PRE FINAL 1 EXAMINATION -2024

HS SECOND YEAR

SUB: MATHEMATICS

TIME: 3 HOURS **FULL MARKS: 100**

33
16
49

1.Answer the following questions 1 X 10 = 10

(i) Relation R in the set $A = \{1,2,3\}$ defined as $R = \{(1,2)\}$.Check whether R is transitive. Justify your answer.

(ii) What is the range of the principal branch of $cosec^{-1}x$ defined on the domain $R - (-1,1)$

9
16
24

(iii) "Diagonal elements of a skew symmetric matrix are always zero"- why ?

(iv) Let A be a matrix of order 3 such that $|A| = -9$. Find the value of $|-3A^{-1}|$

(v) Find the sum, if elements of a row (or column) are multiplied with cofactors of any other row(or column).

(vi) Differentiate sin^2x w. r. t e^{sinx}

(vii) $\int_{-1}^{1} sin^5xcos^4xdx = ?$

(viii) The number of arbitrary constants in the particular solution of a differential equation of third order is/are?

$\vec{a}\cdot\frac{\hat{i}}{|\hat{i}|} = 3$

(ix) The projection of a line on the axes are 3, 4 and $2\sqrt{6}$. Find the length of the line.

$\vec{a}\cdot\frac{\hat{j}}{|\hat{j}|} = 4$

(x) Find the unit vector in the direction of the vector $\vec{a} + \vec{b}$ where $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$

2. Show that the relation R in the set $A = \{x \in Z: 0 \le x \le 12\}$, given by $R = \{(a,b): |a-b| \text{ is a multiple of } 4\}$ is an equivalence relation. 4

$\vec{a}\cdot\frac{\hat{k}}{|\hat{k}|} = 2\sqrt{}$

Or

Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2): T_1 \text{is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4,5, T_2 with sides 5,12,13 and T_3 with sides 6,8,10. Which triangles among T_1, T_2 are T_3 are related? 4

3. Solve $tan^{-1}2x + tan^{-1}3x = \frac{\pi}{4}$ 4

Or

Prove that $3sin^{-1}x = sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ 4

4. Find the value of X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$ 4

Or

Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants and find k if $D(k, 0)$ is a point such that area of triangle ABD is 3sq units. 4

5. Find the relationship between a and b so that the function f defined by 4

$$f(x) = \begin{cases} ax + 1, if\ x \le 3 \\ bx + 3, if\ x > 3 \end{cases}$$

Is continuous at x = 3.

$-1/2 < x < 1/2$

$-1/2 < sin\theta < 1/2$

OR

For what values of a and b, the function defined as 4

$-\pi/6 < \theta < \pi/6$

$$f(x) = \begin{cases} 5, \text{ if } x \le 2 \\ ax + b, \text{ if } 2 < x < 10 \\ 21, \text{ if } x \ge 10 \end{cases}$$

is a continuous function.

$\frac{d e^{sinx}}{dx} = e^{sinx}\cos x \quad \frac{d\, sin^2x}{dx} = 2 sinx \cos x$

6. Differentiate the following $x^y + y^x = 1$ 4

OR

If $y = 3\cos(logx) + 4\sin(logx)$, Show that $x^2y_2 + xy_1 + y = 0$ 4

7. Answer the following question (any one) 4

(a) $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx.$ (b) $\int (sin^{-1}x)^2dx.$

8. Answer the following question (any one) 4

(a) $\int_0^{\pi/4} \log(1 + tanx)\, dx$ (b) $\int \sqrt{x^2 + a^2}dx$

9. Solve (any one) 4

(a) $e^x tanydx + (1 - e^x)sec^2ydy = 0$ (b) $x\frac{dy}{dx} + 2y = x^2logx$

10. Solve the differential equation $(x^2 + xy)dy = (x^2 + y^2)dx$ 4

Or

Find the equation of a curve passing through the point (0,1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x coordinate and the product of the x coordinate and y coordinate of that point. 4

11. Show that the vector of magnitude $\sqrt{51}$ which makes equal angles with the vectors $\vec{a} = \frac{1}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$, $\vec{b} = \frac{1}{5}(-4\hat{\jmath} - 3\hat{k})$ and $\vec{c} = \hat{\jmath}$ is $-5\hat{\imath} + \hat{\jmath} + 5\hat{k}$ 4

12. Find both the maximum and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0,3]$

13. A die is thrown three times. Events A and B are defined as below:

A : 4 on the third throw

B : 6 on the first and 5 on the second throw.

Find the probability of A given that B has already occurred. 4

Or

Write the theorem of total probability. Using multiplication rule of probability prove the theorem of total probability. 4

14. Answer any one of the followings

(a) (i) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$ 4

(ii) If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix. 2

Or

(b) (i) What do you mean by singular and non- singular matrices. 2

(ii) The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method. 4

15. A square piece of tin of side 18cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box.What should be the side of the square to be cut off so that the volume of the box is the maximum possible . 6

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. 6

16. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line x = a, find the value of a.

Or

Find the area of the region bounded by the line $y = 3x + 2, x - axis$ and the ordinates $x = -1$ and $x = 1$. 6

17. Answer the followings

(a) (i) State and proof triangular inequality. 4

(ii) Find the vector area of the triangle having the points $A(1,1,1), B(1,2,3)$ and $C(2,3,1)$ as its vertices. 2

18. Find the Cartesian equation of a line passing through a given point and parallel to a given line

19. Solve the following Linear Programming Problems graphically (any one) 6

(a) Maximise and Minimise $Z = x + 2y$

Subject to the constraints :

$x + 2y \geq 100\,,\ 2x - y \leq 0\,, 2x + y \leq 200\,, x \geq 0, y \geq 0$

(b) Minimise $Z = 3x + 5y$ 6

Subject to the constraints :

$x + 3y \geq 3\,,\ x + y \geq 2\,, x \geq 0, y \geq 0$

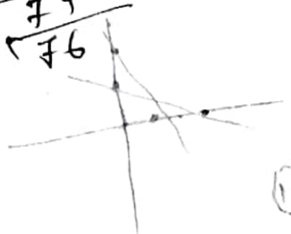
20. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by trains, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train? 6

Or

Assume that the chance of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga? 6

$p(T) = \frac{3}{10}$

$\frac{64}{45}$ / 109



$(x^2 + xy)\,dy = x^2 dx + y^2 dx$

$\frac{x^2+y^2}{x^2+xy}\,dx$

$\vec{a} + \vec{b} \geq \vec{a}$