IIIT-H

VIRTUAL LABS

Manual for Reflection and Refraction of Waves

Introduction:

We have discussed in detail about the waves propagation in various type of medium i.e. lossy, lossless, conductor and insulator and the various factor associated with it like wave number, propagation constant etc. Now, its time to study about the reflection of the plane wave at oblique incidence and later we will discuss about the special case of normal incidence.

Reflection

When a plane wave encounters a change in medium, some or all of it may propagate into the new medium or be reflected from it. The part that enters the new medium is called the transmitted portion and the other the reflected portion. The part which is reflected has a very simple rule governing its behavior. Make the following construction:

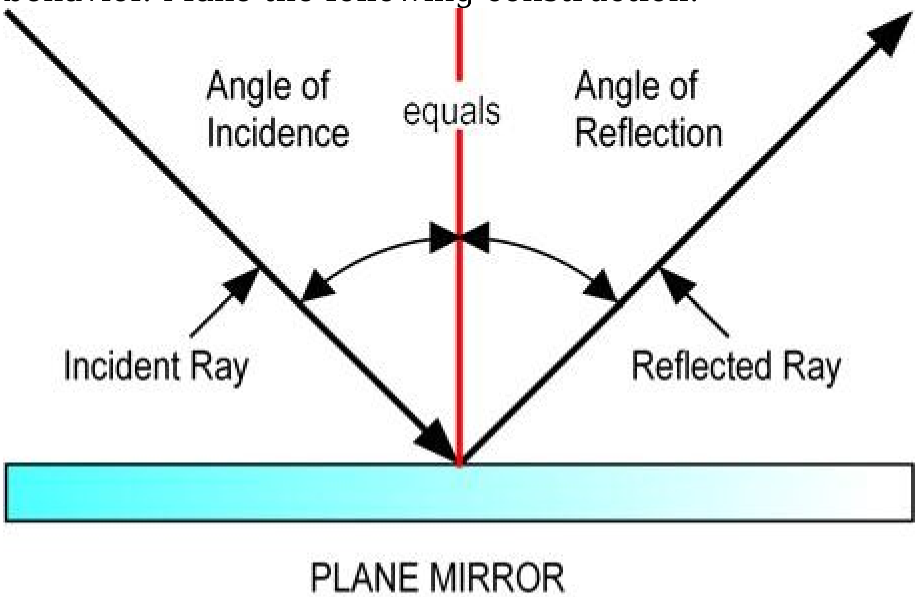


Fig 1. Reflection of a plave wave from the mirror

Angle of Incidence = the angle between the direction of propagation and a line perpendicular to the boundary, on the same side of the surface. Angle of Reflection = the angle between the direction of propagation of the reflected wave and a line perpendicular to the boundary, also on the same side of the surface.

Law of reflection:

If the reflecting surface is very smooth, the reflection of light that occurs is called specular or regular reflection. The laws of reflection are as follows:
1. The incident ray, the reflected ray and the normal to the reflection surface at the point of the incidence lie in the same plane.
2. The angle which the incident ray makes with the normal is equal to the angle which the reflected ray makes to the same normal.

Objectives:

The main objectives of this experiment are the following:

1. To find out how a wave travels from one material to another.

2. To find out how a parallel polarized wave gets reflected and transmitted at boundary.
3. To find out how a perpendicular polarized wave gets reflected and transmitted at boundary.
4. To calculate and observe the critical angle in both the cases.

Theory:

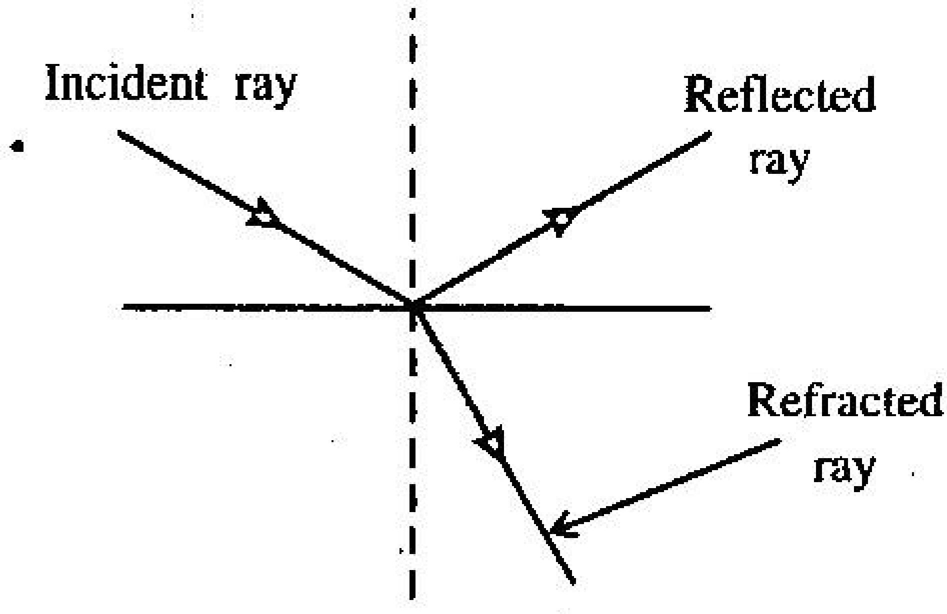
We consider the reflection of the plane wave on the lossless media. The general form of the uniform wave plane is

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_o \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$
$$= \text{Re}\,[E_o e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)}]$$

Where r is the radius vector and k is the wave number or the propagation vector and is always in the direction of the propagation of wave. The magnitude of k is related to "ω" according to the dispersion relation .

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu\varepsilon$$

As E,H and k are mutually orthogonal, while E and H lies on the same plane.



Considering the reflection of the EM wave as shown in the figure above, both the incident and the reflection ray are in medium 1 while the refracted wave is in medium 2.

$$\mathbf{E}_i = \mathbf{E}_{io} \cos (k_{ix}x + k_{iy}y + k_{iz}z - \omega_i t)$$

$$\mathbf{E}_r = \mathbf{E}_{ro} \cos (k_{rx}x + k_{ry}y + k_{rz}z - \omega_r t)$$

$$\mathbf{E}_t = \mathbf{E}_{to} \cos (k_{tx}x + k_{ty}y + k_{tz}z - \omega_t t)$$

Since the tangential component of E must be; continuous at the: boundary z = 0.

$$\mathbf{E}_i(z = 0) + \mathbf{E}_r(z = 0) = \mathbf{E}_t(z = 0)$$

The only way this boundary condition will be satisfied by the waves :

1. $\omega_i = \omega_r = \omega_t = \omega$
2. $k_{ix} = k_{rx} = k_{tx} = k_x$
3. $k_{iy} = k_{ry} = k_{ty} = k_y$

1 implies that the frequency is unchanged. Conditions 2 and 3 require that the tangential components of the propagation vectors be continuous. This means that the propagation vectors ki, kt, and kr must all lie in the plane of incidence. Thus, by conditions 2 and 3,

$$k_i \sin \theta_i = k_r \sin \theta_r$$

$$k_i \sin \theta_i = k_t \sin \theta_t$$

In case of loseless medium,

$$k_i = k_r = \beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_t = \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

Thus, it is proved that angel of incidence is equal to angle of reflection. Also, from the above equation, we can say that

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{k_i}{k_t} = \frac{\mu_2}{\mu_1} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$$

or $n_1 \sin \theta_i = n_2 \sin \theta_t$

This is called as snell's law where n1 and n2 are the refractive index of the medium 1 and 2 respectively. Two special cases are considered, one in which E is perpendicular to the plane of incidence and the other in which it is parallel to the plane of incidence.

1) **Parallel Polarization**

In Parallel polarization E lies in the plane of incidence. In medium 1

$$\mathbf{E}_{is} = E_{io}(\cos\theta_i\, \mathbf{a}_x - \sin\theta_i\, \mathbf{a}_z)\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\mathbf{H}_{is} = \frac{E_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\, \mathbf{a}_y$$

$$\mathbf{E}_{rs} = E_{ro}(\cos\theta_r\, \mathbf{a}_x + \sin\theta_r\, \mathbf{a}_z)\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\mathbf{H}_{rs} = -\frac{E_{ro}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\, \mathbf{a}_y$$

and in medium 2 the transmitted field is given by,

$$\mathbf{E}_{ts} = E_{to}(\cos\theta_t\, \mathbf{a}_x - \sin\theta_t\, \mathbf{a}_z)\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\mathbf{H}_{ts} = \frac{E_{to}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\, \mathbf{a}_y$$

As angle of incidence is equal to angle of reflection and tangential component of E and H are continous at boundary z=0, so

$$(E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to}$$

Expressing Ero and Eto in terms of Eio , we obtain

$$\Gamma_{\|} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

and

$$\tau_{\|} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Thus, we can say that

$$E_{ro} = \Gamma_{\|} E_{io}$$

Using simple trigonometric conversion, it can be shown that

$$1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

Brewster angle

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B_\parallel}$$

Solving the above equation with trigonometric manipulations, we will get

$$\tan\theta_{B_\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

the following result

2) **Perpendicular Polarization**

In this, E is perpendicular to the plane of incidence or H is parallel to the plane of incidence

$$\mathbf{E}_{is} = E_{io}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\,\mathbf{a}_y$$

$$\mathbf{H}_{is} = \frac{E_{io}}{\eta_1}(-\cos\theta_i\,\mathbf{a}_x + \sin\theta_i\,\mathbf{a}_z)\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\mathbf{E}_{rs} = E_{ro}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\,\mathbf{a}_y$$

$$\mathbf{H}_{rs} = \frac{E_{ro}}{\eta_1}(\cos\theta_r\,\mathbf{a}_x + \sin\theta_r\,\mathbf{a}_z)\,e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

while the transmitted field in medium 2 are given by :

$$\mathbf{E}_{ts} = E_{to}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\,\mathbf{a}_y$$

$$\mathbf{H}_{ts} = \frac{E_{to}}{\eta^{2}}(-\cos\theta_t\,\mathbf{a}_x + \sin\theta_t\,\mathbf{a}_z)\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

In similar way as that of parallel polarization, we can derive other equations also

$$\Gamma_{\perp} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

and

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Fresnal's equation for the perpendicular polarization is $E_{ro} = \Gamma_\perp E_{io}$

It can be easily shown that $1 + \Gamma_\perp = \tau_\perp$

and $\tan\theta_{B_\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$

Procedure:

This experiment consists of two stages and each stage will teach you a new concept.

The experiment was designed in a way, so that you can quickly change the parameters and observe the results. This makes you to have a more clear picture of the concepts.

Start the experiment by pressing *start* button

- **STAGE 1**:
    1. In this experiment, you are going to learn about how a wave travels from one material to another. In the first stage, you are going to learn about how a parallel polarized wave gets reflected and transmitted at boundary.
    2. The incident wave contains electric and magnetic fields (shown in red and yellow colors). Now, if you click on the screen, incident wave starts and hits the boundary (at the center of screen). Here you need to observe the direction of electric and magnetic fields in the reflectes wave and the refracted wave with respect to incident wave.
    3. You can use the slider "change camera view" (at the bottom of window) to zoom in and zoom out. At the bottom of experiment window, you have slider for each parameter (like Î¼1, Î¼2, Îµ1, etc) So, using those parameters you can define the medium in which wave travels.
    4. See the output parameters like Amplitude of reflected and refracted wave. Try to calculate critical angle (using snell's law) and cross check your answers. Do this with various values of parameters and write a report of your observations. After completing this, move on to the next stage by clicking on "Next" button provided at the top of the experiment window.

- <u>**STAGE 2**</u>:
    1. This stage is not different from previous stage. In the previous stage you will learn about parallel polarization. In this stage, you will learn about perpendicular polarization.(Observe the change in planes of electric field and magnetic field)
    2. Perform the experiment as you have done in stage 1, and write a report of your observations.
    3. At any point of time, you can move on to the previous stage by clicking on "Back" button provided at the top of the experiment window.