COMPARING THE PERFORMANCE OF THE CORRELATION COEFFICIENTS FOR THE BIVARIATE MIXTURE DISTRIBUTIONS

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Title

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ABSTRACT

This research aimed to compare the performance of five correlation coefficients for the bivariate mixture distributions. They are: Pearson product-moment correlation, Spearman rank correlation, maximal information coefficient, bias corrected distance correlation, and Chartterjee's new correlation. The simulation study was conducted to determine the estimate values of type I error rate and power of the test. The sample sizes employed in this study are 10, 20, 40, 60, and 100. The correlation parameters of each bivariate mixture distribution are 0.2, 0.4, 0.6, and 0.8. The findings indicated that Pearson product-moment correlation and Spearman rank correlation are able to control type I error rate in all cases. Whereas the maximal information coefficient, bias corrected distance correlation, and Chartterjee's new correlation are able to do so when the sample size is larger than 10. In almost all situations, Pearson product-moment correlation has the highest power, with the exception of a few where the mixture of Weibull distribution is involved. In addition, Pearson product-moment correlation, Spearman rank correlation, and bias corrected distance correlation seem to have similar power.

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Chapter 1
Introduction

1. Statements and significance of the problems

Correlation coefficients seem to be one of the most used tools in terms of statistical analysis. Almost all fields, including medical science, social and political sciences, engineering, economics, engineering etc., use it to assess how strongly variables are related. Whether the relevant variables are related and, if so, how strongly they are associated, are frequently of interest to data analysts. There are various statistical methods that are used to measure the strength of association and to test the independence of variables. The Pearson product-moment correlation coefficient (Pearson, 1895) is widely used nowadays due to its simple formulas and ease of understanding, even for those with no statistical background. But the Pearson product-moment correlation coefficient has its own assumptions. When one would like to use the Pearson product-moment correlation coefficient, one needs to make sure that both variables meet these conditions: both variables are interval or ratio variables and are well approximated by a normal distribution, and their joint distribution is bivariate normal (Bolboaca & Jäntschi, 2006). Data analysts struggle to find data that satisfies the aforementioned requirements, the Spearman rank correlation was introduced to help when data violate the assumptions of the Pearson product-moment correlation coefficient. Due to the assumptions underlying the Spearman rank correlation coefficient, there are still limitations to its use. The data must be on an ordinal scale, and the value of one variable must be monotonically related to the value of the other variable. Another issue is that it uses the rank of data to measure the strength of association between variables of interest, not the real value of data. Due to these issues, statisticians introduced more correlation coefficients such as the distance correlation coefficient (Székely et al., 2007), maximal information coefficient, or often called MIC (Reshef et al., 2011), Chartterjee's new correlation coefficient (Chatterjee, 2021) etc.

Data analysts always want their data to be sampled from normal distribution, and linear form is thought to be the best case for variable relationships. But things are never that simple. Some types of random data, particularly those derived from natural sources such as wind speed , rain fall, etc., are likely to be drawn from more than one distribution, often called mixture distribution. A mixture distribution is a weighted summation of two or more distributions, which can be the same or different types of distribution. In real-world data, some common mixture distributions are mixture of normal distribution, mixture of gamma distribution, and mixture of Weibull distribution, etc. As an example, Suhaila & Jemain (2007) studied about fitting of daily rainfall amounts in Peninsular Malaysia using several types of exponential distributions. The results indicated that the mixture distributions are better than the single distributions in modeling rainfall amounts, where the mixed Weibull is identified as the most appropriate model for the majority of sites in Peninsular Malaysia.

The question is, which methods should one use when one needs to measure the strength of association or to perform statistical tests to determine the independence of variables when they are drawn from mixture of normal distribution, mixture of gamma distribution, or mixture of Weibull distribution? There are several studies that are interested in this kind of problem, but only mixture normal distribution was investigated. For example, in the study of Bishara & Hittner (2012). The mixtures of gamma, and Weibull are not of interest to the previous researchers. In this study, we are going to investigate the performance of each method in various sizes of sample within each distribution.

2. Objectives

To compare the performance of Pearson product-moment correlation coefficient, Spearman rank correlation coefficient, distant correlation coefficient, maximal information coefficient, and Chartterjee's new correlation coefficient when random variables are sampled from mixture of normal distribution, mixture of gamma distribution, or mixture of Weibull distribution.

3. Contribution to knowledge

To gain an understanding of which methods of correlation coefficient should be used when random variables are generated from mixture of normal distribution, mixture of gamma distribution, or mixture of Weibull distribution, with various sizes of samples in order to get the most accurate result.

4. Scope of study

1. Correlation coefficients of interest are:
    - Pearson product-moment correlation coefficient
    - Spearman rank correlation coefficient
    - Bias corrected distant correlation coefficient
    - Maximal information coefficient
    - Chartterjee's new correlation coefficient
2. Distributions of interest include:
    - Mixture of normal distribution
    - Mixture of gamma distribution
    - Mixture of Weibull distribution
3. The study will focus on five sample sizes: 10, 20, 30, 50, and 100.
4. Four levels of correlation coefficients, including 0.2, 0.4, 0.6, and 0.8, will be explored.

Chapter 2
Related theory and literature reviews

1. Theory

In this study, we aim to investigate the performance of Pearson product-moment correlation, Spearman rank correlation coefficient, maximal information coefficient, distant correlation coefficient, and Chartterjee's new correlation coefficient when data are being drawn from some type of mixture distributions. Theories related to this study are shown below.

1.1 Correlation coefficients

Pearson correlation coefficient

Pearson product-moment correlation is a measure of strength of the linear relationship between two variables and it can be used to determine the direction of relationship too. It scores ranges between -1 and 1, and it describes the degree to which one variable is linearly related to another. According to Bolboaca & Jäntschi (2006), to use Pearson product-moment correlation coefficient both variables should meet the conditions below :

1. are interval or ratio variables and are well approximated by a normal distribution,
2. their joint distribution is bivariate normal.

To calculate the coefficient of correlation of sample data x and y sized n, the Pearson product-moment correlation coefficient can be estimated by

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

When one needs to perform the hypothesis testing for association, the t test with based statistic t can be used.

$$t = r\sqrt{\frac{n-2}{1-r^2}}$$

The null hypothesis is rejected when $|t|$ is greater than the critical value from a t distribution with $n-2$ degree of freedom

Spearman rank correlation coefficient

A Spearman coefficient is a Pearson product-moment correlation coefficient that is calculated using the ranks rather than the values of each variable (Schober et al., 2018). It is a non-parametric correlation measure that determines whether there is a monotonic relationship between two variables. The advantages of Spearman rank correlation is that, Spearman rank correlation coefficient does not require the assumptions about :

1. the frequency distribution of the variables,
2. the relationship between variable is linear or not,
3. the scale of variables.

(Bolboaca & Jäntschi, 2006)
A random sample of n pairs

$$(x_1, y_2), (x_2, y_2), \cdots, (x_n, y_n)$$

is drawn from a bivariate population. The Spearman rank correlation coefficient when tied data is not presented is obtained as

$$r_s = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2-1)}$$

And when tied data presents, the Spearman rank correlation can be calculated by

$$r_s = \frac{\sum_{i=1}^{n} R_i S_i - n\left(\frac{n+1}{n}\right)^2}{\sqrt{\left[\sum_{i=1}^{n} R_i^2 - n\left(\frac{n+1}{n}\right)^2\right]\left[\sum_{i=1}^{n} S_i^2 - n\left(\frac{n+1}{n}\right)^2\right]}}$$

where $D_i = R_i - S_i$, R_i is the rank of x_i and S_i is the rank of y_i (Gibbons & Chakraborti, 2014). r_s takes value between -1 and 1. If both variables monotonically change in the same direction, Spearman rank correlation coefficient will be positive. However, if the variables monotonically change in the opposite direction, Spearman rank correlation coefficient will be negative (da Costa, 2015).

When one needs to perform the hypothesis testing for association, t test is recommended when $n \geq 10$ (Hays, 1988). The based statistic is

$$t = r_s\sqrt{\frac{n-2}{1-r_s^2}}$$

The null hypothesis is rejected when $|t|$ is greater than the critical value from a t distribution with $n-2$ degree of freedom.

Maximal information coefficient

In 2011, Reshef et al. (2011) proposed maximal information coefficient(MIC). MIC is being used to measure the strength of relationship between two variables.It captures a wide range of associations, both functional and non, with a score for functional relations that generally equates to the coefficient of determination R^2 of the relative regression function (Reshef et al., 2011). It takes value between zero to one and has two main properties :

1. Generality : with a sufficiently large sample size, it can identify linear and nonlinear associations,

2. Equitability : MIC gives similar scores to relationship which are corrupted with the same level of noise regardless of the relationship type.

MIC converges to one for noiseless functional relationships as sample size increases and to zero for statistically independent variables as probability approaches one. MIC makes no assumptions regarding the distribution of the variables, and the association between the two could not be linear. (Reshef et al., 2011). For two vectors x and y, MIC can be estimated by

$$MIC = \max\left\{\frac{I(x,y)}{log_2 \min\{n_x, n_y\}}\right\}$$

where $I(x,y)$ is mutual information:

$$\begin{aligned} I(x,y) &= H(x) + H(y) - H(x,y) \\ &= \sum_{i=1}^{n_x} p(x_i) log_2 \frac{1}{p(x_i)} + \sum_{j=1}^{n_y} p(y_j) log_2 \frac{1}{p(y_j)} - \sum_{i=1}^{n_x}\sum_{j=1}^{n_y} p(x_i, y_j) log_2 \frac{1}{p(x_i, y_j)} \end{aligned}$$

where $p(x,y)$ is the joint probability distribution of x and y, $p(x)$ and $p(y)$ are marginal distribution of X and Y respectively, n_x and n_y are the number of bins for partitioning $x - axis$ and $y - axis$ respectively and $n_x n_y < B(n)$, $B(n) = n^{0.6}$ (Zhang et al., 2014).

If one needs to perform the hypothesis test, one might needs to use the permutation test in order to find the $p - value$ as there is not any asymptotic theory for MIC.

The basic steps to find the $p - value$ via permutation test are followed as below :

1. calculate the $MIC(x,y)$ of original data,
2. set the number of replications, say n here,
3. fix the variable x and re-sample the variable y, set to be y^*,
4. calculate the MIC of x and re-sampled y (y^*), $MIC(x, y^*)$,
5. record the $MIC(x, y^*)$,
6. repeat step $3, 4$ and 5, n times,
7. then the $p - value$ of the test can be calculated by,

$$p - value = \frac{\text{number of } MIC(x, y^*) \geq MIC(x,y)}{\text{number of replications}}$$

Bias corrected distance correlation coefficient

Székely et al. (2007) proposed a distance correlation, denoted as $\mathcal{R}$. It is known as an association measure between random variables or random vectors and takes range from 0 to 1. It equals zero if and only if two random variables or random vectors are independent that means, if $\mathcal{R} = 0$ can identify the independence of the variables. The distance correlation of sample data x and y is computed by

$$\mathcal{R}^2(x,y) = \begin{cases} \dfrac{\nu_n^2(x,y)}{\sqrt{\nu_n^2(x)\nu_n^2(y)}}, & \nu_n^2(x)\nu_n^2(y) > 0 \\ 0, & \nu_n^2(x)\nu_n^2(y) = 0 \end{cases}$$

where $\mathcal{R}_n(x,y)$ is empirical distance correlation for a sample of size n and
and $\nu^2(x,y)$ is distance covaraince, can be computed by

$$
\begin{aligned}
\nu_n^2(x,y) &= \frac{1}{n^2}\sum_{i,j=1}^{n} A_{i,j}B_{i,j} \\
a_{ij} &= \left|x_i - x_j\right| \\
\bar{a}_{i.} &= \frac{1}{n}\sum_{j=1}^{n} a_{ij} \\
\bar{a}_{.j} &= \frac{1}{n}\sum_{i=1}^{n} a_{ij} \\
\bar{a}_{..} &= \frac{1}{n^2}\sum_{i,j=1}^{n} a_{ij} \\
A_{i,j} &= a_{ij} - \bar{a}_{i.} - \bar{a}_{.j} - \bar{a}_{..}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
b_{ij} &= \left|y_k - y_l\right| \\
B_{i,j} &= b_{ij} - \bar{b}_{i.} - \bar{b}_{.j} - \bar{b}_{..} \\
k,l &= 1,2,\cdots,n
\end{aligned}
$$

Also,

$$
\begin{aligned}
\nu_n^2(X) = \nu_n^2(x,x) &= \frac{1}{n^2}\sum_{i,j=1}^{n} A_{ij}^2 \\
\nu_n^2(Y) = \nu_n^2(y,y) &= \frac{1}{n^2}\sum_{i,j=1}^{n} B_{ij}^2
\end{aligned}
$$

$|.|$ is Euclidean distance

Székely & Rizzo (2013) developed the distance correlation to be the bias corrected distance correlation by modify distance covariance to be an unbiased.

Given,

$$
A_{i,j}^{*} = \begin{cases} \dfrac{n}{n-1}\left(A_{i,j} - \dfrac{a_{ij}}{n}\right), & i \neq j \\ \dfrac{n}{n-1}\left(\bar{a}_{i.} - \bar{a}_{..}\right), & i = j \end{cases}
$$

Similarly,

$$
B_{i,j}^{*} = \begin{cases} \dfrac{n}{n-1}\left(B_{i,j} - \dfrac{b_{ij}}{n}\right), & i \neq j \\ \dfrac{n}{n-1}\left(\bar{b}_{i.} - \bar{b}_{..}\right), & i = j \end{cases}
$$

The unbias distance covaraince can be computed by

$$\nu_n^*(x,y) = \frac{1}{n-3}\left(\sum_{i,j=1}^{n} A_{i,j}^* B_{i,j}^* - \sum_{i=1}^{n} \frac{n}{n-2} A_{i,i}^* B_{i,i}^*\right)$$

And the bias corrected distance correlation is

$$\mathcal{R}_n^*(x,y) = \frac{\nu_n^*(x,y)}{\sqrt{\nu_n^*(x,x)\nu_n^*(y,y)}}$$

if $\nu_n^*(x,x)\nu_n^*(y,y) > 0$, and otherwise $\mathcal{R}_n^*(x,y) = 0$

Székely & Rizzo (2013) also proposed a t test for distance correlation, but the condition is that the data must be in high dimension. For one-dimensional data, one may need to use permutation test which is costly and consume more time. To solve this problem, Shen et al. (2022) introduced a chi squared test of distance correlation.

The chi square test of distance correlation consumes less time and have a good performance almost as good as the expensive permutation test. If C is bias corrected distance correlation coefficient, $C = \mathcal{R}_n^*(x,y)$, then the null hypothesis is rejected when the significance level is greater than $1 - F_{\chi_1^2 - 1}(nC)$. That means, the null hypothesis is rejected when

$$P\left(\chi_1^2 - 1 > nC\right) < \alpha$$

where χ_1^2 is chi square distribution with degree freedom of 1, and α is level of significance.

Chartterjee's new correlation coefficient

The Chatterjee's new correlation coefficient has been recently developed by Sourav Chatterjee (2021). Let (X, Y) be a pair of random variables, where Y is not a constant. Let $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ be iid. pair with the same law as (x, y). Suppose that $x_i's$ and $y_i's$ have no ties. Rearrange the data as $\left(x_{(1)}, y_{(1)}\right), \cdots, \left(x_{(n)}, y_{(n)}\right)$ such that $x_{(1)} \leq \cdots \leq x_{(n)}$. Let r_i be the rank of $Y_{(i)}$, that is, the number of j such that $y_j \leq y_i$, the new correlation coefficient is simply defined as:

$$\xi_n(x,y) := 1 - \frac{3\sum_{i=1}^{n-1} |r_{i+1} - r_i|}{n^2 - 1}$$

And when $X_i's$ have ties, then $\xi_n(x,y)$ is defined as :

$$\xi_n(x,y) := 1 - \frac{n\sum_{i=1}^{n-1} |r_{i+1} - r_i|}{2\sum_{i=1}^{n} l_i(n - l_i)}$$

where l_i is the number of j such that $y_{(j)} > y_{(i)}$

$\xi_n(x,y)$ has a simple asymptotic theory under independence when testing is desired.

Suppose that X and Y are independence and Y is continuous. Then as $n \to \infty$

$$\sqrt{n}\xi_n(x,y) \to N\left(0, \frac{2}{5}\right)$$

When the hypothesis testing is performed, The null hypothesis is being rejected when

$$P\left(Z > \frac{\sqrt{n}\xi_n(x,y)}{\sqrt{\frac{2}{5}}}\right) < \alpha$$

where Z is standard normal distribution, and α is level of significance.

In numerical studies, Chatterjee (2021) showed that the convergence is roughly valid even for n as small as 20.

1.2 Mixture distributions

Mixture of normal distribution

The probability density function of a mixture of normal distributions is a weighted sum of k normal distribution functions with different parameters that are weighted by λ. The function is written as

$$p(x) = \sum_{i=1}^{k} \lambda_i f\left(x; \mu_i, \sigma_i^2\right) \qquad \text{where} \qquad k = 2, 3, \cdots$$

When mixture of two normal distribution are of interest, the probability function of mixture normal distribution will be

$$\begin{aligned} p\left(x; \lambda, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2\right) &= \lambda f\left(x; \mu_1, \sigma_1^2\right) + (1-\lambda) f\left(x; \mu_2, \sigma_2^2\right) \\ &= \lambda \left\{ \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2} \right\} + (1-\lambda) \left\{ \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2} \right\} \end{aligned}$$

where

x is random variable

λ is weight parameter

μ_i is mean parameter , $i = 1, 2$

σ_i^2 is variance parameter , $i = 1, 2$

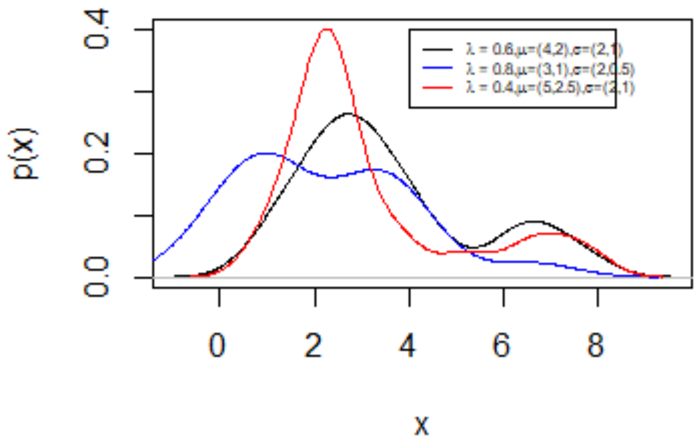

Figure 1 Density plot of mixture of normal distribution

Mixture of gamma distribution

The probability density function of a mixture of gamma distributions is a weighted sum of k gamma distribution functions with different parameters that are weighted by λ. The function is written as

$$p(x) = \sum_{i=1}^{k} \lambda_i f(x; \alpha_i, \beta_i) \qquad \text{where} \qquad k = 2, 3, \cdots$$

When mixture of two gamma distribution are of interest, the probability function of mixture gamma distribution will be

$$\begin{aligned} p(x; \lambda, \alpha_1, \alpha_2, \beta_1, \beta_2) &= \lambda f(x; \alpha_1, \beta_1) + (1-\lambda) f(x; \alpha_2, \beta_2) \\ &= \lambda \frac{1}{\Gamma(\alpha_1) \beta_1^{\alpha_1}} x^{\alpha_1 - 1} exp\left(-\frac{x}{\beta_1}\right) + (1-\lambda) \frac{1}{\Gamma(\alpha_2) \beta_2^{\alpha_2}} x^{\alpha_2 - 1} exp\left(-\frac{x}{\beta_2}\right) \end{aligned}$$

where

x is random variable , $x > 0$

λ is weight parameter ,

α_i is shape parameter , $i = 1, 2$, $\alpha_i > 0$

β_i is scale parameter , $i = 1, 2$,$\beta_i > 0$

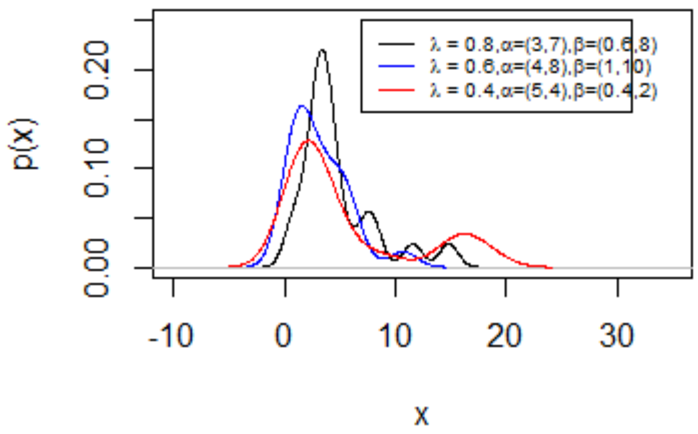

Figure 2 Density plot of mixture of gamma distribution

Mixture of Weibull distribution

The probability density function of a mixture of Weibull distributions is a weighted sum of m Weibull distribution functions with different parameters that are weighted by w. The function is written as

$$p(x) = \sum_{i=1}^{m} w_i f(x; k_i, \lambda_i) \quad \text{where} \quad m = 2, 3, \cdots$$

When mixture of two Weibull distribution are of interest, the probability function of mixture normal distribution will be

$$\begin{aligned} p\left(x; w, k_1, k_2, \lambda_1, \lambda_2\right) &= \lambda f\left(x; k_1, \lambda_1\right) + (1-w) f\left(x; k_2, \lambda_2\right) \\ &= w \frac{k_1}{\lambda_1} \left(\frac{x}{\lambda_1}\right)^{k_1 - 1} e^{-\left(\frac{x}{\lambda_1}\right)^{k_1}} + (1-w) \frac{k_2}{\lambda_2} \left(\frac{x}{\lambda_2}\right)^{k_2 - 1} e^{-\left(\frac{x}{\lambda}\right)^{k_2}} \end{aligned}$$

where

x_i is random variable , $x \geq 0$

w is weight parameter

k_i is shape parameter , $i = 1, 2$

λ_i is scale parameter , $i = 1, 2$

1.3 Study the performances of the tests

In this study, we aim to study the performance of each method of correlation coefficients when data are drawn from some types of mixture distribution. The ability to control type I error rate and the power of the test will be used to evaluate performance.

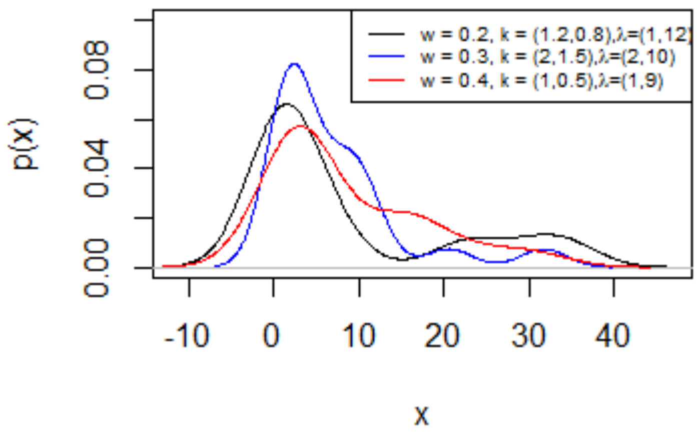

Figure 3 Density plot of mixture of Weibull distribution

Ability to control type I error rate

In hypothesis testing, there might be 2 types of errors that happen when we draw the conclusion. Type I error is one of them, it happens when the null hypothesis is rejected when it is actually true. Type I error rate can be estimated by

$$\hat{\alpha} = \frac{\sum_{i=1}^{n} M_i}{n} \qquad M_i = \begin{cases} 0, & \text{Fail to reject null hypothesis} \\ 1, & \text{Reject null hypothesis} \end{cases}$$

where

M_i is the result of the ith test (reject or fail to reject the null hypothesis)

n is number of replications

Bradley (1978) introduced a liberal criterion such that, when the level of significance is 0.05, then the estimated type I error rate should be in the range of [0.025, 0.075], and if it is out of the range, that means that the test is not robust.

If the tests are robust, then the power of the test will be used to find the best one.

Power of the test

The power of the test is the probability of rejecting the null hypothesis when it is false, denoted as $(1-\beta)$, and can be estimated by

$$\widehat{(1-\beta)} = \frac{\sum_{i=1}^{n} M_i}{n} \qquad M_i = \begin{cases} 0, & \text{Fail to reject null hypothesis} \\ 1, & \text{Reject null hypothesis} \end{cases}$$

where

M_i is the result of the ith test (reject or fail to reject the null hypothesis)

n is number of replications

2. Literature reviews

Székely et al. (2007) introduced a distance covariance, $\nu(x, y)$ and the distance correlation coefficient, $\mathcal{R}(x, y)$. $\mathcal{R}(x, y)$ is able to quantify the relationship between X and Y that is not jointly normal distributed and non-linear, making it appears better than the classical correlation coefficient.

Székely & Rizzo (2013) developed a distance covariance to be unbiased, $\nu^*(x, y)$ and introduced the biase corrected distance correlation, $\mathcal{R}^*(x, y)$. The t test of distance correlation in the case that random variables or random vectors are in high dimension was also proposed in the study.

Shen et al. (2022) proposed the chi squared test of distance correlation. The bias corrected distance correlation was used to perform the chi square test. The study found that, it works better than t test and almost as good as the permutation test but it consumes less time. The chi squared test also work well with one-dimensional data.

Reshef et al. (2011) presented a measure of dependence for variable relationships: the maximal information coefficient (MIC). A wide range of associations, both functional and non-functional, are captured by MIC, which for functional relationships provides a score that nearly equates the coefficient of determination R^2 of the data relative to the regression function. MIC can detect various types of relationship including linear, non-linear, functional and non-functional relationships.

Chatterjee (2021) presented a new coefficient of correlation, $\xi_n(x, y)$. $\xi_n(x, y)$ has three main properties, (a) it is as simple as classical coefficients, (b) it is a consistent estimator of some measure of dependence which is 0 if and only if the variables are independent and 1 if and only if one is a measurable function of the other, and (c) it has a simple asymptotic theory under the hypothesis of independence, like the classical coefficients.

Bishara & Hittner (2012) studied about testing the significance of a correlation with non-normal data with 12 different methods such as Pearson correlation (t-test, Z-test), Spearman correlation (t-test, Exact test), transformation methods(Box-Cox, Yeo-J, Arcsine, RIN), resampling methods (Permutation, Boot-Uni,Boot-Bi and Boot-Bi-BCa). The data were drawn from 6 different distributions such as normal distribution, Weibull distribution, chi-square distribution with 1 degree of freedom, the bimodal (mixture of two normal distribution) and long-tailed shape (mixture of two normal distribution). The result indicated that, when sample size is at least 20, the RIN transformation approach may occasionally be helpful when correlations between non-normal variables need to be assessed for significance. This approach may in many cases increase power while maintaining type I error. For smaller samples of non-normal data, the permutation test may sometimes be more advantageous than commonly recommended alternatives.

Tuğran et al. (2015) used simulation methods to compare correlation coefficients with regard to type 1 error rate and power of the tests. Pearson, Spearman, Kendal Tau, Permutation-based, and Winsorized were compared when data are drawn from bivariate normal distribution, bivariate t-distribution and bivariate log-normal distribution. The Winsorized correlation coefficient and Permutation-based Pearson correlation performed better than the others, although Permutation-based Pearson correlation becomes more computationally difficult as sample sizes increase.

Deebani & Kachouie (2020) compared Pearson's correlation, Spearman's coefficient, Distance correlation, Maximal Information Coefficient(MIC), Maximal Correlation. But distribution families were not of interest. Instead, 7 different shapes of relationships, such as random, linear, polynomial of fourth order, exponential, parabolic, sinusoidal with varying frequency,sinusoidal with fixed frequency, and circle were investigated. Three levels of noise (low, medium and high) were added to the relationship in order to investigate the true signal. The findings indicated that, in noiseless relationships, MIC beats all correlation. On the other hand, it has a large standard error and its performance varies with the level of noise. MIC obtains larger values than distance correlation, which is more accurate.

Chapter 3
Research methodology

In this study, we aim to compare the performance of 5 correlation coefficients when data is drawn from the 2 components mixture distribution. The steps of this study are followed as below

1. Data generating algorithm

Ruscio & Kaczetow (2008) introduced an iterative algorithm which can be used to generate specified correlated non-normal data. As Bishara & Hittner (2012) explained, this algorithm's workflow is followed as below:

1. Firstly, X_0 and Y_0 are individually generated with the specified non-normal distribution shapes, in this case X_0 and Y_0 are mixture distributions.
2. With an intermediate correlation coefficient, bivariate normal X_1 and Y_1 are generated. The target correlation value is the first intermediate correlation coefficient value that is tried.
3. X_0 and Y_0 are used to replace X_1 and Y_1, and they are replaced in such a way that the rank orders of corresponding variables are preserved. It might inflate or deflate observed correlation coefficient as both variables are no longer bivariate normal
4. Based on the difference between the target and observed correlations, a new intermediate correlation is selected (modified to be either higher or lower than the initial intermediate correlation).
5. The process then repeats iteratively: The new intermediate correlation is used to to generate bivariate normal X_2 and Y_2, then non-normal X_0 and Y_0 replace X_2 and Y_2, and a new intermediate correlation is generated .
6. The observe correlation tends to move closer to the target correlation over repeated iterations.
7. The algorithm stops when it fails to reduce the root-mean-square residual correlation on five consecutive iterations.

This algorithm can be used for nearly any desire distribution shape. It can produces almost no sampling error of the result data; if the target correlation is 0.5 then it will produce a correlation of almost exactly 0.5 for every sample, even with a small sample size. The Ruscio and Kaczetow algorithm produced intended population correlation coefficient accurately to at least 2 decimal places, and it did so in every single scenario. The algorithm produced both the intended correlation and the inteded marginal distribution shapes (Bishara & Hittner, 2012).

To generate data from the mixture distributions, package used is "mixR" (Yu, 2021). Package used to find the value of correlation coefficient and perform independence test of Pearson product-moment and Spearman rank correlation coefficient is "stats". The value of correlation coefficient and perform independence test of Maximal information coefficient, Distance correlation coefficient are calculated by package ”AlterCorr” (Pazos, 2022). Lastly, to find the the value of correlation coefficient and perform independence test of Charterjee’s new correlation coefficient, package ”XICOR” (Chartterjee & Susan, 2020) is used. All packages used in this study are freely available in R Core Team (2021).

2. Scope of data simulation

1. Distributions of data:

   As this study takes an interest in real world application, the variables of each distributions were chosen as shown in Table 1.

   – In mixture of normal and mixture of gamma distribution cases, their variables take value from Koca et al. (2019) which studied about assessing wind energy potential using finite mixture distributions. The weight parameter for the mixture of the gamma distribution scored between 0.05 and 0.95, whereas the weight for the mixture of the normal scored between 0.1 and 0.9, according to the study. The shape parameters for the mixture of gamma distribution’s first component varied from 3.2 to 5, while the values for the second component were in the range of 3.5 to 12.2. The scale parameters for the first component’s components were between 0.4 and 2.5 and between 0.6 and 8.15 for the second. The first component of mixture of normal distribution’s mean parameter took a value around 1 to 9.5, and the second component scored around 3.5 to 12. The first component’s standard deviation ranged from 0.2 to 3.7, and the second’s from 1.2 to 4.5.

   – For the variables of mixture of Weibull distribution take values from the work of (Suhaila & Jemain, 2007), which is the study about fitting daily rainfall amount in Peninsular Malaysia. The result showed that the weight parameter ranged from 0.1 to 0.2. The shape parameters has value around 1 to 1.5 for the first component and around 0.8 for the second component. And lastly, the scale parameters took value around 1 for the first component and from 11 to 16 for the second component.

2. Five sample sizes were chosen: 10, 20, 30, 50, and 100.
3. Four correlation coefficient between the two variables are: 0.2, 0.4, 0.6, 0.8.
4. Each pair of distribution will be generated following each sample size and correlation coefficient
5. Level of significance set to be 0.05.
6. In the event that a permutation test is used to find the $p - value$, 100 replications will be produced.

Table 1 Distributions of interest and their variables

Distributions	Parameters			
	First component		Second component	
Mixture of normal (1)	λ	0.68	$1-\lambda$	0.32
	μ_1	6.9	μ_2	3.5
	σ_1	2.7	σ_2	1.28
Mixture of normal (2)	λ	0.12	$1-\lambda$	0.88
	μ_1	0.82	μ_2	5
	σ_1	0.25	σ_2	2.5
Mixture of normal (3)	λ	0.46	$1-\lambda$	0.54
	μ_1	9.4	μ_2	4.5
	σ_1	3.7	σ_2	1.8
Mixture of gamma (1)	λ	0.95	$1-\lambda$	0.05
	α_1	20	α_2	4.4
	β_1	2.3	β_2	1
Mixture of gamma (2)	λ	0.57	$1-\lambda$	0.43
	α_1	3.5	α_2	12.2
	β_1	0.64	β_2	1.4
Mixture of gamma (3)	λ	0.82	$1-\lambda$	0.18
	α_1	4.1	α_2	7.2
	β_1	0.84	β_2	8.1
Mixture of Weibull	w	0.2	$1-w$	0.8
	k_1	1.5	k_2	0.8
	λ_1	1	λ_2	15

3. Process of the study

3.1 Study the ability to control type I error of correlation coefficients

1. Generate the data in each situation following the null hypothesis
2. Calculate the correlation coefficients and the $p-value$ of the following five methods :
   - Pearson product-moment correlation coefficient,
   - Spearman rank correlation coefficient,
   - maximal information coefficient,

  - bias corrected distance correlation coefficient,
  - and Charterjee's new correlation coefficient.
3. Test the following hypothesis

$$\begin{aligned} H_0 : \rho &= 0 \\ H_1 : \rho &\neq 0 \end{aligned}$$

4. The null hypothesis is rejected when $p - value < 0.05$, and one fail to reject it when $p - value \geq 0.05$.
5. Repeat step 1 to step 4 for 10,000 times.
6. Count the number of null hypothesis rejections.
7. Estimate the value of type I error rate $\hat{\alpha}$ by

$$\hat{\alpha} = \frac{\text{Number of null hypothesis rejections}}{\text{Number of replications}}$$

8. Compare the estimated type I error rate with Bradley's criterion. When $\alpha = 0.05$, if the tests have ability to control type I error, the estimated value should be in the range of $[0.025, 0.075]$. If there are tests that have an estimated type I error stays in the range above, one might have to use the power of the tests to compare the performance of the tests instead.

3.2 Study power of the test

1. Generate the data in each situation following the alternative hypothesis.
2. Calculate the correlation coefficients and the $p - value$ of the following five methods :
  - Pearson product-moment correlation coefficient,
  - Spearman rank correlation coefficient,
  - maximal information coefficient,
  - bias corrected distance correlation coefficient,
  - and Charterjee's new correlation coefficient.
3. Test the following hypothesis

$$\begin{aligned} H_0 : \rho &= 0 \\ H_1 : \rho &\neq 0 \end{aligned}$$

4. The null hypothesis is rejected when $p - value < 0.05$, and one fail to reject it when $p - value \geq 0.05$.
5. Repeat step 1 to step 4 for 10000 times.
6. Count the number of null hypothesis rejections.
7. Estimate the power of the test $(1 - \beta)$ by

$$\widehat{1 - \beta} = \frac{\text{Number of null hypothesis rejections}}{\text{Number of replications}}$$

8. Compare the power of the test of all 5 correlation coefficients. The higher the power of the test, the better the performance of the test.

Chapter 4

Results

In this study we aim to investigate the performance of five methods of correlation coefficients such as Pearson product-moment correlation, Spearman rank correlation coefficient, maximal information coefficient, bias corrected distance correlation, and the new correlation coefficient of Chartterjee. Some notations to the result is followed by

- MN1 indicates the mixture of normal distribution (1)
- MN2 indicates the mixture of normal distribution (2)
- MN3 indicates the mixture of normal distribution (3)
- MG1 indicates the mixture of gamma distribution (1)
- MG2 indicates the mixture of gamma distribution (2)
- MG3 indicates the mixture of gamma distribution (3)
- MW indicates the mixture of Weibull distribution
- r indicates the Pearson product-moment correlation coefficient
- r_s indicates the Spearman rank correlation coefficient
- MIC indicates maximal information coefficient
- $\mathcal{R}^*$ indicates bias corrected distance correlation
- ξ_n indicates Chartterjee's new correlation coefficient

The results are broken down into two sections in this chapter.

1. Ability to control type I error of each test, the result is decided by Bradley's liberal criterion.
2. The estimated power of the test

1. Controlling type I error

The ability to control type I error rate of each methods when data are sampled from the mixture of normal distribution, mixture of gamma distribution, and mixture of Weibull distribution will be considered by the Bradley's liberal criterion. When the level of significant is 0.05, then the estimated value of α should be fallen in the range $[0.025, 0.075]$.

The Table 2-11 is the result of estimated type I error rate of each size of samples.

- From the Table 2 and 3, when sample size equals to 10 ($n = 10$), the $t-$test of Pearson-product-moment correlation and Spearman ranked correlation are able to control type I error rate as they produced the estimated type I error rate very close to 0.05. For the bias corrected distance correlation, the estimated type I error rate that are out of control are MN1-MG2, MG1-MG3, and MG3-MG3 pairs. While Chartterjee's new correlation coefficient can control type I error rate for some pairs such as MN2-MG1, MN2-MG3, MN3-MG2, MG1-MW, MG2-MG2, MG2-MG3, and MG3-MW. And the poorest one is MIC which can not control the type I error rate in every pair of data.

- From the Table 4-11, when sample size equals to 20, 30, 50, 100, all fives methods are able to control type I error rate.
- The average value of correlations found by the five methods differs in every sample sizes. Pearson product-moment, Spearman rank, bias corrected distance correlation and Chartterjee's new correlation found correlations that are close to the true value, while MIC gave a higher value than the true correlation.

Table 2 The estimated type I error when one variable is sampled from mixture of normal distribution and $n = 10$

Distributions	**Methods**									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MN1-MN1	-0.0015	0.0492	-0.0015	0.0599	0.3308	**0.0084**	0.0005	0.0717	0.0012	**0.0227**
MN1-MN2	0.0030	0.0499	0.0038	0.0545	0.3284	**0.0098**	0.0005	0.0711	0.0000	**0.0208**
MN1-MN3	0.0069	0.0511	0.0061	0.0564	0.3273	**0.0086**	-0.0006	0.0707	-0.0002	**0.0241**
MN1-MG1	0.0001	0.0504	0.0003	0.0534	0.3261	**0.0083**	-0.0014	0.0720	0.0006	**0.0236**
MN1-MG2	-0.0001	0.0517	-0.0020	0.0588	0.3307	**0.0072**	0.0012	**0.0753**	0.0030	**0.0222**
MN1-MG3	0.0005	0.0519	-0.0003	0.0590	0.3287	**0.0084**	0.0006	0.0719	0.0005	**0.0236**
MN1-MW	0.0024	0.0484	0.0040	0.0558	0.3251	**0.0082**	-0.0020	0.0676	-0.0009	**0.0237**
MN2-MN2	-0.0002	0.0507	0.0016	0.0529	0.3255	**0.0093**	-0.0027	0.0672	-0.0030	**0.0216**
MN2-MN3	-0.0031	0.0500	-0.0021	0.0542	0.3270	**0.0083**	0.0008	0.0725	-0.0003	**0.0223**
MN2-MG1	0.0014	0.0494	0.0017	0.0519	0.3283	**0.0096**	-0.0010	0.0742	-0.0010	0.0280
MN2-MG2	-0.0043	0.0542	-0.0029	0.0593	0.3285	**0.0090**	0.0009	0.0748	0.0020	**0.0240**
MN2-MG3	0.0030	0.0485	0.0029	0.0540	0.3273	**0.0085**	0.0001	0.0699	0.0001	0.0248
MN2-MW	-0.0015	0.0489	0.0004	0.0583	0.3292	**0.0102**	0.0023	0.0738	0.0042	**0.0243**
MN3-MN3	0.0031	0.0479	0.0059	0.0548	0.3272	**0.0099**	-0.0007	0.0712	-0.0009	**0.0224**
MN3-MG1	0.0011	0.0509	0.0011	0.0523	0.3287	**0.0094**	0.0024	0.0731	0.0008	**0.0222**
MN3-MG2	0.0022	0.0469	0.0012	0.0520	0.3265	**0.0084**	-0.0011	0.0718	0.0018	0.0251
MN3-MG3	-0.0031	0.0464	-0.0054	0.0486	0.3270	**0.0078**	-0.0035	0.0661	-0.0017	**0.0200**
MN3-MW	-0.0014	0.0498	-0.0019	0.0557	0.3270	**0.0084**	0.0004	0.0720	0.0007	**0.0239**

Note: Bolded value means the estimated type I error is out of the range of Bradley's liberal criterion

Table 3 The estimated type I error when one variable is sampled from mixture of gamma or mixture of Weibull distribution and $n = 10$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MG1-MG1	-0.0004	0.0521	0.0007	0.0559	0.3281	**0.0085**	0.0009	0.0722	0.0017	**0.0237**
MG1-MG2	-0.0026	0.0457	-0.0018	0.0523	0.3265	**0.0075**	-0.0012	0.0706	0.0008	**0.0237**
MG1-MG3	0.0021	0.0522	0.0027	0.0574	0.3291	**0.0109**	0.0008	**0.0752**	0.0009	**0.0247**
MG1-MW	0.0000	0.0519	0.0008	0.0527	0.3280	**0.0087**	-0.0017	0.0683	0.0015	0.0254
MG2-MG2	-0.0032	0.0518	-0.0021	0.0603	0.3315	**0.0107**	0.0020	0.0740	0.0013	0.0259
MG2-MG3	-0.0070	0.0511	-0.0068	0.0538	0.3283	**0.0081**	0.0005	0.0738	-0.0017	0.0246
MG2-MW	-0.0021	0.0483	-0.0035	0.0546	0.3302	**0.0092**	-0.0019	0.0725	-0.0011	**0.0214**
MG3-MG3	0.0034	0.0526	0.0063	0.0547	0.3276	**0.0092**	0.0012	**0.0787**	-0.0010	**0.0230**
MG3-MW	0.0004	0.0449	0.0012	0.0530	0.3268	**0.0070**	-0.0014	0.0695	-0.0006	0.0246
MW-MW	0.0040	0.0581	0.0046	0.0515	0.3287	**0.0081**	-0.0003	0.0651	-0.0006	**0.0221**

Note: **Bolded** value means the estimated type I error is out of the range of Bradley's liberal criterion

Table 4 The estimated type I error when at least one variable is sampled from normal distribution and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MN1-MN1	0.0011	0.0516	0.0028	0.0532	0.3593	0.0498	-0.0004	0.0562	-0.0003	0.0399
MN1-MN2	0.0003	0.0502	0.0011	0.0505	0.3595	0.0518	0.0006	0.0579	0.0002	0.0399
MN1-MN3	-0.0007	0.0465	0.0004	0.0472	0.3576	0.0475	-0.0016	0.0534	-0.0023	0.0380
MN1-MG1	0.0021	0.0507	0.0023	0.0491	0.3570	0.0475	-0.0010	0.0561	0.0005	0.0399
MN1-MG2	-0.0035	0.0485	-0.0044	0.0477	0.3573	0.0467	-0.0007	0.0538	-0.0019	0.0370
MN1-MG3	0.0009	0.0488	-0.0005	0.0493	0.3569	0.0469	-0.0005	0.0565	0.0001	0.0359
MN1-MW	-0.0005	0.0476	-0.0020	0.0524	0.3606	0.0492	0.0002	0.0559	0.0022	0.0399
MN2-MN2	0.0040	0.0521	0.0052	0.0538	0.3586	0.0503	0.0008	0.0582	0.0017	0.0415
MN2-MN3	-0.0018	0.0508	0.3588	0.0494	0.3588	0.0498	-0.0003	0.0547	0.0011	0.0388
MN2-MG1	-0.0012	0.0502	-0.0011	0.0490	0.3580	0.0520	-0.0001	0.0563	0.0011	0.0412
MN2-MG2	-0.0043	0.0497	-0.0032	0.0481	0.3573	0.0481	-0.0005	0.0572	-0.0006	0.0379
MN2-MG3	0.0010	0.0522	0.0016	0.0528	0.3576	0.0475	0.0002	0.0584	0.0009	0.0394
MN2-MW	0.0007	0.0482	0.0016	0.0478	0.3576	0.0471	0.0002	0.0537	-0.0002	0.0396
MN3-MN3	0.0057	0.0516	0.0064	0.0555	0.3589	0.0492	0.0014	0.0597	0.0009	0.0405
MN3-MG1	0.0003	0.0486	0.0018	0.0491	0.3581	0.0533	0.0001	0.0550	0.0009	0.0383
MN3-MG2	0.0035	0.0501	0.0022	0.0518	0.3592	0.0510	0.0002	0.0545	0.0024	0.0381
MN3-MG3	0.0011	0.0507	0.0004	0.0483	0.3595	0.0505	-0.0001	0.0552	-0.0003	0.0377
MN3-MW	0.0018	0.0536	0.0029	0.0516	0.3583	0.0487	0.0009	0.0585	0.0007	0.0394

Table 5 The estimated type I error when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MG1-MG1	0.0033	0.0514	0.0011	0.0503	0.3596	0.0502	0.0006	0.0555	0.0012	0.0409
MG1-MG2	-0.0016	0.0487	-0.0008	0.0494	0.3574	0.0492	0.0002	0.0560	-0.0006	0.0400
MG1-MG3	0.0022	0.0523	0.0024	0.0533	0.3586	0.0478	0.0005	0.0563	0.0000	0.0368
MG1-MW	0.0016	0.0528	0.0010	0.0509	0.3570	0.0474	0.0000	0.0539	-0.0008	0.0341
MG2-MG2	0.0025	0.0484	0.0019	0.0471	0.3587	0.0488	-0.0003	0.0545	0.0005	0.0392
MG2-MG3	-0.0011	0.0495	-0.0004	0.0497	0.3576	0.0488	-0.0002	0.0586	0.0011	0.0394
MG2-MW	0.0008	0.0489	0.0004	0.0474	0.3578	0.0453	-0.0004	0.0552	-0.0003	0.0386
MG3-MG3	0.0023	0.0486	-0.0002	0.0515	0.3586	0.0495	-0.0001	0.0566	0.0005	0.0416
MG3-MW	-0.0051	0.0458	-0.0039	0.0500	0.3556	0.0442	-0.0015	0.0537	-0.0007	0.0357
MW-MW	0.0007	0.0546	0.0001	0.0522	0.3591	0.0506	0.0003	0.0528	0.0024	0.0391

Table 6 The estimated type I error when at least one variable is sampled from normal distribution and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MN1-MN1	-0.0019	0.0526	-0.0015	0.0502	0.2596	0.0478	-0.0006	0.0509	0.0013	0.0430
MN1-MN2	0.0020	0.0500	0.0007	0.0472	0.2587	0.0459	-0.0004	0.0492	-0.0018	0.0414
MN1-MN3	-0.0014	0.0474	-0.0022	0.0483	0.2585	0.0484	-0.0003	0.0509	-0.0001	0.0424
MN1-MG1	0.0039	0.0526	0.0048	0.0519	0.2596	0.0478	0.0010	0.0510	0.0008	0.0454
MN1-MG2	0.0012	0.0459	0.0014	0.0473	0.2597	0.0511	0.0003	0.0508	0.0001	0.0442
MN1-MG3	-0.0032	0.0515	-0.0033	0.0542	0.2611	0.0523	0.0005	0.0553	-0.0004	0.0422
MN1-MW	0.0037	0.0494	0.0037	0.0541	0.2592	0.0479	-0.0001	0.0552	-0.0009	0.0463
MN2-MN2	0.0013	0.0503	0.0019	0.0510	0.2591	0.0478	0.0003	0.0545	-0.0017	0.0393
MN2-MN3	0.0001	0.0476	0.0000	0.0509	0.2589	0.0488	-0.0001	0.0496	-0.0010	0.0489
MN2-MG1	-0.0002	0.0504	-0.0016	0.0516	0.2600	0.0503	0.0004	0.0505	0.0009	0.0438
MN2-MG2	0.0020	0.0561	0.0023	0.0545	0.2601	0.0499	0.0007	0.0569	0.0010	0.0419
MN2-MG3	0.0006	0.0521	0.0017	0.0509	0.2604	0.0519	0.0005	0.0540	0.0010	0.0435
MN2-MW	-0.0005	0.0499	0.0006	0.0507	0.2980	0.0504	-0.0001	0.0495	0.0007	0.0489
MN3-MN3	0.0028	0.0535	0.0026	0.0556	0.2604	0.0517	0.0008	0.0587	0.0008	0.0440
MN3-MG1	-0.0019	0.0522	-0.0027	0.0516	0.2598	0.0479	0.0004	0.0558	0.0002	0.0443
MN3-MG2	-0.0029	0.0470	-0.0027	0.0503	0.2597	0.0461	-0.0007	0.0503	-0.0006	0.0416
MN3-MG3	-0.0012	0.0514	-0.0021	0.0531	0.2612	0.0522	0.0010	0.0537	0.0010	0.0426
MN3-MW	0.0038	0.0518	0.0033	0.0511	0.2593	0.0485	-0.0002	0.0517	-0.0008	0.0440

Table 7 The estimated type I error when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MG1-MG1	0.0010	0.0512	-0.0008	0.0506	0.2586	0.0470	0.0000	0.0520	-0.0006	0.0410
MG1-MG2	-0.0022	0.0479	-0.0015	0.0486	0.2589	0.0477	-0.0009	0.0513	0.0009	0.0481
MG1-MG3	0.0028	0.0455	0.0037	0.0464	0.2593	0.0497	-0.0011	0.0473	-0.0012	0.0427
MG1-MW	-0.0009	0.0521	0.0007	0.0511	0.2593	0.0467	0.0003	0.0544	0.0011	0.0445
MG2-MG2	-0.0004	0.0504	-0.0006	0.0540	0.2599	0.0513	0.0001	0.0544	0.0013	0.0463
MG2-MG3	-0.0027	0.0489	-0.0019	0.0481	0.2612	0.0502	0.0002	0.0520	0.0018	0.0446
MG2-MW	0.0001	0.0463	0.0004	0.0463	0.2584	0.0462	-0.0009	0.0491	-0.0009	0.0407
MG3-MG3	-0.0008	0.0490	-0.0014	0.0506	0.2596	0.0494	-0.0002	0.0518	0.0002	0.0432
MG3-MW	0.0010	0.0494	0.0011	0.0499	0.2592	0.0478	-0.0001	0.0508	-0.0001	0.0457
MW-MW	0.0036	0.0513	0.0053	0.0509	0.2593	0.0491	0.0004	0.0498	-0.0007	0.0450

Table 8 The estimated type I error when at least one variable is sampled from normal distribution and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MN1-MN1	-0.0012	0.0484	-0.0002	0.0481	0.2977	0.0504	-0.0002	0.0484	0.0006	0.0463
MN1-MN2	0.0003	0.0491	0.0003	0.0497	0.2977	0.0489	0.0000	0.0499	0.0014	0.0472
MN1-MN3	-0.0008	0.0484	-0.0013	0.0501	0.2976	0.0523	0.0000	0.0506	0.0000	0.0452
MN1-MG1	-0.0007	0.0455	0.0000	0.0466	0.2971	0.0488	-0.0003	0.0458	0.0003	0.0498
MN1-MG2	0.0012	0.0459	0.0014	0.0473	0.2597	0.0511	0.0003	0.0508	0.0001	0.0442
MN1-MG3	-0.0047	0.0488	-0.0052	0.0510	0.2969	0.0496	-0.0002	0.0506	-0.0016	0.0457
MN1-MW	-0.0038	0.0478	-0.0039	0.0475	0.2972	0.0484	-0.0001	0.0510	0.0003	0.0502
MN2-MN2	0.0001	0.0502	-0.0004	0.0503	0.2969	0.0448	0.0000	0.0487	-0.0004	0.0473
MN2-MN3	-0.0003	0.0470	-0.0006	0.0480	0.2963	0.0473	-0.0005	0.0493	0.0000	0.0444
MN2-MG1	0.0009	0.0540	0.0009	0.0546	0.2978	0.0500	0.0003	0.0517	-0.0002	0.0483
MN2-MG2	-0.0001	0.0471	-0.0007	0.0494	0.2974	0.0509	-0.0001	0.0502	-0.0004	0.0478
MN2-MG3	0.0010	0.0524	0.0003	0.0515	0.2981	0.0514	0.0003	0.0515	0.0004	0.0515
MN2-MW	-0.0005	0.0499	0.0006	0.0507	0.2980	0.0504	-0.0001	0.0495	0.0007	0.0489
MN3-MN3	-0.0019	0.0486	-0.0023	0.0501	0.2975	0.0503	-0.0003	0.0482	0.0012	0.0505
MN3-MG1	0.0022	0.0483	0.0022	0.0496	0.2988	0.0460	-0.0003	0.0505	-0.0004	0.0478
MN3-MG2	0.0017	0.0497	0.0014	0.0492	0.2978	0.0505	-0.0005	0.0485	-0.0009	0.0445
MN3-MG3	-0.0012	0.0514	-0.0021	0.0531	0.2612	0.0522	0.0010	0.0537	0.0010	0.0426
MN3-MW	0.0007	0.0493	-0.0008	0.0485	0.2964	0.0425	0.0003	0.0495	0.0005	0.0476

Table 9 The estimated type I error when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MG1-MG1	-0.0006	0.0520	-0.0004	0.0537	0.2979	0.0505	0.0006	0.0533	0.0002	0.0457
MG1-MG2	0.0002	0.0519	-0.0006	0.0526	0.2979	0.0555	-0.0001	0.0508	-0.0002	0.0471
MG1-MG3	0.0021	0.0485	0.0017	0.0480	0.2968	0.0465	0.0001	0.0468	0.0017	0.0486
MG1-MW	-0.0008	0.0503	-0.0008	0.0514	0.2977	0.0595	0.0001	0.0478	-0.0011	0.0448
MG2-MG2	0.0003	0.0514	-0.0005	0.0518	0.2968	0.0490	0.0006	0.0529	0.0001	0.0439
MG2-MG3	0.0002	0.0473	0.0006	0.0502	0.3003	0.0520	0.0002	0.0502	-0.0008	0.0435
MG2-MW	0.0001	0.0494	0.0016	0.0534	0.2978	0.0520	0.0005	0.0517	0.0011	0.0480
MG3-MG3	-0.0004	0.0472	-0.0009	0.0491	0.3004	0.0620	-0.0002	0.0468	0.0006	0.0489
MG3-MW	0.0020	0.0494	0.0004	0.0477	0.2972	0.0500	-0.0001	0.0487	-0.0001	0.0469
MW-MW	-0.0003	0.0478	-0.0001	0.0510	0.2988	0.0495	-0.0004	0.0469	0.0022	0.0508

Table 10 The estimated type I error when at least one variable is sampled from normal distribution and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MN1-MN1	-0.0004	0.0526	0.0001	0.0538	0.2367	0.0493	0.0000	0.0502	0.0000	0.0508
MN1-MN2	-0.0019	0.0483	-0.0023	0.0480	0.2366	0.0494	0.0001	0.0481	0.0001	0.0509
MN1-MN3	-0.0030	0.0485	-0.0030	0.0463	0.2369	0.0482	0.0000	0.0459	-0.0007	0.0515
MN1-MG1	-0.0014	0.0540	-0.0014	0.0570	0.2373	0.0560	0.0000	0.0550	-0.0004	0.0510
MN1-MG2	-0.0012	0.0480	-0.0005	0.0480	0.2977	0.0490	-0.0001	0.0460	-0.0008	0.0490
MN1-MG3	0.0016	0.0508	0.0013	0.0530	0.2362	0.0482	0.0001	0.0480	0.0005	0.0461
MN1-MW	-0.0008	0.0443	0.0005	0.0479	0.2360	0.0440	-0.0002	0.0433	0.0001	0.0478
MN2-MN2	0.0015	0.0508	0.0013	0.0504	0.2368	0.0455	0.0002	0.0485	-0.0016	0.0475
MN2-MN3	-0.0012	0.0487	-0.0008	0.0486	0.2375	0.0450	-0.0001	0.0471	-0.0006	0.0507
MN2-MG1	-0.0013	0.0492	-0.0016	0.0511	0.2368	0.0487	-0.0001	0.0477	0.0000	0.0470
MN2-MG2	0.0017	0.0483	0.0013	0.0484	0.2377	0.0480	-0.0001	0.0432	0.0005	0.0526
MN2-MG3	-0.0016	0.0486	-0.0021	0.0480	0.0555	0.2378	0.0000	0.0461	-0.0008	0.0491
MN2-MW	-0.0025	0.0508	-0.0021	0.0495	0.2372	0.0515	0.0000	0.0491	-0.0001	0.0469
MN3-MN3	0.0006	0.0442	0.0013	0.0473	0.0480	0.2380	-0.0003	0.0432	-0.0008	0.0481
MN3-MG1	0.0005	0.0503	0.0005	0.0485	0.2366	0.0520	-0.0001	0.0479	-0.0009	0.0469
MN3-MG2	-0.0020	0.0479	-0.0018	0.0471	0.2375	0.0475	-0.0001	0.0458	0.0002	0.0515
MN3-MG3	-0.0014	0.0500	-0.0017	0.0504	0.2377	0.0490	0.0001	0.0488	-0.0002	0.0474
MN3-MW	-0.0008	0.0517	-0.0005	0.0504	0.2375	0.0525	0.0000	0.0478	0.0005	0.0507

Table 11 The estimated type I error when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$	average	$\hat{\alpha}$
MG1-MG1	-0.0017	0.0504	-0.0006	0.0488	0.2376	0.0545	0.0001	0.0466	0.0005	0.0516
MG1-MG2	-0.0004	0.0489	-0.0006	0.0495	0.2371	0.0530	0.0000	0.0471	0.0010	0.0471
MG1-MG3	0.0013	0.0487	0.0012	0.0446	0.2369	0.0540	-0.0004	0.0425	0.0000	0.0485
MG1-MW	-0.0001	0.0522	-0.0007	0.0505	0.2369	0.0515	0.0002	0.0499	-0.0004	0.0501
MG2-MG2	-0.0008	0.0520	-0.0010	0.0504	0.2376	0.0525	0.0002	0.0486	0.0002	0.0519
MG2-MG3	-0.0004	0.0507	-0.0005	0.0497	0.2369	0.0446	0.0001	0.0472	0.0000	0.0492
MG2-MW	0.0005	0.0491	0.0017	0.0524	0.2383	0.0633	0.0000	0.0498	-0.0002	0.0464
MG3-MG3	0.0010	0.0503	0.0000	0.0512	0.2361	0.0473	0.0001	0.0499	0.0011	0.0010
MG3-MW	0.0011	0.0464	0.0022	0.0464	0.2363	0.0426	-0.0003	0.0444	-0.0011	0.0481
MW-MW	-0.0001	0.0480	0.0009	0.0488	0.2365	0.0467	0.0000	0.0479	0.0003	0.0514

2. Power of the test

In the preceding part, we looked at the type I error rate for each approach using samples of varying sizes and bivariate mixed distributions of various types. The study revealed that only MIC failed to control type I error rate when sample size was equal to 10, whereas Chertterjee's new correlation excelled in doing so. And all methods are reliable when the sample size is at least 20. Therefore, for $n = 10$, MIC and some cases of Chertterjee's new correlation will be excluded in order to study the power of each test.

Table 12–21 displays the outcome of evaluating each test's power in each scenario where the true correlation of the bivariate mixture is 0.2. And the findings indicated that

- Every test appears to perform poorly when the sample size is 10, 20, 30, and 50, as indicated by the very low estimated powers. The asymptotic methods of Chertterjee's new correlation have the lowest estimated powers, while the Pearson product-moment, Spearman rank correlation, and chi-square tests of distance correlation perform nearly equally well.
- The estimated power of the tests increased when the sample size was increased to 100, but it remained within an unsatisfactory range. In almost all cases, Pearson product-moment correlation outperformed Spearman rank correlation, with the exception of those with a mixture of Weibull distribution, when Spearman rank correlation outperformed it. The power of the chi square test for distance correlation increased as well, but it is still inferior to the approaches mentioned above. MIC and Charterjee's new correlation still has a relatively low power.
- In very size of sample, it is obvious that the average value of correlations found by the five approaches differs. Pearson product-moment and Spearman rank correlations are similar to the true value, but MIC gave a higher value than the true correlation, and bias corrected distance correlation and Chartterjee's new correlation yielded a lower value than the true correlation.

Table 12 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.2 and $n = 10$

Distributions	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.1934	0.0898	0.1765	0.0882	-	-	0.0294	**0.1119**	-	-
MN1-MN2	0.1889	0.0872	0.1749	0.0874	-	-	0.0303	**0.1121**	-	-
MN1-MN3	0.1932	0.0908	0.1816	0.0862	-	-	0.0286	**0.1089**	-	-
MN1-MG1	0.1906	0.0826	0.1744	0.0804	-	-	0.0252	**0.0998**	-	-
MN1-MG2	0.1907	0.0865	0.1774	**0.0869**	-	-	-	-	-	-
MN1-MG3	0.1907	0.0863	0.1795	0.0815	-	-	0.0277	**0.1062**	-	-
MN1-MW	0.2017	0.1034	0.2105	0.0979	-	-	0.0347	**0.1153**	-	-
MN2-MN2	0.1917	0.0859	0.1776	0.0883	-	-	0.0295	**0.1116**	-	-
MN2-MN3	0.1913	0.0859	0.1801	0.0872	-	-	0.0284	0.1068	-	-
MN2-MG1	0.1972	0.0846	0.1824	0.0879	-	-	0.0307	**0.1080**	0.0134	0.0303
MN2-MG2	0.1954	0.0844	0.1813	0.0832	-	-	0.0291	**0.1090**	-	-
MN2-MG3	0.1899	0.0858	0.1798	0.0871	-	-	0.0304	**0.1108**	0.0141	0.0312
MN2-MW	0.2010	0.0858	0.2117	0.0962	-	-	0.0372	**0.1163**	-	-
MN3-MN3	0.1916	0.0931	0.1803	0.0849	-	-	0.0257	**0.1045**	-	-
MN3-MG1	0.1855	0.0861	0.1736	0.0856	-	-	0.0284	**0.106**	-	-
MN3-MG2	0.1884	0.0939	0.1773	0.0831	-	-	0.0305	**0.1085**	0.0150	0.0322
MN3-MG3	0.1881	0.0894	0.1773	0.0870	-	-	0.0260	**0.1020**	-	-
MN3-MW	0.1956	0.1023	0.2085	0.0986	-	-	0.0344	**0.1135**	-	-

Note: Bolded value is the highest power of the test in each case

Table 13 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.2 and $n = 10$

Distributions	**Methods**									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.1890	0.0864	0.1723	0.0806	-	-	0.0269	**0.1050**	-	-
MG1-MG2	0.1895	0.0856	0.1766	0.0844	-	-	0.0260	**0.1042**	-	-
MG1-MG3	0.1947	0.0855	0.1812	**0.0859**	-	-	-	-	-	-
MG1-MW	0.2000	0.0894	0.2102	0.0965	-	-	0.0342	**0.1164**	0.0225	0.0378
MG2-MG2	0.1891	0.0869	0.1755	0.0817	-	-	0.0280	**0.1077**	0.0156	0.0330
MG2-MG3	0.1972	0.0936	0.1776	0.0867	-	-	0.0306	**0.1137**	0.0166	0.0317
MG2-MW	0.2042	0.0966	0.2113	0.0988	-	-	0.0348	**0.1145**	-	-
MG3-MG3	0.1968	0.0903	0.1834	0.0904	-	-	-	-	-	-
MG3-MW	0.2030	0.1059	0.2127	0.0976	-	-	0.0333	**0.1116**	0.0178	0.0343
MW-MW	0.2149	**0.1554**	0.2383	0.1119	-	-	0.0420	0.1270	-	-

Note: Bolded value is the highest power of the test in each case

Table 14 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.2 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.1957	**0.1384**	0.1840	0.1259	0.3728	0.0694	0.0310	0.1330	0.0176	0.0541
MN1-MN2	0.1930	**0.1346**	0.1820	0.1263	0.3752	0.0695	0.0299	0.1325	0.0195	0.0578
MN1-MN3	0.2000	**0.1466**	0.1917	0.1340	0.3749	0.0671	0.0319	0.1363	0.0225	0.0626
MN1-MG1	0.1947	**0.1335**	0.1837	0.1281	0.3754	0.0746	0.0306	0.1313	0.0185	0.0597
MN1-MG2	0.1950	**0.1385**	0.1844	0.1261	0.3748	0.0720	0.0303	0.1305	0.0204	0.0577
MN1-MG3	0.1990	0.1375	0.1924	0.1311	0.3762	0.0686	0.0325	**0.1401**	0.0212	0.0610
MN1-MW	0.2049	0.1572	0.2202	**0.1663**	0.3815	0.0804	0.0384	0.1521	0.0284	0.0685
MN2-MN2	0.1974	**0.1377**	0.1852	0.1310	0.3734	0.0709	0.0303	0.1345	0.0187	0.0585
MN2-MN3	0.1985	0.1350	0.1893	0.1292	0.3749	0.0739	0.0306	**0.1362**	0.0199	0.0571
MN2-MG1	0.1937	**0.1305**	0.1822	0.1218	0.3749	0.0733	0.0280	0.1232	0.0179	0.0567
MN2-MG2	0.1915	**0.1342**	0.1813	0.1233	0.3735	0.0682	0.0287	0.1245	0.0177	0.0566
MN2-MG3	0.1996	**0.1404**	0.1918	0.1317	0.3777	0.0754	0.0330	0.1390	0.0206	0.0629
MN2-MW	0.2051	0.1483	0.2219	**0.1618**	0.3818	0.0825	0.0392	0.1542	0.0254	0.0655
MN3-MN3	0.1935	**0.1405**	0.1864	0.1259	0.3747	0.0704	0.0297	0.1297	0.0187	0.0573
MN3-MG1	0.1959	**0.1415**	0.1883	0.1330	0.3766	0.075	0.0316	0.1354	0.0211	0.0570
MN3-MG2	0.1979	**0.1457**	0.1877	0.1324	0.3743	0.0715	0.0310	0.1334	0.0208	0.0597
MN3-MG3	0.1969	**0.1423**	0.1900	0.1283	0.3748	0.0695	0.0309	0.1285	0.0187	0.0547
MN3-MW	0.2015	**0.1684**	0.2185	0.1610	0.3801	0.0752	0.0377	0.1485	0.0248	0.0608

Note: Bolded value is the highest power of the test in each case

Table 15 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.2 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.1961	0.1344	0.1849	0.1258	0.3741	0.0693	0.0314	**0.2172**	0.0182	0.0577
MG1-MG2	0.1951	0.1382	0.1844	0.1286	0.3730	0.0653	0.0304	**0.2178**	0.0184	0.0558
MG1-MG3	0.1972	0.1382	0.1877	0.1299	0.3737	0.0646	0.0301	**0.2264**	0.0185	0.0564
MG1-MW	0.2029	0.1411	0.2206	**0.1605**	0.3826	0.0820	0.0388	0.1515	0.0279	0.0660
MG2-MG2	0.1927	0.1344	0.1826	0.1227	0.3732	0.0661	0.0292	**0.2246**	0.0184	0.0578
MG2-MG3	0.1958	0.1375	0.1864	0.1247	0.3740	0.0688	0.0304	**0.2247**	0.0198	0.0577
MG2-MW	0.2045	0.1526	0.2216	**0.1648**	0.3809	0.0763	0.0381	0.1551	0.0261	0.0666
MG3-MG3	0.1982	0.1412	0.1891	0.1319	0.3751	0.0702	0.0316	**0.2267**	0.0195	0.0587
MG3-MW	0.2049	0.1659	0.2176	0.1560	0.3813	0.0784	0.0398	**0.2293**	0.0261	0.0690
MW-MW	0.2057	0.1864	0.2431	**0.1912**	0.3865	0.0867	0.0424	0.1650	0.0346	0.0742

Note: Bolded value is the highest power of the test in each case

Table 16 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.2 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.1988	**0.1919**	0.1907	0.1828	0.2770	0.0885	0.0320	0.1758	0.0205	0.0651
MN1-MN2	0.1967	**0.1831**	0.1871	0.1689	0.2760	0.0843	0.0303	0.1645	0.0199	0.0690
MN1-MN3	0.1957	**0.1919**	0.1892	0.1792	0.2773	0.0900	0.0309	0.1661	0.0184	0.0690
MN1-MG1	0.1977	**0.1855**	0.1886	0.1766	0.2762	0.0842	0.0309	0.1689	0.0187	0.0677
MN1-MG2	0.1974	**0.1859**	0.1887	0.1764	0.2756	0.0852	0.0305	0.1627	0.0196	0.0660
MN1-MG3	0.1968	**0.1879**	0.1914	0.1757	0.2766	0.0828	0.0318	0.1705	0.0198	0.0717
MN1-MW	0.2017	0.2017	0.2219	**0.2251**	0.2843	0.1045	0.0376	0.1972	0.0253	0.0788
MN2-MN2	0.1964	**0.1906**	0.1868	0.1752	0.2761	0.0857	0.0306	0.1733	0.0202	0.0676
MN2-MN3	0.1999	**0.1933**	0.1933	0.1832	0.2776	0.0885	0.0321	0.1749	0.0197	0.0688
MN2-MG1	0.1958	**0.1876**	0.1854	0.1703	0.2774	0.0880	0.0304	0.1643	0.0185	0.0657
MN2-MG2	0.1954	**0.1816**	0.1865	0.1690	0.2765	0.0843	0.0301	0.1647	0.0201	0.0668
MN2-MG3	0.1985	**0.1922**	0.1924	0.1782	0.2797	0.0942	0.0324	0.1743	0.0224	0.0716
MN2-MW	0.2026	0.1968	0.2250	**0.2276**	0.2841	0.1007	0.0390	0.2020	0.0279	0.0778
MN3-MN3	0.1962	**0.1883**	0.1932	0.1775	0.2779	0.0854	0.0316	0.1642	0.0203	0.0685
MN3-MG1	0.1991	**0.1874**	0.1915	0.1785	0.2767	0.0868	0.0314	0.1697	0.0204	0.0669
MN3-MG2	0.1964	**0.1904**	0.1887	0.1741	0.2764	0.0831	0.0300	0.1656	0.0180	0.0684
MN3-MG3	0.1972	**0.1899**	0.1926	0.1831	0.2772	0.0858	0.0322	0.1736	0.0200	0.0712
MN3-MW	0.2046	0.2135	0.2231	**0.2245**	0.2834	0.1056	0.0388	0.1999	0.0273	0.0829

Note: Bolded value is the highest power of the test in each case

Table 17 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.2 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.1989	0.1849	0.1885	0.1768	0.2767	0.0857	0.0314	**0.2740**	0.0199	0.0693
MG1-MG2	0.1973	0.1842	0.1867	0.1690	0.2765	0.0822	0.0308	**0.2734**	0.0199	0.0685
MG1-MG3	0.1960	0.1833	0.1888	0.1757	0.2776	0.0837	0.0315	**0.2884**	0.0207	0.0696
MG1-MW	0.2068	0.2023	0.2272	**0.2332**	0.2850	0.1039	0.0411	0.2121	0.0297	0.0803
MG2-MG2	0.1947	0.1860	0.1857	0.1744	0.2763	0.0873	0.0308	**0.2807**	0.0198	0.0698
MG2-MG3	0.1979	0.1863	0.1891	0.1753	0.2769	0.0841	0.0314	**0.2807**	0.0221	0.0698
MG2-MW	0.2044	0.2085	0.2254	**0.2352**	0.2860	0.1106	0.0402	0.2029	0.0279	0.0813
MG3-MG3	0.1985	**0.1936**	0.1913	0.1799	0.2792	0.0741	0.0335	0.2872	0.0211	0.0892
MG3-MW	0.2028	0.2090	0.2212	0.2243	0.2838	0.1028	0.0390	**0.2923**	0.0285	0.0822
MW-MW	0.2042	0.2307	**0.2473**	0.2715	0.2891	0.1142	0.0434	0.2122	0.0322	0.0872

Note: Bolded value is the highest power of the test in each case

Table 18 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.2 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.1992	**0.2965**	0.1917	0.2788	0.3157	0.0960	0.0321	0.2582	0.0218	0.0858
MN1-MN2	0.1996	**0.2944**	0.1919	0.2749	0.3155	0.0986	0.0321	0.2496	0.0211	0.0798
MN1-MN3	0.1962	**0.2857**	0.1909	0.2739	0.3137	0.0880	0.0311	0.2464	0.0212	0.0784
MN1-MG1	0.1992	**0.2953**	0.1898	0.2656	0.3146	0.0931	0.0311	0.2458	0.0203	0.0773
MN1-MG2	0.1996	**0.2947**	0.1927	0.2749	0.3152	0.0990	0.0323	0.2550	0.0205	0.0815
MN1-MG3	0.1996	**0.2958**	0.1953	0.2802	0.3158	0.1001	0.0325	0.2598	0.0223	0.0859
MN1-MW	0.2050	0.3109	0.2271	**0.3643**	0.3233	0.1169	0.0401	0.3051	0.0296	0.1001
MN2-MN2	0.1984	**0.2898**	0.1901	0.2685	0.3148	0.0968	0.0313	0.2497	0.0211	0.0836
MN2-MN3	0.2004	**0.2965**	0.1954	0.2861	0.3161	0.0979	0.0327	0.2611	0.0204	0.0829
MN2-MG1	0.1978	**0.2911**	0.1894	0.2666	0.3131	0.0880	0.0312	0.2485	0.0207	0.0822
MN2-MG2	0.1975	**0.2868**	0.1891	0.2654	0.3152	0.1004	0.0310	0.2502	0.0206	0.0838
MN2-MG3	0.1988	**0.2902**	0.1937	0.2754	0.3162	0.1012	0.0325	0.2503	0.0211	0.0826
MN2-MW	0.2020	0.2942	0.2284	**0.3640**	0.3234	0.1224	0.0393	0.3004	0.0298	0.1007
MN3-MN3	0.2001	**0.2991**	0.1962	0.2820	0.3167	0.1006	0.0326	0.2555	0.0210	0.0844
MN3-MG1	0.1976	**0.2873**	0.1914	0.2719	0.3158	0.1011	0.0315	0.2505	0.0209	0.0805
MN3-MG2	0.1980	**0.2926**	0.1923	0.2832	0.3159	0.0959	0.0319	0.2587	0.0222	0.0832
MN3-MG3	0.2021	0.3075	0.2267	**0.3626**	0.3232	0.1178	0.0400	0.3052	0.0294	0.1009
MN3-MW	0.2024	0.5196	0.2277	**0.6296**	0.2631	0.1783	0.0396	0.5233	0.0307	0.1337

Note: Bolded value is the highest power of the test in each case

Table 19 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.2 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.1970	0.2861	0.1865	0.2580	0.3132	0.0893	0.0305	**0.3323**	0.0197	0.0806
MG1-MG2	0.2003	0.2921	0.1912	0.2672	0.3152	0.0932	0.0320	**0.3342**	0.0189	0.0789
MG1-MG3	0.2013	0.2989	0.2266	**0.3617**	0.3220	0.1177	0.0393	0.3043	0.0302	0.1011
MG1-MW	0.2023	0.2920	0.2303	**0.3711**	0.3239	0.1215	0.0407	0.3122	0.0306	0.1028
MG2-MG2	0.1989	0.2937	0.1907	0.2713	0.3153	0.097	0.0317	**0.3392**	0.0205	0.0787
MG2-MG3	0.1991	**0.2884**	0.1930	0.2731	0.3136	0.0940	0.0322	0.2557	0.0199	0.0819
MG2-MW	0.2004	0.2918	0.2267	**0.3542**	0.3197	0.1061	0.0385	0.2914	0.0292	0.1018
MG3-MG3	0.1969	0.2861	0.1958	0.2785	0.3162	0.0961	0.0322	**0.3481**	0.0217	0.0840
MG3-MW	0.2028	0.2090	0.2255	**0.3553**	0.2838	0.1028	0.0390	0.2923	0.0285	0.0822
MW-MW	0.2014	0.3089	0.2499	**0.4294**	0.3281	0.1390	0.0435	0.3173	0.0362	0.1145

Note: Bolded value is the highest power of the test in each case

Table 20 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.2 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.2009	**0.5257**	0.1941	0.4959	0.2547	0.1266	0.0322	0.4539	0.0221	0.1068
MN1-MN2	0.1998	**0.5225**	0.1928	0.4901	0.2575	0.1500	0.0321	0.4527	0.0219	0.1039
MN1-MN3	0.1996	**0.5183**	0.1955	0.5058	0.2557	0.1446	0.0321	0.4525	0.0230	0.1026
MN1-MG1	0.1980	**0.5119**	0.1899	0.476	0.2549	0.1335	0.0313	0.4448	0.0211	0.1014
MN1-MG2	0.2000	**0.5254**	0.1934	0.4968	0.2557	0.1349	0.0324	0.4554	0.0220	0.0996
MN1-MG3	0.2021	0.3075	0.2267	**0.3626**	0.3232	0.1178	0.0400	0.3052	0.0294	0.1009
MN1-MW	0.2015	0.5183	0.2297	**0.6341**	0.2641	0.1867	0.0399	0.5315	0.0311	0.1317
MN2-MN2	0.1992	**0.5165**	0.1922	0.4875	0.2556	0.1320	0.0317	0.4463	0.0212	0.0981
MN2-MN3	0.1978	**0.5155**	0.1944	0.4968	0.2565	0.1363	0.0316	0.4469	0.0234	0.1071
MN2-MG1	0.1993	**0.5196**	0.1917	0.4867	0.2550	0.1327	0.0318	0.4541	0.0216	0.1005
MN2-MG2	0.1979	**0.5139**	0.1908	0.4837	0.2550	0.1289	0.0315	0.4471	0.0217	0.1015
MN2-MG3	0.1995	**0.5187**	0.1954	0.5045	0.2561	0.1423	0.0329	0.4612	0.0221	0.1050
MN2-MW	0.2005	0.5245	0.2306	**0.6439**	0.2637	0.1827	0.0395	0.5392	0.0307	0.1395
MN3-MN3	0.1978	**0.5155**	0.1944	0.4968	0.2565	0.1363	0.0316	0.4469	0.0234	0.1071
MN3-MG1	0.1985	**0.5183**	0.1941	0.4956	0.2563	0.1369	0.0324	0.4541	0.0229	0.1098
MN3-MG2	0.1985	**0.5111**	0.1937	0.4895	0.2565	0.1442	0.0320	0.4522	0.0220	0.1057
MN3-MG3	0.1991	**0.5193**	0.1975	0.5123	0.2567	0.1356	0.0332	0.4692	0.0230	0.1054
MN3-MW	0.1977	**0.5098**	0.1957	0.5025	0.2568	0.1439	0.0318	0.4457	0.0227	0.1042

Note: Bolded value is the highest power of the test in each case

Table 21 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.2 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.1993	**0.5216**	0.1915	0.4890	0.2557	0.1407	0.0321	0.4605	0.0225	0.1047
MG1-MG2	0.1991	**0.5131**	0.1917	0.4841	0.2553	0.1359	0.0319	0.4507	0.0215	0.1040
MG1-MG3	0.1986	**0.5198**	0.1939	0.4951	0.2554	0.1352	0.0322	0.4569	0.0217	0.1033
MG1-MW	0.2023	0.5284	0.2314	**0.6514**	0.2644	0.1882	0.0407	0.5574	0.0312	0.1365
MG2-MG2	0.2006	**0.5271**	0.1938	0.4941	0.2564	0.1349	0.0326	0.4575	0.0231	0.1098
MG2-MG3	0.1999	**0.5250**	0.1957	0.5044	0.2590	0.1500	0.0334	0.4670	0.0222	0.1084
MG2-MW	0.2015	0.5219	0.2293	**0.6419**	0.2655	0.1777	0.0399	0.5435	0.0296	0.1285
MG3-MG3	0.1984	**0.5157**	0.1965	0.5120	0.2558	0.1323	0.0329	0.4653	0.0223	0.1034
MG3-MW	0.2015	0.5183	0.2284	**0.6316**	0.2615	0.1693	0.0401	0.5426	0.0310	0.1322
MW-MW	0.2007	0.5184	0.2294	**0.6449**	0.2635	0.1875	0.0398	0.5401	0.0309	0.1342

Note: Bolded value is the highest power of the test in each case

Table 22–31 displays the outcome of evaluating each test's power in each scenario where the true correlation of the bivariate mixture is 0.4. And the findings indicated that

- The estimated power is low when the sample size is small ($n = 10$, 20, and 30). Chi square test of distance correlation, Pearson product-moment, and Spearman rank all have almost the same power. However, the Pearson produc- moment seemed to work better when $n = 20$ and 30, with the exception of situations where a mixture of Weibull distribution was present. The chi square test of distance correlation appeared to have a greater value when $n = 10$. Chartterjee's new correlation and MIC had the poorest performance compared to the three mentioned above.
- When $n = 50$ and $n = 100$, the chi square test of distance, Pearson product-moment, and Spearman rank correlation have sufficient power. When $n = 100$, the three tests have very high power of the test. While MIC and Chartterjee's new correlation performed poorly with low power. Pearson product-moment fared best in practically every scenario, whereas Spearman did better than Pearson product-moment when the mixture of wiebull distribution is involved.
- In very size of sample, it is clear that the average value of correlations that were detected by the five methods are different. The correlation detected by Pearson product-moment and Spearman rank are close to the true value while MIC resulted lower value than the true correlation followed by bias corrected distance correlation and Chartterjee's new correlation.

Table 22 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.4 and $n = 10$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.3811	0.2171	0.3547	0.1945	-	-	0.1128	**0.2199**	-	-
MN1-MN2	0.3795	0.2146	0.3541	0.1934	-	-	0.1136	**0.2294**	-	-
MN1-MN3	0.3851	0.2236	0.3616	0.1963	-	-	0.1179	**0.2326**	-	-
MN1-MG1	0.3783	0.2114	0.3512	0.1911	-	-	0.1123	**0.2215**	-	-
MN1-MG2	0.3830	**0.2118**	0.3524	0.1874	-	-	-	-	-	-
MN1-MG3	0.3845	0.2197	0.3621	0.1953	-	-	0.1136	**0.2261**	-	-
MN1-MW	0.4085	0.2529	0.4188	0.2573	-	-	0.1475	**0.2728**	-	-
MN2-MN2	0.3793	0.2127	0.3531	0.1934	-	-	0.1135	**0.2269**	-	-
MN2-MN3	0.3827	0.2144	0.3601	0.1954	-	-	0.1161	**0.2272**	-	-
MN2-MG1	0.3787	0.1990	0.3493	0.1877	-	-	0.1098	**0.2159**	0.0597	0.0612
MN2-MG2	0.3848	0.2157	0.3558	0.1956	-	-	0.1143	**0.2286**	-	-
MN2-MG3	0.3848	0.2157	0.3558	0.1956	-	-	0.1143	**0.2286**	0.0608	0.0641
MN2-MW	0.4141	0.2359	0.4286	0.2676	-	-	0.1515	**0.2745**	-	-
MN3-MN3	0.3834	0.2259	0.3651	0.2005	-	-	0.1168	**0.2302**	-	-
MN3-MG1	0.3848	0.2186	0.3607	0.2016	-	-	0.1153	**0.2289**	-	-
MN3-MG2	0.3783	0.2183	0.3561	0.1952	-	-	0.1156	**0.2286**	0.0621	0.0682
MN3-MG3	0.3788	0.2179	0.3557	0.1917	-	-	0.1137	**0.2230**	-	-
MN3-MW	0.4013	0.2621	0.4105	0.2471	-	-	0.1409	**0.2624**	-	-

Note: **Bolded** value is the highest power of the test in each case

Table 23 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.4 and $n = 10$

Distributions	**Methods**									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.3817	0.2181	0.3551	0.1955	-	-	0.1136	**0.2275**	-	-
MG1-MG2	0.3816	0.2149	0.3538	0.1947	-	-	0.1153	**0.2327**	-	-
MG1-MG3	0.3851	**0.2132**	0.3625	0.1947	-	-	-	-	-	-
MG1-MW	0.4108	0.2318	0.4271	0.2652	-	-	0.1526	**0.2800**	0.0876	0.093
MG2-MG2	0.3825	0.2125	0.3518	0.1870	-	-	0.1146	**0.2269**	0.0597	0.0696
MG2-MG3	0.3880	0.2243	0.3639	0.1976	-	-	0.1202	**0.2352**	0.0639	0.0651
MG2-MW	0.4095	0.2465	0.4241	0.2613	-	-	0.1494	**0.2742**	-	-
MG3-MG3	0.3861	**0.2236**	0.3607	0.1954	-	-	-	-	-	-
MG3-MW	0.4038	0.2537	0.4133	0.2474	-	-	0.1440	**0.2716**	0.0829	0.0794
MW-MW	0.3961	**0.2918**	0.4326	0.2675	-	-	0.1466	0.2689	-	-

Note: Bolded value is the highest power of the test in each case

Table 24 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.4 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.3923	**0.4319**	0.3715	0.3852	0.4246	0.1512	0.1228	0.3792	0.0755	0.1367
MN1-MN2	0.3928	**0.4303**	0.3734	0.3932	0.4261	0.1559	0.1235	0.3889	0.0764	0.1362
MN1-MN3	0.3949	**0.4355**	0.3777	0.3976	0.4278	0.1581	0.1252	0.3906	0.0768	0.1386
MN1-MG1	0.3908	**0.4314**	0.3701	0.3887	0.4240	0.1497	0.1227	0.3869	0.0757	0.1394
MN1-MG2	0.3901	**0.4283**	0.3691	0.3838	0.4240	0.1510	0.1214	0.3871	0.0749	0.1313
MN1-MG3	0.3933	**0.4328**	0.3773	0.4007	0.4260	0.1539	0.1253	0.3907	0.0753	0.1375
MN1-MW	0.4084	0.4664	0.4390	**0.5232**	0.4526	0.1959	0.1537	0.4648	0.1059	0.1942
MN2-MN2	0.3926	**0.4307**	0.3724	0.3901	0.4264	0.1504	0.1247	0.3866	0.0772	0.1399
MN2-MN3	0.3923	**0.4333**	0.3759	0.3969	0.4272	0.1567	0.1236	0.3937	0.0760	0.1385
MN2-MG1	0.3912	**0.4292**	0.3714	0.3890	0.4252	0.1574	0.1232	0.3862	0.0766	0.1379
MN2-MG2	0.3924	**0.4303**	0.3728	0.3868	0.4247	0.1525	0.1226	0.3861	0.0761	0.1328
MN2-MG3	0.3924	**0.4303**	0.3728	0.3868	0.4247	0.1525	0.1226	0.3861	0.0761	0.1328
MN2-MW	0.4125	0.4749	0.4494	**0.5464**	0.4597	0.2193	0.1601	0.4911	0.1106	0.2033
MN3-MN3	0.3926	**0.4420**	0.3778	0.4032	0.4279	0.1541	0.1245	0.3779	0.0777	0.141
MN3-MG1	0.3950	**0.3950**	0.3799	0.3799	0.4284	0.1568	0.1264	0.1264	0.0805	0.0805
MN3-MG2	0.3899	**0.4320**	0.3736	0.3967	0.4258	0.1514	0.1239	0.3906	0.0756	0.1341
MN3-MG3	0.3937	**0.4330**	0.3814	0.4058	0.4298	0.1579	0.1273	0.3980	0.0785	0.1380
MN3-MW	0.4082	0.4644	0.4341	**0.5124**	0.4516	0.2046	0.1546	0.4564	0.1028	0.1881

Note: Bolded value is the highest power of the test in each case

Table 25 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.4 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.3946	**0.4365**	0.3685	0.3875	0.4221	0.1433	0.1231	0.3845	0.0737	0.1311
MG1-MG2	0.3907	**0.4289**	0.3696	0.3846	0.4245	0.1497	0.1225	0.3902	0.0739	0.1362
MG1-MG3	0.3984	**0.4450**	0.3844	0.4130	0.4300	0.1603	0.1327	0.4123	0.0809	0.1443
MG1-MW	0.4088	0.4561	0.4494	**0.5504**	0.4588	0.2165	0.1607	0.4923	0.1101	0.2016
MG2-MG2	0.3931	**0.4309**	0.3723	0.3869	0.4223	0.1485	0.1227	0.3781	0.0744	0.1375
MG2-MG3	0.3926	**0.4276**	0.3754	0.3884	0.4265	0.1559	0.1261	0.3905	0.0760	0.1408
MG2-MW	0.4056	0.4603	0.4355	**0.5192**	0.4507	0.2003	0.1527	0.4737	0.1053	0.1883
MG3-MG3	0.3903	**0.4277**	0.3728	0.3898	0.4236	0.1469	0.1223	0.3873	0.0739	0.1297
MG3-MW	0.4066	0.4562	0.4348	**0.5117**	0.4503	0.2012	0.1493	0.4571	0.1032	0.1844
MW-MW	0.4022	0.4456	0.4471	**0.5430**	0.4572	0.2122	0.1564	0.4154	0.1097	0.2008

Note: Bolded value is the highest power of the test in each case

Table 26 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.4 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.3937	**0.6067**	0.3757	0.5618	0.3314	0.2367	0.1240	0.5306	0.0799	0.1848
MN1-MN2	0.3975	**0.6239**	0.3800	0.5760	0.3335	0.2433	0.1272	0.5525	0.0825	0.1880
MN1-MN3	0.3952	**0.6107**	0.3824	0.5776	0.3340	0.2452	0.1276	0.5530	0.0830	0.1901
MN1-MG1	0.3927	**0.6091**	0.3757	0.5593	0.3319	0.2350	0.1241	0.5389	0.0800	0.1793
MN1-MG2	0.3949	**0.6137**	0.3778	0.5598	0.3337	0.2451	0.1268	0.5456	0.0805	0.1851
MN1-MG3	0.3958	**0.6073**	0.3824	0.5783	0.3338	0.2470	0.1279	0.5488	0.0843	0.1935
MN1-MW	0.4051	0.6347	0.4440	**0.7217**	0.3633	0.3458	0.1565	0.6391	0.1132	0.2625
MN2-MN2	0.3947	**0.6096**	0.3777	0.5646	0.3319	0.2413	0.1253	0.5408	0.0802	0.1850
MN2-MN3	0.3945	**0.6096**	0.3829	0.5775	0.3357	0.2576	0.1269	0.5483	0.0824	0.1879
MN2-MG1	0.3961	**0.6191**	0.3779	0.5640	0.3331	0.2384	0.1268	0.5478	0.0810	0.1908
MN2-MG2	0.3974	**0.6211**	0.3799	0.5679	0.3332	0.2442	0.1281	0.5506	0.0816	0.1887
MN2-MG3	0.3962	**0.6078**	0.3838	0.5758	0.3346	0.2445	0.1291	0.5567	0.0825	0.1856
MN2-MW	0.4092	0.6494	0.4554	**0.7438**	0.3692	0.3692	0.1604	0.6652	0.1187	0.2831
MN3-MN3	0.3969	**0.6095**	0.3881	0.5911	0.3376	0.2566	0.1299	0.4475	0.0848	0.1938
MN3-MG1	0.3958	**0.6165**	0.3841	0.5797	0.3352	0.2511	0.1297	0.5571	0.0831	0.1923
MN3-MG2	0.3948	**0.6109**	0.3813	0.5753	0.3353	0.2556	0.1274	0.5444	0.0818	0.1883
MN3-MG3	0.3935	**0.6007**	0.3834	0.5758	0.3350	0.2454	0.1273	0.5434	0.0816	0.1861
MN3-MW	0.4075	0.6193	0.4425	**0.7142**	0.3615	0.3410	0.1562	0.6261	0.1102	0.2643

Note: Bolded value is the highest power of the test in each case

Table 27 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.4 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.3924	**0.6060**	0.3756	0.5523	0.3308	0.2343	0.1244	0.4510	0.0779	0.1842
MG1-MG2	0.3948	**0.6120**	0.3768	0.5618	0.3336	0.2422	0.1271	0.5503	0.0809	0.1789
MG1-MG3	0.3975	**0.6201**	0.3866	0.5833	0.3356	0.2544	0.1319	0.5662	0.0844	0.1907
MG1-MW	0.4118	0.6484	0.4579	**0.7494**	0.3706	0.3696	0.1658	0.6761	0.1212	0.2920
MG2-MG2	0.3941	**0.6038**	0.3762	0.5612	0.3323	0.2408	0.1260	0.4460	0.0806	0.1847
MG2-MG3	0.3947	**0.6100**	0.3799	0.5717	0.3354	0.2468	0.1288	0.5527	0.0839	0.1856
MG2-MW	0.4069	0.6270	0.4447	**0.7205**	0.3652	0.3503	0.1583	0.6495	0.1137	0.2723
MG3-MG3	0.3945	**0.6074**	0.3807	0.5710	0.3342	0.2491	0.1284	0.4514	0.0815	0.1874
MG3-MW	0.4036	0.6102	0.4376	**0.7070**	0.3593	0.3293	0.1515	0.6270	0.1098	0.2574
MW-MW	0.4014	0.5725	0.4550	**0.7432**	0.3699	0.3604	0.1584	0.4742	0.1190	0.2841

Note: Bolded value is the highest power of the test in each case

Table 28 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.4 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.3975	**0.8352**	0.3820	0.7940	0.3716	0.3111	0.1276	0.7653	0.0864	0.2649
MN1-MN2	0.3961	**0.8294**	0.3816	0.7973	0.3718	0.3096	0.1270	0.7656	0.0843	0.2586
MN1-MN3	0.3955	**0.8223**	0.3857	0.8053	0.3729	0.3163	0.1285	0.7665	0.0876	0.2636
MN1-MG1	0.3969	**0.8410**	0.3813	0.8020	0.3706	0.3173	0.1283	0.7743	0.0857	0.2663
MN1-MG2	0.3969	**0.8344**	0.3814	0.7995	0.3711	0.3193	0.1274	0.7705	0.0834	0.2530
MN1-MG3	0.3949	**0.8309**	0.3837	0.8082	0.3732	0.3160	0.1283	0.7741	0.0860	0.2628
MN1-MW	0.4065	0.8470	0.4519	**0.9212**	0.4063	0.4773	0.1598	0.8642	0.1209	0.3952
MN2-MN2	0.3966	**0.8343**	0.3822	0.7994	0.3714	0.3105	0.1276	0.7681	0.0845	0.2565
MN2-MN3	0.3956	0.8379	0.3862	0.8080	0.4052	0.4685	0.1274	0.7719	0.0859	0.2584
MN2-MG1	0.3971	**0.8333**	0.3808	0.7938	0.4028	0.4575	0.1276	0.7707	0.0845	0.2560
MN2-MG2	0.3960	**0.8337**	0.3810	0.7963	0.4065	0.4740	0.1275	0.7675	0.0842	0.2549
MN2-MG3	0.3971	**0.8356**	0.3884	0.8150	0.4034	0.4655	0.1316	0.7838	0.0880	0.2710
MN2-MW	0.4065	0.8627	0.4608	**0.9353**	0.4045	0.4640	0.1623	0.8849	0.1245	0.4112
MN3-MN3	0.3953	**0.8233**	0.3873	0.8101	0.3747	0.3248	0.1287	0.5096	0.0871	0.2662
MN3-MG1	0.3975	**0.8354**	0.3881	0.8094	0.4037	0.4715	0.1305	0.7807	0.0868	0.2709
MN3-MG2	0.3976	**0.8307**	0.3860	0.8094	0.4061	0.4590	0.1286	0.7724	0.0860	0.2596
MN3-MG3	0.3966	**0.8274**	0.3883	0.8141	0.3983	0.4470	0.1303	0.7813	0.0884	0.2644
MN3-MW	0.4054	0.8267	0.4485	**0.9200**	0.4038	0.4460	0.1587	0.8503	0.1189	0.3885

Note: Bolded value is the highest power of the test in each case

Table 29 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.4 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.3974	**0.8356**	0.8312	0.7991	0.3720	0.3151	0.1289	0.5065	0.0857	0.2598
MG1-MG2	0.3970	**0.8448**	0.3812	0.8040	0.4012	0.4655	0.1293	0.7829	0.0854	0.2630
MG1-MG3	0.3982	**0.8395**	0.3886	0.8139	0.3760	0.3407	0.1331	0.7864	0.0874	0.2668
MG1-MW	0.3980	**0.8399**	0.3887	0.8147	0.3729	0.3080	0.1331	0.7939	0.0887	0.2720
MG2-MG2	0.3961	**0.8284**	0.3811	0.7981	0.3708	0.3053	0.1279	0.5016	0.08417	0.2587
MG2-MG3	0.3962	**0.8303**	0.3861	0.8093	0.3750	0.3273	0.1309	0.7784	0.0872	0.2667
MG2-MW	0.4055	0.8410	0.4495	**0.9190**	0.4011	0.4640	0.1578	0.8609	0.1178	0.3920
MG3-MG3	0.3959	**0.8272**	0.3850	0.8027	0.3737	0.3218	0.1308	0.5115	0.0886	0.273
MG3-MW	0.4021	0.9086	0.4447	**0.9124**	0.4013	0.4533	0.1546	0.8507	0.1159	0.3782
MW-MW	0.3998	0.7528	0.4616	**0.9322**	0.4079	0.4811	0.1608	0.5315	0.1237	0.4119

Note: Bolded value is the highest power of the test in each case

Table 30 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.4 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.3968	**0.9859**	0.3846	0.9815	0.3145	0.5572	0.1281	0.9705	0.0879	0.4129
MN1-MN2	0.3983	**0.9871**	0.3856	0.9808	0.3143	0.5671	0.1287	0.9729	0.0891	0.4144
MN1-MN3	0.3975	**0.9859**	0.3891	0.9827	0.3151	0.5677	0.1292	0.9733	0.0908	0.4253
MN1-MG1	0.3976	**0.9872**	0.3852	0.9779	0.3130	0.5480	0.1295	0.9710	0.0878	0.4076
MN1-MG2	0.3990	**0.9862**	0.3862	0.9813	0.3167	0.5886	0.1301	0.9712	0.0884	0.4121
MN1-MG3	0.3992	**0.9858**	0.3906	0.9824	0.3184	0.5873	0.1327	0.9753	0.0909	0.4203
MN1-MW	0.4038	0.9872	0.4563	**0.9983**	0.3479	0.7663	0.1616	0.9934	0.1266	0.6286
MN2-MN2	0.3921	**0.9859**	0.3795	0.9782	0.3089	0.5672	0.1276	0.9698	0.0885	0.4170
MN2-MN3	0.3992	**0.9879**	0.3924	0.9851	0.3157	0.5540	0.1313	0.9746	0.0921	0.4377
MN2-MG1	0.3997	**0.9878**	0.3863	0.9797	0.3470	0.7765	0.1305	0.9741	0.0894	0.4140
MN2-MG2	0.3985	**0.9881**	0.3866	0.9818	0.3147	0.5606	0.1304	0.9743	0.0885	0.4171
MN2-MG3	0.3988	**0.9853**	0.3916	0.9821	0.3169	0.5806	0.1326	0.9748	0.0913	0.4236
MN2-MW	0.4053	0.9912	0.4672	**0.9980**	0.3499	0.7760	0.1653	0.9957	0.1314	0.6547
MN3-MN3	0.3985	**0.9842**	0.3930	0.9815	0.3161	0.5553	0.1313	0.9700	0.0923	0.4324
MN3-MG1	0.3973	**0.9886**	0.3910	0.9836	0.3160	0.5701	0.1315	0.9778	0.0898	0.4163
MN3-MG2	0.4014	0.9865	0.3929	**0.9870**	0.3180	0.5815	0.1327	0.9755	0.0907	0.4250
MN3-MG3	0.4008	0.9890	0.3958	**0.9895**	0.3185	0.5815	0.1345	0.9875	0.0931	0.4500
MN3-MW	0.4019	0.9833	0.4517	**0.9973**	0.3446	0.7498	0.1596	0.9907	0.1209	0.5955

Note: Bolded value is the highest power of the test in each case

Table 31 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.4 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.3982	**0.9879**	0.3832	0.9794	0.3146	0.5593	0.1299	0.9731	0.0863	0.3959
MG1-MG2	0.3985	**0.9895**	0.3857	0.9827	0.3459	0.7585	0.1310	0.9790	0.0898	0.4196
MG1-MG3	0.3987	**0.9873**	0.3933	0.9842	0.3200	0.5593	0.1354	0.9775	0.0929	0.4358
MG1-MW	0.4033	**0.9919**	0.4678	0.9990	0.3525	0.7810	0.1681	0.9952	0.1318	0.6561
MG2-MG2	0.3995	**0.9868**	0.3861	0.9816	0.3149	0.5608	0.1307	0.5478	0.0891	0.4124
MG2-MG3	0.3992	**0.9867**	0.3901	0.9835	0.3154	0.5587	0.1330	0.9766	0.0898	0.4180
MG2-MW	0.4004	0.9871	0.4503	**0.9972**	0.3470	0.7753	0.1567	0.9928	0.1223	0.6027
MG3-MG3	0.4001	**0.9861**	0.3909	0.9824	0.3171	0.5733	0.1337	0.5663	0.0918	0.4283
MG3-MW	0.4029	0.9824	0.4497	**0.9971**	0.3421	0.7373	0.1571	0.9912	0.1212	0.5996
MW-MW	0.4014	0.9562	0.4659	**0.9990**	0.3523	0.7970	0.1634	0.5860	0.1323	0.6421

Note: Bolded value is the highest power of the test in each case

Table 32–41 displays the outcome of evaluating each test's power in each scenario where the true correlation of the bivariate mixture is 0.6. And the findings indicated that

- When $n = 10$, the chi square distance, Pearson product-moment correlation, and Spearman rank correlation produced better power than the true correlation ia 0.2 and 0.4, respectively, but remained in unsatisfactory value. Pearson product-moment works better in almost every scenario except those involving a mixture of Weibull distributions. Even when the true correlation is 0.6, MIC and Chartterjee's new correlation have lesser power than the others.
- When $n = 20$, the power of all tests increased, but only Pearson product-moment and Spearman rank correlation produced acceptable results. In the absence of mixture of Weibull distributions, Pearson product-moment performed better than Spearman rank.
- All tests generated satisfactory results with $n = 30$, 50, and 100, with Pearson product-moment and Spearman rank being the best methods. In any instance, Pearson product-moment has greater power than Spearman rank except when a mixture of Weibull distributions is involved.
- In very size of sample, it is clear that the average value of correlations that were detected by the five methods are different. The correlation detected by Pearson product-moment and Spearman rank are close to the true value while MIC resulted lower value than the true correlation followed by bias corrected distance correlation and Chartterjee's new correlation.

Table 32 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.6 and $n = 10$

Distributions	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.5789	**0.4936**	0.5412	0.4279	-	-	0.2666	0.4581	-	-
MN1-MN2	0.5779	**0.4900**	0.5408	0.4179	-	-	0.2663	0.4580	-	-
MN1-MN3	0.5797	**0.4981**	0.5467	0.4351	-	-	0.2756	0.4662	-	-
MN1-MG1	0.5821	**0.4880**	0.5429	0.4236	-	-	0.2733	0.4665	-	-
MN1-MG2	0.5783	**0.4886**	0.5361	0.4114	-	-	-	-	-	-
MN1-MG3	0.5800	**0.4905**	0.5461	0.4282	-	-	0.2723	0.4575	-	-
MN1-MW	0.6174	0.5527	0.6364	**0.5899**	-	-	0.3498	0.5752	-	-
MN2-MN2	0.5777	**0.4809**	0.5400	0.4198	-	-	0.2706	0.4639	-	-
MN2-MN3	0.5825	**0.4895**	0.5500	0.4403	-	-	0.2782	0.4778	-	-
MN2-MG1	0.5799	**0.4875**	0.5433	0.4260	-	-	0.2708	0.4657	0.1439	0.1538
MN2-MG2	0.5798	**0.4855**	0.5399	0.4219	-	-	0.2696	0.4648	-	-
MN2-MG3	0.5820	**0.4936**	0.5491	0.4319	-	-	0.2745	0.4735	0.1504	0.1569
MN2-MW	0.6330	0.5791	0.6619	**0.6354**	-	-	0.3738	0.6313	-	-
MN3-MN3	0.5755	**0.4816**	0.5444	0.4189	-	-	0.2690	0.4589	-	-
MN3-MG1	0.5875	**0.4984**	0.5559	0.4429	-	-	0.2788	0.4718	-	-
MN3-MG2	0.5787	**0.4825**	0.5461	0.4284	-	-	0.2715	0.4629	0.1464	0.1575
MN3-MG3	0.5796	**0.4862**	0.5466	0.4287	-	-	0.2730	0.4609	-	-
MN3-MW	0.6073	0.5409	0.6175	0.5459	-	-	0.3346	**0.5514**	-	-

Note: Bolded value is the highest power of the test in each case

Table 33 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.6 and $n = 10$

Distributions	**Methods**									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.5759	**0.4861**	0.5321	0.4103	-	-	0.2645	0.4586	-	-
MG1-MG2	0.5786	**0.4843**	0.5416	0.4242	-	-	0.2705	0.4626	-	-
MG1-MG3	0.5852	**0.4932**	0.5515	0.4334	-	-	-	-	-	-
MG1-MW	0.6290	0.5643	0.6598	**0.6379**	-	-	0.3807	0.6346	0.2241	0.2817
MG2-MG2	0.5804	**0.4881**	0.5409	0.4276	-	-	0.2728	0.4688	0.1456	0.1565
MG2-MG3	0.5804	**0.4930**	0.5446	0.4295	-	-	0.2760	0.4724	0.1467	0.1604
MG2-MW	0.6193	0.5560	0.6352	0.5901	-	-	0.3510	**0.5903**	-	-
MG3-MG3	0.5805	**0.4962**	0.5486	0.4363	-	-	-	-	-	-
MG3-MW	0.6032	0.5257	0.6172	0.5537	-	-	0.3272	**0.5565**	0.1930	0.2243
MW-MW	0.5859	0.5119	0.6042	**0.5356**	-	-	0.3110	0.4963	-	-

Note: Bolded value is the highest power of the test in each case

Table 34 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.6 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.5927	**0.8355**	0.5654	0.7831	0.5243	0.3637	0.2899	0.6233	0.1791	0.3644
MN1-MN2	0.5904	**0.8347**	0.5649	0.7807	0.5244	0.3643	0.2885	0.6306	0.1803	0.3666
MN1-MN3	0.5904	**0.8305**	0.5688	0.7915	0.5268	0.3676	0.2897	0.6383	0.1823	0.3673
MN1-MG1	0.5895	**0.8388**	0.5629	0.7780	0.5233	0.3619	0.2895	0.6412	0.1810	0.3651
MN1-MG2	0.5926	**0.8319**	0.5631	0.7747	0.5231	0.3634	0.2893	0.6338	0.1797	0.3576
MN1-MG3	0.5913	**0.8298**	0.5691	0.7891	0.5275	0.3679	0.2949	0.6356	0.1831	0.3757
MN1-MW	0.6152	0.8780	0.6599	**0.9214**	0.5948	0.5379	0.6599	0.7374	0.2536	0.5675
MN2-MN2	0.5895	**0.8312**	0.5626	0.7802	0.5233	0.3632	0.2874	0.6309	0.1786	0.3588
MN2-MN3	0.5900	**0.8345**	0.5708	0.7883	0.5286	0.3753	0.2909	0.6484	0.1841	0.3723
MN2-MG1	0.5905	**0.8372**	0.5641	0.7817	0.5236	0.3626	0.2907	0.6403	0.1788	0.3586
MN2-MG2	0.5911	**0.8370**	0.5644	0.7834	0.5234	0.3648	0.2897	0.6351	0.1796	0.3661
MN2-MG3	0.5961	**0.8490**	0.5756	0.8015	0.5322	0.3801	0.3010	0.6444	0.1885	0.3833
MN2-MW	0.6239	0.9159	0.6866	**0.9494**	0.6170	0.5944	0.3832	0.7887	0.2759	0.6245
MN3-MN3	0.5859	**0.8086**	0.5664	0.7869	0.5248	0.3630	0.2870	0.6322	0.1823	0.3729
MN3-MG1	0.5897	**0.8396**	0.5713	0.7958	0.5305	0.3823	0.2945	0.6507	0.1852	0.3758
MN3-MG2	0.5897	**0.8239**	0.5671	0.7869	0.5265	0.3670	0.2896	0.6399	0.1833	0.3731
MN3-MG3	0.5906	**0.8270**	0.5735	0.7990	0.5319	0.3801	0.2948	0.6368	0.1857	0.3814
MN3-MW	0.6095	0.8447	0.6461	**0.9079**	0.5854	0.5147	0.3522	0.7051	0.2433	0.5355

Note: Bolded value is the highest power of the test in each case

Table 35 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.6 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.5918	**0.8378**	0.5617	0.7774	0.5242	0.3620	0.2910	0.6249	0.1807	0.3696
MG1-MG2	0.5903	**0.8374**	0.5653	0.7876	0.5235	0.3608	0.2918	0.6353	0.1801	0.3686
MG1-MG3	0.5955	**0.8467**	0.5779	0.8040	0.5318	0.3816	0.3039	0.6500	0.1883	0.3886
MG1-MW	0.6224	0.8992	0.6878	0.9496	0.6203	0.5935	0.3937	0.7872	0.2776	0.6227
MG2-MG2	0.5900	**0.8357**	0.5597	0.7721	0.5206	0.3556	0.2879	0.6399	0.1756	0.3535
MG2-MG3	0.5902	**0.8320**	0.5666	0.7835	0.5265	0.3726	0.2924	0.6438	0.1809	0.3652
MG2-MW	0.6121	0.8722	0.6561	**0.9189**	0.5939	0.5370	0.3590	0.7428	0.2494	0.5557
MG3-MG3	0.5912	**0.8298**	0.5694	0.7888	0.5269	0.3641	0.2944	0.6483	0.1827	0.3665
MG3-MW	0.6069	0.8449	0.6447	**0.9079**	0.5820	0.4998	0.3457	0.7160	0.2412	0.5325
MW-MW	0.5965	0.7692	0.6316	**0.8888**	0.5712	0.4792	0.3295	0.6567	0.2304	0.5003

Note: Bolded value is the highest power of the test in each case

Table 36 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.6 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.5942	**0.9522**	0.5716	0.9310	0.4405	0.6000	0.2939	0.6821	0.1913	0.5063
MN1-MN2	0.5941	**0.9558**	0.5730	0.9360	0.4407	0.6020	0.2936	0.6911	0.1931	0.5145
MN1-MN3	0.5913	**0.9499**	0.5739	0.9335	0.4439	0.6063	0.2936	0.6964	0.1940	0.5147
MN1-MG1	0.5921	**0.9565**	0.5713	0.9358	0.4390	0.6061	0.2933	0.6979	0.1911	0.5110
MN1-MG2	0.5939	**0.9527**	0.5708	0.9303	0.4419	0.6022	0.2947	0.6911	0.1919	0.5140
MN1-MG3	0.5949	**0.9532**	0.5764	0.9373	0.4444	0.6151	0.2993	0.6999	0.1951	0.5229
MN1-MW	0.6136	0.9725	0.6686	**0.9910**	0.5196	0.8104	0.3686	0.7856	0.2710	0.7475
MN2-MN2	0.5938	**0.9532**	0.5715	0.9293	0.4409	0.6033	0.2940	0.6898	0.1919	0.5098
MN2-MN3	0.5931	**0.9542**	0.5804	0.9416	0.4475	0.6248	0.2962	0.7032	0.1974	0.5339
MN2-MG1	0.5962	**0.9571**	0.5740	0.9319	0.4437	0.6093	0.2986	0.6974	0.1942	0.5151
MN2-MG2	0.5954	**0.9572**	0.5727	0.9357	0.4405	0.6058	0.2959	0.6944	0.1926	0.5127
MN2-MG3	0.5969	**0.9618**	0.5825	0.9454	0.4489	0.6290	0.3042	0.6982	0.1985	0.5305
MN2-MW	0.6189	0.9848	0.6949	**0.9945**	0.5453	0.8556	0.3868	0.8341	0.2959	0.8089
MN3-MN3	0.5927	**0.9447**	0.5770	0.9386	0.4456	0.6206	0.2955	0.6895	0.1962	0.5304
MN3-MG1	0.5940	**0.9563**	0.5799	0.9417	0.4473	0.6192	0.3005	0.712	0.1987	0.5339
MN3-MG2	0.5929	**0.9519**	0.5744	0.9334	0.4424	0.6129	0.2933	0.6975	0.1936	0.5144
MN3-MG3	0.5924	**0.9463**	0.5788	0.9376	0.4467	0.6162	0.2981	0.6977	0.1980	0.5279
MN3-MW	0.6095	0.9561	0.6529	**0.9834**	0.5068	0.7816	0.3565	0.7577	0.2568	0.7162

Note: Bolded value is the highest power of the test in each case

Table 37 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.6 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.5932	**0.9556**	0.5668	0.9284	0.4370	0.5902	0.2927	0.682	0.1878	0.4937
MG1-MG2	0.5940	**0.9589**	0.5714	0.9306	0.4415	0.6009	0.2979	0.6936	0.1914	0.5077
MG1-MG3	0.5960	**0.9617**	0.5818	0.9402	0.4474	0.6212	0.3070	0.7089	0.1995	0.5398
MG1-MW	0.6169	0.9819	0.6948	**0.9951**	0.5456	0.8642	0.3952	0.8296	0.2948	0.8096
MG2-MG2	0.5933	**0.9551**	0.5688	0.9300	0.4388	0.5984	0.2929	0.6973	0.1900	0.5022
MG2-MG3	0.5958	**0.9563**	0.5762	0.9368	0.4445	0.6145	0.3007	0.7019	0.1959	0.5217
MG2-MW	0.6094	0.9687	0.6667	**0.9872**	0.5188	0.8071	0.3625	0.7912	0.2692	0.7480
MG3-MG3	0.5938	**0.9505**	0.5770	0.9365	0.4453	0.6123	0.3009	0.7043	0.1946	0.5174
MG3-MW	0.6072	0.9569	0.6482	**0.9837**	0.5016	0.7729	0.3481	0.7694	0.2530	0.6999
MW-MW	0.5964	0.8959	0.6405	**0.9821**	0.4944	0.7546	0.3376	0.7098	0.2468	0.6853

Note: Bolded value is the highest power of the test in each case

Table 38 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.6 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.5961	**0.9981**	0.5770	0.9957	0.4780	0.765	0.2964	0.7271	0.1998	0.7109
MN1-MN2	0.5943	**0.9966**	0.5778	0.9936	0.4795	0.7611	0.2968	0.7365	0.2023	0.7142
MN1-MN3	0.5955	**0.9968**	0.5823	0.9959	0.4818	0.7702	0.2988	0.7409	0.2044	0.7204
MN1-MG1	0.5963	**0.9976**	0.5791	0.9957	0.4798	0.7662	0.3003	0.7437	0.2023	0.7153
MN1-MG2	0.5977	**0.9977**	0.5781	0.9953	0.4794	0.7618	0.2996	0.7373	0.2021	0.7141
MN1-MG3	0.5960	**0.9969**	0.5816	0.9947	0.4811	0.7763	0.3024	0.7445	0.2043	0.7230
MN1-MW	0.6123	0.9992	0.6784	**1.0000**	0.5587	0.9351	0.3754	0.8211	0.2874	0.9239
MN2-MN2	0.5971	**0.9982**	0.5786	0.9959	0.4790	0.7668	0.2986	0.7351	0.2034	0.7192
MN2-MN3	0.5964	**0.9964**	0.5867	0.9947	0.4855	0.7842	0.3007	0.7454	0.2081	0.7384
MN2-MG1	0.5962	**0.9971**	0.5779	0.9949	0.4791	0.7605	0.3002	0.7431	0.2016	0.7122
MN2-MG2	0.5953	**0.9974**	0.5768	0.9948	0.4789	0.7584	0.2983	0.7396	0.2019	0.7116
MN2-MG3	0.5980	**0.9987**	0.5875	0.9959	0.4854	0.7846	0.3075	0.7422	0.2101	0.7415
MN2-MW	0.6111	0.9995	0.6991	**0.9999**	0.5780	0.9557	0.3850	0.8635	0.3074	0.9489
MN3-MN3	0.5946	**0.9971**	0.5823	0.9955	0.4819	0.7765	0.2985	0.7309	0.2061	0.7295
MN3-MG1	0.5989	**0.9978**	0.5900	0.9964	0.4879	0.7924	0.3078	0.7583	0.2114	0.7488
MN3-MG2	0.5962	**0.9967**	0.5818	0.9954	0.4821	0.7737	0.2993	0.7411	0.2042	0.7169
MN3-MG3	0.5970	0.9964	0.5857	**0.9967**	0.4832	0.7786	0.3027	0.7471	0.2078	0.7333
MN3-MW	0.6072	**0.9966**	0.6600	**0.9996**	0.5440	0.9156	0.3610	0.7982	0.2700	0.8917

Note: **Bolded** value is the highest power of the test in each case

Table 39 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.6 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.5981	**0.9977**	0.5763	0.9952	0.4783	0.7694	0.3009	0.7255	0.2017	0.7117
MG1-MG2	0.5973	**0.9982**	0.5797	0.9956	0.4795	0.7684	0.3031	0.7410	0.2026	0.7151
MG1-MG3	0.5996	**0.9983**	0.5924	0.9964	0.4899	0.8007	0.3154	0.7537	0.2128	0.7463
MG1-MW	0.6108	0.9995	0.7027	**0.9999**	0.5821	0.9635	0.3969	0.8599	0.3106	0.9537
MG2-MG2	0.5972	**0.9975**	0.5768	0.9947	0.4778	0.7628	0.2997	0.7429	0.2006	0.7091
MG2-MG3	0.5976	**0.9981**	0.5825	0.9951	0.4836	0.7752	0.3051	0.7432	0.2053	0.7236
MG2-MW	0.6089	0.9982	0.6741	**0.9998**	0.5558	0.9361	0.3672	0.8280	0.2829	0.9209
MG3-MG3	0.5947	**0.9971**	0.5816	0.9959	0.4816	0.7723	0.3029	0.7461	0.2055	0.7215
MG3-MW	0.6064	0.9982	0.6558	**0.9994**	0.5390	0.911	0.3507	0.8077	0.2677	0.8949
MW-MW	0.5998	0.9828	0.6482	**0.9995**	0.5331	0.8994	0.3447	0.7548	0.2599	0.8742

Note: **Bolded** value is the highest power of the test in each case

Table 40 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.6 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.5979	**1.0000**	0.5834	**1.0000**	0.4248	0.9771	0.3010	**1.0000**	0.2103	0.9314
MN1-MN2	0.5982	**1.0000**	0.5835	**1.0000**	0.4238	0.9752	0.3005	**1.0000**	0.2115732	0.9349
MN1-MN3	0.5981	**1.0000**	0.5877	**1.0000**	0.4258	0.9773	0.3024	**1.0000**	0.2135	0.9374
MN1-MG1	0.6004	**1.0000**	0.5865	**1.0000**	0.4256	0.9789	0.3063	**1.0000**	0.2131	0.9391
MN1-MG2	0.5982	**1.0000**	0.5824	**1.0000**	0.4242	0.9767	0.3018	**1.0000**	0.2103	0.9315
MN1-MG3	0.5984	**1.0000**	0.5884	**1.0000**	0.4277	0.9744	0.3070	**1.0000**	0.2154	0.9446
MN1-MW	0.6072	**1.0000**	0.6838	**1.0000**	0.5054	0.9995	0.3782	**1.0000**	0.2984	0.9971
MN2-MN2	0.5980	**1.0000**	0.5829	**1.0000**	0.4261	0.9807	0.3011	**1.0000**	0.2101	0.9283
MN2-MN3	0.5982	**1.0000**	0.5916	**1.0000**	0.4298	0.9787	0.3034	**1.0000**	0.2163	0.9400
MN2-MG1	0.5990	**1.0000**	0.5843	**1.0000**	0.4231	0.9700	0.3043	**1.0000**	0.2123	0.9369
MN2-MG2	0.5981	**1.0000**	0.5827	**1.0000**	0.4237	0.9712	0.3020	**1.0000**	0.2103	0.9304
MN2-MG3	0.5979	**1.0000**	0.5915	**1.0000**	0.4294	0.9817	0.3089823	**1.0000**	0.2166	0.9409
MN2-MW	0.6072	**1.0000**	0.7059	**1.0000**	0.5320	1.0000	0.3868	**1.0000**	0.3204	0.9995
MN3-MN3	0.5973	**1.0000**	0.5903	**1.0000**	0.4311	0.9767	0.3037	**1.0000**	0.2158	0.9416
MN3-MG1	0.5989	**1.0000**	0.5933	**1.0000**	0.4292	0.9800	0.3092	**1.0000**	0.2184	0.9432
MN3-MG2	0.5975	**1.0000**	0.5875	**1.0000**	0.4279	0.9821	0.3028	**1.0000**	0.2133	0.9372
MN3-MG3	0.5970	**1.0000**	0.5902	**1.0000**	0.4292	0.9793	0.3056	**1.0000**	0.2159	0.9431
MN3-MW	0.6045	**1.0000**	0.6643	**1.0000**	0.4916	0.9987	0.3619	**1.0000**	0.2791	0.9926

Note: **Bolded** value is the highest power of the test in each case

Table 41 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.6 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.5976	**1.0000**	0.5784	**1.0000**	0.4182	0.9640	0.3014	**1.0000**	0.2057	0.9229
MG1-MG2	0.5982	**1.0000**	0.5841	**1.0000**	0.4248	0.9747	0.3060	**1.0000**	0.2114	0.9363
MG1-MG3	0.5989	**1.0000**	0.5943	**1.0000**	0.4259	0.9941	0.3152	**1.0000**	0.2186	0.9416
MG1-MW	0.6071	**1.0000**	0.7088	**1.0000**	0.5264	**1.0000**	0.3998	**1.0000**	0.3230	0.9992
MG2-MG2	0.5983	**1.0000**	0.5814	**1.0000**	0.4211	0.9713	0.3024	**1.0000**	0.2083	0.9293
MG2-MG3	0.5982	**1.0000**	0.5870	**1.0000**	0.4286	0.9813	0.3071	**1.0000**	0.2136	0.9373
MG2-MW	0.6064	**1.0000**	0.6801	**1.0000**	0.4992	0.9986	0.3696	**1.0000**	0.2941	0.9962
MG3-MG3	0.5978	**1.0000**	0.5886	**1.0000**	0.4265	0.9773	0.3079	**1.0000**	0.2149	0.9398
MG3-MW	0.6028	**1.0000**	0.6612	**1.0000**	0.4888	0.9986	0.3525	**1.0000**	0.2763	0.9933
MW-MW	0.6013	**1.0000**	0.6537	**1.0000**	0.4790	0.9960	0.3489	**1.0000**	0.2691	0.9899

Note: **Bolded** value is the highest power of the test in each case

Table 42–51 displays the outcome of evaluating each test's power in each scenario where the true correlation of the bivariate mixture is 0.8. And the findings indicated that

- When $n = 10$, Charterjee's new correlation still have a low power, while chi square test for distance correlation, Pearson product-moment and Spearman rank started to have high power even with this small size of sample. $t-$test of Pearson product-moment worked best in almost every case except some cases that mixture of Weibull distribution is involved.
- When $n = 20$ and 30, all test gave acceptable power, and Pearson product-moment produced highest power in almost all scenarios except some cases which have mixture of Weibull distribution involved because its power somehow lower or equal to Spearman rank and the chi square test of distance correlation.
- When $n = 50$, Pearson product-moment and Spearman rank outperformed the other three tests as the estimated powers equal to 1.0000 which is the highest value of possible power of the tests. However, the chi square test of distance correlation, MIC, and Chartterjee's new correlation also gave 1.0000 power in some cases.
- When $n = 100$, all five methods have equal power of the test which equal to 1.0000.
- In very size of sample, it is clear that the average value of correlations that were detected by the five methods are different. The correlation detected by Pearson product-moment and Spearman rank are close to the true value while MIC resulted lower value than the true correlation followed by bias corrected distance correlation and Chartterjee's new correlation.

Table 42 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.8 and $n = 10$

Distributions	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.7851	**0.8720**	0.7395	0.7892	-	-	0.5236	0.8064	-	-
MN1-MN2	0.7842	**0.8730**	0.7423	0.7906	-	-	0.5288	0.8216	-	-
MN1-MN3	0.7837	**0.8658**	0.7425	0.7886	-	-	0.5248	0.8083	-	-
MN1-MG1	0.7886	**0.8841**	0.7451	0.7966	-	-	0.5373	0.8265	-	-
MN1-MG2	0.7826	**0.8686**	0.7380	0.7827	-	-	-	-	-	-
MN1-MG3	0.7843	**0.8737**	0.7436	0.7988	-	-	0.5319	0.8244	-	-
MN1-MW	0.8477	**0.9770**	0.8792	0.9746	-	-	0.7306	0.9749	-	-
MN2-MN2	0.7811	**0.8637**	0.7394	0.7840	-	-	0.5249	0.8101	-	-
MN2-MN3	0.7862	**0.8808**	0.7519	0.8086	-	-	0.5374	0.8361	-	-
MN2-MG1	0.7831	**0.8763**	0.7405	0.7890	-	-	0.5266	0.8149	0.2933	0.419
MN2-MG2	0.7853	**0.8726**	0.7400	0.7866	-	-	0.5286	0.8158	-	-
MN2-MG3	0.7899	**0.8848**	0.7526	0.8099	-	-	0.5455	0.8374	0.3061	0.4521
MN2-MW	0.8628	**0.9988**	0.9334	0.9976	-	-	0.8054	0.9978	-	-
MN3-MN3	0.7824	**0.8555**	0.7443	0.7952	-	-	0.5269	0.8090	-	-
MN3-MG1	0.7898	**0.8880**	0.7545	0.8148	-	-	0.5438	0.8337	-	-
MN3-MG2	0.7826	**0.8670**	0.7423	0.7938	-	-	0.5298	0.8218	0.2946	0.4252
MN3-MG3	0.7815	**0.8652**	0.74392	0.7947	-	-	0.5253	0.8181	-	-
MN3-MW	0.8298	0.9352	0.8392	**0.9356**	-	-	0.6654	0.9331	-	-

Note: Bolded value is the highest power of the test in each case

Table 43 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.8 and $n = 10$

Distributions	**Methods**									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.7821	**0.8678**	0.7349	0.7771	-	-	0.5202	0.8063	-	-
MG1-MG2	0.7876	**0.8846**	0.7429	0.7899	-	-	0.5343	0.8227	-	-
MG1-MG3	0.7930	**0.8927**	0.7589	0.8197	-	-	-	-	-	-
MG1-MW	0.8583	0.9920	0.9278	**0.9959**	-	-	0.8108	0.9968	0.5344	0.9379
MG2-MG2	0.7840	**0.8686**	0.7334	0.7694	-	-	0.5200	0.8025	0.2868	0.4078
MG2-MG3	0.7863	**0.8742**	0.7437	0.7890	-	-	0.5342	0.8209	0.2955	0.4322
MG2-MW	0.8367	**0.9714**	0.8717	0.9708	-	-	0.7051	0.9703	-	-
MG3-MG3	0.7854	**0.8723**	0.7443	0.7978	-	-	-	-	-	-
MG3-MW	0.8209	0.9305	0.8363	**0.9349**	-	-	0.6464	0.9342	0.3970	0.6731
MW-MW	0.7813	0.8085	0.7829	**0.8605**	-	-	0.5647	0.8167	-	-

Note: Bolded value is the highest power of the test in each case

Table 44 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.8 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.7917	**0.9950**	0.7613	0.9872	0.6898	0.7545	0.5483	0.9001	0.3517	0.8060
MN1-MN2	0.7939	**0.9957**	0.7677	0.9895	0.6933	0.7553	0.5542	0.9888	0.3612	0.8255
MN1-MN3	0.7928	**0.9938**	0.7701	0.9880	0.6998	0.7757	0.5549	0.9074	0.3640	0.8308
MN1-MG1	0.7963	**0.9974**	0.7711	0.9923	0.7019	0.7771	0.5631	0.9919	0.3644	0.8364
MN1-MG2	0.7909	**0.9965**	0.7604	0.9874	0.6892	0.7520	0.5469	0.9884	0.3516	0.8096
MN1-MG3	0.7931	**0.9964**	0.7688	0.9897	0.7011	0.7776	0.5598	0.9898	0.3620	0.8251
MN1-MW	0.8345	0.9999	0.9010	**1.0000**	0.8565	0.9802	0.7357	1.0000	0.5510	0.9955
MN2-MN2	0.7925	**0.9957**	0.7642	0.9888	0.6952	0.7664	0.5525	0.9061	0.3575	0.8209
MN2-MN3	0.7945	**0.9974**	0.7780	0.9928	0.7086	0.7908	0.5603	0.9185	0.3722	0.8455
MN2-MG1	0.7935	**0.9961**	0.7684	0.9892	0.6968	0.7692	0.5577	0.9055	0.3611	0.8255
MN2-MG2	0.7934	**0.9961**	0.7660	0.9885	0.6966	0.7639	0.5561	0.9084	0.3605	0.8282
MN2-MG3	0.7975	**0.9976**	0.7783	0.9920	0.7093	0.7954	0.5719	0.9209	0.3734	0.8483
MN2-MW	0.8401	**1.0000**	0.9495	**1.0000**	0.9219	0.9982	0.7865	**1.0000**	0.6557	0.9999
MN3-MN3	0.7913	**0.9936**	0.7678	0.9900	0.6974	0.7722	0.5527	0.9056	0.3606	0.8259
MN3-MG1	0.7960	**0.9984**	0.7800	0.9937	0.7101	0.7941	0.5714	0.9209	0.3745	0.8542
MN3-MG2	0.7932	**0.9952**	0.7717	0.9915	0.7025	0.7802	0.5563	0.9106	0.3640	0.8309
MN3-MG3	0.7914	**0.9966**	0.7710	0.9926	0.7033	0.7825	0.5556	0.9061	0.3637	0.8288
MN3-MW	0.8236	0.9990	0.8628	**0.9999**	0.8083	0.9451	0.6809	0.9720	0.4863	0.9761

Note: Bolded value is the highest power of the test in each case

Table 45 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.8 and $n = 20$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.7953	**0.9960**	0.7622	0.9879	0.6918	0.7580	0.5542	0.9036	0.3548	0.8113
MG1-MG2	0.7949	**0.9970**	0.7691	0.9900	0.6993	0.7698	0.5633	0.9100	0.3617	0.8254
MG1-MG3	0.7987	**0.9983**	0.7849	0.9937	0.7168	0.8105	0.5865	0.9239	0.3810	0.8669
MG1-MW	0.8362	**1.0000**	0.9455	**1.0000**	0.9169	0.9980	0.8006	0.9994	0.6457	0.9999
MG2-MG2	0.7928	**0.9949**	0.7621	0.9879	0.6920	0.7660	0.5515	0.8996	0.3547	0.8119
MG2-MG3	0.7936	**0.9952**	0.7682	0.9894	0.6976	0.7737	0.5599	0.9091	0.3613	0.8263
MG2-MW	0.8246	0.9999	0.8920	**1.0000**	0.8460	0.9739	0.7073	0.9893	0.5352	0.9921
MG3-MG3	0.7928	**0.9950**	0.7682	0.9878	0.6997	0.7738	0.5596	0.9104	0.3623	0.8270
MG3-MW	0.8154	0.9993	0.8576	**0.9995**	0.8012	0.9361	0.6553	0.9762	0.4783	0.9716
MW-MW	0.7930	0.9829	0.8083	**0.9967**	0.7412	0.8498	0.5880	0.9051	0.4113	0.9103

Note: Bolded value is the highest power of the test in each case

Table 46 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.8 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.7960	**0.9999**	0.7718	0.9996	0.6275	0.9601	0.5583	0.9270	0.3782	0.9444
MN1-MN2	0.7952	**1.0000**	0.7748	0.9997	0.6296	0.9453	0.5583	0.9994	0.3827	0.9457
MN1-MN3	0.7942	**0.9999**	0.7771	0.9995	0.6344	0.9611	0.5602	0.9291	0.3839	0.9494
MN1-MG1	0.7951	**0.9999**	0.7779	0.9998	0.6351	0.9642	0.5659	0.9995	0.3846	0.9510
MN1-MG2	0.7951	**1.0000**	0.7709	0.9999	0.6248	0.9577	0.5571	0.9999	0.3770	0.9447
MN1-MG3	0.7964	**0.9999**	0.7787	0.9998	0.6362	0.9614	0.5680	0.9996	0.3863	0.9531
MN1-MW	0.8270	**1.0000**	0.9086	**1.0000**	0.8160	0.9998	0.7340	**1.0000**	0.5846	**1.0000**
MN2-MN2	0.7955	**0.9998**	0.7733	0.9996	0.6299	0.9630	0.5597	0.9291	0.3798	0.9458
MN2-MN3	0.7948	**0.9999**	0.7866	0.9998	0.6453	0.9690	0.5648	0.9408	0.3957	0.9587
MN2-MG1	0.7961	**0.9998**	0.7773	**0.9998**	0.6330	0.9621	0.5665	0.9310	0.3851	0.9508
MN2-MG2	0.7960	**1.0000**	0.7742	0.9996	0.6291	0.9586	0.5619	0.9307	0.3803	0.9456
MN2-MG3	0.7973	**1.0000**	0.7857	0.9997	0.6451	0.9683	0.5770	0.9382	0.3953	0.9596
MN2-MW	0.8308	**1.0000**	0.9555	**1.0000**	0.8965	**1.0000**	0.7798	**1.0000**	0.6952	**1.0000**
MN3-MN3	0.7956	**1.0000**	0.7779	0.9998	0.6353	0.9614	0.5624	0.9292	0.3858	0.9497
MN3-MG1	0.7972	**1.0000**	0.7887	0.9996	0.6471	0.9686	0.5777	0.9385	0.3998	0.9642
MN3-MG2	0.7953	**0.9999**	0.7785	0.9997	0.6355	0.9620	0.5619	0.9348	0.3858	0.9492
MN3-MG3	0.7971	**0.9997**	0.7814	0.9996	0.6391	0.9660	0.5662	0.9272	0.3910	0.9560
MN3-MW	0.8203	**1.0000**	0.8710	**1.0000**	0.7587	0.9979	0.6865	0.9810	0.5172	0.9991

Note: Bolded value is the highest power of the test in each case

Table 47 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.8 and $n = 30$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.7951	**0.9998**	0.7692	0.9995	0.6239	0.9546	0.5586	0.9278	0.3752	0.9404
MG1-MG2	0.7968	**1.0000**	0.7775	0.9999	0.6357	0.9638	0.5703	0.9336	0.3846	0.9544
MG1-MG3	0.8000	**1.0000**	0.7943	**1.0000**	0.6547	0.9738	0.5934	0.9410	0.4057	0.9694
MG1-MW	0.8277	**1.0000**	0.9508	**1.0000**	0.8866	**1.0000**	0.7954	0.9996	0.6830	**1.0000**
MG2-MG2	0.7952	**1.0000**	0.7700	0.9995	0.6262	0.9550	0.5591	0.9240	0.3768	0.9438
MG2-MG3	0.7950	0.9998	0.7757	0.9996	0.6348	0.9623	0.5665	0.9309	0.3824	0.9484
MG2-MW	0.8191	**1.0000**	0.9001	**1.0000**	0.8026	0.9997	0.7057	0.9919	0.5681	0.9996
MG3-MG3	0.7956	**0.9998**	0.7764	0.9997	0.6322	0.9617	0.5655	0.9324	0.3838	0.9488
MG3-MW	0.8123	**1.0000**	0.8655	**1.0000**	0.7514	0.9971	0.6572	0.9834	0.5068	0.9985
MW-MW	0.7945	0.9983	0.8162	**1.0000**	0.6817	0.9845	0.5973	0.9281	0.4358	0.9837

Note: Bolded value is the highest power of the test in each case

Table 48 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.8 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.7961	**1.0000**	0.7782	**1.0000**	0.6607	0.9959	0.5611	0.9433	0.3951	0.9954
MN1-MN2	0.7978	**1.0000**	0.7830	**1.0000**	0.6646	0.9533	0.5655	**1.0000**	0.4020	0.9963
MN1-MN3	0.7977	**1.0000**	0.7854	**1.0000**	0.6693	0.9970	0.5673	0.9444	0.4044	0.9968
MN1-MG1	0.7983	**1.0000**	0.7860	**1.0000**	0.6713	0.9553	0.5744	**1.0000**	0.4052	0.9958
MN1-MG2	0.7984	**1.0000**	0.7807	**1.0000**	0.6654	0.9986	0.5666	**1.0000**	0.3988	0.9950
MN1-MG3	0.7989	**1.0000**	0.7866	**1.0000**	0.6733	0.9980	0.5752	**1.0000**	0.4061	0.9970
MN1-MW	0.8190	**1.0000**	0.9146	**1.0000**	0.8505	**1.0000**	0.7334	**1.0000**	0.6116	**1.0000**
MN2-MN2	0.7972	**1.0000**	0.7808	**1.0000**	0.6648	0.9944	0.5648	0.9434	0.3984	0.9954
MN2-MN3	0.7972	**1.0000**	0.7939	**1.0000**	0.6800	0.9973	0.5705	0.9523	0.4159	0.9976
MN2-MG1	0.7981	**1.0000**	0.7851	**1.0000**	0.6690	0.9970	0.5726	0.9444	0.4045	0.9966
MN2-MG2	0.7971	**1.0000**	0.7818	**1.0000**	0.6656	0.9955	0.5674	0.9452	0.4001	0.9961
MN2-MG3	0.7989	**1.0000**	0.7938	**1.0000**	0.6793	0.9977	0.5829	0.9516	0.4150	0.9974
MN2-MW	0.8212	**1.0000**	0.9604	**1.0000**	0.9392	**1.0000**	0.7743	**1.0000**	0.7271	**1.0000**
MN3-MN3	0.7967	**1.0000**	0.7850	**1.0000**	0.6690	0.9958	0.5668	0.9456	0.4036	0.9956
MN3-MG1	0.7979	**1.0000**	0.7951	**1.0000**	0.6793	0.9971	0.5810	0.9535	0.4161	0.9970
MN3-MG2	0.7963	**1.0000**	0.7850	**1.0000**	0.6690	0.9971	0.5651	0.9461	0.4042	0.9957
MN3-MG3	0.7983	**1.0000**	0.7882	**1.0000**	0.6725	0.9972	0.5714	0.9433	0.4082	0.9967
MN3-MW	0.8150	**1.0000**	0.8770	**1.0000**	0.7915	**1.0000**	0.6878	0.9850	0.5398	**1.0000**

Note: Bolded value is the highest power of the test in each case

Table 49 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.8 and $n = 50$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.7965	**1.0000**	0.7767	**1.0000**	0.6609	0.9958	0.5650	0.9434	0.3936	0.9940
MG1-MG2	0.7982	**1.0000**	0.7856	**1.0000**	0.6699	0.9972	0.5768	0.9480	0.4040	0.9972
MG1-MG3	0.8005	**1.0000**	0.8016	**1.0000**	0.6883	0.9980	0.5989	0.9537	0.4254	0.9982
MG1-MW	0.8187	**1.0000**	0.9561	**1.0000**	0.9297	**1.0000**	0.7919	0.9998	0.7141	**1.0000**
MG2-MG2	0.7965	**1.0000**	0.7767	**1.0000**	0.6589	0.9957	0.5628	0.9382	0.3932	0.9962
MG2-MG3	0.7988	**1.0000**	0.7858	**1.0000**	0.6695	0.9958	0.5748	0.9476	0.4047	0.9967
MG2-MW	0.8135	**1.0000**	0.9061	**1.0000**	0.8390	**1.0000**	0.7047	0.9938	0.5938	**1.0000**
MG3-MG3	0.7970	**1.0000**	0.7838	**1.0000**	0.6669	0.9964	0.5708	0.9497	0.4023	0.9956
MG3-MW	0.8079	**1.0000**	0.8723	**1.0000**	0.7839	**1.0000**	0.6561	0.9880	0.5313	**1.0000**
MW-MW	0.7960	**1.0000**	0.8235	**1.0000**	0.7162	0.9993	0.6025	0.9447	0.4557	0.9994

Note: Bolded value is the highest power of the test in each case

Table 50 The estimated power of th test when at least one variable is sampled from mixture of normal distribution with true correlation of 0.8 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MN1-MN1	0.7982	**1.0000**	0.7839	**1.0000**	0.6104584	**1.0000**	0.5661	**1.0000**	0.4091	**1.0000**
MN1-MN2	0.7989	**1.0000**	0.7881	**1.0000**	0.6146	**1.0000**	0.5677	**1.0000**	0.4149	**1.0000**
MN1-MN3	0.7984	**1.0000**	0.7899	**1.0000**	0.6168	**1.0000**	0.5700	**1.0000**	0.4169	**1.0000**
MN1-MG1	0.7992	**1.0000**	0.7924	**1.0000**	0.6208	**1.0000**	0.5787	**1.0000**	0.4194	**1.0000**
MN1-MG2	0.7986	**1.0000**	0.7852	**1.0000**	0.6117	**1.0000**	0.5682	**1.0000**	0.4102	**1.0000**
MN1-MG3	0.7990	**1.0000**	0.7921	**1.0000**	0.6195	**1.0000**	0.5780	**1.0000**	0.4204	**1.0000**
MN1-MW	0.8115	**1.0000**	0.9191	**1.0000**	0.8176	**1.0000**	0.7321	**1.0000**	0.6300	**1.0000**
MN2-MN2	0.7986	**1.0000**	0.7864	**1.0000**	0.6130	**1.0000**	0.5684	**1.0000**	0.4125	**1.0000**
MN2-MN3	0.7984	**1.0000**	0.7998	**1.0000**	0.6306	**1.0000**	0.5736	**1.0000**	0.4296	**1.0000**
MN2-MG1	0.7990	**1.0000**	0.7910	**1.0000**	0.6179	**1.0000**	0.5762	**1.0000**	0.4177	**1.0000**
MN2-MG2	0.7993	**1.0000**	0.7879	**1.0000**	0.6149	**1.0000**	0.5722	**1.0000**	0.4139	**1.0000**
MN2-MG3	0.7994	**1.0000**	0.7992	**1.0000**	0.6288	**1.0000**	0.5865	**1.0000**	0.4289	**1.0000**
MN2-MW	0.8122	**1.0000**	0.9644	**1.0000**	0.9210	**1.0000**	0.7698	**1.0000**	0.7509	**1.0000**
MN3-MN3	0.7979	**1.0000**	0.7892	**1.0000**	0.6164	**1.0000**	0.5696	**1.0000**	0.4161	**1.0000**
MN3-MG1	0.7989	**1.0000**	0.8025	**1.0000**	0.6338	**1.0000**	0.5863	**1.0000**	0.4336	**1.0000**
MN3-MG2	0.7988	**1.0000**	0.7912	**1.0000**	0.6186	**1.0000**	0.5703	**1.0000**	0.4192	**1.0000**
MN3-MG3	0.7985	**1.0000**	0.7927	**1.0000**	0.6211	**1.0000**	0.5737	**1.0000**	0.4198	**1.0000**
MN3-MW	0.8080	**1.0000**	0.8819	**1.0000**	0.7490	**1.0000**	0.6876	**1.0000**	0.5564	**1.0000**

Note: Bolded value is the highest power of the test in each case

Table 51 The estimated power of the test when at least one variable is sampled from mixture of gamma or mixture of Weibull distribution with true correlation of 0.8 and $n = 100$

Distributions	Methods									
	r		r_s		MIC		$\mathcal{R}^*$		ξ_n	
	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$	average	$\widehat{1-\beta}$
MG1-MG1	0.7985	**1.0000**	0.7817	**1.0000**	0.6067	**1.0000**	0.5686	**1.0000**	0.4063	**1.0000**
MG1-MG2	0.7994	**1.0000**	0.7911	**1.0000**	0.6189	**1.0000**	0.5809	**1.0000**	0.4188	**1.0000**
MG1-MG3	0.8005	**1.0000**	0.8074	**1.0000**	0.6404	**1.0000**	0.6029	**1.0000**	0.4399	**1.0000**
MG1-MW	0.8111	**1.0000**	0.9611	**1.0000**	0.9131	**1.0000**	0.7916	**1.0000**	0.7396	**1.0000**
MG2-MG2	0.7981	**1.0000**	0.7830	**1.0000**	0.6087	**1.0000**	0.5680	**1.0000**	0.4091	**1.0000**
MG2-MG3	0.7979	**1.0000**	0.7888	**1.0000**	0.6148	**1.0000**	0.5752	**1.0000**	0.4151	**1.0000**
MG2-MW	0.8073	**1.0000**	0.9114	**1.0000**	0.8037	**1.0000**	0.7034	**1.0000**	0.6136	**1.0000**
MG3-MG3	0.8079	**1.0000**	0.9096	**1.0000**	0.8063	**1.0000**	0.7047	**1.0000**	0.6136	**1.0000**
MG3-MW	0.8112	**1.0000**	0.8863	**1.0000**	0.7513	**1.0000**	0.6667	**1.0000**	0.5581	**1.0000**
MW-MW	0.8281	**1.0000**	0.7953	**1.0000**	0.6742	**1.0000**	0.6062	**1.0000**	0.4711	**1.0000**

Note: Bolded value is the highest power of the test in each case

Chapter 5
Conclusion and discussion

1. Conclusion

The goal of this study is to compare the performance of five correlation coefficient methods when the data is sampled from bivariate mixture distribution. Pearson product-moment correlation, Spearman rank correlation, maximal information coefficient, bias corrected distance correlation, and Chartterjee's new correlation are the five methods. The mixed distributions of interest are the mixtures of normal, gamma, and Weibull distributions. Five different sample sizes were investigated: 10, 20, 30, 50, and 100. The bivariate mixture's true correlations are 0.2, 0.4, 0.6, and 0.8. In this investigation, the significance level was set at 0.05. The outcome revealed that

When type I error rate was investigated:

- Among the five methods, only MIC that can not control type I error rate in every pairs of data while Chartterjee's new correlation coefficient and bias corrected distance correlation happened to lose control the type I error rate in some cases too when $n = 10$.
- When n is not smaller than 20, all five methods are able to control type I error in every scenarios as the estimated type I error rate fell into the Bradley's liberal criterion.

When power of the test was investigated:

- The Pearson product-moment correlation performed the best among the five in almost all cases. In the cases where the true correlation coefficient was small and the sample was small, the Pearson product-moment correlation did not have very good performance, but was still one of the methods that performed best. The Pearson product-moment correlation started to perform better when the sample size increased, but is still insufficient when the true correlation is small. For medium and large correlation coefficients, the Pearson product-moment correlation had very good performance and had the highest power except in some cases where a mixture of Weibull distribution was involved.
- The Spearman rank correlation performed admirably, almost as well as the Pearson product-moment correlation. Just like with Pearson product-moment, Spearman rank performed poorly when sample size and real correlation were small, but improved as sample size increased but remained insufficient. When the true correlation was large enough, the sample size was not an issue for the Spearman rank to demonstrate its power. When dealing with a mixture of Weibull distributions, the Spearman rank outscored the Pearson product-moment in almost every pair.
- The bias corrected distance correlation performed quite well, not as well as the two previously mentioned, but better than the maximum information coefficient and Chartterjee's new correlation coefficient. In small sample sizes, the bias corrected distance correla-

tion performed as well as Pearson product-moment and Spearman rank correlations, and when $n = 10$ and the true correlation was small, the bias corrected distance correlation performed best in almost all cases, despite having an unacceptable power. With medium and large true correlation, the bias corrected correlation's powers were sufficient even with the small size of sample just like the Pearson product-moment and Spearman rank correlation.

– Alongside Chartterjee's new correlation, the maximal information coefficient did the worst. To get good performance, MIC and Chartterjee's new correlation required highly correlated data. In the medium true correlation cases, MIC and Chartterjee's new correlation required a large sample size to perform as well as the three previously mentioned. With a large sample size and a small true correlation, MIC and Chartterjee's new correlation performed quite poorly.

Pearson product-moment and Spearman rank found the value that was most comparable to the actual correlation of the bivariate mixture by considering the average value of correlation in each scenario that was detected by each method. When the true correlation was 0 and 0.2, the MIC detected a value greater than the trues, whereas the other four values were nearly true. Only Pearson product-moment and Spearman rank recognized values that were closer to the true correlation than the other three methods in the cases of medium and high true correlations.

2. Discussion

From the results showed above, the estimated type I error rate of Pearson product-moment and Spearman rank correlation are found to be under control which is the same as the results of Bishara & Hittner (2012). In term of power, Pearson product-moment, Spearman rank, and bias corrected distance correlation also outperformed MIC and Chartterjee's new correlation, while Pearson product-moment is found to be the best one in almost every cases except when one of the distribution involved in bivariate is Weibull mixture distribution which Spearman or bias corrected distance correlation are better than Pearson product-moment correlation and the values are not different from the study of Bishara & Hittner (2012). MIC and Chartterjee's new correlation had low power of the test comparing to distance correlation when the relationship type is linear, Chatterjee (2021) also found that in simulation part too.

The Pearson product-moment correlation and Spearman rank correlation fared better than the others in identifying the correlation coefficient because the simulated data are configured to have a linear form relationship. When true correlation is 0 or the relationship is generated randomly, MIC gave higher value which is the same as the results in simulation study part of Reshef et al. (2011) and Deebani & Kachouie (2020).

In conclusion, Pearson product-moment and Spearman rank are far more reliable than recently developed correlation coefficients in terms of linear form relationships, even when used with data from a mixture distribution. We can also see that the power of these correlations is not significantly impacted by the types of distribution. Therefore, when a relationship is linear, it is advisable to utilize the Pearson product-moment or Spearman rank correlation to find the correlation or perform the test.

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