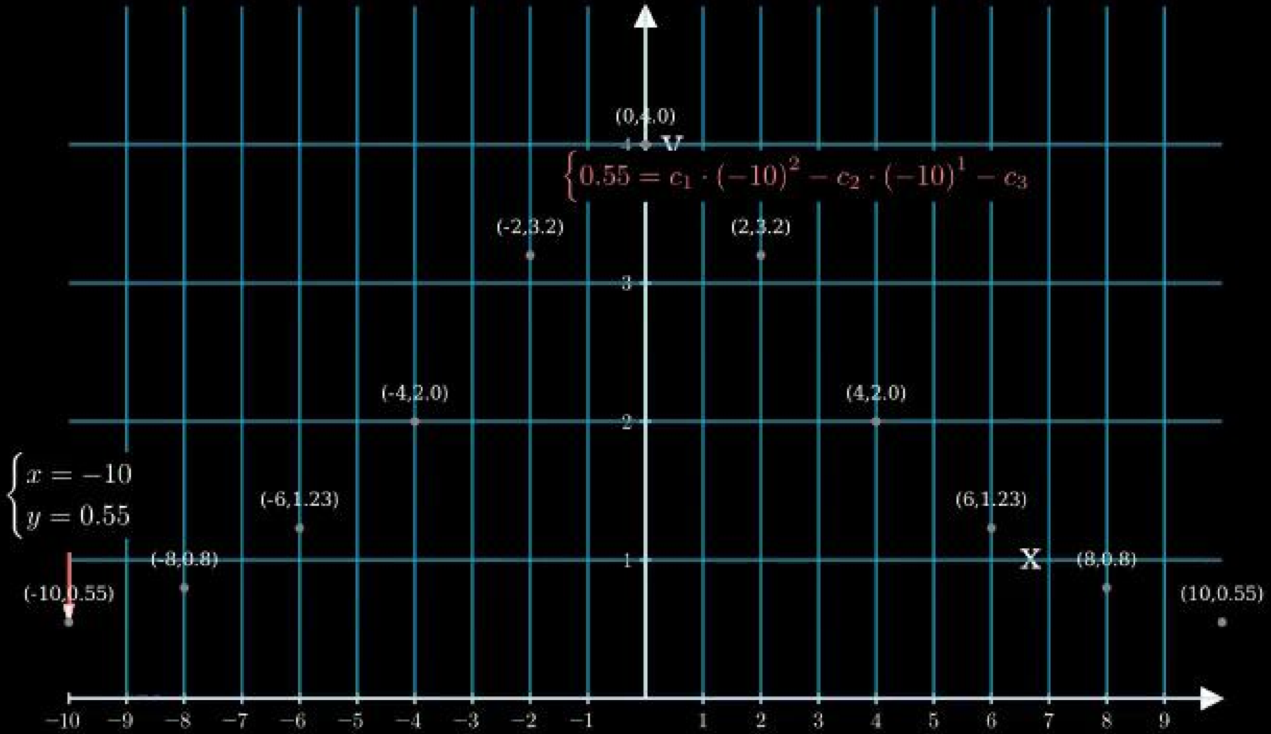
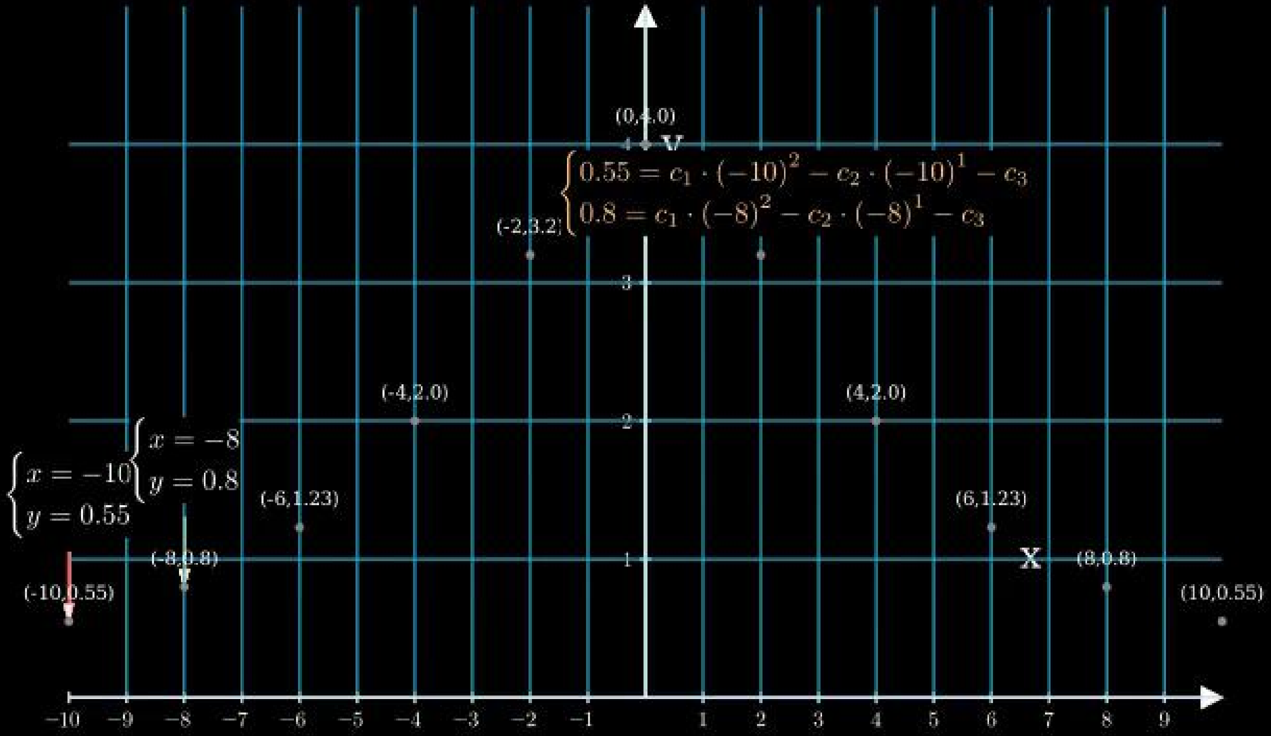
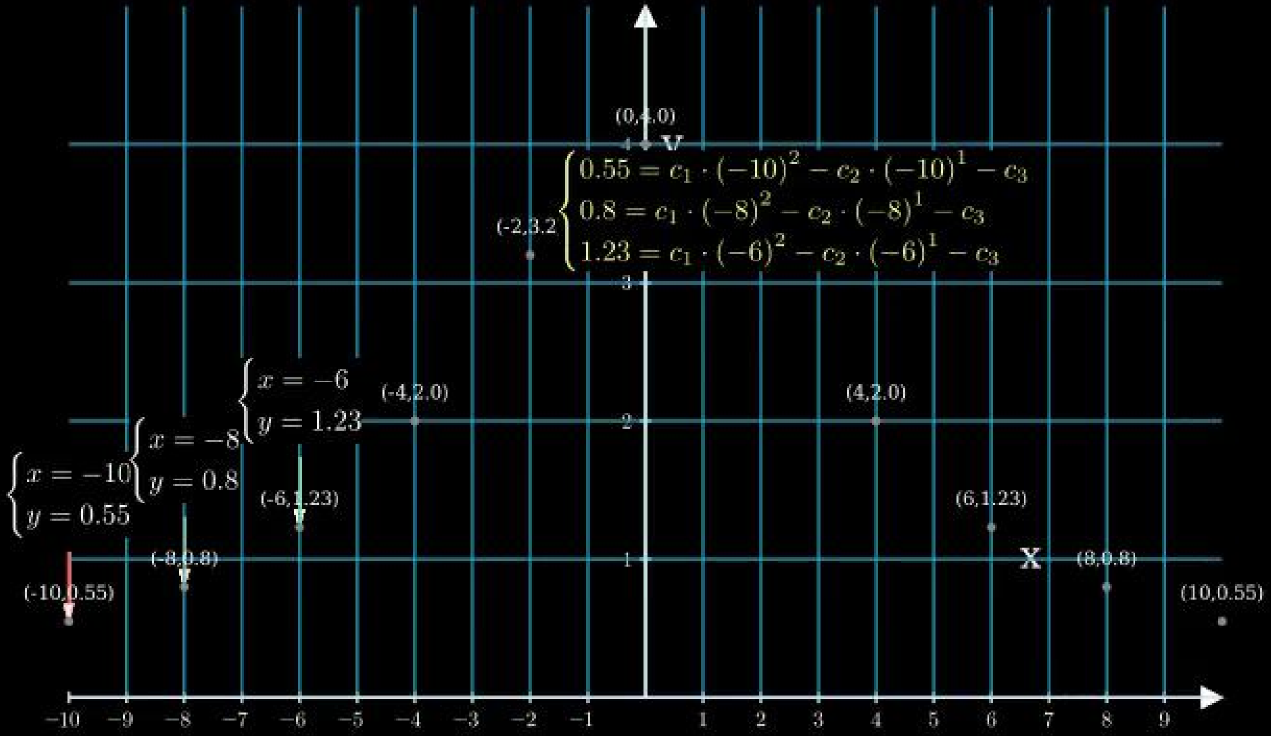
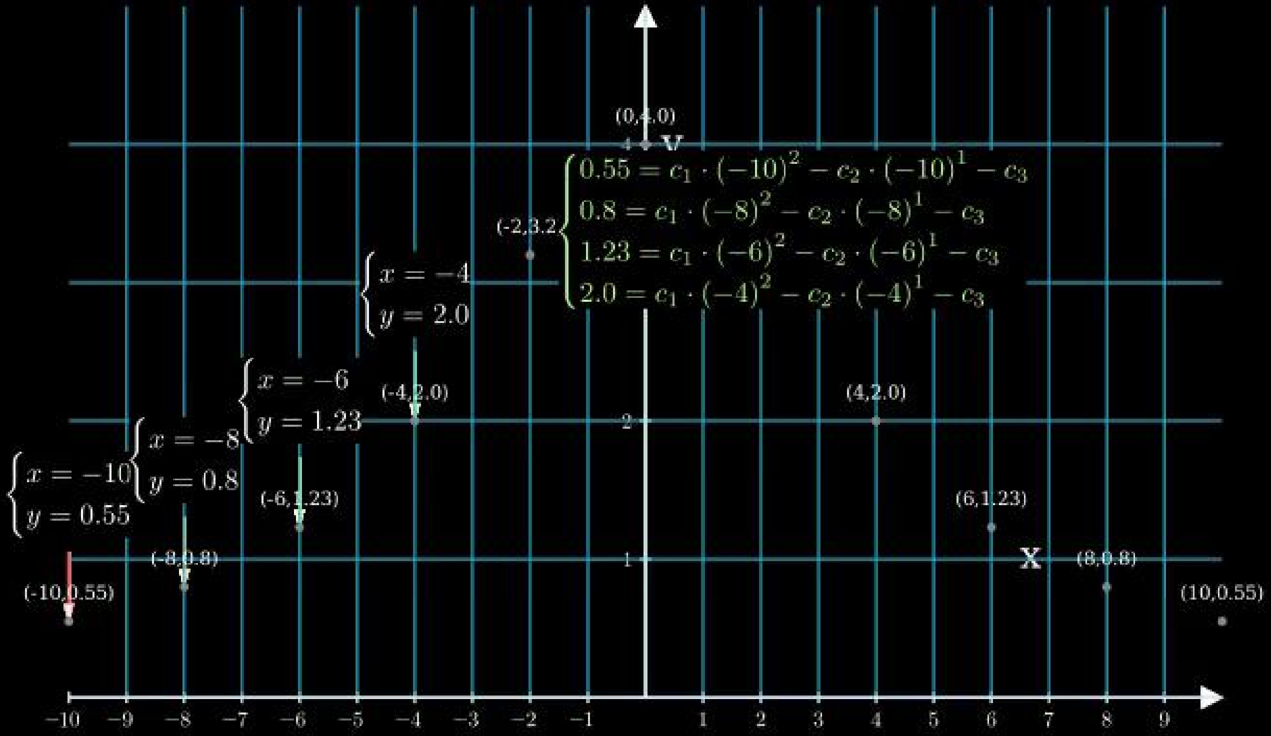


(0,4.0)
Y
$\{0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
(-2,3.2)
(2,3.2)
(-4,2.0)
(4,2.0)
$x = -10$
$y = 0.55$
(-6,1.23)
(6,1.23)
(-8,0.8)
X
(8,0.8)
(-10,0.55)
(10,0.55)

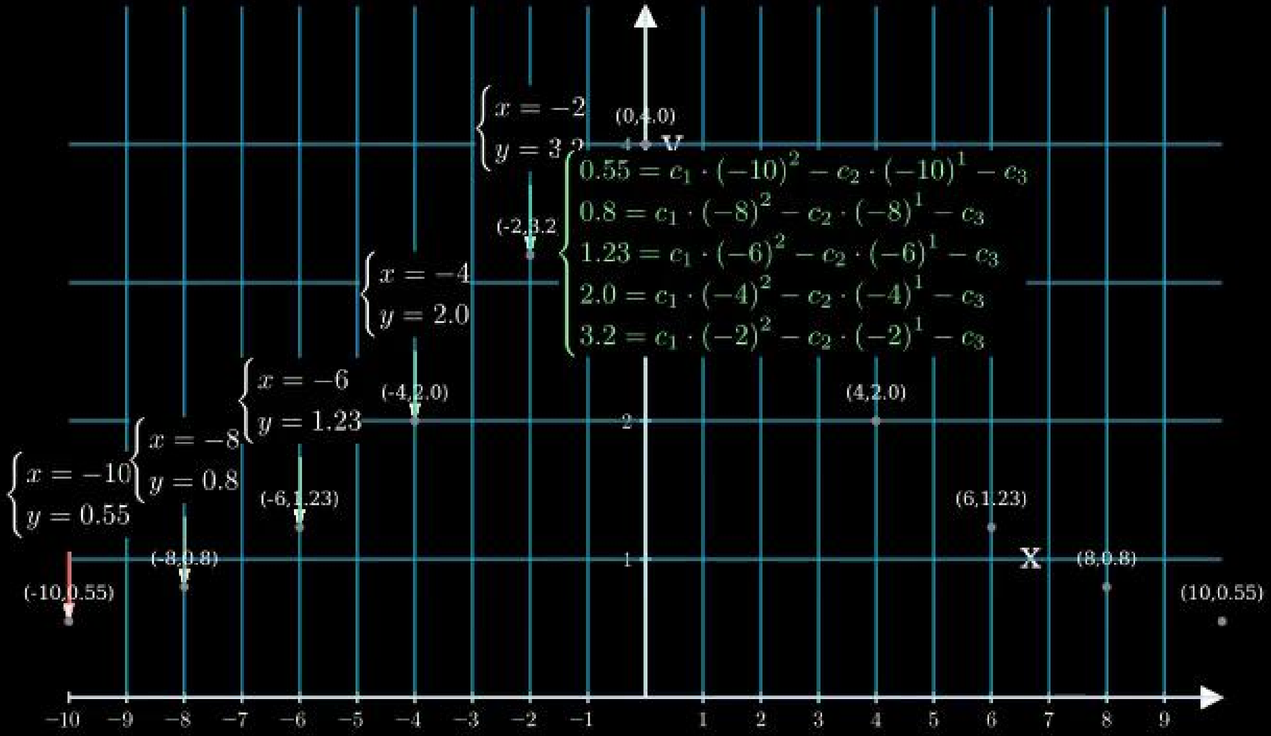

(0,4.0)
Y
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
(-2,3.2)
(-4,2.0)
(4,2.0)
$x = -8$
$y = 0.8$
$x = -10$
$y = 0.55$
(-6,1.23)
(6,1.23)
(-8,0.8)
X
(8,0.8)
(-10,0.55)
(10,0.55)
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9
1 2 3 4



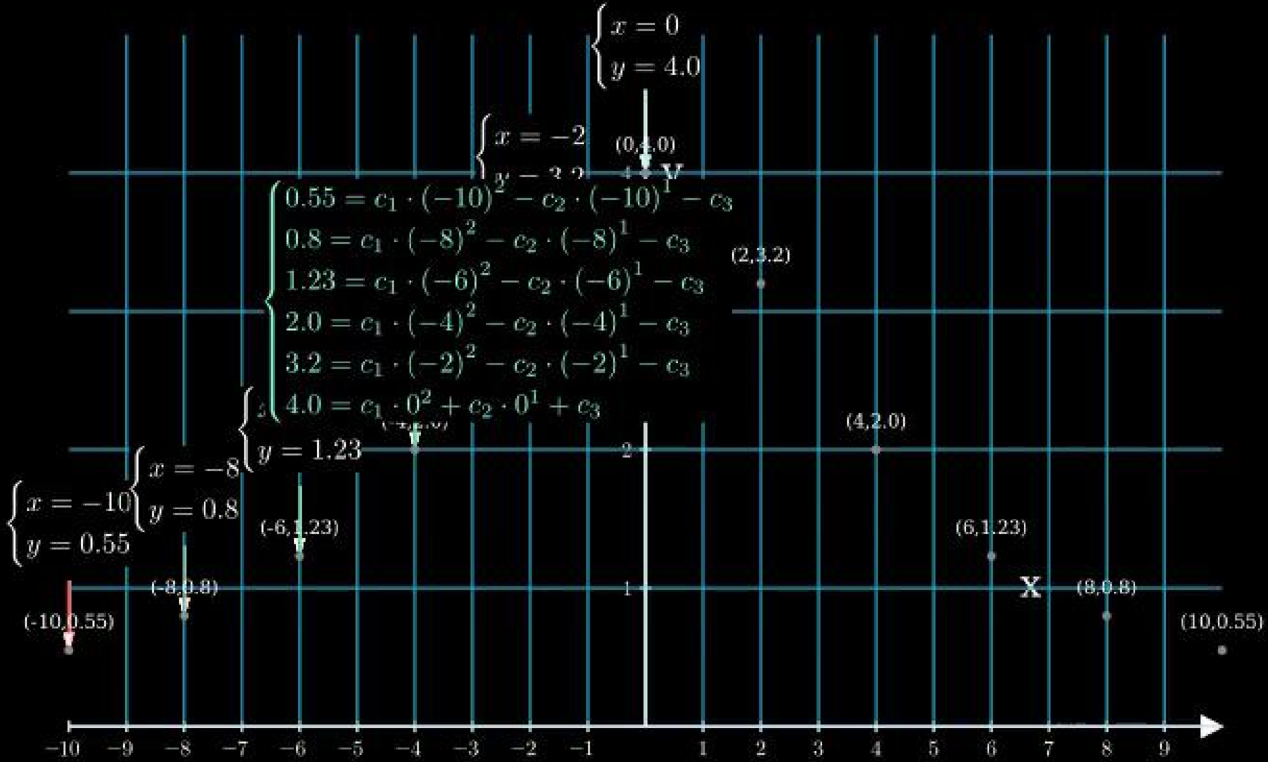
(0,4,0)
y
0.55 = c1 · (−10)^2 − c2 · (−10)^1 − c3
0.8 = c1 · (−8)^2 − c2 · (−8)^1 − c3
1.23 = c1 · (−6)^2 − c2 · (−6)^1 − c3
(-2,3.2
x = −6
y = 1.23
(-4,2.0)
(4,2.0)
x = −8
y = 0.8
x = −10
y = 0.55
(-6,1.23)
(6,1.23)
(-8,0.8)
(8,0.8)
x
(-10,0.55)
(10,0.55)
1
2
3
4
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9



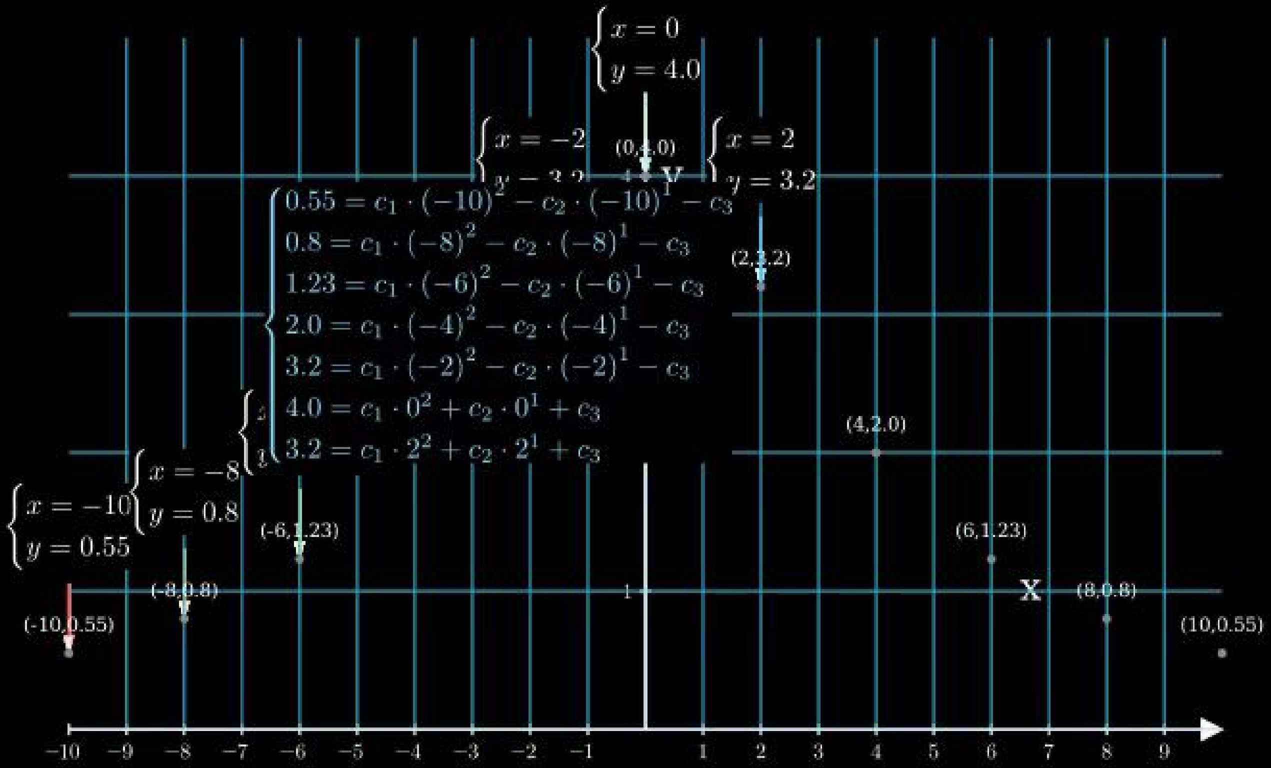
(0,4.0)
y
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
$1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3$
$2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3$
(-2,3.2)
$x = -4$, $y = 2.0$
$x = -6$, $y = 1.23$
$x = -8$, $y = 0.8$
$x = -10$, $y = 0.55$
(-4,2.0)
(4,2.0)
(-6,1.23)
(6,1.23)
(-8,0.8)
(8,0.8)
(-10,0.55)
(10,0.55)
x
4
2
1
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9



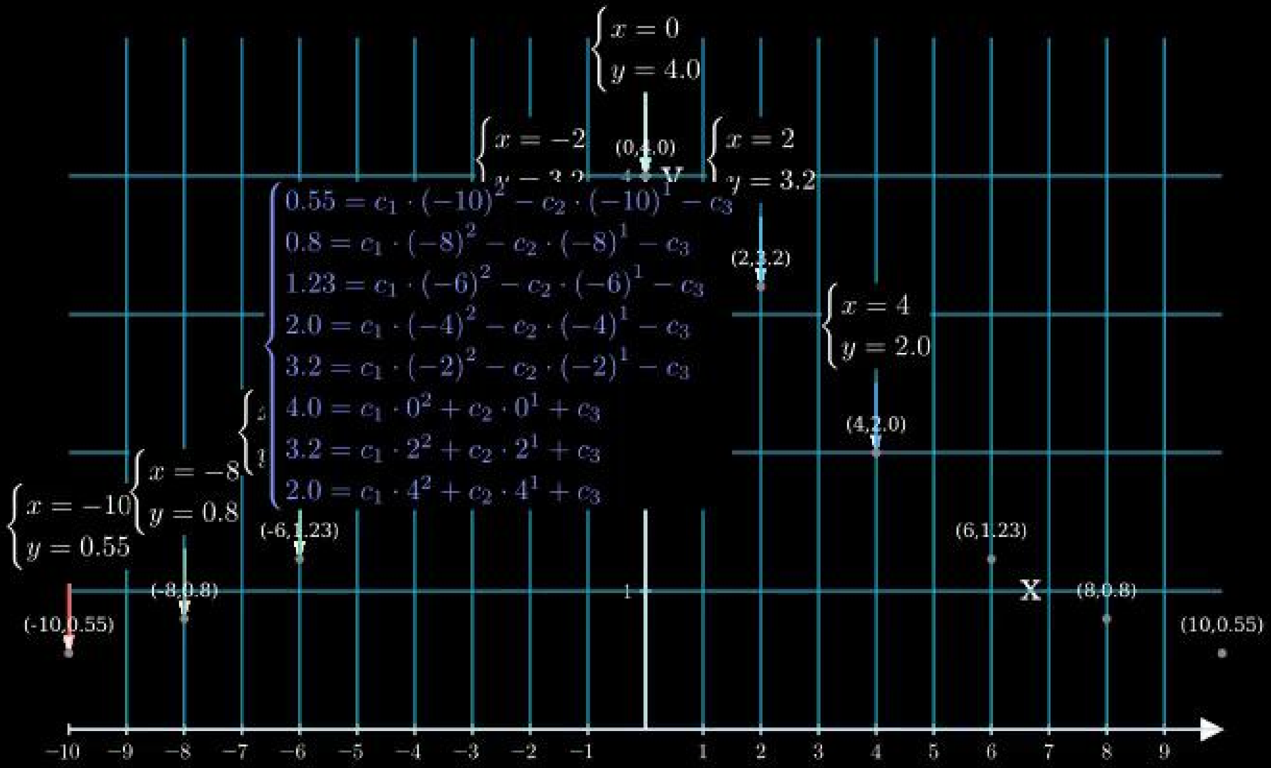
$x = -2$
$y = 3.2$
(0,4.0)
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
$1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3$
$2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3$
$3.2 = c_1 \cdot (-2)^2 - c_2 \cdot (-2)^1 - c_3$
$x = -4$
$y = 2.0$
$x = -6$
$y = 1.23$
$x = -8$
$y = 0.8$
$x = -10$
$y = 0.55$
(-4,2.0)
(4,2.0)
(-6,1.23)
(6,1.23)
(-8,0.8)
(8,0.8)
(-10,0.55)
(10,0.55)
X
2
1
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9

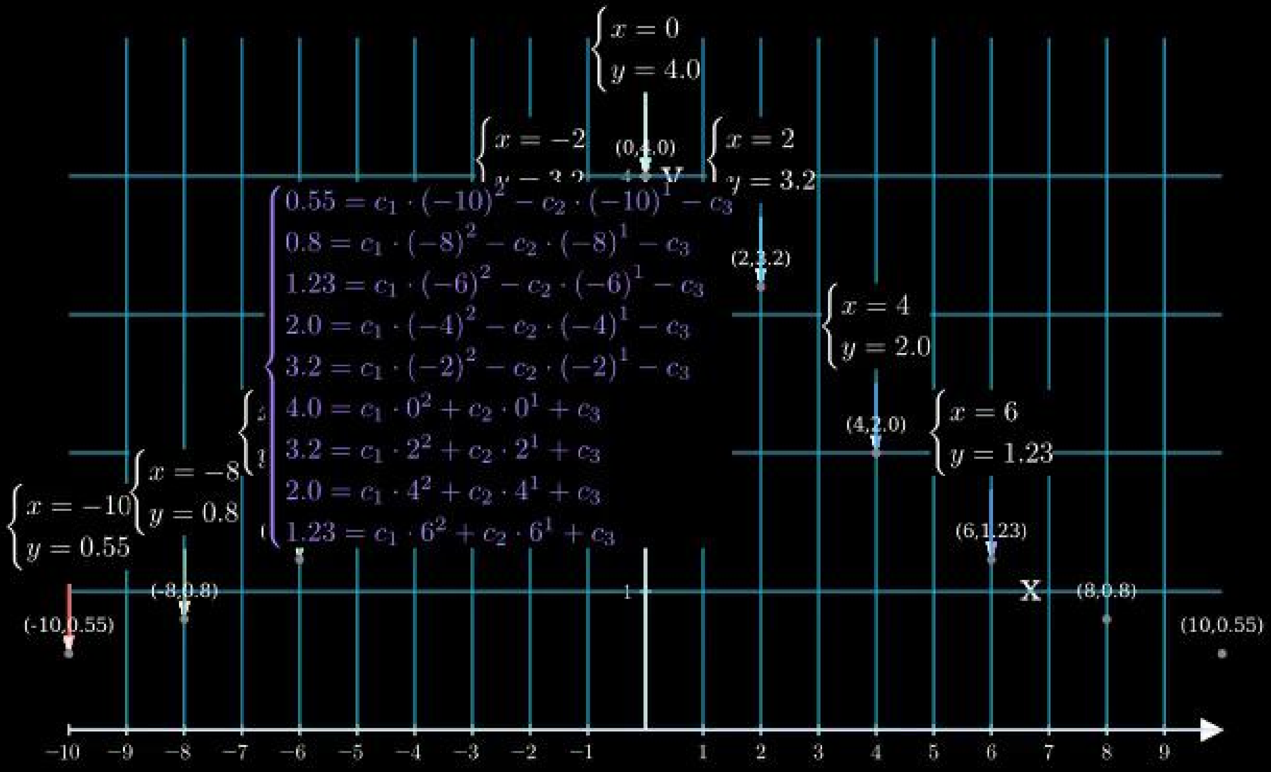


$\begin{cases} x = 0 \\ y = 4.0 \end{cases}$
$x = -2$
(0,4.0)
Y
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
$1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3$
$2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3$
$3.2 = c_1 \cdot (-2)^2 - c_2 \cdot (-2)^1 - c_3$
$4.0 = c_1 \cdot 0^2 + c_2 \cdot 0^1 + c_3$
(2,3.2)
(4,2.0)
$y = 1.23$
$\begin{cases} x = -8 \\ y = 0.8 \end{cases}$
$\begin{cases} x = -10 \\ y = 0.55 \end{cases}$
(-6,1.23)
(6,1.23)
X
(-8,0.8)
(8,0.8)
(-10,0.55)
(10,0.55)
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9

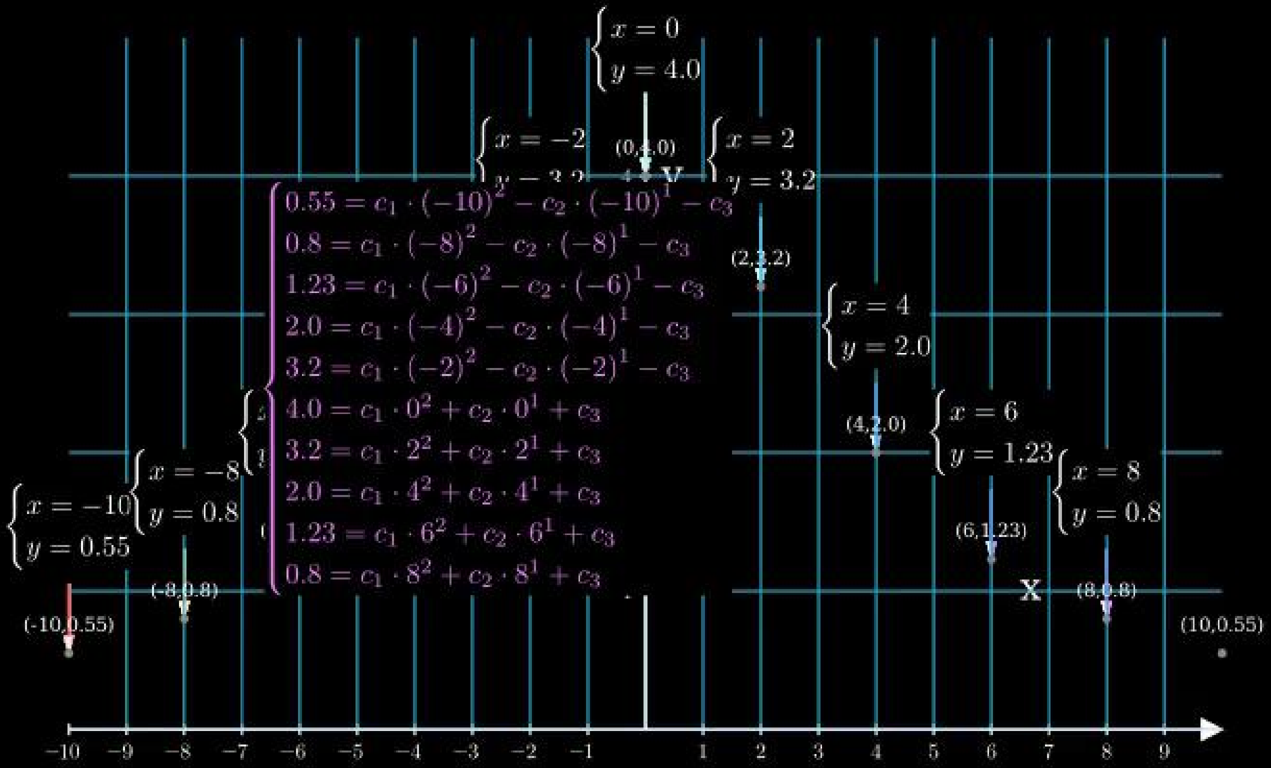


x = 0
y = 4.0
x = −2
y = 3.2
(0,4.0)
Y
x = 2
y = 3.2
0.55 = c1 · (−10)^2 − c2 · (−10)^1 − c3
0.8 = c1 · (−8)^2 − c2 · (−8)^1 − c3
1.23 = c1 · (−6)^2 − c2 · (−6)^1 − c3
2.0 = c1 · (−4)^2 − c2 · (−4)^1 − c3
3.2 = c1 · (−2)^2 − c2 · (−2)^1 − c3
4.0 = c1 · 0^2 + c2 · 0^1 + c3
3.2 = c1 · 2^2 + c2 · 2^1 + c3
(2,3.2)
(4,2.0)
x = −8
y = 0.8
x = −10
y = 0.55
(-6,1.23)
(6,1.23)
(-8,0.8)
X
(8,0.8)
(-10,0.55)
(10,0.55)
4
1
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9

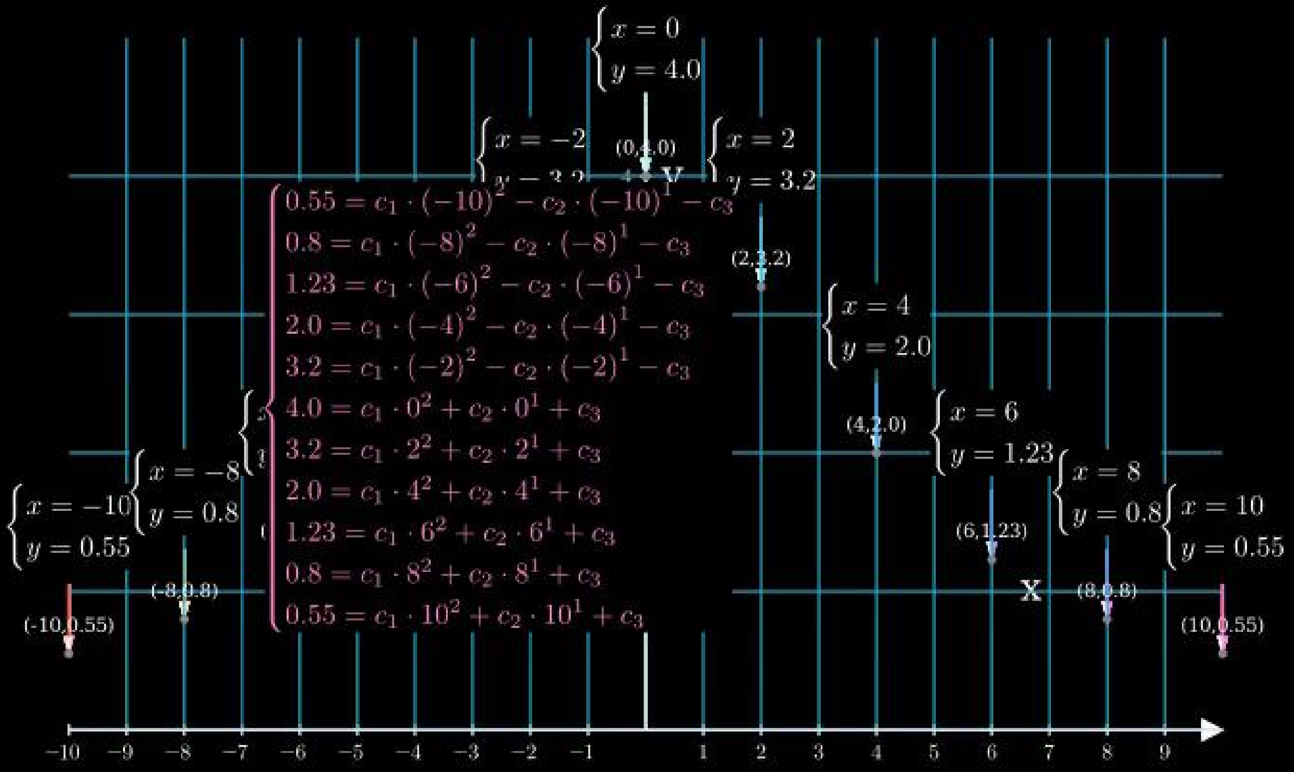

x = 0
y = 4.0
x = −2
x = 2
y = 3.2
(0,4.0)
Y
0.55 = c_1 · (−10)^2 − c_2 · (−10)^1 − c_3
0.8 = c_1 · (−8)^2 − c_2 · (−8)^1 − c_3
1.23 = c_1 · (−6)^2 − c_2 · (−6)^1 − c_3
2.0 = c_1 · (−4)^2 − c_2 · (−4)^1 − c_3
3.2 = c_1 · (−2)^2 − c_2 · (−2)^1 − c_3
4.0 = c_1 · 0^2 + c_2 · 0^1 + c_3
3.2 = c_1 · 2^2 + c_2 · 2^1 + c_3
2.0 = c_1 · 4^2 + c_2 · 4^1 + c_3
(2,3.2)
x = 4
y = 2.0
(4,2.0)
x = −8
y = 0.8
x = −10
y = 0.55
(-6,1.23)
(6,1.23)
X
(-8,0.8)
(8,0.8)
(-10,0.55)
(10,0.55)
1
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9



$x = 0$, $y = 4.0$
$x = -2$
$x = 2$, $y = 3.2$
(0,4.0)
y
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
$1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3$
$2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3$
$3.2 = c_1 \cdot (-2)^2 - c_2 \cdot (-2)^1 - c_3$
$4.0 = c_1 \cdot 0^2 + c_2 \cdot 0^1 + c_3$
$3.2 = c_1 \cdot 2^2 + c_2 \cdot 2^1 + c_3$
$2.0 = c_1 \cdot 4^2 + c_2 \cdot 4^1 + c_3$
$1.23 = c_1 \cdot 6^2 + c_2 \cdot 6^1 + c_3$
(2,3.2)
$x = 4$, $y = 2.0$
(4,2.0)
$x = 6$, $y = 1.23$
(6,1.23)
$x = -8$, $y = 0.8$
$x = -10$, $y = 0.55$
(-8,0.8)
(-10,0.55)
x
(8,0.8)
(10,0.55)
1
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9



$x = 0$
$y = 4.0$
$x = -2$
(0,4.0)
$x = 2$
$y = 3.2$
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
$1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3$
$2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3$
$3.2 = c_1 \cdot (-2)^2 - c_2 \cdot (-2)^1 - c_3$
$4.0 = c_1 \cdot 0^2 + c_2 \cdot 0^1 + c_3$
$3.2 = c_1 \cdot 2^2 + c_2 \cdot 2^1 + c_3$
$2.0 = c_1 \cdot 4^2 + c_2 \cdot 4^1 + c_3$
$1.23 = c_1 \cdot 6^2 + c_2 \cdot 6^1 + c_3$
$0.8 = c_1 \cdot 8^2 + c_2 \cdot 8^1 + c_3$
(2,3.2)
$x = 4$
$y = 2.0$
(4,2.0)
$x = 6$
$y = 1.23$
$x = 8$
$y = 0.8$
(6,1.23)
$x = -8$
$y = 0.8$
$x = -10$
$y = 0.55$
(-8,0.8)
(-10,0.55)
X
(8,0.8)
(10,0.55)
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9



$x = 0$, $y = 4.0$
(0,4.0)
$x = -2$
$x = 2$, $y = 3.2$
(2,3.2)
$x = 4$, $y = 2.0$
(4,2.0)
$x = 6$, $y = 1.23$
(6,1.23)
$x = 8$, $y = 0.8$
(8,0.8)
$x = 10$, $y = 0.55$
(10,0.55)
$x = -8$, $y = 0.8$
(-8,0.8)
$x = -10$, $y = 0.55$
(-10,0.55)
$0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3$
$0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3$
$1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3$
$2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3$
$3.2 = c_1 \cdot (-2)^2 - c_2 \cdot (-2)^1 - c_3$
$4.0 = c_1 \cdot 0^2 + c_2 \cdot 0^1 + c_3$
$3.2 = c_1 \cdot 2^2 + c_2 \cdot 2^1 + c_3$
$2.0 = c_1 \cdot 4^2 + c_2 \cdot 4^1 + c_3$
$1.23 = c_1 \cdot 6^2 + c_2 \cdot 6^1 + c_3$
$0.8 = c_1 \cdot 8^2 + c_2 \cdot 8^1 + c_3$
$0.55 = c_1 \cdot 10^2 + c_2 \cdot 10^1 + c_3$
Y
X
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9

$$\begin{cases} 0.55 = c_1 \cdot (-10)^2 - c_2 \cdot (-10)^1 - c_3 \\ 0.8 = c_1 \cdot (-8)^2 - c_2 \cdot (-8)^1 - c_3 \\ 1.23 = c_1 \cdot (-6)^2 - c_2 \cdot (-6)^1 - c_3 \\ 2.0 = c_1 \cdot (-4)^2 - c_2 \cdot (-4)^1 - c_3 \\ 3.2 = c_1 \cdot (-2)^2 - c_2 \cdot (-2)^1 - c_3 \\ 4.0 = c_1 \cdot 0^2 + c_2 \cdot 0^1 + c_3 \\ 3.2 = c_1 \cdot 2^2 + c_2 \cdot 2^1 + c_3 \\ 2.0 = c_1 \cdot 4^2 + c_2 \cdot 4^1 + c_3 \\ 1.23 = c_1 \cdot 6^2 + c_2 \cdot 6^1 + c_3 \\ 0.8 = c_1 \cdot 8^2 + c_2 \cdot 8^1 + c_3 \\ 0.55 = c_1 \cdot 10^2 + c_2 \cdot 10^1 + c_3 \end{cases}$$

$$\begin{pmatrix} (-10)^2 & (-10)^1 & 1 \\ (-8)^2 & (-8)^1 & 1 \\ (-6)^2 & (-6)^1 & 1 \\ (-4)^2 & (-4)^1 & 1 \\ (-2)^2 & (-2)^1 & 1 \\ 0^2 & 0^1 & 1 \\ 2^2 & 2^1 & 1 \\ 4^2 & 4^1 & 1 \\ 6^2 & 6^1 & 1 \\ 8^2 & 8^1 & 1 \\ 10^2 & 10^1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0.55 \\ 0.8 \\ 1.23 \\ 2.0 \\ 3.2 \\ 4.0 \\ 3.2 \\ 2.0 \\ 1.23 \\ 0.8 \\ 0.55 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} (-10)^2 & (-10)^1 & 1 \\ (-8)^2 & (-8)^1 & 1 \\ (-6)^2 & (-6)^1 & 1 \\ (-4)^2 & (-4)^1 & 1 \\ (-2)^2 & (-2)^1 & 1 \\ 0^2 & 0^1 & 1 \\ 2^2 & 2^1 & 1 \\ 4^2 & 4^1 & 1 \\ 6^2 & 6^1 & 1 \\ 8^2 & 8^1 & 1 \\ 10^2 & 10^1 & 1 \end{pmatrix}}_{\vec{A}} \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} 0.55 \\ 0.8 \\ 1.23 \\ 2.0 \\ 3.2 \\ 4.0 \\ 3.2 \\ 2.0 \\ 1.23 \\ 0.8 \\ 0.55 \end{pmatrix}}_{\vec{b}}$$

$$\underbrace{\begin{pmatrix} (-10)^2 & (-10)^1 & 1 \\ (-8)^2 & (-8)^1 & 1 \\ (-6)^2 & (-6)^1 & 1 \\ (-4)^2 & (-4)^1 & 1 \\ (-2)^2 & (-2)^1 & 1 \\ 0^2 & 0^1 & 1 \\ 2^2 & 2^1 & 1 \\ 4^2 & 4^1 & 1 \\ 6^2 & 6^1 & 1 \\ 8^2 & 8^1 & 1 \\ 10^2 & 10^1 & 1 \end{pmatrix}}_{\vec{A}} \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} 0.55 \\ 0.8 \\ 1.23 \\ 2.0 \\ 3.2 \\ 4.0 \\ 3.2 \\ 2.0 \\ 1.23 \\ 0.8 \\ 0.55 \end{pmatrix}}_{\vec{b}}$$

$$\boxed{\vec{A}\vec{x} = \vec{b}}$$

$$A^T \underbrace{\begin{pmatrix} (-10)^2 & (-10)^1 & 1 \\ (-8)^2 & (-8)^1 & 1 \\ (-6)^2 & (-6)^1 & 1 \\ (-4)^2 & (-4)^1 & 1 \\ (-2)^2 & (-2)^1 & 1 \\ 0^2 & 0^1 & 1 \\ 2^2 & 2^1 & 1 \\ 4^2 & 4^1 & 1 \\ 6^2 & 6^1 & 1 \\ 8^2 & 8^1 & 1 \\ 10^2 & 10^1 & 1 \end{pmatrix}}_{\vec{A}} \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\vec{x}} = A^T \underbrace{\begin{pmatrix} 0.55 \\ 0.8 \\ 1.23 \\ 2.0 \\ 3.2 \\ 4.0 \\ 3.2 \\ 2.0 \\ 1.23 \\ 0.8 \\ 0.55 \end{pmatrix}}_{\vec{b}}$$

$$\boxed{\vec{A}^T\vec{A}\vec{x}} = \vec{A}^T\vec{b}$$

$$\underbrace{\begin{pmatrix} (-10)^2 & (-8)^2 & (-6)^2 & (-4)^2 & (-2)^2 & 0^2 & 2^2 & 4^2 & 6^2 & 8^2 & 10^2 \\ (-10)^1 & (-8)^1 & (-6)^1 & (-4)^1 & (-2)^1 & 0^1 & 2^1 & 4^1 & 6^1 & 8^1 & 10^1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}}_{\vec{A}^T} \underbrace{\begin{pmatrix} (-10)^2 & (-10)^1 & 1 \\ (-8)^2 & (-8)^1 & 1 \\ (-6)^2 & (-6)^1 & 1 \\ (-4)^2 & (-4)^1 & 1 \\ (-2)^2 & (-2)^1 & 1 \\ 0^2 & 0^1 & 1 \\ 2^2 & 2^1 & 1 \\ 4^2 & 4^1 & 1 \\ 6^2 & 6^1 & 1 \\ 8^2 & 8^1 & 1 \\ 10^2 & 10^1 & 1 \end{pmatrix}}_{\vec{A}} \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\vec{x}}$$

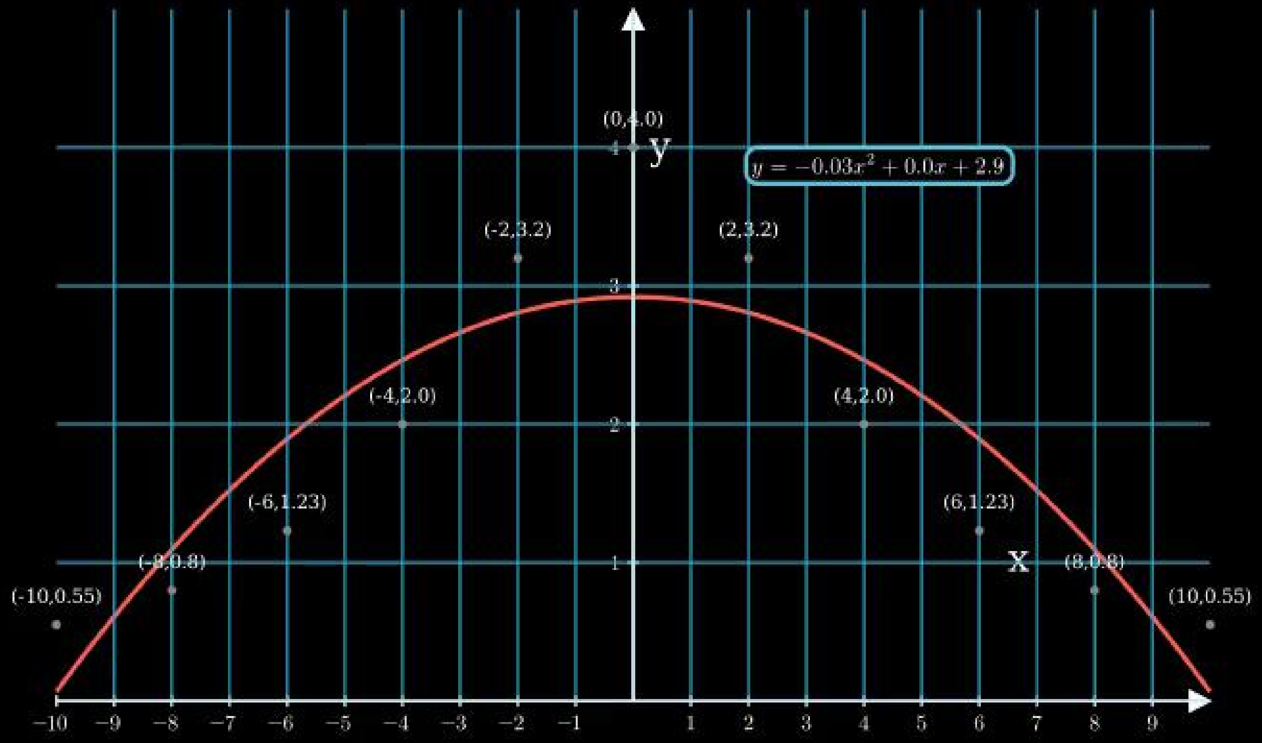
$$\vec{A}^T\vec{A}\vec{x}\boxed{=\vec{A}^T\vec{b}}$$

$$= \underbrace{\begin{pmatrix} (-10)^2 & (-8)^2 & (-6)^2 & (-4)^2 & (-2)^2 & 0^2 & 2^2 & 4^2 & 6^2 & 8^2 & 10^2 \\ (-10)^1 & (-8)^1 & (-6)^1 & (-4)^1 & (-2)^1 & 0^1 & 2^1 & 4^1 & 6^1 & 8^1 & 10^1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}}_{\vec{A}^T} \underbrace{\begin{pmatrix} 0.55 \\ 0.8 \\ 1.23 \\ 2.0 \\ 3.2 \\ 4.0 \\ 3.2 \\ 2.0 \\ 1.23 \\ 0.8 \\ 0.55 \end{pmatrix}}_{\vec{b}}$$

$$\begin{pmatrix} 11 & 0 & 440 \\ 0 & 440 & 0 \\ 440 & 0 & 31328 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 19.56 \\ 0.0 \\ 390.96 \end{pmatrix}$$

$$\begin{cases} c_1 = -0.03 \\ c_2 = 0.0 \\ c_3 = 2.92 \end{cases}$$

*värden på konstanter har avrundats till två decimaler.



y
x
$y = -0.03x^2 + 0.0x + 2.9$
(0,4.0)
(-2,3.2)
(2,3.2)
(-4,2.0)
(4,2.0)
(-6,1.23)
(6,1.23)
(-8,0.8)
(8,0.8)
(-10,0.55)
(10,0.55)
1
2
3
4
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9

$$\boxed{\vec{A}\vec{x} = \vec{b}}$$

$$\boxed{\vec{A}^T \vec{A}\vec{x} = \vec{A}^T \vec{b}}$$

Minsta kvadratmetoden

Konstruera ett system av ekvationer för att interpolera över önskat antal punkter. (se tidigare i videon) Skriv om systemet på matrisform, där $\vec{A}$ är koefficientmatris och $\vec{b}$ är högerledsmatris. Låt $\vec{x}$ representera de sökta koefficienterna för interpolationspolynomet du vill skapa. Ekvationssystemet som ska lösas ges då av:

$$\boxed{\vec{A}^T\vec{A}\vec{x} = \vec{A}^T\vec{b}}$$