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Assignment 5

1. Evaluate the following λ expression

- a. $((\lambda x. \lambda y. (y \ x) \ \lambda p. \lambda q. p) \ \lambda i. i)$
 $(\lambda y. (y \ \lambda p. \lambda q. p) \ \lambda i. i)$
 $\lambda i. i \ \lambda p. \lambda q. p$
 $\lambda p. \lambda q. p$
- b. $((((\lambda x. \lambda y. \lambda z. ((x \ y) \ z) \ \lambda f. \lambda a. (f \ a)) \ \lambda i. i) \ \lambda j. j)$
 $((\lambda y. \lambda z. ((\lambda f. \lambda a. (f \ a) \ y) \ z) \ \lambda i. i) \ \lambda j. j)$
 $((\lambda f. \lambda a. (f \ a) \ \lambda i. i) \ \lambda j. j)$
 $(\lambda a. (\lambda i. i \ a) \ \lambda j. j)$
 $(\lambda i. i \ \lambda j. j)$
 $\lambda j. j$
- c. $(\lambda h. ((\lambda a. \lambda f. (f \ a) \ h) \ h) \ \lambda f. (f \ f))$
 $(\lambda h. (\lambda f. (f \ h) \ h) \ \lambda f. (f \ f))$
 $(\lambda h. (h \ h) \ \lambda f. (f \ f))$
 $(\lambda f. (f \ f) \ \lambda f. (f \ f))$
 $(\lambda f. (f \ f) \ \lambda f. (f \ f))$
 $(\lambda f. (f \ f) \ \lambda f. (f \ f))$
...Infinite Loop
- d. $((\lambda p. \lambda q. (p \ q) \ (\lambda x. x \ \lambda a. \lambda b. a)) \ \lambda k. k)$
 $((\lambda p. \lambda q. (p \ q) \ (\lambda a. \lambda b. a)) \ \lambda k. k)$
 $(\lambda q. ((\lambda a. \lambda b. a) \ q) \ \lambda k. k)$
 $((\lambda a. \lambda b. a) \ \lambda k. k)$
 $(\lambda b. \lambda k. k)$

- e. $(((\lambda f.\lambda g.\lambda x.(f (g x)) \lambda s.(s s)) \lambda a.\lambda b.b) \lambda x.\lambda y.x)$
 $(((\lambda g.\lambda x.(\lambda s.(s s) (g x))) \lambda a.\lambda b.b) \lambda x.\lambda y.x)$
 $(\lambda x.(\lambda s.(s s) (\lambda a.\lambda b.b x)) \lambda x.\lambda y.x)$
 $(\lambda s.(s s) (\lambda a.\lambda b.b \lambda x.\lambda y.x))$
 $(\lambda s.(s s) (\lambda b.b))$
 $(\lambda b.b \lambda b.b)$
 $\lambda b.b$

2. Define a function:

- a. `def make triplet = λf.λs.λt.λfunc.(((func f) s) t)`
b. `def triplet first = λfirst.λsecond.λthird.first`
c. `def triplet second = λfirst.λsecond.λthird.second`
d. `def triplet third = λfirst.λsecond.λthird.third`

3. Use α conversion to ensure unique names in the expressions in each of the following λ expressions:

- a. $\lambda x.\lambda y.(\lambda x.y \lambda y.x)$
 $\lambda x.\lambda y.(\lambda a.y \lambda b.x)$
- b. $\lambda x.(x (\lambda y.(\lambda x.x y) x))$
 $\lambda x.(x (\lambda y.(\lambda a.a y) x))$
- c. $\lambda a.(\lambda b.a \lambda b.(\lambda a.a b))$
 $\lambda a.(\lambda b.a \lambda c.(\lambda x.x c))$
- d. $(\lambda free.bound \lambda bound.(\lambda free.free bound))$
 $(\lambda free.bound \lambda a.(\lambda b.b a))$
- e. $\lambda p.\lambda q.(\lambda r.(p (\lambda q.(\lambda p.(r q)))) (q p))$
 $\lambda p.\lambda q.(\lambda r.(p (\lambda a.(\lambda p.(r a)))) (q p))$

4. Define a λ calculus representation for implication:

```
def implies =  $\lambda x.\lambda y.((x\ y)\ \text{true})$ 
```

5. Define a λ calculus representation for equivalence:

```
def equiv =  $\lambda x.\lambda y.((x\ y)\ (\text{not}\ y))$ 
```

6. Write a function that finds the product of the numbers between n and one:

```
rec prod n =  
  if isone n then 1  
  else prod (mult n (pred n))
```