

For a random variable X , if $\log X$ (natural logarithm, i.e., base e) follows a normal distribution, then we say that it follows a Lognormal distribution. In other words, we say that X follows a lognormal distribution with parameters μ (a real number connected to the location of the distribution) and σ (a positive number connected to the spread of the distribution), if $\log X \sim N(\mu, \sigma^2)$. Therefore, the cumulative distribution function (CDF) for a lognormal distribution can be written as

$$F(t) = P(X \leq t) = P(\log X \leq \log t) = \Phi\left(\frac{\log t - \mu}{\sigma}\right)$$

with Φ indicating the CDF of a standard normal distribution.

Mr. Aberforth Dumbledore has been studying the concepts of the lognormal distribution and has learnt that the population mean of this distribution is $\exp\left(\mu + \frac{\sigma^2}{2}\right)$, and the population variance is $(\exp(\sigma^2) - 1) \exp(2\mu + \sigma^2)$. He has also got to know that the probability density function (PDF) of lognormal distribution is given by (defined for positive values of x)

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x - \mu)^2}{2\sigma^2}\right).$$

For better understanding of the applicability of lognormal distribution, Mr. Aberforth has collected data from 400 random households from the village of Godric's Hollow and wants to utilise a lognormal distribution for modelling the monthly income (in lakhs of sickles). He knows from reputable sources that for households in Godric's Hollow, the true value of the parameter σ in the lognormal distribution (equivalently, standard deviation of log-income) can be taken as 0.2, but the parameter μ (i.e., mean of log-income) is unknown. Detailed descriptive statistics of the data on monthly income (lakhs of sickles) are shown below.

Mean	2.46
Standard Error	0.07
Median	1.65
Mode	1.04
Standard Deviation	2.36
Sample Variance	5.56
Kurtosis	16.79
Skewness	3.24
Range	24.61
Minimum	0.18
Maximum	24.79
Sum	2455.52
Count	400