Closed oriented 3-manifolds are equivalence classes of plane graphs *

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Abstract

A *blink* is a plane graph with an arbitrary bipartition of its edges. As a consequence of a recent result of Martelli, we show that the homeomorphisms classes of closed oriented 3-manifolds are in 1-1 correspondence with classes of blinks. Two blinks are equivalent if they are linked by a finite sequence of local moves, where each move is one of a concrete list of 64 moves: they organize in 8 types, each beeing essentlially the same move on 8 simply related configurations. The size of the list can be substantially decreased at the cost of loosing symmetry.

1 Introduction: knots, links, 3-manifolds and Martelli's moves

In a recent paper B. Martelli [3] presented a reformulation of the Fenn-Rourke version ([1]) of Kirby's calculus [2]. Our objective here is to further reformulate Martelli's moves so as to obtain a calculus of blinks, which is an exact combinatorial counterpart for factorizing homeomorphisms of closed orientable 3-manifolds. This goal is desirable because it has the consequence that each 3-manifold become a subtle class of plane graphs. Our exposition is complete and elementary seeking to reach both audiences: topologists and combinatorialists.

A *knot* is an embedding of a circle, an $\mathbb{S}^1$, into $\mathbb{R}^3$ or $\mathbb{S}^3$. A *link with k components* is an embedding of a disjoint union of k copies of $\mathbb{S}^1$ into $\mathbb{R}^3$ or $\mathbb{S}^3$. In this way, a knot is a link with one component.

Knots and links can be presented by their *decorated general position projections* into a fixed plane $\mathbb{R}^2$. *General position* means that in the image of the link there is no triple points and that at each neighborhood of each double is the transversal crossing of two segments of the link, named *strands*. *Decorated* means that we keep the information of which strand is the upper one, usually by removing a piece of the lower strand. In this paper we use another way to decorate the link projections: the images of the link components are thich black curves and the upper strands are indicated by a thinner white segment inside the thich black curve at the crossing.

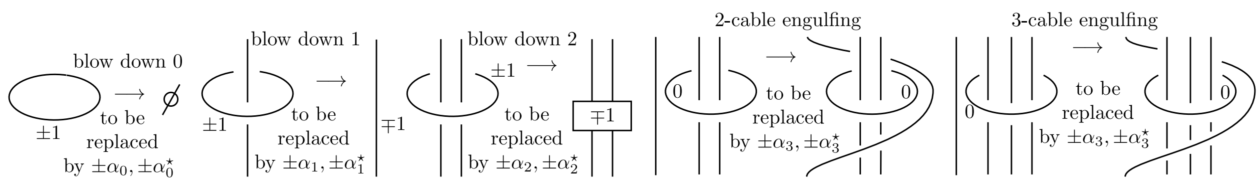

Figure 1: Martelli's calculus on fractional framed links

References

[1] R. Fenn and C. Rourke. On Kirby's calculus of links. *Topology*, 18(1):1–15, 1979.

[2] R. Kirby. A calculus for framed links in S^3. *Inventiones Mathematicae*, 45(1):35–56, 1978.

[3] B. Martelli. A finite set of local moves for kirby calculus. *Journal of Knot Theory and Its Ramifications*, 21(14), 2012.

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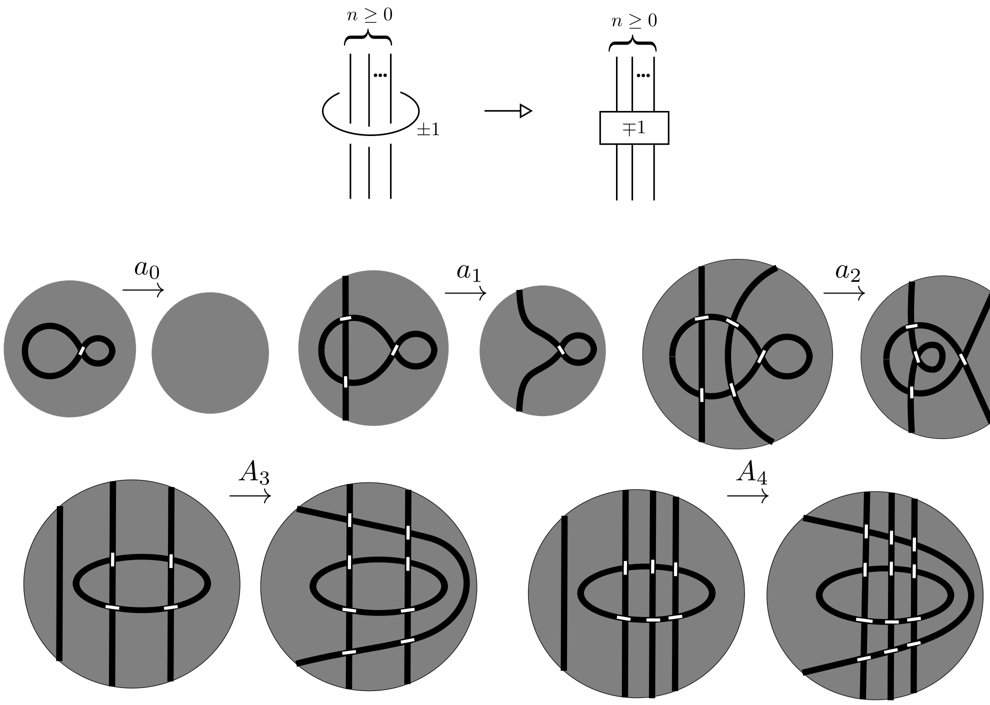

Figure 2: Fenn-Rourke infinite 1-parametrized blown-down moves and Martelli's moves in blackboard framed form

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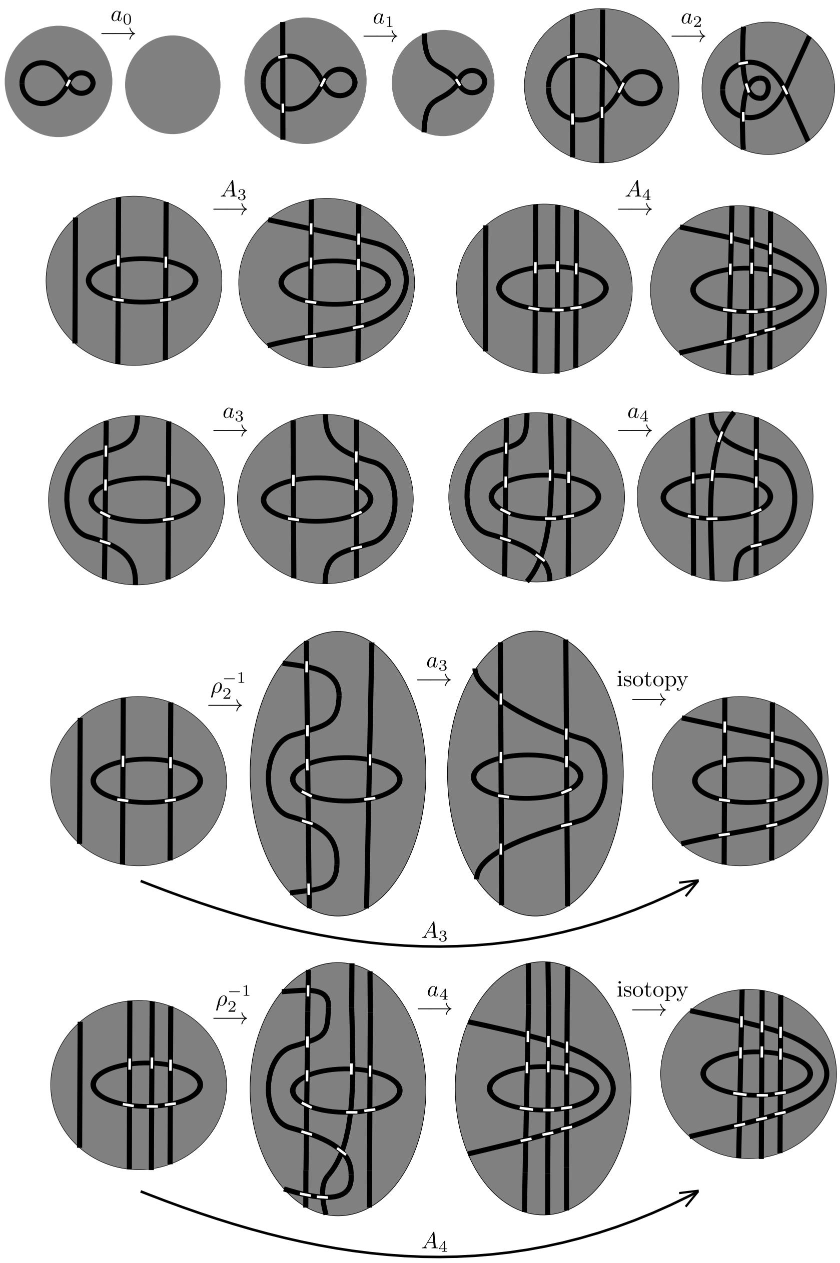

Figure 3: The minimum blink, minimum link and minimum gem inducing the binary tetrahedral space

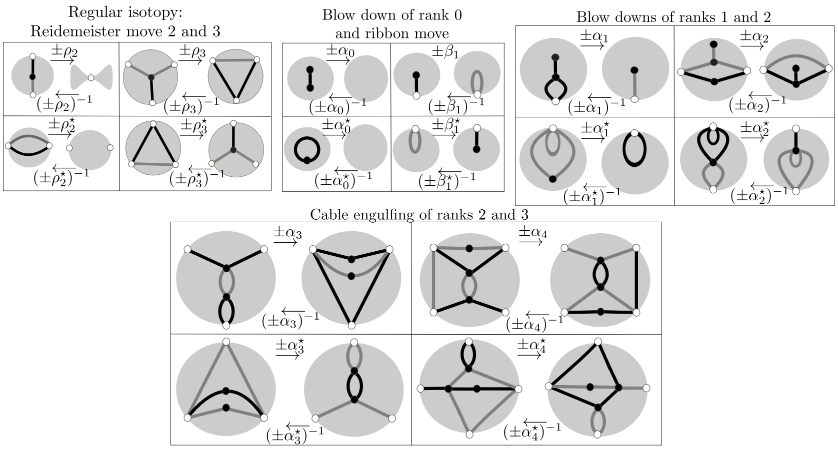

Figure 4: Blink-coin reformulation of Martelli's moves