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VIRTUAL LABS

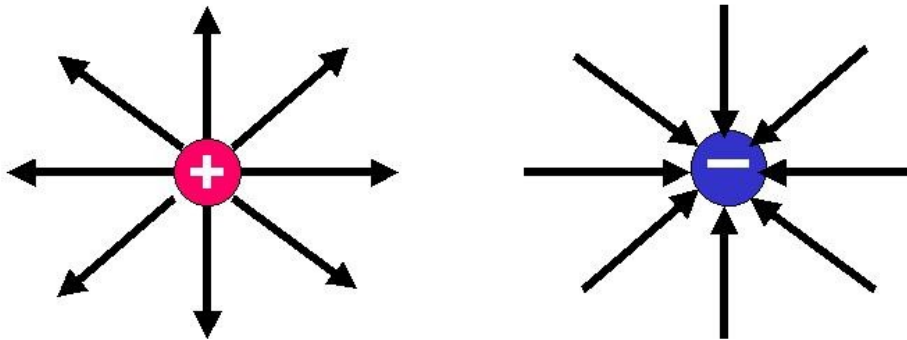
Manual for Electrostatic Field I

(Electrostatic Fields for point charge, infinite line charge and finite line charge)

## **Introduction:**

In 1<sup>st</sup> experiment using Coulomb's law we have seen force due to point charges on other. In this experiment we will estimate electric fields due to various charge distributions.

Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge. The electric field is radially outward from a positive charge and radially inward toward a negative point charge.

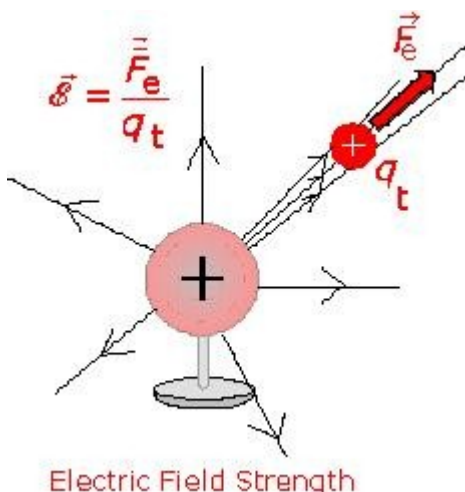


$$\mathbf{E} = \frac{\mathbf{F}}{q} \text{ where,}$$

$\mathbf{F}$  is the electric force experienced by the particle

$q$  is its charge

$\mathbf{E}$  is the electric field wherein the particle is located.



In the above figure  $q_t$  is the unit test charge to evaluate the Electric Field.

$\vec{F}_e$  is the force on  $q_t$  due to the charge at centre.

Now let us evaluate the electric field due to point charge and uniformly distributed infinite and finite line charges ( $\rho_L$  C/m) .

## **Objectives:**

The main objectives of this experiment are the following:

1. To measure the electric field due to a point charge at any point in the space. Plot the graph of electric field as a function of distance and charge. Also observe the distribution of electric field as a function of distance.
2. To measure the electric field due to a uniformly distributed infinite line charge at any point in the space. Plot the graph of electric field as a function of distance and line charge density. Observe the difference between the plots of point charge. Also observe the distribution of electric field as a function of distance.
3. To measure the electric field due to a uniformly distributed finite line charge at any point in the space. Plot the graph of electric field as a function of distance and line charge density. Observe the difference between the plots of infinite line charge.

## **Theory:**

### 1. Electric Field due to a Point Charge

Based on Coulomb's Law, for interacting point charges, the contribution to the E-field at a point in space due to a single, discrete charge located at another point in space is given by the following

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

where

q is the charge of the particle creating the electric force,

r is the distance from the particle with charge q to the E-field evaluation point,



$\hat{\mathbf{r}}$  is the unit vector pointing from the particle with charge q to the E-field evaluation point,

$\epsilon$  is the electric constant.

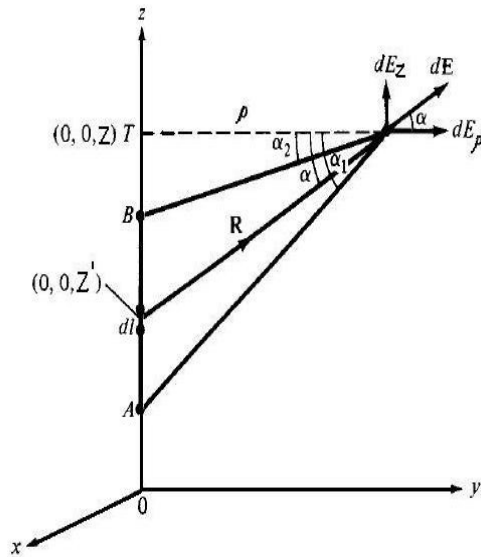
The total E-field due to a quantity of point charges,  $n_q$ , is simply the superposition of the contribution of each individual point charge.

$$\mathbf{E} = \sum_{i=1}^{n_q} \mathbf{E}_i = \sum_{i=1}^{n_q} \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

### 2. Electric field due to Line Charge

Consider a line charge element dQ with uniform charge density  $\rho_L$  extending from A to B along z-axis. The charge element dQ associated with the element  $dl = dz$  of the line is  $dQ = \rho_L dl$ . Hence the total charge is  $Q = \int \rho_L dl$ . (line

charge)



The electric field at any arbit point can be given by :

$$\mathbf{E} = \int \frac{\rho_l d\mathbf{l}}{4\pi\epsilon_0 r^2}$$

Let the field point be  $(x,y,z)$  and source point be  $(x',y',z')$   
 $dl=dz'$

$$\mathbf{R} = (x,y,z)-(0,0,z')$$

$$\mathbf{R} = x\mathbf{a}_x + y\mathbf{a}_y + (z-z')\mathbf{a}_z$$

$$\mathbf{R} = \rho\mathbf{a}_\rho + (z-z')\mathbf{a}_z$$

$$R^2=|\mathbf{R}|^2,$$

$$\mathbf{a}_R/R^2 = \mathbf{R}/|\mathbf{R}|^3$$

Hence

$$\mathbf{E} = \frac{-\rho}{4\pi\epsilon_0} \left( \int \rho \sec^2 \alpha [\cos \alpha \mathbf{a}_\rho + \sin \alpha \mathbf{a}_z] \frac{d\alpha}{\rho^2 \sec^2 \alpha} \right)$$

Thus for finite line charge :

$$\mathbf{E} = (\rho_L/4\pi\epsilon_0\rho)[-(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_z]$$

So for an infinite line charge, point B is at  $(0,0,\infty)$  and A at  $(0,0,-\infty)$  so that  $\alpha_1 = \pi/2$ ,  $\alpha_2 = -\pi/2$ ; and z component vanishes. so E is

$$\mathbf{E} = (\rho_L/2\pi\epsilon_0)\mathbf{a}_{\text{rho}} .$$

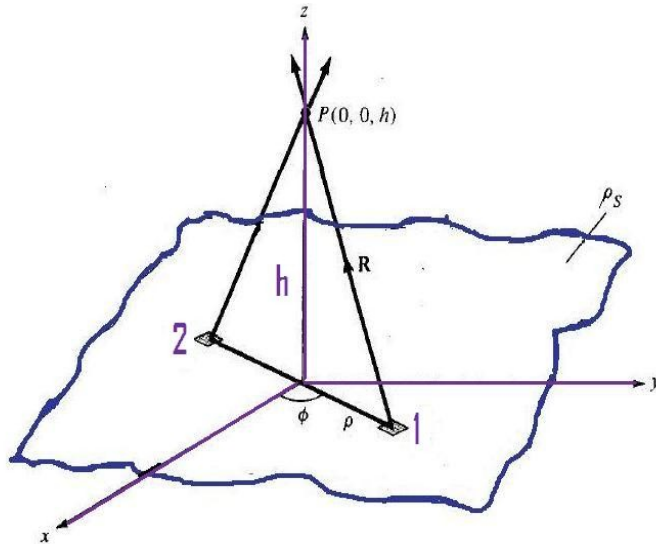
Here E is obtained for infinite line charge along z-axis so that  $\rho$  and  $\mathbf{a}_{\text{rho}}$  have their usual meaning. If line is not along z-axis,  $\rho$  is perpendicular distance from the line to the point of interest and  $\mathbf{a}_\rho$  is a unit vector along distance directed from line charge to field point.

### 3. Electric field due to a Surface Charge:

Consider an infinite sheet of charge in the xy-plane with uniform charge density  $\rho_s$ . The charge associated with an elemental area  $dS$  is

$$dQ = \rho_s dS$$

So the total charge is  $Q = \int \rho_s dS$



Thus the contribution to the  $E$  at point  $P(0,0,h)$  by the elemental surface is

$$dE = dQ a_R / 4\pi\epsilon_0 R^2.$$

$$R = \rho(-a_\rho) + ha_z, \quad R = |R| = [\rho^2 + h^2]^{1/2}$$

$$a_R = R/R, dQ = \rho_s dS = \rho_s \rho d\phi d\rho$$

$$E = \int dE_z$$

$$E = \rho_s a_z / 2\epsilon_0$$

This is only z-component of Electric field if the charge is in x-y plane. For infinite sheet of charge

$$E = \rho_s a_n / 2\epsilon_0$$

For a parallel plate capacitor  $E = \rho_s a_n / \epsilon_0$ .

### **Procedure:**

This experiment consists of 3 stages. Each stage depicts the electric field of the charge distribution considered. Changing the sliders, you can see the change in  $E$  field. Also the corresponding values related to these are shown in the panel just above the sliders. Also the graphs related to each experiment

are shown in the right panel.

In each of the experiments observe the symmetric field surfaces (the surface over which the electric field would be equal) formed by each of the distributions.

Start the experiment by pressing *start* button

- **STAGE 1:**

1. In the first stage we show E field due to a point Charge. Vary the charge using slider.
2. The electric field at any point can be known by pressing at that point using mouse click.
3. Also the electric field distribution at a distance  $r$  can be seen by moving the distance( $r$ ) slider.
4. Notice the direction of electric field with negative charge.
5. Click on next button to move to next stage.

- **STAGE 2**

1. During this stage we consider a uniformly distributed infinite line charge.
2. Observe it's Electric field by changing its linear charge density using slider provided.
3. Observe the direction of electric field by clicking at any point on the screen.
4. Also calculate the electric field distribution at a distance  $r$  from the line charge by using the distance slider provided in the bottom panel
5. Observe the direction of electric field changes when a transition occurs from positive to negative charge occurs.
6. Find out how electric field is related to linear charge density.

- **STAGE 3**

1. During this stage we consider a uniformly distributed finite line charge and observe it's Electric field by changing its linear charge density.
2. Observe the direction of electric field by clicking at any point on the screen.
3. We can also observe the electric field distribution at a distance  $r$  from the line charge by using the distance slider provided in the bottom panel.
4. Here also the direction of electric field changes when a

transition occurs from positive to negative charge. Observe how electric field at any point is related to linear charge density.