Transparency conversion from sRGB to Linear color space

https://github.com/sotanmochi/TransparencyConverter

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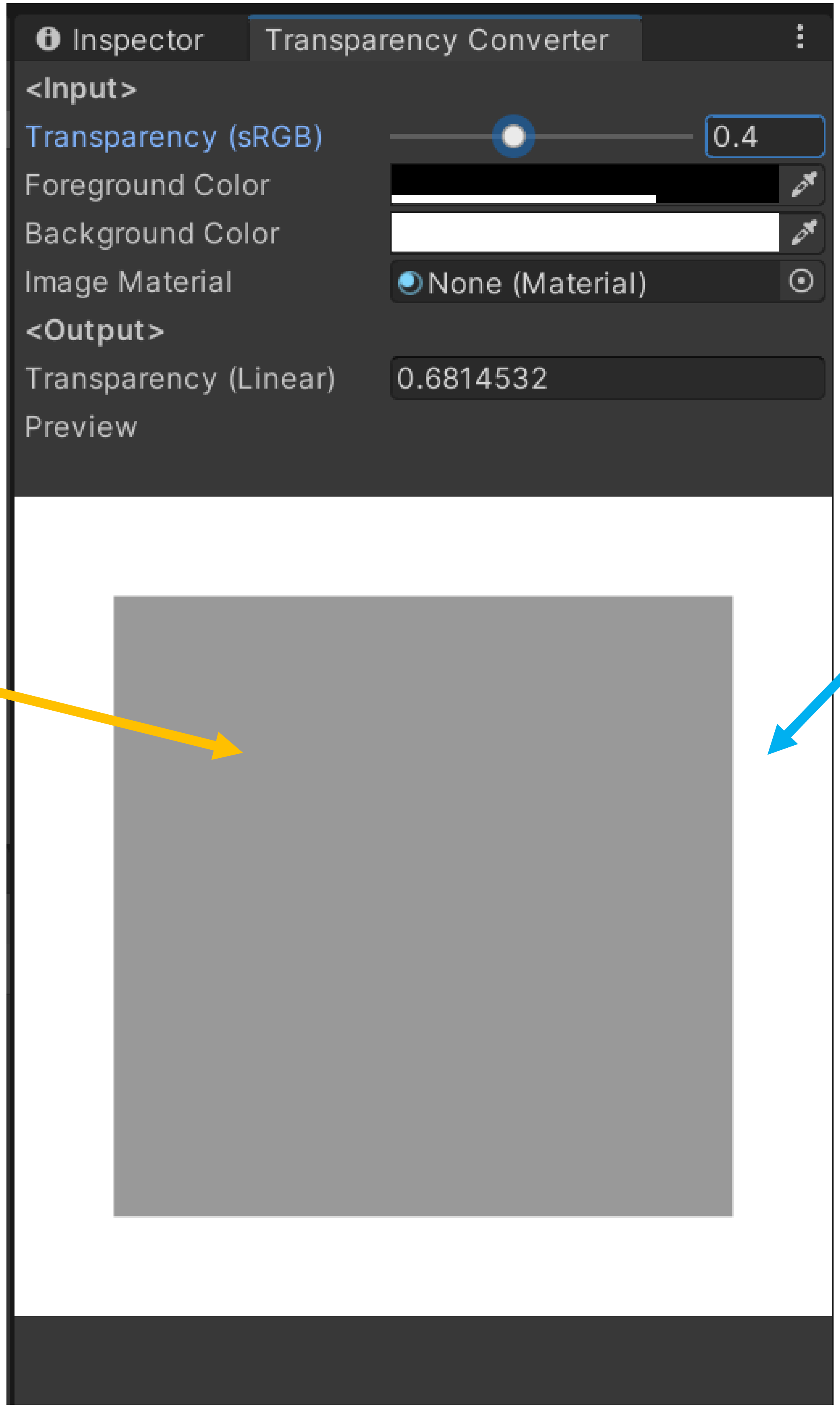
Latest update: 2022/05/08

Create: 2022/04/24

Transparency conversion from sRGB color space to linear color space

Foreground (Transparent)
+
Background (Opaque)

$$(R_F, G_F, B_F, \alpha)$$
$$+$$
$$(R_B, G_B, B_B, 1)$$



Background (Opaque)

$$(R_B, G_B, B_B, 1)$$

Transparency conversion from sRGB color space to linear color space

$$C_{output} = C_F\alpha + C_B(1-\alpha)$$

where $0 \leq c_F \leq 1$, $0 \leq c_B \leq 1$ and $0 \leq \alpha \leq 1$.

$$C_{linear} = C_F\alpha_{linear} + C_B(1-\alpha_{linear})$$

$$C_{sRGB} = C_F\alpha_{sRGB} + C_B(1-\alpha_{sRGB})$$

$$C_{linear} = T(C_{sRGB})$$

where $T(x)$ is transfer function from sRGB color space to linear color space.

$$C_F\alpha_{linear} + C_B(1-\alpha_{linear}) = T(C_{sRGB})$$

$$\alpha_{linear} = T'(C_F, C_B, \alpha_{sRGB}) = \frac{-C_B + T(C_{sRGB})}{C_F - C_B}$$

where $C_F \neq C_B$ and $C_{sRGB} = C_F\alpha_{sRGB} + C_B(1-\alpha_{sRGB})$.

$$C_{output} = C_F\alpha + C_B(1-\alpha)$$

where $0 \le c_F \le 1$, $0 \le c_B \le 1$ and $0 \le \alpha \le 1$.

$$R_{output} = R_F\alpha + R_B(1-\alpha)$$
$$G_{output} = G_F\alpha + G_B(1-\alpha)$$
$$B_{output} = B_F\alpha + B_B(1-\alpha)$$

$$\alpha_{linear} = T'(R_F, R_B, \alpha_{sRGB}) = T'(G_F, G_B, \alpha_{sRGB}) = T'(B_F, B_B, \alpha_{sRGB})$$

$$\boxed{\begin{aligned}\alpha_{linear} &= T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B) \\ &= \frac{1}{3}[T'(R_F, R_B, \alpha_{sRGB}) + T'(G_F, G_B, \alpha_{sRGB}) + T'(B_F, B_B, \alpha_{sRGB})]\end{aligned}}$$

Considering the constraint $0 \le \alpha_{linear} \le 1$

$$\alpha_{linear} = \frac{T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B) - T''_{MIN}}{T''_{MAX} - T''_{MIN}}$$

When $T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B) = T''_{MAX} \;\rightarrow\; \alpha_{linear} = 1$

When $T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B) = T''_{MIN} \;\rightarrow\; \alpha_{linear} = 0$

When $T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B) = (T''_{MAX} + T''_{MIN})/2 \;\rightarrow\; \alpha_{linear} = 0.5$

$T(R_F, R_B, \alpha_{sRGB}), T(G_F, G_B, \alpha_{sRGB}), T(B_F, B_B, \alpha_{sRGB})$ are monotonically increasing for α_{sRGB} .

The function $T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B)$ is monotonically increasing for α_{sRGB} .

where $R_F \neq R_B, G_F \neq G_B, B_F \neq B_B$

$$T''_{MAX} = T'(R_F, R_B, 1) + T'(G_F, G_B, 1) + T'(B_F, B_B, 1)$$

$$T''_{MIN} = T'(R_F, R_B, 0) + T'(G_F, G_B, 0) + T'(B_F, B_B, 0)$$

$$T''_{MAX} = T'(R_F, R_B, 1) + T'(G_F, G_B, 1) + T'(B_F, B_B, 1)$$

$$= \frac{T(R_F) - R_B}{R_F - R_B} + \frac{T(G_F) - G_B}{G_F - G_B} + \frac{T(B_F) - B_B}{B_F - B_B}$$

$$T''_{MIN} = T'(R_F, R_B, 0) + T'(G_F, G_B, 0) + T'(B_F, B_B, 0)$$

$$= \frac{T(R_B) - R_B}{R_F - R_B} + \frac{T(G_B) - G_B}{G_F - G_B} + \frac{T(B_B) - B_B}{B_F - B_B}$$

$$T''_{MAX} - T''_{MIN} = \frac{T(R_F) - T(R_B)}{R_F - R_B} + \frac{T(G_F) - g(G_B)}{G_F - G_B} + \frac{T(B_F) - T(B_B)}{B_F - B_B}$$

$$T''_{MAX} - T''_{MIN} = \frac{\big(T(R_F) - T(R_B)\big)(G_F - G_B)(B_F - B_B)}{(R_F - R_B)(G_F - G_B)(B_F - B_B)}$$

$$+ \frac{(R_F - R_B)(T(G_F) - T(G_B))(B_F - B_B)}{(R_F - R_B)(G_F - G_B)(B_F - B_B)}$$

$$+ \frac{(R_F - R_B)(G_F - G_B)((T(B_F) - T(B_B))}{(R_F - R_B)(G_F - G_B)(B_F - B_B)}$$

$$T''_{MAX} - T''_{MIN} = \frac{\big(T(R_F) - T(R_B)\big)(G_F - G_B)(B_F - B_B)}{(R_F - R_B)(G_F - G_B)(B_F - B_B)} + \frac{(R_F - R_B)(T(G_F) - T(G_B))(B_F - B_B)}{(R_F - R_B)(G_F - G_B)(B_F - B_B)} + \frac{(R_F - R_B)(G_F - G_B)(T(B_F) - T(B_B))}{(R_F - R_B)(G_F - G_B)(B_F - B_B)}$$

$$T''(\alpha_{sRGB}, R_F, R_B, G_F, G_B, B_F, B_B) - T''_{MIN} = \frac{\big(T(R_{sRGB}) - T(R_B)\big)(G_F - G_B)(B_F - B_B)}{(R_F - R_B)(G_F - G_B)(B_F - B_B)} + \frac{(R_F - R_B)(T(G_{sRGB}) - T(G_B))(B_F - B_B)}{(R_F - R_B)(G_F - G_B)(B_F - B_B)} + \frac{(R_F - R_B)(G_F - G_B)(T(B_{sRGB}) - T(B_B))}{(R_F - R_B)(G_F - G_B)(B_F - B_B)}$$

$$R_{sRGB} = R_F \alpha_{sRGB} + R_B(1 - \alpha_{sRGB})$$
$$G_{sRGB} = G_F \alpha_{sRGB} + G_B(1 - \alpha_{sRGB})$$
$$B_{sRGB} = B_F \alpha_{sRGB} + B_B(1 - \alpha_{sRGB})$$

$$\alpha_{linear} = \frac{\big(T(R_{sRGB}) - T(R_B)\big)(G_F - G_B)(B_F - B_B) + (R_F - R_B)(T(G_{sRGB}) - T(G_B))(B_F - B_B) + (R_F - R_B)(G_F - G_B)(T(B_{sRGB}) - T(B_B))}{\big(T(R_F) - T(R_B)\big)(G_F - G_B)(B_F - B_B) + (R_F - R_B)(T(G_F) - T(G_B))(B_F - B_B) + (R_F - R_B)(G_F - G_B)(T(B_F) - T(B_B))}$$

where $R_F \neq R_B, G_F \neq G_B, B_F \neq B_B$

$$R_{sRGB} = R_F\alpha_{sRGB} + R_B(1 - \alpha_{sRGB})$$

$$G_{sRGB} = G_F\alpha_{sRGB} + G_B(1 - \alpha_{sRGB})$$

$$B_{sRGB} = B_F\alpha_{sRGB} + B_B(1 - \alpha_{sRGB})$$