TD - Week 1: Solutions

October 2025

Solutions

Problem 1: Product Mix Problem - FORMULATION

Let x_A = number of units of product A, x_B = number of units of product B

Objective Function:

$$\text{Maximize } Z = 40x_A + 90x_B$$

Constraints:

$$\begin{aligned}
x_A &\geq 0.8(x_A + x_B) && \text{(Sales volume constraint)} \\
x_A &\leq 110 && \text{(Maximum A units)} \\
2x_A + 4x_B &\leq 300 && \text{(Raw material constraint)} \\
x_A, x_B &\geq 0 && \text{(Non-negativity)}
\end{aligned}$$

Simplified Form:

Simplifying the first constraint: $x_A \geq 0.8x_A + 0.8x_B \Rightarrow 0.2x_A \geq 0.8x_B \Rightarrow x_A \geq 4x_B$

$$\begin{aligned}
x_A - 4x_B &\geq 0 \\
x_A &\leq 110 \\
2x_A + 4x_B &\leq 300 \quad \text{or} \quad x_A + 2x_B \leq 150 \\
x_A, x_B &\geq 0
\end{aligned}$$

Problem 2: Investment Problem - FORMULATION

Let x_A = amount invested in A (\$), x_B = amount invested in B (\$)

Objective Function:

$$\text{Maximize } Z = 0.05x_A + 0.08x_B$$

Constraints:

$$\begin{aligned}
x_A + x_B &= 5000 && \text{(Total investment)} \\
x_A &\geq 0.25(x_A + x_B) = 1250 && \text{(At least 25\% in A)} \\
x_B &\leq 0.50(x_A + x_B) = 2500 && \text{(At most 50\% in B)} \\
x_A &\geq 0.5x_B && \text{(A at least half of B)} \\
x_A, x_B &\geq 0
\end{aligned}$$

Simplified Form:

$$\begin{aligned}
x_A + x_B &= 5000 \\
x_A &\geq 1250 \\
x_B &\leq 2500 \\
x_A &\geq 0.5x_B \quad \text{or} \quad 2x_A - x_B \geq 0 \\
x_A, x_B &\geq 0
\end{aligned}$$

Problem 3: Advertising Problem - FORMULATION

Let x_R = minutes of radio advertising, x_T = minutes of TV advertising

Objective Function:

$$\text{Maximize } Z = x_R + 25x_T \quad \text{(effectiveness)}$$

Constraints:

$$\begin{aligned} 15x_R + 300x_T &\leq 10000 && \text{(Budget)} \\ x_R &\geq 2x_T && \text{(Radio at least twice TV)} \\ x_R &\leq 400 && \text{(Maximum radio time)} \\ x_R, x_T &\geq 0 \end{aligned}$$

Simplified Form:

$$\begin{aligned} 15x_R + 300x_T &\leq 10000 \quad \text{or} \quad x_R + 20x_T \leq 666.67 \\ x_R - 2x_T &\geq 0 \\ x_R &\leq 400 \\ x_R, x_T &\geq 0 \end{aligned}$$

Problem 4: Factory Production Problem - FORMULATION

Let x_A, x_B, x_C = number of units of products A, B, C

Objective Function:

$$\text{Maximize } Z = 40x_A + 50x_B + 60x_C$$

Constraints:

$$2x_A + x_B + 2x_C \leq 100 \quad \text{(Labor)}$$

$$x_A + 2x_B + 3x_C \leq 120 \quad \text{(Machine)}$$

$$3x_A + 2x_B + 4x_C \leq 150 \quad \text{(Raw Material)}$$

$$x_A, x_B, x_C \geq 0$$

Note: This problem has 3 variables and requires the Simplex method or computational tools for solution.

Problem 5: Diet Problem - FORMULATION

Let x_1, x_2, x_3, x_4 = units of foods F1, F2, F3, F4

Objective Function:

$$\text{Minimize } Z = 2x_1 + 3x_2 + x_3 + 4x_4$$

Constraints:

$$\begin{aligned} 10x_1 + 15x_2 + 5x_3 + 20x_4 &\geq 50 && \text{(Protein)} \\ 5x_1 + 10x_2 + 5x_3 + 10x_4 &\leq 30 && \text{(Fat)} \\ 20x_1 + 5x_2 + 30x_3 + 10x_4 &\geq 60 && \text{(Carbs)} \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Note: This problem has 4 variables and requires the Simplex method or computational tools for solution.

Problem 6: Product Mix Problem - GRAPHICAL METHOD (Solution to Problem 1)

Given model:

$$\text{Maximize } Z = 40x_A + 90x_B$$

Subject to:

$$\begin{aligned} x_A - 4x_B &\geq 0 \\ x_A &\leq 110 \\ x_A + 2x_B &\leq 150 \\ x_A, x_B &\geq 0 \end{aligned}$$

Step 1: Plot the constraints

$$\begin{aligned} &x_A = 4x_B && \text{(sales constraint)} \\ &x_A = 110 && \text{(upper limit for A)} \\ &x_A + 2x_B = 150 && \text{(raw material constraint)} \end{aligned}$$

Step 2: Identify feasible region

The feasible region lies in the area satisfying:

$$x_A \geq 4x_B,\ x_A \leq 110,\ x_A + 2x_B \leq 150,\ x_A, x_B \geq 0$$

Step 3: Determine corner points

(1) $(x_A, x_B) = (0, 0)$ Intersection of axes

(2) $(x_A, x_B) = (110, 0)$ From $x_B = 0$

(3) $(x_A, x_B) = (110, 20)$ From $x_A = 110,\ x_A + 2x_B = 150$

(4) $(x_A, x_B) = (100, 25)$ From $x_A = 4x_B,\ x_A + 2x_B = 150$

Step 4: Evaluate the objective function

$$
\begin{aligned}
(0,0): &\quad Z = 40(0) + 90(0) = 0 \\
(110,0): &\quad Z = 40(110) + 90(0) = 4400 \\
(110,20): &\quad Z = 40(110) + 90(20) = 4400 + 1800 = 6200 \\
(100,25): &\quad Z = 40(100) + 90(25) = 4000 + 2250 = 6250
\end{aligned}
$$

Step 5: Optimal solution

The maximum value occurs at:

$$(x_A, x_B) = (100, 25)$$

$$Z_{\text{max}} = 6250$$

Interpretation: Produce 100 units of Product A and 25 units of Product B to obtain the maximum profit of \$6250.

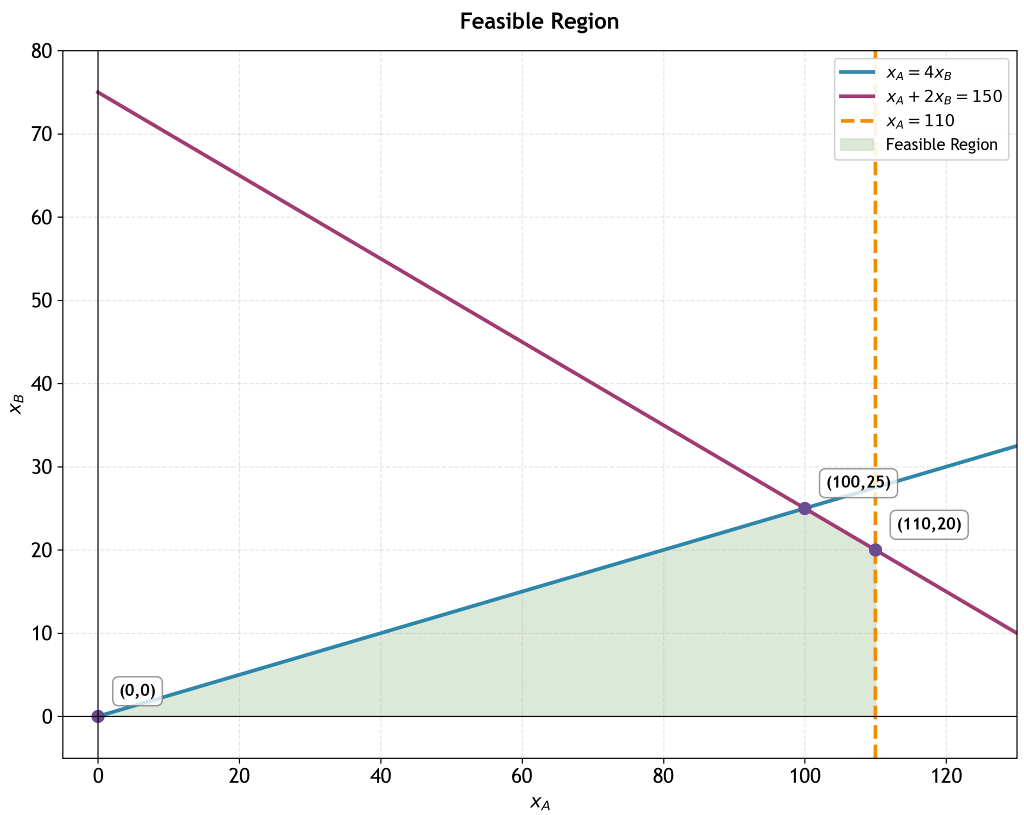

Figure 1: Solution to Problem 1.

Problem 7: Investment Problem - GRAPHICAL METHOD (Solution to Problem 2)

Given model:

$$\text{Maximize } Z = 0.05x_A + 0.08x_B$$

Subject to:

$$\begin{aligned} x_A + x_B &= 5000 \\ x_A &\geq 1250 \\ x_B &\leq 2500 \\ 2x_A - x_B &\geq 0 \\ x_A, x_B &\geq 0 \end{aligned}$$

Step 1: Plot the constraints

$$\begin{aligned} &x_A + x_B = 5000 && \text{(total investment)} \\ &x_A = 1250 && \text{(minimum investment in A)} \\ &x_B = 2500 && \text{(maximum investment in B)} \\ &x_B = 2x_A && \text{(A at least half of B)} \end{aligned}$$

Step 2: Identify feasible region

The feasible region lies on the budget line $x_A + x_B = 5000$, satisfying:

$$x_A \geq 1250, \quad x_B \leq 2500, \quad 2x_A - x_B \geq 0$$

Step 3: Determine feasible segment

$$(x_A, x_B) \in [(2500, 2500), (5000, 0)]$$

Step 4: Evaluate the objective function along feasible segment

$$Z = 0.05x_A + 0.08x_B$$

$$\text{At } (2500, 2500): \quad Z = 0.05(2500) + 0.08(2500) = 325$$
$$\text{At } (5000, 0): \quad Z = 0.05(5000) + 0.08(0) = 250$$

Step 5: Optimal solution

$$(x_A, x_B) = (2500, 2500), \quad Z_{\text{max}} = 325$$

Interpretation: Invest \$2500 in A and \$2500 in B to achieve the maximum return of \$325.

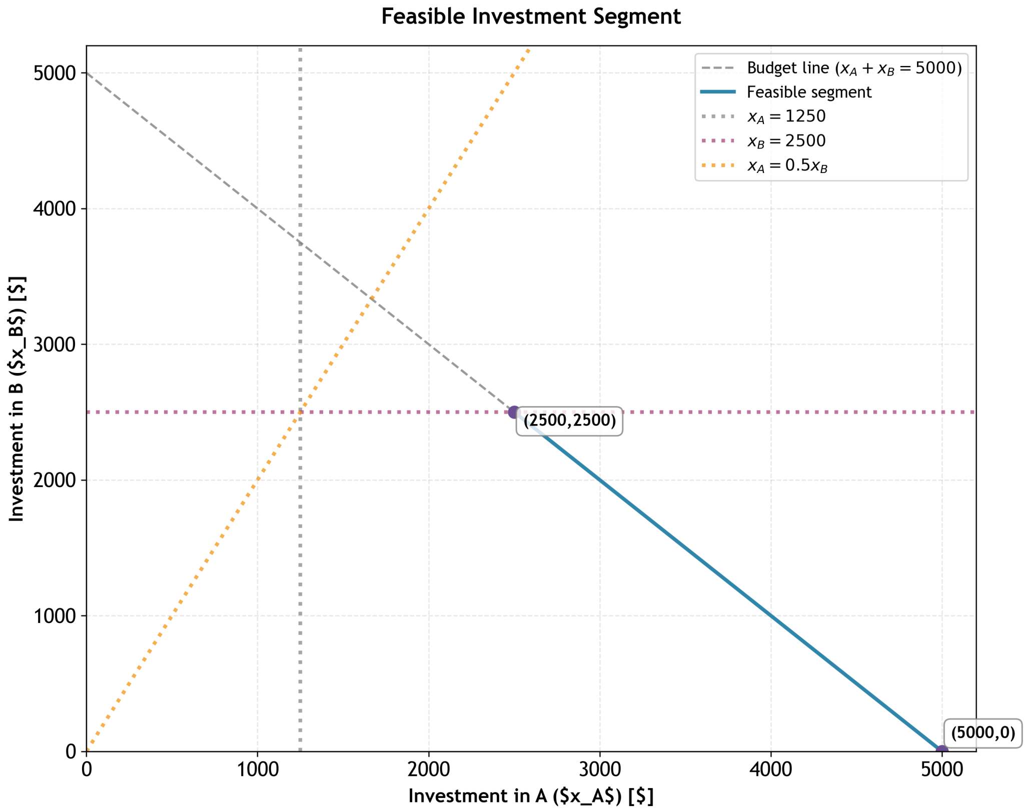

Figure 2: Solution to Problem 2.

Problem 8: Advertising Problem - GRAPHICAL METHOD (Solution to Problem 3)

Given model:

$$\text{Maximize } Z = x_R + 25x_T$$

Subject to:

$$\begin{aligned} 15x_R + 300x_T &\leq 10000 \\ x_R &\geq 2x_T \\ x_R &\leq 400 \\ x_R, x_T &\geq 0 \end{aligned}$$

Step 1: Plot the constraints

$$\begin{aligned} &x_R + 20x_T = 666.67 && \text{(budget line)} \\ &x_R = 2x_T && \text{(radio at least twice TV)} \\ &x_R = 400 && \text{(maximum radio time)} \end{aligned}$$

Step 2: Identify feasible region

The feasible region is the area satisfying:

$$x_R + 20x_T \leq 666.67, \quad x_R \geq 2x_T, \quad x_R \leq 400, \quad x_R, x_T \geq 0$$

Step 3: Determine corner points

(1) $(x_R, x_T) = (0, 0)$ Intersection of axes

(2) $(x_R, x_T) = (400, 13.33)$ Intersection of $x_R = 400$ and budget line $x_R + 20x_T = 666.67$

(3) $(x_R, x_T) = (60.6, 30.3)$ Intersection of $x_R = 2x_T$ and budget line $x_R + 20x_T = 666.67$

Step 4: Evaluate the objective function

$$\begin{aligned} &(0, 0): && Z = 0 + 25(0) = 0 \\ &(400, 13.33): && Z = 400 + 25(13.33) = 733.3 \\ &(60.3, 30.3): && Z = 60.6 + 25(30.3) = 818.1 \end{aligned}$$

Step 5: Optimal solution

The maximum value occurs at:

$$(x_R, x_T) = (60.3, 30.3), \quad Z_{\text{max}} \approx 818.1$$

Interpretation: Allocate 60.6 minutes to radio advertising and 30.3 minutes to TV advertising to achieve maximum effectiveness of 818.1.

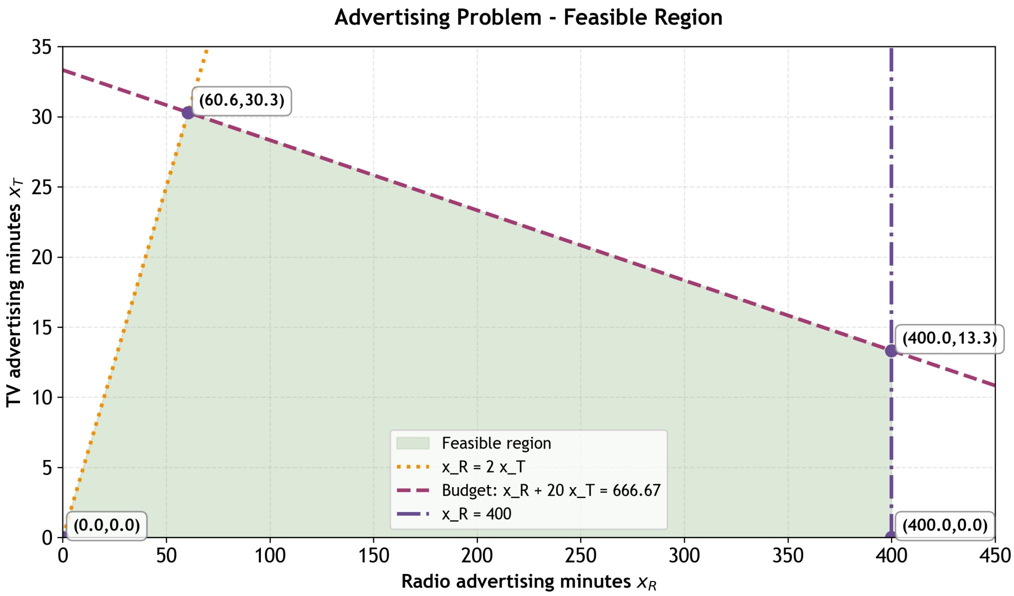

Figure 3: Solution to Problem 3.

Problem 9: Day Trader Investment - FORMULATION

Let x_B = amount invested in blue chips (\$), x_H = amount invested in high tech (\$)

Objective Function:

$$\text{Minimize } Z = x_B + x_H$$

Constraints:

$$\begin{aligned} 0.10x_B + 0.25x_H &\geq 10000 && \text{(Minimum yield)} \\ x_H &\leq 0.6(x_B + x_H) && \text{(High tech limit)} \\ x_B, x_H &\geq 0 \end{aligned}$$

Simplified Form:

Simplifying the second constraint:

$$x_H \leq 0.6x_B + 0.6x_H \Rightarrow 0.4x_H \leq 0.6x_B \Rightarrow x_H \leq 1.5x_B$$

Or equivalently: $2x_H - 3x_B \leq 0$

$$\begin{aligned} 0.10x_B + 0.25x_H \geq 10000 \quad &\text{or} \quad 2x_B + 5x_H \geq 200000 \\ -3x_B + 2x_H \leq 0 \quad &\text{or} \quad 3x_B - 2x_H \geq 0 \\ x_B, x_H \geq 0 \end{aligned}$$

Graphical Method

Step 1: Plot the constraints

$$\begin{aligned} &2x_B + 5x_H = 200000 && \text{(minimum yield, feasible region above)} \\ &x_H = 1.5x_B && \text{(high tech limit, feasible region below)} \\ &x_B = 0,\ x_H = 0 && \text{(non-negativity)} \end{aligned}$$

Step 2: Identify feasible region

The feasible region lies in the first quadrant satisfying:

$$2x_B + 5x_H \geq 200000, \quad x_H \leq 1.5x_B$$

Step 3: Determine corner points

(1) $(x_B, x_H) = (21052.63, 31578.95)$ Intersection of $2x_B + 5x_H = 200000$ and $x_H = 1.5x_B$

(2) $(x_B, x_H) = (100000, 0)$ Intersection with $x_H = 0$

Step 4: Evaluate the objective function

$$\begin{aligned}(21052.63, 31578.95): &\quad Z = 21052.63 + 31578.95 \approx 52631.58\\ (100000, 0): &\quad Z = 100000 + 0 = 100000\end{aligned}$$

Step 5: Optimal solution

$$(x_B, x_H) \approx (21053, 31579), \quad Z_{\text{min}} \approx 52632$$

Interpretation: Invest $21,053 in blue chips and $31,579 in high tech to achieve the minimum total investment of $52,632 while satisfying the yield and allocation constraints.

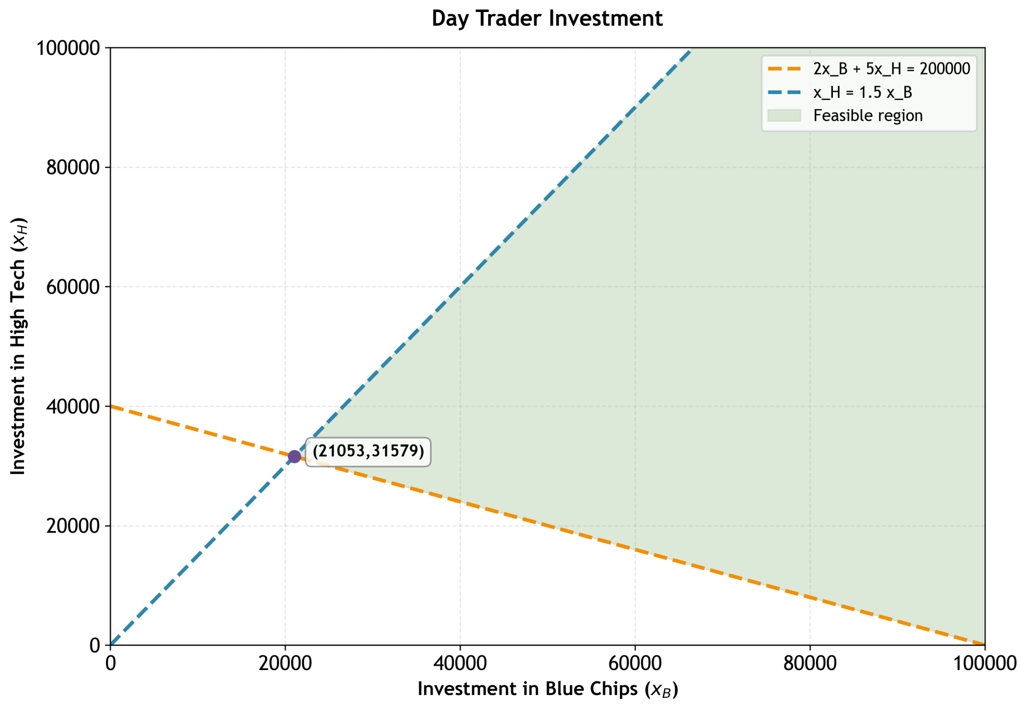

Figure 4: Solution to Problem 9.

Problem 10: Top Toys Advertising - FORMULATION

Let x_R = number of radio ads, x_T = number of TV ads

Reach Function:

- Radio: First ad reaches 5000, each additional ad reaches 2000 more

  Total reach = $5000 + 2000(x_R - 1) = 3000 + 2000x_R$ for $x_R \geq 1$

- TV: First ad reaches 4500, each additional ad reaches 3000 more

  Total reach = $4500 + 3000(x_T - 1) = 1500 + 3000x_T$ for $x_T \geq 1$

Objective Function:

$$\text{Maximize } Z = (3000 + 2000x_R) + (1500 + 3000x_T) = 4500 + 2000x_R + 3000x_T$$

Or equivalently (since 4500 is constant): Maximize $Z = 2000x_R + 3000x_T$

Constraints:

$$\begin{aligned}
300x_R + 2000x_T &\leq 20000 && \text{(Budget)} \\
x_R, x_T &\geq 1 && \text{(At least one each)} \\
300x_R &\leq 16000 && \text{(Radio max 80\%)} \\
2000x_T &\leq 16000 && \text{(TV max 80\%)}
\end{aligned}$$

Simplified Form:

$$\begin{aligned}
300x_R + 2000x_T \leq 20000 \quad &\text{or} \quad 3x_R + 20x_T \leq 200 \\
x_R &\geq 1 \\
x_T &\geq 1 \\
x_R &\leq 53.33 \\
x_T &\leq 8 \\
x_R, x_T &\geq 0
\end{aligned}$$

Graphical Method

Step 1: Identify feasible region

The feasible region satisfies all constraints above. The corner points are:

A: $(x_R, x_T) = (53.33, 1)$ intersection of $x_R = 53.33$ and $x_T = 1$

B: $(x_R, x_T) = (13.33, 8)$ intersection of budget line $3x_R + 20x_T = 200$ and $x_T = 8$

C: $(x_R, x_T) = (53.33, 2)$ intersection of budget line and $x_R = 53.33$

D: $(x_R, x_T) = (1, 8)$ intersection of $x_R = 1$ and $x_T = 8$

E: $(x_R, x_T) = (1, 1)$ intersection of axes

Step 2: Evaluate objective function at corner points

$$A : Z = 2000(53.33) + 3000(1) = 109,660$$
$$B : Z = 2000(13.33) + 3000(8) = 53,660$$
$$C : Z = 2000(53.33) + 3000(2) = 111,660$$
$$D : Z = 2000(1) + 3000(8) = 26,000$$
$$E : Z = 2000(1) + 3000(1) = 5,000$$

Step 3: Optimal solution

The maximum value occurs at:

$$(x_R, x_T) = (53.33, 2), \quad Z_{\text{max}} = 111,660$$

Interpretation: Allocate 53 radio ads and 2 TV ads (rounding to integers) to maximize reach for the advertising campaign.

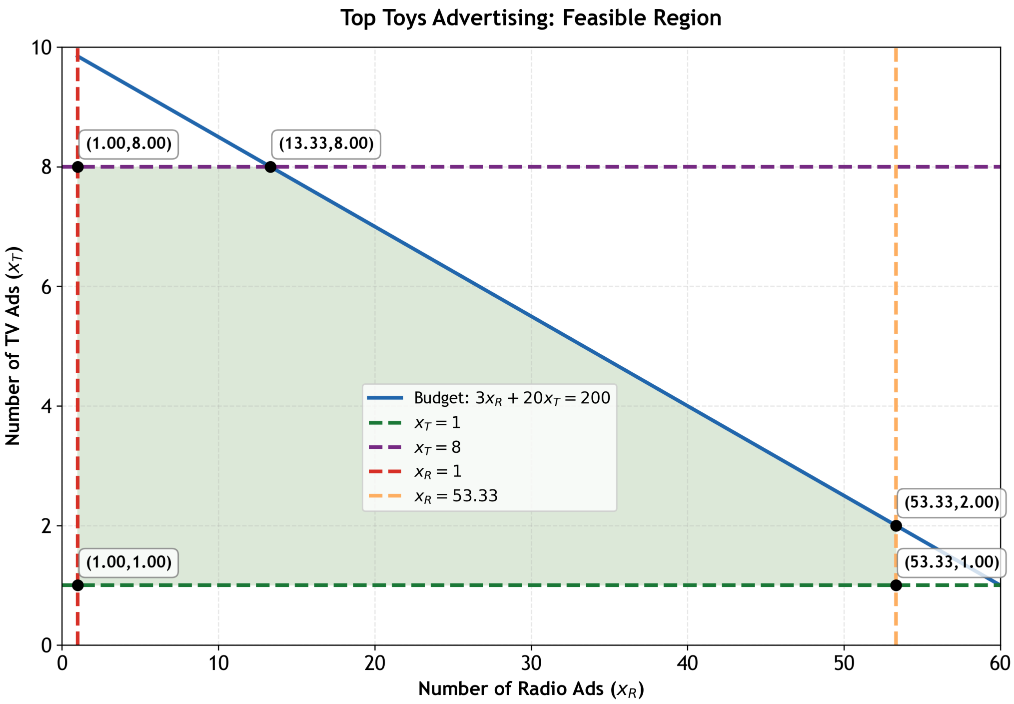

Figure 5: Solution to Problem 10.

Problem 11: Furniture Company - FORMULATION

Let x_C = number of chairs, x_D = number of desks

Department Capacities:

- Sawing: 200 chairs OR 80 desks per day
- Chair Assembly: 120 chairs per day
- Desk Assembly: 60 desks per day
- Painting: 150 chairs OR 110 desks per day

Objective Function:

$$\text{Maximize } Z = 50x_C + 100x_D$$

Constraints:

Using the conversion: if a department can produce A units of product 1 OR B units of product 2, then:

$$\frac{x_1}{A} + \frac{x_2}{B} \leq 1$$

$$\begin{aligned}
\frac{x_C}{200} + \frac{x_D}{80} &\leq 1 && \text{(Sawing)} \\
x_C &\leq 120 && \text{(Chair assembly)} \\
x_D &\leq 60 && \text{(Desk assembly)} \\
\frac{x_C}{150} + \frac{x_D}{110} &\leq 1 && \text{(Painting)} \\
x_C, x_D &\geq 0
\end{aligned}$$

Simplified Form:

Multiplying through to eliminate fractions:

$$\begin{aligned}
x_C + 2.5x_D &\leq 200 && \text{(Sawing)} \\
x_C &\leq 120 && \text{(Chair assembly)} \\
x_D &\leq 60 && \text{(Desk assembly)} \\
11x_C + 15x_D &\leq 1650 && \text{(Painting)} \\
x_C, x_D &\geq 0
\end{aligned}$$

Graphical Method

Step 1: Plot the constraints

$$\begin{aligned}
&x_C + 2.5x_D = 200 && \text{(Sawing)}\\
&x_C = 120 && \text{(Chair Assembly)}\\
&x_D = 60 && \text{(Desk Assembly)}\\
&11x_C + 15x_D = 1650 && \text{(Painting)}
\end{aligned}$$

Step 2: Identify feasible region

The feasible region satisfies all constraints:

$$x_C + 2.5x_D \leq 200, \quad x_C \leq 120, \quad x_D \leq 60, \quad 11x_C + 15x_D \leq 1650, \quad x_C, x_D \geq 0$$

Step 3: Determine corner points

$(0,0)$	Intersection of axes
$(0,60)$	Intersection of $x_D = 60$ and axes
$(50,60)$	Intersection of Sawing and Desk Assembly
$(90,44)$	Intersection of Sawing and Painting (optimal)
$(120,22)$	Intersection of Chair Assembly and Painting
$(120,0)$	Intersection of Chair Assembly and axes

Step 4: Evaluate the objective function

$$\begin{aligned}
(0,0):&\quad Z = 50(0) + 100(0) = 0\\
(0,60):&\quad Z = 50(0) + 100(60) = 6{,}000\\
(50,60):&\quad Z = 50(50) + 100(60) = 8{,}500\\
(90,44):&\quad Z = 50(90) + 100(44) = 8{,}900\\
(120,22):&\quad Z = 50(120) + 100(22) = 8{,}200\\
(120,0):&\quad Z = 50(120) + 100(0) = 6{,}000
\end{aligned}$$

Step 5: Optimal solution

The maximum value occurs at:

$$(x_C, x_D) = (90, 44), \quad Z_{\text{max}} = 8,900$$

Interpretation: Produce 90 chairs and 44 desks per day to achieve the maximum profit of 8,900.

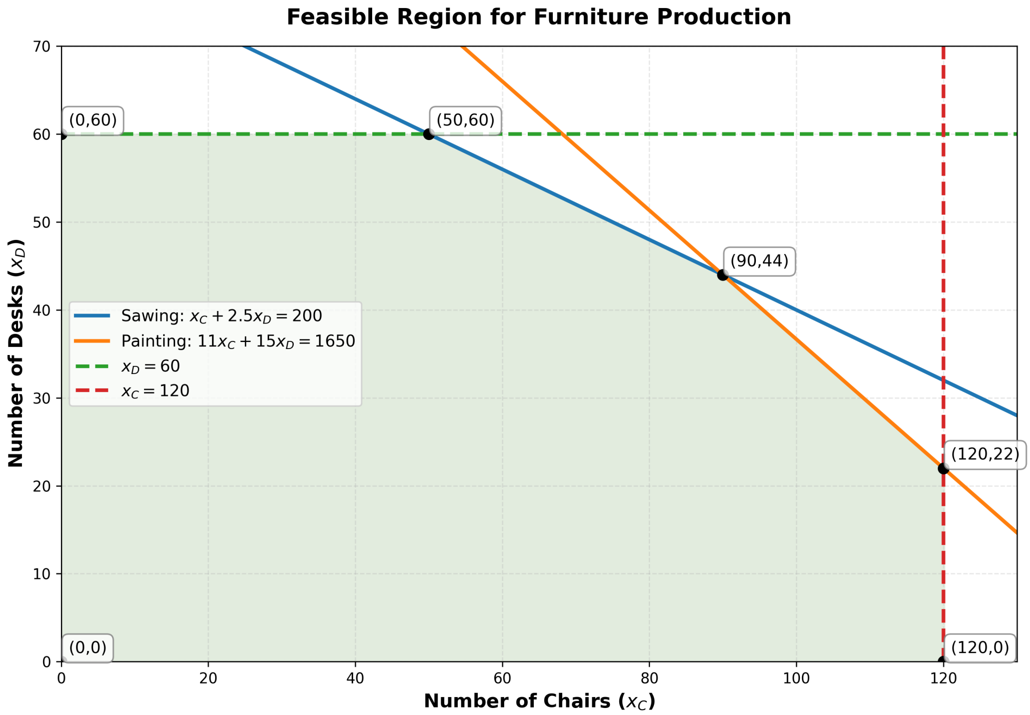

Figure 6: Solution to Problem 11.