Derivation of formula for transparency converter

https://github.com/sotanmochi/TransparencyConverter

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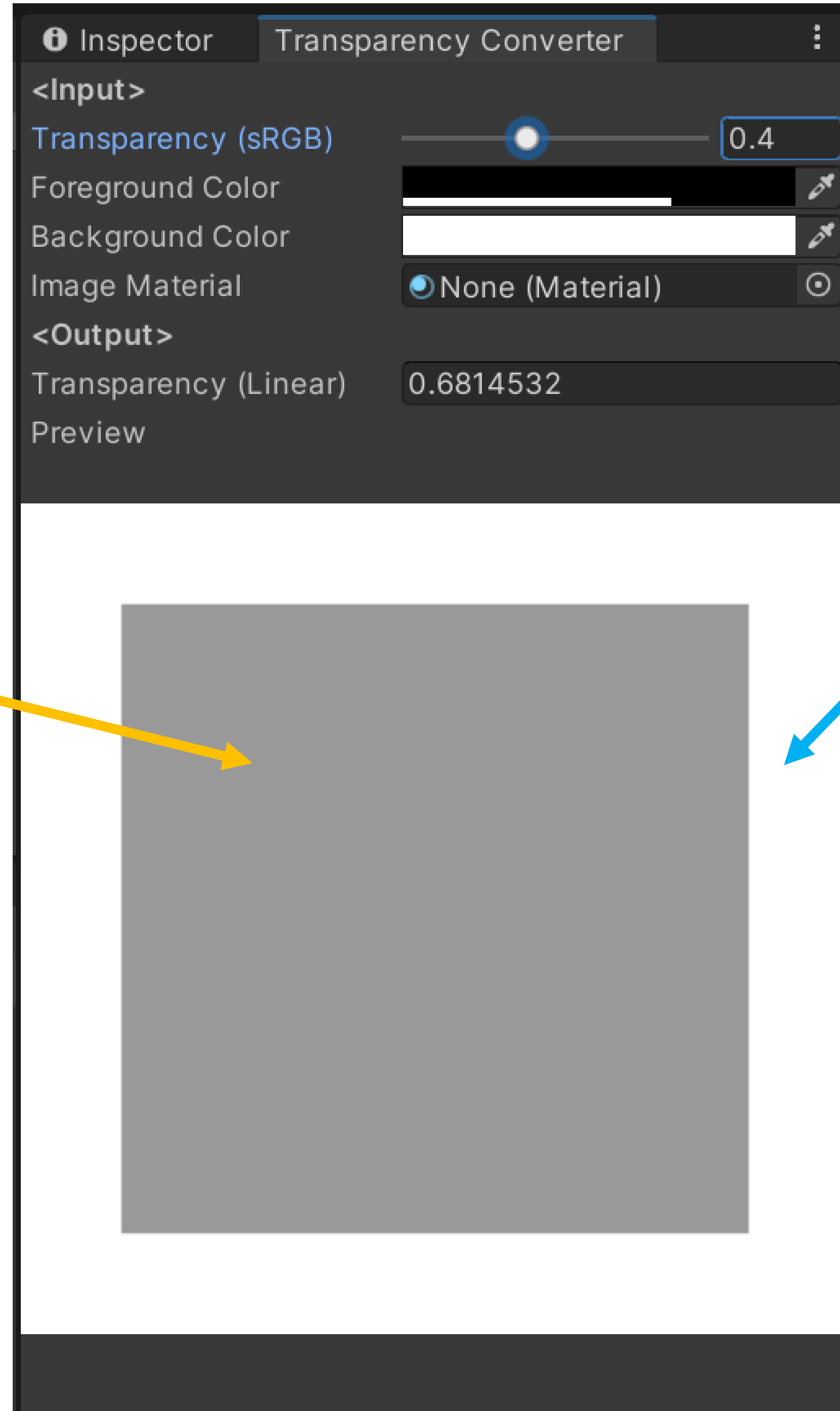
Transparency conversion from sRGB color space to linear color space



Foreground (Transparent)
+
Background (Opaque)

$$(R_F, G_F, B_F, \alpha)$$
$$+$$
$$(R_B, G_B, B_B, 1)$$

Background (Opaque)

$$(R_B, G_B, B_B, 1)$$

Transparency conversion from sRGB color space to linear color space

$$C_{output} = C_F\alpha + C_B(1-\alpha)$$

where $0 \le c_F \le 1$, $0 \le c_B \le 1$ and $0 \le \alpha \le 1$.

$$C_{linear} = C_F\alpha_{linear} + C_B(1-\alpha_{linear})$$

$$C_{sRGB} = C_F\alpha_{sRGB} + C_B(1-\alpha_{sRGB})$$

$$C_{linear} = g(C_{sRGB})$$

where $g(x)$ is conversion function from sRGB color space to linear color space.

$$C_F\alpha_{linear} + C_B(1-\alpha_{linear}) = g(C_{sRGB})$$

$$\alpha_{linear} = f(C_F, C_B, \alpha_{sRGB}) = \frac{-C_B + g(C_{sRGB})}{C_F - C_B}$$

where $C_F \ne C_B$ and $C_{sRGB} = C_F\alpha_{sRGB} + C_B(1-\alpha_{sRGB})$.

$$\alpha_{linear} = f(C_F, C_B, \alpha_{sRGB}) = \frac{-C_B + g(C_{sRGB})}{C_F - C_B}$$

where $C_F \neq C_B$ and $C_{sRGB} = C_F\alpha_{sRGB} + C_B(1 - \alpha_{sRGB})$.

Considering the following constraints.

$$0 \leq \alpha_{linear} \leq 1$$

$$\boxed{\begin{aligned} \alpha_{linear} &= \frac{f(C_F, C_B, \alpha_{sRGB}) - f_{MIN}}{f_{MAX} - f_{MIN}} \\ f(C_F, C_B, \alpha_{sRGB}) &= \frac{-C_B + g(C_{sRGB})}{C_F - C_B} \end{aligned}}$$

$$f(C_F, C_B, \alpha_{sRGB}) = f_{MAX} \rightarrow \alpha_{linear} = 1$$

$$f(C_F, C_B, \alpha_{sRGB}) = f_{MIN} \rightarrow \alpha_{linear} = 0$$

$$f(C_F, C_B, \alpha_{sRGB}) = (f_{MAX} + f_{MIN})/2 \rightarrow \alpha_{linear} = 0.5$$

The conversion function $g(x)$ is monotonically increasing function.

When $C_F > C_B$, the funcfion $g(C_{sRGB}) = g\left(C_F \alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\right)$ is monotonically increasing for α_{sRGB} .

When $C_F < C_B$, the funcfion $g(C_{sRGB}) = g\left(C_F \alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\right)$ is monotonically decreasing for α_{sRGB} .

The funcfion $f(C_F, C_B, \alpha_{sRGB})$ is monotonically increasing for all α_{sRGB} .

$$f_{MAX} = f(C_F, C_B, 1) = \frac{-C_B + g(C_F)}{C_F - C_B}$$

$$f_{MIN} = f(C_F, C_B, 0) = \frac{-C_B + g(C_B)}{C_F - C_B}$$

$$\alpha_{linear} = \frac{f(C_F, C_B, \alpha_{sRGB}) - f_{MIN}}{f_{MAX} - f_{MIN}}$$

$$\alpha_{linear} = \frac{\dfrac{-C_B + g(C_F\alpha_{sRGB} + C_B(1 - \alpha_{sRGB}))}{C_F - C_B} - \dfrac{-C_B + g(C_B)}{C_F - C_B}}{\dfrac{-C_B + g(C_F)}{C_F - C_B} - \dfrac{-C_B + g(C_B)}{C_F - C_B}}$$

$$= \frac{\dfrac{g\big(C_F\alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\big) - g(C_B)}{C_F - C_B}}{\dfrac{g(C_F) - g(C_B)}{C_F - C_B}}$$

$$= \frac{g\big(C_F\alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\big) - g(C_B)}{g(C_F) - g(C_B)}$$

Therefore,

$$\alpha_{linear} = \frac{g\left(C_F\alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\right) - g(C_B)}{g(C_F) - g(C_B)}$$

where $0 \le c_F \le 1, \;\; 0 \le c_B \le 1, \;\; 0 \le \alpha_{sRGB} \le 1$
and $g(x)$ is conversion function from sRGB color space to linear color space.

Transparency conversion from sRGB color space to linear color space

$$\alpha_{linear} = \frac{g\left(C_F \alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\right) - g(C_B)}{g(C_F) - g(C_B)}$$

where $0 \leq c_F \leq 1,\ \ 0 \leq c_B \leq 1,\ \ 0 \leq \alpha_{sRGB} \leq 1$
and $g(x)$ is conversion function from sRGB color space to linear color space.

When $g(x) = x^{2.2}$,

$$\alpha_{linear} = \frac{\left(C_F \alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\right)^{2.2} - C_B^{2.2}}{C_F^{2.2} - C_B^{2.2}}$$

Special cases

$$\alpha_{linear} = 1 - (1 - \alpha_{sRGB})^{2.2}$$ for $C_F = 0$ (black), $C_B = 1$ (white)

$$\alpha_{linear} = \alpha_{sRGB}^{2.2}$$ for $C_F = 1$ (white), $C_B = 0$ (black)

Transparency conversion from sRGB color space to linear color space

$$\alpha_{linear} = \frac{g\left(C_F \alpha_{sRGB} + C_B(1 - \alpha_{sRGB})\right) - g(C_B)}{g(C_F) - g(C_B)}$$

where $0 \leq c_F \leq 1,\ \ 0 \leq c_B \leq 1,\ \ 0 \leq \alpha_{sRGB} \leq 1$
and $g(x)$ is conversion function from sRGB color space to linear color space.

For the following $g(x)$ (Refer to https://en.wikipedia.org/wiki/SRGB#Transformation)

$$g(x) = \begin{cases} \dfrac{x}{12.92}, & x \leq 0.04050 \\ \left(\dfrac{x + 0.055}{1.055}\right)^{2.4}, & x > 0.04050 \end{cases}$$

$$\alpha_{linear} = \begin{cases} \alpha_{sRGB}, & C_{sRGB} \leq 0.04050 \\ \dfrac{\left(\dfrac{C_F \alpha_{sRGB} + C_B(1 - \alpha_{sRGB}) + 0.055}{1.055}\right)^{2.4} - \left(\dfrac{C_B + 0.055}{1.055}\right)^{2.4}}{\left(\dfrac{C_F + 0.055}{1.055}\right)^{2.4} - \left(\dfrac{C_B + 0.055}{1.055}\right)^{2.4}}, & C_{sRGB} > 0.04050 \end{cases}$$

Transparency conversion from sRGB color space to linear color space

$$\alpha_{linear} = \begin{cases} \alpha_{sRGB}, & C_{sRGB} \leq 0.04050 \\ \dfrac{\left(\dfrac{C_F\alpha_{sRGB} + C_B(1-\alpha_{sRGB}) + 0.055}{1.055}\right)^{2.4} - \left(\dfrac{C_B + 0.055}{1.055}\right)^{2.4}}{\left(\dfrac{C_F + 0.055}{1.055}\right)^{2.4} - \left(\dfrac{C_B + 0.055}{1.055}\right)^{2.4}}, & C_{sRGB} > 0.04050 \end{cases}$$

where $C_{sRGB} = C_F\alpha_{sRGB} + C_B(1-\alpha_{sRGB})$.

Special cases

For $C_F = 0$ (black) and $C_B = 1$ (white),

$$\alpha_{linear} = \begin{cases} \alpha_{sRGB}, & 1-\alpha_{sRGB} \leq 0.04050 \\ \dfrac{1}{0.9991662}\left[1 - \left(\dfrac{1-\alpha_{sRGB} + 0.055}{1.055}\right)^{2.4}\right], & 1-\alpha_{sRGB} > 0.04050 \end{cases}$$

For $C_F = 1$ (white) and $C_B = 0$ (black),

$$\alpha_{linear} = \begin{cases} \alpha_{sRGB}, & \alpha_{sRGB} \leq 0.04050 \\ \dfrac{1}{0.9991662}\left[\left(\dfrac{\alpha_{sRGB} + 0.055}{1.055}\right)^{2.4} - 0.0008338\right], & \alpha_{sRGB} > 0.04050 \end{cases}$$