

TD - Week 5

October 2025

Questions

Instructions:

- **Maximization Problems (1-5):** Transform the profit matrix to cost matrix using $c_{ij} = p_{max} - p_{ij}$. Find initial solution using specified method, then test optimality with MODI. Report final solution using ORIGINAL profit values.
- **Degeneracy Problems (6-10):** Find initial solution. If degenerate (occupied cells $< m + n - 1$), add ε to appropriate independent cells. Then apply MODI to test optimality.
- **Time Minimization (11-15):** Treat time as the objective (like cost). Find initial solution and optimize using MODI. Report total time.
- **Transshipment (16-25):** Calculate buffer, apply to all nodes, construct cost matrix with self-loops (cost=0) and prohibited routes (cost=M). Use Least Cost Method starting with self-loops. Report only actual shipments (ignore self-loops) and total cost.

Part A: Maximization Problems

1. Three distribution centers serve four retail zones. The profit per unit (in \$100) is given below. Find the allocation that maximizes total profit using **Least Cost Method** on the

transformed matrix, then verify optimality with MODI.

	Zone A	Zone B	Zone C	Zone D	Supply
DC_1	12	8	15	10	80
DC_2	14	11	9	13	100
DC_3	10	13	12	11	70
Demand	60	70	50	70	

2. A logistics company wants to maximize revenue (in \$1000s per container) by shipping from three ports to three destinations. Use **VAM** on transformed matrix and optimize with MODI.

	City P	City Q	City R	Supply
$Port_1$	25	18	22	50
$Port_2$	20	24	19	60
$Port_3$	23	21	26	40
Demand	45	55	50	

3. Four factories supply three warehouses. Efficiency ratings (higher is better) are shown. Transform and solve to maximize total efficiency using **Least Cost Method**, then apply MODI. Note: Supply exceeds demand.

	W_1	W_2	W_3	Supply
F_1	16	20	14	70
F_2	18	15	19	80
F_3	13	21	17	60
F_4	19	16	22	90
Demand	100	90	80	

4. A pharmaceutical company distributes medicine from plants to hospitals. Profit per unit (\$) is given. Maximize profit using **VAM transformation** and verify with MODI.

	H_1	H_2	H_3	H_4	Supply
P_1	45	38	50	42	120
P_2	40	47	35	48	150
P_3	52	41	46	39	130
Demand	100	120	90	90	

5. Agricultural cooperatives maximize profit by selling rice (profit in \$1000 per ton) to different markets. Transform to minimization, solve with **Least Cost**, and optimize with MODI.

	Market ₁	Market ₂	Market ₃	Supply
Coop ₁	8	12	9	100
Coop ₂	11	7	10	140
Coop ₃	9	10	13	110
Demand	120	130	100	

Part B: Degeneracy Problems

6. Find initial solution using **North-West Corner Method**. Handle degeneracy if it occurs, then apply MODI to find optimal solution.

	D_1	D_2	D_3	D_4	Supply
S_1	7	4	9	6	60
S_2	5	8	3	11	80
S_3	10	6	7	4	60
Demand	60	50	70	20	

7. Use **Least Cost Method** to find initial solution. If degenerate, add ε appropriately and optimize with MODI.

	W_1	W_2	W_3	Supply
P_1	8	6	10	100
P_2	5	9	7	100
P_3	12	4	8	50
Demand	100	50	100	

8. Apply **VAM** to obtain initial solution. Check for degeneracy and resolve. Then use MODI to reach optimal solution.

	C_1	C_2	C_3	C_4	Supply
F_1	11	7	13	9	70
F_2	8	10	6	12	90
F_3	14	9	11	7	40
Demand	50	70	40	40	

9. Find initial solution using **North-West Corner**. This problem will be degenerate. Add ε to maintain $m + n - 1$ allocations, then optimize with MODI.

	M_1	M_2	M_3	Supply
S_1	9	12	7	80
S_2	6	8	11	80
S_3	13	5	9	40
Demand	80	80	40	

10. Use **Least Cost Method**. Degeneracy is likely. Handle it properly and apply MODI until optimal.

	D_1	D_2	D_3	D_4	Supply
S_1	15	10	18	12	50
S_2	13	16	9	14	100
S_3	11	14	17	10	50
Demand	50	50	50	50	

Part C: Time Minimization

11. Emergency medical supplies must be delivered from three warehouses to four hospitals. Time (in hours) is given. Minimize total delivery time using **Least Cost Method** and MODI.

	H_1	H_2	H_3	H_4	Supply
W_1	3.5	2.8	4.2	3.0	80
W_2	2.5	3.6	2.9	4.1	100
W_3	4.0	3.2	2.4	3.5	70
Demand	60	70	60	60	

12. Technicians are dispatched from three service centers to four job sites. Travel time (minutes) is shown. Minimize total travel time using **VAM** and optimize with MODI.

	Site A	Site B	Site C	Site D	Supply
Center₁	25	35	20	30	50
Center₂	30	22	28	32	60
Center₃	18	27	33	24	40
Demand	35	40	45	30	

13. Construction materials delivery time (hours) from quarries to project sites. Minimize total time. Balance if needed, use **Least Cost**, then MODI.

	Project ₁	Project ₂	Project ₃	Supply
Quarry ₁	4.5	3.8	5.2	120
Quarry ₂	3.2	4.9	3.6	100
Quarry ₃	5.1	3.5	4.0	130
Demand	140	110	100	

14. Fire trucks must reach emergency locations. Response time (minutes) is critical. Minimize total response time using **VAM** and MODI.

	Loc ₁	Loc ₂	Loc ₃	Loc ₄	Supply
Station ₁	12	8	15	10	40
Station ₂	9	14	7	11	50
Station ₃	13	10	12	8	30
Demand	30	35	25	30	

15. Data packets routed through servers. Latency (milliseconds) must be minimized. Use **Least Cost Method** and optimize with MODI. Supply exceeds demand.

	Server _A	Server _B	Server _C	Supply
Node ₁	45	38	52	200
Node ₂	41	47	36	250
Node ₃	50	42	44	180
Demand	180	220	200	

Part D: Transshipment Problems

16. Two factories ship through one hub to two customers. Direct shipments are not allowed. Find the optimal transshipment plan.

Supply/Demand: F1: 150, F2: 200, C1: 180, C2: 170

Transportation Costs (\$):

Route	Cost	Route	Cost
F1 to H	6	H to C1	8
F2 to H	5	H to C2	7

17. Three suppliers send goods through two transshipment points to three destinations. Find the minimum cost allocation.

Supply/Demand: S1: 100, S2: 120, S3: 80, D1: 90, D2: 110, D3: 100

Transportation Costs (\$):

From	T1	T2	From	D1	D2	D3
S1	4	6	T1	5	7	6
S2	5	3	T2	8	4	6
S3	7	4				

18. Two plants ship through three distribution centers to four retailers. Direct plant-to-retailer shipments are expensive (\$15-\$20). Set up and solve the transshipment problem.

Supply/Demand: P1: 250, P2: 300, R1: 120, R2: 150, R3: 140, R4: 140

Plant to DC Costs (\$):

	DC1	DC2	DC3
P1	7	5	8
P2	6	9	5

DC to Retailer Costs (\$):

	R1	R2	R3	R4
DC1	4	6	5	7
DC2	5	3	6	4
DC3	7	5	3	6

19. Agricultural products from two farms go through a central market before reaching three processing plants. Farms cannot ship directly to plants. Solve using the transshipment method.

Supply/Demand (tons): Farm A: 180, Farm B: 220, P1: 150, P2: 130, P3: 120

Transportation Costs (\$/ton):

Route	Cost	From	P1	P2	P3
Farm A to M	3	M	5	6	4
Farm B to M	4				

20. Three warehouses distribute through two regional hubs to four stores. Find the optimal flow and total cost.

Supply/Demand: W1: 140, W2: 160, W3: 120, S1: 100, S2: 110, S3: 90, S4: 120

Warehouse to Hub Costs (\$):

	H1	H2
W1	8	6
W2	7	9
W3	5	7

Hub to Store Costs (\$):

	S1	S2	S3	S4
H1	4	6	5	7
H2	6	4	8	5

21. Electronics manufacturer: Two factories ship to three distribution centers, which then supply four retail chains. Direct factory-to-retail shipments are prohibited. Calculate buffer, construct the full table, and solve.

Supply/Demand: F1: 300, F2: 350, R1: 150, R2: 180, R3: 160, R4: 160

Factory to DC Costs (\$):

	DC1	DC2	DC3
F1	12	10	14
F2	11	13	10

DC to Retail Costs (\$):

	R1	R2	R3	R4
DC1	7	9	6	8
DC2	8	6	10	7
DC3	9	8	7	9

22. Petroleum distribution: Two refineries ship to three regional depots which supply four

gas stations. Direct refinery-to-station costs are prohibitive (\$20+). Apply the transshipment method.

Supply/Demand: R1: 500, R2: 600, G1: 250, G2: 280, G3: 270, G4: 300

Refinery to Depot Costs (\$):

	D1	D2	D3
R1	10	8	12
R2	9	11	8

Depot to Gas Station Costs (\$):

	G1	G2	G3	G4
D1	6	7	5	8
D2	7	5	9	6
D3	8	6	7	5

23. Pharmaceutical supply chain: Three manufacturing plants use two consolidation centers to reach five hospitals. Direct plant-to-hospital shipment is not feasible. Solve the complete transshipment problem.

Supply/Demand: M1: 200, M2: 250, M3: 180, H1: 120, H2: 140, H3: 130, H4: 110, H5: 130

Plant to Consolidation Center Costs (\$):

	C1	C2
M1	7	9
M2	6	8
M3	10	7

Center to Hospital Costs (\$):

	H1	H2	H3	H4	H5
C1	5	6	4	7	5
C2	6	4	7	5	8

24. International shipping: Four exporters ship through three ports to five importers. Direct exporter-to-importer costs are prohibitive (\$40+). Calculate buffer = 680, construct the full matrix, and solve.

Supply/Demand: E1: 150, E2: 180, E3: 160, E4: 190, I1: 130, I2: 150, I3: 140, I4: 120, I5: 140

Exporter to Port Costs (\$):

	P1	P2	P3
E1	18	22	20
E2	17	19	23
E3	21	18	20
E4	19	21	17

Port to Importer Costs (\$):

	I1	I2	I3	I4	I5
P1	12	14	11	15	13
P2	13	11	16	12	14
P3	15	13	12	14	10

25. Complex logistics network: Two production sites, three transshipment hubs, and four demand centers. Hubs can redistribute among themselves at \$3. Direct production-to-demand costs are expensive (\$18+). Model with buffer = 600, include all possible routes, and solve.

Supply/Demand: PS1: 280, PS2: 320, DC1: 140, DC2: 160, DC3: 150, DC4: 150

Production Site to Hub Costs (\$):

	Hub A	Hub B	Hub C
PS1	9	7	11
PS2	8	10	7

Hub to Demand Center Costs (\$):

	DC1	DC2	DC3	DC4
Hub A	6	8	5	7
Hub B	7	5	9	6
Hub C	8	6	7	5

Hub-to-Hub Redistribution Cost: \$3 (between any two hubs)