

TIME: 3 HOURS

FULL MARKS: 100

1 x 10 = 10

1. Answer the following questions.

- (a) Find the number of all possible matrices of order  $3 \times 3$  with each entry 0 or 1. 2<sup>9</sup>
- (b) What do you mean by diagonal of a matrix.
- (c) If a matrix has 24 elements, what are the possible orders it can have?
- (d) Define continuity of the function at a particular point.
- (e) Find the derivative of  $\sin(x^0)$  with respect to  $x$ .
- (f) Find the antiderivative of  $e^{3x}$  by the method of inspection.
- (g)  $\int \frac{dx}{e^{2x}} = ?$
- (h) What are the direction cosines of X axis, Y axis and Z axis?
- (i) When  $g(x) = h^x$  is called exponential function?
- (j) Let A and B are two events such that  $P(A) \neq 0$ . Find  $P(B/A)$  if

(a) A is a subset of B

(b)  $A \cap B = \Phi$ 

2. (a) Construct a  $3 \times 4$  matrix, whose elements are given by  $a_{ij} = \frac{1}{2} | -3i + j |$  2

- (b) Construct a  $3 \times 2$  matrix, whose elements are given by  $a_{ij} = \frac{1}{2} (i - 3j)^2$  2

3. Find the value of a, b, c, d from the given equations

2+2=4

$$a) \begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

$$b) \begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

4. Find the values of x, y, z and w if

2+2=4

$$a) 3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$b) \text{ Find X and Y if } 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{ and } 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

5. Show that  $f(x) = 2x - |x|$  is continuous at  $x = 0$ .

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$$\begin{array}{r} 15 \\ 24 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 20 \\ 22 \\ \hline 42 \end{array}$$

$$z = \tan x$$

$$dz = \sec^2 x dx$$

6. Find the points at which the function  $f(x) = \frac{3x+7}{x^2-5x+6}$  is continuous. 4
7. Find  $\frac{dy}{dx}$  (i)  $y = 2\sqrt{\cot x^2}$  (ii)  $y = x^a + a^x + a^a$ , for some fixed  $a > 0$  2 + 2 = 4
8. Find the value of  $\int \frac{1}{1+\tan x} dx$  4
9. Find the value of  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$  4
10. Find the value of (i)  $\int \frac{\sin \theta}{\sin(\theta+a)} d\theta$  (ii)  $\int \frac{2-3\sin x}{\cos^2 x} dx$  2 + 2 = 4
11. Find the value of (i)  $\int \tan x dx$  (ii)  $\int \sec x dx$  2 + 2 = 4
12. For any three vectors  $\vec{a}, \vec{b}, \vec{c}$  prove that  $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$  4
13. A couple has two children, 2+2=4
- (a) Find the probability that both children are males, if it is known that at least one of the children is male.
- (b) Find the probability that both children are females, if it is known that the elder child is a female.
14. (a) If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , for  $-1 < x < 1$ , prove that  $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$  4

$$\frac{P(E' \cap F) + P(E \cap F)}{P(F)}$$

- (b) Find  $\frac{dy}{dx}$  if  $y = \sqrt{e^{\sqrt{5x+1}}}$  2

$$= \frac{P}{P(F)}$$

15. Find the value of (i)  $\int \frac{e^{2x}-1}{e^{2x}+1} dx$  (ii)  $\int \cot x \log \sin x dx$  3+3=6
16. Establishes the relation between direction cosine of a line. 6
17. Find the value of a)  $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$  b)  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$  4 + 2 = 6
18. Find  $\frac{dy}{dx}$  (i)  $y = \cos x^3 \cdot \sin^2 x^5$  (ii)  $\sin^2 y + \cos x \cdot y = a$  4 + 2 = 6
19. (a) Let E and F are two events associated with a random experiment. Define  $P(E/F)$ . Find  $P(E/F)$  when  $P(F)=0$ . Write multiplication rule of probability for three events E, F and G associated with a random experiment 3
- (b) Given that two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4. 3

$$\frac{P(E' \cap F)}{P(F)}$$

$$1 - \frac{P(E \cap F)}{P(F)}$$

20. Let E and F are two events associated with a sample space S. Prove that

a)  $P(S/F) = 1$

b)  $P((A \cup B)/F) = P(A/F) + P(B/F) - P((A \cap B)/F)$

c)  $P(E'/F) = 1 - P(E/F)$

$$= \frac{P((1-E) \cap F)}{P(F)} = \frac{P(E' \cap F)}{P(F)}$$

$E' = 1 - E$

$$\cancel{2-5x+6=0}$$

$$\cancel{x^2-6x+x+6=0}$$

$$\cancel{x(x-6)}$$

$$1 + 3 + 2 = 6$$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

$$(x-3)(x-2) = 0$$

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$x = 2, 3$