TEST 2 EXAMINATION 2024

SUB: MATHEMATICS

YEAR: H.S 2^{ND} YEAR

TIME: 3 HOURS | FULL MARKS: 100

1. Answer the following questions. **1 x 10 = 10**

(a) What do you mean by skew symmetric matrix? Give an example.

(b) Give an example of two non-zero and non-identity matrix which satisfies the commutative law.

(c) Where is the function $f(x) = \sec x$ is discontinuous ?

(d) Differentiate $\sin x^2$ w.r.t x^2

$\frac{\sin x \cos a + \cos x \sin a}{\sin x \cos a - \cos x \sin a}$

(e) $\sin|x|$ is a continuous function at x belongs to

(a) R^+ (b) R^- (c) R (d) $R - \{0\}$

(f) If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$, such that $f(1) = 0$. Then find $f(x)$.

(g) If $\vec{a} = 5\hat{\imath} - 2\hat{\jmath} + \sqrt{7}\,\hat{k}$ then find the direction cosines of $\vec{a}$.

$m \times P = ~~P \times P~~ \; P \times K$

(h $\int \frac{dx}{\sqrt{a^2-x^2}}$ =?

3×2

2×4

(i) If $\vec{p}$ is a unit vector and $(\vec{p} - \vec{q}).(\vec{p} + \vec{q}) = 8$,then find $|\vec{q}|$.

(j) A pair of dice is rolled. Find the probability of obtaining an even prime number on each die.

2. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$ then show that $(AB)C = A(BC)$.

$\log|x+\sqrt{x^2+a^2}|$

3. Find the values of x,y,z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$. 4

$\log|$

4. Find "a" and "b" so that the function f is continuous $f(x)= \begin{cases} 1 & ,x \le 3 \\ ax + b & ,3 < x < 5 \\ 7 & ,x \ge 5 \end{cases}$ 4

OR

Find the value of k if $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & , -1 \le x < 0 \\ \frac{2x+1}{x-1} & , 0 \le x \le 1 \end{cases}$ is continuous at x=0 4

$\frac{x}{a} =$

5. Determine if f defined by $f(x)= \begin{cases} x^2 \sin\frac{1}{x} & , \text{if } x \ne 0 \\ 0 & , \text{if } x = 0 \end{cases}$ is a continuous function? 4

6. If $\cos y = x\cos(a + y)$, with $\cos a \ne 1$,prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ 4

7. Evaluate $\int \frac{dx}{\sqrt{x^2+a^2}}$ 4

8. Evaluate , $\int \cos 2x \cos 4x \cos 6x \, dx$ **OR** $\int \frac{dx}{\cos(x-a)\cos(x-b)}$ 4

9. Evaluate $\int \frac{\sin(x+a)}{\sin(x-a)} dx$

$\frac{1}{a}\tan^{-1}\frac{x}{a}$

10. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$.

11. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively, in the ratio 2 : 1

(i) internally (ii) externally **2+2=4**

12. In a hostel, 60% of the students read Hindi newspaper,40% read English newspaper and 20% read both Hindi and English newspapers .A student is selected at random.

(i) Find the probability that she reads neither Hindi nor English newspapers. **2**

(ii) If she reads English newspaper, find the probability that she reads Hindi newspaper. **2**

OR

A person has undertaken a construction job .The probabilities are 0.65 that there will be strike,0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time. **4**

13. Of the students in a college ,it is known that 60% reside in hostel and 40% are day scholars(not residing in hostel).Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade ,what is the probability that the student is a hostlier ? **4**

65
32
130
195
2080

35
80 4
00
280

14. If $A = \begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$ **6**

15. For any square matrix A with real number entries ,show that $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **3 + 3 = 6**

16. (i) If $y = \frac{\log(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}$, then prove that $(1 + x^2)\frac{dy}{dx} + xy = 1$ **4**

(ii) Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$ **2**

17. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

OR

Find the maximum and minimum values of

(i) $f(x) = x^5 - 5x^4 + 5x^3 - 1$, $-4 \le x \le 4$ **4**

(ii) $f(x) = 3\sin x + 5$ **2**

$(1+x^2)\frac{dy}{dx} = 1 - xy$

$\frac{dy}{dx} = \frac{1-xy}{1+x^2}$

18. (i) $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ (ii) $\int \sin^3 x \cos^2 x\, dx$ **4+2=6**

19. (i) If $\vec{p}, \vec{q}$ and $\vec{r}$ are mutually perpendicular vectors of equal magnitudes ,show that the vector $\vec{p} + \vec{q} + \vec{r}$ is equally inclined to $\vec{p}, \vec{q}$ and $\vec{r}$. **4**

(ii) If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}$-$\vec{b}$+$3\vec{c}$ **2**

20.(i) If $\hat{\imath} + \hat{\jmath} + \hat{k}, 2\hat{\imath} + 5\hat{\jmath}, 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\hat{\imath} - 6\hat{\jmath} - \hat{k}$ are the position vectors of points A,B,C and D respectively, then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

(ii) Show that the points $A(1,2,7), B(2,6,3)$ and $C(3,10,-1)$ are collinear. **4 + 2 = 6**

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