

# TD - Week 5

October 2025

## Questions

### Instructions:

- **Maximization Problems (1-5):** Transform the profit matrix to cost matrix using  $c_{ij} = p_{max} - p_{ij}$ . Find initial solution using specified method, then test optimality with MODI. Report final solution using ORIGINAL profit values.
- **Degeneracy Problems (6-10):** Find initial solution. If degenerate (occupied cells  $< m + n - 1$ ), add  $\varepsilon$  to appropriate independent cells. Then apply MODI to test optimality.
- **Time Minimization (11-15):** Treat time as the objective (like cost). Find initial solution and optimize using MODI. Report total time.
- **Transshipment (16-25):** Calculate buffer, apply to all nodes, construct cost matrix with self-loops (cost=0) and prohibited routes (cost=M). Use Least Cost Method starting with self-loops. Report only actual shipments (ignore self-loops) and total cost.

### Part A: Maximization Problems

1. Three distribution centers serve four retail zones. The profit per unit (in \$100) is given below. Find the allocation that maximizes total profit using Least Cost Method on the

transformed matrix, then verify optimality with MODI.

	Zone A	Zone B	Zone C	Zone D	Supply
DC <sub>1</sub>	12	8	15	10	80
DC <sub>2</sub>	14	11	9	13	100
DC <sub>3</sub>	10	13	12	11	70
Demand	60	70	50	70	

2. A logistics company wants to maximize revenue (in \$1000s per container) by shipping from three ports to three destinations. Use VAM on transformed matrix and optimize with MODI.

	City P	City Q	City R	Supply
Port <sub>1</sub>	25	18	22	50
Port <sub>2</sub>	20	24	19	60
Port <sub>3</sub>	23	21	26	40
Demand	45	55	50	

3. Four factories supply three warehouses. Efficiency ratings (higher is better) are shown. Transform and solve to maximize total efficiency using Least Cost Method, then apply MODI. Note: Supply exceeds demand.

	W <sub>1</sub>	W <sub>2</sub>	W <sub>3</sub>	Supply
F <sub>1</sub>	16	20	14	70
F <sub>2</sub>	18	15	19	80
F <sub>3</sub>	13	21	17	60
F <sub>4</sub>	19	16	22	90
Demand	100	90	80	

4. A pharmaceutical company distributes medicine from plants to hospitals. Profit per unit (\$) is given. Maximize profit using VAM transformation and verify with MODI.

	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>	Supply
P <sub>1</sub>	45	38	50	42	120
P <sub>2</sub>	40	47	35	48	150
P <sub>3</sub>	52	41	46	39	130
Demand	100	120	90	90	

5. Agricultural cooperatives maximize profit by selling rice (profit in \$1000 per ton) to different markets. Transform to minimization, solve with **Least Cost**, and optimize with MODI.

	Market <sub>1</sub>	Market <sub>2</sub>	Market <sub>3</sub>	Supply
Coop <sub>1</sub>	8	12	9	100
Coop <sub>2</sub>	11	7	10	140
Coop <sub>3</sub>	9	10	13	110
Demand	120	130	100	

## Part B: Degeneracy Problems

6. Find initial solution using **North-West Corner Method**. Handle degeneracy if it occurs, then apply MODI to find optimal solution.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	7	4	9	6	60
S <sub>2</sub>	5	8	3	11	80
S <sub>3</sub>	10	6	7	4	60
Demand	60	50	70	20	

7. Use **Least Cost Method** to find initial solution. If degenerate, add  $\varepsilon$  appropriately and optimize with MODI.

	W <sub>1</sub>	W <sub>2</sub>	W <sub>3</sub>	Supply
P <sub>1</sub>	8	6	10	100
P <sub>2</sub>	5	9	7	100
P <sub>3</sub>	12	4	8	50
Demand	100	50	100	

8. Apply **VAM** to obtain initial solution. Check for degeneracy and resolve. Then use MODI to reach optimal solution.

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Supply
F <sub>1</sub>	11	7	13	9	70
F <sub>2</sub>	8	10	6	12	90
F <sub>3</sub>	14	9	11	7	40
Demand	50	70	40	40	

9. Find initial solution using **North-West Corner**. This problem will be degenerate. Add  $\varepsilon$  to maintain  $m + n - 1$  allocations, then optimize with MODI.

	$M_1$	$M_2$	$M_3$	Supply
$S_1$	9	12	7	80
$S_2$	6	8	11	80
$S_3$	13	5	9	40
Demand	80	80	40	

10. Use **Least Cost Method**. Degeneracy is likely. Handle it properly and apply MODI until optimal.

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	15	10	18	12	50
$S_2$	13	16	9	14	100
$S_3$	11	14	17	10	50
Demand	50	50	50	50	

## Part C: Time Minimization

11. Emergency medical supplies must be delivered from three warehouses to four hospitals. Time (in hours) is given. Minimize total delivery time using **Least Cost Method** and MODI.

	$H_1$	$H_2$	$H_3$	$H_4$	Supply
$W_1$	3.5	2.8	4.2	3.0	80
$W_2$	2.5	3.6	2.9	4.1	100
$W_3$	4.0	3.2	2.4	3.5	70
Demand	60	70	60	60	

12. Technicians are dispatched from three service centers to four job sites. Travel time (minutes) is shown. Minimize total travel time using **VAM** and optimize with MODI.

	Site A	Site B	Site C	Site D	Supply
$Center_1$	25	35	20	30	50
$Center_2$	30	22	28	32	60
$Center_3$	18	27	33	24	40
Demand	35	40	45	30	

13. Construction materials delivery time (hours) from quarries to project sites. Minimize total time. Balance if needed, use **Least Cost**, then MODI.

	Project <sub>1</sub>	Project <sub>2</sub>	Project <sub>3</sub>	Supply
Quarry <sub>1</sub>	4.5	3.8	5.2	120
Quarry <sub>2</sub>	3.2	4.9	3.6	100
Quarry <sub>3</sub>	5.1	3.5	4.0	130
Demand	140	110	100	

14. Fire trucks must reach emergency locations. Response time (minutes) is critical. Minimize total response time using **VAM** and MODI.

	Loc <sub>1</sub>	Loc <sub>2</sub>	Loc <sub>3</sub>	Loc <sub>4</sub>	Supply
Station <sub>1</sub>	12	8	15	10	40
Station <sub>2</sub>	9	14	7	11	50
Station <sub>3</sub>	13	10	12	8	30
Demand	30	35	25	30	

15. Data packets routed through servers. Latency (milliseconds) must be minimized. Use **Least Cost Method** and optimize with MODI. Supply exceeds demand.

	Server <sub>A</sub>	Server <sub>B</sub>	Server <sub>C</sub>	Supply
Node <sub>1</sub>	45	38	52	200
Node <sub>2</sub>	41	47	36	250
Node <sub>3</sub>	50	42	44	180
Demand	180	220	200	

## Part D: Transshipment Problems

16. Two factories ship through one hub to two customers. Direct shipments are not allowed. Find the optimal transshipment plan.

**Supply/Demand:** F1: 150, F2: 200, C1: 180, C2: 170

**Transportation Costs (\$):**

Route	Cost	Route	Cost
F1 to H	6	H to C1	8
F2 to H	5	H to C2	7

17. Three suppliers send goods through two transshipment points to three destinations. Find the minimum cost allocation.

**Supply/Demand:** S1: 100, S2: 120, S3: 80, D1: 90, D2: 110, D3: 100

**Transportation Costs (\$):**

From	T1	T2	From	D1	D2	D3
S1	4	6	T1	5	7	6
S2	5	3	T2	8	4	6
S3	7	4				

18. Two plants ship through three distribution centers to four retailers. Direct plant-to-retailer shipments are expensive (\$15-\$20). Set up and solve the transshipment problem.

**Supply/Demand:** P1: 250, P2: 300, R1: 120, R2: 150, R3: 140, R4: 140

**Plant to DC Costs (\$):**

	DC1	DC2	DC3
P1	7	5	8
P2	6	9	5

**DC to Retailer Costs (\$):**

	R1	R2	R3	R4
DC1	4	6	5	7
DC2	5	3	6	4
DC3	7	5	3	6

19. Agricultural products from two farms go through a central market before reaching three processing plants. Farms cannot ship directly to plants. Solve using the transshipment method.

**Supply/Demand (tons):** Farm A: 180, Farm B: 220, P1: 150, P2: 130, P3: 120

**Transportation Costs (\$/ton):**

Route	Cost	From	P1	P2	P3
Farm A to M	3	M	5	6	4
Farm B to M	4				

20. Three warehouses distribute through two regional hubs to four stores. Find the optimal flow and total cost.

**Supply/Demand:** W1: 140, W2: 160, W3: 120, S1: 100, S2: 110, S3: 90, S4: 120

**Warehouse to Hub Costs (\$):**

	H1	H2
W1	8	6
W2	7	9
W3	5	7

**Hub to Store Costs (\$):**

	S1	S2	S3	S4
H1	4	6	5	7
H2	6	4	8	5

21. Electronics manufacturer: Two factories ship to three distribution centers, which then supply four retail chains. Direct factory-to-retail shipments are prohibited. Calculate buffer, construct the full table, and solve.

**Supply/Demand:** F1: 300, F2: 350, R1: 150, R2: 180, R3: 160, R4: 160

**Factory to DC Costs (\$):**

	DC1	DC2	DC3
F1	12	10	14
F2	11	13	10

**DC to Retail Costs (\$):**

	R1	R2	R3	R4
DC1	7	9	6	8
DC2	8	6	10	7
DC3	9	8	7	9

22. Petroleum distribution: Two refineries ship to three regional depots which supply four

gas stations. Direct refinery-to-station costs are prohibitive (\$20+). Apply the transshipment method.

**Supply/Demand:** R1: 500, R2: 600, G1: 250, G2: 280, G3: 270, G4: 300

**Refinery to Depot Costs (\$):**

	D1	D2	D3
R1	10	8	12
R2	9	11	8

**Depot to Gas Station Costs (\$):**

	G1	G2	G3	G4
D1	6	7	5	8
D2	7	5	9	6
D3	8	6	7	5

23. Pharmaceutical supply chain: Three manufacturing plants use two consolidation centers to reach five hospitals. Direct plant-to-hospital shipment is not feasible. Solve the complete transshipment problem.

**Supply/Demand:** M1: 200, M2: 250, M3: 180, H1: 120, H2: 140, H3: 130, H4: 110, H5: 130

**Plant to Consolidation Center Costs (\$):**

	C1	C2
M1	7	9
M2	6	8
M3	10	7

**Center to Hospital Costs (\$):**

	H1	H2	H3	H4	H5
C1	5	6	4	7	5
C2	6	4	7	5	8

24. International shipping: Four exporters ship through three ports to five importers. Direct exporter-to-importer costs are prohibitive (\$40+). Calculate buffer = 680, construct the full matrix, and solve.

**Supply/Demand:** E1: 150, E2: 180, E3: 160, E4: 190, I1: 130, I2: 150, I3: 140, I4: 120, I5: 140

**Exporter to Port Costs (\$):**

	P1	P2	P3
E1	18	22	20
E2	17	19	23
E3	21	18	20
E4	19	21	17

**Port to Importer Costs (\$):**

	I1	I2	I3	I4	I5
P1	12	14	11	15	13
P2	13	11	16	12	14
P3	15	13	12	14	10

25. Complex logistics network: Two production sites, three transshipment hubs, and four demand centers. Hubs can redistribute among themselves at \$3. Direct production-to-demand costs are expensive (\$18+). Model with buffer = 600, include all possible routes, and solve.

**Supply/Demand:** PS1: 280, PS2: 320, DC1: 140, DC2: 160, DC3: 150, DC4: 150

**Production Site to Hub Costs (\$):**

	Hub A	Hub B	Hub C
PS1	9	7	11
PS2	8	10	7

**Hub to Demand Center Costs (\$):**

	DC1	DC2	DC3	DC4
Hub A	6	8	5	7
Hub B	7	5	9	6
Hub C	8	6	7	5

**Hub-to-Hub Redistribution Cost:** \$3 (between any two hubs)