Numerical exer 2 - Neural

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1 Leaky I&F for coupled neurons

Let's consider we have the leaky integrate and fire model for a single neuron,

$$\tau_m \frac{dV(t)}{dt} = E_L - V(t) + R_m I(t)$$

where E_L is the rest potential for no input current, and τ_m the relaxation of the membrane. Then, let's say we put another neuron, j, and we link it to our first one. Then, the intensity received by our neuron is given by the contributions of the external current and also the spikes of j, so $I(t) = I_{ext}(t) + I_j(t)$. The input intensity from j is given by the postsynaptic potential, so

$$I_j(t) = -R_s(t - t_j)(V_i(t) - E_s)$$

where E_s is the reversal potential of the synapse. For excitatory synapses, this potential is higher than the resting potential, making $I_j(t)$ positive and depolarizing the neuron. Inhibitory synapses have the reversal potential similar to the resting, so when the neuron depolarizes it tries to go again to the initial state, inhibiting the signal. Note that the potential we are using is the V_i, this is, the action potential of the receiver neuron. The effect of the pulse of the presynaptic one is encoded in the form of the resistance of the synapse, $R_s(t)$. This varies with time, since when presynaptic potential release neurotransmitters, then R_s decreases, but when no pulse arrives to the synapse, R_s increases. The time t_j is the time at which the neuron j pulsed and released the neurotransmitters. Then, we can write

$$R_s(t) = R_{s0} P(t) = \frac{R_{s0} P_{max}}{\tau_s} t e^{1 - t/\tau_s}$$

where τ_s is the relaxation time for the number of neurotransmitters, and P_{max} the maximum probability of having channels open. Rewriting $\overline{g} = 1/(R_{s0} R_m)$ the conductance of membrane and synapsis, we finally have an equation for the coupled neuron i:

$$\tau_m \frac{dV_i(t)}{dt} = E_L - V(t) - \overline{g} P(t - t_j)(V_i(t) - E_s) - R_m I_e$$

where we have used a constant intensity $I_{ext}(t) = I_e$. If now i also have a synapsis to j, then we have the same equation for both neurons. In the case of more neurons, we only have to change $P(t - t_j) \to \sum_j P(t - t_j)$ so every input neuron can reduce the membrane conductance through the neurotransmitter input.

2 Simulations

We use a RK4 method to simulate the equations for the model. We have two neurons, both connected to each other, so it is a system of two equations. Every step we compute the value for the voltages V_1 and V_2, and check if they are greater than the threshold. If, say, $V_1 \geq V_{thres}$ at time t, then we do $V_1 = V_{rest}$ and

and set $t_1 = t$. Next step, when we compute $dV_2(t)/dt$, we will have the term $P(t - t_1) = P(0) = P_{max}$. At the end of the step, we increase $t = t + h$, so as time goes $P(t - t_1) \to 0$. We use a value $h = 0.001$ for the integration.

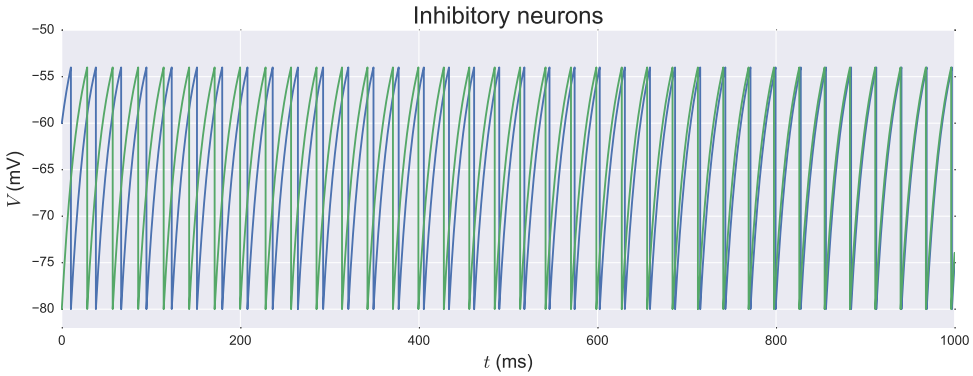

Figure 1: Inhibitory neurons for $t_f = 1$ s. We can see that at the end, they pulse in phase.

For the initial conditions, we use $V_{01} \neq V_{02}$ to avoid a synchronization between the neurons from the very beginning. Then, we run the program for inhibitory neurons ($E_s = -80$ mV, Figure 1) to see if they synchronize. Indeed, they do after approximately a second. Also, we can see that if we increase $\overline{g}$, this time reduces, since the coupling between both neurons increases and they have more effect on each other. On the other hand, when we use excitatory neurons ($E_s = 0$ mV, Figure 2) at the end they are out of phase, even if we start with $V_{01} \simeq V_{02}$. The larger the difference, the sooner the signals happens to be completely out of phase. However if both are equal the system keeps synchronized.

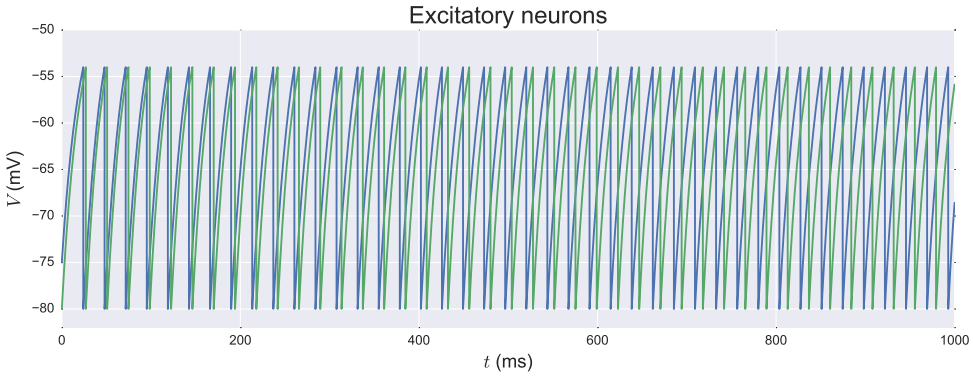

Figure 2: Excitatory neurons for $t_f = 1$ s. Even if they start with very similar initial conditions, they end out of phase. In this case $V_{01} = -75$ mV and $V_{02} = -80$ mV.

Another interesting thing is to see what happens when a excitatory neuron couples to an inhibitory one (Figure 3). Then the pattern looks to be random. However, if we compute the difference $V_1 - V_2$ and plot it against time, then it is easier to see that there is kind of oscillations, so looks like both signals could have some sort of regularity.

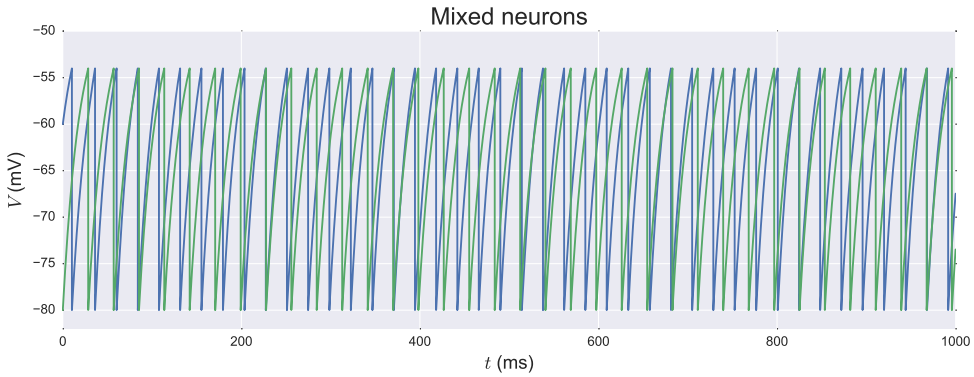

Figure 3: Inhibitory (green) and excitatory (blue) neurons coupled together. In $t = 200$ ms and $t = 800$ ms we can see a similar patter that repeats.

Then we perform a Fourier Transform over V_1 and V_2. This allows us to see that, for the most important peaks, the spectra is similar, but it is displaced in a few hertzs (Figure 4). There is few peaks with very similar frequency. In general, we can see that the inhibitory is pulsating at an higher frequency. This is logical since the input that controls this neuron is excitatory, making it pulsing fast. The synapse in the other direction is inhibitory, making the other to pulse at a slower rate. For this reason, it is interesting to study the coherence of the function. We see that it has a clear maximum for a fast oscillation mode, near 3 Hz. However, the maximum value of the coherence is very low, so we cannot infer that one neuron is driving the other, but they have a constant feedback.

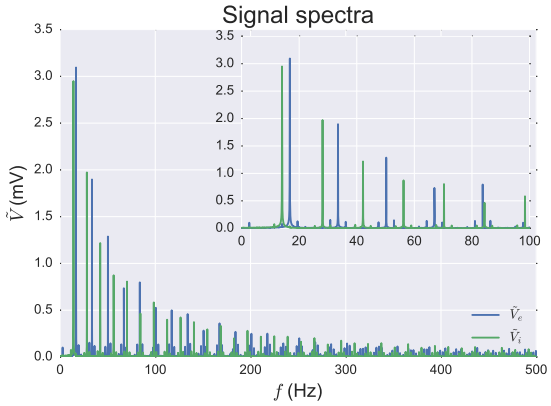

Figure 4: Inhibitory (green) and excitatory (blue) FFT of the signals and a zoom in the first 100 Hz. The FFT was done for a $t_f = 10$ seconds signal. We can see that most important peaks are the fast oscillations.

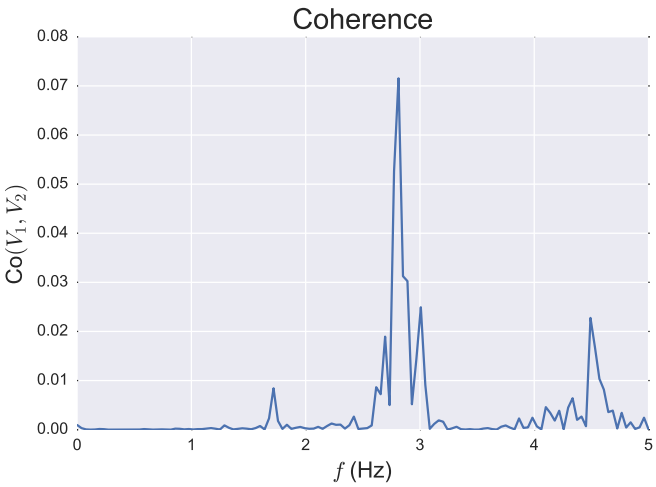

Figure 5: Coherence of V_1 with V_2. It is possible to see that the coherence has a peak, but its value is very low.