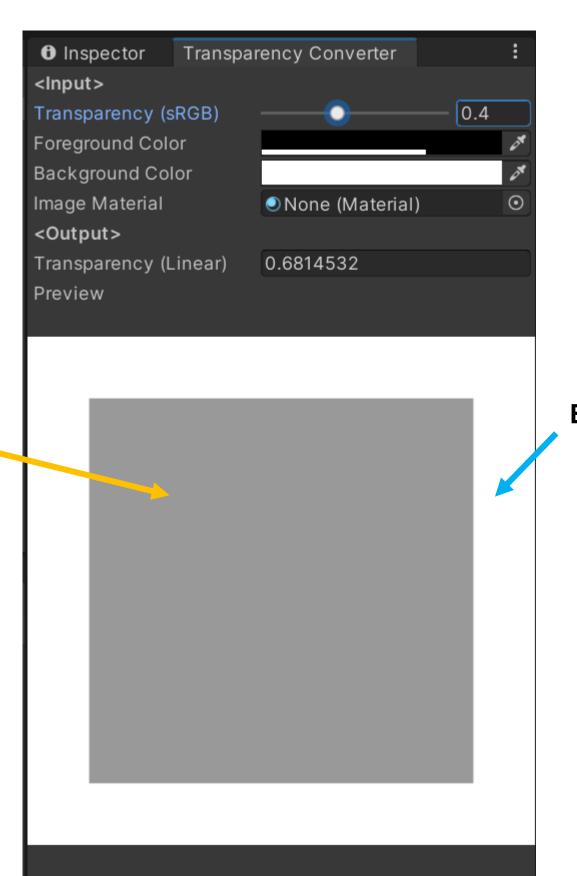
Derivation of formula for transparency converter

https://github.com/sotanmochi/TransparencyConverter

2022/04/24 Soichiro Sugimoto



Foreground (Transparent)

Background (Opaque)

$$(R_f, G_f, B_f, \alpha) + (R_b, G_b, B_b, 1)$$

Background (Opaque)

 $(R_b, G_b, B_b, 1)$

$$C_{output} = C_f \alpha + C_b (1 - \alpha)$$

where C is R, G, and B.

$$C_{linear} = C_f \alpha_{linear} + C_b (1 - \alpha_{linear})$$

$$C_{SRGB} = C_f \alpha_{SRGB} + C_b (1 - \alpha_{SRGB})$$

$$C_{linear} = g(C_{sRGB})$$

where g(x) is conversion function from sRGB color space to linear color space.

$$C_f \alpha_{linear} + C_b (1 - \alpha_{linear}) = g(C_{sRGB})$$

$$\alpha_{linear} = \frac{-C_b + g(C_{sRGB})}{C_f - C_b}$$
 where $C_f \neq C_b$ and $C_{sRGB} = C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})$.

$$\alpha_{linear} = \frac{-C_b + g(C_{SRGB})}{C_f - C_b}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})$.

For
$$g(x) = x^{2.2}$$
,

$$\alpha_{linear} = \frac{-C_b + \left(C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})\right)^{2.2}}{C_f - C_b}$$

where $C_f \neq C_b$.

Special cases

$$\alpha_{linear} = 1 - (1 - \alpha_{SRGB})^{2.2}$$
 for $C_f = 0$ (black), $C_b = 1$ (white)

$$\alpha_{linear} = \alpha_{SRGB}^{2.2}$$
 for $C_f = 1$ (white), $C_b = 0$ (black)

$$\alpha_{linear} = \frac{-C_b + g(C_{SRGB})}{C_f - C_b}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})$.

For the following g(x) (Refer to https://en.wikipedia.org/wiki/SRGB#Transformation)

$$g(x) = \begin{cases} \frac{x}{12.92}, & x \le 0.04050\\ \left(\frac{x + 0.055}{1.055}\right)^{2.4}, & x > 0.04050 \end{cases}$$

$$\alpha_{linear} = \begin{cases} \frac{C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})}{12.92}, & C_{sRGB} \leq 0.04050\\ \left(\frac{C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB}) + 0.055}{1.055}\right)^{2.4}, & C_{sRGB} > 0.04050 \end{cases}$$

$$\alpha_{linear} = \begin{cases} \frac{C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})}{12.92}, & C_{sRGB} \leq 0.04050\\ \left(\frac{C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB}) + 0.055}{1.055}\right)^{2.4}, & C_{sRGB} > 0.04050 \end{cases}$$

where $C_f \neq C_b$ and $C_{sRGB} = C_f \alpha_{sRGB} + C_b (1 - \alpha_{sRGB})$.

Special cases

For $C_f = 0$ (black) and $C_b = 1$ (white),

$$\alpha_{linear} = \begin{cases} \frac{1 - \alpha_{sRGB}}{12.92}, & 1 - \alpha_{sRGB} \le 0.04050\\ \frac{1 - \alpha_{sRGB} + 0.055}{1.055} \end{cases}^{2.4}, & 1 - \alpha_{sRGB} > 0.04050 \end{cases}$$

For $C_f = 1$ (white) and $C_b = 0$ (black),

$$\alpha_{linear} = \begin{cases} \frac{\alpha_{sRGB}}{12.92}, & \alpha_{sRGB} \leq 0.04050\\ \frac{\alpha_{sRGB} + 0.055}{1.055} \end{cases}^{2.4}, & \alpha_{sRGB} > 0.04050$$