1 Conceptual Understanding

Question 1: Definition of Mixed Strategy

(a) Define what a mixed strategy is and explain how it differs from a pure strategy.

(b) Give three real-world examples where players would use mixed strategies rather than pure strategies.

(c) Explain why a mixed strategy is represented as a probability distribution.

Question 2: The Indifference Principle

Consider the following statement:

"In a mixed strategy Nash equilibrium, each player must be indifferent between all pure strategies in their support."

(a) Explain why this statement is true.

(b) What would happen if a player strictly preferred one pure strategy over another? Would they still mix?

(c) Whose indifference condition do you use to find Player 1's mixing probability?

Question 3: Nash's Theorem

(a) State Nash's Existence Theorem regarding mixed strategies.

(b) Why is this theorem important for game theory?

(c) Give an example of a game that has no pure strategy Nash equilibrium but has a mixed strategy Nash equilibrium.

2 Expected Utility Calculations

Question 4: Computing Expected Utilities

Consider the following game:

		Player 2 L	Player 2 R
2*Player 1	U	$(5,3)$	$(1,4)$
	D	$(2,1)$	$(4,5)$

Suppose Player 2 plays L with probability $q = 0.6$ and R with probability $1 - q = 0.4$.

(a) Calculate Player 1's expected utility from playing U.

(b) Calculate Player 1's expected utility from playing D.

(c) What is Player 1's best response to Player 2's mixed strategy?

(d) If Player 1 plays a mixed strategy with probability $p = 0.5$ on U, what is Player 1's overall expected utility?

Question 5: Expected Utility Against Pure Strategies

In the same game from Question 4:

(a) If Player 2 plays the pure strategy L (i.e., $q = 1$), calculate Player 1's expected utilities for U and D.

(b) If Player 2 plays the pure strategy R (i.e., $q = 0$), calculate Player 1's expected utilities for U and D.

(c) Based on your calculations, can you determine Player 1's best response to each of Player 2's pure strategies?

3 Finding Mixed Strategy Nash Equilibria

Question 6: Matching Pennies Variant

Consider the following zero-sum game:

		Player 2	
		H	T
2*Player 1	H	$(2,-2)$	$(-1,1)$
	T	$(-1,1)$	$(1,-1)$

(a) Verify that no pure strategy Nash equilibrium exists.

(b) Find Player 1's optimal mixing probability p^* (probability of playing H).

(c) Find Player 2's optimal mixing probability q^* (probability of playing H).

(d) Calculate the expected payoff for each player at the mixed strategy Nash equilibrium.

Question 7: Battle of the Sexes

A husband and wife want to spend the evening together, but they have different preferences:

		Wife	
		Opera	Football
2*Husband	Opera	$(2,1)$	$(0,0)$
	Football	$(0,0)$	$(1,2)$

(a) Find all pure strategy Nash equilibria.

(b) Find the mixed strategy Nash equilibrium. Specifically:

(i) Find the probability p^* that Husband plays Opera

(ii) Find the probability q^* that Wife plays Opera

(c) Calculate the expected payoffs for both players in the mixed strategy equilibrium.

(d) Compare the expected payoffs from the mixed equilibrium with the payoffs from the pure equilibria. What do you observe?

Question 8: Asymmetric Coordination Game

Consider the following game:

		Player 2 A	Player 2 B
2*Player 1	A	$(4, 2)$	$(0, 0)$
	B	$(0, 0)$	$(2, 4)$

(a) Identify all pure strategy Nash equilibria.

(b) Set up and solve the indifference conditions to find the mixed strategy Nash equilibrium.

(c) Verify that your solution satisfies $0 < p^*, q^* < 1$.

(d) Calculate expected payoffs at the mixed equilibrium.

Question 9: No Pure Equilibrium Game

Consider this game:

		Player 2 L	Player 2 R
2*Player 1	U	$(3, 1)$	$(0, 3)$
	D	$(1, 4)$	$(2, 0)$

(a) Use the underline method to verify that no pure strategy Nash equilibrium exists.

(b) Find the mixed strategy Nash equilibrium:

(i) What is p^* (probability Player 1 plays U)?

(ii) What is q^* (probability Player 2 plays L)?

(c) Verify your answer by checking that each player is indifferent at these probabilities.

Question 10: Three-Strategy Game

Consider the following game where Player 1 has three strategies:

		Player 2	
		L	R
3*Player 1	U	$(3, 2)$	$(0, 3)$
	M	$(2, 4)$	$(2, 1)$
	D	$(0, 3)$	$(4, 2)$

(a) Check if any of Player 1's strategies are strictly dominated.

(b) After eliminating dominated strategies (if any), find all Nash equilibria in the reduced game.

(c) If a mixed strategy equilibrium exists where Player 1 mixes between only two strategies, identify which two and find the equilibrium.

4 Support and Best Responses

Question 11: Support of Mixed Strategies

(a) Define the support of a mixed strategy.

(b) In a mixed strategy Nash equilibrium, what property must all strategies in the support satisfy?

(c) Consider a mixed strategy $\sigma = (0.3, 0.7, 0)$ over three pure strategies $\{A, B, C\}$. What is the support of this strategy?

(d) Can a strictly dominated strategy ever be in the support of a mixed strategy Nash equilibrium? Explain why or why not.

Question 12: Best Response Correspondence

Consider the game from Question 6 (Matching Pennies Variant).

(a) For each possible value of q (Player 2's probability of playing H), determine Player 1's best response:

(i) What is $\mathrm{BR}_1(q)$ when $q > q^*$?

(ii) What is $\mathrm{BR}_1(q)$ when $q < q^*$?

(iii) What is $\mathrm{BR}_1(q)$ when $q = q^*$?

(b) Sketch or describe the best response correspondence for Player 1.

(c) Explain how the mixed strategy Nash equilibrium corresponds to the intersection of best response correspondences.

5 Dominated Strategies and Equilibrium

Question 13: IESDS and Mixed Strategies

Consider this game:

		Player 2 L	M	R
3*Player 1	U	$(5, 1)$	$(3, 2)$	$(1, 5)$
	M	$(3, 3)$	$(4, 4)$	$(2, 3)$
	D	$(2, 2)$	$(1, 5)$	$(6, 1)$

(a) Use iterated elimination of strictly dominated strategies (IESDS) to reduce this game.

(b) In the reduced game, find all Nash equilibria (both pure and mixed).

(c) Explain why we can eliminate dominated strategies before looking for mixed strategy equilibria.

Question 14: Weakly Dominated Strategies

Consider:

		Player 2 L	R
2*Player 1	U	$(3, 3)$	$(3, 0)$
	D	$(0, 0)$	$(2, 2)$

(a) Identify any weakly dominated strategies.

(b) Find all Nash equilibria without eliminating weakly dominated strategies.

(c) Find all Nash equilibria after eliminating weakly dominated strategies.

(d) What do you observe? Can eliminating weakly dominated strategies remove Nash equilibria?

6 Applications and Extensions

Question 15: Rock-Paper-Scissors

Consider the classic Rock-Paper-Scissors game with the following payoff matrix:

		Player 2		
		Rock	Paper	Scissors
3*Player 1	Rock	$(0,0)$	$(-1,1)$	$(1,-1)$
	Paper	$(1,-1)$	$(0,0)$	$(-1,1)$
	Scissors	$(-1,1)$	$(1,-1)$	$(0,0)$

(a) Explain why no pure strategy Nash equilibrium exists.

(b) Using symmetry arguments, what should the mixed strategy Nash equilibrium be?

(c) Verify your answer by setting up the indifference conditions.

(d) What is each player's expected payoff in equilibrium?

Question 16: Penalty Kicks in Soccer

A striker and goalkeeper face each other in a penalty kick. The striker can kick left (L) or right (R), and the goalkeeper can dive left or right. The payoffs represent the probability (in percentage) of scoring:

		Goalkeeper	
		Dive L	Dive R
2*Striker	Kick L	$(50,50)$	$(90,10)$
	Kick R	$(90,10)$	$(50,50)$

(a) Find the mixed strategy Nash equilibrium.

(b) What is the striker's expected scoring percentage in equilibrium?

(c) If the striker has a stronger left foot and the payoffs change to $(60,40)$ when both go left, how does the equilibrium change?

Question 17: Market Entry Game

An incumbent firm (Player 1) can choose to Fight or Accommodate a potential entrant (Player 2). The entrant can choose to Enter or Stay Out:

		Entrant	
		Enter	Stay Out
2*Incumbent	Fight	$(-1, -1)$	$(2, 0)$
	Accommodate	$(1, 1)$	$(2, 0)$

(a) Find all pure strategy Nash equilibria.

(b) Does a mixed strategy Nash equilibrium exist? If so, find it.

(c) Interpret the mixed strategy equilibrium in the context of market entry.

(d) Which equilibrium is more realistic or likely to occur? Explain.

Question 18: Inspection Game

A worker (Player 1) can either Work or Shirk. A manager (Player 2) can either Inspect or Not Inspect:

		Manager	
		Inspect	Not Inspect
2*Worker	Work	$(3, -1)$	$(3, 0)$
	Shirk	$(0, -2)$	$(5, -5)$

(a) Verify that no pure strategy Nash equilibrium exists.

(b) Find the mixed strategy Nash equilibrium:

(i) Probability worker chooses to Work

(ii) Probability manager chooses to Inspect

(c) Calculate expected payoffs for both players.

(d) What happens to the equilibrium if the cost of inspection decreases (manager's payoff for Inspect increases)?

7 Advanced Problems

Question 19: Multiple Equilibria Analysis

Consider the following game:

		Player 2 X	Player 2 Y
2*Player 1	A	$(5, 5)$	$(0, 0)$
	B	$(0, 0)$	$(4, 4)$

(a) Find all Nash equilibria (pure and mixed).

(b) Create a table comparing the payoffs from each equilibrium.

(c) Which equilibrium is:

(i) Payoff dominant?

(ii) Risk dominant?

(d) If the players could communicate before playing, which equilibrium would you expect them to coordinate on? Why?

(e) If there is uncertainty about what the other player will do, which equilibrium might be safer to play?

Question 20: Comprehensive Problem

Consider the following game between two tech companies deciding whether to invest in Research (R) or Marketing (M):

		Company 2 R	Company 2 M
2*Company 1	R	$(3, 3)$	$(5, 1)$
	M	$(1, 5)$	$(2, 2)$

(a) Identify all pure strategy Nash equilibria using the underline method.

(b) Find the mixed strategy Nash equilibrium. Show all work including:

(i) Setting up indifference conditions

(ii) Solving for mixing probabilities

(iii) Calculating expected payoffs

(c) For each equilibrium found, discuss:

(i) Which company benefits more?

(ii) Is it a Pareto efficient outcome?

(iii) Which equilibrium would you predict in reality?

(d) Suppose Company 1 can commit to a strategy before Company 2 chooses. What should Company 1 do? How does this change the game?

Additional Notes for Students

Tips for Success

General Approach to Mixed Strategy Problems:

1. **Always check for pure strategy equilibria first** using the underline method
2. **Eliminate dominated strategies** to simplify the problem
3. **Set up indifference conditions** correctly:
   - Your mixing probabilities make your opponent indifferent
   - Opponent's mixing probabilities make you indifferent
4. **Solve the algebra carefully** and check your work
5. **Verify your solution**:
   - Check that $0 \leq p, q \leq 1$
   - Verify that players are actually indifferent at these probabilities
   - Confirm no player wants to deviate

Common Mistakes to Avoid

- **Don't** try to make yourself indifferent — you make your opponent indifferent
- **Don't** forget to check if pure strategy equilibria exist first
- **Don't** assume mixed strategies always give higher payoffs (usually they're lower!)
- **Don't** skip the verification step
- **Don't** round too early in calculations — keep fractions when possible

Key Formulas

Expected Utility (2 strategies):

$$EU_i(s_i, \sigma_{-i}) = q \cdot u_i(s_i, s^1_{-i}) + (1 - q) \cdot u_i(s_i, s^2_{-i})$$

Indifference Condition:

$$EU_i(s^1_i, \sigma^*_{-i}) = EU_i(s^2_i, \sigma^*_{-i})$$

Mixed Strategy Nash Equilibrium:

$$\sigma^* = (\sigma^*_1, \sigma^*_2, \ldots, \sigma^*_n) \text{ where no player can improve by deviating}$$

Good luck with your practice!