A family of accessibility measures

Introduction

Accessibility, a central concept in transport geography, planning, and engineering, is conceptually related to human mobility and the opportunity landscape. In simple terms, it is defined as the "potential of opportunities for interaction" (Hansen 1959). The range of opportunities considered in accessibility analysis has grown to include employment [REFS], health care [REFS], green spaces [REFS], schools [REFS], social contacts [REFS], emergency services [e.g., shelters, REFS], and many more. In other words, accessibility analysis is today used to understand the potential to reach opportunities that are important to people (Ferreira and Papa 2020). Compared to earlier measures of performance used in transportation (e.g., VKT, PKT, etc.) that benchmark the amount of movement, accessibility brings a more holistic understanding of transportation and land use systems combined (S. L. Handy and Niemeier 1997). However, the focus of transportation planning has historically prioritized mobility and treated accessibility as a by-product of movement, leading to problematic consequences (S. Handy 2020). With the rise of an automobility mono-culture (Miller 2011; Lavery, Páez, and Kanaroglou 2013), recent decades of mobility-based practice (e.g., road and highway expansion) have been marked by increased travel cost (time, effort, energy, financial cost), often with limited impact on accessibility levels (e.g., S. Handy 2002; Páez et al. 2010). In response, transportation planning practice has begin to adopt accessibility planning approaches, via accessibility planning tools (Silva et al. 2017; Paez et al. 2013) or through the traditional four-step transportation demand model (Ortúzar and Willumsen 2011). However, institutional and conceptual barriers remain, many of which relate to the diversity of accessibility definitions (S. Handy 2020) and interpretability and communicability of the outputs (Geurs and van Wee 2004; Ferreira and Papa 2020).

In adopting the perspective that accessibility is an evolution of mobility conceptually, this paper aims to re-unify accessibility measures with its spatial interaction-based roots. It proposes to do so by mapping accessibility onto Wilson's family of spatial interactions models (Wilson 1971). In this, we aim not only to clarify accessibility's interpretation as a measure of performance, but also to encourage future developments that draw from its gravity modelling siblings, and spatial interaction history. To this end, we begin by tracing the development of spatial interaction from Ravenstein (1889) to its split into accessibility research from Hansen

(1959) to today. After this, we derive different types of accessibility measures, following the family of gravity models namely: unconstrained, total constrained, origin- and destination-constrained, composite destination-constrained, and doubly constrained. Then, we will provide empirical examples of the types of measures. Finally, we will discuss the use of each measure and broadly the implications of ignoring the role of constrains in accessibility analysis.

Background

The inquiry into spatial interaction traces to Newton's Universal Gravitation

The patterns of people's movement in space have been a subject of scientific inquiry for centuries. From as far back as Henry C. Carey's *Principles of Social Science* (Carey 1858) there was already a concern with the scientific study of human interaction over space. It is in this work where Carey states that "man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance" (McKean 1883, 37–38), which shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton's Law of Universal Gravitation posited in *Principia Mathematica* of 1687 and expressed in 1.

$$F_{ij} \propto \frac{M_i M_j}{D_{ij}^2} \tag{1}$$

To be certain, the expression above, a proportionality, is one of the most famous in all of science. In brief, it states that the force of attraction F between a pair of bodies i and j is directly *proportional* to the product of their masses M_i and M_j, and inversely *proportional* to the square of the distance between them D_{ij}. Direct proportionality only means that as the product of the masses increases, so does the force. Likewise, inverse proportionality only means that as the distance increases, the force decreases. To quantify the magnitude of the force, an empirical factor, say G, is required to convert the proportionality into an equality, which ensures that the force F matches empirical observations of the force of attraction between masses. Ultimately, the equation for the force is as seen in Equation 2, where G is a proportionality constant obtained empirically:

$$F_{ij} = G\frac{M_i M_j}{D_{ij}^2} \tag{2}$$

Newton's initial estimate of G was based on a speculation that the mean density of earth was between five or six times that of water, an assumption that Hutton's experiments of 1778 validated (Hutton 1778, 783). Still, it took over a century from the publication of *Principia* to refine the estimate of this constant to within 1% accuracy, with Cavendish's 1798 experiment (Cavendish 1798).

Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Following Carey's *Principles* of 1858, research into human spatial interaction continued in different contexts. In the late 1880s Ravenstein proposed his "Laws of Migration", based on his empirical analysis of migration flows in various countries (Ravenstein 1885, 1889). In these works, Ravenstein posited a directly proportional relationship of migration flows with the size of the destination (i.e., centres of commerce and industry), and an inversely proportional relationship between the size of flows and separation between origin and destination. These propositions are similar to Newton's gravitational laws. Over time, other researchers discovered similar relationships. For example, Reilly et al. (1929) formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations. Later, Zipf proposed a $\frac{P_1 P_2}{D}$ hypothesis for the case of goods movement by railways Zipf (1946c), intercity personal movement Zipf (1946b),and information Zipf (1946a). The $\frac{P_1 P_2}{D}$ hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

A common theme among these early researchers of human spatial interaction is that a proportionality constant similar to G in Equation 2 is missing. Of the researchers cited, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly's hypothesis was expressed as follows:

$$B_a = \frac{(P_a \cdot P_T)^N}{D_{aT}^n} \tag{3}$$

where B_a is the amount of business drawn to a from T, P_a and P_T are the populations of the two settlements, and D_{aT} is the distance between them. Quantity N was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of $n = 2$.

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

$$C^2 = \frac{P_1 \cdot P_2}{D} \tag{4}$$

where C is the volume of goods exchanged between 1 and 2, P_1 and P_2 are the populations of the two settlements, and D_{12} is the distance between them.

After Carey, it is in Stewart's work on the principles of demographic gravitation that we find the strongest connection yet to Newton's law (Stewart 1948). We suspect that this may relate to academic backgrounds; Stewart was a physicist, whereas Ravenstein, Reilly, and Zipf were social scientists. Besides awareness of preceding research, as he cites Reilly and Zipf as predecessors in the analysis of human spatial interaction, Stewart appears to have been the first author to express his theorized relationships for human spatial interaction with a proportionality constant G, as follows:

$$F = G\frac{(N_1\mu_1)(N_2\mu_2)}{d_{12}^2} \tag{5}$$

where:

- F is the *demographic force*
- N_1 and N_2 are the numbers of people of in groups 1 and 2
- μ_1 and μ_2 are so-called *molecular weights*
- d_{12}^2 is the distance between 1 and 2
- And finally G, a constant that Stewart "left for future determination" (1948, 34)

In addition to demographic force, Stewart defined a measure of "population potential" of 2 with respect to 1 as follows:

$$V_1 = G\frac{M_2}{d} \tag{6}$$

For a system with more than 2 population bodies, Stewart formulated the population potential at i as:

$$V_i = \int \frac{D}{r} ds \tag{7}$$

where D is population density over an infinitesimal area ds, and r is the distance to i. In more contemporary terms, this can be rewritten as:

$$V_i = \sum_j \frac{M_j}{d_{ij}} \tag{8}$$

which astute readers will identify as our modern definition of accessibility.

Stewart's formulation of demographic force, developed in the context of what he called "social physics" (Stewart 1947), was problematic. It had issues with inconsistent mathematical notation. For example, e.g., in Stewart (1948), G is used as a proportionality constant for both *demographic force* and *demographic energy* on p. 34 . Perhaps more seriously, Stewart's work was permeated by a view of humans as particles following physical laws, but with ideas that were afflicted by unscientific racism. For instance, the molecular weight μ was presumed to be one for the average American, but "presumably…much less than one….for an Australian aborigine" [p. 35]. Stewart's ideas about "social physics" fell out of favour among social scientists, but not before influencing the emerging field of accessibility research.

In summary: from the 1850s to the 1940s, researchers attempted to theoretically and empirically characterise human spatial interaction as some force of attraction F that is directly proportion to the masses M of a pair of locations (i and j) and inversely proportional by some sort of separation distance D_{ij}. Those that captured the relationship empirically, approached the concept with different expressions, some including a proportionality constant and some not, but all tying back to the Newtonian gravity analogy.

Hansen's gravity-based accessibility

From Stewart (1948), we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature (Hansen 1959). In his seminal paper, Hansen defined accessibility, i.e., "the potential of opportunities for interaction" [p.73], as "a generalization of the population-over-distance relationship or 'population potential' concept developed by Stewart (Stewart 1948)" [p.73]. As well as being a student of city and regional planning at Massachusetts Institute of Technology, Hansen was also an engineer with the the Bureau of Roads likely preoccupied with applied research useful to evaluate the power of transportation to shape land uses. Hansen (1959) focused on Stewart (1948)'s *population potential* (expressed in Equation 8), leaving Stewart's other formulaic contributions and objectionable aspects of "social physics" behind. Hansen (1959) recast Equation 8 to reflect accessibility, a model of human behaviour useful in capturing regularities in mobility patterns . In this equation, Hansen (1959) replaced M_j with *opportunities* to derive *opportunity potential*, or more accurately, *potential of opportunities for interaction* as follows:

$$S_i = \sum_j O_j \cdot d_{ij}^{\beta} \tag{9}$$

Alternatively, a more general form of this expression to account for different impedance functions:

$$S_i = \sum_j O_j \cdot f(d_{ij}) \tag{10}$$

S_i in Equation 9 is is a relative measure of the accessibility at i, O_j is the size of the activity at j and d_{ij} is the distance or travel time between i and j with β describing the effect size. Today, Hansen is is frequently cited as the father of modern accessibility analysis due to his work in Hansen (1959; e.g., Reggiani and Martín 2011), and the Hansen-type accessibility is commonly referred to today as the gravity-based accessibility measure.

Of note, however is that between Stewart (1948) and Hansen (1959) the proportionality constant G in Equation 2 vanished. There is some evidence that Hansen (1959) was aware of the importance of this constant as he wrote about *direct* and *inversely proportional* relationships between opportunities in one area, opportunities at another, and their separation distance. Furthermore, those reading Hansen (1959) could consult Stewart (1948) in which the proportionality constant G was set to 1 and that "G [was] left for future determination: a suitable choice of other units can reduce it to unity".

From Hansen-type accessibility to accessibility today

Since Hansen (1959), accessibility analysis has been widely employed across planning disciplines, but to our knowledge, developments have not been concerned with explicitly determining a proportionality constant G. G continues to be implicitly set to 1 even when the fundamental relationship in accessibility is proportionality (e.g., $ S_{i} G j O_j f(d{ij})$ and not equality (for instance, see the formula for accessibility at the top of Figure 1 in Wu and Levinson 2020). Without a proportionality constant, however, the units of S_i remain unclear: the unit of 'potential of opportunity for interaction' is left free to change as β is calibrated for the population center with no discussion of a constant to balance the units.

To overview recent accessibility literature development, the majority of studies develop on; the breath and geographic case study selection of the opportunity O_j studied and potentially impacted population []; the accuracy of parameters that capture distance-decay $f(c_{ij})$ []; and additions to the opportunity O_j attributes such as competitive demand considerations (Stouffer 1960; Weibull 1976; Shen 1998; Allen and Farber 2020; Soukhov et al. 2024). Overall, due to the diverse range of questions accessibility measures have been applied to help clarify, a plethora of variations in Equation 10 are present in the literature.

However, it has been raised that there is a difficulty in interpreting gravity-based accessibility's meaning as anything other than a relative index of potential interaction (higher accessibility/lower accessibility) []. This interpretation plagues this type of analysis, as implications beyond high-or-low access hotspots are often desired. What does the accessibility value mean? Difficultly in interpreting the output's tangible meaning contributes to resistance to its application in practice (Ferreira and Papa 2020). As reviewed, quantity and variations on Equation 10 also compounds the issue of interpretability across studies.

A testament to this inconsistent interpretation are the unsettled classifications generated by influential review works of the accessibility literature; for instance, S. L. Handy and Niemeier (1997) classifies studies into gravity- and utility-based measures, Geurs and Wee (2004) introduces further classification by expanding categories into potential- (or gravity), utility-, location-, and person- based measures with different components. Wu and Levinson (2020) demonstrate that most or all of these concepts can be unified into its main features: the relationship between travel-cost and corresponding reachable opportunities. Indeed, these authors demonstrate how previous classifications are special cases of travel-cost or opportunities constraints. From the initial Newtonian analogy, we interpret Wu and Levinson (2020)'s work as offering a positive contribution by circling back to re-unify mass M and separation distance D; however, an important component is still missing, the proportionality constant G.

The aim of this paper's focus is on the proportionality constant. We wish to demonstrate the importance of transiting from proportionality to equality in Equation 10, in order to balance the units of accessibility into more interpretable values with clear meanings. To address this matter, we need not reinvent the wheel: instead we refer to the spatial interaction model as developed by Wilson (1971). In essence, in what follows, we reunite the accessibility indicators literature with the spatial interaction literature as developed by Wilson (1971) based on entropy maximization concepts. Although Wilson's approach is based on a different conceptual foundation than the old reference to Newtonian gravity, the work succeeded at identifying the steps from proportionality to equality to yield variations of proportionality constants: including the one that eluded Stewart (1948) and has been ignored in almost all subsequent accessibility research.

Gravity-based accessibility's sibling: the gravity model

Today's gravity-based accessibility measures (Equation 10) share the same early 20th century Newtonian-based origins as the gravity model, the most widely used model in the study of spatial interaction (Ortúzar and Willumsen 2011; Wilson 1971). However, in some ways the gravity model and spatial interaction continued to evolve independently from Equation 10, the former being developed into formalized transportation demand modelling practice to 'predict-and-provide', and the later taken up to describe land-use and transportation interactions by geography and social science- adjacent researchers and planners.

There are plenty of insights to be shared across these sibling branches; they are both spatial interaction models From the perspective of gravity-based accessibility measure, this paper's focus is on the proportionality constant of the classic gravity model, which takes the following form:

$$T_{ij} = kO_i D_j f(c_{ij}) \tag{11}$$

Where T_{ij} represents a n x m matrix of flows between an n number of origins i and an m number of destinations j, O_i and D_j is a vector of mass attributes representing origin i and destination j respectively, and $f(c_{ij})$ is a n x m matrix representing some function of travel cost c_{ij} which reflects travel impedance. In other words, T_{ij} explicitly measures *interaction* as a unit of trips and the single proportionality constant k ensures the total sum of T_{ij} represents the total flows in the data. It should be noted that the early analogy with the gravitational law proved too simplistic, and improvements to the gravity model (Equation 11) are the use of total trip ends (number of originating trips from zone i are O_i and ending trips at zone j are D_j) instead of total populations and separation distance calibrated by a power n (such as P_i, P_j, and n in Equation 4, Equation 3 and Equation 5). Finally, and of most important note in this paper, k is a scale parameter that makes the overall equation proportional to the rate characteristic of the modeled phenomenon.

Traditionally, the development of the gravity model has put an emphasis on the interpretability of the resulting units (Kirby 1970; Wilson 1967, 1971). As such, Equation 11 can take account of any additional information that can be built into the model to constrain the *interaction* variable T_{ij}. As mentioned, $T_i j$ is in the unit of trips, so additional knowledge can be the total trip flows O_i originating at each i and the total trip flows D_j terminating at j. These pieces of information can constrain T_{ij} as follows, ensuring it reflects this knowledge:

$$\sum_j T_{ij} = O_i \tag{12}$$

$$\sum_i T_{ij} = D_j \tag{13}$$

Using these constraints, the gravity model Equation 11 is typically distinguished in four cases, cases that are mathematically and conceptually consistent, so they have continued to be useful in classifying the gravity model in trip distribution modelling to this day (Ortúzar and Willumsen 2011):

- neither Equation 12 or Equation 13 hold – the unconstrained case;
- Equation 12 holds – the production constrained case (origin-constrained, a singly-constrained case);
- Equation 13 holds – the attraction constrained case (destination-constrained, a singly-constrained case);
- Equation 12 and Equation 13 both hold simultaneously – the production-attraction constrained case (doubly-constrained).

In each of these four cases, the single proportionality constant k is replaced by two sets of balancing factors A_i and B_j to ensure the case-specific constraints are satisfied. For the doubly-constrained case:

$$T_{ij} = A_i O_i B_j D_j f(c_{ij}) \tag{14}$$

And the balancing factors equal:

$$A_i = \frac{1}{\sum_j B_j D_j f(c_{ij})} \tag{15}$$

$$B_j = \frac{1}{\sum_j A_i O_j f(c_{ij})} \tag{16}$$

For the singly-constrained cases, origin-constrained case replaces B_j with 1 for all j and A_i is Equation 15, and the the destination-constrained case replaces A_i with 1 for all i and B_j is Equation 16. In a way, the singly-constrained and doubly-constrained cases are variants of the unconstrained case.

Entropy maximising: the gravity-model as statistical averages

Instead of the Newtonian gravity analogy, entropy maximisation is a popular model building approach taken in the spatial interaction studies to generate the gravity model. Namely, it is mathetmatically regorouse and offers a sensible interpretation of the resulting mode as a statistical averaging procedure. This allows the measure to be effective in describing behaviour in large populations but less effective in smaller decision making units in the region of interest.

The basis of the approach is to accept that there are macro, meso, and micro states – where all micro states that are consistent with our information about macro states are equally likely to occur. For the *interaction* variable of trips T_{ij}, as shown in Wilson (1971), the number of micro states $W\{T_{ij}\}$ within a meso state T_{ij} can be expressed as:

$$W\{T_{ij}\} = \frac{T!}{\prod_{ij} T_{ij}!} \tag{17}$$

Using the maximizing entropy approach, we must find a set of T_{ij} that maximizes Equation 17 subject to our sensible macro-state constraints Equation 12, Equation 12 and the following on total cost C constraint:

$$\sum_{ij} T_{ij} c_{ij} = C \tag{18}$$

The natural logarithm of Equation 17 is maximized under Equation 12, Equation 12 and Equation 18 for mathematical convenience, as both lnW and T_{ij} share the same maximum (Wilson 1971; Ortúzar and Willumsen 2011). The result is:

$$T_{ij} = A_i O_i B_j D_j exp(-\beta c_{ij}) \tag{19}$$

Where,

$$A_i = \frac{1}{\sum_j B_j D_j \exp(-\beta c_{ij})} \tag{20}$$

and

$$B_j = \frac{1}{\sum_j A_i O_j \exp(-\beta c_{ij})} \tag{21}$$

Results of maximisation are identical to Equation 14, Equation 15, and Equation 16 except the general $f(c_{ij})$ is $exp(-\beta c_{ij})$. From this approach, the gravity model can be interpretted to say that given the total number of individual trips (micro level) statistically represented at origin and destination zones (meso level), the cost of travelling (micro level, represented at the meso level), and the fixed total cost of travelling in the region (number of trips and cost of travel at the macro level). There is then a most probable distribution of trips between zones (meso-states) that stasify all the macro-state constraints.

A family of accessibility measures

Referring to the preceding section, the entropy maximizing method can be used to derive an equivalent family of measures, but for the case of accessibility where the *interaction* variable is V_{ij}, represent *potential interaction*, i.e., the number of potential reachable opportunities. T_{ij} in Equation 11 can be replaced with V_{ij} and presented as follows:

$$V_{ij} = kO_iD_jf(c_{ij}) \tag{22}$$

Where V_{ij} represents a n x m matrix of potential reachable opportunities between an n number of origins i and an m number of destinations j, O_i and D_j is a vector of mass attributes representing origin i and destination j respectively, and $f(c_{ij})$ is a n x m matrix representing some function of travel cost c_{ij} which reflects travel impedance. In other words, V_{ij} explicitly measures *potential interaction* as a unit of number of potentially reachable opportunities and the single proportionality constant k ensures the total sum of V_{ij} represents the total potential reachable opportunities in the data. Finally, and of most important note in this paper, k is a scale parameter that makes the overall equation proportional to the rate characteristic of the modeled phenomenon.

Like with Equation 11, we distinguish Equation 22 into FIVE cases based on versions of the gravity-model building constraints (Equation 12, Equation 13, and Equation 18). To inform each case of Equation 22 with constraints that reflect knowledge of the macro-state, the total trip flow O_i originating at each i and the total trip flows D_j terminating at j are used. These pieces of information can constrain V_{ij} as follows, ensuring it reflects this macro-level knowledge, i.e., the sum of the number of potential reachable opportunities.

$$\sum_j V_{ij} = O_i \tag{23}$$

Where sum of all potentially reachable opportunities for all j zones in the region is equal to the opportunities originating O_i at each i in the region.

$$\sum_i V_{ij} = D_j \tag{24}$$

Where sum of all potentially reachable opportunities for all i zones in the region is equal to the opportunities ending D_i at each j in the region.

$$\sum_i \sum_j V_{ij}c_{ij} = C \tag{25}$$

Where the sum of the travel cost of each potentially reached opportunity in the region is equal to a total travel cost C.

For all FIVE cases, Equation 25 holds but are differentiated like: - neither Equation 23 or Equation 24 hold – the unconstrained case; - Equation 23 holds – the production constrained case (origin-constrained, a singly-constrained case); - Equation 24 holds – the attraction constrained case (destination-constrained, a singly-constrained case); - Equation 23 and Equation 24 both hold simultaneously – the production-attraction constrained case (doubly-constrained). - Equation 23 or Equation 24 partly hold – hybrid production constrained case

Using the maximizing entropy approach… For each case…

Unconstrained case

Neither Equation 23 or Equation 24 hold, so $V_i j'$ has no opportunity constraints!

$$V'_{ij} = kO_iD_jf(c_{ij}) \tag{26}$$

Where k is two sets of balancing factors, A'_i and B'_j, and both are equal to 1.

Singly-constrained case (origin-constrained)

Spatial availability; origin-constrained.

Origins in human spatial interaction: "intervening opportunities and competing migrants" Stouffer (1940) and Stouffer (1960), "axiomatically" Weibull (1976), "for multiple-modes, as a rate" Shen (1998); for multiple-modes, as the number of opportunities" Soukhov et al. (2024)

$$V''_{ij} = kO_iD_jf(c_{ij}) \tag{27}$$

$$V''_{ij} = O_j \frac{F^p_{ij} \cdot F^c_{ij}}{\sum_{i=1}^K F^p_{ij} \cdot F^c_{ij}}$$

Population balancing factor:

$$F^p_{ij} = \frac{P^{\alpha}_{i\in r}}{\sum_{i=1}^K P^{\alpha}_{i\in r}}$$

Travel cost balancing factor:

$$F^c_{ij} = \frac{f(c_{ij})}{\sum_{i=1}^K f(c_{ij})}$$

Singly-constrained case (destination-constrained)

Origins are in retail models, decision model for shops to optimally spatially locate to max. their potential consumer interaction. Starting with reilly's law of gravitational retail (1931), formalized later in Huff (1964) and Lakshmanan and Hansen (1965). Spatial availability as another example.

Doubly constrained case (origin-destination constrained)

(only when the units of the marginals are the same, therefore of more limited application) ex. Horner (2004) and Allen and Farber (2020)

Hybrid singly-constrained case:

Balanced floating catchment areas (not truly doubly constrained)

Empirical examples

Use the TTS2016R package to illustrate every measure.

Discussion

When to use each of these measures?

Conclusions

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