

A family of accessibility measures: bringing practical interpretation to accessibility analysis

A FAMILY OF ACCESSIBILITY MEASURES: BRINGING PRACTICAL  
INTERPRETATION TO ACCESSIBILITY ANALYSIS

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*A Thesis Submitted to the School of Graduate Studies in the Partial Fulfillment  
of the Requirements for the Degree Doctor of Philosophy*

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## **Lay Abstract**

(150 words or less).

The aim of transportation systems is to connect people and opportunities (i.e., jobs, services). However, traditionally transportation planning has focused on mobility (distances travelled), instead of accessibility (how many opportunities can be reached). Experts have been calling for a shift from mobility-based methods to access-based ones, but this change hasn't fully happened yet for a variety of reasons. One challenge is methodological: the lack of clear units for measuring accessibility. This thesis aims to help address this gap by outlining how accessibility methods relate to dominant mobility-based techniques, and how the concept of 'constraints' can be borrowed from these techniques to re/introduce units for accessibility measures.



# Abstract

Transportation systems play a fundamental role in urban contexts by facilitating access between people and various social and economic opportunities. Spatial place-based accessibility, otherwise known as accessibility, can be defined as the potential to spatially interact with opportunities. For decades, transportation planning research and practice have relied on mobility-based estimates and indicators. These have been oriented to reflect realized movement (e.g., kilometers traveled, emissions released, vehicles passing through an intersection) as opposed to potential movement (e.g., the number of opportunities that can be reached). However, in recent years, there have been calls to move from mobility-based methods to access-based ones, but a few barriers remain. One significant issue is methodological—namely, the lack of clarity in the interpretation of conventional accessibility measures' scores. An emerging approach is to link these scores to outcomes, but this thesis proposes a preceding step: clarifying the units of accessibility. In this spirit, this thesis is organized into six chapters. First, a review of how accessibility literature largely diverged from the spatial interaction literature is detailed. This lays the foundation for how the addition of proportionality constants that return the units to the measure and balance them to reflect known constraints in the system may be useful. Second, using a synthetic example, the total constrained accessibility measure is introduced. This measure is formally proportional to unconstrained accessibility (i.e., Hansen-type accessibility measure commonly used in literature) but with results consistently expressed in units of opportunities. Next, using the same synthetic example, the single constrained accessibility measure is introduced. This measure considers competition for opportunities and could be understood as related to the popular measures of competitive accessibility. Multimodal extensions as well as the expression of these measure in terms of 'market potential' (i.e., accessible population at a zone as opposed to accessible opportunities at a zone) is also introduced. Third, motivation, data and methods of an empirical example of the parkland area and population in the City of Toronto are detailed. All members of the family of accessibility measures introduced in this thesis are compared and contrasted in the subsequent chapters using this empirical example. Fourth, a unimodal case considering a binary impedance function and all population as walking is presented for accessible parkland area and accessible population (to parkland) in Toronto. Differences in the unconstrained (i.e., conventional) and constrained measures are detailed. Fifth, a multimodal case of accessible parkland area and accessible population (to parkland area) is detailed, considering walking, cycling, transit, and car modes. Similarly, differences in the unconstrained (i.e., conventional) and constrained measures (are detailed. Sixth, this final chapter summarizes key points across all chapters, provides a thorough discussion on how constrained accessibility can be used to communicate for the purpose of equitable transportation policy, and details potential future lines of research.



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# **Declaration of Authorship**

I, Anastasia Soukhov, declare that this thesis titled, *A family of accessibility measures: bringing practical interpretation to accessibility analysis* and the work presented in it are my own. I confirm that:

I did most of the research.

Also the writing.

Sometimes I cried.

But mostly I had fun.

# Preface - Introduction

## 0.1 This dissertation's evolution

This work has come to be non-chronologically. In the summer of 2020, a largely ‘virtual’ summer for many of us, I began working with Dr. Antonio Paez on a reading course as a Masters of Applied Science student under the supervision of Dr. Moataz Mohammed. At that point, I was nearing the end of my Master’s program, writing a MAsc. thesis aimed at providing guidance on right-sizing passenger transportation sustainability policies based on vehicle technologies well-to-wheel life cycle operating conditions e.g., (Soukhov, Foda, & Mohamed, 2022; Soukhov & Mohamed, 2022). Coming from transportation engineering, I had to take on a new research process in engaging with this new research. Antonio, who had recently developed a deep appreciation for open and reproducible science and the R universe, combined with a passion for historical analysis and use of data for advocacy, had worked with students before me to gather information on public schools in Hamilton. A public school in his neighborhood, along with several others across the city, was closing, and he was keen to explore the impacts these closures would have.

It was within this context that I began the reading course. I found myself intuitively working through the available data, building upon my beginner R skills, estimating additional data to supplement the analysis, and crafting research questions that felt sufficiently satisfying; questions that blended policy impacts, spatial analysis, and active transportation implications of a change in school-seats. Spatial accessibility, or the potential to spatially interact with school-seats, as an indicator that could answer these questions. Accessibility, conventionally defined as the sum of opportunities  $O_j$  weighted by the travel impedance function  $f(c_{ij})$  of some travel cost  $c_{ij}$  for a set of zones of origin  $i \dots I$  and destination  $j \dots J$  in Equation 1, takes the following general form:

$$S_i = \sum_i O_j f(c_{ij}) \quad (1)$$

Driven by my engineering training, I wanted as precise a solution with an interpretable meaning at the highest spatial resolution possible. For instance: how had the number of potentially reachable school-seats change for households? Looking back now, that’s a high expectation from a place-based accessibility metric, but it was my engineering background that was informing this desire and fixation on units. I knew the number of school-seats in Hamilton for each study year, the number of students, and I knew

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that, intuitively, the school-seat accessibility for each household should decrease within proximity to schools. However, conventional methods produced results that were difficult to interpret. Moreover, competitive measures, such as those often applied using Floating Catchment Areas (FCA), had their own inflationary issues (i.e., discussed in Antonio Paez, Higgins, & Vivona, 2019).

This challenge led towards developing a more intuitive approach, one that involved establishing proportional allocation factors. These factors allocate opportunities proportionally based on a zone's population relative to the region's total population, as well as the zone's travel impedance to other zones relative to the travel impedance to all zones from that zone. The balancing factors ensure that each parcel is assigned a portion of the total school-seats in the region. This intuitive approach ensured the school-seats from each school is proportionally allocated to each zone based on the zone's relative population and travel impedance *and* each value summed to the total number of opportunities in the region. This method of competitive accessibility, which preserves its units, was introduced as "spatial availability" (Soukhov, Paez, Higgins, & Mohamed, 2023).

Chris Higgins, an early collaborator, helped us realize "spatial availability" is *singly-constrained* from the perspective of opportunities, akin to the singly-constrained spatial interaction model in Wilson (1971). We adopted this conceptualisation, without scrutinizing the Wilson's framework (at the time), and published "spatial availability", a singly-constrained competitive accessibility measure in *PLOS ONE* (Soukhov et al., 2023). An open data paper demonstrating my growing expertise in R and reproducible methods in *Environment and Planning B: Urban Analytics and City Science* (Soukhov & Páez, 2023) then followed, followed by a multimodal extension of the spatial availability measure the year after (Soukhov, Tarriño-Ortiz, Soria-Lara, & Páez, 2024). Along with years of thinking about the equity and justice implications of transportation systems summarized in the (forthcoming) review of the literature Soukhov, Aitken, Palm, Farber, & Paez (2025), the first work -that started back in my MSc during that reading course- the article assessing the change in active (and motorized) school-seat accessibility after 10 years of school closures was published in *Networks and Spatial Economics* (Soukhov, Higgins, Páez, & Mohamed, 2025).

However, in the past few months, I returned to Chris' original comment: how similar is the spatial availability balancing factor to the single constraint in the spatial interaction model? It turns out, they can be simplified to be identical. This means that the most popular competitive accessibility measure —the 2SFCA— can also be seen as proportional to the singly-constrained expression in Wilson (NOTE: in Soukhov et al. (2023), the mathematical equivalence between spatial availability per capita and 2SFCA is established). And, requiring only information about the total number of opportunities in the region and non-competitive in its definition, a 'total constraint', fitting within Wilson's framework, can also be established. The total constraint ensures the zonal accessibility values are proportional to the total in the region, share the same

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scaled magnitude as conventional accessibility in Equation 1, but maintains units of the accessibility concept , i.e. the *number of accessible opportunities*.

Thematically, this thesis begins with this forthcoming (and submitted) work of Soukhov, Pereira, Higgins, & Paez (2025), which details the precedent, logic, and intuition of weaving ‘constraints’ from the spatial interaction model into measures of potential for spatial interaction: both the singly-constrained version (as previously published) and, for the first time, introducing the total constraint are included.

In summary, this this dissertation is based on the following publications (listed in chronological order of their publication):

- Soukhov, A., Pereira, R. H. M., Higgins, C. H. & Paez, A. (2025). A family of accessibility measures derived from spatial interaction principles. (Forthcoming - submitted to PLOS ONE).
- Soukhov, A., Higgins, C. D., Páez, A., & Mohamed, M. (2025). Ten Years of School Closures and Consolidations in Hamilton, Canada and the Impact on Multimodal Accessibility. Networks and Spatial Economics. <https://doi.org/10.1007/s11067-025-09677-z>
- Soukhov, A., Aitken, I. T., Palm, M., Farber, S., & Paez, A. (2025). Searching for fairness standards in the transportation literature. (Forthcoming - submitted to *Transportation*). An extended version, as a report, can be found here: <https://mobilizingjustice.ca/wp-content/uploads/2024/01/just-transportation-1.pdf>
- Soukhov, A., Tarriño-Ortiz, J., Soria-Lara, J. A., & Páez, A. (2024). Multimodal spatial availability: A singly-constrained measure of accessibility considering multiple modes. PLOS ONE, 19(2), e0299077. <https://doi.org/10.1371/journal.pone.0299077>
- Soukhov, A., & Páez, A. (2023). TTS2016R: A data set to study population and employment patterns from the 2016 Transportation Tomorrow Survey in the Greater Golden Horseshoe area, Ontario, Canada. Environment and Planning B: Urban Analytics and City Science, 23998083221146781. <https://doi.org/10.1177/23998083221146781>
- Soukhov, A., Paez, A., Higgins, C. D., & Mohamed, M. (2023). Introducing spatial availability, a singly-constrained measure of competitive accessibility. PLOS ONE, 1–30. <https://doi.org/10.1371/journal.pone.0278468>

## 0.2 The importance of accessibility; and conceptual issues interpreting ‘unconstrained’ access

Transportation is a special kind of land-use that compresses space and time, enabling connections, and hence potential interaction, between populations in different zones.

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But, as posed in S. Handy (2020), “is accessibility an idea whose time has finally come?” This concept has been discussed in the literature for decades as the ulterior goal of transportation systems. Accessibility doesn’t express mobility directly but instead captures the potential for meaningful mobility — the reachability of opportunities. Despite the utility of accessibility measures in both literature and practice to identify areas of high and low opportunity access, the interpretability of foundational accessibility measures, such as the Hansen-type accessibility measure (Hansen, 1959) (as in Equation 1 ), can be challenging (Karst T. Geurs & van Wee, 2004a; E. J. Miller, 2018; Santana Palacios & El-geneidy, 2022).

By definition, accessibility is the product of the sum of opportunities that can be reached through some travel impedance, meaning each zonal value is sensitive to both inputs in a way that isn’t constrained by any known system. For example, consider comparing the accessibility values for a region between two years where the number of opportunities remained constant but the travel behaviour dramatically changed. In this case, both years would have travel impedance functions of different forms. As a result, the zone values would have different units, one in opportunities-weighted-travel-impedance-value-from-form-1 and another in units of opportunities-weighted-travel-impedance-value-from-form-2. These values would exist within different ranges, and while they could be normalized and compared, this process introduces bias. Consider also, when comparing values between regions, the number of opportunities varies with the size of the region (e.g., larger regions generally have more opportunities than smaller ones). For this reason, raw accessibility scores aren’t necessarily comparable without some form of normalization, which again introduces bias. As with many spatial issues, these problems are compounded by the Modifiable Areal Unit Problem (MAUP) and Modifiable Temporal Unit Problem (MTUP), where the zonal and temporal units may differ between comparison periods, further obscuring the results.

Aside from temporal and regional comparisons, gleaned interpretation of raw ‘opportunities-weighted-travel-impedance-value’ scores within the same temporal and spatial region is challenging. To move closer to an intuitive interpretation, a balancing factor that is sensitive to the known information about the transportation system can be introduced. This would allow the zonal value of ‘how many opportunities can be accessed’ to be more plainly contemplated - as the units of travel impedance is set aside. In this way, linking accessibility values to something else meaningful (e.g., outcomes) can become more straightforward and interpretable.

### 0.3 Aims

The primary contribution of this dissertation is demonstrating how the units of accessibility -the number of opportunities that can potentially be reached- can be preserved within the zonal values of the accessibility measure. This contribution offers a practical way to enhance accessibility as a planning tool, making it more interpretable and potentially more useful for decision-making.

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This dissertation offers four key contributions. First, it defines accessibility as the *potential* for spatial interaction by explicitly linking it to the spatial interaction literature, and providing a review of how the two streams of literature diverged. Next, it explores how spatial interaction modelling literature maintained ‘units’ in outputs by introducing system and zonal constraints through balancing factors, and shows how accessibility can be reformulated using these same constraints.

Second, the total constraint is introduced, which is conceptually equivalent in magnitude to the Hansen-type accessibility measure (Equation 1) but is expressed in units of ‘accessible opportunities’. This approach can help ensure that accessibility measures remain more consistent and comparable across different areas and time periods, addressing the issue of unit discrepancies that often arise in accessibility analyses.

Third, the single constraint is introduced, which accounts for competition among populations for access to opportunities. This constraint can be understood as a competitive measure, and offering an additional layer of constraints (e.g., ensuring opportunities at a destination are proportionally distributed to reachable origins based on relative population and travel impedance, as well as maintaining the total constraint). It can also be seen as a more general version of the Two Step Floating Catchment Area (2SFCA) method (Luo & Wang, 2003a), before it is divided by zonal population. In this way, singly-constrained accessibility can offer a more accurate understanding of localized competition for opportunities.

Fourth, the multi-modal extension of both total- and singly- constrained accessibility is specified. Specifically, these extensions demonstrate how these measures can be adapted to consider different groups with different travel impedance functions (i.e., be it multiple modes of transport or populations with different travel likelihoods) accessing the same opportunities. This multi-modal extension can be used to interpret the accessibility gap between groups, illustrating how many more opportunities can be reached by one group compared to another.

## 0.4 A practical case study: Toronto and publically owned and operated green space

The magnitude of opportunities are reachable, and the distribution of this value spatially or among people, is often the preoccupation of those who use accessibility measures. An advantage of the *constrained* accessibility measures, is in their interpretation. They can be used in place of a static metrics of opportunity per zone or opportunity per capita in a zone metric (e.g., the number of jobs located in a census block, or the number of hospital beds per person in a neighbourhood). Instead, constrained accessibility measures yield a value of the number of *accessible* opportunities per zone or per capita (in a zone), cleanly folding in the assumptions about how the population may interact with that opportunity (transportation-land-use interaction), but still allowing the value itself to be understood like a level of service metric.

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To demonstrate the practical advantage constrained accessibility's interpretation, a case study reporting the accessibility to green space that is owned and operated by the City of Toronto will be detailed. Green space is important to well being... XYZ. Furthermore, the City is undergoing benchmarking of access to different opportunity types. From this perspective, greenspace is an important public service to benchmark using accessibility, as its use is related to its ease of reach [].

Green space is an interesting opportunity, as there are a variety of types with different qualities that may impact how important population-competition considerations are. For instance, smaller neighbourhood parks typically only attract people residing in proximity to these parks, and the furniture may get congested - meaning competition may matter, but only locally. However, data reflection the number of people that visit these parks is more likely to be unavailable. On the other hand, large legacy parks with natural attractions may attract people from further distances, so congestion of the park may be more of a pertinent consideration.

With the aim of showcasing constrained accessibility's practical interpretation, the calculation and comparison between total constrained and singly constrained accessibility will be outlined in this work. Constrained accessibility for each zonal origin will be calculated using the total constraint assuming one travel impedance function and a multi-modal extension, showing how constrained accessibility, expressed in units of accessible opportunities, can be aggregated across different spatial and temporal scales. This aggregation allows for a clearer communication of the policy implications of transportation and land-use decisions, making accessibility measures more communicable for urban planning and decision-making.

## 0.5 Overview of methods

Firstly, this entire thesis adopts principles of open and reproducible methods; using R and RStudio workflows, everything from the manuscript, code, and the final manuscript is transparent and freely available for all to replicate [X].

### 0.5.1 Accessibility metrics

The dissertation begins with the demonstration of the unconstrained accessibility (Equation 1), then total constrained accessibility, and singly-constrained accessibility. Framing of the constrained and unconstrained accessibility methods in the spatial interaction literature.

### 0.5.2 Data sources and travel time estimates

- 1) Parks - from City of Toronto portal.
- 2) Canadian census for Toronto CMA. Population weighted centroids. Census data at the DA. Neighbourhoods.

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- 3) Empirical travel flows from the 2022 TTS for leisure trips for different modes (as included in the updated version of the TTS2016R package (text from “*TTS2016R: A data set to study population and employment patterns from the 2016 Transportation Tomorrow Survey in the Greater Golden Horseshoe area, Ontario, Canada*” is included)
  - 4) Using modified OSM Toronto network, routed r5r travel times for multiple modes.

## 0.6 Chapters outline

This dissertation is divided into six chapters. The first provides the conceptual and historical framework of this thesis as written in the first half of the manuscript titled “*Family of accessibility measures derived from spatial interaction principles*” submitted to *PLOS ONE*. This chapter outlines the evolution of the spatial interaction literature, how accessibility literature diverged, and introduces the total and single constraints using numeric examples.

The second chapter outlines the family of accessibility measures (unconstrained, total constrained, singly constrained, and doubly constrained accessibility) using a simple toy example. It is the second part of the manuscript “Family of accessibility measures derived from spatial interaction principles” \_ submitted to *PLOS ONE*. As well as additional writing detailing the context for and the formulation of a multimodal extension of each measure, inspired writing from the published “*Multimodal spatial availability: A singly-constrained measure of accessibility considering multiple modes*” paper.

The third chapter details the empirical example of parkland accessibility in Toronto developed in chapters four and five of this dissertation. Namely the origins, the different zonal levels of aggregation, qualities about the destinations, and the routing and trip length assumptions are detailed. The unconstrained and constrained accessibility measures used in the chapters four and five are briefly summarised in context with the empirical example.

The fourth chapter calculates a unimodal example of parkland accessibility and accessible population (to parkland area) for the city of Toronto, using the unconstrained, total and singly constrained accessibility. Results using different measures are compared, discussed and contextualized. The discussion is inspired by the unimodal focus of the published paper “*Introducing spatial availability, a singly-constrained measure of competitive accessibility*”.

The fifth chapter builds on the preceding chapter, and extends the example to consider multiple modes: walking, as well as cycling, transit and car. Multimodal accessibility to parkland (and to population from parkland) is detailed. Results are compared to the unimodal example, and compared, discussed and contextualised. Discussion of competitive gains is inspired by the published paper “*Ten Years of School Closures and Consolidations in Hamilton, Canada and the Impact on Multimodal Accessibility*” .

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The sixth chapter concludes the work. the open and reproducibility of methods learned is discussed (text from “*TTS2016R: A data set to study population and employment patterns from the 2016 Transportation Tomorrow Survey in the Greater Golden Horseshoe area, Ontario, Canada*” is included), how constrained measures can be interpreted, how this interpretation can aid policy makers, and how policy makers can ultimately use this to plan for inequities (some text pulled from “*Searching for fairness standards in the transportation literature*”). Future research directions (?)

# Chapter 1

## CHP 1 - A common history between accessibility and spatial interaction principles

### 1.1 Chapter overview

This chapter consists of text from the first half of the “*Family of accessibility measures derived from spatial interaction principles*” authored by Anastasia Soukhov, Rafael H. M. Pereira, Christopher D. Higgins, and Antonio Páez and submitted to *PLOS ONE* in May 2025. This portion of the submitted manuscript reviews the historical background of spatial interaction modelling in the context of accessibility, the use of proportionality constants—linked to empirical information about the system—to maintain units, and the divergence of accessibility’s practice in applying this approach. Overall: this chapter provides the historical context for the introduction of the family of measures, providing supporting motivation for the formulation of the measures and their use in various applications throughout this dissertation.

### 1.2 Introduction

In the early nineteenth century, industrializing cities grew rapidly: so did traffic congestion and the development of transportation planning practice focused primarily on mobility. In this newly founded practice, access to destinations was treated as a by-product of movement. Following the rise of the automobile and significant investments in transportation infrastructure after World War II, this mobility-oriented approach has led to some problematic outcomes. Specifically, the car became seen as the ultimate mobility tool, helping to foster the development of low-density, single-use residential neighborhoods and entrenching an automobility mono-culture within transportation systems (Lavery, Páez, & Kanaroglou, 2013; H. J. Miller, 2011). Decades of planning for this automobility mono-culture, still often characterized by road and highway expansion, have been marked by increased travel costs and environmental burdens, with limited impact on enhancing the ease with which people can reach destinations (Steven Farber &

Páez, 2011; S. Handy, 2002; Antonio Páez, Mercado, Farber, Morency, & Roorda, 2010). In response, transportation researchers have increasingly advocated for the adoption of accessibility as a planning criterion, in contrast to traditional mobility-oriented transportation planning approaches which translate into indicators that benchmark movement (e.g., vehicle kilometres traveled, intersection through traffic, etc.) which are not necessarily linked to improved accessibility (El-Geneidy & Levinson, 2022; S. Handy, 2020; A. Paez, Moniruzzaman, Bourbonnais, & Morency, 2013; Silva, Bertolini, Te Brömmelstroet, Milakis, & Papa, 2017).

Accessibility may be defined as the “potential of opportunities for [spatial] interaction” (Hansen, 1959), in contrast to mobility which is simply, movement in itself or ‘spatial interaction’. Mobility has been the basis of traditional transportation planning approaches (Ortúzar & Willumsen, 2011). While accessibility brings a more holistic understanding of combined transportation and land use systems, incorporating both the explicit consideration of destination as opportunity and the concept of *potential* (S. L. Handy & Niemeier, 1997).

The growing interest in accessibility has been accompanied by a boom in scholarly research using different methods and focusing on different research contexts; it has grown to include studies of access to employment (Grengs, 2010a; e.g., Karst & Van Eck, 2003; Merlin & Hu, 2017; Antonio Páez, Farber, Mercado, Roorda, & Morency, 2013; Tao, Zhou, Lin, Chao, & Li, 2020), health care (Boisjoly, Moreno-Monroy, & El-Geneidy, 2017; Delamater, 2013; e.g., Luo & Wang, 2003a; Antonio Páez et al., 2010; Pereira, Braga, et al., 2021; Wan, Zou, & Sternberg, 2012), green spaces (Liang, Yan, & Yan, 2024; Reyes, Paez, & Morency, 2014; Rojas, Paez, Barbosa, & Carrasco, 2016), schools (Marques, Wolf, & Feitosa, 2021; Romanillos & Garcia-Palomares, 2018; e.g., Williams & Wang, 2014), social contacts (S. Farber, Neutens, Miller, & Li, 2013; S. Farber, Páez, & Morency, 2012; e.g., Neutens, Witlox, Van de Weghe, & De Maeyer, 2007), and regional economic analysis (Gutierrez, Condeco-Melhorado, Lopez, & Monzon, 2011; Lopez, Gutierrez, & Gomez, 2008; Ribeiro, Antunes, & Páez, 2010; e.g., R. Vickerman, Spiekermann, & Wegener, 1999) among many other domains of application. In other words, accessibility analysis is used today to broadly understand the potential to reach (or spatially interact with) opportunities that are important to people (Ferreira & Papa, 2020). However, despite its growth in popularity in scholarly works, challenges remain with respect to the more widespread adoption of accessibility in planning practice. For instance, the diversity of accessibility definitions has been flagged by van Wee (2016), S. Handy (2020), and Kapatsila, Palacios, Grisé, & El-Geneidy (2023). Further, difficulties in the interpretability and communicability of outputs has also been noticed by many authores, including Karst T. Geurs & van Wee (2004b), van Wee (2016), and Ferreira & Papa (2020).

The adoption of accessibility in planning practice is not necessarily made easier when potential adopters have to contend with a plethora of definitions, each seemingly more sophisticated but less intuitive than the last (Kapatsila et al., 2023). At a high level, Karst T. Geurs & van Wee (2004b) identify four families of accessibility measures:

infrastructure-, place-, person-, and utility-based. Of the place-based family, which is the focus of this paper, the menu has grown to include gravity-based accessibility (e.g., Hansen, 1959; Pirie, 1979), cumulative opportunities (Pirie, 1979; e.g., Wachs & Kumagai, 1973; Ye, Zhu, Yang, & Fu, 2018), modified gravity (e.g., Schuurman, Berube, & Crooks, 2010), 2-Step Floating Catchment Areas (e.g., Luo & Wang, 2003a), Enhanced 2-Step Floating Catchment Areas (e.g., Luo & Qi, 2009), 3-Stage Floating Catchment Areas (e.g., Wan et al., 2012), Modified 2-Step Floating Catchment Areas (e.g., Delamater, 2013), inverse 2-Step Floating Catchment Areas (e.g., F. Wang, 2021), and n-steps Floating Catchment Areas (Liang et al., 2024). How is a practitioner to choose among this myriad options? What differences in accessibility scores should matter, and how should they be communicated? [see van Wee (2016); p. 14].

Here, we seek to address the breath of place-based accessibility's definitions and lack of interpretability by demonstrating that mobility-oriented models—such as the commonly used gravity-based accessibility (e.g., Hansen, 1959) and the spatial interaction model (e.g., Wilson, 1971)—are rooted in the same spatial interaction modeling foundation. In fact, we propose that accessibility can be specified using spatial interaction principles as a *family of measures*, akin to the family of spatial interaction models introduced in Wilson (1971) that can be defined using balancing factors that constrains values based on known information about the land-use.

This work aims to offer three contributions: (1) it introduces a family of accessibility measures within the principles of spatial interaction; (2) it formally defines three *constrained* accessibility measures by reintroducing Wilson-analogous balancing factors. These measures are the total constrained accessibility measure, the singly constrained accessibility measure, and the doubly constrained measure, which are explicitly connected to popular measures such as the Hansen-type accessibility (Hansen, 1959), the popular competition approach of the 2-Step Floating Catchment Area (2SFCA) (Luo & Wang, 2003a; Q. Shen, 1998a), and the concept of market potential (C. D. Harris, 1954; R. W. Vickerman, 1974). These Wilson-analogous balancing factors introduced are also defined in order of increasing restrictiveness to shed light on the role of *potential* in accessibility and access. (3) This work also demonstrates that the introduction of balancing factors makes accessibility measures easier to interpret and to communicate by restoring the measurement units to the resulting raw accessibility values. Each zonal and zonal flow value from a constrained accessibility measure is always in interpretable units, namely the number of ‘opportunities for spatial interaction’ or ‘population for spatial interaction’. This is in contrast to conventional accessibility measures, particularly gravity based measures, that yield values in units of ‘opportunities weighted by some representation of travel friction’.

To achieve these stated objectives, this paper contends that accessibility research must reconnect with its spatial interaction origins. Particularly, we argue that an important aspect of spatial interaction modelling—namely, constraining the results to match empirical observations—was never effectively reincorporated into accessibility analysis. Empirical constraints were embraced by early spatial interaction literature following the work

of Wilson (1971), but this stream of literature tended to flow separately from research inspired by Hansen (1959)’ accessibility. The application of Wilson (1971)’s empirical constraints supported the development of various spatial interaction models that remain relevant in research and practice today (Ortúzar & Willumsen, 2011). However, the same cannot be said of the contemporary accessibility literature, where empirical constraints were not explicitly adopted. We argue that the absence of empirical constraints (and their attendant proportionality constants) has contributed to some of accessibility analysis’ interpretability issues; for instance, the fuzziness of insights beyond simple proportional statements like ‘higher-than’ or ‘lower-than’ (E. J. Miller, 2018). Moreover, without constraints we lose track of what accessibility measures, that is, the number of opportunities that can be spatially interacted with. Without a clear sense of measurement units, comparability between accessibility measures, across cities, and transport modes, may also become compromised.

Fortunately, accessibility and spatial interaction modelling literature share common headwaters, and the latter has given careful attention to measurement units and their interpretability. It is by looking to the past that we believe accessibility analysis can newly wade into the future. We continue this work in the following section by tracing the development of accessibility from its origins in spatial interaction: from the Newtonian gravitational expression in Ravenstein (1889) through to the seminal accessibility work of Hansen (1959). We then present evidence for a narrative highlighting the marked divergence between accessibility and spatial interaction modelling research after the work of Wilson (1971). Next, we hark back to Wilson (1971)’s spatial interaction models, and use them to derive a family of accessibility measures based on analogous constraints. We illustrate members of this family of constrained accessibility measures with a simple numerical example. We then conclude by discussing the uses of these measure and their interpretation.

### 1.3 Newtonian’s roots of human spatial interaction research

The patterns of people’s movement in space have been a subject of scientific inquiry for at least a century and a half. From as far back as Henry C. Carey’s *Principles of Social Science* (Carey, 1858), a concern with the scientific study of human spatial interaction can be observed. It was in this work where Carey stated that “*man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance*”. (McKean, 1883, pp. 37–38). This statement shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton’s Law of Universal Gravitation, first posited in 1687’s *Principia Mathematica* and expressed as in Equation 1.1.

$$F_{ij} \propto \frac{M_i M_j}{D_{ij}^2} \quad (1.1)$$

To be certain, the expression above is one of proportionality, and is also the most famous in all of science. It states that the force of attraction  $F$  between a pair of bodies  $i$  and  $j$  is directly *proportional* to the product of their masses  $M_i$  and  $M_j$ , and inversely *proportional* to the square of the distance between them  $D_{ij}$ . Direct proportionality means that as the product of the masses increases, so does the force. Likewise, inverse proportionality means that as the distance increases, the force decreases. Equation 1.1, however, does not quantify the magnitude of the force. To do so, an empirical constant, a.k.a. the gravitational constant, is required to convert the proportionality into an equality, ensuring that values of the force  $F$  in Equation 1.1 match the observed force of attraction between masses. In other words, Equation 1.1 needs to be *constrained* using empirical data. Ultimately, the equation for the force is as seen in Equation 1.2, where  $G$  is an empirically calibrated proportionality constant:

$$F_{ij} = G \frac{M_i M_j}{D_{ij}^2} \quad (1.2)$$

Newton's initial estimate of  $G$  was based on a speculation that the mean density of earth was between five or six times that of water, an assumption that received support after Hutton's experiments of 1778 (Hutton, 1778, p. 783). Still, it took over a century from the publication of *Principia* to refine the estimate of the proportionality constant to within 1% accuracy, with Cavendish's 1798 experiment (Cavendish, 1798).

## 1.4 Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Since the 1880s to the 1940s, a number of researchers theoretically and empirically attempted to characterise human spatial interaction as some force of attraction  $F$  that is directly proportion to the masses  $M_i$  and  $M_j$  and inversely proportional by their separation distance. This concept was captured with different expressions, but all tie back to the same Newtonian gravity analogy, although not all of them included a proportionality constant in their formulation.

Following Carey's *Principles* of 1858, research into human spatial interaction continued in different contexts. In the late 1880s, Ravenstein proposed some "Laws of Migration" based on his empirical analysis of migration flows in various countries (Ravenstein, 1885, 1889). In these works, Ravenstein posited 1) a directly proportional relationship between migration flows and the size of destinations (i.e., centres of commerce and industry), and 2) an inversely proportional relationship between the size of flows and the separation between origins and destinations. As with Carey, these propositions echo Newton's gravitational laws. Over time, other researchers discovered similar relationships. For example, Reilly (1929) formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations that could be understood as 'potential trade territories'. Later, Zipf proposed a  $\frac{P_1 P_2}{D}$  hypothesis for the case of information (Zipf, 1946a), intercity personal movement (Zipf, 1946b), and goods movement

by railways (Zipf, 1946c). The  $\frac{P_1 P_2}{D}$  hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

A common feature of these early investigations of human spatial interaction is that a proportionality constant similar to  $G$  in Equation 1.2 was never considered. Of the researchers cited above, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly's hypothesis was presented in the following form:

$$B_a = \frac{(P_a \cdot P_T)^N}{D_{aT}^n} \quad (1.3)$$

where  $B_a$  is the amount of business drawn to  $a$  from  $T$ ,  $P_a$  and  $P_T$  are the populations of the two settlements, and  $D_{aT}$  is the distance between them. Quantity  $N$  was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of  $n = 2$ .

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

$$C^2 = \frac{P_1 \cdot P_2}{D_{12}} \quad (1.4)$$

where  $C$  is the volume of goods exchanged between 1 and 2,  $P_1$  and  $P_2$  are the populations of the two settlements, and  $D_{12}$  is the distance between them.

After Carey, it is in Stewart's work on the principles of demographic gravitation that we find the strongest connection yet to Newton's law (Stewart, 1948). This may relate to academic backgrounds; where Stewart was a physicist while Ravenstein, Reilly, and Zipf were social scientists. Besides awareness of preceding research (he cites both Reilly and Zipf as predecessors in the analysis of human spatial interaction), Stewart appears to have been the first author to express his theorized relationships for human spatial interaction with a proportionality constant  $G$ , as follows:

$$F = G \frac{(\mu_1 N_1)(\mu_2 N_2)}{d_{12}^2} = G \frac{M_1 \cdot M_2}{d_{12}^2} \quad (1.5)$$

Where:

- $F$  is the *demographic force*
- $N_1$  and  $N_2$  are the numbers of people of in groups 1 and 2
- $\mu_1$  and  $\mu_2$  are so-called *molecular weights*, the attractive weight of groups 1 and 2
- $M_1 = \mu_1 N_1$  and  $M_2 = \mu_2 N_2$  are the demographic masses at 1 and 2
- $d_{12}^2$  is the distance between 1 and 2

- And finally  $G$ , a constant that Stewart “left for future determination” (1948, p. 34)

In addition to demographic force, Stewart defined a measure of the “population potential” of group 2 with respect to group 1. In other words, the potential number of people from location 2 that could visit location 1, as follows:

$$V_1 = G \frac{M_2}{d_{12}} \quad (1.6)$$

For a system with more than two population bodies, Stewart formulated the population potential at  $i$  as follows (after arbitrarily assuming that  $G = 1$ ):

$$V_i = \int \frac{D}{r} ds \quad (1.7)$$

where  $D$  is the population density over an infinitesimal area  $ds$  and  $r$  is the distance to  $i$ . In Equation 1.7,  $D \cdot ds$  gives an infinitesimal count of the population, say  $dm$ , and so, after discretizing space, Equation 1.7 can be rewritten as:

$$V_i = \sum_j M_j d_{ij}^{-1} \quad (1.8)$$

Alerted readers will notice that Equation 1.8, with some re-organization of terms, is formally equivalent to our modern definition of accessibility popularized by Hansen (1959) in the late 1950s.

Stewart’s formulation of demographic force, developed in the context of what he called “social physics” (Stewart, 1947), was problematic. It had issues with inconsistent mathematical notation. More seriously though, Stewart’s work was permeated by a view of humans as particles following physical laws, but tinted by unscientific and racist ideas. For instance, he assumed that the molecular weight  $\mu$  of the average American was one, but “presumably... much less than one....for an Australian aborigine”

p.35

. Stewart’s ideas about “social physics” soon fell out of favour among social scientists, but not before influencing the nascent field of accessibility research, as detailed next.

## 1.5 Hansen’s gravity-based accessibility to today

From Stewart (1948), we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature (Hansen, 1959). In this seminal paper, Hansen defined accessibility as “the potential of opportunities for interaction...

a generalization of the population-over-distance relationship or *population potential* concept developed by Stewart (1948)” (p. 73). As well as being a student of city and regional planning at the Massachusetts Institute of Technology, Hansen was also an engineer with the Bureau of Roads, and preoccupied with the power of transportation to shape land uses in a very practical sense. Hansen (1959) focused on Stewart (1948)’s *population potential* (expressed in Equation 1.8, and not on the other formulaic contributions and objectionable aspects of “social physics”. Hansen (1959) recast Stewart’s population potential to reflect accessibility, a model of human behaviour useful to capture regularities in mobility patterns. Hansen (1959) replaced  $M_j$  in Equation 1.8 with *opportunities* to derive an *opportunity potential*, or more accurately, a *potential of opportunities for interaction* as follows:

$$S_i = \sum_j \frac{O_j}{d_{ij}^\beta} \quad (1.9)$$

A contemporary rewriting of Equation 1.9 accounts for a variety of impedance functions beyond the inverse power  $d^{-\beta}$ :

$$S_i = \sum_j O_j \cdot f(d_{ij}) \quad (1.10)$$

$S_i$  in Equation 1.9 is a measure of the accessibility from site  $i$ . This is a function of  $O_j$  (the mass of opportunities at  $j$ ),  $d_{ij}$  (the cost, e.g., distance or travel time, incurred to reach  $j$  from  $i$ ), and  $\beta$  (a parameter that modulates the friction of cost). Today, Hansen is frequently cited as the father of modern accessibility analysis (e.g., Reggiani & Martín, 2011), and Hansen-type accessibility is commonly referred to as the gravity-based accessibility measure.

However, Hansen’s use of Stewart’s *population potential* measure included one crucial omission that afflicts the literature to this day. The omission is that between Stewart (1948) and Hansen (1959), the proportionality constant  $G$  in Equation 1.6 vanished. This constant was not explicitly addressed in Hansen (1959) and accessibility research continues to evolve without it. Since Hansen (1959), accessibility analysis has been widely used in numerous disciplines but, to our knowledge, the proportionality constant has remained forgotten, with no notable developments to explicitly acknowledge or determine it.

The omission of this constant generates a fundamental problem for the measurement unit of accessibility estimates, which undermines the interpretation, communication and comparability of accessibility analysis. Those reading Hansen (1959) must recall that Stewart (1948) had set the proportionality constant  $G$  to 1, with a note that “ $G$

*was*

left for future determination: a suitable choice of other units can reduce it to unity” [p. 34]. In practice, the persistent omission of the constant in accessibility analyses means that  $G$  continues to be implicitly set to 1, even when the fundamental relationship in accessibility is proportionality (e.g.,  $S_i \propto \sum_j g(O_j) f(d_{ij})$ ) and not equality (for instance, see the formula for accessibility at the top of Figure 1 in Wu & Levinson, 2020). The direct consequence is that without a proportionality constant, the units of  $S_i$  remain unclear: the unit of “potential of opportunity for interaction” is left free to change as  $\beta$  is calibrated. For example, if  $c_{ij}$  is distance in meters, it will be number of opportunities per  $m^\beta$  when  $f(c_{ij}) = d^{-\beta}$  but number of opportunities per  $e^{-\beta \cdot m}$  when  $f(c_{ij}) = e^{-\beta \cdot m}$ . This undermines the comparability of accessibility metrics with different decay functions, and renders their results difficult to understand and communicate. The Hansen-style accessibility estimates found in the literature, therefore, are better thought of as ordinal measures of potential that can only be interpreted in terms of higher and lower accessibility, but which has not palpable meaning (E. J. Miller, 2018).

## 1.6 Wilson’s family of spatial interaction models

On the other side of the Atlantic, Alan G. Wilson was developing related, yet parallel work. In his groundbreaking study (Wilson, 1971), Wilson defined a general spatial interaction model as follows. While accessibility was characterised as an associated concept of ‘potential’, the primary focus was on modelling observed spatial interaction:

$$T_{ij} = k W_i^{(1)} W_j^{(2)} f(c_{ij}) \quad (1.11)$$

The model in Equation 1.11 posits a quantity  $T_{ij}$  that represents a value in a matrix of flows of size  $n \times m$ , that is, between  $i = 1, \dots, n$  origins and  $j = 1, \dots, m$  destinations. The quantities  $W_i^{(1)}$  and  $W_j^{(2)}$  are proxies for the masses at  $i = 1, \dots, n$  origins and  $j = 1, \dots, m$  destinations. The super-indices (1) and (2) are meant to indicate that these masses could be different things, i.e.,  $W_i^{(1)}$  could be populations, and  $W_j^{(2)}$  hectares of park space. Finally,  $f(c_{ij})$  is some function of travel cost  $c_{ij}$  which reflects travel impedance. In this way,  $T_{ij}$  explicitly measures *interaction* in the unit of trips, and the role of  $k$  is to ensure that the system-wide sum of  $T_{ij}$  represents the total flows in the data. In other words,  $k$  is a scale parameter that makes the overall amount of flows identical to the magnitude of the phenomenon being modeled. In other words, it balances the units, in a conceptually similar sense as the gravitational constant in Newton’s Law of Universal Gravitation.

Traditionally, the development of the spatial interaction model put an emphasis on the interpretability of the results (Kirby, 1970; Wilson, 1967, 1971). But instead of relying on the heuristic of Newtonian gravity (e.g., some interaction between a mass at  $i$  and a mass at  $j$  separated by some distance), Wilson’s approach was to maximise the entropy of the system. Entropy maximisation in this case achieves stable results as a statistical average that represents the population. The approach works by assuming

undifferentiated individual interactions, and assessing their probabilities of making a particular journey. The result of Equation 1.11 then is a statistical average (Senior, 1979; Wilson, 1971).

To ensure that  $T_{ij}$  in Equation 1.11 is in the unit of trips (our unit of origin-destination spatial interaction), additional knowledge about the system is required. At the very least, this framework assumes that the total number of trips in the system  $T$  is known, and therefore:

$$\sum_i \sum_j T_{ij} = T \quad (1.12)$$

Additional information can be introduced. For example, when information is available about the total number of trips produced by each origin,  $W_i^{(1)}$  is represented as  $O_i$  and the following constraint can be used:

$$\sum_j T_{ij} = O_i \quad (1.13)$$

Alternatively, if there is information available about the total number of trips attracted by each destination,  $W_j^{(2)}$  is represented as  $D_j$  and the following constraint can be used:

$$\sum_i T_{ij} = D_j \quad (1.14)$$

It is also possible to have information about both  $O_i$  and  $D_j$ , in which case both constraints could be imposed on the model at once.

Using information about the system that satisfy these constraints fully, partially or not at all, a family of spatial interaction models can be derived based on Equation 1.11.  $K$  is specified depending on the applied constraint(s). In the framework introduced in Wilson (1971), three constrained versions are specified: the first being a case where results only match the total volume of interaction, the second being a singly constrained case, and the third a doubly constrained case.

In the first, Equation 1.13 and Equation 1.14 do not hold. In practical terms, this means that the total number of trips predicted by the model must be equal to sum of all flows from origins  $i$  to destinations  $j$ . The balancing constant  $K$  in this case is (see Cliff, Martin, & Ord, 1974; A. S. Fotheringham, 1984):

$$K = \frac{T}{\sum_i \sum_j T_{ij}} \quad (1.15)$$

In the second case, only one of Equation 1.13 or Equation 1.14 hold. The resulting models are, in Wilson’s terms, singly constrained. When only Equation 1.13 holds, entropy maximisation leads to the following production-constrained model:

$$T_{ij} = A_i O_i W_j^{(2)} f(c_{ij}) \quad (1.16)$$

Notice how, in this model, the proxy for the mass at the origin  $W_i^{(1)}$  is replaced with  $O_i$ , representing what we know about the system, the spatial interaction outbound flow, i.e., outbound trips produced at  $i$ . Also, there is no longer a single system-wide proportionality constant, but rather a set of proportionality constants (i.e., balancing factors) specific to origins. For this model, the balancing factors ensure that Equation 1.13 is satisfied, meaning that the sum of predicted flows from one origin going to all destinations must equal the known mass at that origin  $O_i$  i.e., the total number of outbound trips. Satisfying this constraint also implicitly fulfills the total constraint (Equation 1.12), since the sum of  $O_i$  values across all origins equals the total number of trips. This model is useful when trips ends are unknown but the number of trips originating from each location is known and the total of these trips represents all trips in the system. The balancing factors for the production-constrained model are solved for each origin  $A_i$ , and according to Wilson are:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \quad (1.17)$$

The attraction-constrained model is similar to the production-constrained model as it is also singly constrained but from the perspective of the mass at the destination. From the attraction-constrained model, the proxy for the mass at the destination  $W_j^{(2)}$  is now replaced with  $D_j$ , representing the spatial interaction inbound flow, i.e., trips attracted at the destination and takes the following form:

$$T_{ij} = B_j D_j W_i^{(1)} f(c_{ij}) \quad (1.18)$$

In this model (Equation 1.18), the balancing factors ensure that Equation 1.14 is satisfied (hence the total constraint Equation 1.12 is as well), meaning that the sum of predicted flows going to one destination from all origins must equal the known mass of that destination  $D_j$  i.e., the total number of inbound trips to  $j$ . This should hold for all destinations. As before, destination-specific proportionality constants (i.e., balancing factors)  $B_j$  were derived by Wilson as:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \quad (1.19)$$

The third case in the family of spatial interaction models is the production-attraction constrained model. In this case, both Equation 1.13 and Equation 1.14 hold simultaneously. These constraints ensure that the sum of predicted flows from one origin to all destination, and the predicted flows going to one destination from all origins must equal the known mass of the origin  $O_i$  and of the destination  $D_j$ . This should hold for all origins and destinations. The resulting model is, in Wilson’s terms, doubly constrained, and takes the following form:

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \quad (1.20)$$

In this model, both proxies for the masses are replaced with the known masses, that is, the trips produced by the origin and the trips attracted by the destination. There are now two sets of mutually dependent proportionality constants:

$$\begin{aligned} A_i &= \frac{1}{\sum_j B_j D_j f(c_{ij})} \\ B_j &= \frac{1}{\sum_i A_i O_i f(c_{ij})} \end{aligned} \quad (1.21)$$

Derivation of these models is demonstrated in detail elsewhere (e.g., Ortúzar & Willumsen, 2011; Wilson, 1967). It is worth noting, however, that although Wilson’s approach is built on a different conceptual foundation than the old reference to Newtonian gravity, the work succeeded at identifying the steps from proportionality to equality to yield variations of proportionality constants, including the one that eluded Stewart (1948) and that has been overlooked in almost all subsequent accessibility research. Why was this key element of spatial interaction models potentially ignored in accessibility research? In the next section we aim to address this question.

## 1.7 Accessibility and spatial interaction modelling: two divergent research streams

The work of Hansen (1959) and Wilson (1971) responded to important developments, in particular a need “to meet the dictates and needs of public policy for strategic land use and transportation planning” (Michael Batty, 1994). These dictates and needs were far from trivial. In the United States alone, the Federal-Aid Highway Act of 1956 authorized the creation of the U.S. Interstate Highway System, with a budget that ultimately exceeded one hundred billion dollars (equivalent to over \$600 billion in 2023) (MDOT, 2007; Weiner, 2016). Spatial interaction modelling was incorporated into institutional modelling practices meant to “predict and provide”, i.e., predict travel demand and supply transportation infrastructure (Kovatch, Zames, et al., 1971; Weiner, 2016). Accessibility, at the time, did not quite have that power, as it did not quantify trips, but rather something somewhat more elusive: it predicts the less tangible “potential” for spatial interaction with opportunities. In this way, where spatial interaction modelling became

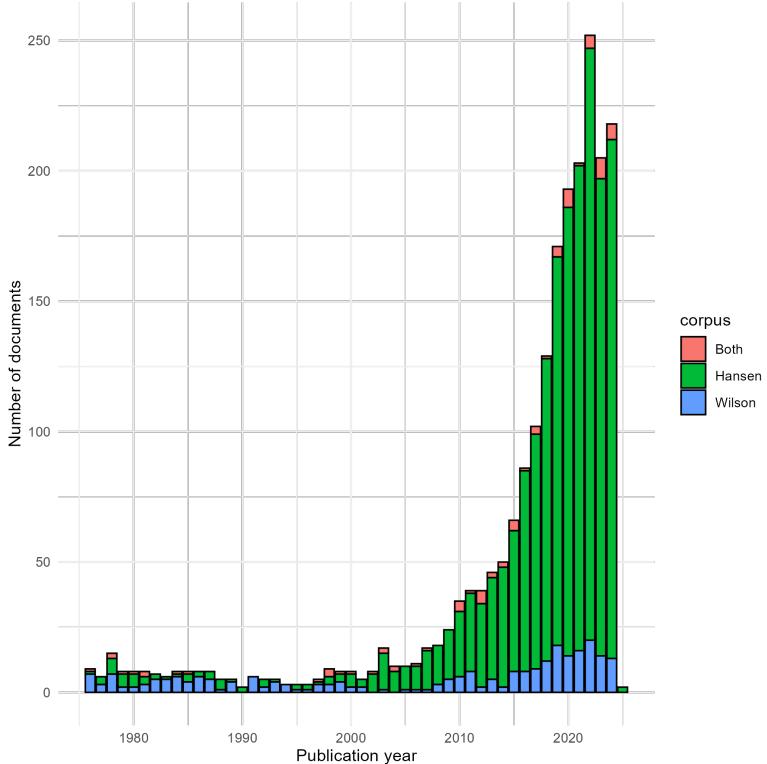


FIGURE 1.1: Historical pattern of publication: documents per year.

a key element of transportation planning practice, accessibility remained a somewhat more academic pursuit, and the two streams of literature only rarely connected.

To illustrate this point, we conducted a bibliographic analysis of the literature that cites Hansen (1959), Wilson (1971), or both. We retrieved all relevant documents using the Web of Science “Cited References” functionality, and the digital object identifiers of Hansen (1959) and Wilson (1971). As a result of this search, we identified 1,788 documents that cite Hansen (1959), only 258 documents that cite Wilson (1971), and 76 that cite both. The earliest document in this corpus dates to 1976 and the most recent is from 2025. The number of documents per year appears in Figure 1.1, where we see the frequency of documents over a span of almost fifty years. In particular, we notice the remarkable growth in the number of papers that cite Hansen (1959) compared to those that cite Wilson (1971) since the year 2000.

As noted, literature that cites both Wilson (1971) and Hansen (1959) are sparse (only 3.6% of the corpus, visualised in pink in Figure 1.1). After reading the works, we can also discern that they too are divergent, with one stream focused on developing accessibility (i.e., *potential for spatial interaction*) and another on spatial interaction. The focuses of these divergent streams contribute this paper’s broader hypothesis that the concepts of accessibility and spatial interaction have remained largely disconnected, and at times,

improperly conflated.

On one hand, the stream of literature focused on spatial interaction models inspired by Wilson (1971) and which cite both Hansen (1959) and Wilson (1971), tends to contribute to understanding how accessibility is interpreted and incorporated in spatial interaction models. These works treat separate spatial interaction and accessibility as a separate but related phenomenon. Specifically, some early works interpret the spatial interaction model's balancing factors (Equation 1.17 or Equation 1.21 as the inverse of Hansen (1959)'s model (A. Stewart Fotheringham, 1981; A. S. Fotheringham, 1985; B. Harris & Wilson, 1978; G. Leonardi, 1978), recognizing it as a “common sense” approach (Morris, Dumble, & Wigan, 1979, p. 99) to including accessibility in the spatial interaction model, though further exploration of its relationship is warranted (M. Batty & March, 1976). Some authors have explored this relationship, for instance as in A. S. Fotheringham (1985) who demonstrates how the spatial interaction model may insufficiently explain spatial patterns, and suggests that explicitly defining destinations' accessibility as a variable within the model may remedy the issue (e.g., the *competition destination* model). Other works used both Hansen (1959) and Wilson (1971)'s framework in conjunction, such as in defining location-allocation problems in operations research (Beaumont, 1981; G. Leonardi, 1978), estimating trip flows (or some other spatial interaction flows) alongside accessibility (e.g., Clarke, Eyre, & Guy, 2002; Grengs, 2004; Türk, 2019), or considering accessibility within spatial interaction models, in line with A. S. Fotheringham (1985)'s demonstration (e.g., Beckers et al., 2022). Other works departed from Hansen (1959)'s definition and aligned with spatial interaction in different ways, such as using micro-economic consumer behaviour concepts to express potential for spatial interaction (Giorgio Leonardi & Tadei, 1984; Morris et al., 1979).

On the other hand, there is another subset of literature that cite both Hansen (1959) and Wilson (1971) that is accessibility-focused. We categorise their citation of Wilson (1971) for three general reasons. Firstly, a group of these works cite Wilson (1971) as attribution for using context-dependent travel cost functions (Ashiru, Polak, & Noland, 2003; Caschili, De Montis, & Trogu, 2015; Chia & Lee, 2020; Grengs, 2015; S. L. Handy & Niemeier, 1997; Kharel, Sharifiasl, & Pan, 2024; Kwan, 1998; Margarida Condeço Melhorado, Demirel, Kompil, Navajas, & Christidis, 2016; Pan, 2013; Pan, Jin, & Liu, 2020; Rau & Vega, 2012; Roblot, Boisjoly, Francesco, & Martin, 2021; Sharifiasl, Kharel, & Pan, 2023; Q. Shen, 1998a; e.g., J. W. Weibull, 1980). These works do not necessarily comment on spatial interaction explicitly. Secondly, another group of this accessibility-focused “both” citing literature *does* associate spatial interaction as defined in Wilson (1971) with accessibility's potential for spatial interaction more explicitly (Giuliano, Gordon, Pan, & Park, 2010; Grengs, 2010a, 2012; Grengs, Levine, Shen, & Shen, 2010; He, Li, Yu, Liu, & Huang, 2017; Levine, Grengs, Shen, & Shen, 2012; Levinson & Huang, 2012; X. Liu & Zhou, 2015; e.g., H. J. Miller, 1999; Naqavi, Sundberg, Västberg, Karlström, & Hugosson, 2023; Ng, Roper, Lee, & Pettit, 2022; Suel et al., 2024; Tong, Zhou, & Miller, 2015; Wu & Levinson, 2020). We agree accessibility and spatial interaction are related topics: accessibility is an expression of its *potential* and the Wilson (1971) paper briefly touches on the concept. However, in some of this literature, Hansen (1959) and

Wilson (1971) are co-cited as both being ‘gravity models’ (Chia & Lee, 2020; Dai, Wan, & Gai, 2017; e.g., S. Liu & Zhu, 2004; Y. Shen, 2019), perhaps revealing the murkiness of the distinction between spatial interaction and the *potential for* spatial interaction in the literature. Thirdly, there is a group of accessibility-focused works that interpret Hansen (1959)’s model as the singly- or doubly- constrained spatial interaction model’s inverse balancing factor (e.g., R. W. Vickerman, 1974). This group cities the earlier spatial interaction works that make this connection and is especially prominent in the investigation of competitive accessibility topics (Albacete, Olaru, Paül, & Biermann, 2017; Allen & Farber, 2020; Alonso, Beamonte, Gargallo, & Salvador, 2014; Chen & Silva, 2013; Curtis & Scheurer, 2010; El-Geneidy & Levinson, 2011; Karst T. Geurs, van Wee, & Rietveld, 2006; e.g., Karst & Van Eck, 2003; Levinson & Wu, 2020; Manaugh & El-Geneidy, 2012; Marwal & Silva, 2022; Mayaud, Tran, Pereira, & Nuttall, 2019; Sahebgharani, Mohammadi, & Haghshenas, 2019; Su & Goulias, 2023; Willigers, Floor, & van Wee, 2007). As outlined in preceding sections, we argue interpreting the singly- or doubly- constrained spatial interaction model’s balancing factor as accessibility yields output values that are similarly plagued by interpretability issues.

Lastly, as an extension of the third reason within this group, only the works of Soukhov et al. (2023) and Soukhov et al. (2024) use Wilson (1971)’s balancing factors as a method for maintaining constraints on opportunities within the context of competitive accessibility. These works introduce the balancing factors as a mechanisms to ensure that opportunities at each destination are proportionally allocated to each zone (based on the proportion of population seeking opportunities and the relative travel impedance). This is to ensure that each zonal accessibility value is the sum of this proportional allocation from each destination, and that all zonal values ultimately sum to the total number of opportunities in the region. However, these balancing factors were deduced intuitively. These works do not explicitly state that the mathematical formulation of the equations are effectively equivalent to Wilson’s singly constrained model (derived from entropy maximization); this equivalence was only discovered in hindsight. These two works also do not discuss other constrained cases, which will be addressed in the next chapter.

## 1.8 Chapter conclusion

In this chapter, the Newtonian roots of human spatial interaction modelling was traced from Ravenstein (1885), to Hansen (1959)’s application of Stewart (1947)’s ‘population potential’ (Equation 1.6) and then to Wilson (1971)’s family of spatial interaction models. We detailed how the balancing factors from Wilson’s family of spatial interaction models are formulated to reflect known system constraints (i.e., the total constraint, and production and attraction constraints in Equations 1.12, 1.13 and 1.14).

Literature that cite both seminal works is then reviewed (Hansen (1959) and Wilson (1971)). It is found that these works are divergent: some are spatial interaction focused, citing Hansen (1959) as the father of the accessibility concept and uses accessibility as a variable within spatial interaction modelling. Another set of work is accessibility focused, and what we may now know as the modern accessibility literature beginning in the early

2000s. These accessibility focused works cite Wilson (1971) for various general reasons, but none (other than Soukhov et al. (2023) and Soukhov et al. (2024)) cite the logic of the balancing factors. Beginning with this logic in the following chapter, balancing factors are introduced akin to those used in Wilson’s family of spatial interaction models. They are formulated to incorporate system-wide or zonal constraints (i.e., knowns) to accessibility. In this way, the following chapter introduces a ‘family of accessibility measures’. Each member and variant of the family is mathematically formulated and solved using a simple toy example.

# Chapter 2

## CHP 2 - A family of accessibility measures

### 2.1 Chapter overview

This chapter consists of text from two journal articles: first, from the second half of the “*Family of accessibility measures derived from spatial interaction principles*” authored by Anastasia Soukhov, Rafael H. M. Pereira, Christopher D. Higgins, and Antonio Páez (submitted to *PLOS ONE* in May 2025) that details the formulation of the family of accessibility measures and solved simple numeric examples, and second, from the manuscript detailing the multimodal extension of spatial availability (Soukhov et al., 2024).

The objective of this chapter is to detail the formulation of the family of accessibility measures which are based on spatial interaction principles. These formulations are used in the next chapters of this dissertation. This chapter first details four cases of the family: unconstrained, total constrained, singly constrained and doubly constrained accessibility measures. Each case has two variants, either accessible ‘opportunities’ (i.e., the way Hansen (1959) is understood) or accessible ‘populations’ (i.e., market potential, the way Reilly (1929) can be understood). An empirical example will be solved using the total constrained and singly constrained cases, so for these cases their multimodal extensions are defined. Lastly, a demonstration of a solved simple toy example is included following the formulation of each case and all the cases are summarised in the conclusion.

This chapter offers two general contributions:

- The formulaic specification of four cases of the family of accessibility measures, along with different variants and multimodal extensions: specifically, the ‘unconstrained’ measure (i.e., Hansen-type measure), the ‘total constrained’ measure (i.e., a constrained version of the Hansen-type measure), the ‘singly constrained’ measure (i.e., related to the popular two step floating catchment approach (2SFCA)), and the ‘doubly constrained’ measure representing realized interactions or ‘access’, effectively equal to the doubly constrained spatial interaction model in formulation.

- Summary of the interpretability advantages of the family using a worked simple toy example, as these constrained accessibility measures yield values in units of the number of potential “opportunities for spatial interaction” or “population for spatial interaction” for each zone and zonal flow.

## 2.2 Outline of the family of accessibility measures

As argued in the preceding chapter, the streams of research on accessibility and spatial interaction modelling have evolved as largely separate streams with little contact since Hansen (1959) and Wilson (1971). This may explain why the constraints and associated balancing factors of spatial interaction models did not cross over to accessibility analysis. This is intriguing since Wilson made an effort to connect developments in spatial interaction modelling to accessibility, noting for instance, the denominator of the proportionality constants specific to origins (Equation 1.17) is the inverse of balancing factor  $A_i$  (Wilson, 1971, p. 10):

$$S_i = \frac{1}{A_i} = \sum_j W_j^{(2)} f(c_{ij}) \quad (2.1)$$

Understanding  $S_i$  as the inverse of Wilson’s  $A_i$  does not uncover any new meaning for  $S_i$  itself. Indeed, mathematically it is true,  $A_i$ ’s role in Wilson’s general model  $T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij})$  is that of a balancing factor  $k$  i.e., keeping units balanced and proportionality based on constraints. Understanding  $S_i$  itself as a balancing factor is not wholly helpful as Hansen and (Stewart before him) defined accessibility as a partial sum of the demographic force  $F$  or the system-wide population potential of opportunities for spatial interaction (i.e.,  $V_i = \sum_j \frac{M_j}{d_{ij}}$  after losing  $G$ ).

Hence, we propose stepping back to introduce a revised definition of accessibility: the *constrained* potential for spatial interaction. Once we bring back Wilson’s proportionality constant  $k$  into the picture, we can define the *potential for spatial interaction* between two locations  $i$  and  $j$  is as follows:

$$V_{ij} = kW_j^{(2)} f(c_{ij}) \quad (2.2)$$

where  $V_{ij}$  is the potential for interaction from  $i$  to  $j$ . The accessibility from origin  $i$  can then be summarised as a partial sum of the potential at  $i$ :

$$V_i = k \sum_j W_j^{(2)} f(c_{ij}) \quad (2.3)$$

Similar to Equation 1.11,  $W_j^{(2)}$  above is the mass at the destination and the sub-indices are for  $i = 1, \dots, n$  origins and  $j = 1, \dots, m$  destinations.

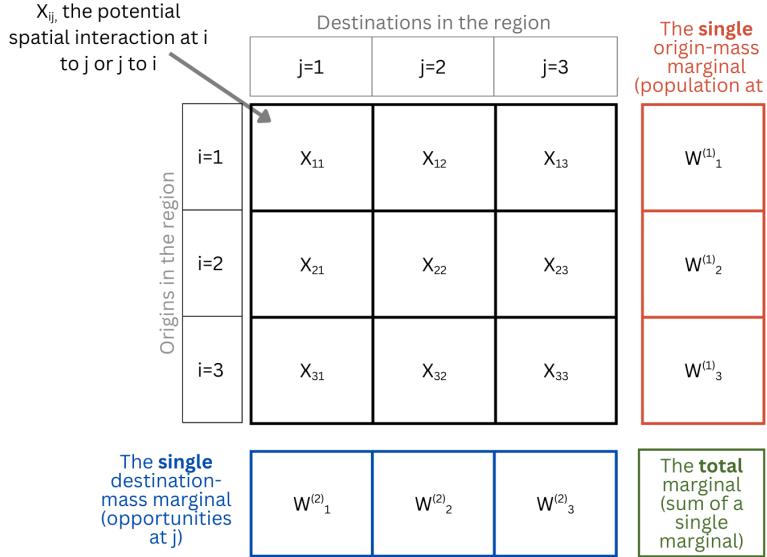


FIGURE 2.1: The family of accessibility measures analytical framework: labelling and associating  $ij$  flows, zonal weights, the single marginals, and the total marginal.

The market potential variant can also be generally defined, which is effectively the transpose of  $i$  and  $j$  in Equation 2.2 and Equation 2.3 as follows:

$$M_{ji} = kW_i^{(1)}f(c_{ij}) \quad (2.4)$$

$$M_j = k \sum_i W_i^{(1)} f(c_{ji}) \quad (2.5)$$

Similar to Equation 1.11,  $W_j^{(2)}$  above is the mass at the destination and the sub-indices are for  $i = 1, \dots, n$  origins and  $j = 1, \dots, m$  destinations.

To detail the anatomy of  $V_{ij}$  and  $M_{ji}$  along with the partial sums of  $V_i$  and  $M_j$ , Figure 2.1 illustrates the accessibility analytical framework we propose using a simple 3 zone system. Each measure's most disaggregate value is  $X_{ij}$ , potential for spatial interaction from  $i$  to  $j$ . The  $X$  is a stand in for the  $ij$  values of all the cases, their variants and multimodal extensions that will be described (i.e.,  $V_{ij}^0, V_{ij}^{m0}, M_{ji}^0, M_{ji}^{m0}, V_{ij}^T, V_{ij}^{mT}, M_{ji}^T, M_{ji}^{mT}, V_{ij}^S, V_{ij}^{mS}, M_{ji}^S, M_{ji}^{mS}$  and  $V_{ij}^D, M_{ji}^D$ ). The single marginals represent the origin-side and destination-side weights of the zones. The total marginal represent the sum of a single marginal.

As an additional overview, we define four distinct members of the constrained accessibility measure family, all delineated based on their constant  $k$ , which takes the form

of either balancing factors  $K^T$ ,  $B_j$ ,  $A_i$  (and their multimodal  $m$  versions) depending on the indicator:

1. **The unconstrained case** with variants  $V_i^0$  and  $M_j^0$  and multimodal extension  $V_i^{m0}$  and  $M_j^{m0}$ . The first variant  $V_i^0$  is equivalent to Hansen (1959)'s formulation and  $M_j^0$  is equivalent to Reilly (1929)'s market potential formulation. Both these variants neglect including a balancing factor, so units of zonal values are in units of 'opportunities-by-travel-impedance-value' and 'population-by-travel-impedance-value'. In this case, no constraints are introduced to ensure that values of marginals are preserved.
2. **The total constrained case** with variant  $V_i^T$  and  $M_j^T$  along with multimodal extensions  $V_i^{mT}$  and  $M_j^{mT}$ . The first variant  $V_i^T$ —the *total constrained accessible opportunities measure*—resembles Hansen (1959)'s formulation but with an additional regional balancing factor  $K^T$  term, defined as the ratio of the total number of opportunities in the region to the total sum of unconstrained accessibility values in the region. In this way,  $K^T$  ensures that each zonal accessibility value is a proportion of the total opportunities in the region (i.e., the total marginal in Figure 2.1), requires no information about the population seeking opportunities, and is in units of 'opportunities' accessible. The second variant  $M_j^T$  is the transpose of  $i$  to  $j$  of the first variant, effectively a constrained version of market potential and is referred to as *total constrained accessible population measure*. In this variant, each zonal accessibility value is a proportion of the total population in the region (maintained by  $\hat{K}^T$ ), requires no information about the opportunities that are sought, and each zonal value is in units of 'population' accessible.
- When conceptualising *potential* for spatial interaction—whether with opportunities or population—the total constrained case reflects the lowest level of restriction and hence maximum potential. The balancing factors  $K^T$  and  $\hat{K}^T$  only ensure that  $V_{ij}^T$  and  $M_{ji}^T$  values end up matching the *total sum* of one of the single marginals, not the individual marginals themselves. For example,  $K^T$  does not guarantee that all the opportunities accessibility at  $V_{i=1}^T$  reflect a proportional allocation of the destination mass (i.e., *number* of opportunities) at  $j = 1, 2, 3$ . In some cases, an allocation of opportunities from a destination  $j$  to  $i = 1$  could *exceed* the number of opportunities at that  $j$  (i.e., meaning that destination  $j$  is very attractive and reachable for an  $i$ , relative other flows in the region). However, what  $K^T$  *does* ensure is that each  $V_{ij}^T$  does not exceed the overall number of opportunities in the region—the total marginal. In other words,  $V_{i=1}^T$  expresses the number of opportunities that origin  $i = 1$  could potentially interact with, as drawn from the entire regional opportunity mass, rather than constrained by individual destination totals. In this way, the *total constrained* case reflects the maximum amount of *potential* while still maintaining interpretable units.
3. **The singly constrained case**, with two variants  $V_i^S$  (opportunity-constrained) and  $M_j^S$  (population-constrained) along with their multimodal extensions  $V_i^{mS}$  and

$M_j^{mS}$ . The first variant is mathematically equivalent to the spatial availability measure (Soukhov et al., 2023), and this variant's per capita form is equivalent to the popular 2SFCA measure (Luo & Wang, 2003a; Q. Shen, 1998b). Both variants can also be defined using either Hansen (1959)'s or the market potential formulation but with balancing factor  $B_j$  for  $V_i^S$  or  $A_i$  for  $M_j^S$ . These balancing factors ensure that the total marginal (green box in Figure 2.1) is maintained *as well as* the values at one of the single marginals (destination-mass marginal for opportunity-constrained and origin-mass marginal for population constrained in Figure 2.1). In this way, the singly constrained case reflects a medium amount of potential, restricted by either values fitting the single destination-mass marginal or the single origin-mass marginal.

- The first variant  $V_i^S$  includes a set of destination-side balancing factors  $B_j$  which ensure opportunities (or destination masses) are allocated to each origin  $i$  based on the population (i.e, origin mass) of the origin  $i$  and travel impedance from that  $j$  to all  $i$ s. Hence,  $B_j$  ensures that each  $V_i^S$  is the product sum of a *proportional share* of opportunities allocated from each destination  $j$  implicitly also being a share of the total opportunities in the region. Similar to the total constrained case, each  $V_i^S$  and  $V_{ij}^S$  accessibility value is expressed in units of 'opportunities' accessible. But unlike the total constrained case, the singly constrained case explicitly incorporates population by allocating a balanced share of a destination's total opportunities to populations at reachable origins.
  - The second variant  $M_j^S$  includes a set of opportunity-side balancing factors  $A_i$  which ensure population (or origin masses) are allocated to each destination  $j$  based on the destination mass of the destination  $j$  and all possible travel impedance from that  $i$  to all  $j$ s.  $A_i$  ensures similar, but transposed, so constraints are satisfied. Specifically, each  $M_j^S$  represents both a share of the total regional population and the sum of balanced proportions of population from all origins, allocated to each destination based on the opportunities sought and travel impedance. Each zonal value is expressed in units of 'population' accessible.
4. **The doubly constrained case.** It is constrained simultaneously by population (origin masses) and opportunities (destination masses), so each  $V_{ij}^D$  (equivalent to  $M_{ji}^D$ ) is expressed in units of 'population-opportunity capacity' that is accessible between  $i$  to  $j$ . This case requires the number of opportunities and population to match, e.g., the analyst must know the matching spatial interaction capacity of the population (demand) and opportunities (supply). By using this one-to-one matching data and the double constraints (i.e.,  $A_i$  and  $B_j$  at once, ensuring both the double constraint maintains both marginals in Figure 2.1), this case restricts the *potential* to spatially interact completely. In other words, the doubly constrained accessibility values reflect the number of predicted interactions between the opportunities and population, effectively, this case is equivalent formulaically as Wilson's doubly constrained spatial interaction model (i.e., *attraction-production*

*constrained*). It can be understood as predicting a value of ‘access’, and not accessibility.

And as a summary: each member of the family of accessibility measure is named, explained in plain language, alongside their balancing factor(s), proportional allocation factor(s), and mathematical equation and value interpretations in Table 2.1.

TABLE 2.1: Summary of family of accessibility measure members, their definitions and interpretations

|  | Unconstrained<br>( $V_i^0$ , $M_j^0$ )  | Total Constrained ( $V_i^T$ , $V_i^{mT}$ ,<br>$M_j^T$ , $M_j^{mT}$ )  | Singly Constrained<br>( $V_i^S$ ,<br>$V_i^{mS}$ , $M_j^S$ , $M_j^{mS}$ )   | Doubly Constrained<br>( $V_{ij}^D$ ,<br>$M_{ij}^D$ )  |
|--|---|---|--|---|
| <b>Measure Equation</b>                            | $V_i^0 = \sum_j W_j^{(2)} f(c_{ij});$<br>$M_j^0 = \sum_i W_i^{(1)} f(c_{ij})$   | $V_i^T = \sum_j \kappa_{ij}^T W_j^{(2)};$<br>$M_j^T = \sum_i \hat{\kappa}_{ji}^T W_i^{(1)}$<br>Multimodal:<br>$V_i^{mT} = \sum_j \kappa_{ij}^{mT} D_j;$<br>$M_j^{mT} = \sum_i \hat{\kappa}_{ji}^{mT} O_i$   | $V_i^S = \sum_j \kappa_{ij}^S D_j;$<br>$M_j^S = \sum_i \hat{\kappa}_{ji}^S O_i$<br>Multimodal:<br>$V_i^{mS} = \sum_j \kappa_{ij}^{mS} W_j^{(2)};$<br>$M_j^{mS} = \sum_i \hat{\kappa}_{ji}^{mS} W_i^{(1)}$  | $V_{ij}^D = A_i B_j O_i D_j f(c_{ij})$  |
| <b>Constraint Explanation and Balancing Factor</b> | No constraints; marginals not equal to any regional or zonal knowns.<br>Multimodal extension, where other modes $m$ are a function of $V_i^0$ and $M_j^0$ is not specified – however, a specific $V_i^0$ and $M_j^0$ value for a mode can be expressed, and is represented as $V_i^{m0}$ and $M_j^{m0}$ | Balancing factors $K^T$ and $\hat{K}^T$ ensures the sum of $ij$ values matches the total marginal, where:<br>$K^T = \frac{D}{\sum_i V_i^0};$<br>$\hat{K}^T = \frac{O}{\sum_j M_j^0}$<br>Multimodal balancing factors ensuring the $ij$ matrix for each $m$ is a proportion of the marginal, where:<br>$K^{mT} = \frac{D}{\sum_m \sum_i V_i^{m0}};$<br>$\hat{K}^{mT} = \frac{O}{\sum_m \sum_j M_j^{m0}}$ | Single balancing factor $B_j$ (for $V_i^S$ ) that ensures the destination-mass marginal is constrained, and $A_i$ (for $M_j^S$ ) ensures the origin-mass marginal is constrained:<br>$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})};$<br>$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})}$<br>Multimodal extension:<br>$B_j^m = \frac{1}{\sum_i W_i^{m(1)} f^m(c_{ij}^m)};$<br>$A_i^m = \frac{1}{\sum_j W_j^{m(2)} f^m(c_{ij})^m}$ | Values reflect both single marginals simultaneously, maintained via $A_i$ and $B_j$ .<br>Multimodal extension, where other modes $m$ are a function of $V_i^0$ and $M_j^0$ is not specified in this work – however, balancing factors that are multimodal could be solved for, but is out of this work’s scope. |
| <b>Proportional Allocation Factor</b>              | None  | Allocates the total marginal as opportunities proportionally based on $\kappa_{ij}^T = K^T f(c_{ij})$ and as population based on $\hat{\kappa}_{ji}^T = \hat{K}^T f(c_{ij})$ .<br>Multimodal extension:<br>$\kappa_{ij}^{mT} = B_j^m W_i^{m(1)} f^m(c_{ij}^m);$<br>$\hat{\kappa}_{ji}^{mT} = A_i^m W_j^{m(2)} f^m(c_{ij}^m)$  | Allocates the single opportunities marginal proportionally based on $\kappa_{ij}^S = B_j W_i^{(1)} f(c_{ij})$ in the case of $V_i^S$ and the single population marginal proportionally based on $\hat{\kappa}_{ji}^S = A_i W_j^{(2)} f(c_{ij})$ ;<br>Multimodal extension:<br>$\kappa_{ij}^{mS} = B_j^m W_i^{m(1)} f^m(c_{ij}^m);$<br>$\hat{\kappa}_{ji}^{mS} = A_i^m W_j^{m(2)} f^m(c_{ij}^m)$                            | —   |
| <b>Interpretation</b>                              | Values in various units depending on the impedance and destination-mass (e.g., “opportunities x decay”) for $V_i^0$ and impedance and origin-mass (e.g., “population x decay”); no total or marginal constraint   | Values reflect a share of total regional opportunities ( $V_i^T$ ) or population ( $M_j^T$ ).   | $V_i^S$ values reflect a share of the opportunities at each destination based on origin population ‘demand’ and impedance; $M_j^S$ values reflect a share of the population at each origin based on destination opportunities ‘supply’ and impedance.  | The spatial interactions between population and opportunities (i.e., access).   |

### **2.2.1 [NEW - from multimodal SA paper] A brief review of multimodal accessibility measures**

Unconstrained accessibility, namely does not consider the opportunities allocated (or population) as finite. To cite an example, Tahmasbi et al. (2019) (Tahmasbi, Mansouri-anfar, Haghshenas, & Kim, 2019) uses Hansen-type accessibility to assess the potential interaction with retail locations by three modes: walking, public transit, and car (i.e.,  $m = w, p, c$ ).  $S_i^m$  is the sum of retail locations  $j$  that can potentially be reached under the travel impedance as calculated for each  $i$  and  $m$ . In other words, for each origin  $i$  three accessibility scores are calculated. Tahmasbi et al. (2019) shows that car travel affords the highest  $S_i^m$  values in the majority of  $i$  i.e., travelers who use a car can potentially reach more retail opportunities than populations using other modes. However, higher  $S_i^m$  values for car do not affect the values of  $S_i^m$  for other modes: in effect, each mode is analysed as if the others did not exist. Since the measure is not constrained, each opportunity is typically counted multiple times within and between modes, and as a result the sum of accessibility is not necessarily a meaningful quantity. The accessibility scores for the modes are often values that are difficult to interpret beyond making statements about relative size.

As another example, Lunke (2022) reports accessibility scores for car in the order of tens of thousands of employment opportunities in the Oslo region. The corresponding scores for transit are lower, but still often in the thousands or tens of thousands. As reported, the ratio of the transit to the car score can be lower than 0.2 (meaning transit gives access to less than 20% of the opportunities than car). But despite the discussion about “sufficient accessibility”, it is unclear what the unconstrained scores mean: is having access to 10,000 jobs by transit insufficient? After all, 10,000 employment opportunities are still plenty of opportunities. These ratios can be found elsewhere in the literature e.g., Figs. 7, 8, and 9 in Páez et al. (2010) (A. Páez, Mercado, Farber, Morency, & Roorda, 2010), Fig. 5 in Páez et al. (2010) (Antonio Páez et al., 2010), Páez et al. (2013) Figs. 6, 7, and 8 in (Antonio Páez et al., 2013). They are useful as relative assessment of when some members of the public are better or worse off than others, but they are silent on how bad is “worse off”.

Besides ratios of accessibility, another way seen in the literature to improve interpretability of scores is to standardise them within a [0-1] range. This adjustment is only helpful insofar as it facilitates relative comparisons, but interpretation of the scores remains challenging because the values are specific to a region and convey no meaning about the magnitude of the scores. In this approach, zones always have values between 0 and 1, but how remarkable is a zone with a low score for pedestrians and a high value for car? And if remarkable, what does the difference in these standardized values mean for planners? By how much should transport systems and land-use configurations be changed to improve conditions? And in what way can these scores be used to track differences over time? Or between regions? These questions lack straightforward answers since certain values will always be relatively ‘low’ or ‘high’, but do not track to a quantity that can be intuitively understood. Presentation or discussion of Hansen-type

accessibility that has been standardised in this way is not uncommon in the literature (Campbell, Rising, Klopp, & Mbilo, 2019; Maharjan, Tilahun, & Ermagun, 2022).

If we understand opportunities to be finite and/or subject to some levels of congestion, it is possible for an accessibility measure to take on a crisper meaning. Accessibility research has a history of considering opportunity competition, especially regarding school-seats, hospital capacity, and employment opportunities (Cui et al., 2020; Grengs, 2010b; Higgs, Zahnow, Corcoran, Langford, & Fry, 2017; Joseph & Bantock, 1982; Kawabata & Shen, 2006; Kelobonye, Zhou, McCarney, & Xia, 2020; Kwok & Yeh, 2004; Li & Wang, 2024; Mao & Nekorchuk, 2013; Merlin & Hu, 2017; Morris et al., 1979; Q. Shen, 1998a; Soukhov et al., 2023; Van Wee, Hagoort, & Annema, 2001; Jörgen W. Weibull, 1976). If one person reaches an opportunity - it is taken: the supply of an opportunity and the demand for that opportunity are the nodes in accessibility analysis. These types of opportunities are unambiguously exclusive.

Amenities are a good example of this. For instance, standards for providing green spaces are often stated in the form of *exclusive access*, in units of amenity per capita. For example, a 2013 planning document for the Ile-de-France region suggested a public green space municipal standard of  $10\ m^2$  *per inhabitant* (Liotta, Kervinio, Levrel, & Tardieu, 2020). Green spaces are not evenly distributed, meaning those who have access to them depends on where they are and how easy they are to reach. This formulation of amenity provision is not unusual. As another example, Natural England recommends a national “accessible natural greenspace standard” such that the minimum supply of space is  $1\ ha$  of statutory local nature reserves per thousand population (Natural England, 2010). Similarly, the World Health Organization (OECD, 2013) recommends that cities provide a minimum of  $9\ m^2$  of green area per inhabitant. For our purposes, standards of this type translate into “how much of this resource is available to one individual that has not been claimed by anyone else?”. Green spaces often have large capacities, but they still have a capacity and it is not the same for a person to have access to  $5\ m^2$  of *uncongested* green space as  $15\ m^2$ . This difference is in fact a matter of justice (Lara-Valencia & García-Pérez, 2015; Liotta et al., 2020). Constraining accessibility is a useful way to evaluate the congested availability of any type of opportunity. As development of sound standards is emphasized in the planning literature, in particular in regards to fairness in transportation (Martens & Golub, 2021), spatial availability analysis can be used to develop and assess standards.

The relevance of the considerations above is put in sharper relief when we think about the use of multiple modes (or heterogeneous populations). If we return to Oslo for a moment (Lunke, 2022), we notice that the places that have high accessibility by transit are also the places that have *very high* accessibility by car (in their Figure 2). Those two populations are going for the same opportunities, and those travelling by transit have fewer to choose from the start. More generally, people in a zone who are advantaged with relatively low cost of travel will have the ability to potentially reach more opportunities than other people. Due to this advantage, through the perspective

of finite opportunities, there are fewer opportunities left for everyone else, especially for those who use modes that are slower or otherwise more expensive.

‘Competitive’ accessibility was the rationale for developing floating catchment area methods (FCA), popularized in Luo & Wang (2003b) who reformulated the work of Q. Shen (1998a) into two steps (although similar, and earlier, developments are found in (Joseph & Bantock, 1982; Jørgen W. Weibull, 1976)). Shen-type accessibility is formulated as:  $a_i^m = \sum_j \frac{O_j f^m(c_{ij}^m)}{\sum_m D_j^m}$  where  $D_j^m$  is the potential demand for opportunities equal to travel impedance weighted population  $\sum_i P_i^m f^m(c_{ij}^m)$  and the remaining variables are repeated in the Hansen-type measure. Shen-type modal accessibility ( $a_i^m$ ) can be understood as a ratio of the travel impedance-weighted supply of opportunities for  $m$ -mode in  $i$  over the travel impedance-weighted demand for opportunities. In this way, it considers competition. That said, the measure remains unconstrained, meaning both population and opportunities are multiply counted (Antonio Paez et al., 2019). In other words, interpretation of the Shen-type accessibility scores between modes is fraught, as it is for Hansen-type measures.

To illustrate, Tao et al. (2020) calculates  $a_i^m$  to jobs for different income-group populations in Shenzhen, China for those using transit and car. Their results indicate that zones with low-income populations have lower  $a_i^m$  than zones with higher-income populations. Further, they show that  $a_i^{\text{transit}}$  is lower than  $a_i^{\text{car}}$  in many zones, arguing that this may further place those zones with lower-income populations at a disadvantage.  $a_i$  and/or  $a_i^m$  are used to compare relative spatial differences in overall competitive accessibility and multimodal competitive accessibility, but because opportunities were doubly counted (entering the sums of both modes), this makes for uneasy interpretations of the differences in  $a_i^m$  between modes. Questions that this approach leaves unaddressed include: what is the impact of competition on the difference in  $a_i^m$  values? How does the impact vary spatially? And what is the interpretation of this difference?

The family of accessibility measures improves on the discussed Hansen-type and Shen-type accessibility approaches by constraining the sum of opportunities, that is, by treating them as finite. This is done by means of proportional allocation factors that follow well established principles of spatial interaction and the gravity model (Wilson, 1971). These principles consider: the cost of travel from different zones for a certain traveling group  $m$  compared to all groups (e.g., some sub-populations face relatively higher or lower costs), and if singly-constrained, the mass effect of  $m$  compared to all other  $ms$  e.g., the mass at different origins or destinations).

## 2.3 Setup of the simple numeric example

Consider a simple region as shown in Figure 2.1, with zone IDs 1, 2 and 3. Each zone is both an origin  $i$  and a destination  $j$ . The following three pieces of information are defined: zonal population and opportunities, zonal cost matrix, and travel impedance functions for three types of travel behaviour. In this example, the population are people,

TABLE 2.2: Simple system with three zones (ID 1, 2 and 3). Population is in 10,000 persons and opportunities in number of physicians

| ID (i or j) | Population <sup>1</sup> | Opportunities <sup>2</sup> |
|-------------|-------------------------|----------------------------|
| 1           | 4                       | 160                        |
| 2           | 10                      | 150                        |
| 3           | 6                       | 180                        |

<sup>1</sup>Population is  $Wi^{(1)}$  when used as a proxy for the mass at the origin, and  $Oi$  when used as a constraint.

<sup>2</sup>Opportunities is  $Wj^{(2)}$  when used as a proxy for the mass at the destination, and  $Dj$  when used as a constraint.

TABLE 2.3: Cost matrix for system with three zones (travel time in minutes)

| Origin ID | Destination ID |    |    |
|-----------|----------------|----|----|
|           | 1              | 2  | 3  |
| 1         | 10             | 30 | 15 |
| 2         | 30             | 10 | 25 |
| 3         | 15             | 25 | 10 |

the opportunities are physicians, and the region can be described by three possible travel behaviours.

Firstly, Table 2.2 summarises the population (in units of 10,000s of people) and the opportunities (the number of physicians) per zone. [^1]. Considering Table 2.2’s values, the Provider-to-Population-Ratio (PPR) in this system is 24.5. For reference, the number of physicians per 10,000 in Canada in 2022 was 24.97 (WHO, 2025). Secondly, to pair the zonal population and opportunity information, the assumed cost of movement (in minutes of travel time) between origins and destinations is as shown in Table 2.3.

From both Table 2.2 and Table 2.3, Zone 3 and 1 can be interpreted as of an urban core with major healthcare institutions and a healthcare cluster on the edge of the city respectively, each with a moderate residing population (with zone 3 having both a higher population and physician count). Zone 2, can be interpreted as a more distant bedroom community, with a relatively high population and fewer physicians. In sum, Zones 1 and 3 are more proximate to each other than to Zone 2, and together match Zone 2’s population while offering more than twice the physician availability.

And lastly, in Equation 2.6 we distinguish accessibility measure values for the following three impedance functions that represent the potential for spatial interaction travel behaviour of the population to opportunities. Accessibility will be calculated three times for each case, one assuming the most decay ( $f_1(c_{ij})$ ), another assuming medium decay ( $f_2(c_{ij})$ ), and a third assuming the least decaying travel behaviour ( $f_3(c_{ij})$ ) for the entire region. A helpful analogy may be tying travel behaviour to the used mode’s mobility potential, i.e., the most decaying travel behaviour ( $f_1(c_{ij})$ ) would assume all travel in the region being done by foot, while calculating accessibility assuming the least decay ( $f_3(c_{ij})$ ) would assume unfettered automobility. Or alternatively, these functions could

represent travel behaviour on snowstorm-affected day ( $f_1(c_{ij})$ ) for the entire region versus a clear, ideal travel day ( $f_3(c_{ij})$ ). As an example of a discussion on how travel behaviour has been considered in accessibility measures cost of travel see A. Paez, Scott, & Morency (2012).

$$\begin{aligned} f_1(c_{ij}) &= \frac{1}{c_{ij}^3} \\ f_2(c_{ij}) &= \frac{1}{c_{ij}^2} \\ f_3(c_{ij}) &= \frac{1}{c_{ij}^{0.1}} \end{aligned} \quad (2.6)$$

Any set of concepts representing population, opportunities, and their associated travel behaviour, whether representing the entire region uniformly (as we will demonstrate) or representing specific subgroups, can be substituted into our simple example, depending on the research question. The purpose of the following simple example is to demonstrate the calculation of each member of the accessibility measure family, interpret the values, and compare them both within and across travel behaviour groups and members of the family of accessibility measures.

## 2.4 Unconstrained accessibility

Setting the balancing factor  $k$  to 1 or omitting it completely in Equation 2.2 results in the unconstrained accessibility case:

$$V_{ij}^0 = 1 * W_j^{(2)} f(c_{ij}) \quad (2.7)$$

In this case, the partial sum of spatial interaction is simply identical to Hansen's accessibility  $S_i$  (Hansen, 1959), the current standard practice in accessibility measurement:

$$V_i^0 = \sum_j V_{ij}^0 = \sum_j W_j^{(2)} f(c_{ij}) = S_i \quad (2.8)$$

The sum of the unconstrained accessibility values for each origin  $V_i^0$  generally does not equal the total number of opportunities  $O$  (e.g.,  $\sum_i V_i^0 \neq O$ ), since arbitrarily setting  $k$  to 1 (or neglecting  $k$  all together) strips the values of any meaningful unit-based interpretation, the units are in ‘summed opportunities by some travel impedance value’. Moreover, comparisons of these  $V_i^0$  values across different contexts such as different impedance functions  $f(c_{ij})$  or varying number of zones exacerbates this issue as the units between  $V_i^0$  values change i.e., comparisons between a value of ‘summed opportunities by a travel impedance value’ to a value of ‘summed opportunities by another travel impedance value’ are not directly interpretable. From this perspective, the raw unconstrained accessibility scores are not intuitively comparable across different contexts and decay functions. They should more appropriately be used as an ordinal variable to

TABLE 2.4: Simple system: unconstrained accessibility

| Origin | $V_i^0$                                       |   |   |
|--------|---|---|---|
|        | $f_1(c_{ij}) = 1/c_{ij}^3$                    | $f_2(c_{ij}) = 1/c_{ij}^2$                    | $f_3(c_{ij}) = 1/c_{ij}^{0.1}$                  |
|        | units: <i>physicians-minute</i> <sup>-3</sup> | units: <i>physicians-minute</i> <sup>-2</sup> | units: <i>physicians-minute</i> <sup>-0.1</sup> |
| 1      | 0.219   | 2.567   | 371.143   |
| 2      | 0.167   | 1.966   | 363.479   |
| 3      | 0.237   | 2.751   | 373.738   |
| Sum    | 0.6233422                                     | 7.283556                                      | 1108.361  |

make comparisons of size (i.e., greater than, less than, equal to), not to calculate ratios or intervals (i.e., the magnitude of differences).

The multimodal version of unconstrained accessibility can be represented as  $V_i^{m0}$  (Equation 3.7) for each mode  $m$  assuming mode-specific impedance functions  $f^m(c_{ij}^m)$ . However, variables of other modes (i.e., summation of all  $m$ , etc.) are not included in the equation, so  $V_i^{m0}$  is effectively equivalent to  $V_i^0$ —except  $m$  makes explicit that there are multiple groups at each  $i$  and those groups have their own  $V_i^0$  values based on  $f^m(c_{ij}^m)$ . There is no consistent approach of representing  $V_i^0$  as a function of multiple modes, other than averaging  $i$  values or other post hoc adjustments that generally do not preserve known properties of the system. From this perspective, as *other m* values are not incorporated into the measure, we assume  $V_i^0$  cannot be made multimodal, and only  $V_i^0$ , as calculated for different  $m$ s is used in this work.

$$V_i^{m0} = \sum_j V_{ij}^{m0} = \sum_j W_j^{(2)} f^m(c_{ij}^m) \quad (2.9)$$

Returning to our numeric example, the calculated unconstrained accessibility  $V_i^0$  for each origin, a sum of all the travel impedance weighted opportunities at each destination ( $\sum_i V_i^0$ ), is displayed in Table 2.4.

As the different impedance functions represent different travel behaviours, comparing the raw unconstrained accessibility values across groups is meaningless beyond notions of higher or lower. For instance, at zone 1 the difference between the least decay ( $f_3(c_{ij})$ ) and most decay ( $f_1(c_{ij})$ ) groups is 370.92, but in what units? These two values are a product of different impedance functions, making the comparison uninterpretable in absolute terms. Likewise, we could compare values within the same travel behaviour scenario across different zones, as they are in the same units, however the issue of unit interpretability will also be apparent. Considering the most decaying scenario  $f_1(c_{ij})$  and zone 1 (the zone with a healthcare cluster at the edge of the city): zone 1 captures 0.0181185 fewer *physicians-minute*<sup>-1</sup> than zone 3 (urban core). Again, the fundamental uninterpretability of what is a *physicians-minute*<sup>-1</sup> or *opportunity-weighted-travel-impedance* unit remains.

Moving onto the market potential variant, it is the transpose ( $i$  to  $j$ ) of  $V_{ij}^0$  and depicted as follows:

$$M_j^0 = \sum_i M_{ji}^0 = \sum_i W_i^{(1)} f(c_{ij}) \quad (2.10)$$

Where  $M_{ji}^0$  is the number of population- $f(c_{ij})$  at each  $j$  from  $i$ . It can be summarised for each  $j$  and hence expressed as  $M_j^0$ .

The multimodal extension of  $M_{ji}^0$  can also be defined as follows:

$$M_j^{m0} = \sum_i M_{ji}^{m0} = \sum_i W_i^{(1)} f^m(c_{ij}^m) \quad (2.11)$$

For the sake of brevity, the simple example is not solved for  $M_j^0$ : results are similarly unconstrained as shown in the unconstrained accessible population demonstration.

In sum, multimodal extensions  $V_i^{m0}$  and  $M_j^{m0}$  for the simple example are also not solved here, but for a different reason. Namely, they are not truly ‘multimodal’, as they are not a function of *all* modes in the system. Instead, one must attempt to adjust the units post-calculation (e.g., scaling, normalization) or select impedance functions to facilitate comparison across scenarios (potentially at the expense of accurately reflecting travel behavior) and such adjustments may introduce bias. Hence, the raw unconstrained accessibility values themselves are challenging to compare due to their units. To enable more meaningful comparison, the following sections will detail the introduction of constraining constants to ensure consistent units across scenarios and demonstrate results on the same numeric example.

## 2.5 Total constrained accessibility

In the total constrained accessibility case, a total balancing factor proportionally adjusts unconstrained zonal accessibility values  $V_i^0$  based on the regional sum of  $V_i^0$  and the total population or opportunities in the region. Alternatively, we reformulate this case using a proportional allocation constant, which allocates opportunities (or population) proportionally based on the the travel impedance and the total population or opportunities in the region. In both formulations, all zonal values become a proportion of a known system total, be it the regional opportunities or regional population depending on the variant.

We define two variants for this case: (a)  $V_i^T$  where accessibility is constrained by the total number of opportunities (total constrained accessible opportunity) and which is interpreted as Hansen’s accessibility with a constraining constant, and (b)  $M_j^T$ , where  $i$  and  $j$  of the first variant is transposed, yielding a measure constrained by the total number of population and to be interpreted as constrained ‘market potential’. Following

these definitions, these variants are extended into their multimodal expressions, considering multiple travel behaviour groups or modes  $m$ .

### 2.5.1 Total constrained accessible opportunities: Hansen's accessibility with a total constraint

Unlike in Equation 2.2, the proportionality constant  $k$  is retained. For the total constrained case, it is represented as  $K^T$ :

$$V_{ij}^T = K^T \cdot W_j^{(2)} \cdot f(c_{ij}) \quad (2.12)$$

In this way, the total constrained accessibility measure now becomes Hansen's accessibility with a balancing factor  $K^T$ :

$$V_i^T = \sum_j V_{ij}^T = K^T \sum_j W_j^{(2)} f(c_{ij}) = K^T \cdot V_i^0 \quad (2.13)$$

Imagine that the only system known is the total number of opportunities  $D$  in the region. Accordingly, the constant we impose in this case ensures the regional sum of total constrained accessibility is equal to the total number of opportunities as follows:

$$\sum_i V_i^T = \sum_i \sum_j V_{ij}^T = D \quad (2.14)$$

This constraint is analogous to the total constraint of Equation 1.12, congruent with Wilson's framework. Given the total number of opportunities in the region, we can then substitute Equation 3.3 in Equation 2.14 to solve for  $K^T$ :

$$K^T = \frac{D}{\sum_i \sum_j V_{ij}^0} = \frac{D}{\sum_i \sum_j W_j^{(2)} f(c_{ij})} \quad (2.15)$$

Which is also congruent with Wilson's framework as it comparable to the total flow spatial interaction model (e.g., Equation 2.11 in Cliff et al. (1974)). Hence, rearranging the equation to have opportunities and the proportional constant distinctly represented, our total constrained accessibility model is:

$$V_i^T = K^T \sum_j W_j^{(2)} f(c_{ij}) = \sum_j W_j^{(2)} \frac{D \cdot f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

Further, we can see that, since  $D$  and  $W_j^{(2)}$  are both in units of opportunities, the proportional allocation factor for the total constrained opportunity case  $\kappa_{ij}^T$  is dimensionless:

TABLE 2.5: Simple system: total constrained accessible opportunities

| Origin | $V_i^T$                     |                             |                                 |
|--------|-----------------------------|-----------------------------|---------------------------------|
|        | $f_1 (c_{ij}) = 1/c_{ij}^3$ | $f_2 (c_{ij}) = 1/c_{ij}^2$ | $f_3 (c_{ij}) = 1/c_{ij}^{0.1}$ |
|        | units: <i>physicians</i>    | units: <i>physicians</i>    | units: <i>physicians</i>        |
| 1      | 172.065                     | 172.672                     | 164.080                         |
| 2      | 131.627                     | 132.247                     | 160.692                         |
| 3      | 186.308                     | 185.081                     | 165.228                         |
| Sum    | 490                         | 490                         | 490                             |

$$\kappa_{ij}^T = \frac{D \cdot f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

and therefore  $V_i^T$  is now in the units of  $W_j^{(2)}$ , that is, the mass at the destination ( $V_i^T = \kappa_{ij}^T \sum_j W_j^{(2)}$ ). The role of  $\kappa_{ij}^T$  in this reformulation of accessibility is to transform between units and to adjust the number of opportunities accessible from  $i$  so they represent a proportion of the total number of opportunities in the region.  $\kappa_{ij}^T$  then assigns opportunities in proportion to the impedance between  $i$  and  $j$ . This is why we refer to  $\kappa_{ij}^T$  as a proportional allocation factor. On the other hand, the proportionality constant  $K^T$  balances the units of  $V_i^0$ , the Hansen-type accessibility values, and is an alternative expression of the total constrained accessibility measure.

Referring back to our simple numeric example, balancing factor  $K^T$  for the most decay travel behaviour scenario  $f_1(c_{ij}) = 1/c_{ij}^3$  would then be:

$$\begin{aligned} K^T &= \frac{D}{\sum_i \sum_j W_j^{(2)} f(c_{ij})} \\ K^T &= \frac{D}{\frac{W_1^{(2)}}{c_{11}^3} + \frac{W_1^{(2)}}{c_{21}^3} + \frac{W_1^{(2)}}{c_{31}^3} + \dots + \frac{W_3^{(2)}}{c_{31}^3} + \frac{W_3^{(2)}}{c_{32}^3} + \frac{W_3^{(2)}}{c_{33}^3}} \\ K^T &= \frac{490}{0.6233422} \\ K^T &= 786.085 \end{aligned}$$

$K^T$  for the lower decay scenarios  $f_2(c_{ij}) = 1/c_{ij}^2$  and  $f_3(c_{ij}) = 1/c_{ij}^{0.1}$  are 67.2748352 and 0.4420944 respectively.

Using the calculated balancing factors for all zones and multiplying them by the unconstrained accessibility value  $V_i^0$ , the total opportunity constrained accessibility values for all zones and different travel behaviour scenarios is presented in Table 2.5.  $\kappa_{ij}^T$  for each zone is not reported, but can be understood to be the unitless proportion of opportunities (of the total opportunities) allocated to each zone based on travel impedance.

In contrast to unconstrained accessibility  $V_i^0$ , imposing a constraint -in this case a total opportunity constraint for this variant- allows for the comparison of differences

and ratios between regions and across different travel behaviour scenarios as well. Each value is effectively in units of physicians, with the impedance units already accounted for by  $\kappa_{ij}^T$ .

Considering the highest decay scenario ( $f_1(c_{ij})$ ), zone 1 (a healthcare cluster at the edge of the city) captures an intermediate amount of physicians (172.0652825) like in the unconstrained accessibility case. However, unlike in the unconstrained case, we can say that this value is out of the 490 physicians in the region, which allows us also to deduce that zone 1 captures 1.3072213 and 0.9235529 times more than zone 2 and 3. Values for the lesser decay ( $f_2(c_{ij})$ ) and lowest decay ( $f_3(c_{ij})$ ) scenarios are calculated separately, with decay scenario values also summing to equal 490 physicians accessible in the region.

One can also directly compare values at a specific zone due to the consistent units. For instance, zone 1 remains intermediate in capturing accessible physicians relative to zones 2 and 3 across scenarios, similar to the unconstrained case. However, the difference between travel behaviour scenarios differ in direction. Specifically, Zone 1 captures 0.6067946 more and 7.9850478 fewer opportunities than the lesser decay scenarios  $f_2(c_{ij})$  and  $f_3(c_{ij})$  respectively. Why?  $\kappa_{ij}^T$  ensures proportional allocation for each travel behaviour scenario. Meaning, while the unconstrained accessibility increases,  $\kappa_{ij}^T$  adjusts the values to remain proportional to the total number of opportunities (490 physicians accessible in the region). As the decay behaviour decreases, more opportunities are accessible for all zones. In the medium decay scenario  $f_2(c_{ij})$ , zone 1 sees a slight increase in values (relative to the highest decay scenario) as the zone can accessible more opportunities relative to increases seen in other zones. However, in the lowest decay scenario, zone 1 sees a decrease, as it is outpaced by increases in other zones - namely zone 2 (recall: zone 2 has the lowest number of opportunities, hence the increases in opportunity gains is much higher in a low decay scenario).

Using the total opportunity constrained formulation of accessibility offers a solution to the unit interpretability issue of Hansen (1959)'s accessibility measure. Intuitively, the use of the constraint illustrates how the differences and ratios of values between zones and decay groups can be compared. This is true for other constrained cases of the family of accessibility measures.

### 2.5.2 Total constrained accessible population: Reilly's potential trade territories with a total constraint

Another variant of the total constrained accessibility measure is the *total constrained accessible population measure*, which represents the transpose of  $i$  to  $j$  of the total constrained accessible opportunities measure. This variant, expressed in Equation 3.4, represents an expression of the concept of market potential (i.e., potential users) as proposed in C. D. Harris (1954) and R. W. Vickerman (1974), and which Reilly earlier referred to as 'potential trade territories' (Reilly, 1929). This unconstrained form of market potential  $M_j^0$  (Equation 3.2), effectively the  $i$  to  $j$  transpose of  $V_{ij}^0$ , has been used in recent research to express the potentially accessible population (i.e., users) as a

result of regional transportation infrastructure investment projects (Condeço-Melhorado & Christidis, 2018; e.g., Gutiérrez, 2001; Holl, 2007). Put another way, market potential can also be thought of as a form of *passive accessibility*, indicating the number of people that can reach each destination. However, like  $V_{ij}^0$ , issues of unit interpretability arise in  $M_j^0$ 's unconstrained form. To address this, the constrained variant, the total constrained accessible population measure  $M_j^T$ , is introduced. To formulate this variant, the total balancing factor  $\hat{K}^T$  is applied to the mass of the *population* at  $i$  ( $W_i^{(1)}$ ) instead of the opportunities at  $j$ .

$$M_j^T = \sum_i M_{ij}^T = \hat{K}^T \sum_i W_i^{(1)} f(c_{ij}) = \hat{K}^T M_j^0 \quad (2.16)$$

Where we impose the total system known as a constraint, i.e., that the total market potential equals the total population  $O$  in the region:

$$\sum_j M_j^T = \sum_i \sum_j M_{ji}^T = O \quad (2.17)$$

Substituting Equation 3.4 in Equation 2.17, and solving for  $\hat{K}^T$ , we obtain:

$$\hat{K}^T = \frac{O}{\sum_i \sum_j M_{ji}^T} = \frac{O}{\sum_i \sum_j W_i^{(1)} f(c_{ij})} \quad (2.18)$$

The constrained market potential then takes the following form:

$$M_j^T = \hat{K}^T \sum_i W_i^{(1)} f(c_{ij}) = \sum_i W_i^{(2)} \frac{O \cdot f(c_{ij})}{\sum_i \sum_j W_i^{(2)} f(c_{ij})}$$

Where the following  $\hat{\kappa}_{ji}^T$  proportional allocation factor is dimensionless:

$$\hat{\kappa}_{ji}^T = \frac{\sum_i O \cdot f(c_{ij})}{\sum_i \sum_j W_i^{(2)} f(c_{ij})}$$

Returning back to the numerical example, the proportionality constant  $\hat{K}^T$  is solved for each travel behaviour scenario, and the market potential of each zone  $M_j^T$  is expressed as units of population (e.g., the number of people accessible from each origin at that destination) in Table 2.6.

Readers may note the difference in trends in accessible population (Table 2.6, immediately above) and the preceding accessible physicians (Table 2.5). In the accessible population Table 2.6, zone 1, 2, 3 represent destinations and the accessibility values reflect the number of accessible people from the vantage of physicians. Zone 1, in its role as

TABLE 2.6: Simple system: total constrained accessible population.

| Destination | $M_i^S$                      |                              |                                 |
|-------------|------------------------------|------------------------------|---------------------------------|
|             | $f_1 (c_{ij}) = 1/c_{ij}^3$  | $f_2 (c_{ij}) = 1/c_{ij}^2$  | $f_3 (c_{ij}) = 1/c_{ij}^{0.1}$ |
|             | units: population in 10,000s | units: population in 10,000s | units: population in 10,000s    |
| 1           | 5.018                        | 5.447                        | 6.598                           |
| 2           | 8.596                        | 7.986                        | 6.717                           |
| 3           | 6.386                        | 6.567                        | 6.684                           |
| Sum         | 20                           | 20                           | 20                              |

a destination, is no longer intermediately-ranked relative to other zones; it now attracts the fewest number of people across all three travel behaviour scenarios. However, similar to the total constrained opportunity case, as travel decay reduces, the availability of population begins to converge (though Zone 1 continues as the lowest-ranked) for similar reasons. As decay reduces, the population’s travel impedance to all zones become more similar, making the relative location of the zones less important and all people in the region more equally accessible. Overall: like the total constrained accessible opportunities case, this variant allows for the interpretation of both ordinal and interval comparisons of the raw values themselves.

### 2.5.3 [NEW] Multimodal extension of total constrained accessible opportunities and population

Having detailed both variants  $V_i^T$  and  $M_j^T$ , their multimodal extensions can now be introduced. This extension is relevant if one is interested in accounting for different travel behaviour groups  $m$  within the same system, i.e., where the resulting total constrained accessibility value is a function of all the  $ms$  in the system. The total constrained multimodal accessible opportunities for each group  $m$  would be:

$$V_i^{mT} = \sum_j V_{ij}^{mT} = K^{mT} W_j^{(2)} f^m(c_{ij}^m) \quad (2.19)$$

Where: -  $V_{ij}^{mT}$  is the number of opportunities that can be accessed at origin zone  $i$  from destination zone  $j$  by mode  $m$ , -  $f^m(c_{ij}^m)$  is the cost of travel  $c_{ij}^m$  by mode  $m$  from  $i$  to  $j$ , - the destination zone attraction mass  $W_j^{(2)}$ ; and -  $K^{mT}$  is the modal balancing factor  $\frac{D}{\sum_m \sum_i \sum_j W_j^{(2)} f^m(c_{ij}^m)}$  that serves to allocate the opportunities in the region and ensures units remain balanced.

Summarising equation 3.9 as a measure of modal-group-specific total constrained accessibility  $V_i^{mT}$  by summing all  $V_{ij}^{mT}$  for a specific  $i$  and  $m$  (i.e.,  $V_i^{mT} = \sum_j V_{ij}^{mT}$ ).  $V_i^{mT}$  can also be summed by mode to equal  $V_i^T$  (i.e.,  $\sum_m V_i^{mT} = V_i^T$ ) and summed across the region to equal  $D$  (i.e.,  $\sum_m \sum_i \sum_j V_{ij}^{mT} = \sum_m \sum_i V_i^{mT} = D$ ).

And like in the unimodal formulation,  $\kappa_{ij}^{mT}$ , a unitless expression of the proportion of  $W_j^{(2)}$  that are allocated to each zone  $i$  for each  $m$ , can also be defined for the origin zone  $i$ , allowing for the expression of total constrained multimodal accessible opportunities at a zone  $i$   $V_i^{mT}$  to be equal to  $\kappa_{ij}^{mT} \sum_j W_j^{(2)}$ :

$$\kappa_{ij}^{mT} = \frac{\sum_j D \cdot f(c_{ij})}{\sum_m \sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

Regarding multimodal market potential, i.e., the  $V_i^{mT}$  transpose of  $i$  and  $j$ .  $M_{ji}^{mT}$ , the multimodal ‘market potential’ or the population accessible by mode can also be defined using the total constrained formulation, with similar parameters as previously defined.

$$M_j^{mT} = \sum_i M_{ji}^{mT} = \hat{K}^{mT} W_j^{(2)} f^m(c_{ij}^m) \quad (2.20)$$

And like in the unimodal formulation of market potential,  $\hat{\kappa}_{ji}^{mT}$ , a unitless expression of the proportion of  $W_i^{(1)}$  that is allocated to each zone  $j$  for each  $m$ , can also be defined for the origin zone  $j$ , allowing for the expression of total constrained multimodal accessible population at a zone  $j$   $M_j^{mT}$  to be equal to  $\hat{\kappa}_{ji}^{mT} \sum_i W_i^{(1)}$ :

$$\hat{\kappa}_{ji}^{mT} = \frac{\sum_i O \cdot f(c_{ij})}{\sum_m \sum_i \sum_j W_i^{(1)} f^m(c_{ij}^m)}$$

Returning to the numeric example, we consider all three travel behaviour groups  $m$  together in a single total constrained accessible opportunities calculation. Specifically, each origin  $i$  has three type of travellers, either those traveling at the highest decay of  $f_1(c_{ij})$ , or medium decay of  $f_2(c_{ij})$ , or lowest decay of  $f_3(c_{ij})$ . In this way, there is only one  $K^{mT}$  for this multimodal system, equal to  $\frac{D}{\sum_m \sum_i \sum_j W_j^{(2)} f^m(c_{ij}^m)} = \frac{490}{0.6233422+7.283556+1108.361} = 0.4389629$ . Note:  $K^{mT}$  is smaller than 1, lower than the majority of higher decay unimodal  $K^T$ . This suggests that the numerator (sum of the destination-side marginal, i.e., opportunities) is smaller than the sum of all the unconstrained accessibility  $V_i^0$  values for each mode across the system. It can be seen that the modal group travelling at  $f_1(c_{ij})$  contributes a lower amount of unconstrained accessibility ( $0.6233422$  physicians-minute $^{-3}$ ), compared to the furthest moving lowest decay  $f_3(c_{ij})$  group ( $1108.3606756$  physicians-minute $^{-0.1}$ ).

Multiplying the system-wide  $K^{mT}$  value by the unconstrained accessibility flows  $V_{ij}^0$  for each group yields a  $V_{ij}^{mT}$  value for each  $m$ . This value can be summarised for each  $m$  at each  $i$  ( $V_i^{mT}$ ) and also summarised for each  $i$  ( $\sum_m V_{ij}^{mT}$ ) as presented in Table ??.

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| Origin | $V_i^{mT}$                 |                            |                                | \text{units: population in 10,000s} |
|--------|----------------------------|----------------------------|--------------------------------|-------------------------------------|
|        | $f_1(c_{ij}) = 1/c_{ij}^3$ | $f_2(c_{ij}) = 1/c_{ij}^2$ | $f_3(c_{ij}) = 1/c_{ij}^{0.1}$ |                                     |
| 1      | 0.096                      | 1.127                      | 162.918                        |                                     |
| 2      | 0.074                      | 0.863                      | 159.554                        |                                     |
| 3      | 0.104                      | 1.208                      | 164.057                        |                                     |
| Sum    | 0.2736241                  | 3.19721                    | 486.5292                       | units: population in 10,000s        |

A few important findings can be discerned from Table ??; Firstly the sum of all  $V_i^{mT}$  values is equal to the known number of physicians in the system, 490. Secondly, the lowest decay group  $f_3(c_{ij})$  captures the vast majority of accessible opportunities (99%), demonstrating the significant travel impedance advantage that  $m$ -travelling  $f_3(c_{ij})$  group has relative to other  $m$  traveling groups competing in the same system. When all  $m$ s are considered in the same system, the advantage that one impedance offers in proportionally allocating more opportunities becomes more apparent. Thirdly, the consideration of  $m$  within the system allows for the comparison of values between origin  $i$  values but also between traveling group –just like in the unimodal version, but with a different meaning. For instance, zone 3 is allocated the most accessible opportunities out of any other zone for each travel group, just like in Table 2.5. However, the magnitude of values are different: whereas in the unimodal case  $c_{ij}$  drives the allocation, in the multimodal case both the  $c_{ij}^m$  and the  $f^m()$  drive the allocation. As  $c_{ij}^m$  is the same across travel groups, their rank in allocation does not shift, but their contribution on account of  $f^m()$  changes.

It is important to highlight that a multimodal comparison, i.e., going beyond a post hoc adjustment after calculation, is not possible with unconstrained accessibility, but is possible using constraints. Total constrained accessibility scores can be summed across all  $m$ s (i.e., column 5 in Table ??) and represented per  $i$ ; representing the number of accessible opportunities at that zone considering all travelling groups  $m$ .

The multimodal total constrained accessible population (i.e., market potential), can also be represented. Each destination  $j$  is attracting the same three type of travelers  $m$ , so  $\hat{K}^{mT}$  for this multimodal system equals to  $\frac{\sum_i \sum_j W_i^{(1)} f^m(c_{ij}^m)}{\sum_m \sum_i \sum_j W_i^{(1)} f^m(c_{ij}^m)} = \frac{20}{0.02450548+0.2856+45.07428} = 0.4406802$ . Note: like  $K^{mT}$ ,  $\hat{K}^{mT}$  is smaller than 1 and lower than the majority of higher decay unimodal  $\hat{K}^T$ . This suggests that the origin-side marginal sum of 20 is smaller than the sum of all the unconstrained accessibility  $M_i^0$ , for each  $m$ , across the entire system.

$M_{ij}^{mT}$  values are summarised for each  $m$  at each  $j$  ( $M_j^{mT}$ ) and also summarised for each  $j$  ( $\sum_m M_{ij}^{mT}$ ) as presented in Table ??.

In Table ??, similar to Table 2.6, travel group  $f_3(c_{ij})$  drives the allocation, with the majority of population being allocated from zones assuming such impedance. Population also is allocated most from zone 3, like in the previous results.

| Destination | $M_j^{mT}$                  |                             |                                 | \text{units: population in 10,000s} |
|-------------|-----------------------------|-----------------------------|---------------------------------|-------------------------------------|
|             | $f_1 (c_{ij}) = 1/c_{ij}^3$ | $f_2 (c_{ij}) = 1/c_{ij}^2$ | $f_3 (c_{ij}) = 1/c_{ij}^{0.1}$ |                                     |
| 1           | 0.003                       | 0.034                       | 6.553                           |                                     |
| 2           | 0.005                       | 0.050                       | 6.671                           |                                     |
| 3           | 0.003                       | 0.041                       | 6.639                           |                                     |
| Sum         | 0.01079908                  | 0.1258583                   | 19.86334                        | units: population in 10,000s        |

## 2.6 Singly constrained accessibility

Similar to the total constrained accessibility measure, the singly constrained case includes a balancing factor that adjusts the unconstrained zonal accessibility values  $V_i^0$  such that a *single* constraint is satisfied. Two variants are defined: (a) the *singly constrained accessible opportunities* case (an alternative formula to spatial availability in Soukhov et al. (2023))  $V_i^S$ , and (b) the *singly constrained accessible population* case, its transpose, or singly constrained market potential  $M_j^S$ .

Unlike the total constraint (i.e., Equation 2.14), the single constraint—as will be defined in Equation 2.21 and Equation 2.29 for the first and second variants—incorporates additional information. In the opportunities-accessible variant, the associated balancing factor constrains the ‘potential’ for spatial interaction by ensuring that only a proportional amount of opportunities at each destination are allocated to ‘demanding’ origins. This allocation is informed by demand, or the relative amount of population, and associated travel impedance connecting the zones. In the population-accessible variant (i.e., the  $i \rightarrow j$  transpose of the first variant), population at each origin is proportionally allocated to destinations based on the share of opportunities and travel impedance.

In both variants, the singly constrained accessibility measure introduces population-based (or opportunity-based) competition at the zonal level, unlike the total constraint, which more simply allocates a fixed regional total of opportunities (or population depending on the variant). In sum, all singly constrained zonal accessibility values are both a proportion of the known regional opportunity (or population) total *and* a sum of a *balanced* proportion of opportunities allocated from each destinations (or population allocated from each origin). Each zonal value remains in units of opportunities accessible (or population accessible).

Following the definition of these two variants, they are extended into their multimodal expressions  $V_i^{mS}$  and  $M_j^{mS}$ , an expression of the value for each travel behaviour group or mode  $m$ . In this extension, multimodal travel impedance function  $f^m(c_{ij}^m)$  and either origin- or destination- side marginals ( $D_j^m$  or  $O_i^m$ ) are defined for each  $m$ . This multimodal extension allocates the variant’s constrained marginal based on the proportion of travel impedance for that  $m$  relative to all impedance in the region *and* the non-constrained (i.e., other) marginal’s mass relative to the total non-constrained marginal mass in the region. The later is not considered in the total constrained multimodal accessibility case. Put another way, the multimodal extension of singly constrained

accessibility allocates the opportunities (or population) at the marginal based on the population’s productive demand (or opportunity’s attractive supply) of each  $m$  as a share of this potential relative to all other  $ms$  in the region.

### 2.6.1 Singly constrained accessible opportunities: spatial availability

To demonstrate this variant formulaically, we begin with the opportunity constraint (Equation 2.21) as our known piece of information. Namely, the sum of accessible opportunities from a destination should equal the number of opportunities  $D_j$  at that destination. As the number of opportunities at each  $j$  are known, it is represented as  $D_j$  instead of  $W_j^{(2)}$  as in the total constrained case. This constraint should hold for all destinations in the region. This is comparable to the single attraction-constraint (Equation 1.14) from Wilson’s framework.

$$\sum_i V_{ij}^S = D_j \quad (2.21)$$

The underlying spatial interaction model is now the attraction-constrained model in Equation 1.18, and our accessibility measure becomes:

$$V_i^S = \sum_j V_{ij}^S = \sum_j B_j D_j W_i^{(1)} f(c_{ij}) \quad (2.22)$$

where  $W_i^{(1)}$  is a measure of the mass at origin  $i$  (i.e., the opportunity-seeking population). The corresponding balancing factor, as per Wilson, is:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \quad (2.23)$$

Introducing the balancing factor in Equation 3.5, we obtain:

$$V_i^S = \sum_j D_j \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \quad (2.24)$$

Further, we define the following proportional allocation factor:

$$\kappa_{ij}^S = \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \quad (2.25)$$

After this, it is possible to rewrite Equation 2.24 as an origin summary expression of proportionally allocated known opportunities (i.e.,  $D_j$ ).

TABLE 2.7: Simple system: singly constrained accessible opportunities

| Origin | Population (units: <i>people<br/>in 10,000s</i> ) | $V_i^S$  |  |  |
|--------|---|--|--|--|
|        |   | $f_1(c_{ij}) = 1/c_{ij}^3$<br>units: <i>physicians</i> | $f_2(c_{ij}) = 1/c_{ij}^2$<br>units: <i>physicians</i> | $f_3(c_{ij}) = 1/c_{ij}^{0.1}$<br>units: <i>physicians</i> |
| 1      | 4   | 133.469  | 122.255  | 98.848   |
| 2      | 10  | 166.781  | 185.096  | 241.877  |
| 3      | 6   | 189.750  | 182.650  | 149.275  |
| Sum    | —   | 490  | 490  | 490  |

$$V_i^S = \sum_j \kappa_{ij}^S D_j \quad (2.26)$$

Soukhov et al. (2023) have shown that the role of  $\kappa_{ij}^S$  is to allocate opportunities  $D_j$  proportionally to the mass at each origin  $i$  and the impedance between  $i$  and  $j$ . As in the total constrained opportunity case,  $\kappa_{ij}^S$  is dimensionless and  $V_i^S$  is in the units of opportunities  $D_j$ . The singly constrained accessibility measure in Equation 2.26 is called spatial availability by Soukhov et al. (2023), because it represents the number of opportunities that can be reached *and* are available, in the sense that accessible opportunities have been proportionally allocated based on relative demand, travel impedance and the regional total number of opportunities, i.e., spatial competition for them has been considered. These authors also show that the following expression (accessibility per capita) is a constrained version of the popular two-stage floating catchment area measure of Q. Shen (1998b) and Luo & Wang (2003a):

$$v_i^S = \frac{V_i^S}{W_i^{(1)}} \quad (2.27)$$

Returning to the simple numeric example, the opportunity-constrained case would yield the following  $B_j$  for  $f_1(c_{ij})$ :

$$\begin{aligned} B_j &= \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \\ B_1 &= \frac{1}{\frac{4}{10^3} + \frac{10}{30^3} + \frac{6}{15^3}} = 162.6506 \\ B_2 &= \frac{1}{\frac{4}{30^3} + \frac{10}{10^3} + \frac{6}{25^3}} = 94.9474 \\ B_3 &= \frac{1}{\frac{4}{10^3} + \frac{10}{25^3} + \frac{6}{10^3}} = 93.9850 \end{aligned}$$

The balancing factors  $B_j$  for the  $f_2(c_{ij})$  decay group for zones 1, 2 and 3 is 12.8571429, 8.7685113 and 10.6635071, and for  $f_3(c_{ij})$  decay group is 0.0672461, 0.0660559 and 0.0663798. Using these these balancing constants, we can calculate the singly constrained opportunity accessibility as presented in Table 2.7.

Imposing the single proportional allocation factor  $\kappa_{ij}^S$  allows for the comparison of differences and ratios of the accessibility values, like previously discussed in the total constrained accessible opportunities case. The proportional allocation factor ensures that resulting values are in units of *physicians*, with the impedance units already accounted for in the allocation process.

However, unlike the total constrained opportunity case,  $\kappa_{ij}^S$  reflects zonal competition based on the mass of the origin, i.e., population. Again, consider the highest decay scenario  $f_1(c_{ij})$ . Under this scenario, zone 1 no longer captures a medium amount of physicians as in the total constrained opportunity case: it now captures the fewest in the region i.e., 133.4687282 at zone 1, 166.7813387 at zone 2, and 189.7499331 at zone 3. Why may zone 1 (healthcare cluster at the edge of the city) capture 27% of the physicians regionally while this same zone captures 35% in the total opportunity constrained case?

This difference is due to the single-opportunity factor  $\kappa_{ij}^S$ 's role in  $V_i^S$ . The only inputs required in the total-opportunity factor  $\kappa_{ij}^T$  is the total number of opportunities in the region  $D$  as well as the associated opportunities at each  $j$  and travel impedance. Opportunities are allocated to  $i$ s, regardless of the mass weights of origin. Whereas the single opportunity constraint  $\kappa_{ij}^S$  requires the population at  $i$  as an input. In fact,  $\kappa_{ij}^S$  is calculated as the proportion of impedance-weighted population at an  $i$  to the sum of impedance-weighted population for the entire region. Hence, since zone 1 has the lowest population in the region, is in close proximity to a more populated zone (zone 3, the ‘urban core’), and is not well connected (in terms of travel impedance) to other zones with opportunities,  $V_i^S$  values, or the number of physicians accessible, is lowest.

Readers may also notice the change in the proportion of opportunities drawn from different zones depending on the travel behaviour scenarios considered. For instance, consider Zone 2 which has the highest population. It is more evident in the  $f_3(c_{ij})$  scenario than in higher decay scenarios that this zone does not have an exceptionally large population for the region - Zone 2 only represents 50% of the population in the three zone region. In this sense, other zones are not *that* disadvantaged, and in this scenario with unfettered travel cost, Zones 1 and 3 also take opportunities from Zone 2 (i.e., Zone 1 and Zone 3 takes 17% and 25% more from Zone 2 between  $f_3(c_{ij})$  and  $f_1(c_{ij})$  scenarios hence  $\kappa_{2,2}^S$  decreases by 42%). Zones 1 and 3 are allocated opportunities at relates similar to their relative population size.

Readers may also notice the change in the proportion of opportunities drawn from different zones depending on the travel behaviour scenarios considered. For instance, Zone 2 has the highest zonal population, representing 50% of the 200,000 regional population. However, due to its high relative travel distance from the other zones, its population is less competitive in capturing opportunities in the high-decay travel scenario ( $f_1(c_{ij})$ ): Under this scenario, zone 2 captures almost exclusively opportunities from its own zone. However, in  $f_3(c_{ij})$ , the scenario with unfettered travel cost, Zone 2 captures by far the most number of physicians. But in this scenario, Zones 1 and 3 also take opportunities from Zone 2 (i.e., Zone 1 and Zone 3 takes 17% and 25% more from Zone 2 between

TABLE 2.8: Simple system: singly constrained accessible opportunities per capita

| Origin | Population (units: people<br>in 10,000s) | $v_i^S$  |  |  |
|--------|--|--|--|--|
|        |  | $f_1 (c_{ij}) = 1/c_{ij}^3$<br>units: physicians per<br>capita | $f_2 (c_{ij}) = 1/c_{ij}^2$<br>units: physicians per<br>capita | $f_3 (c_{ij}) = 1/c_{ij}^{0.1}$<br>units: physicians per<br>capita |
| 1      | 4  | 33.367   | 30.564   | 24.712   |
| 2      | 10                                       | 16.678   | 18.510   | 24.188   |
| 3      | 6  | 31.625   | 30.442   | 24.879   |

$f_3(c_{ij})$  and  $f_1(c_{ij})$  scenarios hence  $\kappa_{2,2}^S$  decreases by 42%). Zones 1 and 3 are allocated opportunities at rates similar to their relative population size.

In this way, the consideration of constrained accessibility *per capita* may be clarifying. Often, accessibility values are reported as raw scores without the consideration for population. But, as we introduced constraints, these constrained accessibility values can be normalized using anything that is relevant to the zone. In Table 2.8, we present per capita accessibility for the numeric example, simply in units of number of physicians accessible per population at each zone. Notably, these per capita rates are equivalent to the 2SFCA values.

This simple example was constructed so that the regional average equals 24.5 physicians per 10,000 people. As distance decay decreases and becomes *relatively* uniform (all zones can reach all zones), the effect of population drives the proportional allocation of opportunities. Consequently, per capita accessibility values begin to stabilise to the regional per capita average (e.g., in the lowest distance decay  $f_3(c_{ij})$ , per capita values are all around 24 physicians accessible per capita).

This trend mirrors how the accessibility values in the total constrained opportunity case stabilises to the  $V_i^T$  regional average (e.g., the accessible opportunities allocated to each of the three zones approaches a third of 490 physicians, or 163.33, under the unfettered mobility scenario  $f_3(c_{ij})$ ). These patterns make intuitive sense: the balancing factors act as regional and/or zonal averaging mechanisms. In distance decay travel behaviour scenarios that are more relatively uniform (i.e., low for all zones like in  $f_3(c_{ij})$ ), what remains is the relative effect of the other variables in the balancing factor. In the total constrained case, this is the proportion of opportunities relative to the regional opportunities, and in the case of the single opportunity constrained case, this is the population at a zone relative to the regional population.

### 2.6.2 [NEW EXAMPLE SOLVED] Singly constrained accessible population: market availability

Similar to Equation 2.18 in transposing the origins and destinations, we can define a *singly constrained* measure of market potential that preserves the known population (i.e., the mass weight at the origin  $W_i^{(1)}$  is now represented by  $O_i$ ). In it's per-capita

expression, i.e., equivalent to 2SFCA, this constrained concept of market potential been used to express “facility crowdedness” as in F. H. Wang (2018).

The underlying spatial interaction model is now the production-constrained model in Equation 1.16, and our market potential measure  $M_j^S$  becomes:

$$M_j^S = \sum_i M_{ji}^S = \sum_i A_i O_i W_j^{(2)} f(c_{ij}) \quad (2.28)$$

In this case, the measure is singly constrained by the population *by origin* (i.e.,  $O_i$ ), like Equation 1.14 from Wilson’s framework:

$$\sum_j M_{ji}^S = O_i \quad (2.29)$$

And the corresponding balancing factor, as per Wilson, is:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \quad (2.30)$$

Following the same logic as in the preceding section on total constrained market potential, one arrives at the following expression:

$$M_j^S = \sum_i \hat{\kappa}_{ji}^S O_i \quad (2.31)$$

with:

$$\hat{\kappa}_{ji}^S = \frac{W_j^{(2)} f(c_{ij})}{\sum_i W_j^{(2)} f(c_{ij})} \quad (2.32)$$

As well, the single (population) constraint in Equation 2.29 ensures that the the total constraint (e.g.,  $\sum_j M_j^S = \sum_i \sum_j M_{ji}^S = O$ ) is maintained.

With these constraints,  $\frac{M_j^S}{O}$  can be interpreted as the proportion of the total population serviced by location  $j$ .

Solving the numeric example, the balancing factors  $A_i$  for zones 1, 2 and 3 for the highest decay group  $f_1(c_{ij})$  is: 4.5685279, 5.9720772 and 4.2192774. For  $f_2(c_{ij})$  decay group is: 0.3896104, 0.5087045 and 0.3634895. And for the lowest decay group  $f_3(c_{ij})$  is: 0.0026944, 0.0027512 and 0.0026757. Using these these balancing constants, we calculate the singly constrained accessible population:

TABLE 2.9: Simple system: singly constrained accessible population

| Dest. | Physicians | $M_j^S$                    |                            |                                |
|-------|------------|----------------------------|----------------------------|--------------------------------|
|       |            | $f_1(c_{ij}) = 1/c_{ij}^3$ | $f_2(c_{ij}) = 1/c_{ij}^2$ | $f_3(c_{ij}) = 1/c_{ij}^{0.1}$ |
|       |            | units: people in 10,000s   | units: people in 10,000s   | units: people in 10,000s       |
| 1     | 160        | 4.478                      | 4.949                      | 6.462                          |
| 2     | 150        | 9.303                      | 8.414                      | 6.174                          |
| 3     | 180        | 6.219                      | 6.638                      | 7.364                          |
| Sum   | —          | 20                         | 20                         | 20                             |

Similar to the total constrained case (Table 2.6), the singly constrained accessible population (Table 2.9) shows that Zone 1 attracts the fewest people under both high travel cost ( $f_1(c_{ij})$ ) and medium decay ( $f_2(c_{ij})$ ) scenarios. This is largely due to its small local population (4 out of 20 units in the region), and the system’s lower ability for other zones to attract away or attract population from other zones). However, under low decay ( $f_3(c_{ij})$ ), where people travel more freely, Zone 1 surpasses Zone 2 in attracting population. Despite having only a medium level of physician supply, Zone 1’s central location—close to both the urban core (Zone 3) and ‘bedroom community’ Zone 2—makes it more competitive in low decay scenarios. In contrast, Zone 2 has the largest local population but the fewest physicians, and its relative isolation boosts its attractiveness only when travel is highly restricted (i.e., under  $f_1(c_{ij})$ ). The difference in results between Zones 1 and 2 across scenarios highlights that in the singly constrained accessible population measure, both travel costs (cost effect) and the supply of physicians (mass effect) influence population allocation. Furthermore, these values could be expressed as the rate of accessible population per opportunity ( $m_j^S$ ), aligning with the per capita logic of  $v_i^S$ .

### 2.6.3 [NEW] Multimodal extension of spatial availability and market availability

$V_i^S$  and  $M_j^S$  can be extended to explicitly incorporate multiple modes (or travel groups)  $m$  like established in the total constrained accessibility case. This extension is relevant if: 1) the singly constrained measure is pertinent, i.e., one has information about both marginals and an intuition that the to-be-constrained marginal should be allocated proportionally based on the matching marginal and associated travel impedance. 2) the analyst is interested in accounting for different travel behaviour groups  $m$  within the same system. In this way, the multimodal extension of the singly constrained accessibility measure is allocates the constrained marginal based on the mass effect (of the matching marginal) and the travel cost effect along multiple  $m$  in the system. The singly constrained multimodal accessible opportunities at each  $i$  from  $j$  for each group  $m$  can be expressed as follows:

$$V_i^{mS} = \sum_j V_{ij}^{mS} = B_j^m D_j W_i^{m(1)} f^m(c_{ij}^m) \quad (2.33)$$

Where the mass at the origin  $W_i^{m(1)}$  corresponds to each  $m$  group and the associated multimodal balancing factor equals  $B_j = \frac{1}{\sum_m \sum_i W_i^{m(1)} f^m(c_{ij}^m)}$ .  $V_{ij}^{mS}$  can be aggregated and expressed as accessible opportunities for each  $m$  at  $i$   $V_i^{mS} = \sum_j B_j D_j W_i^{m(1)} f^m(c_{ij}^m)$ , and even for each  $i$  (across  $m$ s) by summing all  $m$ s ( $\sum_m V_i^{mS}$ ). These aggregations remain balanced, as all aggregation of  $ij$  values ensure the opportunity-side single constraint is maintained (i.e.,  $\sum_m \sum_i \sum_j V_{ij}^{mS} = \sum_m \sum_i V_i^{mS} = D$  as well as  $\sum_m \sum_i V_{ij}^{mS} = D_j$ ).

Like in the unimodal singly constrained accessibility formulation,  $\kappa_{ij}^{mS}$  a unitless expression of the proportion of  $D_j$  that is allocated to each zone  $i$  for each  $m$ . It can also be defined for the origin zone  $i$ , allowing for the expression of singly constrained multimodal accessible opportunities at a zone  $i$  to be  $V_i^{mS} = \sum_j \kappa_{ij}^{mS} D_j$ , where  $\kappa_{ij}^{mS}$  is:

$$\kappa_{ij}^{mS} = \frac{W_i^{m(1)} f^m(c_{ij}^m)}{\sum_m \sum_i W_i^{m(1)} f^m(c_{ij}^m)} \quad (2.34)$$

Regarding the multimodal market potential variant, i.e., the  $V_i^{mS}$  transpose of  $i$  and  $j$ .  $M_{ji}^{mS}$ , the multimodal accessible population from opportunity types  $m$ , and can also be defined for each  $j$  to  $i$  flow as:

$$M_{ji}^{mS} = \sum_i M_{ji}^{mS} = A_i^m O_i W_j^{m(2)} f^m(c_{ij}^m) \quad (2.35)$$

And like in the unimodal formulation of market potential, the multimodal balancing factor  $A_i^m = \frac{1}{\sum_j W_j^{m(2)} f^m(c_{ij}^m)}$  can be defined as a unitless factor  $\hat{\kappa}_{ji}^{mS}$  that serves to proportionally allocates the marginal populations at each origin  $O_i$  to each zone  $j$  for each  $m$ .  $\hat{\kappa}_{ji}^{mS}$  allows for  $M_j^{mT}$  to be equal to  $\sum_i \hat{\kappa}_{ji}^{mS} O_i$  where:

$$\hat{\kappa}_{ji}^{mS} = \frac{W_j^{m(2)} f^m(c_{ij}^m)}{\sum_m \sum_j W_j^{m(2)} f^m(c_{ij}^m)}$$

Returning to the numeric example, we need two additional pieces of information: the proportion of population (i.e., origin mass) that travel by each travel impedance scenario for the destination-constrained case (singly constrained accessible opportunities) and the proportion of the opportunities (i.e., destination mass) that are reached by each travel impedance scenario for the origin-constrained case (singly constrained accessible population).

For the singly constrained multimodal accessible opportunities case. We presume that the in Zone 3, the split between  $f_1(c_{ij})$ ,  $f_2(c_{ij})$ , and  $f_3(c_{ij})$  is 70:20:10—a zone in the urban centre with the majority of people travelling locally, with only 30% of the population traveling further and even further distances. For Zone 1—on the edge of the

TABLE 2.10: Simple system: singly constrained multimodal accessible opportunities. Destination Zone 1, 2, and 3 have a (50:30:20), (20:30:50) and (70:20:10) split in  $m$  respectively.

| Origin | $V_i^{mS}$                         |                                    |  | $\sum_m V_i^{mS}$ |
|--------|------------------------------------|------------------------------------|--|-------------------|
|        | $f_1(c_{ij}) = \frac{1}{c_{ij}^3}$ | $f_2(c_{ij}) = \frac{1}{c_{ij}^2}$ | $f_3(c_{ij}) = \frac{1}{c_{ij}^{0.1}}$ | For all groups    |
|        | units: physicians                  | units: physicians                  | units: physicians                      | units: physicians |
| 1      | 0.093                              | 0.653                              | 61.971                                 | 62.717            |
| 2      | 0.067                              | 1.194                              | 378.330                                | 379.592           |
| 3      | 0.210                              | 0.696                              | 46.785                                 | 47.691            |
| Sum    | 0.3706339                          | 2.543451                           | 487.0859                               | 490               |

urban core– the split is 50:30:20, more people travel from further distances. For Zone 2–the more distance bedroom community–people tend to travel further distances, with a split of 20:30:50.

Beginning with the singly constrained multimodal accessible opportunities case. Each destination is associated with a calculated  $B_j^m$ : 0.2138794, 0.1995429 and 0.2110702 for zones 1, 2 and 3 respectively. Note, these values are larger than the unimodal  $B_j$  for the lowest decay group (i.e., mean of 0.0665606 across the  $j$ s), but not by much compared to the unimodal  $B_j$  for  $f_1(c_{ij})$  (mean of 128.4634991) and  $f_2(c_{ij})$  (mean of 10.7630538). This is notable since the balancing factor directly reflects how potentially energetic the system is over all (i.e, the denominator of  $B_j$ ) relative to what's being attracted to the zone (the numerator). We know all  $m$  are considered, and proportions of the population (mass at origins) demand for opportunities based on each one of the  $m$ s, zones with a higher proportion of ‘energetic’ population (i.e., share of  $f_3(c_{ij})$  in our case) will be allocated a higher proportion of opportunities.

In fact, we can examine the sum of  $\kappa_{ij}^m$  values for the proportion that is allocated from each  $j$  to the  $m = f_3(c_{ij})$  populations at each  $i$ . As  $f_3(c_{ij}) = 1/c_{ij}^0.1$ , exceptionally less steep than  $= 1/c_{ij}^2$  and  $= 1/c_{ij}^3$ , each destination allocates practically *all* (99%) of its opportunities to populations at these zones. The singly constrained opportunity accessibility for each  $m$  group is demonstrated accordingly Table 2.10.

As can be observed in Table 2.10, indeed  $f_3(c_{ij})$  group is allocated about 99% of the total 490 opportunities. This *may* be reasonable if the  $m$ -origin-mass split at each  $i$  was predominately  $f_3(c_{ij})$ , but recall, it is assumed in Zone 1, 2 and 3 the split is 20% (of the 40,000), 50% (of the 100,000) and 10% (of the 60,000); meaning 118,000 of the 200,000 people in the system (or 59%) capture 99% of the accessible opportunities. Put another way, the  $f_3(c_{ij})$  group has access to a mean 77.0347963 physicians per 10,000 capita while  $f_2(c_{ij})$  and  $f_1(c_{ij})$  only have access to 0.5074515 and 0.0434215 physicians per capita (recall, the system average (and Canada’s average PPR (WHO, 2025)) is 24.5 physicians per 10,000 population). In our simple example: the cost effect appears to be significantly more influential than the impact of the population mass.

From the perspective of singly constrained multimodal market potential, or market

TABLE 2.11: Simple system: singly constrained multimodal accessible population. Destination Zone 1, 2, and 3 have a (10:20:70), (70:20:10) and (33:33:33) split in  $m$  respectively.

| Dest. | $M_j^{mS}$                         |                                    |  | $\sum_m M_j^{mS}$        |
|-------|------------------------------------|------------------------------------|--|--------------------------|
|       | $f_1(c_{ij}) = \frac{1}{c_{ij}^3}$ | $f_2(c_{ij}) = \frac{1}{c_{ij}^2}$ | $f_3(c_{ij}) = \frac{1}{c_{ij}^{0.1}}$ | For all groups           |
|       | units: people in 10,000s           | units: people in 10,000s           | units: people in 10,000s               | units: people in 10,000s |
| 1     | 0.001                              | 0.017                              | 11.852                                 | 11.870                   |
| 2     | 0.008                              | 0.025                              | 1.621                                  | 1.654                    |
| 3     | 0.003                              | 0.039                              | 6.434                                  | 6.476                    |
| Sum   | 0.01205932                         | 0.08153133                         | 19.90641                               | 20                       |

*availability* we can ask “what is the amount of potential physician-seeking population that is accessible at a destination, considering three different travel behaviours to physicians?”. As this case is singly-constrained on the side of the population allocation, we don’t know how they travel. However, we do know the split in how attractive opportunities are, hence the proportion of  $m$  that accesses them. Let’s assume that the destination zone 1—on the edge of the urban core—has plenty of specialist physicians, catering to people all over the region: hence a split of 10:20:70 between  $f_1(c_{ij})$ ,  $f_2(c_{ij})$ , and  $f_3(c_{ij})$  is assumed. Zone 2—the more distance bedroom community—contains physicians that are mostly used locally (e.g., general practitioners) that aren’t necessarily attractive to the full region, so the split is opposite of Zone 1, at 70:20:10. Lastly, Zone 3—a zone in the urban center—is presumed to contain an equal mix: a third for each  $m$ .

Each origin is associated with a calculated  $A_i^m$ : 0.0068463, 0.0073718 and 0.006905 for zones 1, 2 and 3 respectively. Like in the case of Table 2.10, these balancing factors are exceptionally low: matching the lower magnitude values of  $A_i$  in the unimodal case. Using these balancing factors to calculate the multimodal market availability in Table 2.11, similar trends as in Table 2.10 can be observed. Namely: the physicians that are assumed to be the most attractive (i.e., to overcome spatial separation by the low decay of  $f_3(c_{ij})$  group) are allocated the majority of ‘potential’ population.

This case (Table 2.11) also makes the role of the single constraint more conceptually clear: though the majority of the population is allocated to destination zones that contain  $f_3(c_{ij})$ -attractive physicians, those zones may not be able to satisfy this demand. For instance, physicians in the  $f_3(c_{ij})$ -group attract, on average, 1070.3984037 people per physician — significantly more than the regional benchmark of  $\frac{10,000}{24.5} \approx 408$  people per physician. In contrast,  $f_2(c_{ij})$  and  $f_1(c_{ij})$  physicians attract only 6.759189 and 0.5801247 people per physician, respectively. This difference illustrates how population is allocated based on the availability of physicians and the impedance-weighted attractiveness of those opportunities, as defined by the travel cost function associated with each  $m$  subset of opportunities.

However, this allocation captures *potential*, as information to adjust/reflect the actual capacity or realized demand is not used. For example, while 187 of the 490 physicians

(about 38%) are assumed to be  $f_3(c_{ij})$ -attractive physicians, this may not mean those physicians *will* or *do* serve a similar share of the population. We are only estimating which opportunities people could access, not which they actually do. Because this measure applies a single constraint (of the marginal population), the total ‘demand’ is fixed at the origin, i.e.,  $M_{ij}^m$  values always sum to the population at each origin  $i$ . Take Zone 1, located at the edge of the urban core — while it has a residential population of 60,000, its potential accessible population is  $1.1869629 \times 10^5$  people. This value is higher than the actual population: mostly because the majority of population from other origins are allocated to this zone due to the concentration of opportunities (i.e., 70% of physicians in the zone are  $f_3(c_{ij})$ -attractive). The high market availability of Zone 1 reflects the attractiveness of this zone – not actual utilization.

In summary, the singly constrained cases describes where populations may go (or where opportunities are allocated, in the opportunity-constrained case), given the spatial distribution of the other marginal and travel impedance. It does not also allocate based on the *other* marginal. To answer this motivation, both population (demand) and opportunity (supply) marginals must be imposed simultaneously. This logic is the foundation of the doubly constrained measure, which matches demand to supply and reveals *realized* accessibility flows or simply *access*.

## 2.7 Doubly constrained accessibility

This accessibility case requires zonal populations and opportunities to match one-to-one, like the doubly constrained spatial interaction model. In this way, doubly constrained accessibility can be thought as “access” or simply spatial interaction (no potential).

To contextualize this point, the total and singly constrained accessibility measures discussed thus far have used either  $O_i$ ,  $D_j$ , or the regional sums of either, but never both simultaneously. For example, when opportunities  $D_j$  are used to constrain Equation 3.5 in the *singly constrained accessible opportunities measure*, the specific mass of the population at origin  $i$  demanding only those opportunities at that  $j$  is unknown. Instead, only the population at  $i$  demanding opportunities in the region is known, and this is more generally represented as  $W_i^{(1)}$  (as in Equation 2.23). Similarly, when the population  $O_i$  is used as a constraint in Equation 3.6 in the *singly constrained accessible population measure*, the mass at the destination is given by  $W_j^{(2)}$  (also in Equation 2.30) since only the mass of opportunities at each  $j$  is known and information on what opportunities are allocated to what zone is unknown.

By contrast, the double-constrained accessibility case requires that both populations and opportunities match. Meaning, opportunities at each destination can be accessed by all populations, while at the same time, the population at each origin can be accessed by all opportunities. However, this requirement is often unintuitive in traditional accessibility analysis. Namely, the distinction between population (origin masses) and opportunities (destination masses) typically represent different entities without a shared

unit of measurement. On the population side, we usually count people; on the opportunity side, we may be referring to physicians, clinics, grocery stores, schools, parks, or libraries. In a few cases, a one-to-one correspondence or their own capacities at which they interact and are interacted with may exist e.g., one person with one job. Another similar opportunity type example is healthcare, e.g., one person and one unit of capacity, e.g. a vaccine shot.

A doubly constrained approach to accessibility calculation needs a one-to-one relationships between population and opportunities to be present. Mathematically, this model requires the simultaneous imposition of both the population- and opportunity-constraints in the preceding singly constrained variants (Equation 2.21 and Equation 2.29), namely the sum of population in all origins should match the sum of opportunities in all destinations (Equation 2.36):

$$\sum_i O_i = \sum_j D_j \quad (2.36)$$

As before, the simultaneous imposition of both constraints ensures the total system constraint is maintained i.e., the sum of all doubly constrained accessibility values  $\sum_i V_i^D = \sum_i \sum_j V_{ij}^D = D$  remains equal to the total number of opportunities in the region  $O$  as shown in Equation 2.14.

As the doubly constrained accessibility measure  $V_{ij}^D$  takes the form of the production-attraction (doubly constrained) spatial interaction model, as shown in Equation 1.20,  $V_i^D$  is as follows:

$$V_{ij}^D = A_i B_j O_i D_j f(c_{ij}) \quad (2.37)$$

where the corresponding balancing factors  $A_i$  and  $B_j$ , as per Wilson, are:

$$A_i = \frac{1}{\sum_j B_j D_j f(c_{ij})}$$

$$B_j = \frac{1}{\sum_i A_i O_i f(c_{ij})}$$

Calibration of the two sets of proportionality constants is accomplished by means of iterative proportional fitting, whereby the values of  $A_i$  are initialized as one for all  $i$  to obtain an initial estimate of  $B_j$ . The values of  $B_j$  are used to update the underlying  $V_{ij}^D$  matrix, before calibrating  $A_i$ . This process continues to update  $A_i$  and  $B_j$  until a convergence criterion is met (see Ortúzar & Willumsen, 2011, pp. 193–195). The proportional allocation factor  $\kappa_{ij}^D$  would then be:

$$\kappa_{ij}^D = \sum_j \frac{1}{\sum_j B_j D_j f(c_{ij})} \frac{1}{\sum_i A_i O_i f(c_{ij})} O_i f(c_{ij})$$

TABLE 2.12: Modified simple system with three zones reflecting matched population and opportunities. Population is in 10,000 persons and opportunities in 10,000 of physician-capacity.

| ID (i or j) | Population | Opportunities |
|-------------|------------|---------------|
| 1           | 4          | 7             |
| 2           | 10         | 5             |
| 3           | 6          | 8             |

One could rewrite Equation 2.37 as an origin summary expression of proportionally allocated opportunities:

$$V_i^D = \sum_j \kappa_{ij}^D D_j \quad (2.38)$$

However,  $V_i^D$  is not wholly helpful, as it will simply equal the origin-mass marginal. In this way, only  $V^D_{ij}$  values are interpretable. Furthermore, unlike in the total and singly constrained cases, the doubly-constrained case does not have an interpretable per capita version. For instance, representing  $V_{ij}^D$  per capita is not meaningful, as the value *already* matches population to opportunities. Following this logic, the market potential form  $M_{ij}^D$  is effectively equivalent to  $V_{ij}^D$ , but can be read with a different interpretation: i.e., the opportunities accessed from  $j$  at an  $i$  vs. the population accessed from  $i$  at a  $j$ . The inputs of ‘opportunities accessed’ and ‘accessed population’ can already be interpreted as inherently being sensitive to both opportunities and population.

To calculate doubly constrained accessibility, the interpretation of the population data and the counts of the opportunity data in the numeric example must be modified. Namely, a count of physician *capacity* per destination is needed instead of just the number of physicians, as used to calculated total and singly constrained cases. We also must be able to clearly state that the population is a count of people seeking opportunities at the new capacities, i.e., the population must reflect the *capacity* of the population to interact with opportunities.

So, this adjusted simple example is summarised in Table 2.12: with the population (in units of 10,000s of people seeking physicians) and the opportunities (in units of 10,000s of physician-capacity) per zone. For the population, we leave this unchanged numerically but theoretically know that each person interacts with one physician capacity (i.e., our opportunities with a *capacity*). Hence, the number of opportunities per destination is new: the example is modified such that the physician-capacity at each zone is an approximately scaled version of the number of destination-side physicians at each zone from the unmodified example (Table 2.2). To emphasize the new definition of ‘provider’ as physician capacity, the new system PPR is simply 1, this is compared to the unmodified example which yields system PPR of 24.5. We keep the same zonal cost matrix, and travel impedance functions for three types of travel behaviour as before (Table 2.3 and Equation 2.6).

TABLE 2.13: Doubly constrained opportunity accessibility for the travel behaviour group with the highest travel decay in the modified simple system.

| Origin ID | Destination ID |            |           | sum |
|-----------|----------------|------------|-----------|-----|
|           | 1              | 2          | 3         |     |
| 1         | 3.235859       | 0.01032226 | 0.7556568 | 4   |
|           | 2.132602       | 4.95932483 | 2.9044391 | 10  |
|           | 1.631539       | 0.03035291 | 4.3399040 | 6   |
| Sum       | —              | 7          | 5         | 8   |

However, despite the modifications to the example, our objective remains the same as in previous cases: to measure accessibility under different travel behavior scenarios. Specifically, we aim to quantify the number of *potential* spatial interactions between the physician-seeking population in each zone  $i$  to the physician capacity in a zone  $j$  i.e.,  $V_{ij}^D$ . The highest decay travel behaviour scenario ( $f_1(c_{ij})$ ) is presented in Table 2.13.

As mentioned, accessibility values are typically interpreted as a summary of the proportionally allocated opportunities at each  $i$ . Hence, in interpreting the doubly constrained accessibility from Table 2.13,  $V_i^D$  values (i.e., the sum of values at all three  $j$  destinations for each origin  $i$ ) would be 4.0018381, 9.9963656, and 6.0017963 physician-capacity accessible for Zones 1, 2 and 3 respectively. This approximately equal to the number of population at each of these zones. Conversely, the market potential  $M_j^D$  interpretation of these values would be 7, 5, and 8 people accessible from Zones 1, 2 and 3 respectively, equal to the number of *opportunities* (physician-capacities accessible) at each of these zones. Notice, the mass weight at the origin equals the mass weight of the destination: this is precisely the function of the double constraint. In other words,  $V_i^D$  is the number of accessed opportunities and  $M_j^D$  is the number of population accessed. For the other two travel behaviour, identical  $V_i^D$  and  $M_j^D$  values are calculated, following the same logic. Hence, the usefulness of the doubly constrained measure lies in the interpretation as  $V_{ij}^D$  values.

For instance, differences in  $V_{ij}^D$  values between travel behaviour scenarios are notable. These values can be directly compared to discuss mass and distance decay impacts. Examining Zone 2 (the bedroom community), Table 2.14 demonstrates these  $i$  to  $j$  access values for this more relatively remote, higher-populated and lower-opportunity rich zone. It can be observed that the number of intrazonal opportunities proportionally allocated decreases as the assumed distance decay decreases e.g., from 4.9593248 to 2.6672837 out of the ~10 opportunities allocated to Zone 2 (a population of 10). Following the intuition discussed in the singly constrained opportunity case, as decay decreases, the mass effect of the population (at origin) and opportunity (at destination) is more evident: zonal opportunities are supplied and zonal populations demand at the weights assigned to these zones, with minimal decay adjustment, reflected by the proportional allocation factors  $\kappa_{ij}^D$ .

TABLE 2.14: Doubly constrained opportunity accessibility for all travel behaviour groups at Zone 2 in the modified simple system.

| Dest. | Population at 2<br>(units: <i>people</i><br>in 10,000s) | Opportunities<br>(units: <i>capacity</i><br>in 10,000s) | $V_{\{ij\}}^D$              |                             |                                 |
|-------|---|---|-----------------------------|-----------------------------|---------------------------------|
|       |   |   | $f_1 (c_{ij}) = 1/c_{ij}^3$ | $f_2 (c_{ij}) = 1/c_{ij}^2$ | $f_3 (c_{ij}) = 1/c_{ij}^{0.1}$ |
| 1     | 10.000  | 7.000   | 2.133                       | 2.272                       | 3.411                           |
| 2     | 10.000  | 5.000   | 4.959                       | 4.766                       | 2.667                           |
| 3     | 10.000  | 8.000   | 2.904                       | 2.958                       | 3.919                           |

Recall, accessibility is defined as “the potential for interaction” and is traditionally presented as a summary zonal measure. In the doubly constrained case, we force the zonal population and zonal opportunities to match one-to-one, hence providing a zonal summary is no longer relevant: the sum of  $V_{ij}^D$  for all  $is$  ends up being equal to the population at the zone. However, if what interests readers is the “potential for interaction” in the case population and opportunities to match one-to-one, reframing the investigation may be needed. In this sense, it would be examining “interaction” (much less room for potential) through the values of  $V_{ij}^D$  e.g., how many opportunities are being allocated to an origin from a destination (or in a transposed sense for  $M_{ji}^D$ ). In this sense, the doubly constrained case can be thought of as an estimate of *realized* accessibility or access: reflecting spatial interaction. It is also formulaically identical to the doubly constrained spatial interaction model, but with specific interpretations of the origin and destination weights as ‘population’ and ‘opportunity’, respectively.

As Wilson explicitly noted, origin and destination weights defined in the spatial interaction model *can* be defined using any unit. Accessibility, however, is often presented and understood as a zonal summary of *potential* for interaction between origins and destinations that contain inherently different units. Arriving at the doubly constrained case through the unconstrained, total, and singly constrained cases make the connection between the *potential* for spatial interaction (accessibility) and *realized* potential for spatial interaction more interpretable. Namely, by demonstrating that these members of the accessibility family all derive from the same root and can be derived from Wilson’s original formulation, this more clearly demonstrates what *potential* is, within the context of spatial interaction modelling.

Namely, *potential* depends on the framing of the masses at the origin and destination, and how similar they are in their units. The appropriate constraint should be decided based on the the input data and their similarity. In increasing unit similarity from the perspective of opportunity-constraint: the total-constraint can be used if population (origin mass) is not known, the single constraint can be used if population is known and matters to *potential* but it does not match the capacity of opportunities, and lastly, the double constraint is appropriate if population is known, matters and population matches the capacity of opportunities. These constraint measures can reflect the opportunities

accessible at each zone, but reflects the assumptions embedded in the input data about the potential to spatially interact.

A multimodal version could be defined by solving the multimodal singly-constrained balancing factors  $A_i^m$  and  $B_j^m$  simultaneously. However, we do not pursue this approach here in this work. As discussed earlier, the doubly-constrained accessibility measure is equivalent to the doubly-constrained spatial interaction model—one that models realized spatial interaction flows rather than ‘potential’. Since accessibility is fundamentally about potential access, not realized flows, this extension falls outside the scope at this moment.

## 2.8 Chapter conclusions

In this chapter, balancing factors are introduced akin to those used in Wilson’s family of spatial interaction models. They are formulated to incorporate system-wide or zonal constraints (i.e., knowns) to accessibility. Four cases of the family are outlined: unconstrained, total constrained, singly constrained and doubly constrained. Variants of each case (i.e., either accessible opportunities, or accessible populations), along with their multimodal extensions, are mathematically formulated and solved assuming a simple toy example.

The constraining constants, depending on the case (i.e., total, singly- or doubly- constrained), restrict the degree of *potential*, linking accessibility (the potential for spatial interaction) with access (spatial interaction) on the same continuum based on the constraint used. This chapter also discussed how popular measures such as the one used in Hansen (1959) and the 2SFCA link into the family of accessibility measures. The family of accessibility measures, as follows.

We first place the popular Hansen-type accessibility measure (Hansen, 1959) within this family of measures as an “unconstrained” case, demonstrating that resulting values cannot be directly compared across different travel scenarios without ad-hoc adjustments. We then show how applying a total constraint balances the units and produces a statistically averaged solution that converges to the regional average for each zone as the decay effect decreases. In other words, the total-constraint model could be a more interpretable alternative for the unconstrained case if population-competition is not relevant and one is interested in capturing the maximum *potential*; specifically, if there is a fixed number of opportunities in the region, and if it makes sense to assume that people accessing proximate opportunities leave fewer for others, *without* considering the population size at the origins.

We then introduce the singly constrained case, which *does* takes into account the population size at the origin in the allocation of opportunities (unlike the total constraint). It is also mathematically equivalent to the spatial availability introduced in Soukhov et al. (2023). In this case, all accessibility values are fixed to sum to a known zonal opportunity-size value (implicitly, the regional total of opportunities), but they are not required to sum to any population-based values at the zone or regional level.

The singly constrained model could be useful if regional competition is a factor and if the acknowledgment that only a finite number of opportunities can be allocated from each destination (with those allocations distributed based on origin population size) is suitable. We also introduce an ‘accessible’ PPR (e.g., opportunities per capita), calculated by dividing each accessibility value by the zonal population. To clarify, this per capita expression of the singly constrained case is equivalent to the 2SFCA (Luo & Wang, 2003a; Q. Shen, 1998b), hence linking this literature back to spatial interaction principles.

Lastly, the doubly constrained case is introduced. In this case, the sums must equal both the regional total and ensure that no zone allocates more opportunities than it has available. Specifically, accessibility values for each  $i-j$  pair must be a proportion of the zonal opportunity and population values simultaneously. For example, the accessibility at zone 1 must equal the sum of opportunities from zones 1, 2, and 3, as well as the sum of the population at zone 1. Satisfying the double constraint means the opportunities and population data must match one-to-one, so working with the accessibility  $i-j$  pair values should be of interest. In this sense, the research question should be concerned with ‘access’ (how many opportunities accessed from  $j$  at  $i$  based on given zonal opportunities and populations) instead of potential spatial interaction (e.g., typically expressed as a zonal summary measure of how many opportunities one could reach (out of a regional total and/or zonal-allocation).

In summary, building on Wilson (1971)’s foundational work, this paper proposed a unified framework for accessibility that is able to account for competition. By reintroducing Wilson’s proportionality constant, the proposed family of constrained accessibility measures restores measurement units to accessibility estimates. This enhancement provides a more interpretable, consistent, and theoretically grounded basis for accessibility analysis, which could help advance the adoption of accessibility-oriented planning.

With the aim of demonstrating how the family of accessibility measures may improve interpretability planning for accessibility-oriented planning, the next chapter outlines an empirical example of the population and parkland in the City of Toronto. This data will be used in later chapters of this dissertation to calculate cases of the family of accessibility measure and demonstrate their interpretability advantages.

# **Chapter 3**

## **CHP 3 - Describing the data and methods: an empirical case study of Toronto's population and parklands**

### **3.1 Overview**

Urban greenspace has many benefits for residents and the environment: it is linked to wellbeing [], higher rates of physical activity [], and contributes to reducing urban island heating as well as mitigating carbon pollution in combating climate change []. Parkland, or greenspace that is owned and operated by a locality, can also be framed as a public service. In this way, the spatial accessibility to parkland should be concerned with ensuring it is distributed in an equitable way.

In this chapter, the empirical example of parkland city of Toronto will be detailed, including (1) the spatial resolution of the zoning system and the residing population, (2) the assumptions associated with calculating parkland entrance points, and the (3) assumed interaction with parkland area . (4) Following the data, the total constrained accessibility measure is briefly summarised. The calculated results are presented and interpreted in a policy-relevant lens: namely, the amount of parkland that is accessible per zone, identification of zones with low and high accessibility.

Notably, this process of methods, data and results presentation was done in consultation with staff from the City of Toronto. They are actively working on developing the Transportation Equity Policy Framework and are partners of the Mobilizing Justice Partnership. All data is openly available, but some has been edited by the city staff (as attributed) or was calculated using assumptions the city staff were agreement with.

## 3.2 Data

### 3.2.1 Origins: the dissemination block and associated census data

The most disaggregated level that census variables (e.g., household income, proportion commute mode) is available is the level of the dissemination area (DA). Census zoning systems are designed by Statistics Canada to represent the population's socioeconomic characteristics as homogeneously, with disaggregated systems (like the DA) requiring to nest within more aggregated systems while remaining a relatively compact geographic size (Statistics Canada, 2021a). Each DA in the Toronto metropolitan area represents population between 441 (Q1) to 836(Q3)) and each representing between 0.0732 sq km (Q1) to 0.2322 sq km (Q3)) in area.

However, population data is available at an even more disaggregated level, the dissemination blocks (DB). The DB is an artifact of the road network (Statistics Canada, 2021b), with between 2 to 5 DBs being typically nested within one DA . The more accurately the origin point represents the known residing population, the more accurately the routed travel time and hence the accessibility results along with its associated assumptions, can be understood. Moreover, the highest level of spatial disaggregation available, is typically the preferred appropriate: to reduce issues associated with the scale effect of the modifiable areal unit problem (MAUP) [] .

For these reasons, points that are representative of the population within each DB was calculated using the methodology described by Statistics Canada (Statistics Canada, 2021c). These representative points are available through Statistics Canada at the DA level, however, DB level points are not. hence, the calculated DB points, or the DB weighted centroids, were generated by the staff at the City of Toronto, and sent to the author on May 2, 2025. The methodology to prepare the points is summarised as follows: (1) if the DB contained no address points, then the geometric centroid of the DB was used, (2) if the DB contains 1 or more address points, then the central point between the dwelling weighted address points was calculated. If this point was not with the DB boundaries, it was manually moved into the DB.

To contextualise the Toronto metropolitan area, the population (per DB) and the associated DA boundaries plus the cities that compose the Toronto metropolitan area are visualised in Figure 3.1.

Two other important zoning systems for the city of Toronto are worth including. First, the city's former municipal boundaries. The modern City of Toronto is an amalgamation of six former municipalities. These boundaries, along with their names, are shown in Figure 3.2, overlaid on DB-level population density.

Another commonly used zoning system, offering a slightly coarser spatial resolution, is the neighbourhood. Toronto contains 158 neighbourhoods, defined by the City staff for planning purposes. These are illustrated in Figure 3.3. In addition to their geographic boundaries, neighbourhoods have also received a classification by the city, occasionally

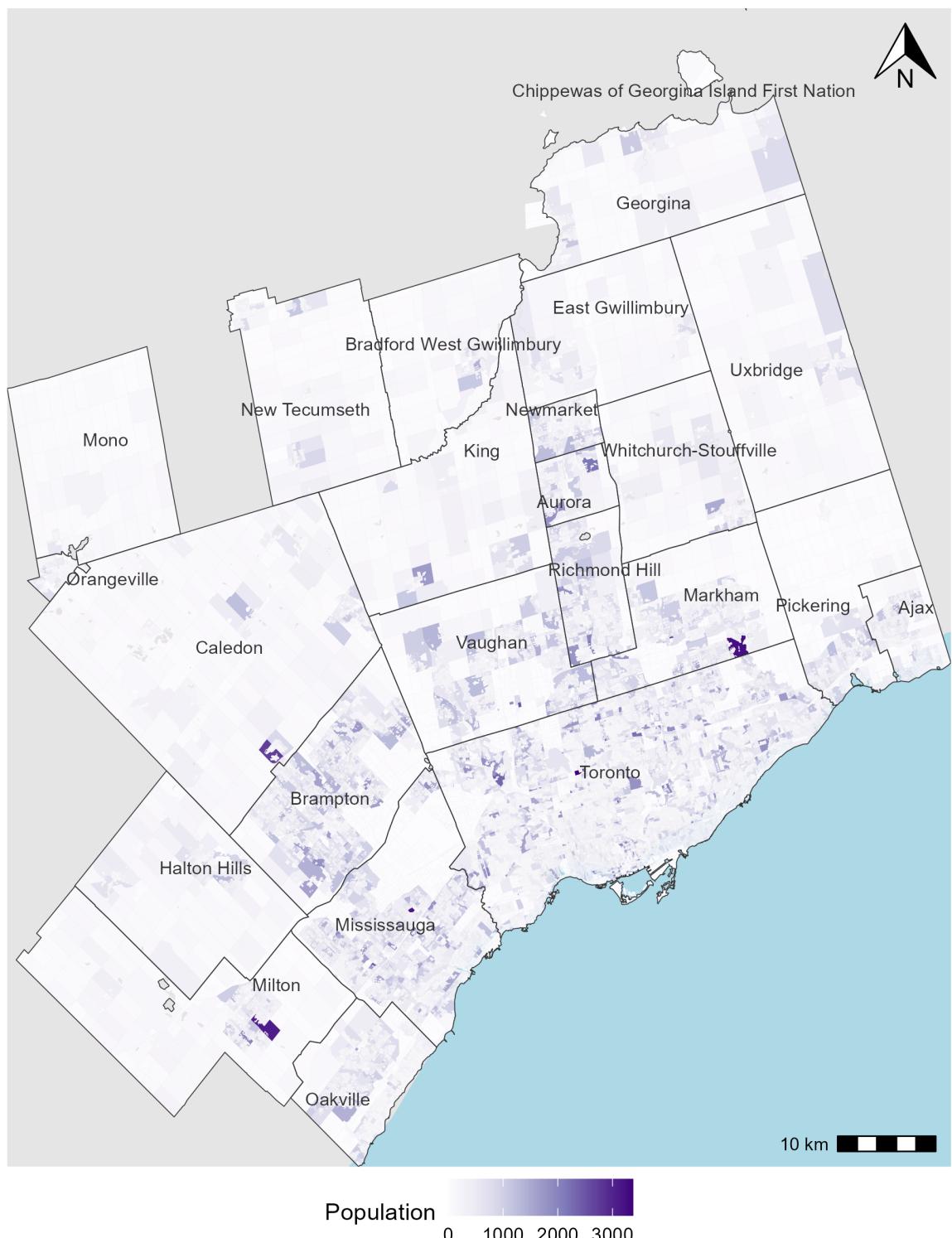


FIGURE 3.1: Map of Toronto CMA with population per DB from the 2021 Census

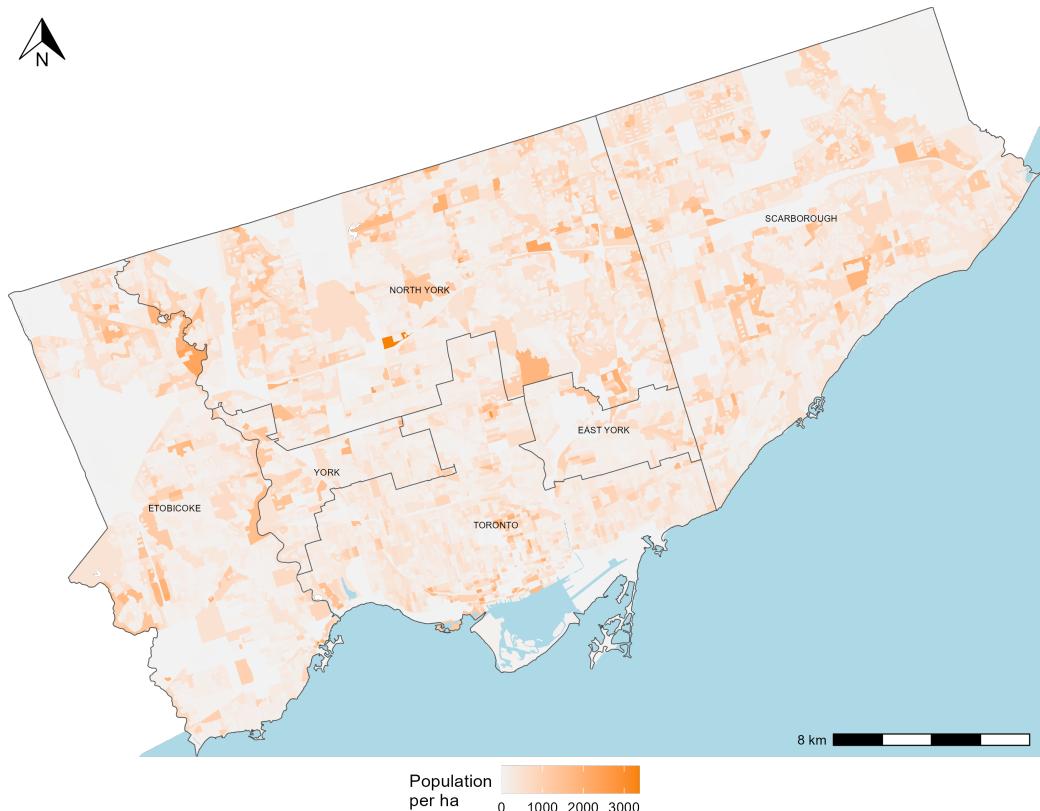


FIGURE 3.2: Map of the City of Toronto’s former municipal boundaries atop DB level population from the 2021 census

used for policy or program initiatives. This classification includes either: a “Neighbourhood Improvement Area” (NIA), an “Emerging Neighbourhood” (EA), or neither. These classifications are based on a composite index developed by the City, incorporating indicators of population marginalization and neighbourhood infrastructure. The index includes factors such as economic opportunity (e.g., unemployment rates), social development (e.g., high school graduation rates), civic participation (e.g., voter turnout), infrastructure (e.g., walkability, availability of community spaces), and health outcomes (e.g., premature mortality) (City of Toronto, 2014, 2024). Overall, between both plots, it is notable that higher residential density is concentrated near the lake in Toronto’s downtown core. Most NIAs are found in the eastern and northwestern parts of the city, and tend to have lower population densities—though a few high-density NIAs also exist downtown.

And lastly, the Figure 3.4 displays the calculated DB weighted centroids, used as the representative point of the origins in the accessibility analysis. There are 13322 points within the City, more spatially clustered in areas with higher population density.

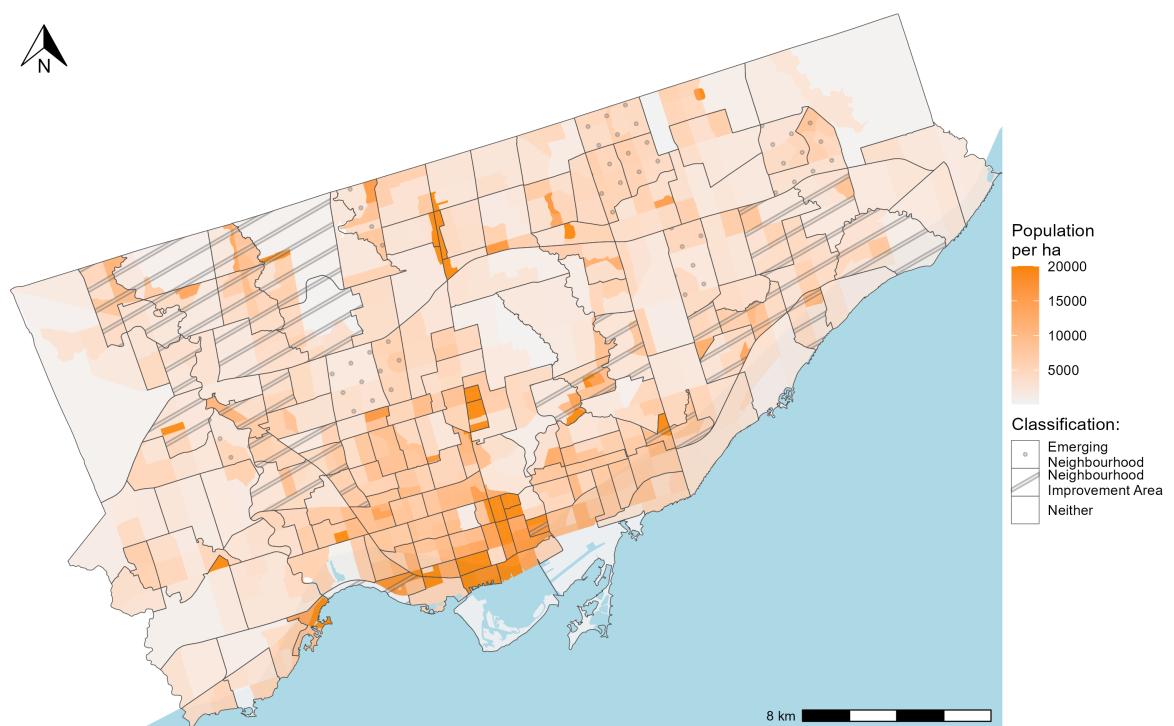


FIGURE 3.3: Map of the 158 Toronto neighbourhoods, with 'Improvement' classification and population density from the 2021 Census

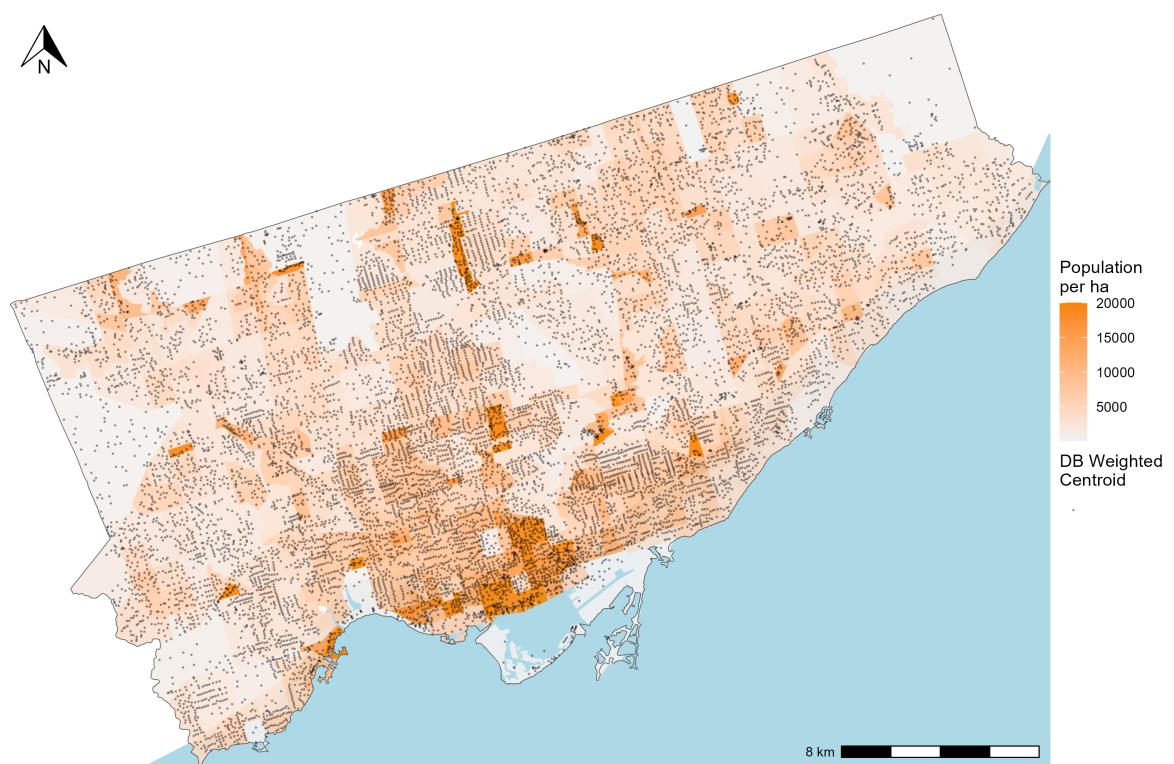


FIGURE 3.4: Map of the City of Toronto’s DB weighted centroids atop the population density from the 2021 Census

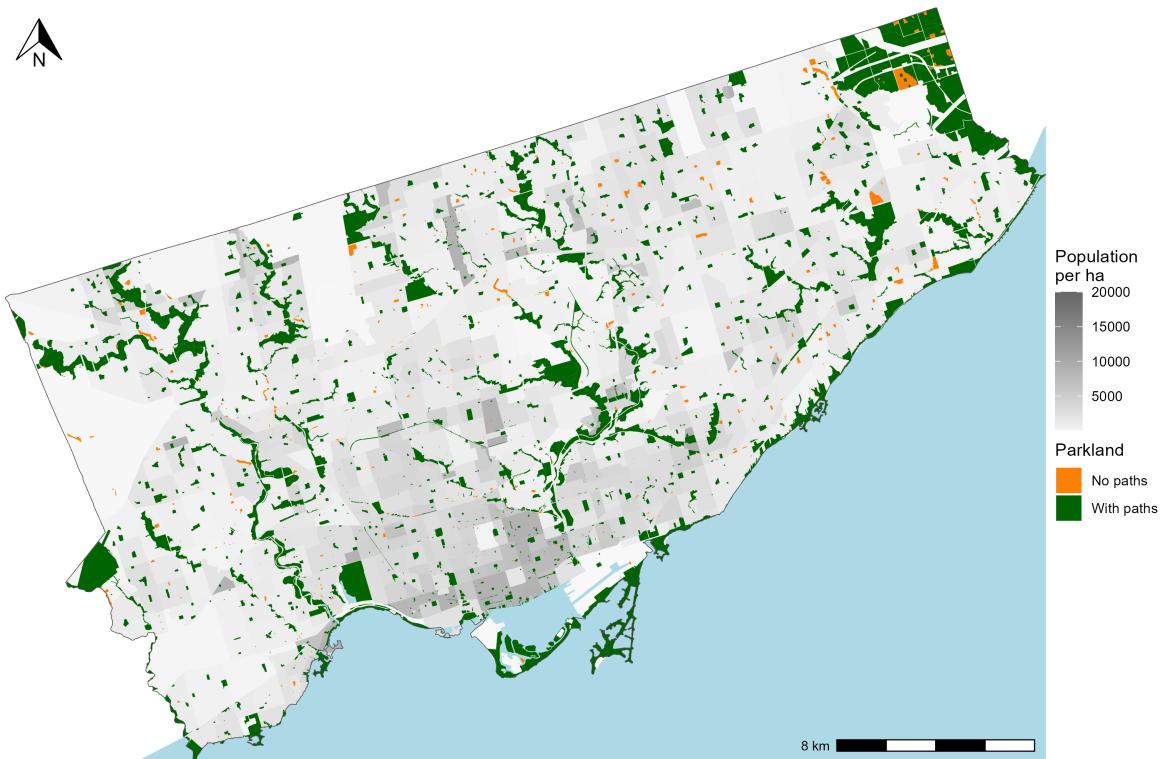


FIGURE 3.5: Map of the City of Toronto’s Parkland (with paths or no paths) atop the population density from the 2021 Census

### 3.2.2 Toronto parkland destinations and normative travel behaviour

Parkland is defined as city operated and/or owned ‘parks’ identified in the greenspaces shapefile available through the city’s official Open Data portal (City of Toronto, 2025). These are the same park assets that are identified as part of the Parkland Strategy report commissioned by the City of Toronto (City of Toronto, 2019, p. 20). This report serves as a guideline for assumptions made regarding park classification and interaction catchments based on park classification. Toronto’s parkland is visualized in Figure 3.5. Of note, other Open Spaces include in the greenspaces shapefile are federally or provincially owned/operated spaces, school yards, cemeteries, and hydro corridors. These spaces are not assumed to be Toronto operated and/or owned parkland, hence are not included in this analysis. These greenspaces are reflected as ‘no population’ DBs.

As retrieved from City of Toronto (2019, p. pg.15), the parkland can be categorized by size: ‘Parkette’ (<0.5 ha) (40% of all parks), ‘Small Park’ (0.5–1.5 ha) (20%), ‘Medium Park’ (1.5–3.0 ha) 16%, ‘Large Park’ (3.0–5.0 ha) 9%, ‘City Park’ (5.0–8.0 ha) 5%, and ‘Legacy Park’ (>8.0 ha) 10%. Interaction with parks can be assumed in a variety of ways, but in this analysis, it is normatively (A. Paez et al., 2012) defined based on the park classification catchments described in City of Toronto (2019) as well as assumptions associated with the mode: as outlined in the following section.

How parks can potentially be accessed is assumed based on their entrances. Entrances are not explicitly available, so they are assumed. They are calculated based on the parkland edge intersection with each path within the park itself or intersecting the park edge. 4 of the the 1607 parks have paths, each with anywhere between 1 to 84 (median of 2) entrances. The remaining 1603 parks do not have paths, and hence their entrances cannot be precisely assumed. Parks without an entrance are significantly smaller in area (i.e., median of 0.24 ha as opposed to the median of area parks with paths which is 1.43 ha). Upon visual inspection using Google Maps streetview, parks with no paths often contain a playground, some sport amenity or gardens within the center. Some of them are unfurnished -containing only mowed grass. For these parks with no internal paths, it is assumed then that these spaces can be entered from any direction and that the geometry centroid is the point of interest calculated and assumed as the entrance point. This point is snapped onto the transportation network based on an origin's shortest path, as will be later described in the routing subsection.

As a visualisation, Figure 3.6 contains a panel of three DAs in Toronto, each displaying the roads, parks, and assumed park entrances. Readers should note that in the first two panels, these parks contain multiple entrance points, corresponding to where the edge of the paths within the parks intersect with the edge of the park boundary. These plots showcase different DAs that represent the diversity in DA size and park composition across the city. The first plot showcases a relatively small DA, near the downtown core of the city featuring high density of population, other amenities and multimodal transportation systems. This DA is unique in the area, as it features a large planned ‘Legacy’ park by the name of Christie Pits. Planned parks are typically square or rectangular and can be accessed from most of the sides. They are also contained within the city in all areal sizes. The second plot contains a larger DA, north of downtown, that contains a few parks and near higher density suburban built form. One is the Roycroft Park Lands, a large ‘City’ (smaller than Legacy), part of the Don River wetlands with maintained trails, enjoyed as nature reserve. Note it’s long shape and minimal entrances. Natural parklands like Roycroft Park Lands are common along wetlands and other preserved natural spaces; they are typically larger in size (Medium, Large, or City parks) hence offer a lot of parkland space to those near their entrances. In the third plot, the DA is also large but with lower density suburban built form, and mostly residential. It contains only one small park: Pleasantview Park. This park contains no internal pathways, a playground in the middle of the park with no other amenities.

### 3.2.3 Origin to destination routing and trip lengths

Routing of travel times was done using the `travel_time_matrix()` function in `{r5r}`, an R package that provides an R-interface for the Java-based R5 Routing engine (Pereira, Saraiva, Herszenhut, Braga, & Conway, 2021). The function was run four time, one for bicycle, car, transit and walking modes, producing four separate travel time matrices. Each origin destination pair has an associated shortest travel time, selected by the function based on all possible origin destination routes given the input road network (and transit schedule, for the transit mode). The road network is an edited version of

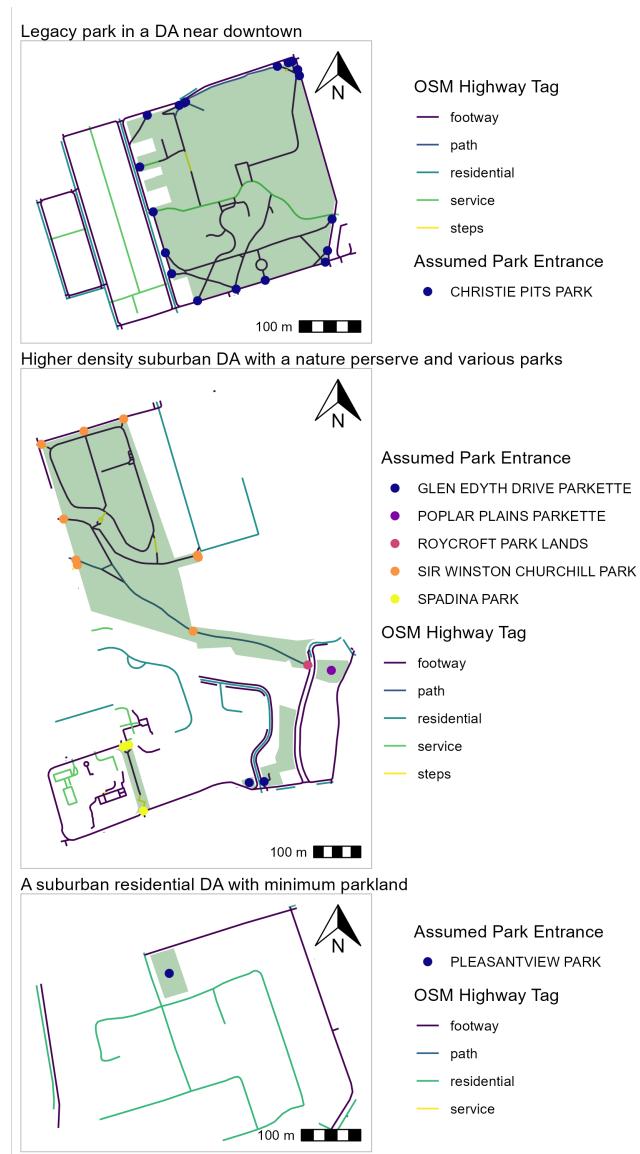


FIGURE 3.6: Three DAs featuring parkland, road network by OSM tag, and park entrances. From top to bottom: planned Legacy park Christie Pits near the downtown core, parks north of the downtown core preserving natural space, and planned park with no paths in a more suburban DA.

OpenStreetMaps (OSM) street network (December 9, 2022) and edited General Transit Feed Specification (GTFS) files (February 4, 2024) for transit operating within the City i.e., GO (regional commuter train and bus service), TTC (local subway, lightrail, and bus network), and the UP Express (regional commuter train line). The files were edited by city staff to more accurately reflect access into TTC subway stations and reflect the transit schedule for the week of February 4 2025.

Concerning the inputs for the ‘travel\_time\_matrix()’ function for all modal travel time calculations: the origins are the DB weighted centroids (13322 locations), the destinations are points representing the 1607 parks and the OSM road network were used. Some parks have multiple pieces, separated by the road network: in total, there are 1958 park pieces, each with between 1 and 68 path entrances (median 3 entrances) or 1 park piece centroid. In total, there are 6274 possible destination points (5724 path entrances and 550 centroids) for each of the 13322 origins.

For the motorized modes, a maximum travel time of 120 minutes was selected. For transit, the GTFS files and a departure time of between 11:00-11:15am (i.e., a 15 minute departure window) on February 8th 2025 was used, to reflect a Saturday afternoon transit schedule. Travel speeds reflect the posted speeds and intersections as gleaned from the OSM road network, and for transit, the scheduled transit arrival times at stops according to the GTFS file. For non-motorized modes, a maximum travel time of 30 minutes was selected, and the default travel speeds of 3.6 km/h for walk and 12 km/h for cycling was assumed. The travel time thresholds of 120 minutes and 30 minutes was set to normatively reflect the likelihood to travel to parks, by mode.

It is worthwhile summarising the multimodal travel time matrices. Notably, within a 120 minutes trip by motorized modes, the majority of DBs can reach all parks by car (with exception to the 6 on the Toronto Islands, inaccessible by car), i.e., the median DB can reach 1601 out of the 1607 parks, while the most isolated can still reach 184 parks. By contrast, within a 120 minute trip or less by transit, a median DB can only reach 1116 parks, with the most central DB reaching 1539 parks meaning 4% of parks are feasibly unreachable by transit for DBs in the City of Toronto. These parks are located at the edges of the city in that are transit poor, as will be demonstrated and discussed in the findings. Comparing this reach to the lower range non-motorized modes, the number of parks reachable by foot or by cycle within 30 minutes from a DB is much lower: a median DB can only reach 15 parks (max. 61 parks for the most central DB) by walking and 86 parks (max. 295 parks for the most central DB) by cycling.

As a final note on routing assumptions, it’s useful to compare travel times to park centroids versus park path entrances. For the 1204 parks with known path entrances, centroids were also computed and travel times from all DBs to all centroids were calculated. This comparison highlights the impact of the R5 routing algorithm, namely, how the routed travel times can differ depending on how the destination points are ‘snapped’ to the nearest road segment. If a point is not already on the road network, R5 snaps it to the nearest network segment, adding a walking time penalty based on distance. This snapping algorithm prioritizes minimizing overall travel time from the origin. Since edge

entrances are typically already on the network, their snapping penalty is minimal. In contrast, centroid points—especially in large and irregular natural parks—may snap to parts of the road network that are unreachable by certain roads (e.g., far enough away from bus stops, or bike lanes), inflating travel times. Hence: when paths within the parks are available from the OSM network, they are used, as it more accurately reflects the points at each the parks can actually be entered from. But when the park has no entrances, it is assumed that the centroid (or the middle of the park itself) is the destination point, and the associated snapping penalty is folded into the calculated travel time. The following Figure 3.7 and Figure 3.8 demonstrate the relationship between the minimum travel time used in the analysis for each parks with path entrances and the travel time if its centroid point was used, along with a 45 degree dashed line representing a perfect linear relationship.

Regarding the motorized modes in Figure 3.7), the relationship appears to be roughly, with centroid times being consistently lower than path entrance times—except for a few parks that fall to the left of the dashed line. Parks with entrance times that are larger than their centroid times have entrances that are not in opportune positions on the network relative to the snapped centroid point, and vice versa for entrances that have *lower* travel times than their centroid points which is often the case. Transit shows a similar trend but with more noise. A few parks exhibit exceptionally high transit times relative to their centroid times. These are typically larger parks where the centroid snaps to a location near access points to the transit system, but the *actual* path entrances are not in proximity to those opportune system access point. Since the road network for cars is more continuous and the system is more evenly accessible (e.g., the majority of roads can be driven on, whereas the transit system can only be entered in specific points), this discrepancy is not observed in the car mode. This comparison underscores the importance of using realistic entrance points especially for larger parks with few entrance points and for modes like transit, which do not provide uniform access into the system.

Figure 3.8 captures the non-motorized modes, which exemplifies a similar pattern as the motorized modes, with centroid travel times begin typically longer than path entrance points in a linear relationship. However, like transit and car, travel by bike and walk demonstrate different levels of noise. As the travel time threshold is 30 minutes, distance can be traversed at a faster rate by bike than by foot, hence differences in distances between the path entrance point and the snapped centroid is less impact on the estimated travel time for bike mode.

### **Normative and empirical lengths of park-bound trips**

A key component of accessibility is how the cost of overcoming spatial separation to reach a destination is considered. As normative goals for planning modal parkland provision have not be explicitly created, example normative trip lengths are defined. This definition is based on discussions with city staff and using distance catchments based on park size classifications from the City of Toronto (2019) report.

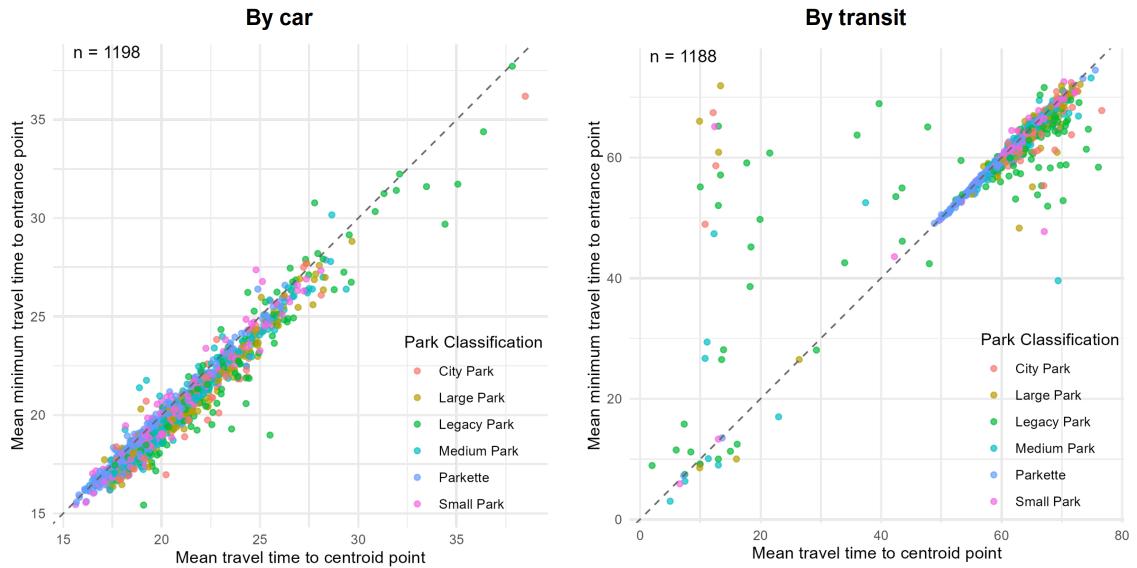


FIGURE 3.7: Scatter plot of car and transit travel times from origins to destinations by destination type for each park (either minimum median travel time to park centroid, or minimum median travel time to park entrance).

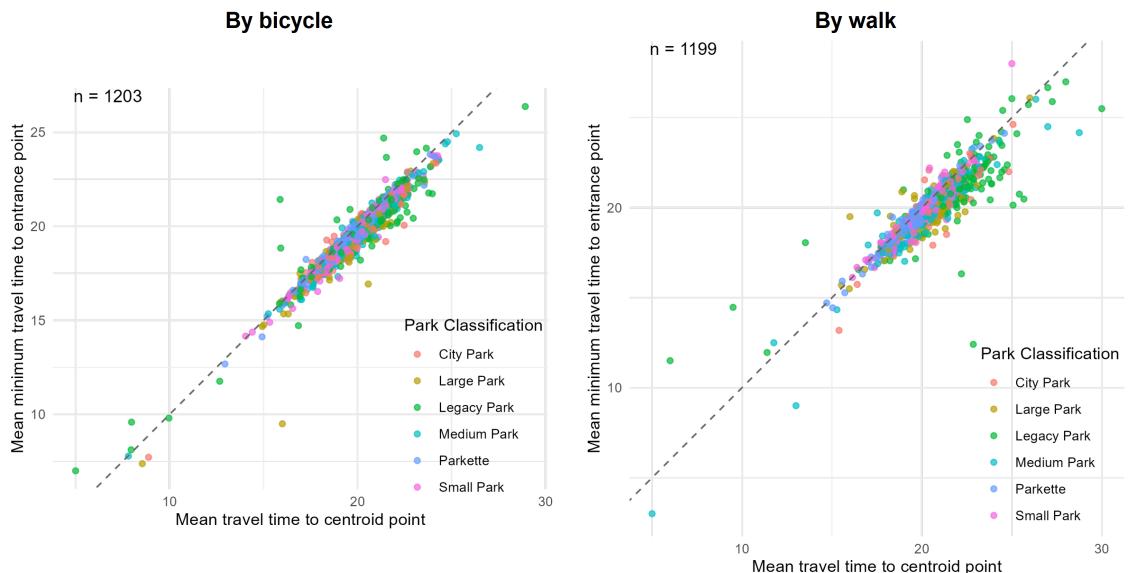


FIGURE 3.8: Scatter plot of cycle and walk travel times from origins to destinations by destination type for each park (either min mean travel time to park centroid, or to min mean travel time to park entrance).

It is assumed that the smaller parks, i.e., parkette (<0.5ha), small parks (0.5-1.0 ha), and medium parks (1.5-3 ha) offer a service coverage of only 0.5km, 1.0km and 1.5km, respectively. Meaning, people are only likely to interact with the park within this travel distance. This assumption has to do with the qualities these parks possess, and the availability of similar parks in the area: namely, these smaller parks are often located in residential neighbourhoods, evenly spatially distributed, and often offer no exceptional amenity that cannot be substituted by another similarly sized park nearby as inferred from City of Toronto (2019). For larger parks, such as large parks (3-5 ha), city parks, and legacy parks, their service catchments are larger as they are more unique, attractive, and harder to substitute parks, as inferred from the City of Toronto (2019) report. Large parks are assumed a service coverage of 3km and city and legacy parks are assumed to be attractive, and hence cover, the whole city.

[1] 17.12376

Hence, normative trip lengths in this dissertation are assumed to vary by park classification (as defined in City of Toronto (2019)), travel mode, and for the sake of communicability—it is only one approach inspired by discussions with city staff. It should be noted that this strategy is not the only possible approach; the City is currently developing a new strategy that may adopt different and more nuanced normative assumptions.

In other words, the following normative travel impedance functions are set based on trip lengths that are *seen* as acceptable and easy to explain. It is assumed that (1) non-motorized travelers are assumed not to travel over 15 minutes to any park, this includes all walking and cycling. For reference, this 0.5 km travel distance (i.e., the ‘catchment’ of parkettes) approximately translates to 8 mins of walking or 3 mins of cycling based on median routed travel speed. (2) transit users can access all types of parks, but based on a negative exponential distance decay function ( $f(c_{ij}) = e^{-0.02c_{ij}}$ ). This function was selected as it yields a median travel time of approximately 30 minutes, meaning that half of the total weight of the travel impedance function—representing the spatial separation between people and parks—comes from trips between 0 to 30 minutes, and the other half comes from trips between 30 to 120 minutes. The function captures the idea that the deterrent effect of distance increases gradually, assigning progressively less ‘spatial interaction’ weight to more distant parks, even though they remain reachable. (3) Car users are assumed *not* to travel to smaller parks (i.e., Parkette, small and medium parks) at all, and as such, these parks are excluded from accessibility results for the car mode. For larger parks (i.e., large, city and legacy parks), car travel impedance is assumed also based on a negative exponential distance decay function, but with parameter  $-0.04$  ( $f(c_{ij}) = e^{-0.04c_{ij}}$ ), twice as steep as the transit impedance function. This reflects a stronger deterrent effect with increasing travel time. Namely, with a median travel time of approximately 17 minutes, trips that are between 0 to 17 minutes are assigned half the weight of the travel impedance function and the remaining half is defused across trips that are between 17 minutes to 120 minutes. Destinations 120 minutes are considered

effectively unreachable, with the function modeled as 0 beyond this point for both transit and car modes.

These assumptions on interaction with parks and trip lengths by mode have equity implications:

- Smaller parks are assumed to only be attractive to, hence accessible for, non-motorized and transit users
- For larger parks which can be reached by all modes, car mode has the largest range and has a steep decay, meaning it may provide favourable spatial separation weight, relative to other modes.
- Travel impedance for transit users is penalized more gradually, assuming broader reach to all park types
- Non-motorized users reach is constrained practically: they are assumed not to travel beyond 15 minutes
- For parks in less densely populated areas, i.e., with travel times by sustainable modes beyond 15 minutes, accessibility will only be present for car users. In sum, individuals in these areas who do not have access to a vehicle are effectively excluded.

To summarise the modal  $m$  travel behaviour based on parkland classification type  $y$  as the resulting travel impedance functions  $f^m(c_{ij}^m)$  in Table 3.1:

| Park Type   | Car                                   | Transit                                   | Cycling                      | Walking                      |
|-------------|---------------------------------------|---|------------------------------|------------------------------|
| Parkette    | 0                                     | $e^{-0.02} \cdot c_{ij}^{\text{transit}}$ | 1 if $c \leq 15$ min, else 0 | 1 if $c \leq 15$ min, else 0 |
| Small Park  | 0                                     | $e^{-0.02} \cdot c_{ij}^{\text{transit}}$ | 1 if $c \leq 15$ min, else 0 | 1 if $c \leq 15$ min, else 0 |
| Medium Park | 0                                     | $e^{-0.02} \cdot c_{ij}^{\text{transit}}$ | 1 if $c \leq 15$ min, else 0 | 1 if $c \leq 15$ min, else 0 |
| Large Park  | $e^{-0.04} \cdot c_{ij}^{\text{car}}$ | $e^{-0.02} \cdot c_{ij}^{\text{transit}}$ | 1 if $c \leq 15$ min, else 0 | 1 if $c \leq 15$ min, else 0 |
| City Park   | $e^{-0.04} \cdot c_{ij}^{\text{car}}$ | $e^{-0.02} \cdot c_{ij}^{\text{transit}}$ | 1 if $c \leq 15$ min, else 0 | 1 if $c \leq 15$ min, else 0 |
| Legacy Park | $e^{-0.04} \cdot c_{ij}^{\text{car}}$ | $e^{-0.02} \cdot c_{ij}^{\text{transit}}$ | 1 if $c \leq 15$ min, else 0 | 1 if $c \leq 15$ min, else 0 |

TABLE 3.1: Normative travel impedance functions by parkland classification  $y$  and mode  $m$

To reiterate, the trip length based travel time behaviour summarised in Table 3.1 are *normative*—they represent a statement about what should be considered a reasonable travel time that defines parkland accessibility (A. Paez et al., 2012). In practice, however, travel behaviour empirically may diverge from these normative statements. For example, empirical data from the 2022 Transportation Tomorrow Survey of trips made for ‘leisure’ purposes by different modes in the Greater Toronto Area region, reveals different patterns (Data Management Group, 2023). Figure 3.9 compares this normative travel behaviour (green lines) from Table 3.1 with the TTS empirically derived curves (dashed lines). For this analysis, the normative curves (i.e., Table 3.1) are used to define accessibility. This will enable the interpretation of results in terms of what *should* be accessible via

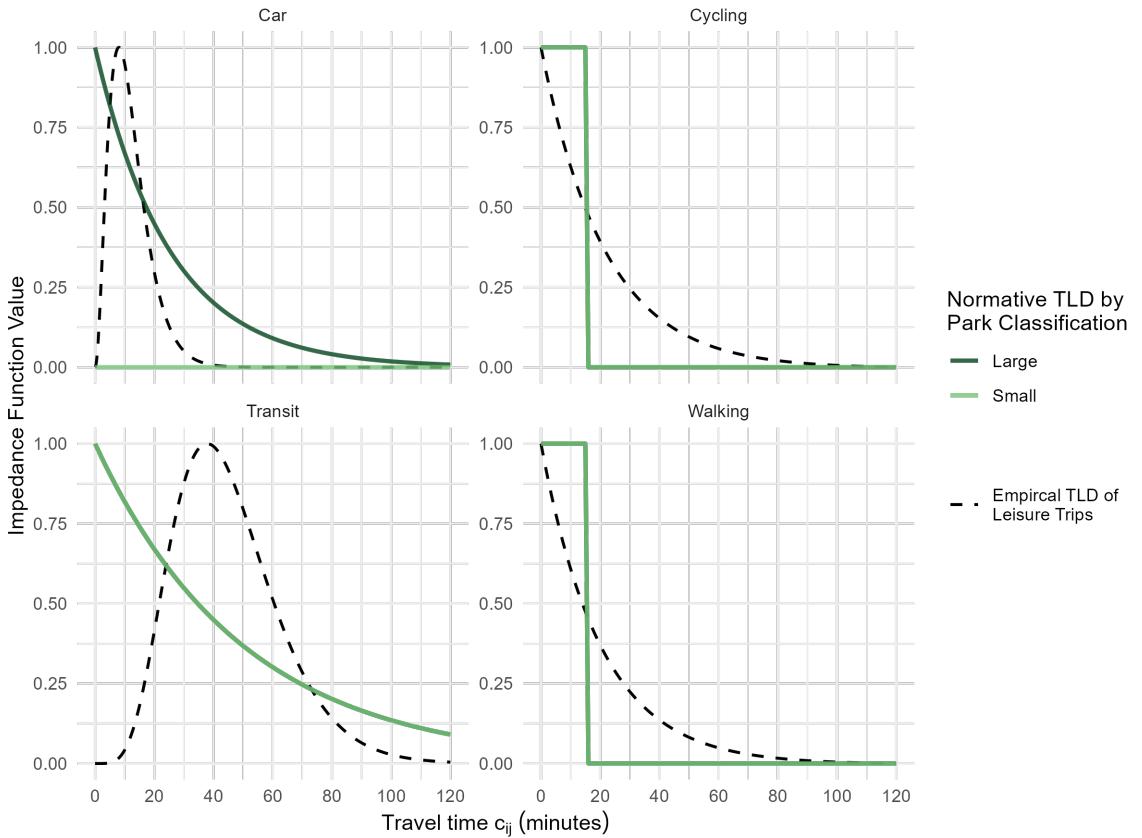


FIGURE 3.9: Scatter plot of cycle and walk travel times from origins to destinations by destination type for each park (either min mean travel time to park centroid, or to min mean travel time to park entrance).

each mode, according to planning goals, rather than what currently is based on observed travel behaviour.

### Origin-side and destination-side mode-share

In the multimodal extension of the singly constrained measure, the proportion of trips from each DB is assumed to follow a given modal split. This is based on the 2022 Transportation Tomorrow Survey (TTS), which includes a sample of 232,555 leisure trips made in the city of Toronto. Trips are grouped by mode as either transit, walking, cycling, or car. The survey data is structured as origin-destination flows and uses the Traffic Analysis Zones (TAZ) system, consisting of 940 zones across the City of Toronto—fewer than the 13,322 DBs used as origins and the 1,607 parks used as destinations—but still representative of the observed travel behaviour in the city to all ‘leisure’ destinations (which include visits to parks, among other leisure activities). Unfortunately, disaggregating further by trip purpose was not available from the TTS.

For the *accessible parkland* measures (i.e., the variant reflecting accessible parkland from an origin zone), origin-side modal splits from the TTS is retained. Each DB, the most granular origin zone used in this analysis, is spatially linked to the TTS zoning system and inherits the modal split of the TAZ it intersects. However, not all DBs fall within TAZs containing leisure trip data: some TAZs have no trips of this type that occur. In such cases (specifically for 131 of the 13,322 DBs), the average modal proportions from the parent Dissemination Area (DA) were used. If no leisure trips were observed at the DA level, the average for the corresponding Census Tract was used.

Through this approach, all DBs are assigned modal proportions that reflect the observed mode split for leisure trips in the TTS. These assigned proportions are shown in Figure 3.10. Notably, areas with high walking shares align closely with the downtown core and other built-up areas of the city, while these same areas display lower car mode shares.

For the *accessible population* variant of the singly constrained measure, mode share at the destination is required. This is interpreted as the proportion of parkland that is accessible by each mode of transportation. Since detailed information about mode-specific park capacities (e.g., availability of parking for cars or bikes) is unavailable, the mode share of trips arriving at destination zones, again drawn from the 2022 TTS leisure trip subset, is used as a proxy.

The method for assigning these mode shares to parks follows a similar approach to that used for the described DBs. First, parks were spatially intersected with TAZs (as destinations), and the modal split of trips arriving in those TAZs was calculated and assigned to the corresponding parks. For parks located in TAZs with no recorded leisure trips, mode share values were backfilled using average proportions from the larger Planning Districts (16 in Toronto) associated with the TAZ zoning system. This was necessary for 67 parks.

Figure 3.11 displays the average mode share of trips arriving at parks, grouped by park size. Similar to Figure 3.10, Figure 3.11 demonstrates that car is the dominant mode. Walking is the second most common mode, although its share decreases as park size increases—while car usage increases. Transit ranks third, and cycling consistently has the lowest share.

### 3.3 Methods: constrained and unconstrained accessibility measures

In this section, we outline the methods outlined in Chapter 2, but specify each variable in terms of parkland as opportunities at destination  $D_j$ , people as population  $O_i$ , and a their multimodal extension. This includes:

For Chapter 4:

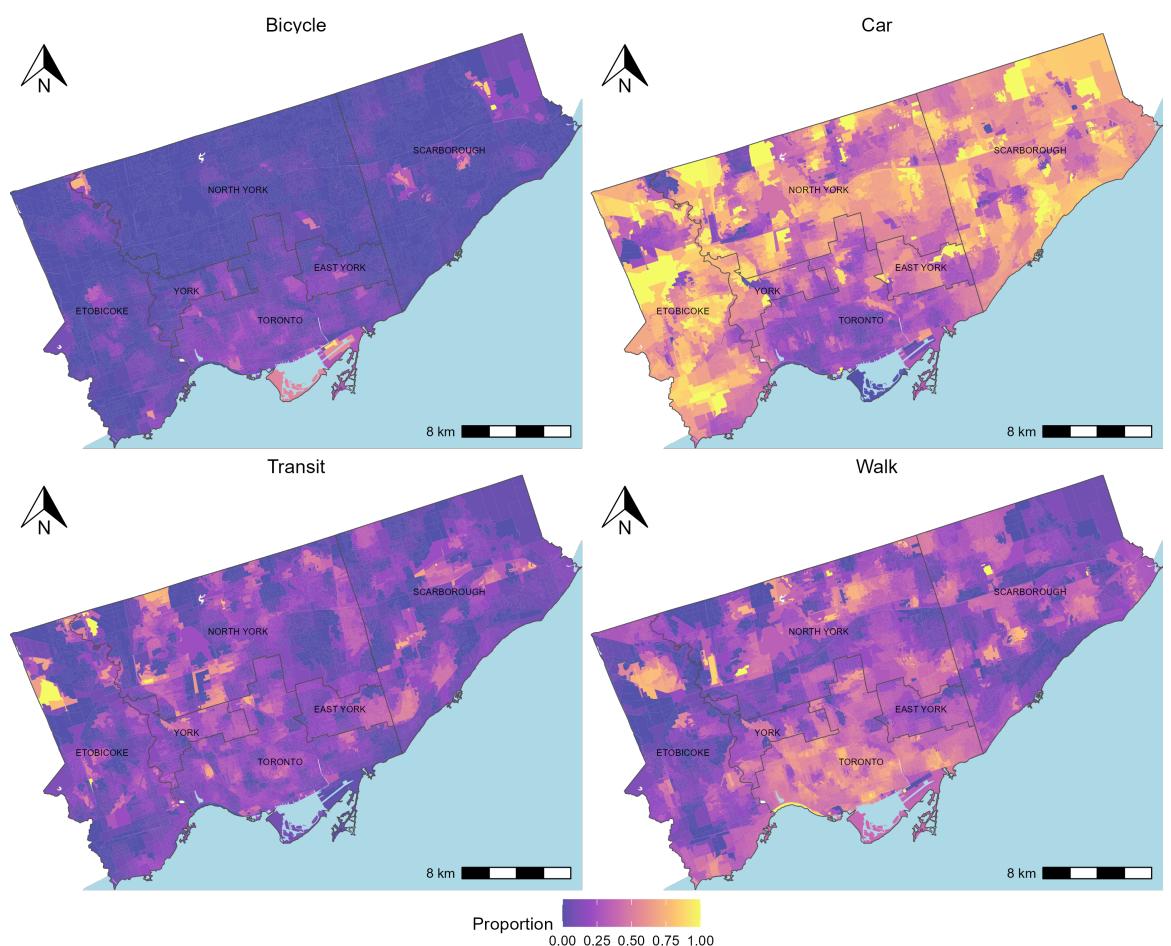


FIGURE 3.10: The proportion of leisure trips by mode on a scale from 0 to 1. Trips are gleaned from the 2022 TTS.

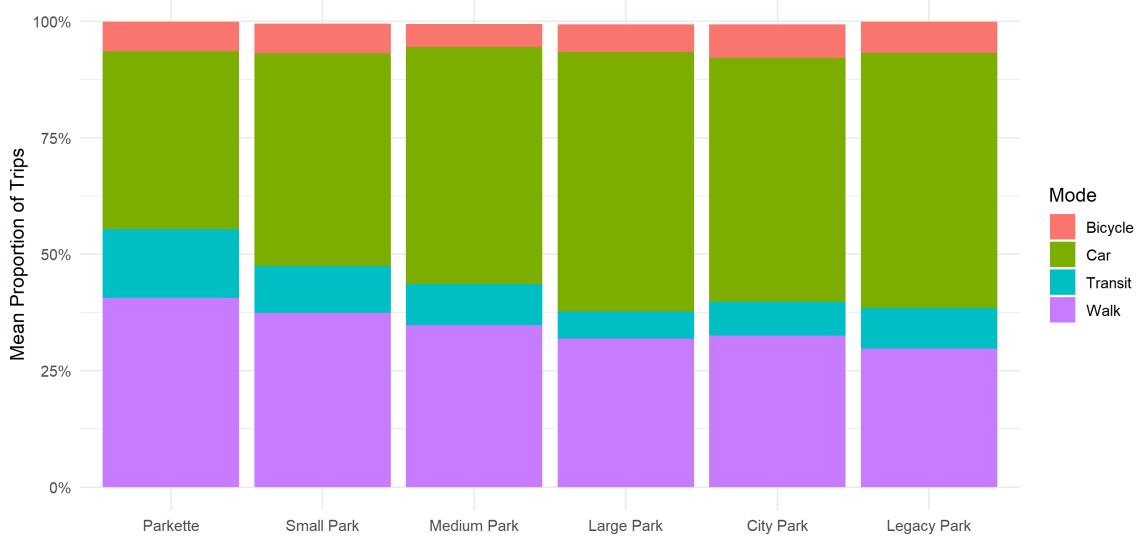


FIGURE 3.11: The average proportion of leisure trips by mode arriving to a destination zone, as summarised by parks and park size. Trips are gleaned from the 2022 TTS.

1. Unimodal total constrained **accessible parkland** summarised per each DB  $V_i^T$ ; All people to all parkland by walking will be assessed: all people will be assumed to be walking. Can be interpreted as “how much parkland area is potentially reachable by the walking-population from this park?”. Will be presented as a ‘parkland provision’ ratio of parkland per walking-population.
2. Unimodal total constrained **accessible population** summarised per park  $M_j^T$ ; the transpose of  $V_i^T$ . Namely, all parkland to all people by walking will be assessed. Can be interpreted as “how much walking-population is potentially reachable from this park?”. Will be presented as a potential population ‘service catchment’ or a ratio of population potential per parkland area.
3. The first two measures will be compared to a unimodal unconstrained **accessible parkland** summarised per each DB  $V_i^0$  and **accessible population** summarised per park  $M_j^0$ , representing the traditional accessibility measure calculation method. Like the other two, all people (assumed to be walking) to all parkland will be assessed. Will be interpreted as “the proportion of parkland area that is potentially reachable by the walking-population from this park” and “the proportion of walking population that is potentially reachable by the park”. No ‘parkland provision’ or ‘service catchment’ ratio can be calculated.

For **Chapter 5**, additional information regarding modes  $m$  is introduced, namely: 1) the population at each origin DB is split to represent ‘walking’, ‘cycling’, ‘transit’ and ‘car’ modes based on observed TTS data, with their associated travel cost  $c_{ij}^m$  to

parkland and travel impedance function  $f^m(c_{ij}^m)$ . And 2) the parkland at each destination is split to reflect the likelihood that they'll be visited by mode. This split is based on the parkland normative classification, namely that small parks are attractive based on walking and cycling impedance, while larger parks are attractive based on all modal impedance. The following measures are solved: 1. Multimodal singly constrained **accessible parkland** summarised per mode and each DB  $V_i^{mS}$ ; All can be interpreted as the proportion of parkland that is accessible as a function of the multimodal population and multimodal travel impedance. 2. Multimodal singly constrained **accessible population** summarised per mode and park destination  $M_j^{mS}$ ; Can be interpreted as the proportion of population that is accessible as a function of the multimodal parkland (i.e., magnetic quality associated with the park) and associated parkland travel impedance. Will also be presented as a potential population ‘service catchment’ or a ratio of population potential per  $m$ -type of parkland area. 3. The first two measures will be compared to unimodal unconstrained **accessible parkland** calculated for each mode summarised for each DB  $V_i^0$  and park  $M_j^0$ , representing the traditional approach to calculating multimodal accessibility measure. Like the other two results. Will be interpreted as “the proportion of parkland area that is potentially reachable by the walking-population from this park”. No ‘parkland provision’ ratio can be calculated.

For both **Chapter 4** and **Chapter 5**, per population and per opportunity ratios at different zonal aggregations for all members (variants and multimodal extensions) are calculated, and their meanings interpreted and compared.

Across all equations, in this empirical example refer to: - Each zone can be either origins  $i$  or destinations  $j$  in the same system. However, in this empirical case  $i$ s and  $j$ s are distinct, but still within the same system (city). Each  $i$  is a dissemination block (DB) and each destination is a park  $j$ . -  $m$  is an index indicating the type of multimodal quality of either the origin-zone-mass (population) or destination-zone-mass (parkland);  $m$  is either “walk”, “cycle”, “transit” and “car”. -  $f(c_{ij})$  is the function of the cost of travel expressed in travel time  $c_{ij}$  from  $i$  to  $j$ . In the multimodal variation,  $f^m(c_{ij}^m)$  is defined for different  $m$ s (e.g., travel time and the travel function from an origin to a destination for “walk” is distinct than “car”) - the destination zone attraction mass  $W_j^{(2)}$  is the number of opportunities (i.e., the parkland in hectares at a park  $j$ ); A multimodal expression of this mass is expressed as  $W_j^{m(2)}$ .  $W_j^{(2)}$  refers to a known marginal in a zone (i.e., parkland area at a  $j$  that is allocated or used for allocation) it is denoted as  $D_j$ , when it refers to the known total marginal sum in the region  $D$  (i.e., total parkland area in the city that is allocated). - the origin zone production mass  $W_i^{(1)}$  is the number of population (i.e., the population at a DB  $i$ ); A multimodal expression of this mass is expressed as  $W_i^{m(1)}$ .  $W_i^{(1)}$  refers to a known marginal in a zone (i.e., population at a  $i$  that is allocated or used for allocation) it is denoted as  $O_i$ , when it refers to the known total marginal sum in the region  $O$  (i.e., total population in the city that is allocated).

### 3.3.1 Methods for Chapter 4: unimodal accessibility

**Unconstrained accessible parkland and population: proportional to the supply of parks and people reachable weighted by travel impedance**

Re-stating 3.1), a summary of  $V_{ij}^0$  for each  $i$  2.7):

$$V_i^0 = \sum_j V_{ij}^0 = \sum_j W_j^{(2)} f(c_{ij}) = S_i \quad (3.1)$$

Where  $V_{ij}^0$  is the number of parkland-area- $f^{\text{walk}}(c_{ij}^{\text{walk}})$  at each  $i$  from  $j$ . It can be summarised for each  $i$  and hence  $V_i^0$  is equivalent to Hansen's accessibility  $S_i$ .

Likewise, 3.2) depicts the the concept of market potential  $M_j^0$  expressed as a transpose of  $V_{ij}^0$ :

$$M_j^0 = \sum_i M_{ji}^0 = \sum_i W_i^{(1)} f(c_{ij}) \quad (3.2)$$

Where  $M_{ji}^0$  is the number of population- $f^{\text{walk}}(c_{ij}^{\text{walk}})$  at each  $j$  from  $i$ . It can be summarised for each  $j$  and hence expressed as  $M_j^0$ .

#### Total constrained accessibility:

This member of measure should be used when information that justifies constraining the opportunities (or population) at each zone is unavailable—leaving only the justification of allocating the regional total (of opportunities in the case of accessible opportunities or population in the case of accessible population).

In the variant of **accessible opportunities** measure, this situation arises when opportunities are not zonally competitive, that is, they can be freely allocated to zones based solely on travel impedance, without accounting for zonal demand of the population. In effect, this measure assumes the population demands the opportunities uniformly across each zone.

Similarly, for **accessible population**, this variant should be applied when population is treated as non-competitive. This measure freely allocates the total population to opportunity zones based on travel impedance, without regard to the capacity or supply of those zones in terms of opportunities. This measure assumes the population is uniformly “supplied” to opportunities throughout the region.

**Accessible parkland: constrained to the total supply of parkland** Re-stating 3.3), the total constrained accessible opportunities measure  $V_i^T$  now becomes Hansen's accessibility with a balancing factor  $K^T$ :

$$V_i^T = \sum_j V_{ij}^T = K^T \sum_j W_j^{(2)} f(c_{ij}) = K^T V_i^0 \quad (3.3)$$

Where: -  $V_{ij}^T$  is the parkland area accessible at  $i$  from  $j$  that can be summarised as the parkland accessible at  $i$   $V_i^T$ ; -  $K^T$  is the balancing factor reflecting the total constraint (Equation 2.14), i.e., ensuring that the sum of all the accessible parkland area is equal to the total parkland in the city ( $\sum_i \sum_j V_{ij}^T = \sum_i V_i^T = D$ )  $\frac{W_j^{(2)}}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$  that serves to proportionally allocate the opportunities  $D$  in the region and ensures units remain balanced.

Put another way:  $V_i^T$  sums all  $V_{ij}^T$  for a specific  $i$  (i.e.,  $V_i^T = \sum_j V_{ij}^T$ ).  $V_i^T$  is linearly proportional to the Hansen-type accessibility measure  $S_i = W_j^{(2)} f(c_{ij})$ . Furthermore,  $V_{ij}^T$  can be written to be equal to  $\kappa_{ij}^T D$  where the dimensionless proportional allocation factor ( $\kappa_{ij}^T = \frac{W_j^{(2)} f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$ ) allocates  $D$  to each  $i$  corresponding to each  $j$ , explicitly communicating that each  $V_{ij}^T$  is a proportion of  $D$ .

**Accessible population: constrained to the total demanding population** Restating 3.4), the total constrained accessible population measure now becomes traditional market potential with a balancing factor  $\hat{K}^T$ :

$$M_j^T = \sum_i M_{ji}^T = \hat{K}^T \sum_i W_i^{(1)} f(c_{ij}) = \hat{K}^T M_j^0 \quad (3.4)$$

Where: -  $M_{ji}^T$  is the number of population accessible at  $j$  from  $i$  that can be summarised as the population accessible at  $j$   $M_j^T$ , -  $\hat{K}^T$  is the balancing factor reflecting the total constraint, i.e., ensuring that the sum of all the accessible population is equal to the population in the city ( $\sum_i \sum_j M_{ji}^T = \sum_j M_j^T = O$ )  $\frac{W_i^{(1)}}{\sum_i \sum_j W_i^{(1)} f(c_{ij})}$  that serves to proportionally allocate the population  $O$  in the region and ensures units remain balanced.

As well,  $M_{ji}^T$  can be written to be equal to  $\hat{\kappa}_{ij}^T O$  where the dimensionless proportional allocation factor ( $\hat{\kappa}_{ij}^T = \frac{W_i^{(1)} f(c_{ij})}{\sum_i \sum_j W_i^{(1)} f(c_{ij})}$ ) allocates  $O$  to each  $j$  corresponding to each  $i$ , explicitly communicating that each  $M_{ji}^T$  is a proportion of  $O$ .

### Singly constrained accessibility:

This member of measure should be used when we know more information: namely, we know that the amount of parkland (or population) should be allocated based on the origin-mass demand (or destination-mass supply). This builds on the ‘total constrained’

measure, where now the opportunities (or population) can be considered competitive (or responding to competition).

So: in the variant of **accessible opportunities**, this variant should be applied when opportunities are seen as zonally competitive, that is, they should be allocated to zones based on the population's zonal demand as well as travel impedance. This measure assumes the zonal demand of the population and distributes zonal opportunities based on this demand (and travel impedance).

Similarly, for **accessible population**, this variant should be applied when population is treated as zonally competitive, that is, population should be allocated to zones based on the zonal parkland supply as well as travel impedance. This variant assumes the zonal population is allocated to zones based on their opportunity “supply” (and travel impedance).

**Accessible parkland: constrained to the total and marginal supply of parkland** Equation 3.5 is re-stated as follows:

$$V_i^S = \sum_j V_{ij}^S = \sum_j B_j D_j W_i^{(1)} f(c_{ij}) \quad (3.5)$$

Where: -  $V_{ij}^S$  is the parkland area accessible considering population demand at  $i$  from  $j$ . It can be summarised as  $V_i^S$ : the parkland accessible at  $i$ ; -  $B_j$  is the balancing factor reflecting the opportunity constraint, i.e., ensuring that the sum of accessible parkland area considering population demand  $V_{ij}^S$  allocated from a specific  $j$  is equal to the parkland from that  $j$  (i.e., Equation 2.21  $\sum_i V_{ij}^S = D_j$ ). This constraint also satisfies the total constraint (Equation 2.14) and ensures units remain balanced.

$V_{ij}^S$  can be written to be equal to  $\kappa_{ij}^S D_j$  where the dimensionless proportional allocation factor ( $\kappa_{ij}^S = \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})}$ ) allocates  $D_j$  to each  $i$ , explicitly communicating that each  $i$  to  $j$  is a proportion of  $D_j$  and implicitly  $D$ .

**Accessible population: constrained to the total and marginal demanding population** Re-stating 3.6), the total constrained accessible population measure now becomes traditional market potential with a balancing factor  $A_i$ :

$$M_j^S = \sum_i M_{ji}^T = \sum_i A_i O_i W_j^{(2)} f(c_{ij}) \quad (3.6)$$

Where: -  $M_{ji}^T$  is the number of population accessible considering parkland supply at  $j$  from  $i$ . It can be summarised as  $M_j^S$ : the population accessible at  $j$ ; -  $A_i$  is the balancing factor reflecting the population constraint, i.e., ensuring that the sum of accessible population considering parkland supply  $V_{ij}^M$  allocated from a specific  $i$  is equal

to the population from that  $i$  (i.e., Equation 2.29  $\sum_j M_{ji}^S = O_i$ ). This constraint also satisfies the total constraint (Equation 2.17) and ensures units remain balanced.

$M_{ji}^S$  can be written to be equal to  $\hat{\kappa}_{ij}^S O_i$  where the dimensionless proportional allocation factor ( $\hat{\kappa}_{ij}^S = \frac{W_j^{(2)} f(c_{ij})}{\sum_j W_j^{(2)} f(c_{ij})}$ ) allocates  $D_i$  to each  $j$ , explicitly communicating that each  $j$  to  $i$  is a proportion of  $O_i$  and implicitly  $O$ .

### 3.3.2 Methods for Chapter 5: Multimodal accessibility

#### Unconstrained accessible parkland and population by multiple modes

Re-stating 3.7):

$$V_i^{m0} = \sum_j V_{ij}^{m0} = \sum_j W_j^{(2)} f^m(c_{ij}^m) \quad (3.7)$$

Where  $V_{ij}^{m0}$  is the amount of parkland-area- $f^m(c_{ij}^m)$  at each  $i$  from  $j$  for a specific  $m$ . It can be summarised for each  $i$  and hence  $V_i^{m0}$  is equivalent to Hansen's accessibility  $S_i$  calculated for a specific mode  $m$ .

Likewise, 3.8) depicts the concept of market potential expressed as a transpose of  $V_{ij}^{m0}$ :

$$M_j^{m0} = \sum_i M_{ji}^{m0} = \sum_i W_i^{(1)} f^m(c_{ij}^m) \quad (3.8)$$

Where  $M_{ji}^{m0}$  is the number of population- $f^m(c_{ij}^m)$  at each  $j$  from  $i$ . It can be summarised for each  $j$  and hence expressed as  $M_j^{m0}$ .

Both multimodal extensions  $V_i^{m0}$  and  $M_j^{m0}$  are not truly 'multimodal', as they are not a function of *all* modes in the system. Instead, an analyst must adjust the units post-calculation (e.g., scaling, normalization) or select impedance functions to facilitate comparison across scenarios (potentially at the expense of accurately reflecting travel behavior). Such adjustments may introduce bias. Hence, the raw unconstrained accessibility values themselves are challenging to compare due to their units. However,  $V_i^{m0}$  and  $M_j^{m0}$  are included as a point of comparison for the multimodal scenarios against the following accessibility measures (total constrained and singly constrained multimodal accessibility).

#### Total constrained multimodal accessibility:

The multimodal extension of this member of measure should be used when we know more information: similar to when one would use the unimodal version, but now this information must be known for multiple modes  $m$ . Namely, the multimodal extension should be used when it is known that the amount of parkland (or population) should

be allocated based on the origin-mass demand (or destination-mass supply) and the matching marginal is qualified by  $m$ -specific travel impedance function  $f^m(c_{ij}^m)$ .

The multimodal extension of the **accessible opportunities** measure variant of this member should be applied when opportunities are seen as zonally competitive, that is, they should be allocated to zones based on a  $m$ -specific split of population's zonal demand as well as its corresponding  $m$  travel impedance. This measure could reflect the number of accessible parkland per  $m$  at an  $i$  based on the allocation of parkland from each  $j$  to the different  $m$ -populations at each  $i$  (and their associated  $m$  travel impedance). In this empirical example, there will be four values per  $i$ , each reflecting the amount of parkland that is accessible for the ‘walking’, ‘cycling’, ‘transit-using’ and ‘car-using’ populations (different  $ms$ ) given their different travel impedance ( $f^m(c_{ij}^m)$ ).

Similarly, the multimodal extension of the **accessible population** measure should be applied when population is treated as zonally competitive, that is, population should be allocated to zones based on the  $m$ -specific supply of parkland and their associated  $m$  travel impedance. In this empirical example, there will be four  $m$  specific values per park  $j$ , each reflecting the amount of population that is accessible for the ‘walking’, ‘cycling’, ‘transit-using’ and ‘car-using’ populations (different  $ms$ ) given their different travel impedance ( $f^m(c_{ij}^m)$ ).

**Accessible parkland by modal-qualities of population: constrained to the total of parkland** Equation 3.9 is restated below:

$$V_i^{mT} = \sum_j V_{ij}^{mT} = K^{mT} W_j^{(2)} f^m(c_{ij}^m) \quad (3.9)$$

Where: -  $V_{ij}^{mT}$  is the parkland area accessible at  $i$  from  $j$  for a specific  $m$  (either walking, cycling, transit or motorist)—a subset of the origin-mass (population) at that  $i$ . It can be summarised as the parkland accessible at  $i$  for a specific  $m$   $V_i^{mT}$ ; -  $K^{mT}$  is the balancing factor reflecting the total constraint (Equation 2.14), i.e., ensuring that the sum of all the accessible parkland area is equal to the total parkland in the city ( $\sum_m \sum_i \sum_j V_{ij}^{mT} = \sum_m \sum_i V_i^{mT} = D$ )  $\frac{W_j^{(2)}}{\sum_m \sum_i \sum_j W_j^{(2)} f^m(c_{ij}^m)}$  that serves to proportionally allocate the opportunities  $D$  in the region and ensures units remain balanced.

$V_{ij}^{mT}$  can be written to be equal to  $\kappa_{ij}^{mT} D$  where the dimensionless proportional allocation factor ( $\kappa_{ij}^{mT} = \frac{W_j^{(2)} f^m(c_{ij}^m)}{\sum_m \sum_i \sum_j W_j^{(2)} f^m(c_{ij}^m)}$ ) allocates  $D$  to each  $m$  at  $i$  corresponding to opportunities at  $j$ , explicitly communicating that each  $V_{ij}^{mT}$  is a proportion of  $D$ .

**Accessible population by modal-qualities of parks: constrained to the total demanding population** Equation 3.10 is restated:

$$M_j^{mT} = \sum_i M_{ji}^{mT} = \hat{K}^{mT} W_j^{(2)} f^m(c_{ij}^m) \quad (3.10)$$

Where: -  $M_{ji}^{mT}$  is the population accessible at  $j$  from  $i$  for a specific  $m$  (either walking, cycling, transit or motorist)—a subset of the destination-mass (population) at that  $i$ . It can be summarised as  $M_j^{mT}$ , the population accessible at  $j$  for a specific  $m$ ; -  $\hat{K}^{mT}$  is the balancing factor reflecting the total constraint (Equation 2.14) but from the perspective of the sum of the total population marginal, i.e., ensuring that the sum of all the accessible population is equal to the total population in the city ( $\sum_m \sum_i \sum_j M_{ji}^{mT} = \sum_m \sum_j M_j^{mT} = O$ )  $\frac{W_i^{(1)}}{\sum_m \sum_i \sum_j W_i^{(1)} f^m(c_{ji}^m)}$  that serves to proportionally allocate the opportunities  $D$  in the region and ensures units remain balanced.

$M_{ji}^{mT}$  can be written to be equal to  $\hat{\kappa}_{ji}^{mT} O$  where the dimensionless proportional allocation factor ( $\hat{\kappa}_{ji}^{mT} = \frac{W_i^{(1)} f^m(c_{ji}^m)}{\sum_m \sum_i \sum_j W_i^{(1)} f^m(c_{ji}^m)}$ ) allocates  $O$  to each  $m$  at  $j$  corresponding to population at  $i$ , explicitly communicating that each  $M_{ji}^{mT}$  is a proportion of  $O$ .

### Singly constrained multimodal accessibility:

This member of measure should be used when we know more information: namely, we know not only the amount of parkland (or population) that should be allocated based on the origin-mass demand (or destination-mass supply) but also specific types of  $m$  at each  $i$  (or  $j$ ) and their associated  $f^m(c_{ij}^m)$  that should dictate this proportional allocation. This extension on the ‘total constrained’ multimodal measure, where now the opportunities (or population) can be considered competitive (or responding to competition) in a multimodal sense.

In the multimodal **accessible opportunities** variant, opportunities are seen as zonally competitive, that is, they should be allocated to origins based on the  $m$ -population’s zonal demand as well as their  $m$  specific travel impedance. In this empirical example, there will four values per  $i$ , each reflecting the amount of parkland that is accessible for the ‘walking’, ‘cycling’, ‘transit-using’ and ‘car-using’ populations (different  $ms$ ) given their different travel impedance ( $f^m(c_{ij}^m)$ ) and the proportion of their  $m$ -populations themselves i.e., DB A will have some X number of parkland accessible by walking, Y by cycling, Z by transit and ZZ by car given their relative travel impedance  $ms$  and amount of  $m$ -using population.

Similarly, for the multimodal **accessible population** variant, population is treated as zonally competitive hence allocated to destinations based on the  $m$ -opportunity’s zonal supply as well as their  $m$  specific travel impedance. In this empirical example, there will be four  $m$  specific values per park  $j$ , each reflecting the amount of population that is accessible for the ‘walking’, ‘cycling’, ‘transit-using’ and ‘car-using’ populations (different  $ms$ ) given their different travel impedance ( $f^m(c_{ij}^m)$ ) and the proportion of their  $m$ -specific parkland themselves, i.e., park A will have some X number of people

accessible by walking, Y by cycling, Z by transit and ZZ by car given their relative travel impedance  $ms$  and amount of  $m$ -attractive parkland.

**Accessible parkland by modal-qualities of population: constrained to the total and marginal supply of parkland** Restating Equation 3.11:

$$V_i^{mS} = \sum_j V_{ij}^{mS} = B_j^m D_j W_i^{m(1)} f^m(c_{ij}^m) \quad (3.11)$$

Where: -  $V_{ij}^{mS}$  is the parkland area accessible at  $i$  from  $j$  for a specific  $m$  (either walking, cycling, transit or motorist)—a subset of the origin-mass (population) considering  $m$ -population demand at  $i$ s and  $m$ -specific travel impedance. -  $B_j^m$  is the  $B_j$  considering  $ms$  and reflects the opportunity constraint, i.e., ensuring that the  $\sum_m V_{ij}^{mS}$  (sum of accessible parkland area considering population demand) allocated from a specific  $j$  is equal to the parkland from that  $j$  (i.e., Equation 2.21  $\sum_m \sum_i V_{ij}^{nS} = D_j$ ). This constraint also satisfies the total constraint (Equation 2.14) and ensures units remain balanced.

$V_{ij}^{mS}$  can be written to be equal to  $\kappa_{ij}^{mS} D_j$  where the dimensionless proportional allocation factor ( $\kappa_{ij}^{mS} = \frac{W_i^{m(1)} f^m(c_{ij}^m)}{\sum_m \sum_i W_i^{m(1)} f^m(c_{ij}^m)}$ ) allocates  $D_j$  to each  $m$  at each  $i$ , explicitly communicating that each  $m$  at each  $i$  to  $j$  is a proportion of  $D_j$  and implicitly all sums to equal  $D$ .

**Accessible population by modal-qualities of parks: constrained to the total and marginal demanding population** Restating Equation 3.12:

$$M_{ji}^{mS} = \sum_i M_{ji}^{mS} = A_i^m O_i W_j^{m(2)} f^m(c_{ij}^m) \quad (3.12)$$

Where: -  $M_{ji}^{mS}$  is the population accessible at  $j$  from  $i$  for a specific  $m$  (either walking, cycling, transit or motorist)—a subset of the destination-mass (opportunity) considering  $m$ -parkland supply at  $js$  and  $m$ -specific travel impedance. -  $A_i^m$  is the  $A_i$  considering  $ms$  and reflects the population constraint, i.e., ensuring that the  $\sum_m M_{ji}^{mS}$  (sum of accessible population considering parkland supply) allocated from a specific  $i$  is equal to the population from that  $i$  (i.e., Equation 2.29  $\sum_m \sum_j M_{ji}^{mS} = O_i$ ). This constraint also satisfies the total constraint (Equation 2.14) and ensures units remain balanced.

$M_{ji}^{mS}$  can be written to be equal to  $\hat{\kappa}_{ji}^{mS} O_i$  where the dimensionless proportional allocation factor ( $\hat{\kappa}_{ji}^{mS} = \frac{W_j^{m(2)} f^m(c_{ij}^m)}{\sum_m \sum_j W_j^{m(2)} f^m(c_{ij}^m)}$ ) allocates  $O_i$  to each  $m$  at each  $j$ , explicitly communicating that each  $m$  at each  $j$  to  $i$  is a proportion of  $O_i$  and implicitly all sums to equal  $O$ .

### 3.3.3 For both Chapter 4 and 5: Representing constrained accessibility as a ratios

Referring to **accessible opportunities**, i.e.,  $V_{ij}^T$  &  $V_{ij}^{mT}$ , and  $V_{ij}^S$  &  $V_{ij}^{mS}$ , are represented per capita and aggregated at different zonal levels to reflect **potential parkland provision**:

Accessible parkland area per capita at a DB:

$$v_i^T = \frac{V_i^T}{O_i} \quad (3.13)$$

Accessible parkland area per  $m$  per capita at a DB:

$$v_i^{mT} = \frac{V_i^{mT}}{O_i} \quad (3.14)$$

This can likewise be applied to market potential. Referring to **accessible population**, i.e., summaries of  $M_{ji}^T$  &  $M_{ji}^{mT}$ , and  $M_{ji}^S$  &  $M_{ji}^{mS}$ , are represented per parkland type and aggregated at different zonal levels to reflect **potential population serviced**.

Accessible population per parkland area at a park:

$$m_j^T = \frac{m_j^T}{D_j} \quad (3.15)$$

Accessible population per  $m$  per parkland area at a park:

$$m_j^{mT} = \frac{m_j^{mT}}{D_j} \quad (3.16)$$

## 3.4 Chapter conclusions

This chapter includes all the City of Toronto population and parkland data included in the accessibility analysis (chapters 4 and 5) as well as the restatement of accessibility equations.

## Chapter 4

# CHP 4 - Comparing unconstrained, total constrained, singly constrained accessibility of Toronto's parklands

### 4.1 Overview

In this chapter, walking accessibility (i.e., unimodal) of parkland and population is presented: where population is assumed to be all people at each DB and parkland is in hectares per park. We adopt the normative impedance function for walking discussed in Chapter 3, i.e., if a park entrance can be reached within 15 minutes of walking than the park is assumed to be reachable and enters the calculation as a 1, and if not, it is assumed full not reachable and enters the accessibility calculation as a 0.

In the first half of the chapter the focus is on *accessible parkland area*. Values of  $V_i^0$ ,  $V_i^T$  and  $V_i^S$  are expressed at the DB level and at the neighbourhood level. Then, the accessible parkland per capita ratio is demonstrated, neighbourhoods are ranked and a discussion is detailed based on ‘potential parkland service provision’. In the second half of the section, *accessible population*  $M_j^0$ ,  $M_j^T$  and  $M_j^S$  is presented per park and per parkland area ratios at the level of the park itself and the neighbourhood is discussed as the ‘potential population served’.

### 4.2 Accessible parkland (for the population)

Figure 4.11 demonstrates that the unconstrained, total constrained (i.e., without population competition) and singly constrained accessible parkland (i.e., with population competition). All three plots are binned by quartile: 0-25th, 25-50th, 50-75th, 75th-98th (the top 2 percentile is exceptionally high for a few DBs: and is clipped and set to the 98th percentile value). White areas are DBs with 0 parkland accessibility or outside the Toronto boundary. Grey DBs have population but have no parkland accessibility, as their representative DB point cannot reach an assumed parkland entrance within 15

minutes of walking. Also of note, the total and singly-constrained are visualised on the same scale as there both represent the same units of “accessible parkland ha” per da, whereas the unconstrained measure is in units of ‘parkland-ha- $\leq$ -15-minutes’ (i.e., opportunities  $x$  impedance function). It is notable that within a 15 minute walk, only 7632.922 hectares of the total 8037.547 hectares of parkland can be reached and hence enters the accessibility calculations (%95). Hence: the sums of the constrained accessibility measures across all DBs equals 7632.922 hectares. Parks that cannot be reached are diverse in their classification and in their entrance point type (assumed centroid or edge entrance) as well as park size, qualities of these unreachable parks is discussed in the next subsection related to ‘accessible population (to parkland)’.

Across all plots in Figure 4.11, white DBs (dissemination blocks with no population nor access) are represented. These areas correspond to the airport lands (Pearson airport) in the northwest; large indoor mall (Sherway Gardens) and the surrounding industrial and rail yard in the southwest; and natural areas in the northeast, including Rouge National Urban Park and the Toronto Zoo, where protected lands, restrictive zoning, and steep terrain limit development. Similar conditions are present in the city’s centre near the Don Valley and associated river system, where floodplains and protected natural space restrict residential development.

Likewise, grey DBs (DBs with no access) are present across all three plots. These DBs are in proximity to white DBs and typically contain restricted pedestrian infrastructure and connections into park that are nearby (i.e., due to steep terrain as in the case in the Don Valley system or lack of parkland in close proximity like near the indoor mall and industrial lands in the southwest). The number of grey DBs are the same in both the unconstrained and total constrained plots. Moreover, these two plots share another commonality: the total constrained plot is proportional (by  $K^T$ ) to the unconstrained plot. As reviewed in Chapter 3, mathematically  $K^T$  is the sum of parkland area that can be walked to within 15 minutes (in ha) divided by the sum of total unconstrained accessibility in the city ( $\frac{7632.922}{509354.5} = 0.01498548$ ). So the total constrained plot can be understood like you would a 15-minute cumulative opportunities measure (i.e., displayed in the unconstrained plot) but in interpretable units. Namely, each DB in the total constrained plot represents the amount of parkland area that can be accessed within a 15 minute walk for that DB out of the total parkland in the area. Put another way, this plot can be understood as if the parkland area in the city (shown in Figure 3.5) was proportionally extended beyond the park boundaries based on the population’s ability to reach it within a walking travel time of 15 minutes or less. It can be thought of as a map reflecting the *accessible* parkland area allocated to a DB. As it is proportional to the unconstrained plot, so it can also be read through the interpretation of *high* and *low* accessibility areas. Spatially, as a 15-minute walking distance to parks is a relatively small area beyond a park, areas of high accessible parkland are on top of or in close proximity to parks, hence this plot mirrors the spatial distribution of the parks themselves (Figure 3.5). DBs within/near large parks are top quartile in accessible parkland area while areas within proximity to smaller parks are in the bottom quartiles.

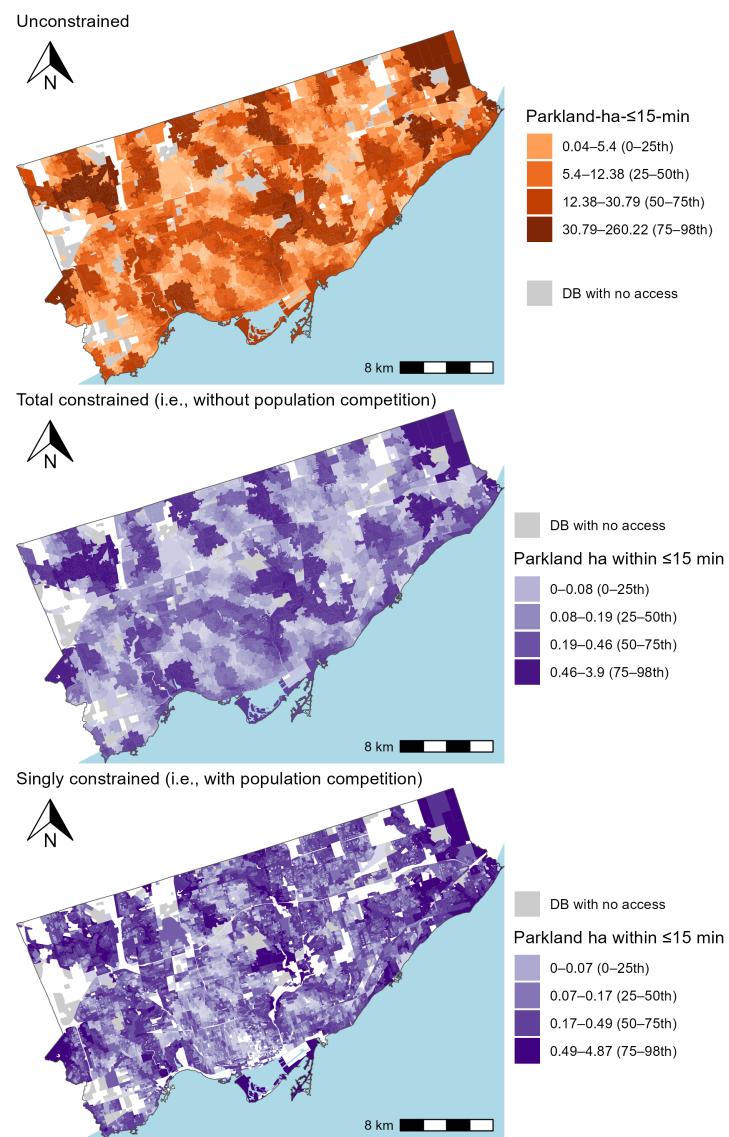


FIGURE 4.1: Map of Toronto CMA with accessibility to parklands per DB.

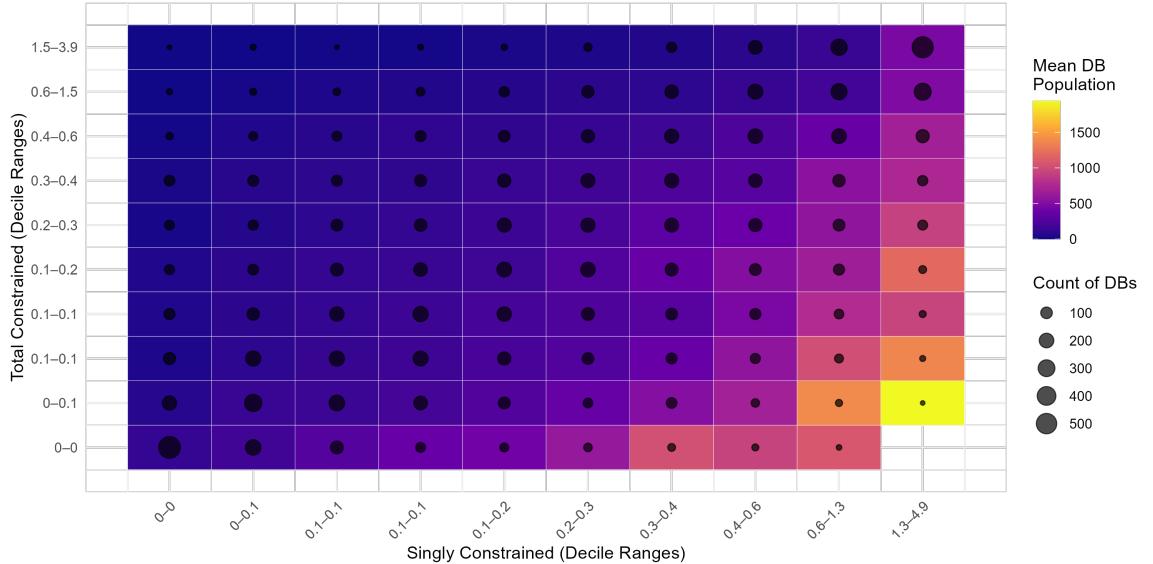


FIGURE 4.2: Heatmap of DB-level population, total constrained and singly constrained accessible parkland binned by decile.

Further inspecting Figure 4.11, the singly constrained plot contains 1.0518895 times more white DBs (i.e., DBs with no population or access) than the constrained plot (1865 DBs instead of out of 1773 DBs out of the total 13322 DBs in the city). This is because this measure considers population in how the proportionality factor allocates parkland area in addition to travel impedance (i.e., if a park can be reached within 15 minutes or not). Hence, if a DB has no population but is within its DB point can reach a park entrance within 15 minutes, it is still allocated no singly constrained accessible due to having no opportunity ‘demand’. This additional consideration allows the singly constrained measure to be considered a competitive accessibility measure, whereas the total constrained measure does not allocate based on population demand. In the same way as the total constrained plot can be interpreted as a plot of parkland area that extends beyond the parks boundaries, the singly constrained plot can also be interpreted this way but based on an allocation that proportionally reflects the 15-minute travel time distance *and* the amount of population in the DB. Visually, singly and total constrained measures appear to mirror some similar trends (i.e., correlation of 0.5771639), however their trends are still divergent. A variable that is included in the proportional allocation factor in the singly constrained measure is DB *population*. DBs with a relatively high population in 15-minute walking proximity to parks always have high singly constrained accessible parkland, whereas this occurrence is only coincidental for the total constrained measure as population is not an input variable. Figure 4.2 reflects the relationship between total constrained, singly constrained and this third variable: population.

In Figure 4.2, DBs with a higher mean population (i.e., pinks, above 500 people

per DB) tend to be assigned high singly constrained accessibility. Conversely, DBs in 15-minute walking proximity but with low population are proportionally allocated less parkland area in the singly constrained measures. This relationship is in part due to the consideration and role of population in the singly constrained measure. DBs in 15-minute walking proximity are allocated proportionally more parkland area if their population is relatively high, and vice versa. Since the total constrained measure does not consider population demand, it is not a function of population. In fact, in this empirical example it appears the opposite relationship tends to be true: with *some* DBs with lower total constrained accessibility calculated for DBs with higher population. These DBs are high population but not that proximate to a relatively large amount of parkland area. However, these same DBs score singly constrained values that are opposite: due to their high population (relative to their neighbourhooding DBs) and their proximity to some parkland area –they are assigned a *high* amount of accessible parkland. These are DBs that are efficient in parkland area accessibility when population competition is considered, but not when population competition is not considered. Furthermore, population competition tends to introduce larger distribution and more unequal distribution of accessibility, as the additional dynamic of proportional allocation of parkland area at the zonal level is introduced (as opposed to simply regional allocation as in the total constrained measure). In fact, the total constrained values tend to be lower than the singly constrained values and larger in range (median is 0.1842466 vs. 0.1706972 with a lower SD of 0.7994873 vs. 0.8944843).

Furthermore: the singly constrain proportionally allocates the area of each park to DBs that can reach it within 15 minutes. Consider a single park: as the 15-minute walking range is spatially small, the DB population size tends not to vary significantly within this range. Hence, the parkland is proportionally allocated evenly. However, what varies is the size of the park. For a large park, with say, few entrances and DBs that have low populations: these DBs will be allocated plenty of parkland ha despite their lower populations, just because they are the only ones that can access this park due to the restrictive 15-minute walk assumption. Their singly constrained values will be *higher* than their total constrained values. Conversely, a small park near many high population DBs has a low amount of hectares to be allocated to the nearby DBs, hence those DBs will have even lower singly constrained accessibility than their total constrained value. This can be observed, for instance, slightly west from the center of the city near the shoreline in Figure 4.11, which is an area near the downtown core relatively dense but relatively green as well. This area has lower quantiles of singly constrained accessibility than totally constrained due to the consideration of *population*. Conversely, further west in the city near the shoreline is more suburban but still green, hence there are pockets that have higher quantiles of singly constrained accessibility than total constrained as population is relatively lower and green space is relatively more plentiful.

Another way to examine the relationship between singly and total constrained accessibility is to sum the  $V_{ij}^S$  and  $V_{ij}^T$  flows to each park and plot them against one another, as shown in Figure 4.3. Recall that the sum of singly constrained flows to a park is equal to its area, since this constraint proportionally allocates only the park's land area

to reachable origins. This plot therefore illustrates how the total constrained measure compares to a park’s actual size. Total constrained accessibility reflects how much parkland is effectively allocated to the origin zone, accounting for both travel impedance and population demand relatively. Parks plotted above the red 1:1 line allocate more accessible parkland than their actual area—indicating these parks are more centrally located or more accessible within the region. Parks below the line allocate less accessible area than they physically contain, suggesting they are more peripheral or less competitive. Interestingly, most parks fall below the line, under-allocating relative to their size. However, a small number—especially among the larger parks—allocate substantially more than their physical area. The top three such parks in each size category are labeled in Figure 4.3. These parks can be interpreted as having ‘over-allocated’ parkland, potentially due to high desirability or centrality. In summary: parks above the line experience greater ‘spatial demand’ than parks below the line. Hence, accessibility flows from parks above the line (and parks really below the line) are especially susceptible to change depending on the constrained accessibility measure used.

Overall, both constrained measures offer more interpretable insights than unconstrained accessibility. The total constrained measure is similar in spirit to commonly used metrics like the 15-minute opportunity, but with a meaningful twist: its units can be interpreted as accessible parkland area (in hectares or any area unit). In contrast, the singly constrained measure also uses the same units, but incorporates both population demand and travel impedance, making allocation a function of both park size and spatial accessibility.

So when should one use singly constrained versus total constrained accessibility? It depends on how the analyst conceptualises the nature of parkland as a resource that is demanded by the population. If park space is considered inexhaustible—meaning that its value does not diminish based on how many people are near it—then the total constrained measure may suffice. For example, a large park like Queen’s Park, surrounded by high population density, may still offer meaningful access to everyone nearby. If everyone (within a 15-minute distance) visited, it is not hard to imagine that the park’s ‘accessibility’ would not change (i.e., it would suffer too much population congestion). However, consider for example a parkette that is near the same highly dense population. As it is smaller in park size, it makes more intuitive sense to see this park as more exhaustive, i.e., if everyone within 15-minutes visited its accessibility would certainly diminish due to population congestion as it is a small park by area. It could be conceptualised that both a large park and a small park are exhaustible: based on the demanding population (in proximity) per parkland *area*. In this sense, the singly constrained approach offers a more nuanced and realistic measure than the total constrained measure. It recognizes that parkland is a finite, exhaustible resource, especially in dense urban areas as well as accounting for the resilience of exhaustibility of a larger park (due to its larger area).

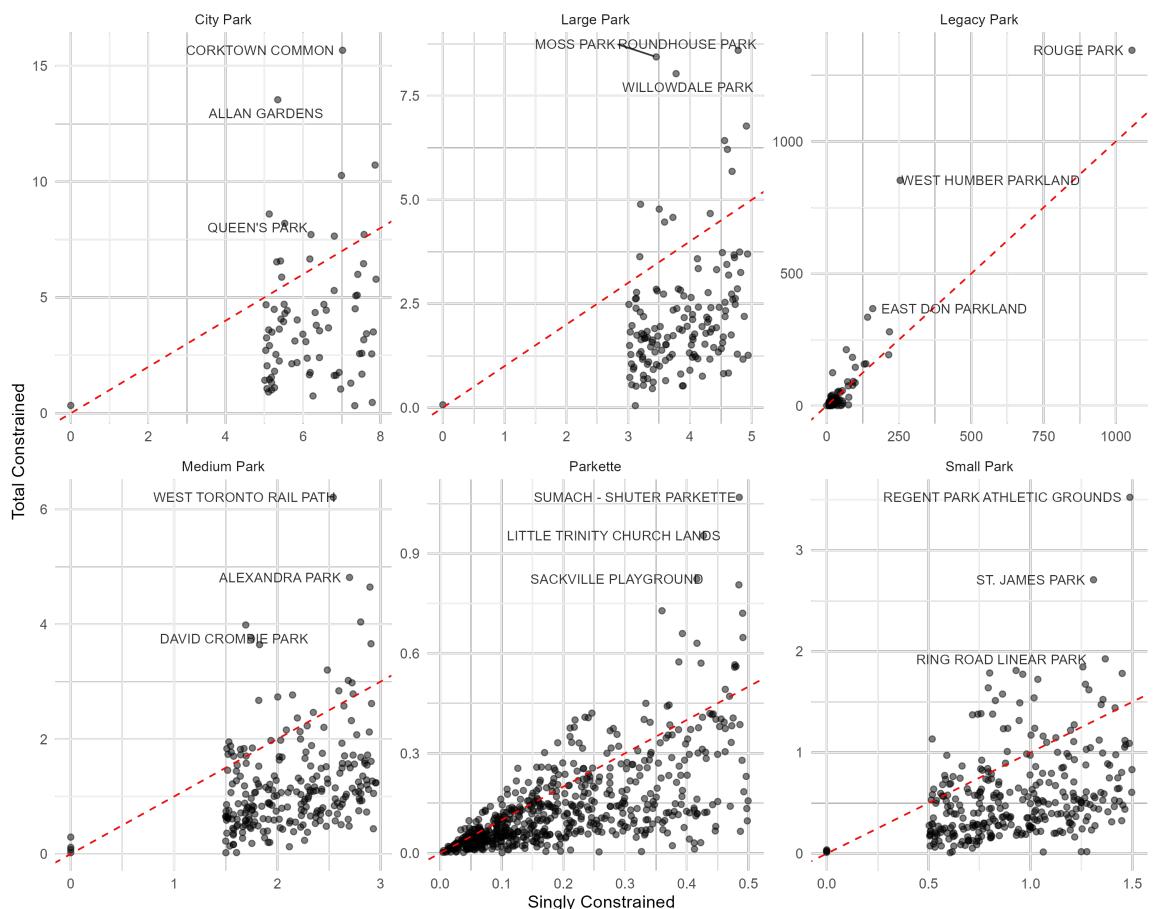


FIGURE 4.3: Heatmap of DB-level population, total constrained and singly constrained accessible parkland binned by decile.

#### **4.2.1 Neighbourhood-level accessible parkland area (to the population within 15-minute walking range)**

Another way of examining the accessibility data is by aggregating it to a spatial zoning system with more meaning: such as the city of Toronto neighbourhoods. These neighbourhoods are City-designated ‘social planning neighbourhoods’ which are used by staff to collect data, plan and analyse service provision (City of Toronto, 2024). The neighbourhoods, as reviewed in Chapter 3, were labelled with Emerging Neighbourhood (EN) and Neighbourhood Improvement Area (NIA) classification. In the Figure 4.4, all  $V_{ij}^0$ ,  $V_{ij}^T$  and  $V_{ij}^S$  values are presented at this level of aggregation.

In the Figure 4.4, as can be seen the unconstrained values are not fixed to any sort of zonal or regional total whereas the sum of total constrained and singly constrained values equal to all the amount of parkland ha accessible within 15-minute. For instance, in the top quantile dark oranges represent 3210 ‘parkland-ha- $\leq$ -15-min’ or greater, now summed at the area of the neighbourhood. Whereas the top quantile in the total constrained measure represent *accessible* 48.11 ha or greater, directly in units of parkland ha that can be accessed out of the total parkland hectares. Summing all DBs within a neighbourhood maintains this total constraint and hence the associated intuition. The same logic can be applied to the singly constrained measure, but with the additional consideration of population competition. And like Figure 4.11, values between the measures in Figure 4.4 share some commonalities: unconstrained and total constrained measures still remain proportional (by  $K^T$ ), and the singly constrained values match total constrained values in DBs with and near population values that are relatively average. Singly constrained values deviate when population is low (i.e., singly constrained values are lower than total constrained) and population is high (i.e., singly constrained values are higher than total constrained values). If the heatmap of these two measures and population was reproduced at the neighbourhood aggregation, the general trends would be similar as at the DB level.

However, the trends in the data are more smoothed out in this aggregation. As shown in Figure 4.5. At both the DB and neighbourhood levels, the total constrained accessibility distribution is right-shifted and more dispersed compared to the singly constrained measure. This indicates that parkland is allocated more evenly under the total constraint, with fewer areas experiencing extremely high or low access. In contrast, the singly constrained measure shows a slightly more peaked and skewed distribution, reflecting localized competition for limited park space. At the DB-level, accessibility measures are more granular and variable, capturing finer differences in proximity and population competition. This results in a wider range and more extreme values, especially under the singly constrained measure, which reflects localized demand. At the neighbourhood-level, aggregating across DBs smooths out some of the variation and reduces some extremes. While this can enhance interpretability at a policy scale, it may also mask pockets of low or high access that could be critical. For this reason, examining patterns at the lowest level of aggregation is best practice, alongside summing to zoning systems that have more significant meaning from a planning perspective (such as

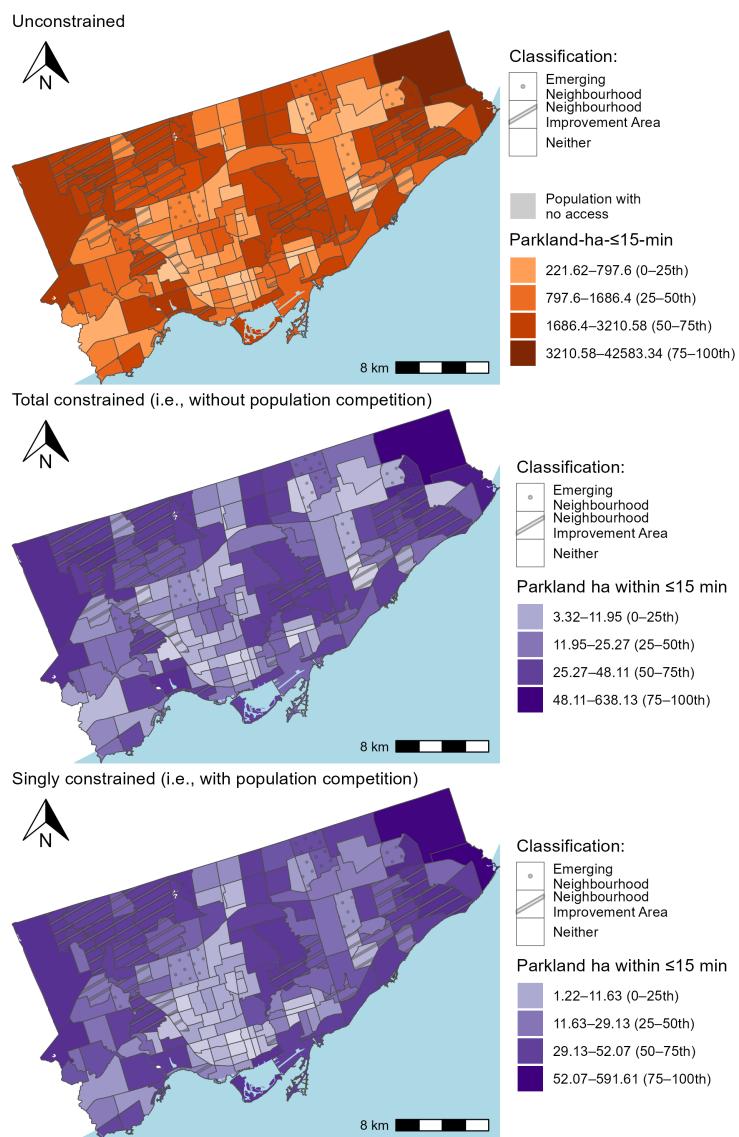


FIGURE 4.4: Map of Toronto CMA with accessibility to parklands per neighbourhood and neighbourhood classification

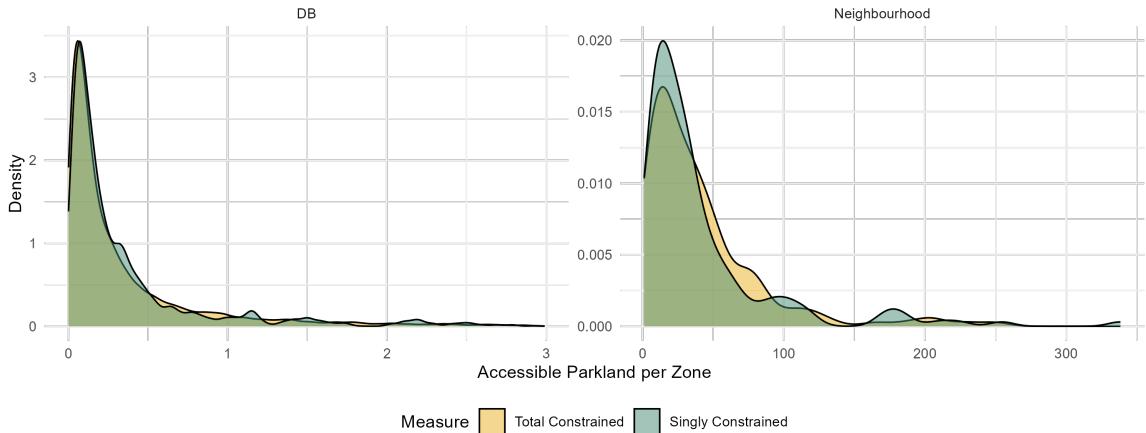


FIGURE 4.5: Density distribution of the constrained accessibility measure values at the dissemination block and neighbourhood block. For both aggregations, the datasets are truncated at top decile for the purpose of visualisation.

Neighbourhood).

Furthermore: since DBs are designed to be the smallest units of population with as homogeneous census profiles as possible (range between 0 to 3375, median of 117 people), much smaller than the city-designed neighbourhoods (range between 6419 to 33690, median of 17024 people), the following question may arise specifically for neighbourhoods: what is the accessible parkland ha per neighbourhood population? This per capita measure is visualised in 4.6. All three accessibility measures are shown, though the unconstrained measure should be interpreted with caution. Because it lacks a population constraint, the units–parkland-hectares $\leq$ 15-minute’ divided by neighbourhood population–can be misleading. While neighbourhood population is a fixed and tangible quantity (a known share of the total population), the amount of ‘parkland-hectares $\leq$ 15-minute’ is not similarly bounded or consistently defined across the region. As a result, the regional sum of the unconstrained measure lacks clear meaning, making comparisons or interpretations less intuitive than with constrained measures. Hence, for a non-competitive yet interpretable alternative to the unconstrained measure, the per capita total constrained accessibility should be considered, as it retains intuitive units while incorporating the intuition of the unconstrained measure.

As can be surmised and as discussed at the level of the DB for the raw scores, depending on the constrained measure used, results can be quite different. However, when inspecting the neighbourhood level values, the median per capita values that consider competition (i.e., singly constrained) yield scores that are higher than the total constrained measures. For EN and NIA, values are 18.4 sq m per capita and 22.2 sq m per capita for NIAs assuming the singly constrained measure and 13 sq m per capita and 20.9 sq m per capita considering the non-competitive measure (i.e., total constrained).

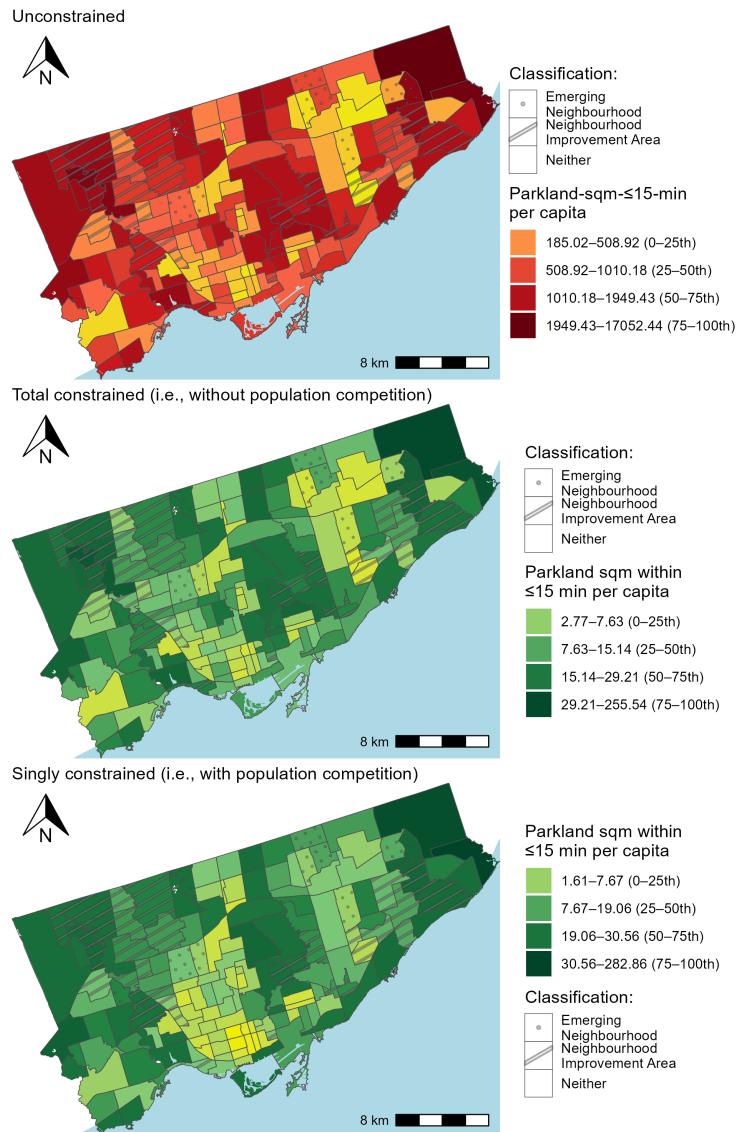


FIGURE 4.6: Map of Toronto CMA with accessibility to parklands per neighbourhood and neighbourhood classification

These neighbourhoods make up 43 of the 158 neighbourhoods in the city, with the neighbourhoods of neither classification also demonstrating a similar trend (i.e., 15.2 sq m per capita considering competition and 13.4 sq m per capita considering the total constrained accessibility per capita). The spatial distribution of the impact of population competition (i.e., difference between singly and total constrained per capita measures) is visualised spatially in Figure 4.7.

From Figure 4.7, the difference in per capita scores between the singly constrained and total constrained measures can be interpreted as the effect of population competition. Positive values (in purple) indicate neighbourhoods with higher per capita accessible parkland when population competition is considered—that is, they are more regionally competitive in capturing parkland access. In contrast, negative values (in red) suggest neighbourhoods that are less competitive once population is factored in, having higher per capita access only when travel impedance is considered alone. These differences have important policy implications. For example, 8 of the 10 EN and 19 of the 33 NIAs have higher per capita access under the total constrained measure, meaning their populations benefit when competition is accounted for. In contrast, neighbourhoods outside of these categories are more likely to be negatively affected—with 64 neighbourhoods showing higher access under the singly constrained measure, compared to 51 that benefit more under total constrained. This suggests that population competition presents NIA and EN as having higher parkland accessibility, in part due to their proximity to a larger amount of parkland area and lower amount of population, while more established (non NIA or EN) neighbourhoods may lose relative access under competitive allocation.

However, the difference in these per capita rates—and in the raw values aggregated at the neighbourhood themselves—are no longer as clearly related to population nor the amount of park space as previous outlined. This is because these values are aggregated to a coarser spatial scale and divided by population, which obscures some of the finer-scale spatial dynamics. Still, they relate closely to the concept introduced in Figure 4.3: the difference between how much parkland is allocated versus how much parkland actually exists.

Under the total constrained measure: neighbourhoods with more parks that allocate *less* than their park area tend to have parks that are below the red line shown in Figure 4.3 and can be perceived as parks with greater “potential” that is not allocated away. However: this difference in allocation (between the actual area and total) results in a *positive* difference in neighbourhood accessibility allocation when calculating the singly constrained neighbourhood value (i.e. purple DBs). And this relationship is roughly linear (Figure 4.8).

While this relationship is not perfectly captured due to difference in zonal aggregations and spatial structure of the travel impedance allocation. Nonetheless, the pattern reinforces a key point: when population demand is not used to constrain the allocation of accessible parkland, as in the total constrained measure, parkland is allocated freely based on travel impedance alone. As a result, centrally located parks over-allocate relative to their actual area, while peripheral parks under-allocate—an imbalance corrected

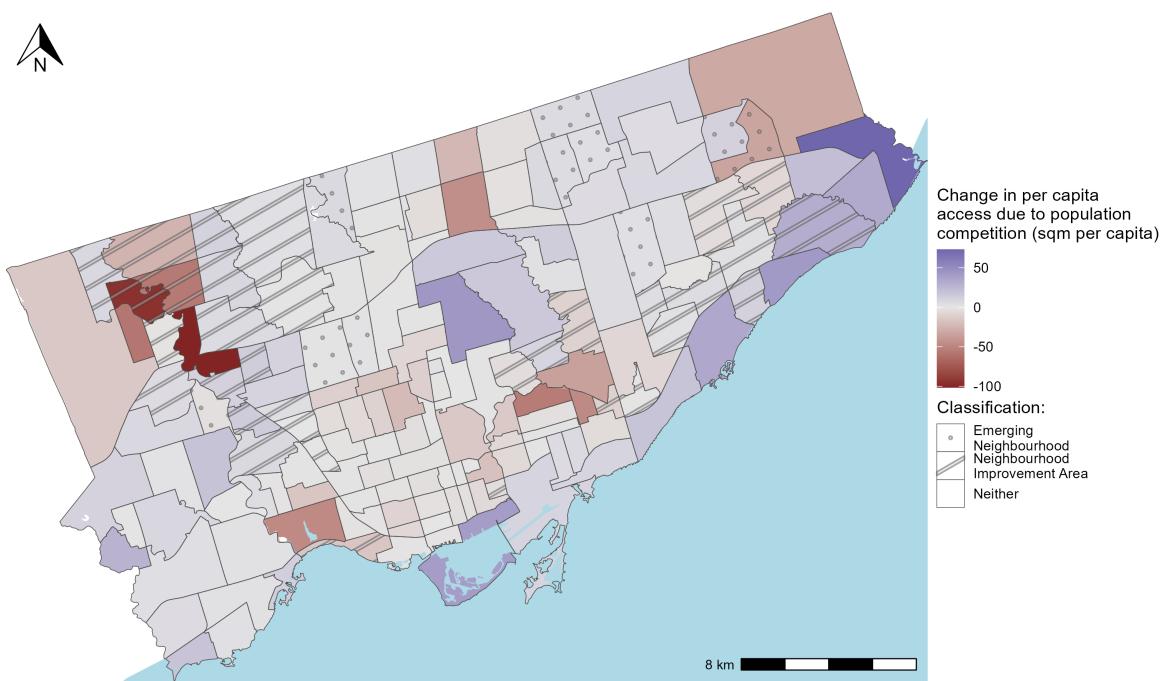


FIGURE 4.7: Difference in per capita constrained accessibility to parklands per neighbourhood values. Namely, singly constrained per capita minus total constrained per capita.

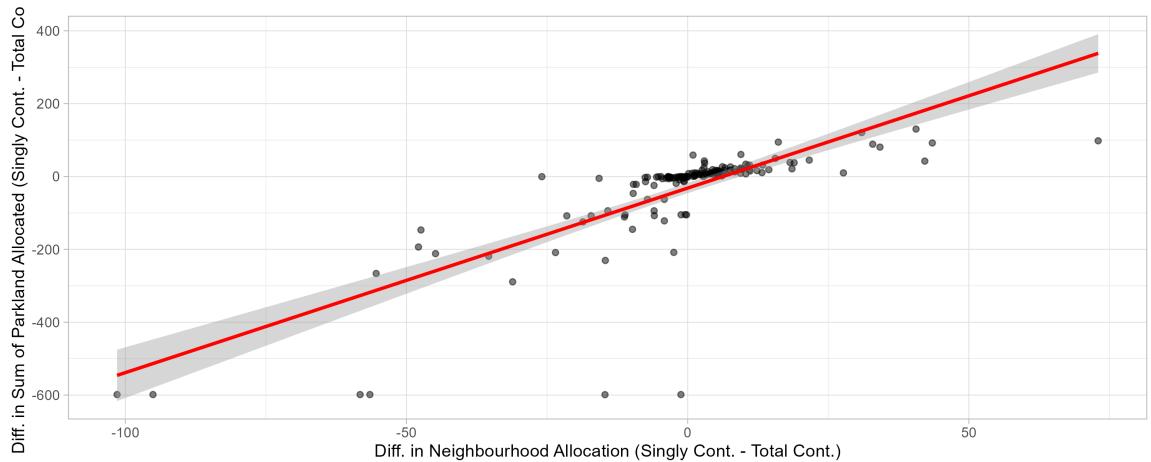


FIGURE 4.8: Difference in 'accessible' parkland allocated from parks in a neighbourhood and the difference in neighbourhood constrained accessibility measure (difference is singly constrained minus total constrained).

when population demand is incorporated in the singly constrained measure. Also note: only 1149 parks out of the total 1581 parks that entered the accessibility calculation are shown in Figure 4.8: parks with differences that match in magnitude. Parks that do not match in magnitude are few, and tend to be very small (likely due to discrepancies due to aggregation inconsistencies across spatial units).

From another examination vantage point: depending on what constrained measure is used, neighbourhoods can dramatically change rank. Of the 158 neighbourhoods, 154 changed rank, with 75 increasing in rank (i.e., higher per capita value when singly constrained is used compared total constrained) and 79 decreasing in rank. The top 10 neighbourhoods that increased rank (purples) and top 10 that decreased rank (reds) are in Figure 4.9. From those familiar with Toronto, top 10 are neighbourhoods with a good amount of parkland, and well residential. These neighbourhoods are like Highland Creek which jumped 85 ranks when using the singly constrained measure. This neighbourhood is relatively low population and contains a high amount of green space that is not easily reached by other populations. Hence: it has relatively low amount of accessible parkspace without considering competition (6.24 sq m per capita) but a high amount when competition is factored into proportional allocation (27.9 sq m per capita). Conversely, neighbourhoods that are centrally located to plenty of parkspace but are also relative proximate to a high amount of population decreased in rank when considering competition, like Cabbagetown-South St. James Town which dropped 73 ranks, from 26.3 sq m per capita in the total constrained measure to 7.65 sq m per capita in singly constrained measures.

Figure 4.10 summarises the change in rank of all neighbourhoods depending on what constrained per capita measure is used. The four neighbourhoods that did not change

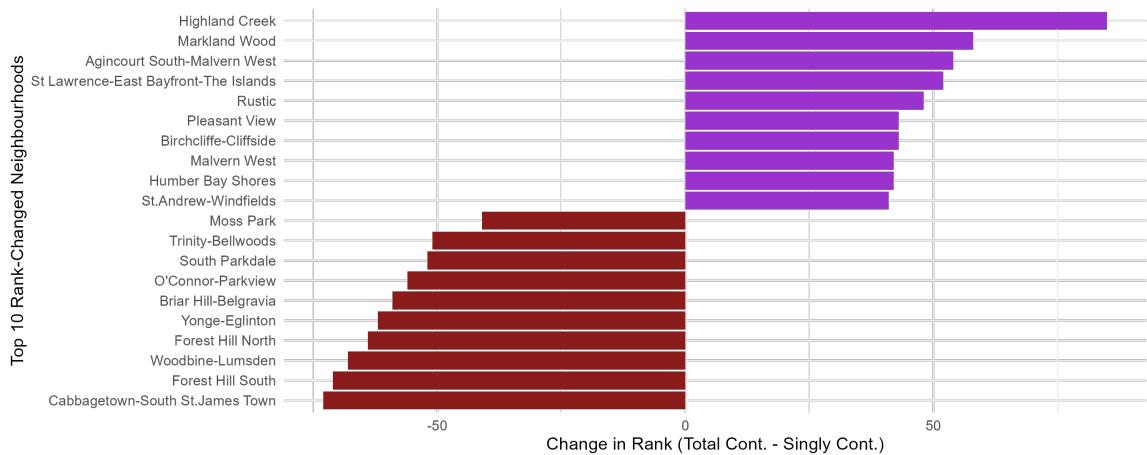


FIGURE 4.9: Change in neighbourhood rank using singly to total constrained accessible parkland measure for top negative and positive change-rank neighbourhoods.

rank are along the x axis in black, the top 10 negative and positive rank-changed neighbourhoods as seen in Figure 4.9 in red and purple respective along with the rest of the neighbourhoods in grey. The smoothed curved trend line demonstrates the general increasing trend. For neighbourhoods with high total constrained ranks, their rank using singly constrained accessibility decreases (i.e., a much lower  $v_i^S$  than  $v_i^T$ ), and as the rank of the neighbourhood decreases considering the total constrained score - their singly constrained scores increase and hence rank higher by these scores (i.e., a higher  $v_i^S$  than  $v_i^T$ ). Overall: neighbourhoods in these extreme ends are especially sensitive to changing rank: i.e., especially neighbourhoods with parks that are central, and especially those that are larger, over-allocated their accessible parkland, while parks in neighbourhoods that are not that central (and/or smaller) under allocated –both these allocation types were ‘corrected’ with the proportional and zonal allocation of the singly constrained accessibility measure. Neighbourhoods with constrained values that are more similar tend to see changes in rank that are more modest and in either directions, in part as a product of rank changes in the extreme cases.

In summary, the total and singly constrained (opportunity-constrained) accessibility measures offer intuitive ways to interpret values at the zonal level, as they are expressed in meaningful units of accessible parkland area. This contrasts with the unconstrained measure, where values—beyond simply being high or low—lack clear interpretability, especially due to the absence of constraints on either population or opportunity. Importantly, the total constrained measure retains proportionality with conventional methods (e.g., 15-minute catchments), but does so while preserving interpretable units. The singly constrained measure goes a step further by incorporating population demand, offering a more restrictive—but theoretically more realistic—representation of access where competition for finite opportunities (like parkland) exists.

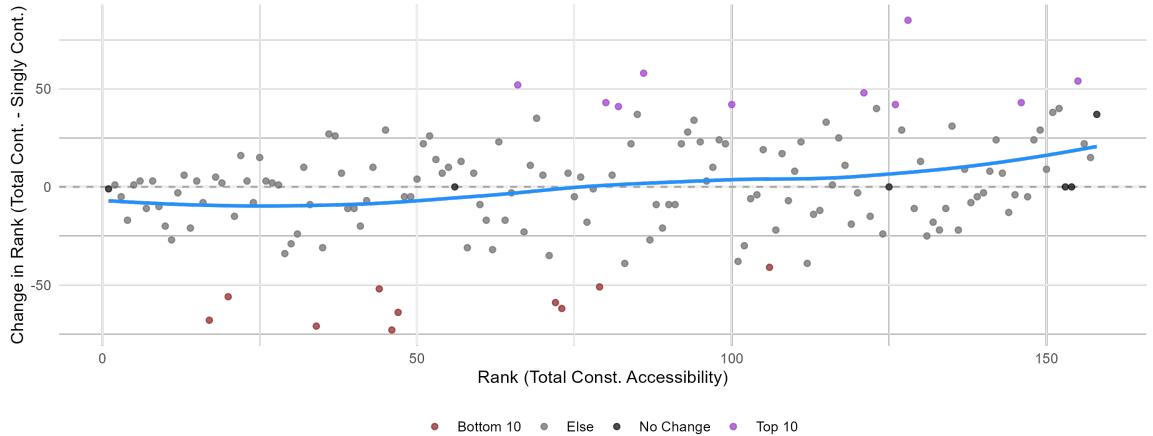


FIGURE 4.10: Change in neighbourhood rank using singly to total constrained accessible parkland measure for all neighbourhoods.

The figures in this section highlight how neighbourhood rankings shift depending on whether a competitive (singly constrained) or non-competitive (total constrained) measure is used. These shifts demonstrate that each measure reflects a different allocation logic, and neither is inherently better. The appropriate choice depends on the analyst's assumptions about the nature of the opportunity:

- Does the opportunity lack scarcity or competition, such that allocation based solely on regional travel impedance (as in the total constrained measure) is appropriate?
- Or, does the opportunity reflect a finite, competitive resource (e.g., parkland area), in which case allocation should also account for local population demand (as in the singly constrained measure)?

If the opportunity is indeed competitive—and most real-world opportunities are—then the analyst must ensure there is adequate zonal information about the population that demands it. In this analysis, a normative assumption is made that all people should have access to the opportunity, making the singly constrained model especially relevant for evaluating spatial equity. However, if using population as done in this empirical example is inappropriate (i.e., all people should not enter the calculation), the results will *differ* hence impacting conclusions made for policy.

### 4.3 Accessible population (to the parkland)

From another analytical vantage point, a policy analyst may be interested in conducting this investigation from the perspective of *accessible population*, i.e., understanding which parks are serving what amount of population. This is essentially the transpose of the accessibility measures discussed in the preceding section: instead of measuring accessible parkland per population zone, accessible population per park is examined.

Since parks serve as the destination zones (with each park represented by its nearest reachable entrance), the lowest level of aggregation for  $M_{ij}^0$ ,  $M_{ij}^T$ , and  $M_{ij}^S$  is at the park level ( $j$ ). Thus, the summaries  $M_j^0$ ,  $M_j^T$ , and  $M_j^S$  represent the population potentially served by each park under the unconstrained, total constrained, and singly constrained measures, respectively. These are shown in Figure 4.11. The unconstrained measure can be interpreted as “market potential,” reflecting pure travel-time-based reachability. The total constrained measure introduces a realistic intuition by allocating the total population in the region to each park by its associated travel impedance, while the singly constrained measure allocates each zonal population considering associated reachable parkland area (i.e., the park supply) as well as associated travel impedance. All three plots are binned into quartiles (0–25th, 25–50th, 50–75th, 75–100th percentile). White areas indicate non-park zones, while grey parks are not accessible to any population within a 15-minute walk. As in the previous section, the unconstrained and total constrained measures remain proportionally equivalent, and both the total and singly constrained results are shown on a common scale to support direct comparison.

Figure 4.11 visualises the unconstrained and total constrained measures that are proportional in their magnitude as maintained by the  $\hat{K}_j^T$  balancing factor, and the singly and total constrained values are placed on the same scale with the singly constrained measure demonstrating a larger range and variance than the total constrained measure. Beyond these commonalities, the constrained values from Figure 4.11 should be interpreted as the accessible population (within 15 minutes of walking) as opposed to the accessible parkland area (within 15 minutes of walking). As well, each constrained measure sums across the region to equal the total amount of population that can reach parks within 15 minutes: 2761103 people.

Unlike the DBs, a spatial unit without as much significance for planning (compared to the neighbourhood), the park itself is worth examining. Due to the proportionality, parks per park size classification can be compared by their ratios of *accessible population* to their park size as shown in Figure 4.12. It can be seen that Parkettes and Small parks serve population at a more efficient rate (i.e., their parkland area) than the larger parks using the total constrained measure. But the moment that their parkland supply is considered (i.e., allocation is based on the area of the park itself and its travel impedance – and not just its proximity to population and associated travel impedance), the intensity of the dynamic changes. In fact, when parkland supply is considered, smaller parks are still more ‘efficient’ than larger parks but not by *that* much. Again: this is because small parks are not allowed to be ‘over’ allocated, despite their centrality to population due to the constrained measure. Conversely, larger parks are *more* allocated to populations though these parks are relatively less centrally located as the populations face lower competition. Depending on how the opportunity type is considered, either measure can be used: but how parks rank and their efficiency of ‘population service’ will differ (as in the transpose expression of *accessible parkland area*).

Furthermore: parks can be ranked by this accessible population per the park’s area rate. This rate expresses the park’s potential ‘population serviced’ per sq m. It can

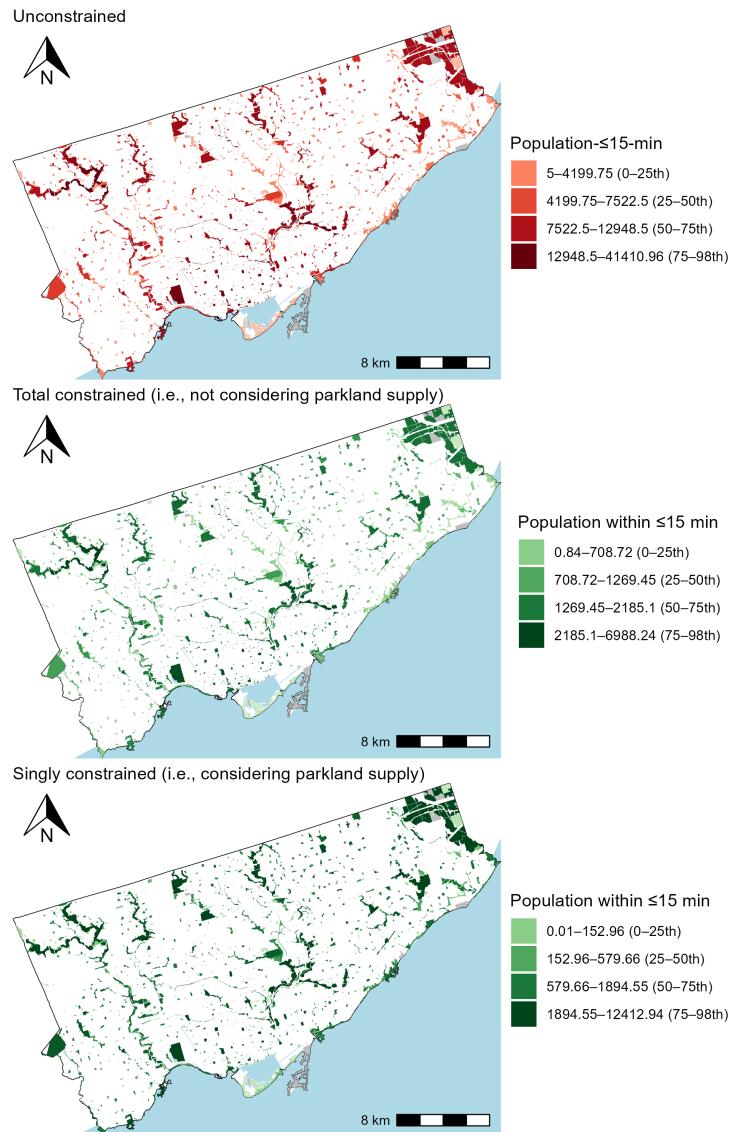


FIGURE 4.11: Map of Toronto CMA with accessibility to population per park.

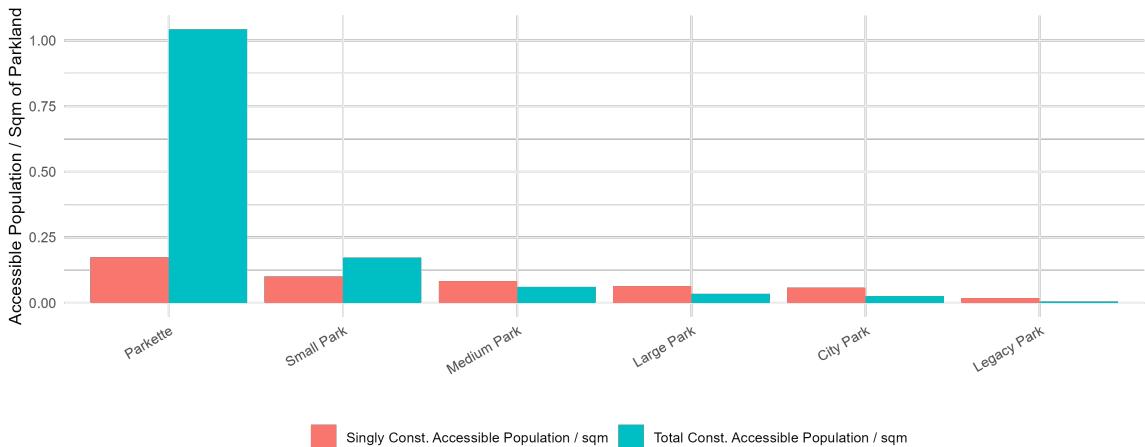


FIGURE 4.12: Constrained accessible population per parkland area by park size classification: total constrained vs. singly constrained.

be used to rank the parks by how centrally they are located near population per area for the city: to determine which parks are located near the highest amount of potential population, i.e., parks with the most population (to reach) potential. These parks should be flagged as parks that should be most concerned with population congestion. Conversely, parks that rank low are of least concern (at a per sq m rate).

On this topic: Figure 4.13 visualises how park rankings shift depending on whether the total or singly constrained measure is used, with parks grouped by size classification. Notably, smaller parks—particularly parkettes, small parks, and some medium parks—often see sharp drops in rank under the singly constrained measure. These parks typically score higher under total constrained accessibility because they are centrally located and close to higher population densities. However, when the supply of parks (typically larger) is considered (i.e., singly constrained), accessibility declines. Examples include Old Mill Parkette, Ormskirk Park, and R.V. Burgess Park, all of which are situated in residential areas near much larger parks.

While a clear uniform trend across all parks does not emerge in Figure 4.13, a pattern among larger parks (Legacy, City, and Large classifications) can be seen. These larger parks tend to have mid to low ranks under the total constrained measure but often climb in rank under the singly constrained measure. For example, McGregor Park sees a significant boost due to its central location to population and competitive supply advantage. In contrast, large parks with initially low total constrained values experience only modest changes (e.g., small increase in rank for Colonel Danforth Park and slight decline for Coxwell Ravine Park). Overall, this suggests that larger parks tend to attract more population simply by virtue of their size, and when park supply is explicitly considered—as in the singly constrained measure—their accessibility increases relative to smaller parks competing in denser park environments.

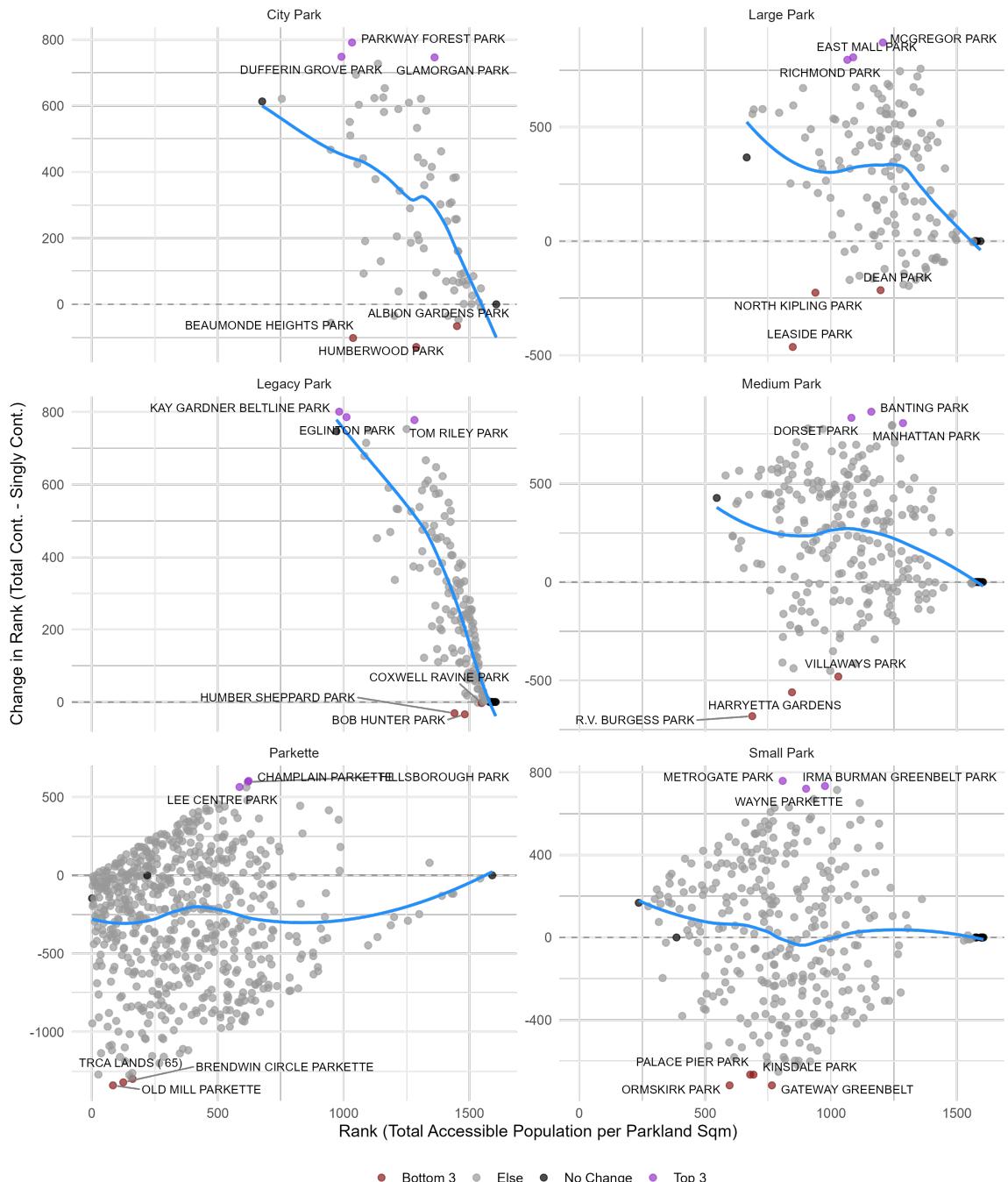


FIGURE 4.13: Change in park rank based on constrained accessible population per parkland area measures by park size classification.

#### **4.3.1 Neighbourhood-level accessible population (to the parkland within 15-minute walking range)**

Moving forward, the neighbourhood can serve as a useful spatial unit for analysis as well, as it aligns well with both population distribution and parkland organization. The majority of parks are fully contained within a single neighbourhood, allowing for meaningful calculation of ratios such as accessible population per unit of parkland. Additionally, from an equity perspective, neighbourhood-level analysis enables examination of how different types of parks serve populations across neighbourhoods with varying characteristics, supporting more targeted planning and policy evaluation.

In the Figure 4.14, all  $M_{ij}^0$ ,  $M_{ij}^T$  and  $M_{ij}^S$  values are aggregated at the level of the neighbourhood. They can also be divided by parkland area in that neighbourhood (as determined by summing and splitting, if needed, parkland area within each neighbourhood) and demonstrated as accessible population per parkland sq m as in Figure 4.15.

Referring to Figure 4.14 and Figure 4.15, the unconstrained measure is once again shown on its own scale, while the constrained measures are displayed using a shared scale. Visually, there is a notable contrast between the total and singly constrained measures: the total constrained map shows higher accessible population values in the downtown core (near the center and shoreline) while the singly constrained map highlights higher values in the northeast and northwest.

The pattern in Figure 4.14 reflects some intuitive spatial dynamics. The downtown area has a dense population and numerous centrally located parks (especially small ones), resulting in high “market potential” under the total constrained measure. However, once parkland supply and population competition are considered (as in the singly constrained measure) these same areas score relatively lower. In contrast, areas in the northeast and northwest, with both substantial parkland and nearby pockets of higher population (and pockets of *no population*), show higher accessibility under the singly constrained measure. These neighbourhoods have a more favorable population-to-parkland ratio than the downtown core (i.e., a higher supply of parkland relative to the population in proximity), leading to higher values.

While these patterns are distinct for the raw neighbourhood level values, normalizing by parkland area (i.e., calculating accessible population per square metre of parkland) reveals greater similarity between the two constrained measures (i.e., Figure 4.15). This suggests that, at the neighbourhood level, accessible population tends to scale with available parkland. Nonetheless, the per-parkland-area ratio may be useful for identifying neighbourhoods where additional parkland would be most efficient, i.e., neighbourhoods with high accessible population per unit of existing parkland.

Compared to the previous subsection, the analysis of *accessible population* per neighbourhood offers a complementary perspective: neighbourhoods that are neither NIA or EN tend to have *higher* accessible population per square metre of parkland. Under the total constrained measure, their median is 0.078 people/sq m, and 0.089 people/sq m under the singly constrained measure. In contrast, EN and NIA neighbourhoods show

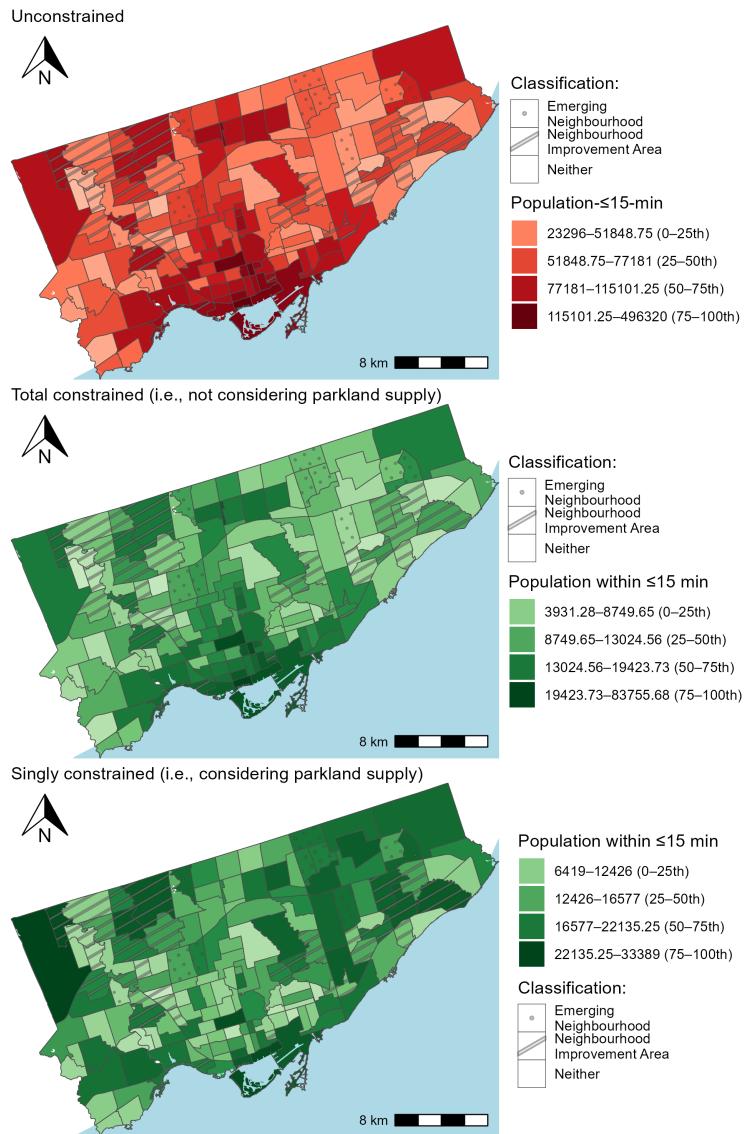


FIGURE 4.14: Map of Toronto CMA with accessibility to accessible population per neighbourhood for unconstrained, total constrained and singly constrained measures.

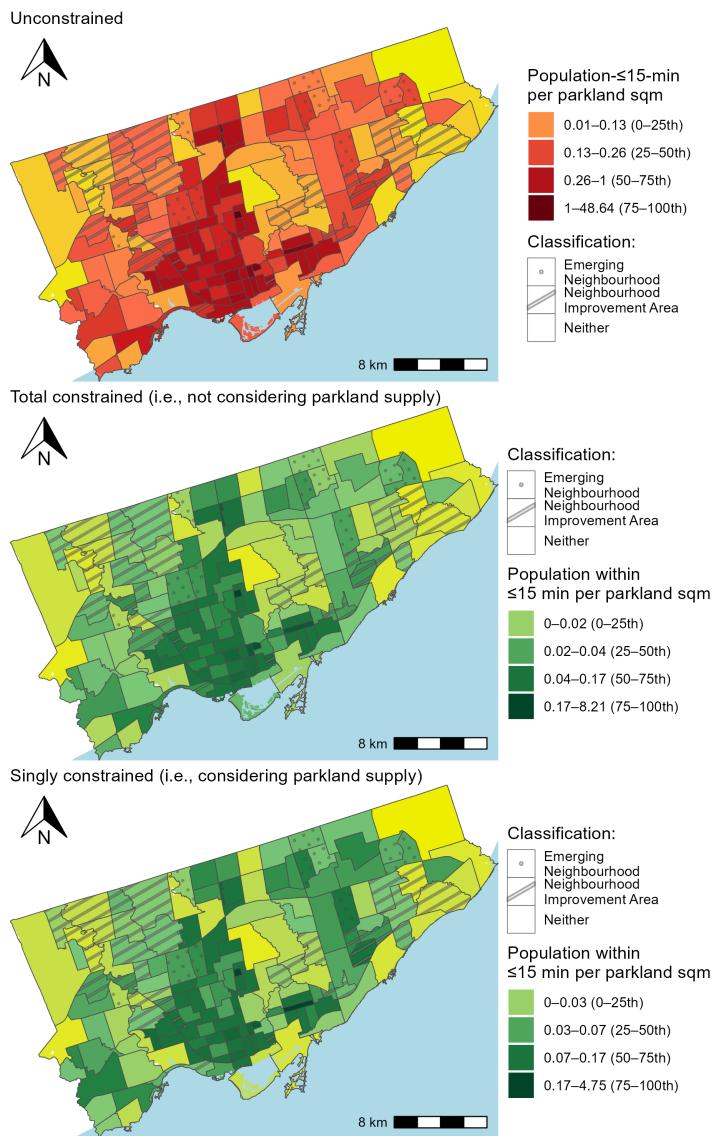


FIGURE 4.15: Map of Toronto CMA with accessibility to accessible population per parkland sqm at level of the neighbourhood for unconstrained, total constrained and singly constrained measures.

much lower medians (total: 0.038 and 0.029, singly: 0.062 and 0.045). This suggests that, from either constrained perspective, parks in non-NIA/EN neighbourhoods are more efficiently reachable. This finding mirrors the per capita neighbourhood *accessible parkland* analysis findings. However, unlike the preceding per capita accessible parkland analysis, the rank ordering of neighbourhood classifications remains more consistent between the two constrained *accessible population* measures. This consistency may be due to the way parkland area is aggregated and allocated across neighbourhood boundaries, as well as how ratios (e.g., population per parkland sq m) are calculated. This reinforces a broader point previously raised: the choice of aggregation unit and denominator strongly influences results, and analysts must be attentive to these decisions when interpreting accessibility metrics.

Ultimately, examining population-constrained accessibility from the park itself *and* from the neighbourhood provides valuable insight for planning of parkland service through the perspective of population potentially served. It allows analysts to: - Understand how many people are served by each park, both in total and per parkland (or summed and normalized by some other zonal unit variable); - Identify over- or under-performing parks. For example, parks near large populations but lacking amenities may represent strategic opportunities for investment to improve service efficiency.

In sum, analyzing the *accessible population* approach can serve as a complimentary way in analyzing parkland service provision.

#### 4.4 Conclusion

This chapter presents the calculated accessibility scores across the following dimensions:

1. Unimodal total constrained **accessible parkland** summarised per origin zone  $V_i^T$  (i.e., DB) and then per neighbourhood. All people (in DBs) to all parkland area (in parks) by a 15-minute walk is assessed. This measure can be interpreted as “how much parkland area is potentially reachable by the walking-population from this zone?”. The measure is also presented as a ‘parkland provision’ ratio of parkland per walking-population.
2. Unimodal total constrained **accessible population** summarised per destination zone  $M_j^T$  (i.e., the park itself) and then per neighbourhood. This measure is the  $i \rightarrow j$  transpose of  $V_i^T$ , and can be interpreted as “how much walking-population is potentially reachable from this park?”. The measure is also presented as a potential population ‘service catchment’ or a ratio of population potential per parkland area at the park level and at the neighbourhood level.
3. Both of the above constrained accessibility measures are compared to the conventional calculation of accessibility, or what is referred to in this work as unimodal unconstrained **accessible parkland** summarised per each DB  $V_i^0$  and **accessible population** summarised per park  $M_j^0$ . These measures assess the same walking network but do not consider any zonal or regional constraints. As such, these

conventional measures only offer insight into the proportion of potential reachability. Unlike the constrained measures, no provision or catchment ratio can be meaningfully calculated for these measures.

Overall, constrained accessibility provides a more interpretable and policy-relevant framework than unconstrained methods. These measures allow analysts to:

- Identify DBs with poor access to parkland. This could be DBs in equity-seeking neighbourhoods (NIA), helpful in highlighting areas that may warrant new parks or improved access.
- Identify parks with low population reach, revealing underutilized assets or areas where connectivity enhancements could expand service reach. This can be seen as a transpose understanding of the first point, but with its interpretation meant for a different communication.

However, the analysis in this chapter has only considered walking accessibility. One key consideration is missing: parks can be reached not only by walking. People travel by transit, cycling, and driving—each with their own distinct behavioural patterns (as discussed in Chapter 3). The unconstrained accessibility measure does not have a consistent methodology in considering multiple mode, but the constrained measures can be formulated to consider these modes. Hence, in the following chapter, the multimodal extension of the measures will be detailed.

# Chapter 5

## CHP 5 - Comparing multimodal total-, singly and unconstrained accessibility of Toronto's parklands

### 5.1 Overview

In this chapter, incorporates multiple modes, expressing multimodal accessibility of parkland from the perspective of parkland area and population. Like the unimodal example, origin and destinations are DB and park entrance points, and their weights are modal population per DB *or* mode-specific parkland area (in hectares) per park. The normative impedance functions for all modes discussed in Chapter 3 are adopted.

In the first half of the chapter the focus is on *accessible multimodal parkland area*. Values of  $V_i^{m0}$ ,  $V_i^{mT}$  and  $V_i^{mS}$  are expressed at the DB level and the neighbourhood aggregation along with a comparison between multimodal accessible parkland area by walking and unimodal accessible parkland area values. Then, the accessible parkland per capita ratio is demonstrated, neighbourhoods are ranked and a discussion is detailed based on ‘potential parkland service provision’.

In the second half of the section, *accessible multimodal population*  $M_j^{m0}$ ,  $M_j^{mT}$  and  $M_j^{mS}$  is presented per park and per parkland area ratios at the level of the park itself and the neighbourhood is discussed as the ‘potential population served’. This section is concluded by a comparison of multimodal parkland and population values aggregated by the 6 former municipal boundaries.

### 5.2 Accessible multimodal parkland (for the population)

First turning to the concept of *accessible multimodal parkland*, the results in this section include those calculated by unconstrained and constrained multimodal measure. Each measure yields different results (i.e., the amount of parkland area that is accessible by

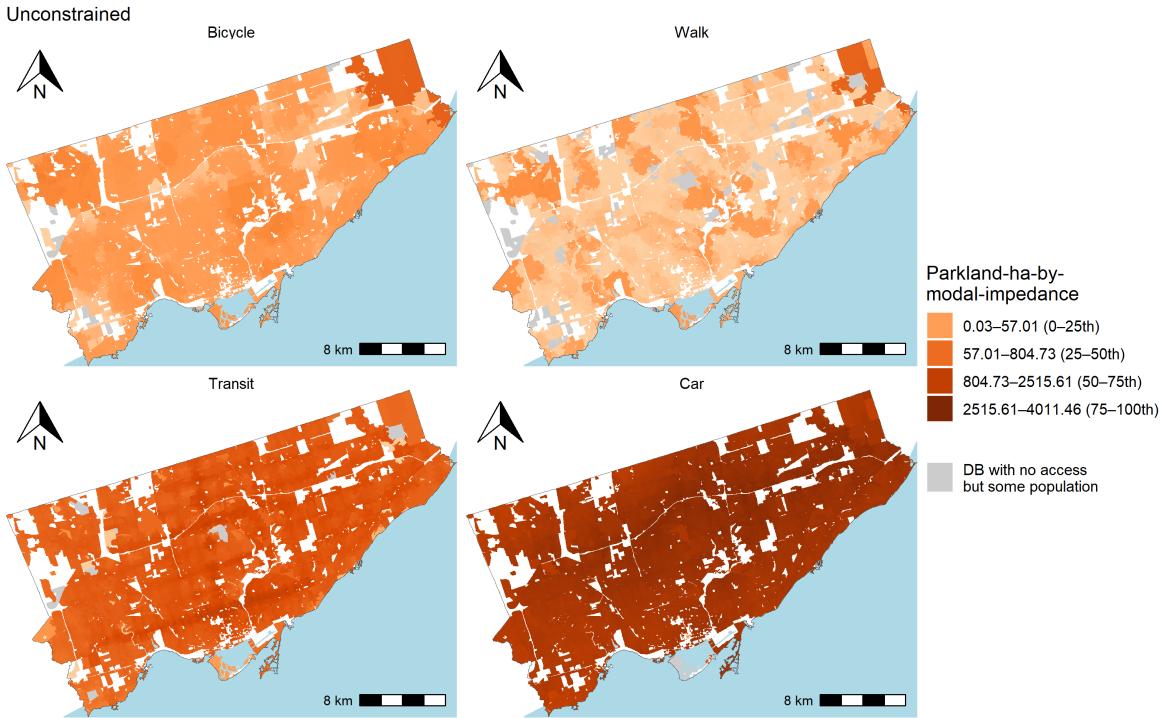


FIGURE 5.1: Accessibility to parkland area per DB as measured by unconstrained accessibility for multiple modes.

a certain mode at a specific zone) depending on the assumptions contained within the accessibility calculation. For the unconstrained measure, the values proportionally reflect the amount of accessible parkland area by mode: as there is not proportionality constant to maintain the resulting values units. For the constrained measures, the values remain a proportion of the regional total and/or the parkland area of each specific park allocated to each origin. Each origin can also be defined by the amount of mode-share present; the use of mode-share as an origin weight is only relevant for the singly constrained accessibility measure, and differences between results will be discussed.

Beginning with unconstrained accessibility, Figure 5.1 displays the unconstrained accessibility to parkland area, calculated for each mode separately. The scale is in quartiles. Also, DBs with no accessibility are shown in white while DBs with some census population but no access are shown in grey. As the impedance functions are assumed to be comparable, results are placed on the same scale across all four plots, but their units are different depending on the mode. For cycling and walking, it can be understood as accessible parkland-ha-within-15-minutes, whereas for transit and car it is parkland-ha- $e^{-0.02}$  and parkland-ha- $e^{-0.04}$ .

In Figure 5.1, it can be observed that many pockets of the city offer no access by

bike, more access spatially for walk but lower, medium amount of access by transit and high amount by car (recall: motorized travelers can travel up to 120 minutes, though with diminishing likelihood and motorists do not access smaller parks). It is notable that in almost all DBs with population, motorists have accessibility. Furthermore, it is notable that there are bands of higher accessibility by transit running east-west and north-south; these higher bands correspond to lines of higher-frequency transit service (e.g., the subway and frequent light rail or bus service). High walking, and to a greater extend cycling accessibility surround areas with no population: these areas are DBs that consist of large park area.

Figure 5.2 displays the accessibility to parkland area (in hectares) per DB as measured by the total constrained multimodal accessibility placed on the same scale coloured by quartiles. Again, these values are *proportional* to the unconstrained measures, but they are explicit in their units: all in accessible parkland area, so explicitly travel impedance functions are taken as comparable as they are all within the  $K^{mT}$  balancing factor (i.e., the sum of all unconstrained accessibility flow in the system across all modes is the denominator while the numerator is the sum of *all* the parkland in the city (8037.547 ha). Together, at least one mode can reach every park unlike in the unimodal case where some parks could not be reached within a 15 minute walk).

As the system is constrained, values in the region can be discussed as proportions of the total parkland area in the region. The majority of parkland area (65%) is calculated to be accessible for car using population, while 31%, 3% and 1% of the remaining parkland is accessible by transit, cycling and walking populations respectively.

Recall: this allocation works by taking the sum of total parkland area, and allocating it to origins based on the mode's relative travel impedance to reachable parkland area. The proportion of the population using the mode does not enter the equation. Hence, as car and transit are the furthest reaching modes these modes dominate allocation. Furthermore: car populations are assumed to not be able to reach smaller parkland area, but despite this assumption, car still dominates allocation, in part because larger parkland area dominates: 9% of the total parkland area is small (i.e., parkettes, small and medium parks) and the remaining 91% is large (i.e., large, city and legacy parks).

The spatial pattern within Figure 5.3 appears quite a bit different than the total constrained measure. Firstly, many DBs in the north part of the city have no cycling mode share for leisure trips (but some census population). In this assessment, as their mode share is 0, they do not ‘demand’ for parkland area hence their accessibility is 0. In this sense, these cyclists contribute to taking parkland area from all other modes more intensely within the downtown core, by introducing more competition. For this reason, while cycling looked not as competitive as walking in the downtown core in the total constrained measure – cycling looks much more competitive, outpacing walking in the singly constrained measure. Variability for transit and car also is more than in the total constrained modes. Particularly, transit looks more competitive than before, out performing car mode (and all other modes) in certain pockets of the downtown. This is due to how proportional allocation works for the singly constrained measure: where

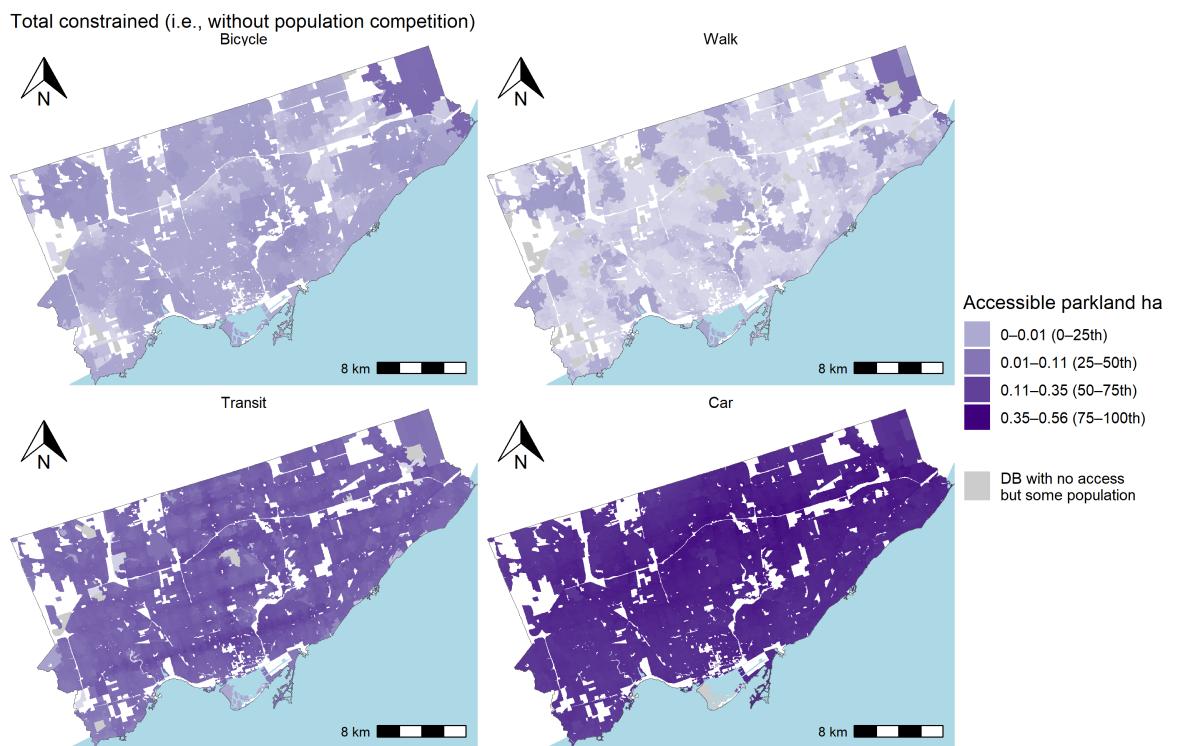


FIGURE 5.2: Accessibility to parkland area per DB as measured by total constrained multimodal accessibility.

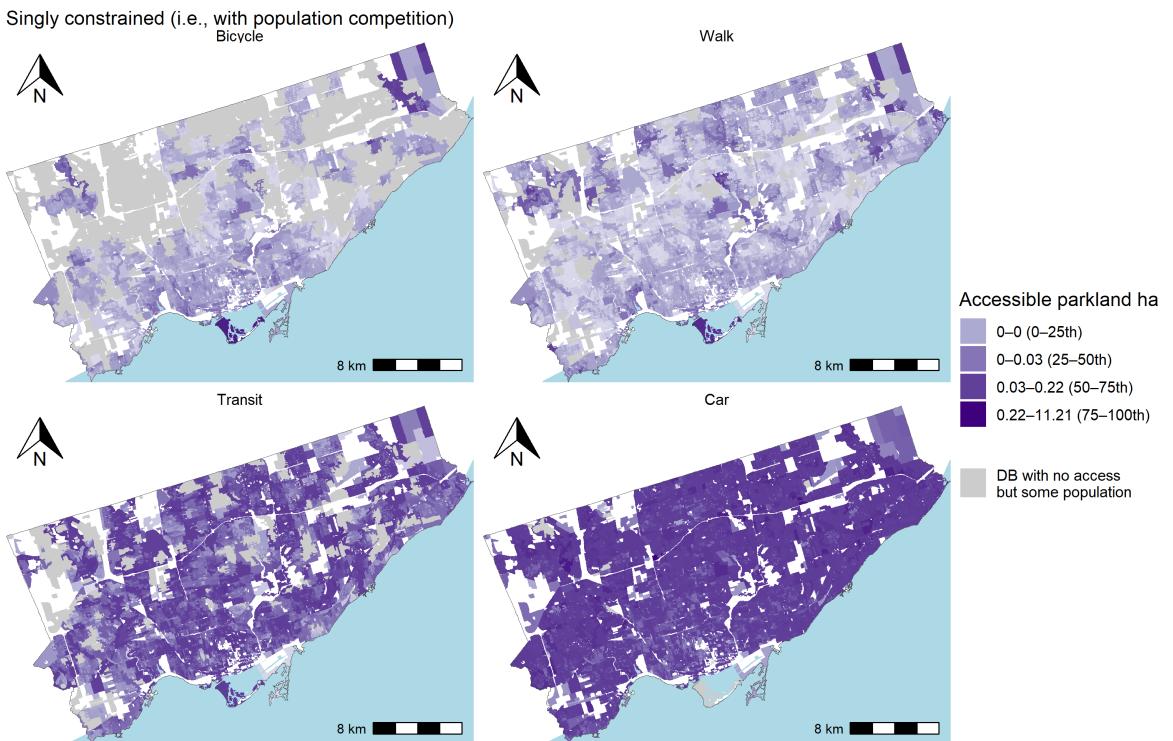


FIGURE 5.3: Accessibility to parkland area per DB as measured by singly constrained multimodal accessibility.

now instead of just the total parkland area being allocated to each mode-group overall, each specifically park’s area is allocated to each mode-group. This introduces another scale of dynamic, namely, the modal populations.

### 5.2.1 By neighbourhood

As done in the unimodal example, constrained accessibility values can be aggregated to another more meaningful zoning system: the neighbourhood. Figure 5.4 illustrates both the total constrained and singly constrained values. The total constrained measure can be interpreted as the unconstrained measure anchored to a known quantity: the total amount of parkland in the city that is reachable by at least one mode given travel impedance (8018.5417763 ha out of 8,037.55 ha). The four plots of modal singly constrained measures also sum to the same total parkland area, but incorporate *modal population* along with zonal allocation of opportunities in a constrained way. All values across both the total and singly constrained measures are presented on a common scale—from 0 to 110 hectares per neighbourhood—along with the quartiles of each measures dataset. This is done as the units of the measures are the same, i.e., the amount of accessible parkland area per mode for each neighbourhood. And same as in Chapter

4, spatial variation is dampened both by aggregation to the neighbourhood scale (as compared to the DB) and by the varying distance-decay characteristics of each mode.

Referring to the total constrained values in Figure 5.4 it can be observed that every neighbourhood is populated with a value, unlike at the DB aggregation. This is due to aggregation, as the DBs are summed per neighbourhood to reflect the accessibility score of that neighbourhood. As the measure is constrained: summing each DB within a neighbourhood still preserves the constraint(s) satisfied by the measure. What is interesting though is, with the singly constrained measure, is that due to the large proportion of DBs with 0 cycling mode-share, there are certain neighbourhoods that too are devoid of cycling mode-share hence no *singly constrained* cycling parkland accessibility is allocated.

Furthermore in the singly constrained plots of Figure 5.4, the singly constrained cycling measure often yields higher values than the total constrained version. This occurs because parkland accessibility is not distributed to unpopulated zones, hence concentrating the parkland access in among fewer areas. A similar pattern is seen for walking mode, although at a smaller scale in magnitude to its relatively higher travel impedance. For transit and car modes, both constrained measures produce high values (often in the top quartile), and due to the coarse spatial scale of aggregation, differences between the two are less visually distinct.

Before disaggregating results, it is important to clarify the relationship between the newly introduced multimodal accessibility measure and the unimodal framing presented in the previous chapter. To enable comparison, parkland area per capita is calculated for each neighbourhood using both the unimodal (walking-only for everyone) and multimodal (walking relative to cycling, car and transit) approaches. While both use the same travel impedance (e.g., 15-minute threshold), the denominators and accessible parkland totals differ: the unimodal measure considers total neighbourhood population and total accessible parkland via walking, whereas the multimodal measure considers only the walking population and the portion of parkland accessible within the full multimodal system. Notably, population is not part of the total constrained measure itself—it only informs the per capita representation. Figure 5.5 shows this relationship across neighbourhoods for both total and singly constrained measures, with outlying neighbourhoods labeled.

Figure 5.5 illustrates the relationship between unimodal and multimodal walking measures for both total and singly constrained accessibility. In both cases, the measures show a roughly linear relationship and similar levels of central clustering: 35% of neighbourhoods fall within the interquartile range for the total constrained measure, and 34% for the singly constrained.

Key differences between the two plots lie in their scales. Unimodal values range from 0 to 285 sqm/person, while multimodal values range from 0 to 20 sqm/person. This is due to differing calculation assumptions: the unimodal measure assumes *all* the population accesses parkland via walking alone, while the multimodal measure accounts for the



FIGURE 5.4: Accessibility to parkland area per neighbourhood summarised per neighbourhood as measured by total constrained multimodal accessibility (top) and singly constrained multimodal accessibility (bottom).

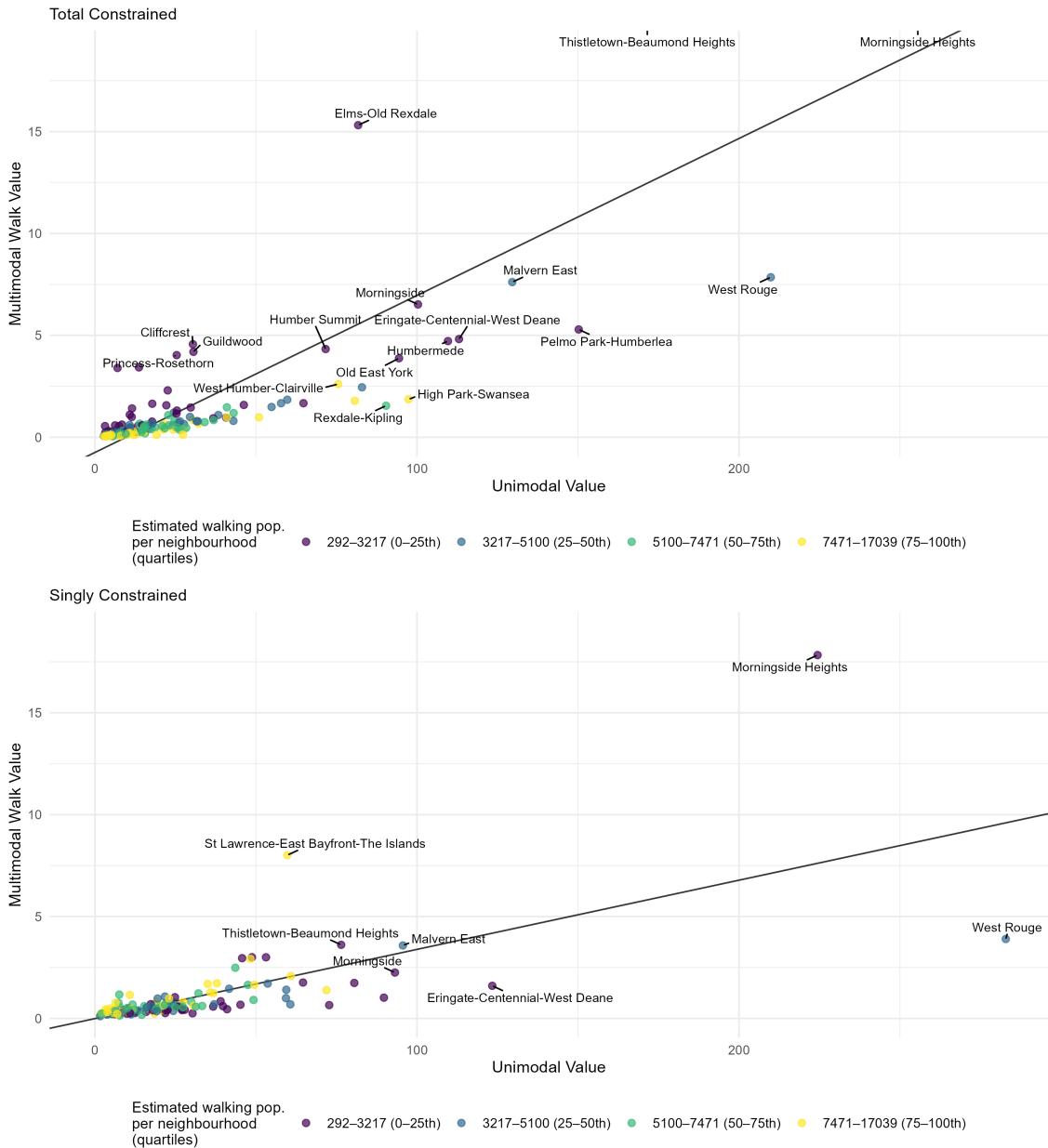


FIGURE 5.5: Neighbourhood-level accessibility to parkland area per capita by walking, comparison between unimodal accessibility and multimodal accessibility ratios. Total constrained (top plot), singly constrained (bottom plot).

travel impedances of other modes (e.g., cycling, transit and car) as well as walking, which reduces the share of parkland that ends up being attributed walking even after being adjusted by the walking-population of that neighbourhood. Faster modes can “out-access” walking, diluting per-capita parkland area accessible for the walking population.

What is notable is despite these differences, the unimodal and multimodal relationship is maintained in both the total constrained and singly constrained measures. This relationship is maintained as they both measures are still informed by the *same* spatial distribution of parkland area and the same impedance function (15-minute cutoff). However: the total constrained comparison yields a stronger correlation ( $R^2 = 0.70$ ) than the singly constrained ( $R^2 = 0.49$ ). As discussed, the singly-constrained measure yields results with more variance due to the more restrictive population and opportunity constraints. In both cases, similar outlying neighbourhoods are common across both measures and tend to be those with extreme population levels (i.e., either top or bottom quartile).

In sum, Figure 5.5 highlights the value of applying constraints: despite differences in how the unimodal and multimodal measures are calculated, the same underlying walking behaviour is used in both measures so results *should* be related: and they are.

Building on the discussion above, it may also be helpful to explore how multimodal walking accessibility compares to other modes before turning to visualisation of the parkland accessibility per capita. Figure 5.6 contains two scatterplots: neighbourhood level multimodal walk per capita value compared to the other three modes for the total constrained and singly constrained measures. Caution is warranted in interpreting these comparisons. In the total constrained measure, per capita values are derived by normalizing total accessible parkland by the neighbourhood population—yet this population allocation factor is not explicitly part of the accessibility calculation itself. By contrast, the singly constrained measure does incorporate modal population directly into the allocation of parkland accessibility. It also accounts for demand from all modes, which introduces additional variation and reflects competitive access to limited park resources. As such, apparent relationships (or the lack thereof) between modes should be interpreted with these methodological differences in mind.

For the total constrained measure in Figure 5.6, other modes—particularly car and transit—tend to exhibit higher parkland per capita values than walking though the hierarchy is somewhat diffuse. There also does not appear to be a pattern in how per capita walking access compares to other modes. However, this changes when considering the singly constrained per capita rates for the different modes where a hierarchy in modal structures becomes more clear as distinct “bands” emerge: across all levels of walking accessibility, car, transit, and cycling consistently register higher per capita values. In other words, at all values of walk - car, transit and cycling consistently score higher values than walk mode *and* are in consistent ranges based on what they do not have not in common: their impedance function,  $i$  to  $j$  travel cost, proportion of modal population at an  $i$ .

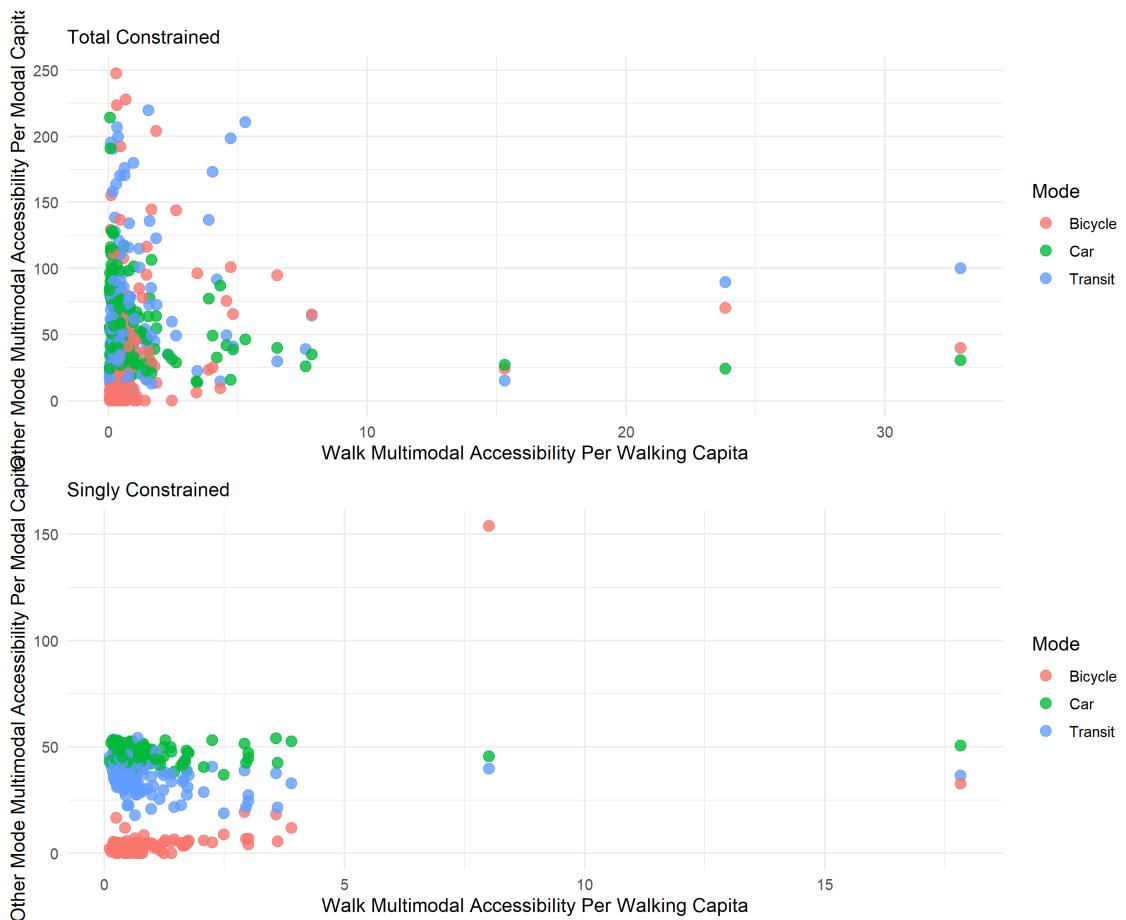


FIGURE 5.6: Neighbourhood-level accessibility to parkland area per capita, comparison of multimodal values between all modes. Total constrained (top plot), singly constrained (bottom plot).

The difference lies in how each measure treats allocation. The total constrained model averages parkland across the region, assigning all parkland to each zone based on the mode's relative travel impedance without accounting for modal population. In contrast, the singly constrained model allocates parkland competitively and proportionally, based on the same travel impedance but also the modal share and the enforcement that only the area available at each park is allocated. No under- or over-allocation of park area is allowed, unlike in the total constrained case. The singly constrained allocation is based on the mode share, which when referring to Figure 3.10 in chapter 3 demonstrates a distinct spatial distribution. Hence, the singly constrained measure also ends up reflecting this distribution, as tied to the distribution of parkland area and associated travel impedance, resulting in multimodal values that are more restricted.

It's also important to reiterate that per capita rates in the total constrained measure are externally imposed—they are not part of the calculation (but are imposed for the purpose of comparison and discussion at this point). In contrast, the singly constrained measure inherently considers modal population in its formulation. As a result, their non-normalised accessibility values (i.e., Figure 5.4), reflect patterns more similar to their per capita values as the ‘capita’ value is reflected in its internal structure. The singly constrained measure and its per capita ratio will tend to be more comparable in trends than non-normalised total constrained values compared to their per capita values.

These scatterplots in Figure 5.6 therefore highlight how introducing normalization (e.g., per capita) can create misleading variability—unless the normalizing variable is already accounted for in the model itself. Overall, under the total constrained measure, travel impedance and the spatial distribution of parks alone drives the allocation of parkland area. As a result, faster modes like car and transit tend to receive greater access, followed by cycling and walking. Per capita values introduce additional variation, as they reflect the underlying mode share of each neighbourhood's population. However, to fully account for modal population as a form of demand, the singly constrained measure should be used.

Turning to the visualisation of per capita values, Figure 5.7 displays neighbourhood-level parkland accessibility for motorized modes, normalized by population and clipped at the 98th percentile for visualization. These patterns are reinforced by the LISA cluster map in Figure 5.8, which identifies statistically significant hotspots of high-high and low-low spatial association.

Referring to the motorised values in Figure 5.7 and Figure 5.8, singly constrained per capita values exhibit clearer patterns of clustering than the total constrained measure. Under the total constrained measure, neighbourhoods with large parkland areas and higher transit access—such as those near the Don Valley or along Line 2—score highly for transit per capita accessibility. However, the singly constrained measure, which allocates parkland area based on both travel impedance and competitive demand from other modes, demonstrates a different trend. In the downtown core, transit often outperforms



FIGURE 5.7: Motorised neighbourhood-level accessibility to parkland area per capita for the City of Toronto.

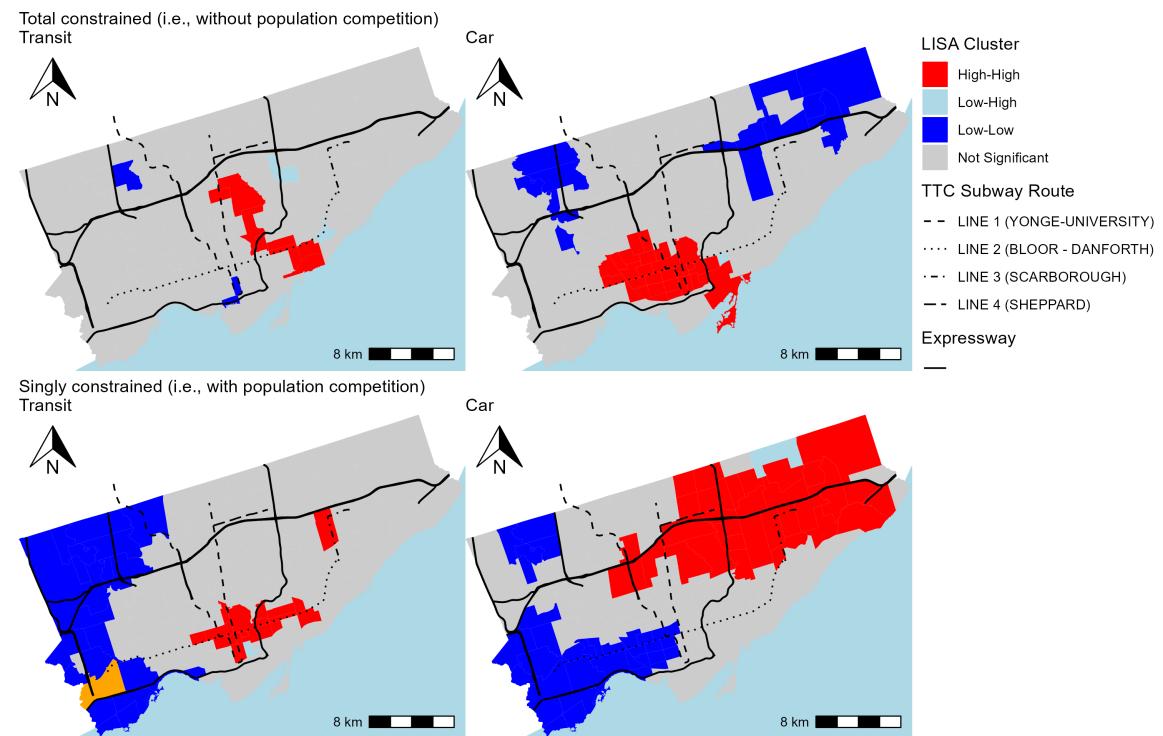


FIGURE 5.8: Local indicator of spatial association by neighbourhood of motorized constrained accessibility to parkland area per capita for the City of Toronto.

car, with these zones capturing more parkland area per capita, and forming many high-high clusters. These patterns do not appear in the total constrained version because this measure lacks population weighting and opportunity constraints.

For car accessibility, the total constrained measure highlights downtown clusters, but these disappear under the singly constrained model—where transit competes more effectively in dense, well-connected areas. Conversely, car accessibility is higher in the northeast, but only in the singly constrained case. This is due to: (1) higher car mode share, (2) presence of large park spaces (which, recall, are the only type of park sizes that cars are assumed to be able to reach), and (3) expressway infrastructure, which reduces travel impedance for motorists. These factors contribute to car accessibility advantages in the northeast of the areas, while the inverse supports transit dominance of parkland area allocation downtown near the subway line. Lastly, the areas with low singly constrained accessibility tend to have: high population density, limited parkland area (especially large parks), and overall relatively higher travel impedance by other modes.

Moving onto non-motorised per capita accessibility: Figure 5.9 demonstrate the values spatially and Figure 5.9 illustrates the associated LISA clusters. Interesting, unlike all three other modes, walking mode follows a similar pattern across both total and singly constrained measures. This is largely a matter of scale: although localized differences appear at the dissemination block DB level (as shown in Figure 5.3), these patterns are muted when aggregated to the neighbourhood level, which spatially exceeds the effective range of the 15-minute walking threshold.

Turning to the singly constrained measure in Figure 5.9 and Figure 5.9, this measure tends to highlight zones where a high proportion of the modal population resides near accessible parkland. However, for walking, this competitive advantage is often outweighed by the lower impedance and higher demand of other modes. For example, even in the downtown core—where walking mode share is relatively high—transit users with extremely low travel impedance, along with motorists and cyclists, dilute walking access due to their broader reach and/or lower travel cost.

Nonetheless, top quartile accessibility values appear in neighbourhoods with an exceptionally high amount of parkland area near residential population—areas where parkland supply exceeds multimodal demand, allowing walking to compete more effectively. Coincidentally: these areas often align with where there is already a high proportion of walking mode-share to leisure trips, hence the singly constrained measure also reflect high values, as well as at the edges of the city. Examples are in the northeast (i.e., Rouge National Urban park, a large natural planned park and creek trail system along with greenspace around the Toronto Zoo) and the northwest (i.e., the Humber river park system) of the city. The Don Valley also stands out as having both high walking mode share and significant parkland availability, reinforcing high accessibility scores in both measures but especially the singly constrained one. Also, many of these areas are near the edge of the boarder hence receive even higher per capita values, partially due to edge effects, where reduced competition inflates accessibility scores.



FIGURE 5.9: Non-motorized neighbourhood-level accessibility to parkland area per capita for the City of Toronto.

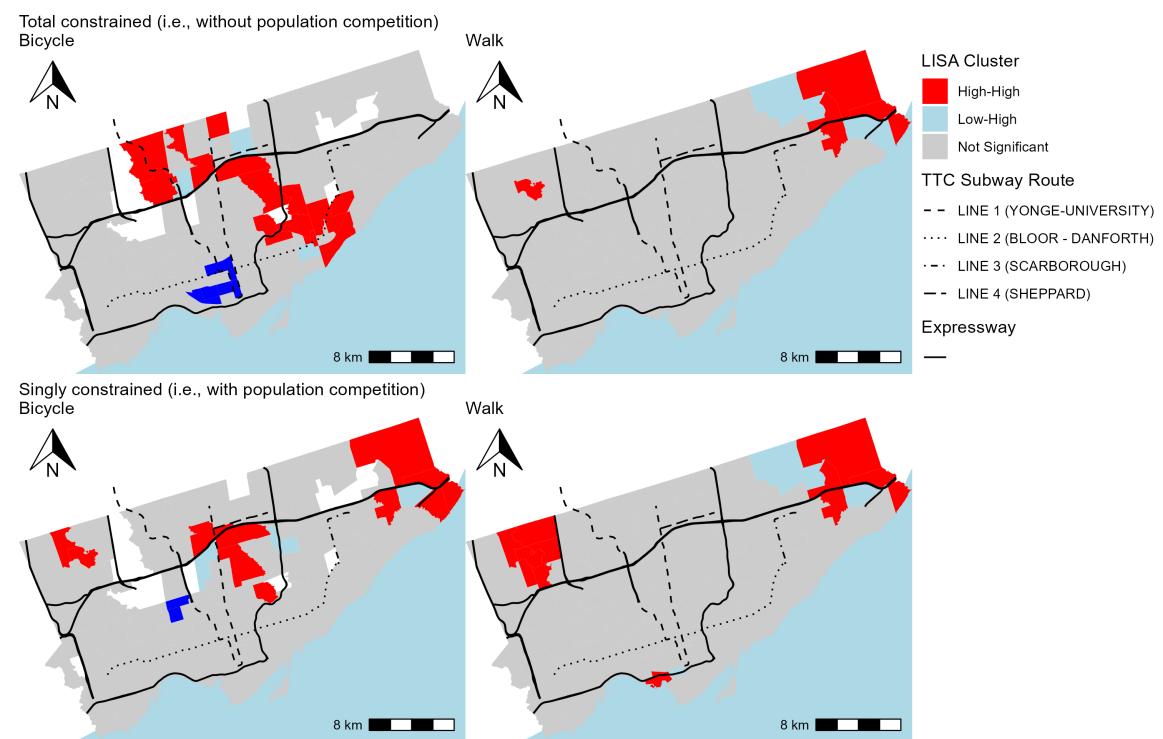


FIGURE 5.10: Local indicator of spatial association by neighbourhood of motorized constrained accessibility to parkland area per capita for the City of Toronto.

Cycling follows a similar spatial logic to walking but benefits from a larger impedance range. Its values are generally higher in magnitude due to broader reach, despite lower modal share. However, some neighbourhoods show zero cycling accessibility due to a lack of suitable cycling infrastructure, which precludes the generation of travel times in the model.

Continuing the analysis, one can disaggregate constrained accessibility totals to examine the composition of parkland accessibility by park type for each neighbourhood. While faceted maps (e.g., by mode and park type) could illustrate spatial patterns, Figure 5.11 instead uses stacked bar plots to emphasize the relative contribution of each park type per mode for each neighbourhood assuming the total constrained measure. This visualisation approach highlights the calculated entropy, or diversity, of accessibility across park types at the neighbourhood level. Entropy is calculated based  $-\frac{\sum x_i \log(x_i)}{\log(N)}$  where  $x_i$  is the proportion of accessibility from parks of a specific park size class  $i$  for a neighbourhood. The  $\log(x_i)$  captures how unevenly (or evenly) accessibility is distributed across classifications, and the denominator  $\log(N)$  normalizes the measure based on the number of park size classifications  $N$  ensuring values range between 0 (complete concentration in a single category) and 1 (equal accessibility across all park sizes).

For motorized modes in Figure 5.11, accessibility is relatively consistent across the city: approximately 70–80% of accessible parkland comes from legacy parks, with the remainder primarily from large and city parks. In contrast, bicycle accessibility is more variable. Neighbourhoods near large parks (often outside the downtown core, as discussed previously) receive a greater share of accessible parkland from larger park size classifications, while those in the downtown core—where smaller parks are more concentrated—show higher proportions from those classifications. Walking accessibility is the most variable, likely due to its tendency to follow more localised trends as discussed (i.e., driven by its short travel range and diversity of travel network and park-population distribution). Although walking mode yields the lowest overall accessible parkland, keep in mind that Figure 5.11 shows the proportional contribution of each park type per neighbourhood. Notably, neighbourhoods with the highest entropy (greater diversity in accessibility by park size) tend to receive a larger share of their walk-accessible parkland from smaller parks. As previously discussed, much of walk-accessible parkland is concentrated in the downtown core, where these smaller parks (and of various classifications with more similar parkland areas) are more prevalent.

It is important to emphasize that the values shown represent proportions within each neighbourhood, not absolute accessibility levels. Motorised modes continue to dominate in total accessible area, while walking contributes the least. In sum: these compositional patterns are largely shaped by the transport network structure and the mode-specific impedance assumptions embedded in each measure.

Figure 5.12 illustrates the compositional diversity of neighbourhoods' singly constrained accessible parkland by mode. Compared to the total constrained measure in



FIGURE 5.11: Compositional diversity of total constrained accessibility to parkland area by mode summarised per neighbourhood. Neighbourhoods ordered from left to right by decreasing proportion of allocated accessible park-size diversity.

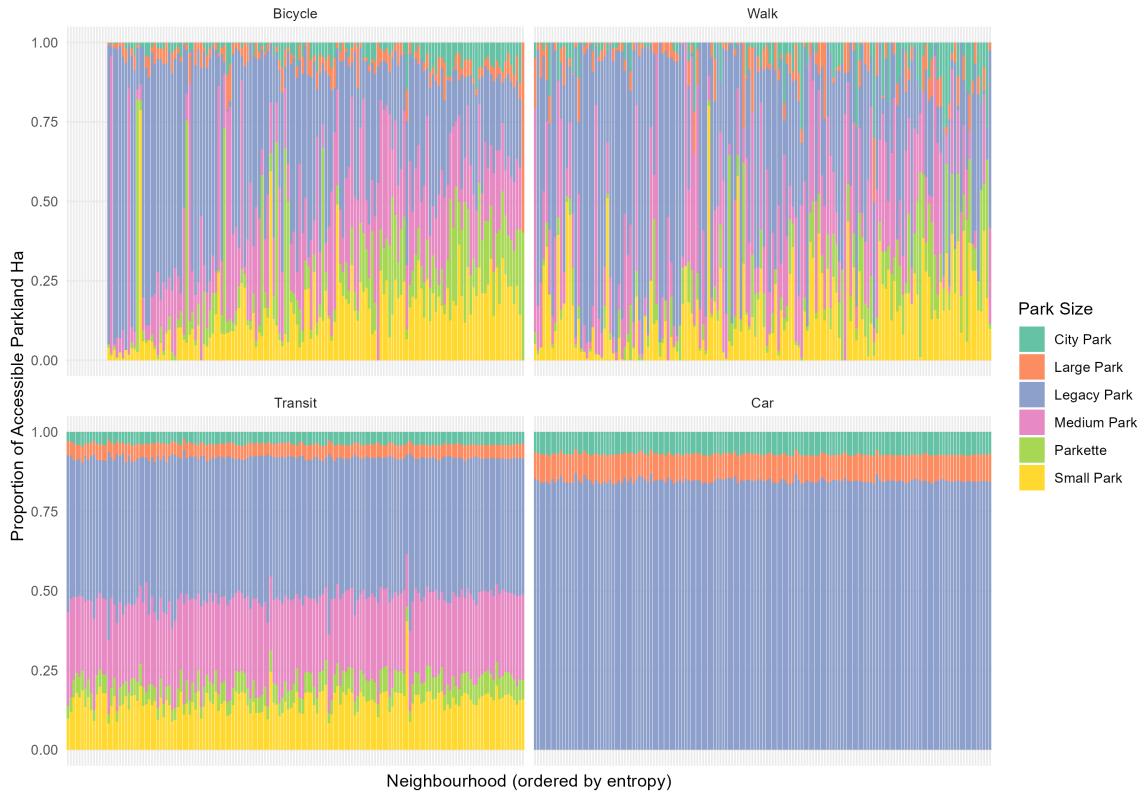


FIGURE 5.12: Compositional diversity of singly constrained accessibility to parkland area by mode summarised per neighbourhood. Neighbourhoods ordered from left to right by decreasing proportion of allocated accessible park-size diversity.

Figure 5.11, the singly constrained version incorporates additional spatial structure—specifically, competition for park space and the enforcement of zonal-level allocation. This ultimately introduces greater variation, especially for non-motorized modes.

Regarding the motorized modes in Figure 5.12, the allocation of parkland area to these modal population main relatively stable in terms of their park size composition, with most neighbourhoods continuing to derive the majority of their accessible parkland from legacy parks.

In contrast, regarding the non-motorized modes in Figure 5.12, walk and bicycle modes (non motorised) access a more balanced mix of park types. This shift reflects the fact that, under the singly constrained approach, parkland can no longer be over-allocated based on attractiveness alone—as was the case with the total constrained measure (see Chapter 4 discussion on over/under-allocation). Instead, each park’s contribution is capped and allocated according to its actual area. As in the total constrained measure, cycling and walking modes still exhibit similar patterns of accessibility park size composition, but their accessibility composition is now less dominated by legacy

parks. Variation across neighbourhoods is shaped both by the spatial reach of each mode (e.g., walking shows more variation than cycling due to its shorter range) and by the binary nature of the impedance function. Unlike motorized modes, which use a continuous decay function allowing access to most parks at varying levels, walking and cycling are constrained by a hard cutoff of 15 minutes, introducing sharper thresholds in park allocation hence more variation in its composition.

### 5.3 Accessible population (to the parkland)

This variant introduces a new framing for the accessibility analysis. Instead of a focus on how well origins by modal population are performing from the perspective of *accessible parkland area*, one can investigate how well the park themselves (or aggregated to some other relevant park zonal level) are performing in serving different modal populations or *accessible multimodal population*, a park's 'market potential'.

Figure 5.13 is the access to people, by mode for the unconstrained measure, and Figure 5.14 for the total constrained measure. Again, they are proportional but the advantage of the constrained measure is that the total sum of accessible population across all modes and parks is 2,794,356, the total sum of the population in 2021 in the Toronto CMA. The focus of discussion will be on the comparison between the multimodal total constrained measure and singly constrained measures. Interestingly: unlike in the accessible parkland examples (preceding section's discussion): transit mode dominates as opposed to car. In the case of accessible population, this means 62% of the population that is accessible to parks is assumed to be transit-using, while 31%, 6% and 1% of the accessible population is represented by car, cycling and walking modes. As allocation of total population is strictly based on the the impedance of mode using populations and not their actual proportions. If we do not know the proportions of modes, this may be all we can do. But we *know* the proportion of transit users and it tends to be about 14% on average from any given origin, or approximately 18% of the total population.

While it is surprising to see transit mode be allocated so much accessible population, this is how the total constrained allocation (Figure 5.14) precisely functions. It is strictly based on the *relative* (to other modes) travel impedance from parks to reachable population zones. Transit, especially higher order transit, tends to be present near more dense populations. Furthermore: there are plenty of park locations (though they tend to be smaller) near these higher density areas which indicate higher transit service areas i.e., the TTC subway and higher order surface transit. Hence, from the perspective of parks location to population-weighted-by-their-modal-impedance, transit is highly efficient in capturing transit-population to parks.

From this perspective: populations that tend to use more car-mode tend to be in lower density zones with fewer parks (albeit larger), coincidentally these populations in these zones allocate fewer population to parks for car mode though car mode has a lower travel impedance than transit. Between accessible multimodal population and accessible multimodal parkland (preceding sections), cycling and walk modes tends to

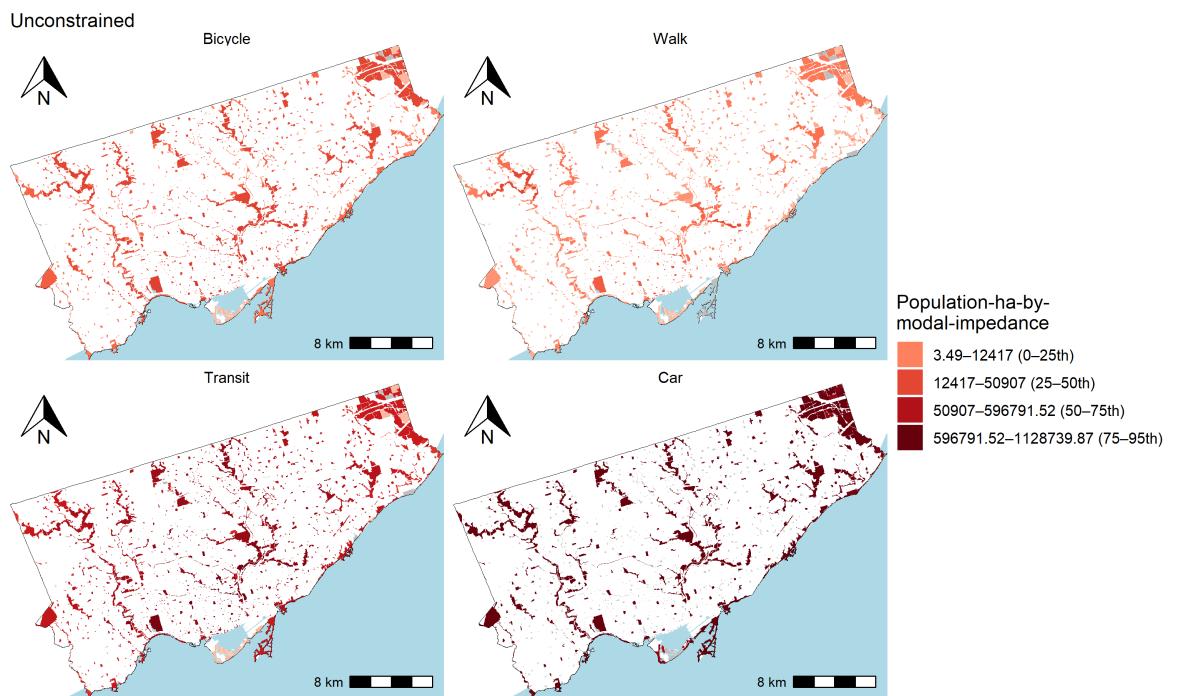


FIGURE 5.13: Multimodal unconstrained accessible population-by-modal-impedance accessibility per park, i.e. market potential of each park.

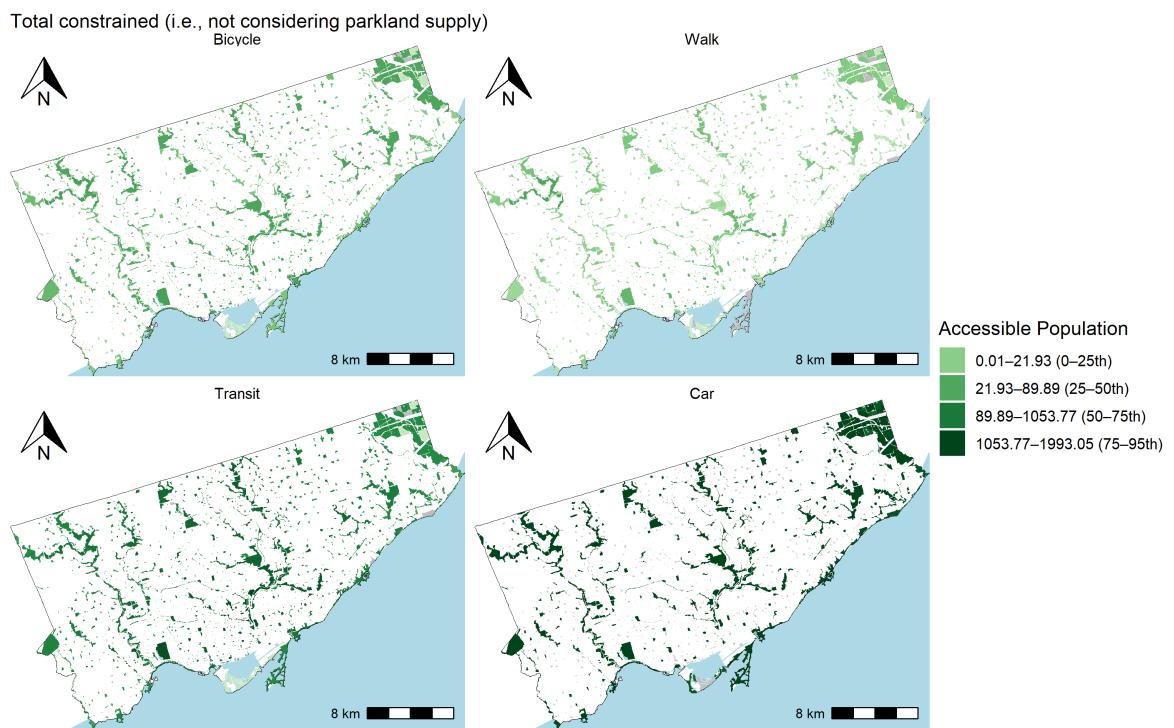


FIGURE 5.14: Total constrained multimodal accessible population by mode per park.

capture similarly low proportions of population. Though, cycling captures two times more (i.e., 6% instead of 3%) accessible population than accessible parkland (for the total constrained measure), for similar reasons discussed as in transit: parks that can be accessed by cycling are located in higher population dense areas of the city.

Again, the total constrained multimodal accessible population measure may represent the most reasonable estimate when park mode-suitability is unknown—that is, when we lack information on how park area is functionally divided by mode (i.e., how much of the park area can be used by a specific mode-traveller group such as parking lot capacity for drivers or bike parking availability). In this analysis, however, we approximate mode-specific parkland accessibility by using the mode shares *to* leisure destinations from the 2022 TTS (i.e., Figure 3.11). These shares are treated as proxies for the population segments most likely to access parkland by each mode, effectively framing parkland area as a mode-specific supply influenced by travel impedance.

Figure 5.15 presents the singly constrained accessible population per park, disaggregated by mode. These results differ notably from those produced by the total constrained measure—similar to the differences observed in the preceding section between total and singly constrained estimates of accessible multimodal parkland area. The key distinction lies in whether competition of parkland supply is explicitly considered.

In Figure 5.15, the singly constrained measure allocates the total population from each origin to the proportion of parkland area (per park) that may be demanded by the incoming modal-split from that origin. This calculation results in the following: that the accessible population is dominated by suitability for the car-using population. Namely, 91% of the accessible population across all parks is calculated to be motorists. With the remaining proportions coming from transit (8%), cyclists (1%) and walking population (1%).

It is important to reiterate the intuition behind this measure. From the perspective of the origin, the motorist mode-share represent 45% of the population at DBs. However, the motorist parkland area suitable represents 56% of the total parkland area. The population at the DB, and the area of the parks, are weighted by the origin-side and destination-side mode shares. Comparing between them is challenging: as they are in fundamentally different units and from different sides of the accessibility analysis (accessible multimodal parkland area vs accessible multimodal population). But despite these differences, car *still* dominates. Why? Because of the intensely high travel impedance, relative to other modes. Though ...

## **5.4 Conclusions: comparing multimodal accessible parkland and accessible population by Toronto neighbourhood**

This section summarises key patterns in multimodal constrained accessibility across Toronto neighbourhoods, focusing on two perspectives:

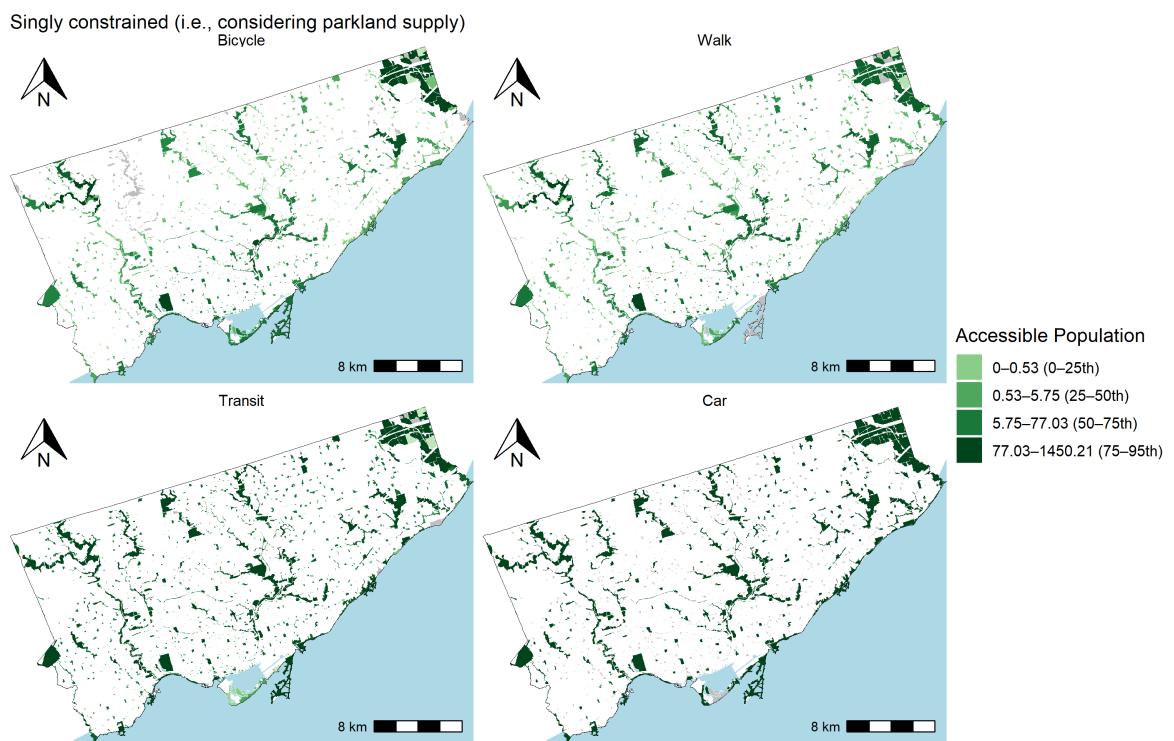


FIGURE 5.15: Singly constrained multimodal accessible population by mode per park.

- Accessible parkland: how total parkland is allocated based on origin-side modal split and travel impedance (i.e., how accessibility varies by mode depending on where people are coming from);
- Accessible population: how total population is allocated to parks based on destination-side modal split and travel impedance (i.e., how many people can access each park, by mode).

To contextualise the results, values are aggregated and discussed by Toronto’s six former municipalities, which are potentially too aggregated for finer planning purposes, offer useful interpretive boundaries. Parks tend to align with these divisions, and each area exhibits distinct spatial and infrastructural characteristics as they recently were their own municipalities. 3.10 shows the boundaries of these former municipalities.

In what follows, we use these groupings to guide interpretation of the accessibility results, while also highlighting neighbourhood-level variation where relevant.

#### 5.4.1 How effectively are the parks serving the population?

If asking this question, one is interested in evaluating how well parks are providing population access to their parkland area. If information about how attractive each park is to different modal groups from the perspective of the origin zone (i.e., mode share travel to parks) is lacking, one should use the *multimodal total constrained opportunities measure*. In this approach, the total parkland area in the region is allocated to each origin zone solely based on travel impedance of each mode relative to all modes in the system. This measure reflects how much accessibility to parkland area a zone has for each type of modal population. This measure assigns higher accessible parkland area values to mode-using populations at an origin for modes with a broader range, such as transit and car.

However, if we do have information about parkland demand by modal group, then we can ask a more refined question: **How effectively is the available parkland supply serving the population?**. In this case, the *multimodal singly constrained opportunities measure* is appropriate. Here, accessibility is allocated competitively, considering both the spatial distribution of the modal population (demand) and its associated travel impedance to allocate only the parkland area available at each park. Results tend to demonstrate higher results for areas with low competition for parkland area: this can be in areas with high modal population proximate to parkland and further from other populations (especially lower impedance populations).

Figure 5.16 summarises these two perspectives across former Toronto municipalities, for the parkland area and population data used in this chapter. This comparison highlights differences in how parkland accessibility is distributed for each measure.

.... some more stuff. about the black point that represents real parkland area per neighbourhood. We can see it does not align with the presence in the muni. why?

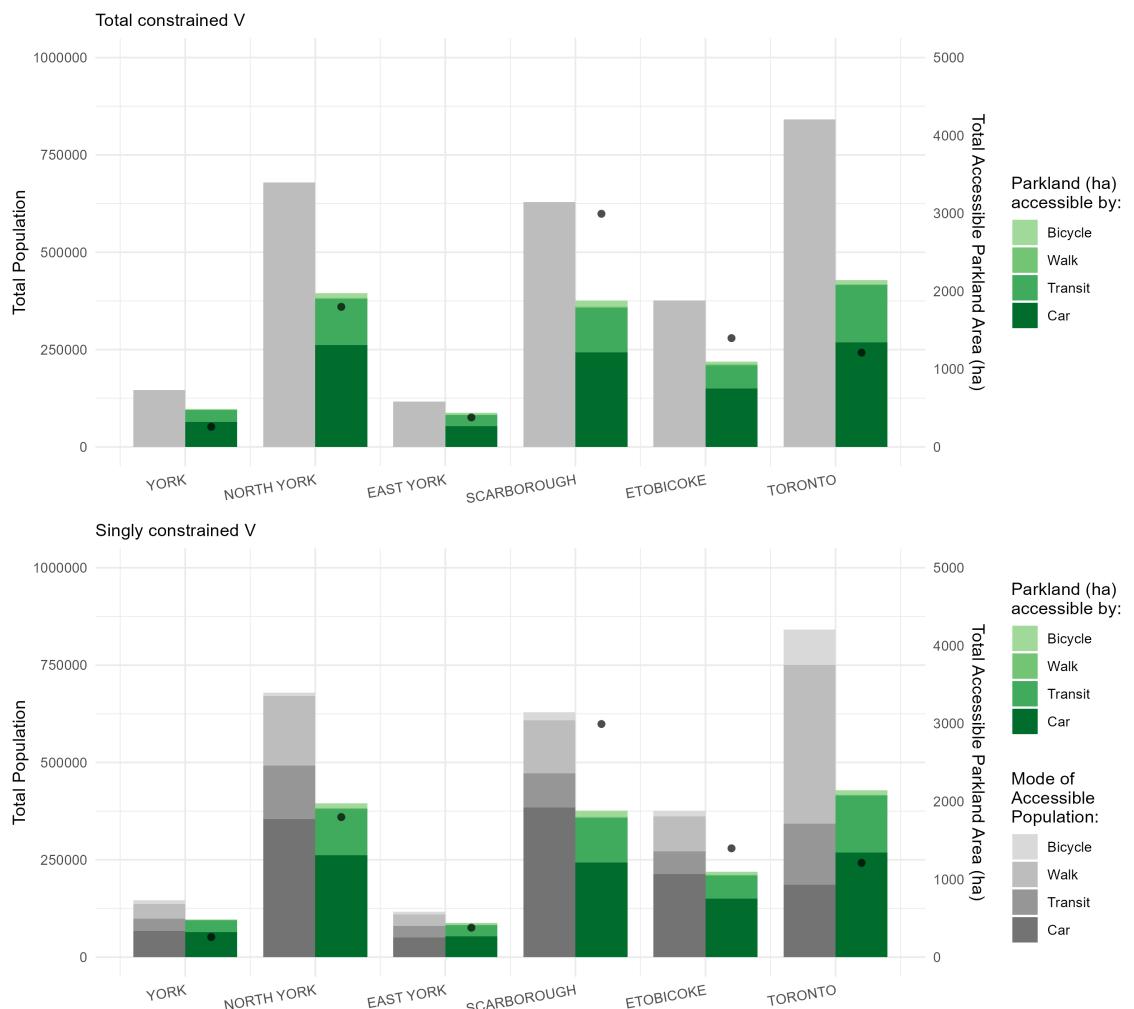


FIGURE 5.16: Multimodal total and singly constrained accessible population summarised by former municipalities and plotted against real parkland area and population within the municipality. The black point represents the sum of total poulation.

because accessible parkland is not the parkland present, but the amount that is allocated to DBs contained in that municipality.

#### **5.4.2 How efficiently is the population within reach of parks?**

When evaluating how efficiently the park provides accessibility to population and the amount of modal-suitable parkland area supply (e.g., a proportion of how well a park accommodates/expects each type of mode group) for the parks is unknown: choose *total constrained accessible population measure*. In this measure, the total population is allocated to each park based on the modal-group's travel impedance. With this measure, one can understand how much population by modal group is within reach of each park and can be aggregated at different levels to reflect this amount (i.e., first plot in Figure 5.17). In this measure, the total amount of population is allocated, with no regard for modal parkland supply –it is solely allocated based on travel impedance from population to parks.

In Toronto, the spatial distribution of parks (though not necessarily the total parkland area) is relatively even. As a result, the accessible population by mode tends to align closely with the actual residential population (as indicated by bars aligning with the black points). Transit offers relatively low travel impedance and is well-connected both to park locations and to areas of population density. Consequently, it yields high levels of accessible population—particularly in municipalities with more well developed transit infrastructure, such as Toronto and York. In contrast, in municipalities where transit service is weaker and car travel has lower impedance, a greater share of the accessible population is captured by the car mode.

However, if population demand *is* known, the question can be augmented: **how efficiently is the population that demands access within reach of parks?** In this case, the singly constrained population measure should be used (see the second plot in Figure 5.17). This produces a notably different distribution. Accessible populations no longer resemble the total population of the municipalities, and transit is no longer the dominant mode.

It is important to remember that in both measures, the total population is constrained. However, its allocation depends on both modal travel impedance and the park-side modal split. The park-side modal split is approximately half car-based overall, but it skews strongly toward car use in areas with large amounts of parkland, and car travel also tends to have exceptionally low impedance.

For example, while Toronto has a large population and good proximity to parks, these parks tend to be smaller and are more aligned with non-car modes. Despite this, Toronto's central location means its population is often allocated to larger, more car-oriented parks located outside the former city boundaries. As a result, Toronto has the lowest accessible population relative to its actual population under this measure.

In contrast, Scarborough exhibits a much higher accessible population than its actual population. It also has a high number of parks and a large amount of parkland area,

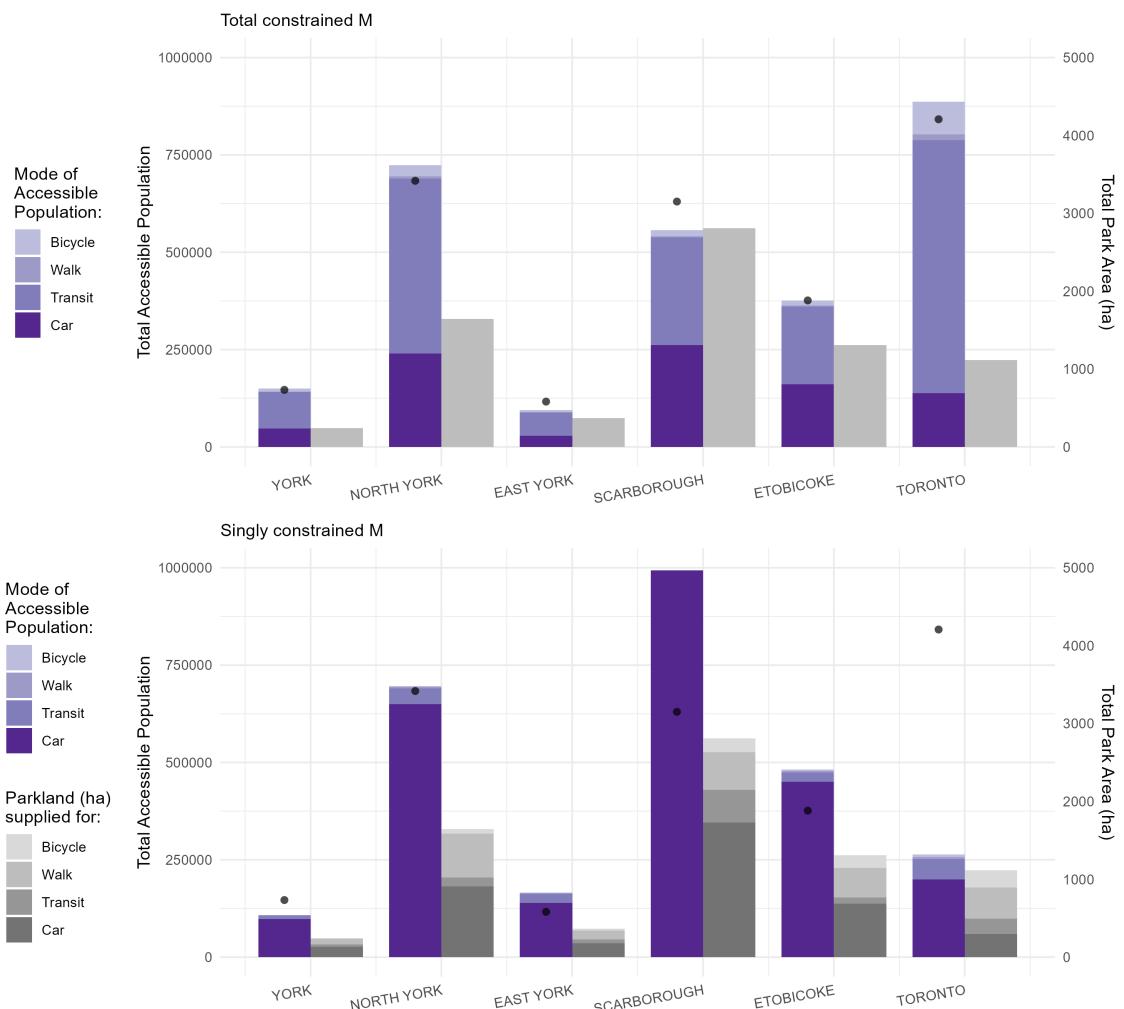


FIGURE 5.17: Multimodal total and singly constrained accessible population summarised by former municipalities and plotted against real parkland area and population within the municipality. The black point represents the sum of total poulation.

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making it a highly competitive municipality when evaluated using singly constrained measure. i.e., Population is effectively “pulled” from across the city toward large, car-accessible parks. This dynamic explains the significant variation in accessible population across municipalities, especially since larger parks (being more car-supplied) more competitively exert draw, while smaller parks are not assumed to attract car-based access....

# Conclusion

## 5.5 An open and reproducible work

- open and reproducibility of methods learned are discussed (text from “*TTS2016R: A data set to study population and employment patterns from the 2016 Transportation Tomorrow Survey in the Greater Golden Horseshoe area, Ontario, Canada*” is included)

## 5.6 Constrained accessibility take aways

- how constrained measures can be interpreted,

## 5.7 Comparision of constrained accessiblitiy to conventional methods

Current accessiblitiy is not standardized. It takes many forms. One of these forms is visualed in figure X. As parkland service pervision. With the constrained of accessiblitiy the unit of the opportunitiy can be interpreted, emasured, and attribtued to an area. then the zonal population of that area can be devided. This can produce a parkland service provision, that includes accessibility – but also can be understood in ways like Figure X. This measure shows plenty of promise for interpretations like this.

Furthermore, - how this interpretation can aid policy makers– per capita, per any other zonal property. comparisions between regions, between times.

## 5.8 Accessibility and equity

As well, - how policy makers can ultimately use this to plan for inequities (some text pulled from “*Searching for fairness standards in the transportation literature*”).

## 5.9 Future research directions (?)

DESTINATIONS – in flux in their competition. - hyper competitive (limited opps + required qualitifcations)– like employment, and post-secondary education - competitive and institional (imited opps creates competitive experiences, open to and essential for

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whole population/subgroup) – like primary, and secondary education, child care centre, healthcare facilities

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