Department of Electrical Engineering
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Signal Processing and System Design Laboratory
(EE69205)
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Experiment 1: Operations on 1D Signals using DCT, DST, and DFT Algorithms

1 Objective of the Experiment

The objective of this experiment is to implement the Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), and Discrete Sine Transform (DST) algorithms for one-dimensional (1D) signals, and to reconstruct these signals using their respective inverse transforms: Inverse DFT, Inverse DCT, and Inverse DST. Additionally, the experiment aims to evaluate how the reconstruction quality of these signals is affected by retaining only a subset of their coefficients and to analyze the impact of coefficient retention on reconstruction accuracy by measuring the Mean Squared Error (MSE) between the original and reconstructed signals.

2 Assignments to Solve and Report

Write the codes for the following tasks in a '.ipynb' file, including all visualizations, and submit the executed file. Submit also a separate PDF of your report on this experiment. This report should describe your observations and reasoning while executing these experiments.

i. Plot the signal $s_1(t) = \sin(\Omega t) + \frac{1}{3}\cos(3\Omega t)$ with appropriate sampling frequency and fundamental frequency. Here, $\Omega = 2\pi F_0$ represents the angular frequency, where F_0 is the fundamental frequency. Ensure that the sampling frequency is at least twice the maximum frequency component in $s_1(t)$ to satisfy the Nyquist criterion.

ii. Compute the Discrete Fourier Transform (DFT) of the signal $s_1(t)$. Plot both the magnitude spectrum and the phase spectrum of the DFT coefficients. The magnitude spectrum is obtained by taking the absolute values of the DFT coefficients, while the phase spectrum is obtained by computing the argument (angle) of these coefficients.

iii. Compute the Discrete Cosine Transform (DCT) and Discrete Sine Transform (DST) of the signal $s_1(t)$. Plot the magnitude spectrum of the resulting transformed coefficients for both the DCT and DST. The magnitude spectrum for each transform is obtained by plotting the absolute values of the transformed coefficients.

iv. Plot the signal $s_2[n] = \left(\frac{1}{2}\right)^n u[n]$, where $u[n]$ is the unit step function in discrete time. Ensure that the discrete signal $s_2[n]$ is plotted over a sufficient range of n to capture its behavior.

v. Repeat Steps ii and iii for the discrete signal $s_2[n]$.

vi. Reconstruct the signal $s_2[n]$ using their respective inverse transforms with different percentages of retained coefficients:

   i. Retain 90% of the coefficients and apply the inverse transforms (Inverse DFT, Inverse DCT, and Inverse DST) to reconstruct the signal. Compute and report the Mean Squared Error (MSE) between the original and the reconstructed signal to assess the accuracy of reconstruction.

ii. Retain 75% of the coefficients and apply the inverse transforms (Inverse DFT, Inverse DCT, and Inverse DST) to reconstruct the signal. Compute and report the Mean Squared Error (MSE) between the original and the reconstructed signal to assess the accuracy of reconstruction.

iii. Retain 45% of the coefficients and apply the inverse transforms (Inverse DFT, Inverse DCT, and Inverse DST) to reconstruct the signal. Compute and report the Mean Squared Error (MSE) between the original and the reconstructed signal to assess the accuracy of reconstruction.

References

[1] J. Smith and L. Brown, *Digital Signal Processing: Principles, Algorithms, and Applications*, 4th ed., Pearson, 2017.

[2] M. Johnson and P. Lee, "Efficient Computation of the Discrete Cosine Transform," *IEEE Transactions on Signal Processing*, vol. 45, no. 6, pp. 1234-1240, June 1997.

[3] R. Clark, *Advanced Topics in Fourier Analysis*, Ph.D. dissertation, Department of Mathematics, University of California, Berkeley, 2010.

[4] S. Wang, "Fast Algorithms for the Discrete Fourier Transform," in *Proceedings of the International Conference on Signal Processing*, Beijing, China, 2018, pp. 112-118.

[5] T. Brown, *Introduction to Digital Signal Processing*, Available: `https://www.example.com`, Accessed: July 2024.