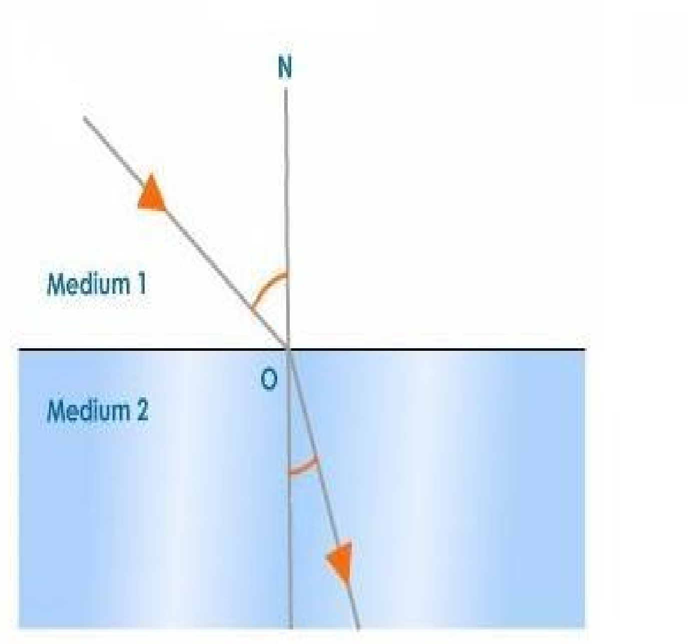
VIRTUAL LABS

Manual for Boundary Conditions

Introduction:

Till now we have considered the existence of Electric field in a homogeneous medium. If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called "Boundary conditions".
These conditions are helpful in determining the field on one side of boundary conditions if the field on the other side is known.



Objectives:

The main objectives of this experiment are the following:

To Determine Electric field intensity and Flux density at boundary of the following mediums:

1)dielectric(ε_{r1}) and dielectric(ε_{r2})

2)dielectric and conductor

Theory:

To determine boundary conditions Maxwell's equations will be used:

$\oint E.dl = 0$
$\oint D.dS = Q_{enclosed}$

E is the Electric field
D is Electric flux density.

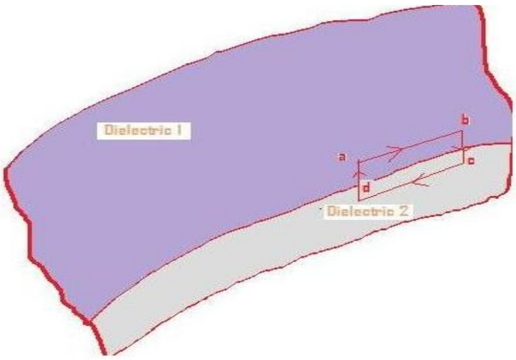
Now decompose the electric field intensity E into two orthogonal components:
$E = E_t + E_n$
E_t is tangential component of Electric field
E_n is normal component of Electric field.

Similarly,
$D = D_t + D_n$
D_t is the tangential component of Electric field intensity
D_n is the normal component of Electric field intensity.

Now let us move to 2 types of boundary conditions:
1) Dielectric - Dielectric boundary conditions.
2) Dielectric - Conductor boundary conditions.

Dielectric - Dielectric boundary conditions

Consider the E field existing in a region of two different dielectric characterized by $\varepsilon_1 = \varepsilon_0\varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_0\varepsilon_{r2}$.



$E_1 = E_{1t} + E_{1n}$
$E_2 = E_{2t} + E_{2n}$

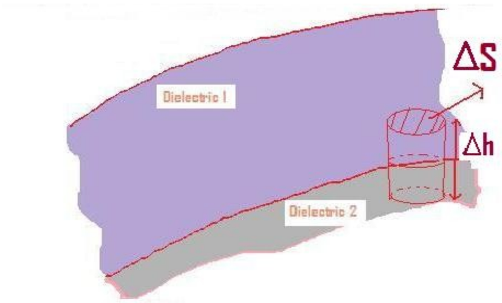
Now the equation
$\oint E.dl = 0$
is applied for a closed path abcda assuming the path is very small with respect to variation in E.

$E_{1t}\triangle w - E_{1n}\triangle h/2 - E_{2n}\triangle h/2 - E_{2t}\triangle w + E_{2n}\triangle h/2 + E_{1n}\triangle h/2$
$E_t = |E_t|$
$E_n = |E_n|$.
As, $\triangle h \to 0$
So, $E_{1t} = E_{2t}$

Thus, the tangential components of E are on the two sides of the boundary. In other words, E_t undergoes no changes and so continuous through the boundary.

And since $D = \varepsilon E = D_t + D_n$.
So,
$D_{1t}/\varepsilon_1 = D_{2t}/\varepsilon_2$
Hence D_t remains discontinuous across the interface.

Now continuity of normal component across the interface will be checked.



From the figure we can see a cylindrical Gaussian surface
$\triangle h \to 0$ gives
$\triangle Q = \rho_s \triangle S = D_{1n} \triangle S - D_{2n} \triangle S$.
Hence,
$D_{1n} - D_{2n} = \rho_s$
ρ_s is the free charge density placed deliberately at the boundary.
D is directed from region 2 to region 1.
If no free charges exists at the interface then $\rho_s = 0$.
So,
$D_{1n} = D_{2n}$

Thus the normal component of D is continuous across the interface; that is D undergoes no change at the boundary. Since $D = \varepsilon E$ so

$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

Hence showing normal component of E is discontinuous at the boundary.Thus

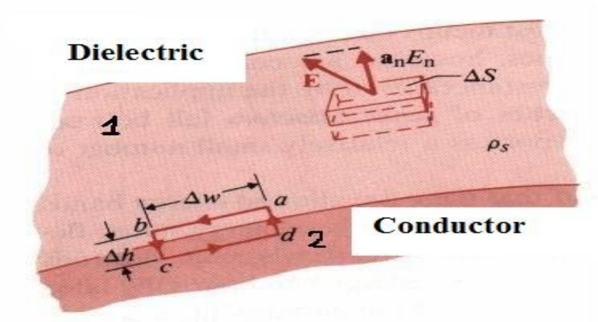
1) $E_{1t} = E_{2t}$

2) $D_{1n} - D_{2n} = \rho_s$

3) $\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

are collectively called the boundary conditions.

Conductor - Dielectric boundary conditions



The conductor is assumed to be perfect. So,
E = 0, inside the conductor surface.
Similar procedures are applied, which were applied for Dielectric - Dielectric interface.
But, the difference is that
E = 0 here
For the path abcda, considering $\triangle h \to 0$, it is found that
$E_t = 0$,
D =0. Hence,
$D_n = \triangle Q / \triangle S = \rho_s$
$D_n = \rho_s$

Procedure:

In this experiment, we will determine the field on one side of boundary conditions if field on the other side is known. The experiment was designed in a way, so that you can quickly change the parameters and observe the results. This makes you to have a more clear picture of the concepts.

Start the experiment by pressing the start button at the top of the window.

Now click on the screen using mouse. You will observe a red colored ray starting from the point and directs towards the origin. After crossing the boundary the ray may diverge from its path with green color.

This ray is nothing but the electric field line. The divergence of the ray is due to change in the medium at the boundary.

Now, your task is to experiment with all possible dielectrics.
So, you have two sliders provided at the bottom of the window. These two stands for the dielectrics.

Notice that if both regions have same dielectric, the ray will not be diverged from its path.