

TD - Week 1: Solutions

October 2025

Solutions

Problem 1: Product Mix Problem - FORMULATION

Let x_A = number of units of product A, x_B = number of units of product B

Objective Function:

$$\text{Maximize } Z = 40x_A + 90x_B$$

Constraints:

$$x_A \geq 0.8(x_A + x_B) \quad (\text{Sales volume constraint})$$

$$x_A \leq 110 \quad (\text{Maximum A units})$$

$$2x_A + 4x_B \leq 300 \quad (\text{Raw material constraint})$$

$$x_A, x_B \geq 0 \quad (\text{Non-negativity})$$

Simplified Form:

Simplifying the first constraint: $x_A \geq 0.8x_A + 0.8x_B \Rightarrow 0.2x_A \geq 0.8x_B \Rightarrow x_A \geq 4x_B$

$$x_A - 4x_B \geq 0$$

$$x_A \leq 110$$

$$2x_A + 4x_B \leq 300 \quad \text{or} \quad x_A + 2x_B \leq 150$$

$$x_A, x_B \geq 0$$

Problem 2: Investment Problem - FORMULATION

Let x_A = amount invested in A (\$), x_B = amount invested in B (\$)

Objective Function:

$$\text{Maximize } Z = 0.05x_A + 0.08x_B$$

Constraints:

$$x_A + x_B = 5000 \quad \text{(Total investment)}$$

$$x_A \geq 0.25(x_A + x_B) = 1250 \quad \text{(At least 25% in A)}$$

$$x_B \leq 0.50(x_A + x_B) = 2500 \quad \text{(At most 50% in B)}$$

$$x_A \geq 0.5x_B \quad \text{(A at least half of B)}$$

$$x_A, x_B \geq 0$$

Simplified Form:

$$x_A + x_B = 5000$$

$$x_A \geq 1250$$

$$x_B \leq 2500$$

$$x_A \geq 0.5x_B \quad \text{or} \quad 2x_A - x_B \geq 0$$

$$x_A, x_B \geq 0$$

Problem 3: Advertising Problem - FORMULATION

Let x_R = minutes of radio advertising, x_T = minutes of TV advertising

Objective Function:

$$\text{Maximize } Z = x_R + 25x_T \quad (\text{effectiveness})$$

Constraints:

$$15x_R + 300x_T \leq 10000 \quad (\text{Budget})$$

$$x_R \geq 2x_T \quad (\text{Radio at least twice TV})$$

$$x_R \leq 400 \quad (\text{Maximum radio time})$$

$$x_R, x_T \geq 0$$

Simplified Form:

$$15x_R + 300x_T \leq 10000 \quad \text{or} \quad x_R + 20x_T \leq 666.67$$

$$x_R - 2x_T \geq 0$$

$$x_R \leq 400$$

$$x_R, x_T \geq 0$$

Problem 4: Factory Production Problem - FORMULATION

Let x_A, x_B, x_C = number of units of products A, B, C

Objective Function:

$$\text{Maximize } Z = 40x_A + 50x_B + 60x_C$$

Constraints:

$$2x_A + x_B + 2x_C \leq 100 \quad (\text{Labor})$$

$$x_A + 2x_B + 3x_C \leq 120 \quad (\text{Machine})$$

$$3x_A + 2x_B + 4x_C \leq 150 \quad (\text{Raw Material})$$

$$x_A, x_B, x_C \geq 0$$

Note: This problem has 3 variables and requires the Simplex method or computational tools for solution.

Problem 5: Diet Problem - FORMULATION

Let x_1, x_2, x_3, x_4 = units of foods F1, F2, F3, F4

Objective Function:

$$\text{Minimize } Z = 2x_1 + 3x_2 + x_3 + 4x_4$$

Constraints:

$$10x_1 + 15x_2 + 5x_3 + 20x_4 \geq 50 \quad (\text{Protein})$$

$$5x_1 + 10x_2 + 5x_3 + 10x_4 \leq 30 \quad (\text{Fat})$$

$$20x_1 + 5x_2 + 30x_3 + 10x_4 \geq 60 \quad (\text{Carbs})$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Note: This problem has 4 variables and requires the Simplex method or computational tools for solution.

Problem 6: Product Mix Problem - GRAPHICAL METHOD (Solution to Problem 1)

Given model:

$$\text{Maximize } Z = 40x_A + 90x_B$$

Subject to:

$$x_A - 4x_B \geq 0$$

$$x_A \leq 110$$

$$x_A + 2x_B \leq 150$$

$$x_A, x_B \geq 0$$

Step 1: Plot the constraints

$$x_A = 4x_B \quad (\text{sales constraint})$$

$$x_A = 110 \quad (\text{upper limit for A})$$

$$x_A + 2x_B = 150 \quad (\text{raw material constraint})$$

Step 2: Identify feasible region

The feasible region lies in the area satisfying:

$$x_A \geq 4x_B, \quad x_A \leq 110, \quad x_A + 2x_B \leq 150, \quad x_A, x_B \geq 0$$

Step 3: Determine corner points

$$(1) (x_A, x_B) = (0, 0) \quad \text{Intersection of axes}$$

$$(2) (x_A, x_B) = (110, 0) \quad \text{From } x_B = 0$$

$$(3) (x_A, x_B) = (110, 20) \quad \text{From } x_A = 110, \quad x_A + 2x_B = 150$$

$$(4) (x_A, x_B) = (100, 25) \quad \text{From } x_A = 4x_B, \quad x_A + 2x_B = 150$$

Step 4: Evaluate the objective function

$$(0, 0) : Z = 40(0) + 90(0) = 0$$

$$(110, 0) : Z = 40(110) + 90(0) = 4400$$

$$(110, 20) : Z = 40(110) + 90(20) = 4400 + 1800 = 6200$$

$$(100, 25) : Z = 40(100) + 90(25) = 4000 + 2250 = 6250$$

Step 5: Optimal solution

The maximum value occurs at:

$$(x_A, x_B) = (100, 25)$$

$$Z_{\max} = 6250$$

Interpretation: Produce 100 units of Product A and 25 units of Product B to obtain the maximum profit of \$6250.

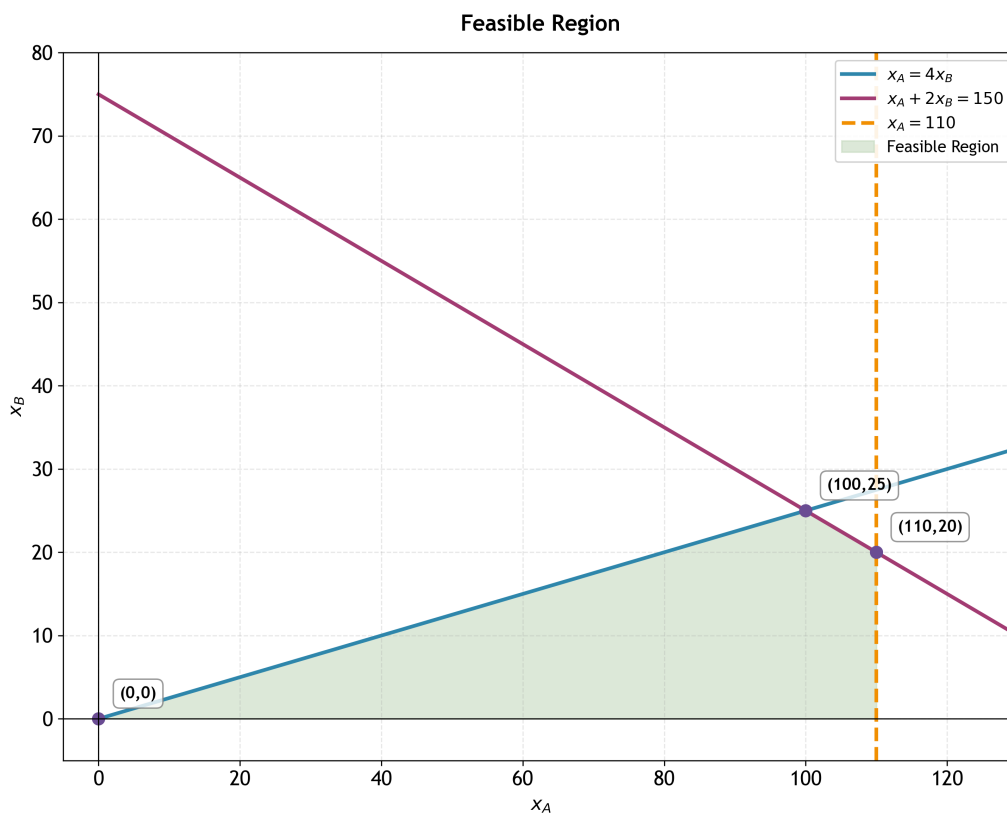


Figure 1: Solution to Problem 1.

Problem 7: Investment Problem - GRAPHICAL METHOD (Solution to Problem 2)

Given model:

$$\text{Maximize } Z = 0.05x_A + 0.08x_B$$

Subject to:

$$x_A + x_B = 5000$$

$$x_A \geq 1250$$

$$x_B \leq 2500$$

$$2x_A - x_B \geq 0$$

$$x_A, x_B \geq 0$$

Step 1: Plot the constraints

$$x_A + x_B = 5000 \quad (\text{total investment})$$

$$x_A = 1250 \quad (\text{minimum investment in A})$$

$$x_B = 2500 \quad (\text{maximum investment in B})$$

$$x_B = 2x_A \quad (\text{A at least half of B})$$

Step 2: Identify feasible region

The feasible region lies on the budget line $x_A + x_B = 5000$, satisfying:

$$x_A \geq 1250, \quad x_B \leq 2500, \quad 2x_A - x_B \geq 0$$

Step 3: Determine feasible segment

$$(x_A, x_B) \in [(2500, 2500), (5000, 0)]$$

Step 4: Evaluate the objective function along feasible segment

$$Z = 0.05x_A + 0.08x_B$$

At $(2500, 2500)$: $Z = 0.05(2500) + 0.08(2500) = 325$

At $(5000, 0)$: $Z = 0.05(5000) + 0.08(0) = 250$

Step 5: Optimal solution

$$(x_A, x_B) = (2500, 2500), \quad Z_{\max} = 325$$

Interpretation: Invest \$2500 in A and \$2500 in B to achieve the maximum return of \$325.

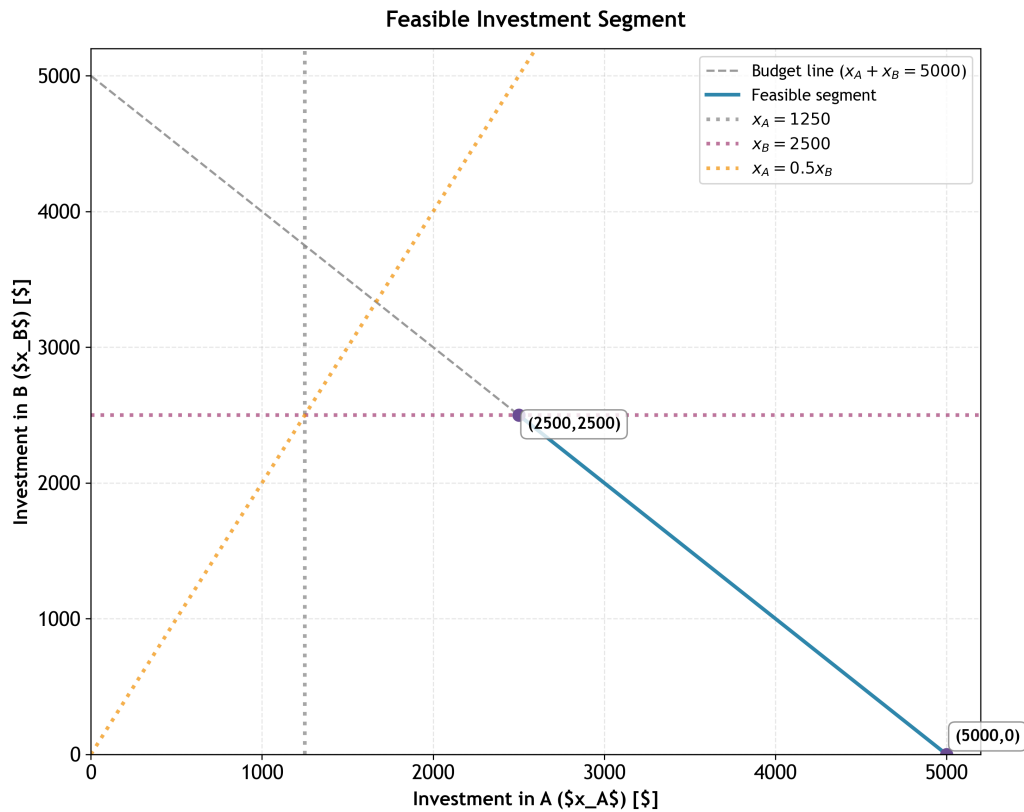


Figure 2: Solution to Problem 2.

Problem 8: Advertising Problem - GRAPHICAL METHOD (Solution to Problem 3)

Given model:

$$\text{Maximize } Z = x_R + 25x_T$$

Subject to:

$$15x_R + 300x_T \leq 10000$$

$$x_R \geq 2x_T$$

$$x_R \leq 400$$

$$x_R, x_T \geq 0$$

Step 1: Plot the constraints

$$x_R + 20x_T = 666.67 \quad (\text{budget line})$$

$$x_R = 2x_T \quad (\text{radio at least twice TV})$$

$$x_R = 400 \quad (\text{maximum radio time})$$

Step 2: Identify feasible region

The feasible region is the area satisfying:

$$x_R + 20x_T \leq 666.67, \quad x_R \geq 2x_T, \quad x_R \leq 400, \quad x_R, x_T \geq 0$$

Step 3: Determine corner points

(1) $(x_R, x_T) = (0, 0)$ Intersection of axes

(2) $(x_R, x_T) = (400, 13.33)$ Intersection of $x_R = 400$ and budget line $x_R + 20x_T = 666.67$

(3) $(x_R, x_T) = (60.6, 30.3)$ Intersection of $x_R = 2x_T$ and budget line $x_R + 20x_T = 666.67$

Step 4: Evaluate the objective function

$$(0, 0) : \quad Z = 0 + 25(0) = 0$$

$$(400, 13.33) : \quad Z = 400 + 25(13.33) = 733.3$$

$$(60.3, 30.3) : \quad Z = 60.6 + 25(30.3) = 818.1$$

Step 5: Optimal solution

The maximum value occurs at:

$$(x_R, x_T) = (60.3, 30.3), \quad Z_{\max} \approx 818.1$$

Interpretation: Allocate 60.6 minutes to radio advertising and 30.3 minutes to TV advertising to achieve maximum effectiveness of 818.1.

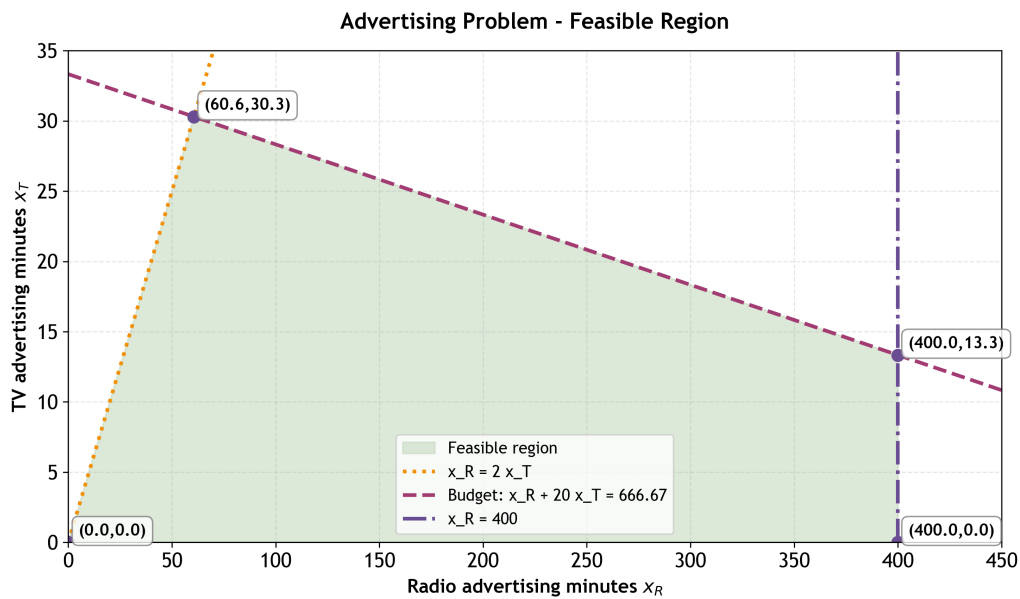


Figure 3: Solution to Problem 3.

Problem 9: Day Trader Investment - FORMULATION

Let x_B = amount invested in blue chips (\$), x_H = amount invested in high tech (\$)

Objective Function:

$$\text{Minimize } Z = x_B + x_H$$

Constraints:

$$0.10x_B + 0.25x_H \geq 10000 \quad (\text{Minimum yield})$$

$$x_H \leq 0.6(x_B + x_H) \quad (\text{High tech limit})$$

$$x_B, x_H \geq 0$$

Simplified Form:

Simplifying the second constraint:

$$x_H \leq 0.6x_B + 0.6x_H \Rightarrow 0.4x_H \leq 0.6x_B \Rightarrow x_H \leq 1.5x_B$$

Or equivalently: $2x_H - 3x_B \leq 0$

$$0.10x_B + 0.25x_H \geq 10000 \quad \text{or} \quad 2x_B + 5x_H \geq 200000$$

$$-3x_B + 2x_H \leq 0 \quad \text{or} \quad 3x_B - 2x_H \geq 0$$

$$x_B, x_H \geq 0$$

Graphical Method

Step 1: Plot the constraints

$$2x_B + 5x_H = 200000 \quad (\text{minimum yield, feasible region above})$$

$$x_H = 1.5x_B \quad (\text{high tech limit, feasible region below})$$

$$x_B = 0, x_H = 0 \quad (\text{non-negativity})$$

Step 2: Identify feasible region

The feasible region lies in the first quadrant satisfying:

$$2x_B + 5x_H \geq 200000, \quad x_H \leq 1.5x_B$$

Step 3: Determine corner points

- (1) $(x_B, x_H) = (21052.63, 31578.95)$ Intersection of $2x_B + 5x_H = 200000$ and $x_H = 1.5x_B$
- (2) $(x_B, x_H) = (100000, 0)$ Intersection with $x_H = 0$

Step 4: Evaluate the objective function

$$(21052.63, 31578.95) : Z = 21052.63 + 31578.95 \approx 52631.58$$

$$(100000, 0) : Z = 100000 + 0 = 100000$$

Step 5: Optimal solution

$$(x_B, x_H) \approx (21053, 31579), \quad Z_{\min} \approx 52632$$

Interpretation: Invest \$21,053 in blue chips and \$31,579 in high tech to achieve the minimum total investment of \$52,632 while satisfying the yield and allocation constraints.

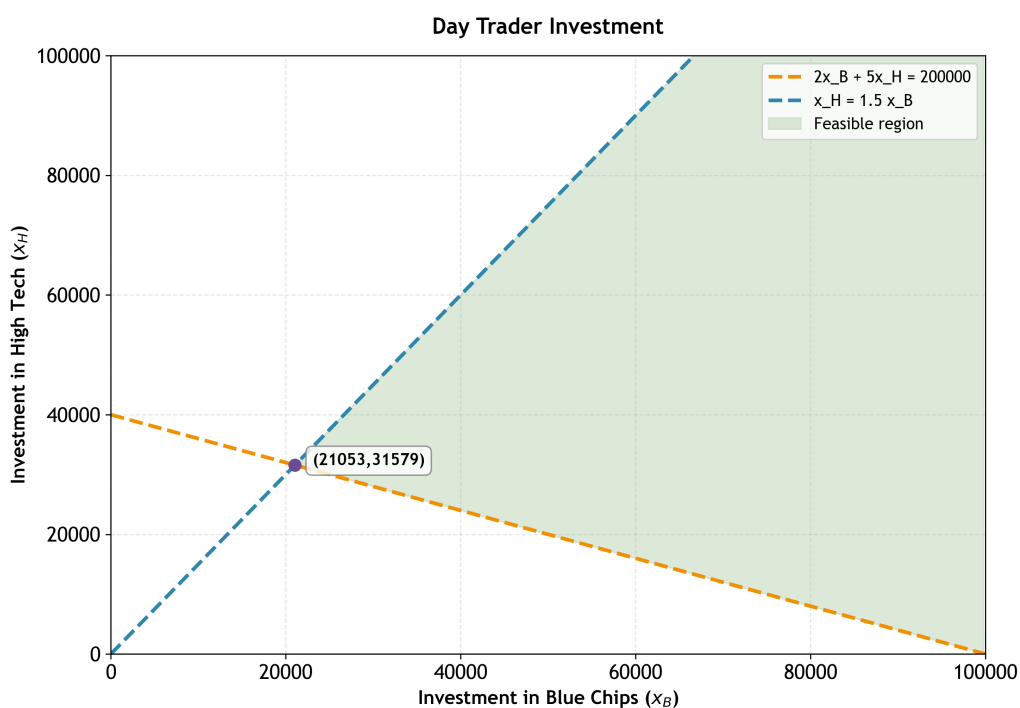


Figure 4: Solution to Problem 9.

Problem 10: Top Toys Advertising - FORMULATION

Let x_R = number of radio ads, x_T = number of TV ads

Reach Function:

- Radio: First ad reaches 5000, each additional ad reaches 2000 more

$$\text{Total reach} = 5000 + 2000(x_R - 1) = 3000 + 2000x_R \text{ for } x_R \geq 1$$

- TV: First ad reaches 4500, each additional ad reaches 3000 more

$$\text{Total reach} = 4500 + 3000(x_T - 1) = 1500 + 3000x_T \text{ for } x_T \geq 1$$

Objective Function:

$$\text{Maximize } Z = (3000 + 2000x_R) + (1500 + 3000x_T) = 4500 + 2000x_R + 3000x_T$$

Or equivalently (since 4500 is constant): Maximize $Z = 2000x_R + 3000x_T$

Constraints:

$$300x_R + 2000x_T \leq 20000 \quad (\text{Budget})$$

$$x_R, x_T \geq 1 \quad (\text{At least one each})$$

$$300x_R \leq 16000 \quad (\text{Radio max 80\%})$$

$$2000x_T \leq 16000 \quad (\text{TV max 80\%})$$

Simplified Form:

$$300x_R + 2000x_T \leq 20000 \quad \text{or} \quad 3x_R + 20x_T \leq 200$$

$$x_R \geq 1$$

$$x_T \geq 1$$

$$x_R \leq 53.33$$

$$x_T \leq 8$$

$$x_R, x_T \geq 0$$

Graphical Method

Step 1: Identify feasible region

The feasible region satisfies all constraints above. The corner points are:

A: $(x_R, x_T) = (53.33, 1)$ intersection of $x_R = 53.33$ and $x_T = 1$

B: $(x_R, x_T) = (13.33, 8)$ intersection of budget line $3x_R + 20x_T = 200$ and $x_T = 8$

C: $(x_R, x_T) = (53.33, 2)$ intersection of budget line and $x_R = 53.33$

D: $(x_R, x_T) = (1, 8)$ intersection of $x_R = 1$ and $x_T = 8$

E: $(x_R, x_T) = (1, 1)$ intersection of axes

Step 2: Evaluate objective function at corner points

$$A : Z = 2000(53.33) + 3000(1) = 109,660$$

$$B : Z = 2000(13.33) + 3000(8) = 53,660$$

$$C : Z = 2000(53.33) + 3000(2) = 111,660$$

$$D : Z = 2000(1) + 3000(8) = 26,000$$

$$E : Z = 2000(1) + 3000(1) = 5,000$$

Step 3: Optimal solution

The maximum value occurs at:

$$(x_R, x_T) = (53.33, 2), \quad Z_{\max} = 111,660$$

Interpretation: Allocate 53 radio ads and 2 TV ads (rounding to integers) to maximize reach for the advertising campaign.

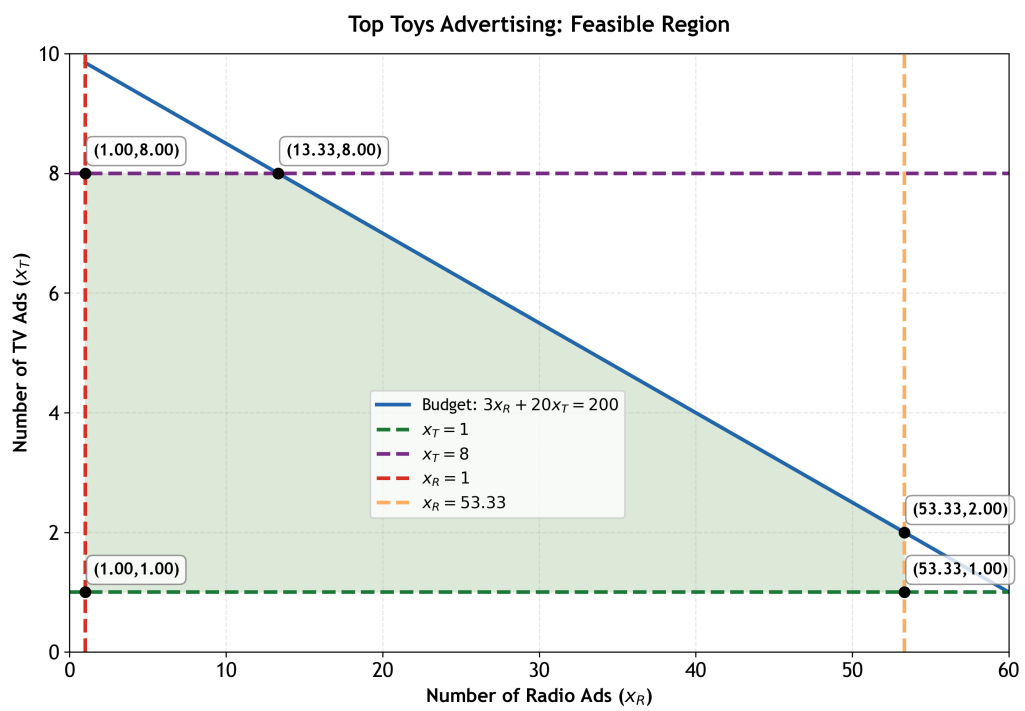


Figure 5: Solution to Problem 10.

Problem 11: Furniture Company - FORMULATION

Let x_C = number of chairs, x_D = number of desks

Department Capacities:

- Sawing: 200 chairs OR 80 desks per day
- Chair Assembly: 120 chairs per day
- Desk Assembly: 60 desks per day
- Painting: 150 chairs OR 110 desks per day

Objective Function:

$$\text{Maximize } Z = 50x_C + 100x_D$$

Constraints:

Using the conversion: if a department can produce A units of product 1 OR B units of product 2, then:

$$\frac{x_1}{A} + \frac{x_2}{B} \leq 1$$

$$\frac{x_C}{200} + \frac{x_D}{80} \leq 1 \quad (\text{Sawing})$$

$$x_C \leq 120 \quad (\text{Chair assembly})$$

$$x_D \leq 60 \quad (\text{Desk assembly})$$

$$\frac{x_C}{150} + \frac{x_D}{110} \leq 1 \quad (\text{Painting})$$

$$x_C, x_D \geq 0$$

Simplified Form:

Multiplying through to eliminate fractions:

$$x_C + 2.5x_D \leq 200 \quad (\text{Sawing})$$

$$x_C \leq 120 \quad (\text{Chair assembly})$$

$$x_D \leq 60 \quad (\text{Desk assembly})$$

$$11x_C + 15x_D \leq 1650 \quad (\text{Painting})$$

$$x_C, x_D \geq 0$$

Graphical Method

Step 1: Plot the constraints

$$\begin{aligned}x_C + 2.5x_D &= 200 && \text{(Sawing)} \\x_C &= 120 && \text{(Chair Assembly)} \\x_D &= 60 && \text{(Desk Assembly)} \\11x_C + 15x_D &= 1650 && \text{(Painting)}\end{aligned}$$

Step 2: Identify feasible region

The feasible region satisfies all constraints:

$$x_C + 2.5x_D \leq 200, \quad x_C \leq 120, \quad x_D \leq 60, \quad 11x_C + 15x_D \leq 1650, \quad x_C, x_D \geq 0$$

Step 3: Determine corner points

(0, 0)	Intersection of axes
(0, 60)	Intersection of $x_D = 60$ and axes
(50, 60)	Intersection of Sawing and Desk Assembly
(90, 44)	Intersection of Sawing and Painting (optimal)
(120, 13)	Intersection of Chair Assembly and Painting
(120, 0)	Intersection of Chair Assembly and axes

Step 4: Evaluate the objective function

$$\begin{aligned}(0, 0) : \quad & Z = 50(0) + 100(0) = 0 \\(0, 60) : \quad & Z = 50(0) + 100(60) = 6,000 \\(50, 60) : \quad & Z = 50(50) + 100(60) = 8,500 \\(90, 44) : \quad & Z = 50(90) + 100(44) = 8,900 \\(120, 13) : \quad & Z = 50(120) + 100(13) = 7,300 \\(120, 0) : \quad & Z = 50(120) + 100(0) = 6,000\end{aligned}$$

Step 5: Optimal solution

The maximum value occurs at:

$$(x_C, x_D) = (90, 44), \quad Z_{\max} = 8,900$$

Interpretation: Produce 90 chairs and 44 desks per day to achieve the maximum profit of 8,900.

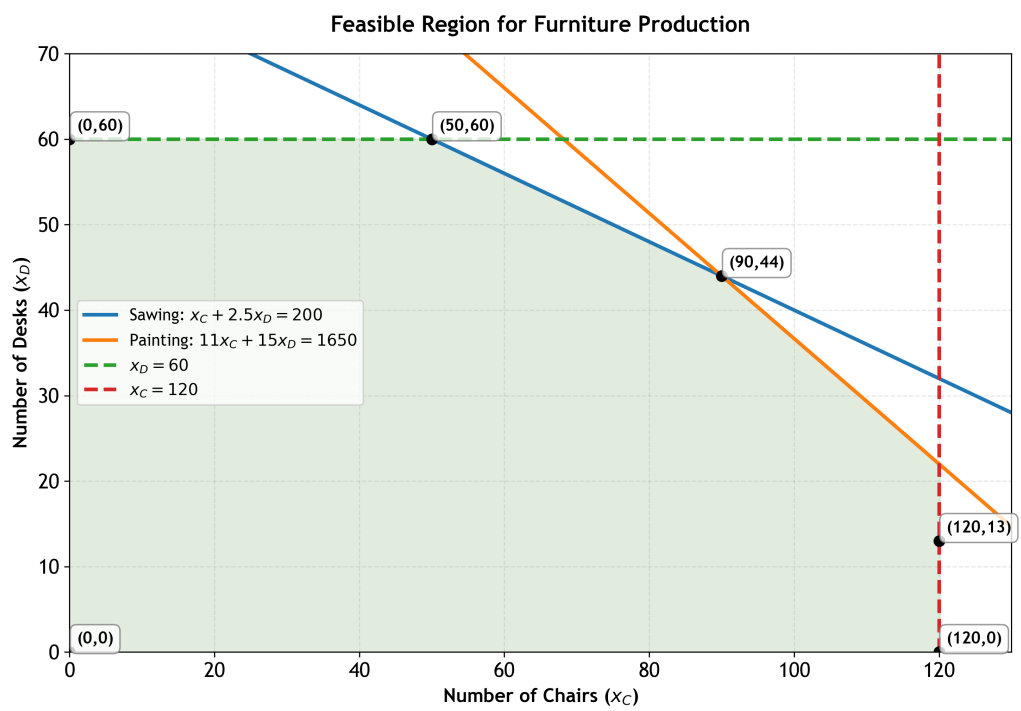


Figure 6: Solution to Problem 11.