

1 Summary of Algorithm Validation Tests

1.1 Introduction

The objective of these tests is to validate the proper functioning of the algorithm by analyzing its ability to correctly estimate the parameters and optimal controls. Three types of tests are conducted: without control, with an evolving control, and with an autoregressive control.

1.2 Data and parameters

The data used in the following tests were generated using the modeling of the observed values $y_i^{(t)}$:

$$y_i^{(t)} = \theta_{c_i} x_i^{(t)} + \sum_{j \neq i, c^*(j)=c^*(i)} \gamma_{ij} x_j^{(t)} + \beta_{c_i} u_{c_i}^{(t)} + \epsilon_i^{(t)}, \quad (1)$$

In the following tests, we set $n = 100$ time series and $T = 50$ timesteps. Moreover, we set the dimensions of time series x and y to 3. Denoting (n, T, d) the dimensions of variables x and y and c the number of clusters, the parameters are of dimension (c, d, d) .

1.3 Test Procedure

In section we explain the different tests conducted and display the results. The latter are averages of several tests conducted with the same settings.

1.3.1 Test without Control

Objective: Verify the proper functioning of the algorithm in the absence of control. **Control:** No control applied, i.e., $u_t = 0, \forall t$.

Analysis:

- **Parameters:** The parameters do not converge toward the ground truth parameters in most tests. This is surprising considering the results we obtain in the other sections. The MSE of the estimated parameters is equal to 8.253
- **Control:** The control does not converge over time. The difference between estimated optimal controls and ground truth controls is significant, with an MSE equals to 7.634.
- **Clusters:** The average MSE of the cluster priors is equals to 0.393. The accuracy of correctly assigned clusters is around 0.54.

1.3.2 Test with Evolving Control

Objective: Study the impact of a progressively evolving control over a time window. **Control:** Control following a deterministic law:

$$u_t = u_0 + \alpha * t \quad (2)$$

where, α is a chosen constant.

Analysis:

- **Parameters:** The parameters converge toward the ground-truth parameters. MSE of the estimated parameters and ground-truth parameters is equal to 1.780

- **Control:** The difference between estimated optimal controls and ground truth controls is close, with an MSE equals to 3.275
- **Clusters:** The cluster priors converge toward the ground-truth values. The average MSE is equals to 0.240. The accuracy of correctly assigned clusters is around 0.70.

1.3.3 Test with Autoregressive Control

Objective: Test the algorithm with a control that continuously varies over time without abrupt changes. **Control:** Autoregressive modeling:

$$u_t = \alpha u_{t-1} + \epsilon_t \tag{3}$$

where $\alpha \in [0.8, 0.99]$ and ϵ_t is a small Gaussian noise.

Analysis:

- **Parameters:** Stable of estimated parameters, with an MSE equals to 2.689.
- **Control:** The difference between estimated optimal controls and ground truth controls is close, with an MSE equals to 3.275
- **Clusters:** The cluster priors do not converge toward the ground-truth values. The average MSE is equals to 0.380. The accuracy of correctly assigned clusters is around 0.62.

1.4 Conclusion

There is a clear issue with the test without control that requires further investigation. While the tests with control provide valuable insights, they are not yet fully satisfactory and can be improved.