IIIT-H

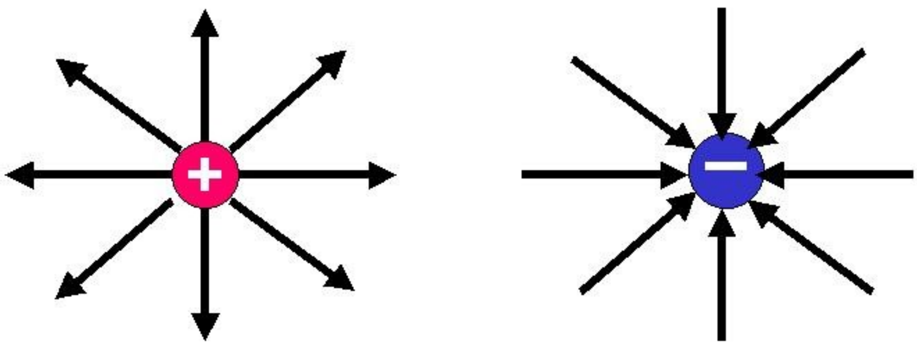
VIRTUAL LABS

Manual for Electrostatic Field

Introduction:

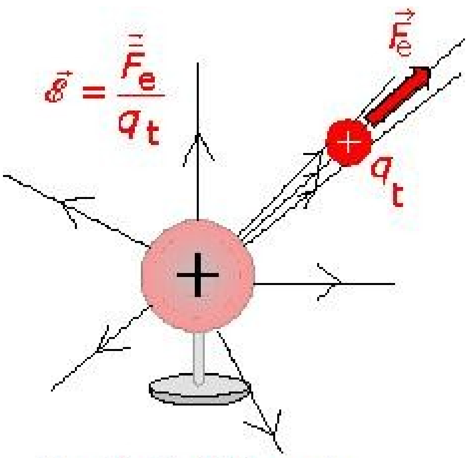
In 1st experiment using Coulomb's law we have seen force due to point charges on other. In this experiment we will estimate electric fields due to various charge distributions.

Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge. The electric field is radially outward from a positive charge and radially in toward a negative point charge.



$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

where,
F is the electric force experienced by the particle
q is its charge
E is the electric field wherein the particle is located.



Electric Field Strength

In the above figure q_t is the unit test charge to evaluate the Electric Field.

F_e is the force on q_t due to the charge at centre.

Objectives:

The main objectives of this experiment are the following:

1. To find electric field due to a Point Charge
2. To find electric field due to a Line Charge
3. To find electric field due to a Surface Charge
4. To find electric field due to a Volume Charge

Theory:

1. Electric Field due to a Point Charge

Based on Coulomb's Law, for interacting point charges, the contribution to the E-field at a point in space due to a single, discrete charge located at another point in space is given by the following

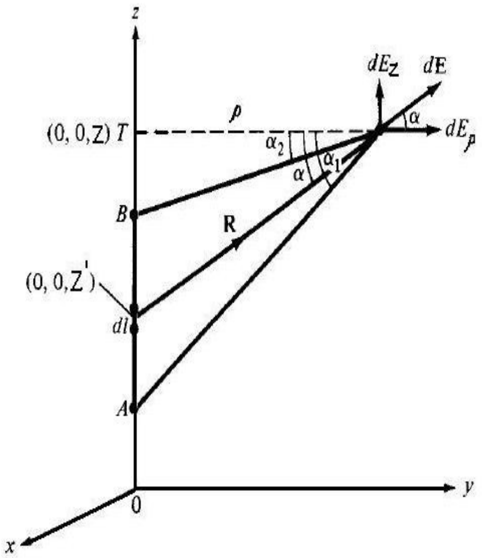
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

where
q is the charge of the particle creating the electric force,
r is the distance from the particle with charge q to the E-field evaluation point, r is the unit vector pointing from the particle with charge q to the E-field evaluation point,
ε is the electric constant.
The total E-field due to a quantity of point charges, n_q, is simply the superposition of the contribution of each individual point charge.

$$\mathbf{E} = \sum_{i=1}^{n_q}\mathbf{E}_i = \sum_{i=1}^{n_q}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i.$$

2. Electric field due to Line Charge

Consider a line charge element dQ with uniform charge density ρ_L extending from A to B along z-axis.The charge element dQ associated with the element dl = dz of the line is $dQ = \rho_L dl$ Hence the total charge is $Q = \int \rho_L dl$.   (line charge)



The electric field at any arbit point can be given by :

$$\mathbf{E} = \int \frac{\rho_l \, \boldsymbol{dl}}{4\pi\epsilon_0 r^2}$$

Let the field point be (x,y,z) and source point be (x',y',z')
dl=dz'
R = (x,y,z)-(0,0,z')
R = $xa_x + ya_y + (z\text{-}z')a_z$
R = $\rho a_\rho + (z\text{-}z')a_z$

$R^2 = |R|^2$,
$a_R/R^2 = R/|R|^3$

Hence

$$E = \frac{-\rho}{4\pi\epsilon_0}\left(\int \rho \sec^2\alpha \left[\cos\alpha \, a_\rho + \sin\alpha \, a_z\right] \frac{d\alpha}{\rho^2 \sec^2\alpha}\right)$$

Thus for finite line charge :

$E = (\rho_L/4\pi\varepsilon_0\rho)[-(\sin\alpha_2 - \sin\alpha_1)a_\rho + (\cos\alpha_2 - \cos\alpha_1)a_z]$

So for an infinite line charge, point B is at (0,0,∞) and A at(0,0,-∞) so that $\alpha_1 = \pi/2$, $\alpha_2 = -\pi/2$; and z component vanishes. so E is

$E = (\rho_L/2\pi\varepsilon_0)a_{rho}$.

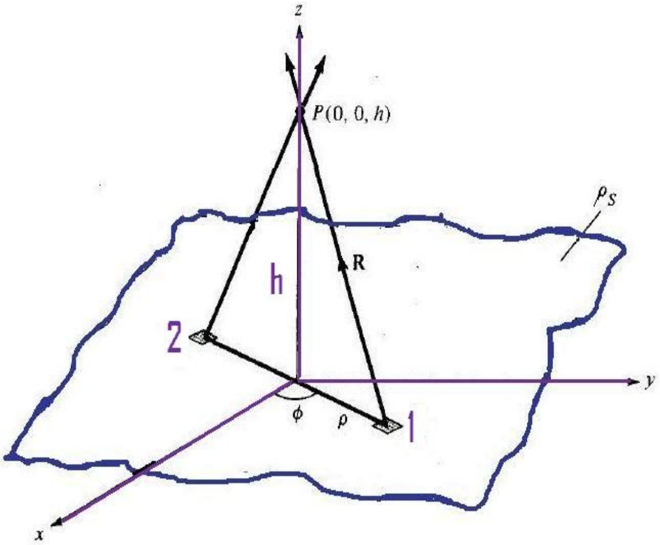
Here E is obtained for infinite line charge along z-axis so that ρ and a_{rho} have their usual meaning. If line is not along z-axis, ρ is perpendicular distance from the line to the point of interest and a_ρ is a unit vector along distance directed from line charge to field point.

3. Electric field due to a Surface Charge:

Consider an infinite sheet of charge in the xy-plane with uniform charge density ρ_s. The charge associated with an elemental area dS is

$dQ = \rho_s\, dS$

So the total charge is $Q = \int \rho_s\, dS$



Thus the contribution to the E at point P(0,0,h) by the elemental surface is

$dE = dQ\, a_R/4\pi\varepsilon_0 R^2$.

$R = \rho(-a_\rho)+ha_z$, $R= |R| = [\rho^2 + h^2]^{1/2}$

$a_R= R/R$,$dQ = \rho_s dS = \rho_s \rho\, d\varphi\, d\rho$

$E= \int dE_z$

$E = \rho_s a_z/2\varepsilon_0$

This is only z-component of Electric field if the charge is in x-y plane. For infinite sheet of charge

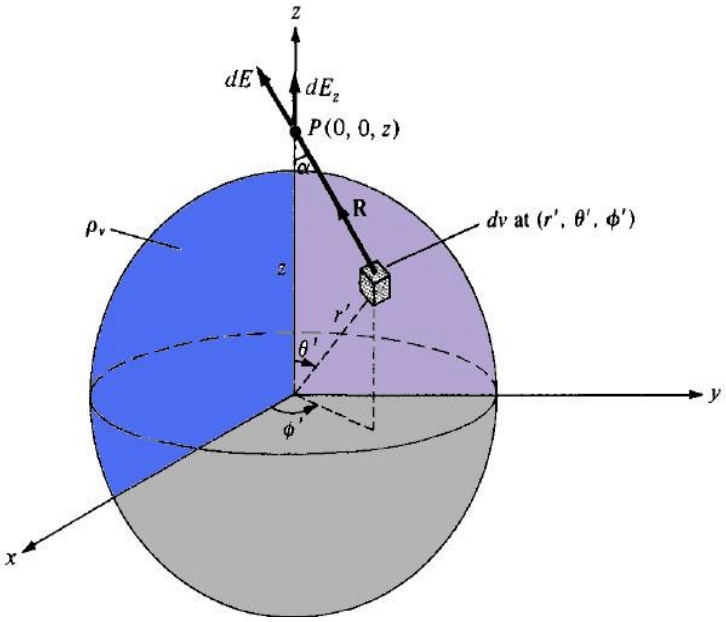
$E = \rho_s a_n/2\varepsilon_0$

For a parallel plate capacitor $E= \rho_s a_n/\varepsilon_0$.

4. Electric field due to Volume Charge Distribution

Let the volume charge distribution with uniform charge density ρ_v, So charge

dQ associated with the elemental volume dv is
$dQ = \rho_v dv$



The total charge in a sphere of radius 'a' is

$Q = \int \rho_v dv = \rho_v \int v$

$Q = \rho_v \, 4\pi \, a^3/3$;

The Electric field at point (0,0,z) is
$dE = \rho_v dv a_R / 4\pi\varepsilon_0 R^2$

$a_R = \cos\alpha \, a_z + \sin\alpha \, a_\rho$.Due to symmetry of charge distribution we have $E_x + E_y = 0$.

$E_z = \int dE\cos\alpha = (\rho_v)/4\pi\varepsilon_0 \int dv \cos\alpha/R^2$
$dv = r'^2 \sin\theta' dr' d\theta' d\varphi'$
$R' = z^2 + r'^2 - 2zr'\cos\theta'$
$r' = z^2 + R^2 - 2zR\cos\alpha$

using all the substitutions and solving the integration the electric field at (0,0,z) is:
$E_z = E = Qa_z / 4\pi\varepsilon_0 z^2$

Due to the symmetry of the charge distribution, the electric field at (0,0,z) is readily obtained from
$E = Q/4\pi\varepsilon_0 r^2$ which is identical to the electric field at the same point due to a point charge Q located at the origin or the center of the spherical charge

distribution.

Procedure:

This experiment consists of 4 stages. Each stage depicts the electric field of the charge distribution considered. Changing the sliders, you can see the change in E field. Also the corresponding values related to these are shown in the panel just above the sliders. Also the graphs related to each experiment are shown in the right panel.
In each of the experiments observe the symmetric field surfaces (the surface over which the electric field would be equal) formed by each of the distributions.

Start the experiment by pressing start button

- STAGE 1:

  In the first stage we show E field due to a point Charge. When the charge under consideration is increased then the electric field at a distance r also increases. The electric field at any point can be known by pressing at that point using mouse click(s). Also the electric field distribution at a distance r can be seen by moving the distance(r) slider. Also you can notice that, as the charge becomes negative the direction of electric field at any point reverses it's direction.

- STAGE 2

  During this stage we consider a uniformly distributed infinite line charge and observe it's Electric field by changing its linear charge density. Here we can see that the direction of electric field at any point is perpendicular to the line charge . This can be observed by clicking at any point on the screen. We can also observe the electric field distribution at a distance r from the line charge by using the distance slider provided in the bottom panel. Here also the direction of electric field changes when a transition occurs from positive to negative charge. We observe that that electric field at any point is directly proportional to its linear charge density.

- STAGE 3:

  In this stage we observe the behaviour of electric field due to an infinite sheet. When the surface charge density on the infinite sheet increases the magnitude of electric field also increases . This can be observed by manipulating the sliders provided. Also an animation is shown in this experiment on how the electric field is behaving at each point in the space. This animation if observed carefully shows us that the electric field due to the infinite sheet is independent of the distance r, it purely depends on the aerial density of the sheet.

- STAGE 4:

In this stage we observe the behaviour of electric field due to a conducting charge. When the volume charge charge density of the conducting charge increases the magnitude of electric field also increases.This can be observed by manipulating the sliders provided. You can observe that inside the conduct charge, elctric feild is zero. So, the charge of conduct sphere is present only on the surface (is shown in experiment by + signs on the surface of conducting sphere.). The conducting sphere behaves similar to point charge, except that inside conducting sphere electric field is zero. This difference can be seen from the graphs shown in right side of window.