Derivation of formula for transparency converter

https://github.com/sotanmochi/TransparencyConverter

2022/04/24

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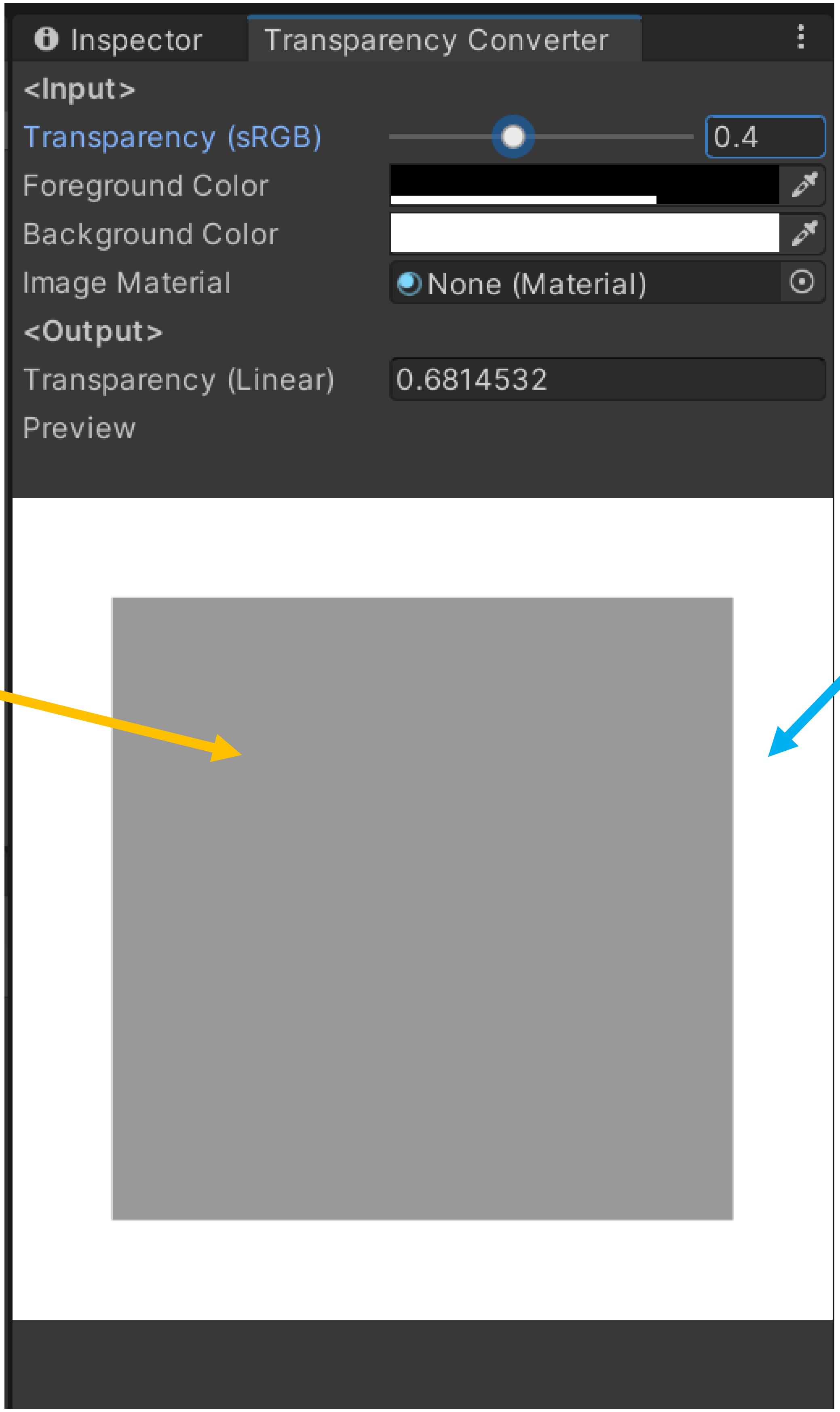
Transparency conversion from sRGB color space to linear color space



Foreground (Transparent)
+
Background (Opaque)

$$(R_f, G_f, B_f, \alpha)$$
$$+$$
$$(R_b, G_b, B_b, 1)$$

Background (Opaque)

$$(R_b, G_b, B_b, 1)$$

Transparency conversion from sRGB color space to linear color space

$$C_{output} = C_f\alpha + C_b(1-\alpha)$$

where C is R, G, and B.

$$C_{linear} = C_f\alpha_{linear} + C_b(1-\alpha_{linear})$$

$$C_{sRGB} = C_f\alpha_{sRGB} + C_b(1-\alpha_{sRGB})$$

$$C_{linear} = g(C_{sRGB})$$

where $g(x)$ is conversion function from sRGB color space to linear color space.

$$C_f\alpha_{linear} + C_b(1-\alpha_{linear}) = g(C_{sRGB})$$

$$\boxed{\alpha_{linear} = \frac{-C_b + g(C_{sRGB})}{C_f - C_b}}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f\alpha_{sRGB} + C_b(1-\alpha_{sRGB})$.

Transparency conversion from sRGB color space to linear color space

$$\alpha_{linear} = \frac{-C_b + g(C_{sRGB})}{C_f - C_b}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f \alpha_{sRGB} + C_b(1 - \alpha_{sRGB})$.

For $g(x) = x^{2.2}$,

$$\alpha_{linear} = \frac{-C_b + \left(C_f \alpha_{sRGB} + C_b(1 - \alpha_{sRGB})\right)^{2.2}}{C_f - C_b}$$

where $C_f \neq C_b$.

Special cases

$\alpha_{linear} = 1 - (1 - \alpha_{sRGB})^{2.2}$ for $C_f = 0$ (black), $C_b = 1$ (white)

$\alpha_{linear} = \alpha_{sRGB}^{2.2}$ for $C_f = 1$ (white), $C_b = 0$ (black)

Transparency conversion from sRGB color space to linear color space

$$\alpha_{linear} = \frac{-C_b + g(C_{sRGB})}{C_f - C_b}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f \alpha_{sRGB} + C_b(1 - \alpha_{sRGB})$.

For the following $g(x)$ (Refer to https://en.wikipedia.org/wiki/SRGB#Transformation)

$$g(x) = \begin{cases} \dfrac{x}{12.92}, & x \leq 0.04050 \\ \left(\dfrac{x + 0.055}{1.055}\right)^{2.4}, & x > 0.04050 \end{cases}$$

$$\alpha_{linear} = \begin{cases} \dfrac{1}{C_f - C_b}\left[-C_b + \dfrac{C_f \alpha_{sRGB} + C_b(1 - \alpha_{sRGB})}{12.92}\right], & C_{sRGB} \leq 0.04050 \\ \dfrac{1}{C_f - C_b}\left[-C_b + \left(\dfrac{C_f \alpha_{sRGB} + C_b(1 - \alpha_{sRGB}) + 0.055}{1.055}\right)^{2.4}\right], & C_{sRGB} > 0.04050 \end{cases}$$

Transparency conversion from sRGB color space to linear color space

$$\alpha_{linear} = \begin{cases} \dfrac{1}{C_f - C_b}\left[-C_b + \dfrac{C_f\alpha_{sRGB} + C_b(1-\alpha_{sRGB})}{12.92}\right], & C_{sRGB} \leq 0.04050 \\ \dfrac{1}{C_f - C_b}\left[-C_b + \left(\dfrac{C_f\alpha_{sRGB} + C_b(1-\alpha_{sRGB}) + 0.055}{1.055}\right)^{2.4}\right], & C_{sRGB} > 0.04050 \end{cases}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f\alpha_{sRGB} + C_b(1-\alpha_{sRGB})$.

Special cases

For $C_f = 0$ (black) and $C_b = 1$ (white),

$$\alpha_{linear} = \begin{cases} 1 - \dfrac{1-\alpha_{sRGB}}{12.92}, & 1-\alpha_{sRGB} \leq 0.04050 \\ 1 - \left(\dfrac{1-\alpha_{sRGB} + 0.055}{1.055}\right)^{2.4}, & 1-\alpha_{sRGB} > 0.04050 \end{cases}$$

For $C_f = 1$ (white) and $C_b = 0$ (black),

$$\alpha_{linear} = \begin{cases} \dfrac{\alpha_{sRGB}}{12.92}, & \alpha_{sRGB} \leq 0.04050 \\ \left(\dfrac{\alpha_{sRGB} + 0.055}{1.055}\right)^{2.4}, & \alpha_{sRGB} > 0.04050 \end{cases}$$