PRE FINAL -2 EXAMINATION 2024

SUB: MATHEMATICS

YEAR: H.S 2ND YEAR

TIME: 3 HOURS FULL MARKS: 100

1. **Answer the following questions.** $1 \times 10 = 10$

(a) Let A={x:1<x<10,x is an odd natural number} and B={y:90<y<100,y is a prime number}

Write the number of relations from A to B.

(b) If the function f={(1,5),(2,6),(3,4)} from the set A={1,2,3} to the set B is invertible,Find the set B

(c) What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}(\sin\frac{2\pi}{3})$

(d) Let A be a skew symmetric matrix of order n. Write down the maximum number of non - zero elements in A.

(e) If A is a 3 x 3 matrix and $|A| = 4$, then the value of $|adj\ A|$.

(f) Find the degree and order of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = (c\frac{d^2y}{dx^2})^{\frac{1}{3}}$

(g) How do you explain $\int_a^b y\,dx$ geometrically.

(h) Find the component of $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ along $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

(i) If a line makes angles α, β and γ with the coordinate axes , find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

(j) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ then what is the value of $\cos(\frac{\theta}{2})$

2. Show that the relation R in $\mathbb{R}$ defined by $R = \{(a, b): a \leq b\}$ is reflexive and transitive but not symmetric. 4

Or

Let $f: N \rightarrow N$ is defined by 4

$$f(n) = \begin{cases} \frac{n+1}{2}; if\ n\ is\ odd \\ \frac{n}{2}; if\ n\ is\ even. \end{cases}$$ For all n∈ N.

Examine if f is bijective.

3. Prove that $cot^{-1}\left\{\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right\} = \frac{x}{2}, x \in \left(0,\frac{\pi}{4}\right)$ $cos2$ 4

Or

Draw the graph of $\sec^{-1} x$ 4

4. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x).F(y) = F(x+y)$ 4

Or

Prove that any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix. 4

5. Prove that if a function f is differentiable at a point C, then it is also continuous at that point. 4

Or

Determine if $f(x)$ defined by $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0. \end{cases}$ is a continuous function at x=0 ?

6. Using parametric form find $\frac{dy}{dx}$ if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$ 4

Or

A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 cm away from the wall ? 4

7. Evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ OR $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ 4

8. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx, a > 0$. Hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x . \cos x} dx$ 2+2=4

Or

(i) Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$ if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$ 2

$\cos 2x = 1 - 2\sin^2 x$

(ii) Find the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$ 2

9. In a bank principal increases continuously at the rate of 5 % per year. In how many years Rs 1000 double itself? 4

Or

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinate of the point. 4

10. Solve the differential equation $2ye^{x/y}dx + \left(y - 2xe^{x/y}\right)dy = 0, where\ x = 0\ and\ y = 1.$

Or

Solve the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$ 4

11. Prove that $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2(\vec{b} \times \vec{a})$. Find the area of parallelogram whose diagonals are the vector $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ $2 + 2 = 4$

12. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1,2,3 or 4, she tosses a coin once and notes whether a head or tail is obtained . If she obtained exactly one head, What is the probability that she threw 1,2,3 or 4 with the die? 4

OR

A and B throw a die alternatively till one of them gets a 6 and wins the game. Find their respective probabilities of winning, if A starts first.

13. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that "the die shows a number greater than 4" given that "there is at least one tail." 4

Or

An electronic assembly consists of two subsystems, say A and B. From previous testing procedures, the following probabilities are assumed to be known:

P(A fails)=0.2 P(B fails alone)=0.15 P(A and B fail)=0.15

Evaluate (i)P(A fails alone) (ii)P(A fails/B has failed)

6

14. If $A = \begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$ 6

Or

(i) Prove that - " Inverse of a square matrix , if it exists , is unique" 3 + 3 =6

(ii) If A and B are invertible matrices of the same order , then prove that $(AB)^{-1}=B^{-1}A^{-1}$

15. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1}. 6

Or

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

16. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$. 6

OR

Find the intervals the function $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$ is 6

(a)Strictly increasing (b) Strictly decreasing

17. Find the area of the region bounded by the triangle whose vertices are (1,0) (2,2) and (3,1) 6

OR

Find the area bounded by the curve $y = |x|x$, x-axis and $x = -1$ and $x = 1$ 6

18. (i) Show that the points A (1, −2, −8), B (5, 0, −2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC. 2+2=4

(ii) Find the values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{\imath} + 4x\hat{\jmath} + \hat{k}$ and $\vec{b} = 7\hat{\imath} - 2\hat{\jmath} + x\hat{k}$ is obtuse 2

19. Find the vector and Cartesian equation of the line passing through the point (1,2,-4) and perpendicular to the lines $\frac{7x-56}{21} = \frac{19+y}{-16} = \frac{10-z}{-7}$ and $\frac{15-x}{-3} = \frac{y-29}{8} = \frac{5-z}{-5}$ 6

Or

Find the relation between direction cosines of a lines 6

20. Solve the L.P.P graphically 6

Maximize $Z = 8000x + 12000y$

Subject to $9x + 12y \leq 180$ $3x + 4y \leq 60$ $x + 3y \leq 30$ $x, y \geq 0$

OR

Minimise $Z = 4x + 6y$

Subject to $4x + 3y \geq 100$ $3x + 6y \geq 80$ $x, y \geq 0$

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