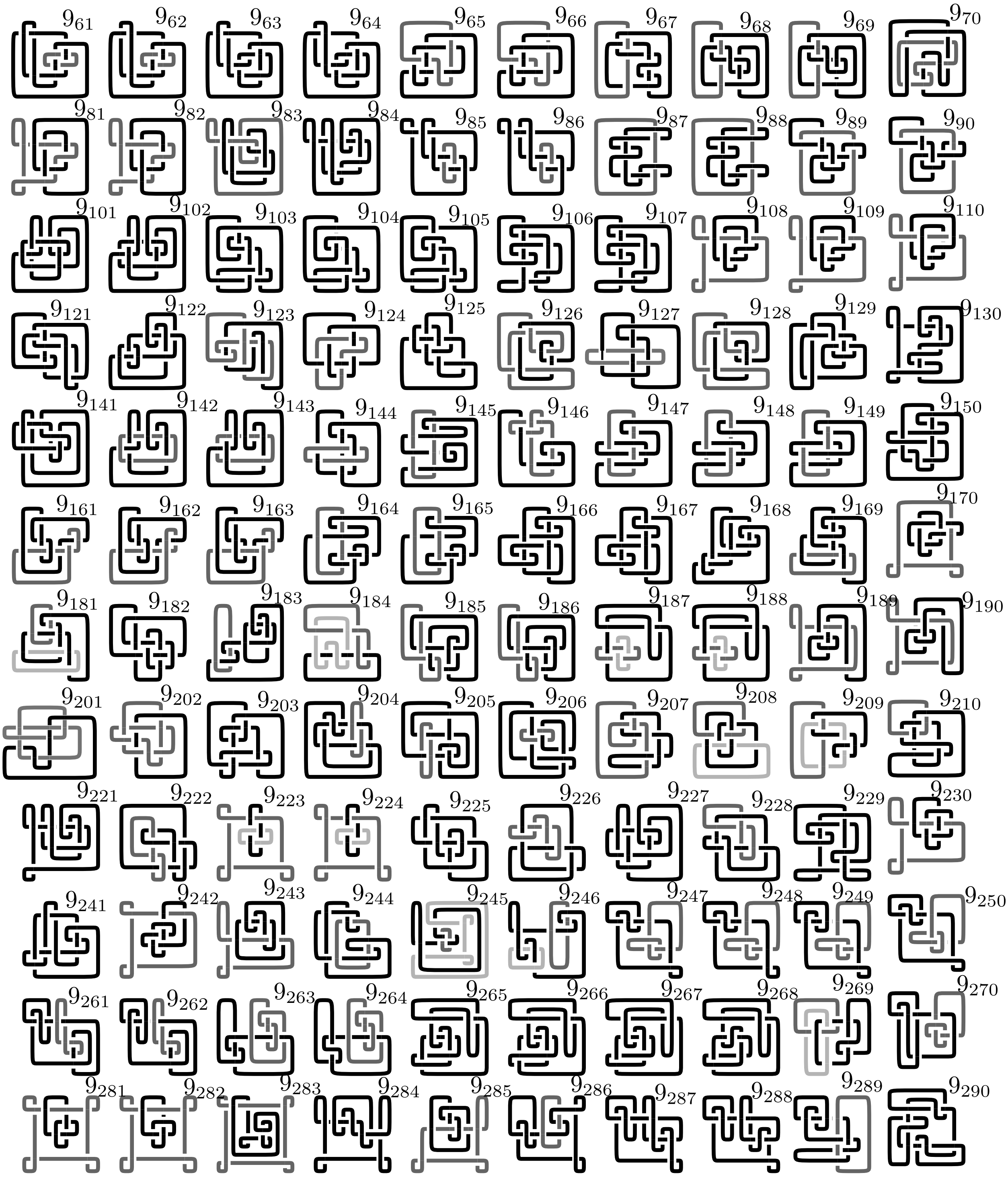
Part 3/4 in terms of blackboard framed links:



Part 4/4 in terms of blinks:



Part 4/4 in terms of blackboard framed links:



8 Conclusion

A closed orientable 3-manifold is denoted *n-small* if it is induced by surgery on a blackboard framed link with at most n crossings. Our bet is that both pairs of 3-manifolds in the 2 first sections of this short note are not homeomorphic. This would mean that the 9-small manifolds are completely classified and that the combinatorial dynamics of Chapter 4 in [14] based on TS-moves which leads to a (small, in the case of hyperbolic 3-manifolds) number of minimal gems, named the *attractor of the 3-manifold* is successful. This induces an efficient algorithm which is capable of classifying topologically all the 3-manifolds given as a blackboard framed link with up to (so far) 9 crossings and maintains live the two Conjectures of page 15 of [14]: the TS- and u^n-moves yield an efficient algorithm to classify n-small 3-manifolds by explicitly displaying homeomorphisms, whenever they exist.

References

[1] P. S. Aleksandroff. *Elementary concepts of topology*, volume 747. Courier Dover Publications, 1961.

[2] P. Alexandroff and D. Hilbert. *Einfachste grundbegriffe der topologie*. Springer Berlin, 1932.

[3] J.A. Bondy and U.S.R. Murty. *Graph theory with applications*. Macmillan London, 1976.

[4] M. Culler, N. M. Dunfield, and J. Weeks. Snappy, a computer program for studying the geometry and topology of 3-manifolds, 2012.

[5] R. Fenn and C. Rourke. On Kirby's calculus of links. *Topology*, 18(1):1–15, 1979.

[6] J. Hempel. *3-Manifolds*, volume 349. Amer Mathematical Society, 1976.

[7] C. D. Hodgson and J. R. Weeks. Symmetries, isometries and length spectra of closed hyperbolic three-manifolds. *Experimental Mathematics*, 3(4):261–274, 1994.

[8] L.H. Kauffman. *Knots and physics*, volume 1. World Scientific Publishing Company, 1991.

[9] L.H. Kauffman and S. Lins. Temperley-Lieb Recoupling Theory and Invariants of 3-manifolds. *Annals of Mathematical Studies, Princeton University Press*, 134:1–296, 1994.

[10] R. Kirby. A calculus for framed links in S^3. *Inventiones Mathematicae*, 45(1):35–56, 1978.

[11] E Klarreich. Getting into shapes: from hyperbolic geometry to cube complexes. *Simons Foundation*, October, 2012.

[12] W.B.R. Lickorish. A representation of orientable combinatorial 3-manifolds. *Annals of Mathematics*, 76(3):531–540, 1962.

[13] L.D. Lins. Blink: a language to view, recognize, classify and manipulate 3D-spaces. *Arxiv preprint math/0702057*, 2007.

[14] S. Lins. *Gems, Computers, and Attractors for 3-Manifolds*. World Scientific, 1995.

[15] S. L. Lins and L. D. Lins. A challenge to 3-manifold topologists and group algebraists. *arXiv:1213.5964v4 [math.GT]*, 2013.

[16] B. Martelli. A finite set of local moves for kirby calculus. *Journal of Knot Theory and Its Ramifications*, 21(14), 2012.

[17] D. Rolfsen. *Knots and links.* American Mathematical Society, 2003.

[18] J. Weeks. SnapPea: a computer program for creating and studying hyperbolic 3-manifolds, 2001.

Sóstenes L. Lins
Centro de Informática, UFPE
Av. Jornalista Anibal Fernandes s/n
Recife, PE 50740-560
Brazil
sostenes@cin.ufpe.br

Lauro D. Lins
AT&T Labs Research
180 Park Avenue
Florham Park, NJ 07932
USA
llins@research.att.com