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## Assignment 5

1. Evaluate the following  $\lambda$  expression

- a.  $((\lambda x. \lambda y. (y \ x) \ \lambda p. \lambda q. p) \ \lambda i. i)$   
 $(\lambda y. (y \ \lambda p. \lambda q. p) \ \lambda i. i)$   
 $\lambda i. i \ \lambda p. \lambda q. p$   
 $\lambda p. \lambda q. p$
- b.  $((((\lambda x. \lambda y. \lambda z. ((x \ y) \ z) \ \lambda f. \lambda a. (f \ a)) \ \lambda i. i) \ \lambda j. j)$   
 $((\lambda y. \lambda z. ((\lambda f. \lambda a. (f \ a) \ y) \ z) \ \lambda i. i) \ \lambda j. j)$   
 $((\lambda f. \lambda a. (f \ a) \ \lambda i. i) \ \lambda j. j)$   
 $(\lambda a. (\lambda i. i \ a) \ \lambda j. j)$   
 $(\lambda i. i \ \lambda j. j)$   
 $\lambda j. j$
- c.  $(\lambda h. ((\lambda a. \lambda f. (f \ a) \ h) \ h) \ \lambda f. (f \ f))$   
 $(\lambda h. (\lambda f. (f \ h) \ h) \ \lambda f. (f \ f))$   
 $(\lambda h. (h \ h) \ \lambda f. (f \ f))$   
 $(\lambda f. (f \ f) \ \lambda f. (f \ f))$   
 $(\lambda f. (f \ f) \ \lambda f. (f \ f))$   
 $(\lambda f. (f \ f) \ \lambda f. (f \ f))$   
...Infinite Loop
- d.  $((\lambda p. \lambda q. (p \ q) \ (\lambda x. x \ \lambda a. \lambda b. a)) \ \lambda k. k)$   
 $((\lambda p. \lambda q. (p \ q) \ (\lambda a. \lambda b. a)) \ \lambda k. k)$   
 $(\lambda q. ((\lambda a. \lambda b. a) \ q) \ \lambda k. k)$   
 $((\lambda a. \lambda b. a) \ \lambda k. k)$   
 $(\lambda b. \lambda k. k)$

- e.  $(((\lambda f.\lambda g.\lambda x.(f (g x)) \lambda s.(s s)) \lambda a.\lambda b.b) \lambda x.\lambda y.x)$   
 $(((\lambda g.\lambda x.(\lambda s.(s s) (g x))) \lambda a.\lambda b.b) \lambda x.\lambda y.x)$   
 $(\lambda x.(\lambda s.(s s) (\lambda a.\lambda b.b x)) \lambda x.\lambda y.x)$   
 $(\lambda s.(s s) (\lambda a.\lambda b.b \lambda x.\lambda y.x))$   
 $(\lambda s.(s s) (\lambda b.b))$   
 $(\lambda b.b \lambda b.b)$   
 $\lambda b.b$

2. Define a function:

- a.  $\text{def make triplet} = \lambda f.\lambda s.\lambda t.\lambda \text{func} .(((\text{func } f) s) t)$   
b.  $\text{def triplet first} = \lambda \text{first}.\lambda \text{second}.\lambda \text{third}.\text{first}$   
c.  $\text{def triplet second} = \lambda \text{first}.\lambda \text{second}.\lambda \text{third}.\text{second}$   
d.  $\text{def triplet third} = \lambda \text{first}.\lambda \text{second}.\lambda \text{third}.\text{third}$

3. Use  $\alpha$  conversion to ensure unique names in the expressions in each of the following  $\lambda$  expressions:

- a.  $\lambda x.\lambda y.(\lambda x.y \lambda y.x)$   
 $\lambda x.\lambda y.(\lambda a.y \lambda b.x)$
- b.  $\lambda x.(x (\lambda y.(\lambda x.x y) x))$   
 $\lambda x.(x (\lambda y.(\lambda a.a y) x))$
- c.  $\lambda a.(\lambda b.a \lambda b.(\lambda a.a b))$   
 $\lambda a.(\lambda b.a \lambda c.(\lambda x.x c))$
- d.  $(\lambda \text{free}.\text{bound } \lambda \text{bound} .(\lambda \text{free}.\text{free bound}))$   
 $(\lambda \text{free}.\text{bound } \lambda a.(\lambda b.b a))$
- e.  $\lambda p.\lambda q.(\lambda r.(p (\lambda q.(\lambda p.(r q)))) (q p))$   
 $\lambda p.\lambda q.(\lambda r.(p (\lambda a.(\lambda p.(r a)))) (q p))$

4. Define a  $\lambda$  calculus representation for implication:

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def implies =  $\lambda x.\lambda y.((x\ y)\ \text{true})$ 
```

5. Define a  $\lambda$  calculus representation for equivalence:

```
def equiv =  $\lambda x.\lambda y.((x\ y)\ ((y\ \text{false})\ \text{true}))$ 
```

6. Write a function that finds the product of the numbers between n and one:

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rec prod n =  
  if isone n then f  
  else prod (mult n (pred n))
```