Game Theory

Problem Set - Dominance & Nash Equilibrium

Problem 1: Finding Dominated Strategies

Consider the following game between two shops setting prices:

	Low	Medium	High
Low	(40, 40)	(60, 35)	(75, 30)
Medium	(35, 60)	(50, 50)	(65, 45)
High	(30, 75)	(45, 65)	(55, 55)

(a) Check if any of Shop 1's strategies are dominated. Create comparison tables to support your answer.
(b) Check if any of Shop 2's strategies are dominated.
(c) What is the reduced game after eliminating dominated strategies?
(d) Find the Nash equilibrium.

Problem 2: IESDS Practice

Consider this game:

	A	B	C	D
W	(5, 4)	(7, 3)	(4, 2)	(3, 4)
X	(6, 4)	(8, 3)	(5, 2)	(4, 4)
Y	(4, 4)	(6, 3)	(3, 2)	(2, 4)
Z	(3, 4)	(5, 3)	(2, 2)	(1, 4)

(a) Use IESDS (Iterated Elimination of Strictly Dominated Strategies). Show each round clearly.
(b) What strategies survive?
(c) Find the Nash equilibrium of the reduced game.
(d) Verify using the underline method.

Problem 3: Advertising Game

Two firms decide whether to advertise:

	Advertise	Don't Advertise
Advertise	(6, 6)	(12, 2)
Don't Advertise	(2, 12)	(8, 8)

(a) Check for dominant strategies for each firm using comparison tables.
(b) Find the Nash equilibrium using the underline method.
(c) Is this a Prisoner's Dilemma? Explain why or why not.

(d) What outcome would both firms prefer? Why can't they achieve it?

Problem 4: Multiple Nash Equilibria

Two friends want to meet but forgot where. Each prefers a different location:

	Beach	Mountains
Beach	(3, 2)	(0, 0)
Mountains	(0, 0)	(2, 3)

(a) Find all Nash equilibria using the underline method.
(b) Verify each equilibrium by checking if either player wants to deviate.
(c) Which equilibrium gives higher total payoff? Which player prefers which equilibrium?
(d) What problem arises from having multiple equilibria? How could communication help?

Problem 5: Battle of the Sexes

A couple wants to spend the evening together, but He prefers Football and She prefers Opera:

	Football	Opera
Football	(3, 2)	(1, 1)
Opera	(0, 0)	(2, 3)

(a) Find all pure strategy Nash equilibria.
(b) Does either player have a dominant strategy? Explain.
(c) Is there an outcome both prefer to the Nash equilibria?
(d) Why might players want to randomize in this game?

Problem 6: Entry Deterrence

A potential entrant decides whether to enter a market. The incumbent decides whether to accommodate or fight:

	Accommodate	Fight
Enter	(2, 2)	(-1, 1)
Stay Out	(0, 5)	(0, 5)

(a) Find all pure strategy Nash equilibria.
(b) Which equilibrium gives higher payoff to the Entrant? Which gives higher payoff to the Incumbent?
(c) If the Incumbent could credibly commit to "always fight," what would happen?
(d) Why is the threat to fight not credible in a simultaneous game?

Problem 7: Technology Adoption

Two companies decide whether to adopt a new technology:

	Adopt	Don't Adopt
Adopt	(10, 10)	(2, 5)
Don't Adopt	(5, 2)	(6, 6)

(a) Find all pure strategy Nash equilibria using the underline method.
(b) Which equilibrium gives the highest total payoff?
(c) Why might the companies fail to reach the best outcome?
(d) How could communication or coordination help?

Problem 8: Saddle Point Practice

For each game below, find the maximin value, minimax value, and determine if a saddle point exists.

Game A:

	C1	C2	C3
R1	3	5	2
R2	4	3	6
R3	2	4	5

Game B:

	C1	C2
R1	6	3
R2	2	8

(a) For Game A, find row minimums and the maximin value.
(b) For Game A, find column maximums and the minimax value.
(c) Does Game A have a saddle point?
(d) Repeat (a)-(c) for Game B.

Problem 9: Classic Prisoner's Dilemma

Two prisoners are interrogated separately:

	Confess	Silent
Confess	(-5, -5)	(-1, -10)
Silent	(-10, -1)	(-2, -2)

(a) Does either prisoner have a dominant strategy? Use comparison tables.
(b) Find the Nash equilibrium.
(c) Is the Nash equilibrium Pareto efficient? Explain.
(d) Why is this called a "dilemma"?

Problem 10: Price Competition

Two gas stations choose pricing strategies:

	Low	Medium	High
Low	(100, 100)	(180, 80)	(200, 60)
Medium	(80, 180)	(150, 150)	(190, 100)
High	(60, 200)	(100, 190)	(120, 120)

(a) Check for dominated strategies using comparison tables.
(b) Use IESDS to reduce the game.
(c) Find the Nash equilibrium.
(d) Would both stations prefer to charge High prices? Why don't they?

Problem 11: Investment Game

Two firms decide investment levels:

	Low	Medium	High
Low	(5, 5)	(7, 6)	(6, 8)
Medium	(6, 7)	(6, 6)	(4, 8)
High	(8, 6)	(8, 4)	(7, 7)

(a) Check for dominated strategies using comparison tables.
(b) Eliminate dominated strategies.
(c) Find all Nash equilibria in the reduced game.
(d) Which equilibrium would you predict and why?

Problem 12: Market Entry

Two firms decide whether to enter a new market:

	Enter	Stay Out
Enter	(1, 1)	(3, 0)
Stay Out	(0, 3)	(0, 0)

(a) Check for dominant strategies.
(b) Find all Nash equilibria using the underline method.
(c) Is there any equilibrium where both firms stay out? Why or why not?
(d) What does this tell us about competitive markets?

Problem 13: Quality Choice

Two restaurants choose quality levels:

	High Quality	Low Quality
High Quality	(8, 8)	(3, 10)
Low Quality	(10, 3)	(5, 5)

(a) Find all Nash equilibria.
(b) Which equilibrium is Pareto efficient?
(c) Is this a Prisoner's Dilemma? Explain.
(d) What coordination mechanism might help achieve the better equilibrium?

Problem 14: R&D Competition

Two tech companies decide on R&D spending:

	Low	Medium	High
Low	(6, 6)	(4, 8)	(2, 7)
Medium	(8, 4)	(7, 7)	(5, 9)
High	(7, 2)	(9, 5)	(8, 8)

(a) Check each strategy for dominance.
(b) Use IESDS if applicable.
(c) Find all Nash equilibria.
(d) Compare the Nash equilibria in terms of total welfare.

Problem 15: Coordination Challenge

Two players must coordinate their actions:

	Strategy A	Strategy B	Strategy C
Strategy A	(10, 10)	(0, 0)	(0, 0)
Strategy B	(0, 0)	(8, 8)	(0, 0)
Strategy C	(0, 0)	(0, 0)	(6, 6)

(a) Find all Nash equilibria.
(b) Which equilibrium would you expect them to choose and why?
(c) What is the coordination problem here?
(d) How might the players establish a focal point?

End of Problem Set