Game Theory Practice Problems

Problem 1: Finding Dominated Strategies

Two competing coffee shops decide on their pricing strategy:

	Low	**Medium**	**High**
Low	(40, 40)	(60, 35)	(75, 30)
Medium	(35, 60)	(50, 50)	(65, 45)
High	(30, 75)	(45, 65)	(55, 55)

(a) Create a comparison table to check if any strategy is dominated for Shop 1.
(b) Check Shop 2's strategies for dominance.
(c) Can you eliminate any strategies? If yes, what is the reduced game?
(d) Find the Nash equilibrium using the underline method.

Problem 2: IESDS Practice

Two players play the following game:

	A	**B**	**C**	**D**
W	(5, 3)	(7, 2)	(4, 1)	(3, 5)
X	(6, 4)	(8, 3)	(5, 2)	(4, 4)
Y	(4, 5)	(6, 4)	(3, 3)	(2, 6)
Z	(3, 2)	(5, 1)	(2, 0)	(1, 3)

(a) Round 1: Check for strictly dominated strategies. Which strategy/strategies can be eliminated?
(b) Round 2: In the reduced game, are there new dominated strategies? Eliminate them.
(c) Continue until no more eliminations are possible. What is the final outcome?
(d) Verify this is a Nash equilibrium using the underline method.

Problem 3: Advertising Game

Two firms choose whether to advertise or not:

	Advertise	**Don't Advertise**
Advertise	(6, 6)	(12, 2)
Don't Advertise	(2, 12)	(8, 8)

(a) Does either firm have a dominant strategy? Use a comparison table.
(b) Find all Nash equilibria using the underline method.
(c) Is this a Prisoner's Dilemma? Explain why or why not.
(d) What outcome would both firms prefer? Why can't they achieve it?

Problem 4: Multiple Nash Equilibria

Two friends want to meet but forgot to decide where. They can go to the Beach or Mountains:

	Beach	**Mountains**
Beach	(3, 2)	(0, 0)
Mountains	(0, 0)	(2, 3)

(a) Find all pure strategy Nash equilibria using the underline method.
(b) Show your work: for each equilibrium, verify that no player wants to deviate.
(c) Which equilibrium do you think they should choose? Why is this a coordination problem?
(d) What would happen if they could communicate before choosing?

Problem 5: Battle of the Sexes

A couple wants to spend the evening together. He prefers Football, she prefers Opera, but both prefer being together to being apart:

	Football	**Opera**
Football	(3, 2)	(1, 1)
Opera	(0, 0)	(2, 3)

(a) Find all pure strategy Nash equilibria.
(b) Explain why neither player has a dominant strategy.
(c) Is there any outcome both players prefer to one of the equilibria?
(d) This game has a mixed strategy equilibrium too. Why might players randomize here?

Problem 6: Penalty Kick Game

A soccer player takes a penalty kick. Kicker chooses Left or Right. Goalie chooses which side to dive. If they choose the same side, goalie saves (payoff 0 for kicker, 1 for goalie). If different sides, kicker scores (payoff 1 for kicker, 0 for goalie).

	Dive Left	**Dive Right**
Kick Left	(0, 1)	(1, 0)
Kick Right	(1, 0)	(0, 1)

(a) Check for pure strategy Nash equilibria using the underline method.
(b) Explain why no pure strategy equilibrium exists.
(c) What strategy should each player use in real life?
(d) Does this remind you of another game from class? Which one?

Problem 7: Basic Mixed Strategy - Matching Pennies

Player 1 wins if both coins match. Player 2 wins if they don't match:

	Heads	**Tails**
Heads	(1, -1)	(-1, 1)
Tails	(-1, 1)	(1, -1)

(a) Verify there is no pure strategy Nash equilibrium.
(b) Let Player 1 play Heads with probability p. Calculate Player 2's expected payoff from playing Heads.
(c) Calculate Player 2's expected payoff from playing Tails.
(d) For Player 2 to be willing to mix, these must be equal. Solve for p.
(e) By symmetry, what is the mixed strategy Nash equilibrium?

Problem 8: Mixed Strategy Practice

	Left	**Right**
Up	(4, 1)	(0, 0)
Down	(0, 0)	(1, 4)

(a) Find all pure strategy Nash equilibria.
(b) Let Player 1 play Up with probability p and Player 2 play Left with probability q. Write Player 2's expected payoff from playing Left: $EU_2(L) = p \cdot 1 + (1-p) \cdot 0$.
(c) Write Player 2's expected payoff from playing Right: $EU_2(R) = ?$
(d) Set $EU_2(L) = EU_2(R)$ and solve for p.
(e) Now find q by making Player 1 indifferent. What is the mixed strategy equilibrium?

Problem 9: Entry Deterrence

A small firm considers entering a market. The large incumbent firm can either accommodate or fight:

	Accommodate	**Fight**
Enter	(2, 2)	(-1, 1)
Stay Out	(0, 5)	(0, 5)

(a) Find all pure strategy Nash equilibria.
(b) Which equilibrium is better for the entrant? For the incumbent?
(c) If the incumbent could commit to "always fight" before the entrant decides, what would happen?
(d) Why is the threat to fight not credible in this simultaneous game?

Problem 10: Technology Adoption

Two companies decide whether to adopt a new technology standard:

	Adopt	**Don't Adopt**
Adopt	(10, 10)	(2, 5)
Don't Adopt	(5, 2)	(6, 6)

(a) Find all pure strategy Nash equilibria.
(b) Which equilibrium gives the highest total payoff?
(c) Explain the coordination problem: why might they fail to reach the best outcome?
(d) How could communication or a third party help solve this problem?

Problem 11: Saddle Point Practice

Find the saddle points (if any) in these games:

Game A:

	C1	**C2**	**C3**
R1	3	5	2
R2	4	3	6
R3	2	4	5

(a) Find the row minimums and identify the maximin.
(b) Find the column maximums and identify the minimax.
(c) Does a saddle point exist? If yes, where?

Game B:

	C1	**C2**
R1	6	3
R2	2	8

(d) Check for a saddle point in Game B using the same method.

Problem 12: Rock-Paper-Scissors

The classic game with payoffs (winner gets 1, loser gets -1, tie gets 0):

	Rock	**Paper**	**Scissors**
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

(a) Check for pure strategy Nash equilibria using the underline method.
(b) Explain intuitively why there should be no pure strategy equilibrium.
(c) By symmetry, what should the mixed strategy equilibrium be?
(d) Verify: if Player 1 plays each option 1/3 of the time, what is Player 2's expected payoff from each pure strategy?

Problem 13: Price Competition

Two gas stations on opposite sides of a highway choose prices:

	Low ($3.00)	**Medium ($3.50)**	**High ($4.00)**
Low	(100, 100)	(180, 80)	(200, 60)
Medium	(80, 180)	(150, 150)	(190, 100)
High	(60, 200)	(100, 190)	(120, 120)

(a) Does either station have a dominated strategy?
(b) Use IESDS if possible. Show each round of elimination.
(c) Find the Nash equilibrium.
(d) Would both stations be better off if they both charged High? Why don't they?

Problem 14: Investment Game

Two firms decide on R&D investment levels:

	Low	**Medium**	**High**
Low	(5, 5)	(3, 7)	(2, 6)
Medium	(7, 3)	(6, 6)	(4, 8)
High	(6, 2)	(8, 4)	(7, 7)

(a) Check each player for dominated strategies using comparison tables.
(b) Are there any dominated strategies? If yes, eliminate them.
(c) Find all Nash equilibria in the resulting game.
(d) Which equilibrium would you predict? Why?

Problem 15: Mixed Strategy Calculation

	Left	**Right**
Top	(2, 3)	(0, 1)
Bottom	(1, 0)	(3, 2)

(a) Find all pure strategy Nash equilibria first.
(b) Let Player 1 play Top with probability p. Calculate Player 2's expected payoff from Left: $EU_2(L) = 3p + 0(1 - p)$.
(c) Calculate Player 2's expected payoff from Right: $EU_2(R) = ?$
(d) Set them equal and solve for p: what probability makes Player 2 indifferent?
(e) Similarly, find probability q (Player 2 plays Left with probability q) that makes Player 1 indifferent.
(f) State the complete mixed strategy Nash equilibrium.