TEXAS TECH UNIVERSITY
Industrial Engineering™

Kidney Paired Donation **O**ptimization **R**eport

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Introduction

Kidney Paired Donation (KPD) is a solution for organ transplants when direct donation isn't possible. Sometimes a person wants to donate a kidney to someone, but medical issues get in the way. One of these issues includes blood type compatibility. KPD helps by matching donors and recipients in a more complex network, allowing more people to receive life-saving kidney transplants. By creating a sophisticated matching network, patients who would otherwise be unable to receive a kidney can now find one through indirect exchanges. This approach transforms the donation landscape from a limited, linear process to a complex, interconnected donation ecosystem.

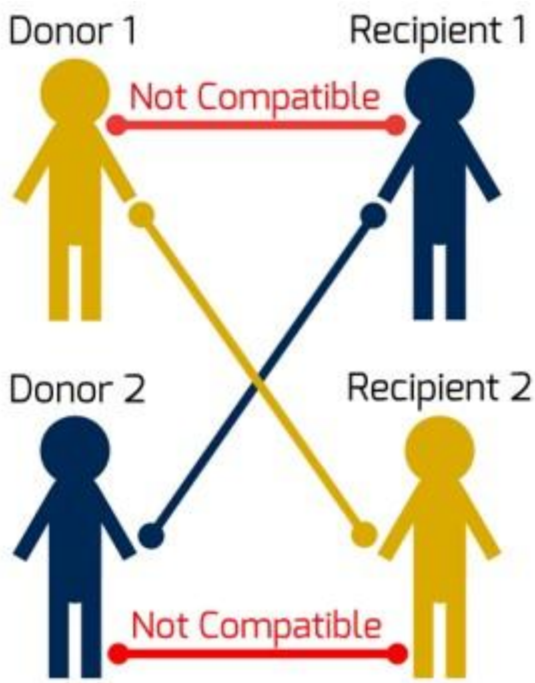

Fig 1: Paired Exchange (source)

Criteria for Kidney Paired Donation

The criteria for kidney paired donation are:

1. **Blood Type Compatibility**: Donors must be compatible with recipient blood types. Below is the compatibility requirements.

- O type donors can donate to all blood types

- A type donors can donate to A and AB recipients
- B type donors can donate to B and AB recipients
- AB type donors can only donate to AB recipients

2. **Other Criteria**

- Maximum cycle length (2-way and 3-way exchanges)
- No donors, recipient pair can participate in multiple cycles
- Each recipient can receive at most one kidney
- Each donor can give at most one kidney

Problem Statement

The goal is to create a smart system that helps more people get kidney transplants by matching incompatible donors and recipients. The system will:

1. Find the best ways to swap kidneys between different patient-donor groups
2. Ensure medical compatibility between donors and recipients
3. Design optimal exchange cycles involving two or three pairs of donors and patients

The key idea is to create "kidney swap chains" where patients who can't receive a kidney from their original donor can still get a transplant by being matched with another compatible donor in the network.

Operations Research Model (in Words and Math)

This Kidney Paired Donation (KPD) optimization model is designed to maximize the total number of kidney transplants. This is done by finding the largest number of disjoint and feasible cycles of size 2 (pairs) and size 3 (triplets) by identifying the best set of compatible donor-patient exchanges. The model works by matching incompatible pairs into cycles, where each donor in a cycle provides a kidney to a recipient in the same cycle. The objective is to select the maximum number of such cycles. The model enforces several constraints: each patient-donor pair can participate in only one cycle, cycles are limited to a maximum of three pairs, and each match must meet compatibility requirements. With these constraints, the model ensures efficient transplant opportunities for as many patients as possible.

Mathematical Formulation:

Sets and Parameters:

- N: Set of all patient-donor pairs (nodes).
- E: Set of all directed edges (i,j) where donor i can donate to recipient j based on compatibility.
- C_2: Set of all 2-cycles (i,j,i) where $(i,j) \in E$ and $(j,i) \in E$.
- C_3: Set of all 3-cycles (i,j,k,i) where $(i,j),(j,k),(k,i) \in E$.

Graph Representation:

- Let $G = (N, E)$ represent the compatibility graph where N is the set of all patient-donor pairs, and E is the set of directed edges representing feasible kidney donations based on compatibility.

Variables:

- x_{ij}: Binary variable, equals 1 if edge (i,j) is included in a cycle; 0 otherwise.
- y_k: Binary variable, equals 1 if cycle $k \in C_2 \cup C_3$ is selected; 0 otherwise.

Objective Function:

$$\text{Maximize} \quad \sum_{k \in C_2 \cup C_3} y_k$$

This represents the maximum number of disjoint cycles selected.

Constraints:

1. **Node Participation:** Each node can participate in at most one cycle:

$$\sum_{k \in C_2 \cup C_3 : i \in k} y_k \leq 1 \qquad \forall_i \in N$$

2. **Edge Validity within Cycles:** A cycle k can only be selected if all edges (i,j) within the cycle are valid:

$$y_k \leq x_{ij} \qquad \forall (i,j) \in k, k \in C_2 \cup C_3$$

3. **Cycle Size Constraints:** Restrict cycles to the maximum size of $k=3$:

$$|c| \leq k + 1 \qquad \forall k \in C_2 \cup C_3$$

4. **Population Constraints:** Ensure that the total number of nodes (patient-donor pairs) in any cycle stays within bounds:

$$\boldsymbol{L \leq \sum_{i \in k} x_{ij} \leq U} \qquad \forall \boldsymbol{k} \in \boldsymbol{C_2} \cup \boldsymbol{C_3}$$

5. **Cycle Structure:** For each selected cycle, all nodes must form a valid cycle structure of size 2 or 3:
   - For 2-cycles: ***(i,j,i)***
   - For 3-cycles: ***(i,j,k,i)***
6. **Binary Constraints:**

$$\boldsymbol{x_{ij}} \in \{\boldsymbol{0}, \boldsymbol{1}\} \qquad \forall(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{E}$$

$$\boldsymbol{y_k} \in \{\boldsymbol{0}, \boldsymbol{1}\} \qquad \forall \boldsymbol{k} \in \boldsymbol{C_2} \cup \boldsymbol{C_3}$$

Explanation of the Process:

This model defines sets of patient-donor pairs N, compatibility edges E, and potential 2-cycle C_2 and 3-cycle exchanges C_3. It builds a compatibility graph G initially with patient-donor pairs and Edges (N, E). It maximizes the objective function that has the binary variable y_k to get maximum number of disjoint cycles that follow the constraints. Constraints include, each node should participate in only one cycle, maintain blood compatibility, and restrict cycle size and others. This effectively creates a matching system that strategically connects incompatible donor-recipient pairs to enable more transplantations.

Python/Gurobi Implementation

The implementation uses key techniques like:

1. Graph Creation:

- Builds a directed graph representing donor-recipient pairs
- Adds edges only between compatible blood types

2. Cycle Detection:

- Finds two-way and three-way cycles in the graph
- Uses **NetworkX** for graph operations

3. Optimization:

- Uses Gurobi solver to maximize exchanges with constraints
- Selects optimal cycles and exchanges

The entire code, data and results are available in my GitHub repo :
https://github.com/soujanyavankudothu/KPD

Experiments

The experiment started with an initial sample of 30 rows and then followed to the complete dataset. The results for both cases were exported to JSON files and visualization of results is done in the notebook.

The optimization model was written, combined, and solved using the Gurobi solver 11.0.3 in Jupyter Notebook, and the coding language used was Python. The model runs on a HP EliteBook x360 1040 G5 with 16GB RAM | CPU model: Intel(R) Core (TM) i7-8650U CPU @ 1.90GHz | Thread count: 4 physical cores, 8 logical processors, using up to 8 threads.

This is solved to optimality with the objective value of 366 edges and 183 cycles in 8,477 seconds.

Donor and Recipient Analysis in the results:

Donor Blood Type Counts: 'AB': 264, 'B': 183, 'A': 183

Recipient Blood Type Counts: 'B': 183, 'A': 183

Pairings: ('B', 'B'): 183, ('A', 'A'): 183, ('AB', 'A'): 133, ('AB', 'B'): 131

Plan:

To optimize Kidney Paired Donation (KPD), we have to develop a transparent, reproducible framework leveraging graph-based modeling. The dataset will be preprocessed to construct a directed graph where nodes represent incompatible donor-patient pairs, and edges indicate potential matches based on compatibility criteria. We have to develop an algorithm to detect cycles of size two and three and an ILP model will be formulated to maximize the number of successful matches while adhering to constraints such as cycle size and exclusivity of pair participation.

The Gurobi solver in python will optimize the matching, and results will be validated. The entire process, including code, visualizations, and outputs, will be documented and exported.

Evaluation of Plan

1. **Extent to Which the Criteria Are Met**:
    - **Matching Criteria**: The plan successfully ensures that the donor's kidney is compatible with the paired patient. This algorithm iterates over incompatible pairs to identify feasible exchanges, maximizing the number of matches.
    - **Cycle Constraints**: The solutions explicitly limit cycles to a maximum size of three.
2. **Optimality**:
    - The optimization routine efficiently identifies the maximum number of transplantations while avoiding prohibited exchanges.

Limitations of the Plans and Analysis

1. **Model Assumptions**:
    - Optimization assumes perfect availability and participation of all donors and patients, which may not always be held true in real world scenarios.
    - Compatibility assessments are based solely on provided data, which might not account for all medical subtleties.
2. **Data Constraints**:
    - The quality of the matching outcomes depends majorly on the accuracy and completeness of the input dataset. Errors or missing data could lead to suboptimal recommendations.
3. **Cycle Size Limitations**:
    - With the limitation to cycles to two or three we are excluding potential larger chains of transplants, which could increase the total number of matches in some cases.
4. **Computational Complexity:**
    - With the increase in the donor-recipient pairs, the complexity of the algorithm increases leading to slower results.

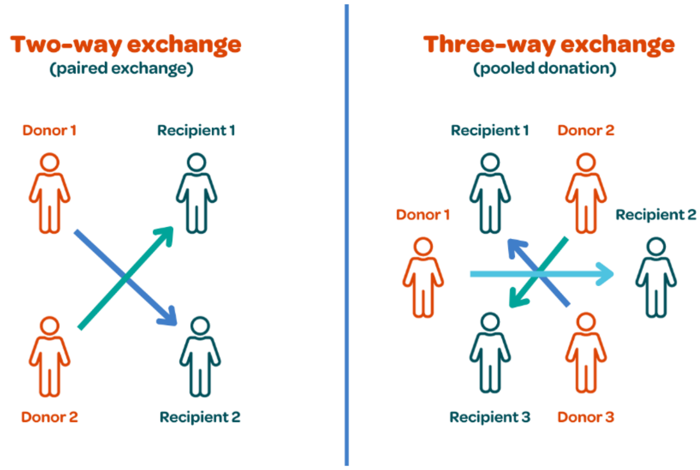

Fig 2: 2-way and 3-way exchange (Source)

Conclusions

The developed Kidney Paired Donation optimization model successfully:

- Creates a framework for identifying kidney exchanges
- Maximizes the number of potential transplantations
- Provides a transparent, reproducible methodology for matching donor-recipient pairs

Technical Appendix

Computational Environment:

- Python 3.10

- NetworkX for graph operations

- Gurobi Optimizer 11.0.3

- Matplotlib for visualization