IIIT-H

VIRTUAL LABS

Manual for Wave Propagation

Introduction:

Till now we have learned about static charges, moving charges which produces Electromagnetic waves, Maxwells equation, faradays law and lot more. Now, we will learn about the propagation of electromagnetic waves which is widely used in TV signals, radar beams, microwaves and all types of communication.
In general, Electromagnetic waves are the means of transporting energy or information and posses the general properties of waves i.e travelling at high velocity etc.We will also deal with the properties of Electromagnetic waves in following media :

1)Free space
2)Lossless dielectric
3)Lossy dielectric
4)Good Conductor

Electromagnetic radiation is a transverse wave meaning that the oscillations of the waves are perpendicular to the direction of energy transfer and travel. An important aspect of the nature of light is frequency. The frequency of a wave is its rate of oscillation and is measured in hertz, the SI unit of frequency, where one hertz is equal to one oscillation per second. Light usually has a spectrum of frequencies which sum together to form the resultant wave. Different frequencies undergo different angles of refraction.

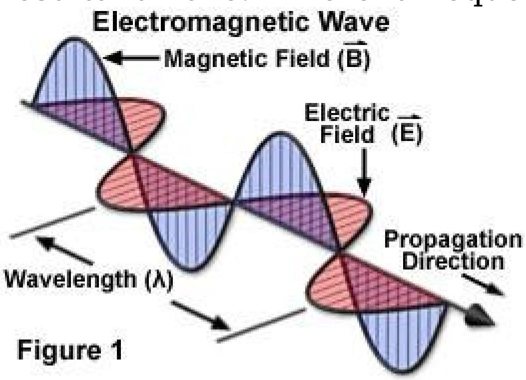

Figure 1

A wave consists of successive troughs and crests, and the distance between two adjacent crests or troughs is called the wavelength. Waves of the electromagnetic spectrum vary in size, from very long radio waves the size of buildings to very short gamma rays smaller than atom nuclei. Frequency is inversely proportional to wavelength, according to the equation:

$$v = f\lambda$$

where v is the speed of the wave (c in a vacuum, or less in other media), f is the frequency and λ is the wavelength.

Objectives:

The main objectives of this experiment are the following:

1. To observe the propagation of wave in various materials
2. To observe the amplitude of E field at a particular location in space as time varies.

Theory:

A wave is a function of both space and time can be defined in partial differential equation of second order.

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0$$

Where u is the wave velocity.

Electromagnetic Wave travelling in z(+ ve) direction.
A wave equation in terms of scalar be expressed in the phasor form in the following manner :

$$E = Ae^{j(\omega t - \beta z)} + Be^{j(\omega t + \beta z)}$$

Where ω is the angular frequency, A and B are real constants.
A negative sign in (ωt ± βz) is associated with a wave propagating in the +z direction (forward travelling or positive-going wave) whereas a positive sign indicates that a wave is travelling in the -z direction (backward travelling or negative going wave).

1. Wave propagation in Lossy Dielectrics :

Now, we will see the EM waves propogation in lossy dielectric. A lossy dielectric is a medium in which an EM wave loses power as it propagates due to poor conduction.
So, We can also say that a lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with as distinct from a lossless dielectric (perfect or good dielectric) in which a = 0.
According to maxwells equation :

$$\nabla \cdot \mathbf{E}_s = 0$$

$$\nabla \cdot \mathbf{H}_s = 0$$

$$\nabla \times \mathbf{E}_s = -j\omega\mu\mathbf{H}_s$$

$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s$$

Taking curl on both sides of the 3rd maxwellâ€™s equation :

$$\nabla \times \nabla \times \mathbf{E}_s = -j\omega\mu \, \nabla \times \mathbf{H}_s$$

If we apply vector identity given below to the above equation,

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

We will get the following equation,

$$\nabla\underbrace{(\nabla \cdot \mathbf{E}_s)}_{0} - \nabla^2\mathbf{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}_s$$

Or in more simplified manner we can say that

$$\nabla^2\mathbf{E}_s - \gamma^2\mathbf{E}_s = 0$$

This is also known as the wave equation, where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

Gamma is the propagation constant, which is a complex number, so we can also write it as

$$\gamma = \alpha + j\beta$$

Value of α and β can be easily expressed by comparison with the above equation.

$$\boxed{\begin{aligned} \alpha &= \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]} \\ \beta &= \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]} \end{aligned}}$$

Since the wave is propagating alone positive z direction this Es will only have x component , Es = Exs (z) ax

Substituting this into the wave equation,

$$\underbrace{\cancel{\frac{\partial^2 E_{xs}(z)}{\partial x^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 E_{xs}(z)}{\partial y^2}}}_{0} + \frac{\partial^2 E_{xs}(z)}{\partial z^2} - \gamma^2 E_{xs}(z) = 0$$

Which is a second order linear homogeneous differential equation, which has the solution

$$\mathbf{E}(z, t) = \text{Re}\,[E_{xs}(z)e^{j\omega t}\mathbf{a}_x] = \text{Re}\,(E_o e^{-\alpha z} e^{j(\omega t - \beta z)}\mathbf{a}_x)$$

Further simplifying, we will get

$$\mathbf{E}(z, t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$

On solving above equation with the maxwellâ€™s equation, we will arrive at the following conclusion i.e

$$\mathbf{H}(z, t) = \text{Re}\,(H_o e^{-\alpha z} e^{j(\omega t - \beta z)}\,\mathbf{a}_y)$$

Where,

$$H_o = \frac{E_o}{\eta}$$

η is a complex quantity known as the intrinsic impedance (in ohms) of the

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\underline{/\theta_\eta} = |\eta|e^{j\theta_\eta}$$

medium.

Substituting this value of intrinsic impedance in the above equations, we get

$$\mathbf{H} = \frac{E_o}{|\eta|}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\,\mathbf{a}_y$$

α is the attenuation factor and β is the wave number, which can be expressed

$$u = \frac{\omega}{\beta}, \qquad \lambda = \frac{2\pi}{\beta}$$

as:

In EM waves, the difference between the phase of E and H is given by θ/2 i.e

$$\frac{|\mathbf{J}_s|}{|\mathbf{J}_{ds}|} = \frac{|\sigma\mathbf{E}_s|}{|j\omega\varepsilon\mathbf{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

Where Js is the Conduction current density , whereas Jd is the displacement current density Jds in the lossy medium , tanO is the loss tangent and O is the loss angle of the medium.

2. Lossless Dielectric :

In a lossless dielectric, a << we.

$$\boxed{\sigma \simeq 0, \qquad \varepsilon = \varepsilon_o\varepsilon_r, \qquad \mu = \mu_o\mu_r}$$

Substituting these value into the above derived equation, we get

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \qquad \lambda = \frac{2\pi}{\beta}$$

And also the intrin

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\,\angle 0^\circ$$

Thus E and H are in phase with each other.

3. Free Space :

In free space e is simply replaced by eo and mue by &mue;$_o$.

$$\boxed{\sigma = 0, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o}$$

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c}$$

$$u = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta}$$

Where c is the speed of light in a vaccum. Here the loss angle is 0 and the intrinsic impedance of free space is given by:

$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \simeq 377\ \Omega$$

4. Good Conductor

Perfect conductor is one in which the conductivity is very high in comparison to the product of angular frequency and permittivity .

$$\boxed{\sigma \simeq \infty, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o\mu_r}$$

In the case of good conductor , the attenuation factor and the wave number are equal and is given by :

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \qquad \lambda = \frac{2\pi}{\beta}$$

And E leads H by 45, as intrinsic impedance is

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

Procedure:

This experiment consists of three stages and each stage will teach you a new concept.

The experiment was designed in a way, so that you can quickly change the parameters and observe the results. This makes you to have a more clear picture of the concepts.

Start the experiment by pressing start button

- STAGE 1:

  In this stage you are going to experiment with various materials and observe the behaviour of waves in those materials. First of all, choose the kind of material from the selection box provided at the bottom of experiment window. Then, choose the kind of wave with which you want to experiment. For example, choose "Lossy" kind of wave. Now, you can set the parameters like Electric Field, frequency and alpha parameters using the sliders provided at the bottom of the experiment window. Measure the Value of E at various points.
  Check whether you make out any difference between Lossy wave and other kinds of waves. If you cannot find out the difference, you should go through the theory once again !!!
  Write a report of your observations. After completing this, move on to the next stage by clicking on "Next" button provided at the top of the experiment window.

- STAGE 2:

  This stage is not different from previous stage. It is just a static view of previous stage. Now you can better measure the value of electric field at each point in the space.
  Do the same as above and write a report of your observations. After

completing this, move on to the next stage by clicking on "Next" button provided at the top of the experiment window.

- STAGE 3:

  Here, you dont see the start button as activated. You can directly do experiment. This stage will help you to measure amplitude of E field at a particular location in space as time varies. The parameters are same as for above stages. Do the same as above and write a report of your observations. If you want to go back to previous stages, click "Back" button provided at the top of the experiment window.