

A family of accessibility measures derived from spatial interaction principles

Introduction

Historically, the focus of modern transportation planning has been to prioritize mobility while treating access to destinations as a by-product of movement. This has had problematic consequences: with the car seen as the ultimate mobility tool, this approach has led to the emergence and continued dominance of an automobility mono-culture (H. J. Miller 2011; Lavery, Páez, and Kanaroglou 2013). Decades of planning for this mono-culture (often characterized by road and highway expansion) have been marked by increased travel cost and environmental burdens, but often having only limited impact on the ease with which people can reach destinations (Steven Farber and Páez 2011; S. Handy 2002; Páez et al. 2010). In response to this situation, transportation researchers have increasingly advocated for the adoption of accessibility as a planning criterion (Silva et al. 2017; Paez et al. 2013; S. Handy 2020) using both positive and normative approaches (Paez, Scott, and Morency 2012; Levine 2020). Accessibility, a central concept in transport geography, planning, and engineering, is conceptually related to human mobility and the opportunity landscape. In simple terms, it is defined as the “potential of opportunities for interaction” (Hansen 1959). Compared to other measures of performance used in transportation that benchmark movement (e.g., VKT, PKT, etc.), accessibility brings a more holistic understanding of transportation and land use systems combined (S. L. Handy and Niemeier 1997).

An ascendant interest in accessibility has been accompanied by a remarkable boom on scholarly research, which has grown to include employment (e.g., Karst and Van Eck 2003; Grengs 2010; Páez et al. 2013; Merlin and Hu 2017; Tao et al. 2020), health care (e.g., Luo and Wang 2003; Páez et al. 2010; Wan, Zou, and Sternberg 2012; Delamater 2013; Boisjoly, Moreno-Monroy, and El-Geneidy 2017; Pereira et al. 2021), green spaces (Reyes, Paez, and Morency 2014; Rojas et al. 2016; Liang, Yan, and Yan 2024), schools (e.g., Williams and Wang 2014; Romanillos and Garcia-Palomares 2018; Marques, Wolf, and Feitosa 2021), social contacts (e.g., Neutens et al. 2007; S. Farber, Páez, and Morency 2012; S. Farber et al. 2013), and regional economic analysis (e.g., R. Vickerman, Spiekermann, and Wegener 1999; Lopez, Gutierrez, and Gomez 2008; Ribeiro, Antunes, and Páez 2010; Gutierrez et al. 2011) among many

other domains of application. In other words, accessibility analysis is used today to broadly understand the potential to reach opportunities that are important to people (Ferreira and Papa 2020). However, despite its growth in popularity in scholarly works, challenges remain to more widespread adoption of accessibility in planning practice. Several barriers to bridging the scholarly-practice accessibility gap have been identified. For example, the diversity of accessibility definitions has been flagged by van Wee (2016), S. Handy (2020), and Kapatsila et al. (2023). Further, difficulties in the interpretability and communicability of outputs has also been noticed by Karst T. Geurs and van Wee (2004), van Wee (2016), and Ferreira and Papa (2020).

Adoption of accessibility in planning practice is not necessarily made easier when potential adopters have to contend with a plethora of definitions, each seemingly more sophisticated but less intuitive than the last (Kapatsila et al. 2023). The menu of accessibility measures has grown to include gravity-based accessibility (e.g., Hansen 1959; Pirie 1979), cumulative opportunities (e.g., Wachs and Kumagai 1973; Pirie 1979; Ye et al. 2018), modified gravity (e.g., Schuurman, Berube, and Crooks 2010), 2-Step Floating Catchment Areas (e.g., Luo and Wang 2003), Enhanced 2-Step Floating Catchment Areas (e.g., Luo and Qi 2009), 3-Stage Floating Catchment Areas (e.g., Wan, Zou, and Sternberg 2012), Modified 2-Step Floating Catchment Areas (e.g., Delamater 2013), inverse 2-Step Floating Catchment Areas (e.g., F. Wang 2021), and n-steps Floating Catchment Areas (Liang, Yan, and Yan 2024). How is a practitioner to choose among this myriad options? What differences in accessibility scores should matter, and how should they be communicated? [see van Wee (2016); p. 14].

In this respect, Wu and Levinson (2020)’s contribution provides a unifying framework to think about accessibility measures, the most general of which they describe as S_i in Equation 1. By considering the key features of accessibility, namely travel-cost $f(c_{ij})$ and the distribution of opportunities $g(O_i)$, Wu and Levinson (2020) demonstrate that a majority of the concepts found scattered throughout the accessibility literature can be seen as particular cases of a general accessibility formula (Equation 1). One needs only to judiciously change the way travel-cost and opportunities are formulated, to derive almost any known accessibility measure. In this way, Wu and Levinson (2020) have taken a considerable step towards clarifying the differences between various accessibility indicators.

$$S_i \propto \sum_j g(O_j) f(c_{ij}) \quad (1)$$

However, there is room to further clarify another relevant aspect: competition. Wu and Levinson (2020)’s unifying framework does not appear to accommodate many (or any) of the floating catchment area methods, a popular approach within the competitive accessibility literature. There have also been more recent developments in the literature: for instance, Soukhov et al. (2023) demonstrate that the introduction of a single constraint is sufficient to turn the general accessibility measure (i.e, Equation 1) into a competitive measure of accessibility in accordance with the 2-Step Floating Catchment Area approach. Moreover, the proportional

allocation balancing factors in Soukhov et al. (2023) apply a singly-constraint to the accessibility measure, akin to the constraints introduced within the spatial interaction modelling framework outlined in Wilson (1971).

Motivated by the challenge of incorporating competition into a unified framework for accessibility, this paper contends that accessibility research must reconnect with its spatial interaction origins. Particularly, we argue that an important aspect of spatial interaction modelling—namely, constraining the results to match empirical observations—was never effectively re-incorporated into accessibility analysis. Empirical constraints were embraced by early spatial interaction literature following the work of Wilson (1971), but this stream of literature tended to flow separately from research inspired by Hansen (1959)’ accessibility. The application of Wilson (1971)’s empirical constraints supported the development of various spatial interaction models that remain relevant in research and practice today (Ortúzar and Willumsen 2011). However, the same cannot be said of the contemporary accessibility literature, where empirical constraints were not explicitly adopted. We argue that the absence of empirical constraints (and their attendant proportionality constants) has contributed to some of accessibility analysis’ interpretability issues; for instance, the fuzziness of insights beyond simple proportional statements like ‘higher-than’ or ‘lower-than’ (E. J. Miller 2018).

These streams of literature share common headwaters. It is by looking to the past that we believe accessibility analysis can newly wade into the future. Hence, this work’s primary focus is to show that the same empirical constraints used in Wilson (1971)’s family of spatial interaction models can be mapped onto accessibility. We begin by tracing the development of accessibility from its origins in spatial interaction, from the Newtonian gravitational expression in Ravenstein (1889) through to the seminal accessibility work of Hansen (1959). We then present evidence for a narrative highlighting the marked divergence between accessibility and spatial interaction modelling research after the work of Wilson (1971). Next, we hark back to Wilson (1971)’s spatial interaction models, and use it to derive a family of accessibility measures based on different types of constraints. We illustrate various members of this family with a simple numerical example and a real world data set. We then conclude by discussing the uses of these measure and their interpretation.

Newtonian’s roots of human spatial interaction research

The patterns of people’s movement in space have been a subject of scientific inquiry for at least a century and a half. From as far back as Henry C. Carey’s *Principles of Social Science* (Carey 1858), a concern with the scientific study of human spatial interaction can be observed. It was in this work where Carey stated that “man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance” (McKean 1883, 37–38). This statement shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton’s Law of Universal Gravitation, first posited in 1687’s *Principia Mathematica* and expressed as in Equation 2.

$$F_{ij} \propto \frac{M_i M_j}{D_{ij}^2} \quad (2)$$

To be certain, the expression above, a proportionality, is one of the most famous in all of science. In brief, it states that the force of attraction F between a pair of bodies i and j is directly *proportional* to the product of their masses M_i and M_j , and inversely *proportional* to the square of the distance between them D_{ij} . Direct proportionality means that as the product of the masses increases, so does the force. Likewise, inverse proportionality means that as the distance increases, the force decreases. Equation 2, however, does not quantify the magnitude of the force. To do so, an empirical constant is required to convert the proportionality into an equality, ensuring that values of the force F in Equation 2 match the observed force of attraction between masses. In other words, Equation 2 needs to be *constrained* using empirical data. Ultimately, the equation for the force is as seen in Equation 3, where G is an empirically calibrated proportionality constant:

$$F_{ij} = G \frac{M_i M_j}{D_{ij}^2} \quad (3)$$

Newton’s initial estimate of G was based on a speculation that the mean density of earth was between five or six times that of water, an assumption that received support after Hutton’s experiments of 1778 (Hutton 1778, 783). Still, it took over a century from the publication of *Principia* to refine the estimate of the proportionality constant to within 1% accuracy, with Cavendish’s 1798 experiment (Cavendish 1798).

Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Following Carey’s *Principles* of 1858, research into human spatial interaction continued in different contexts. In the late 1880s, Ravenstein proposed some “Laws of Migration” based on his empirical analysis of migration flows in various countries (Ravenstein 1885, 1889). In these works, Ravenstein posited 1) a directly proportional relationship between migration flows and the size of destinations (i.e., centres of commerce and industry), and 2) an inversely proportional relationship between the size of flows and the separation between origins and destinations. As with Carey, these propositions echo Newton’s gravitational laws. Over time, other researchers discovered similar relationships. For example, Reilly (1929) formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations. Later, Zipf proposed a $\frac{P_1 P_2}{D}$ hypothesis for the case of information (Zipf 1946a), intercity personal movement (Zipf 1946b), and goods movement by railways (Zipf 1946c). The $\frac{P_1 P_2}{D}$ hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

A common feature of these early investigations of human spatial interaction is that a proportionality constant similar to G in Equation 3 was never considered. Of the researchers cited above, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly's hypothesis was presented in the following form:

$$B_a = \frac{(P_a \cdot P_T)^N}{D_{aT}^n} \quad (4)$$

Where B_a is the amount of business drawn to a from T , P_a and P_T are the populations of the two settlements, and D_{aT} is the distance between them. Quantity N was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of $n = 2$.

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

$$C^2 = \frac{P_1 \cdot P_2}{D_{12}} \quad (5)$$

where C is the volume of goods exchanged between 1 and 2, P_1 and P_2 are the populations of the two settlements, and D_{12} is the distance between them.

After Carey, it is in Stewart's work on the principles of demographic gravitation that we find the strongest connection yet to Newton's law (Stewart 1948). This may relate to academic backgrounds; where Ravenstein, Reilly, and Zipf were social scientists, Stewart was a physicist. Besides awareness of preceding research (he cites both Reilly and Zipf as predecessors in the analysis of human spatial interaction), Stewart appears to have been the first author to express his theorized relationships for human spatial interaction with a proportionality constant G , as follows:

$$F = G \frac{(\mu_1 N_1)(\mu_2 N_2)}{d_{12}^2} = G \frac{M_1 \cdot M_2}{d_{12}^2} \quad (6)$$

Where:

- F is the *demographic force*
- N_1 and N_2 are the numbers of people of in groups 1 and 2
- μ_1 and μ_2 are so-called *molecular weights*
- $M_1 = \mu_1 N_1$ and $M_2 = \mu_2 N_2$ are the demographic masses at 1 and 2
- d_{12}^2 is the distance between 1 and 2
- And finally G , a constant that Stewart "left for future determination" (1948, 34)

In addition to demographic force, Stewart defined a measure of the “population potential” of 2 with respect to 1 as follows:

$$V_1 = G \frac{M_2}{d_{12}} \quad (7)$$

For a system with more than two population bodies, Stewart formulated the population potential at i as follows (after arbitrarily assuming that $G = 1$):

$$V_i = \int \frac{D}{r} ds \quad (8)$$

where D is the population density over an infinitesimal area ds and r is the distance to i . In Equation 8, $D \cdot ds$ gives an infinitesimal count of the population, say dm , and so, after discretizing space, Equation 8 can be rewritten as:

$$V_i = \sum_j \frac{M_j}{d_{ij}} \quad (9)$$

Alerted readers will notice that Equation 9 is formally equivalent to our modern definition of accessibility.

Stewart’s formulation of demographic force, developed in the context of what he called “social physics” (Stewart 1947), was problematic. It had issues with inconsistent mathematical notation. More seriously though, Stewart’s work was permeated by a view of humans as particles following physical laws, but tinted by unscientific ideas that were unadulterated racism. For instance, he assumed that the molecular weight μ of the average American was one, but “presumably...much less than one....for an Australian aborigine” [p. 35]. Stewart’s ideas about “social physics” soon fell out of favour among social scientists, but not before influencing the nascent field of accessibility research, as detailed next.

Hansen’s gravity-based accessibility to today

From Stewart (1948), we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature (Hansen 1959). In his seminal paper, Hansen defined accessibility as “the potential of opportunities for interaction... a generalization of the population-over-distance relationship or *population potential* concept developed by Stewart (1948)” (p. 73). As well as being a student of city and regional planning at Massachusetts Institute of Technology, Hansen was also an engineer with the Bureau of Roads, and preoccupied with the power of transportation to shape land uses in a very practical sense. Hansen (1959) focused on Stewart (1948)’s *population potential* (expressed in Equation 9), leaving Stewart’s other formulaic contributions and objectionable aspects of “social physics” behind. Hansen (1959) recast Stewart’s population potential to reflect accessibility, a model of

human behaviour useful to capture regularities in mobility patterns. Hansen (1959) replaced M_j in Equation 9 with *opportunities* to derive an *opportunity potential*, or more accurately, a *potential of opportunities for interaction* as follows:

$$S_i = \sum_j \frac{O_j}{d_{ij}^\beta} \quad (10)$$

A contemporary rewriting of Equation 10 accounts for a variety of impedance functions beyond the inverse power $d^{-\beta}$:

$$S_i = \sum_j O_j \cdot f(d_{ij}) \quad (11)$$

S_i in Equation 10 is a measure of the accessibility of site i . This is a function of O_j (the mass of opportunities at j), d_{ij} (the cost, e.g., distance or travel time, incurred to reach j from i), and β (a parameter that modulates the friction of cost). Today, Hansen is frequently cited as the father of modern accessibility analysis (e.g., Reggiani and Martín 2011), and Hansen-type accessibility is commonly referred to as the gravity-based accessibility measure.

Of note, however is that between Stewart (1948) and Hansen (1959) the proportionality constant G in Equation 7 vanished. There is some evidence that Hansen (1959) was aware of the importance of this constant as he wrote about *directly* and *inversely proportional* relationships when discussing population, opportunities, and their separation in space. At any rate, those reading Hansen (1959) must recall that Stewart (1948) had set the proportionality constant G to 1, with a note that “ G [was] left for future determination: a suitable choice of other units can reduce it to unity”.

After Hansen (1959), accessibility analysis has been widely used in numerous disciplines but, to our knowledge, the proportionality constant has remained forgotten, with no notable developments to explicitly determine it. In this way, G continues to be implicitly set to 1, even when the fundamental relationship in accessibility is proportionality (e.g., $S_i \propto \sum_j g(O_j)f(d_{ij})$) and not equality (for instance, see the formula for accessibility at the top of Figure 1 in Wu and Levinson 2020). Alas, without a proportionality constant, the units of S_i remain unclear: the unit of “potential of opportunity for interaction” is left free to change as β is calibrated. For example, if c_{ij} is distance in meters, it will be number of opportunities per m^β when $f(c_{ij}) = d^{-\beta}$ but number of opportunities per $e^{-\beta \cdot m}$ when $f(c_{ij}) = e^{\beta \cdot m}$. Hansen-style accessibility, therefore, is better thought of as an ordinal measure of potential that can only be interpreted in terms of higher and lower accessibility (E. J. Miller 2018).

Wilson's family of spatial interaction models

In his groundbreaking work, Wilson (1971) defined a general spatial interaction model as follows:

$$T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij}) \quad (12)$$

The model in Equation 12 posits a quantity T_{ij} that represents a value in a matrix of flows of size $n \times m$, that is, between $i = 1, \dots, n$ origins and $j = 1, \dots, m$ destinations. The quantities $W_i^{(1)}$ and $W_j^{(2)}$ are proxies for the masses at $i = 1, \dots, n$ origins and $j = 1, \dots, m$ destinations. Finally, $f(c_{ij})$ is the $i - j$ element of an $n \times m$ matrix representing some function of travel cost c_{ij} which reflects travel impedance. In this way, T_{ij} explicitly measures *interaction* in the unit of trips, and the role of the single proportionality constant k is to ensure that the system-wide sum of T_{ij} represents the total flows in the data. In other words, k is a scale parameter that makes the overall amount of flows identical to the magnitude of the phenomenon being modeled.

Traditionally, development of the spatial interaction model put an emphasis on the interpretability of the results (Kirby 1970; Wilson 1967, 1971). But instead of relying on the heuristic of Newtonian gravity (e.g., some interaction between a mass at i and a mass at j separated by some distance), Wilson's approach was to maximise the entropy of the system. Entropy maximisation in this case achieves stable results as a statistical average that represents the population. The approach works by assuming undifferentiated individual interactions, and assessing their probabilities of making a particular journey. The result of Equation 12 then is a statistical average (Wilson 1971; Senior 1979).

To ensure that T_{ij} in Equation 12 is in the unit of trips, additional knowledge about the system is required. At the very least, this framework assumes that the total number of trips in the system T is known, and therefore:

$$\sum_i \sum_j T_{ij} = T \quad (13)$$

Additional information can be introduced. For example, when information is available about the total number of trips produced by each origin (O_i), the following constraint can be used:

$$\sum_j T_{ij} = O_i \quad (14)$$

Alternatively, if there is information available about the total number of trips attracted by each destination (D_j), the following constraint can be used:

$$\sum_i T_{ij} = D_j \quad (15)$$

It is also possible to have information about both O_i and D_j , in which case both constraints could be imposed on the model.

Building on this, a family of spatial interaction models can be derived. In the framework introduced in Wilson (1971), this includes the “unconstrained” (which we refer to as the total flow constrained) model in Equation 12. The proportionality constant k in this case is (see Cliff, Martin, and Ord 1974; A. S. Fotheringham 1984):

$$k = \frac{T}{\sum_i \sum_j T_{ij}} \quad (16)$$

When only one of Equation 14 or Equation 15 holds, the resulting models are, in Wilson’s terms, singly-constrained. Entropy maximisation leads to the following production-constrained model:

$$T_{ij} = A_i O_i W_j^{(2)} f(c_{ij}) \quad (17)$$

Notice how, in this model, the proxy for the mass at the origin $W_i^{(1)}$ is replaced by the actual mass, as measured by the trips produced O_i . Also, there is no longer a single system-wide proportionality constant, but rather a set of proportionality constants (i.e., balancing factors) specific to origins. According to Wilson, these constants, namely A_i , are:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \quad (18)$$

The attraction-constrained model, in turn, takes the following form:

$$T_{ij} = B_j D_j W_i^{(1)} f(c_{ij}) \quad (19)$$

Notice how now the proxy for the mass at the destination $W_j^{(2)}$ is replaced by the actual mass, as measured by the trips attracted D_j . As before, destination-specific proportionality constants (i.e., balancing factors) B_j were derived by Wilson as:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \quad (20)$$

When both Equation 14 and Equation 15 hold, the resulting model is, in Wilson’s terms, doubly-constrained, and takes the following form:

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \quad (21)$$

In this model, both proxies for the masses are replaced with the known masses, that is, the trips produced by origin and the trips attracted by destination. And, now there are two sets of mutually dependent proportionality constants:

$$\begin{aligned} A_i &= \frac{1}{\sum_j B_j D_j f(c_{ij})} \\ B_j &= \frac{1}{\sum_i A_i O_i f(c_{ij})} \end{aligned} \tag{22}$$

Derivation of these models is demonstrated in detail elsewhere (e.g., Ortúzar and Willumsen 2011; Wilson 1967). It is worth noting, however, that although Wilson’s approach is built on a different conceptual foundation than the old reference to Newtonian gravity, the work succeeded at identifying the steps from proportionality to equality to yield variations of proportionality constants, including the one that eluded Stewart (1948) and that has been ignored in almost all subsequent accessibility research. Why was this key element of spatial interaction models ignored in accessibility research? In the next section we aim to address this question.

Accessibility and spatial interaction modelling: two divergent research streams

The work of Hansen (1959) and Wilson (1971) responded to important developments in the Anglo-American world at the time, in particular a need “to meet the dictates and needs of public policy for strategic land use and transportation planning” (Michael Batty 1994). Said dictates and needs were far from trivial. In the United States alone, the Federal-Aid Highway Act of 1956 authorized the creation of the U.S. Interstate Highway System, with a budget that ultimately exceeded one hundred billion dollars (Weiner 2016; MDOT 2007). Spatial interaction modelling, with its ability to quantify trips, was incorporated into institutional modelling practices meant to “predict and provide”, i.e., predict travel demand and supply transportation infrastructure (Kovatch, Zames, et al. 1971; Weiner 2016). Accessibility at the time did not quite have that power, as it did not quantify trips, but rather something somewhat more elusive: the less tangible “potential for interaction”. In this way, where spatial interaction modelling became a key element of transportation planning practice, accessibility remained a somewhat more academic pursuit, and the two streams of literature only rarely connected.

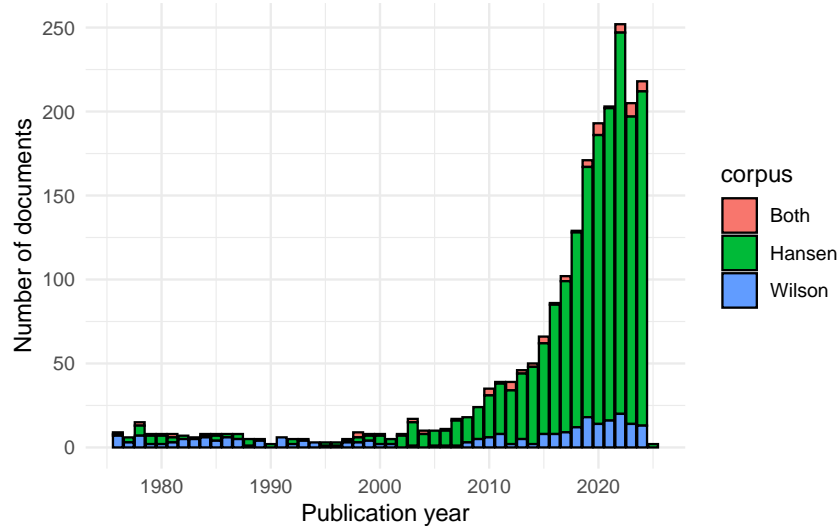


Figure 1: Historical pattern of publication: documents per year.

To illustrate this point, we conducted a bibliographic analysis of the literature that cites Hansen (1959), Wilson (1971), or both. We retrieved all relevant documents using the Web of Science “Cited References” functionality, and the digital object identifiers of Hansen (1959) and Wilson (1971). As a result of this search we identified 1,788 documents that cite Hansen (1959), 258 documents that cite Wilson (1971), and 76 that cite both. The earliest document in this corpus dates to 1976 and the most recent is from 2025. The number of documents per year appears in Figure 1, where we see the frequency of documents over a span of almost fifty years. In particular, we notice the remarkable growth in the number of papers that cite Hansen (1959) compared to those that cite Wilson (1971) or both.

After compiling this corpus of documents, we used the `{bibliometrix}` package (Aria and Cuccurullo 2017) to create a bibliographical coupling matrix. In this matrix, two documents are coupled if they share at least one common reference, and the strength of the coupling increases with the number of references that the documents have in common. Figure 2 presents the results, where each symbol represents a document. The distance between symbols on the plot indicates the strength of bibliographical coupling: documents that are more strongly coupled (i.e., those sharing more references) are plotted closer to each other.

Table 1: Average coupling strength within and between groups of documents. The average coupling strength is the number of references shared on average by a pair of documents.

corpus	Hansen	Wilson	Both
Hansen (n = 1788)	1.45	-	-
Wilson (n = 258)	0.09	1.31	-
Both (n = 76)	1.65	1.23	3.36

Average coupling for the pooled set of documents (n = 2122) is 1.18

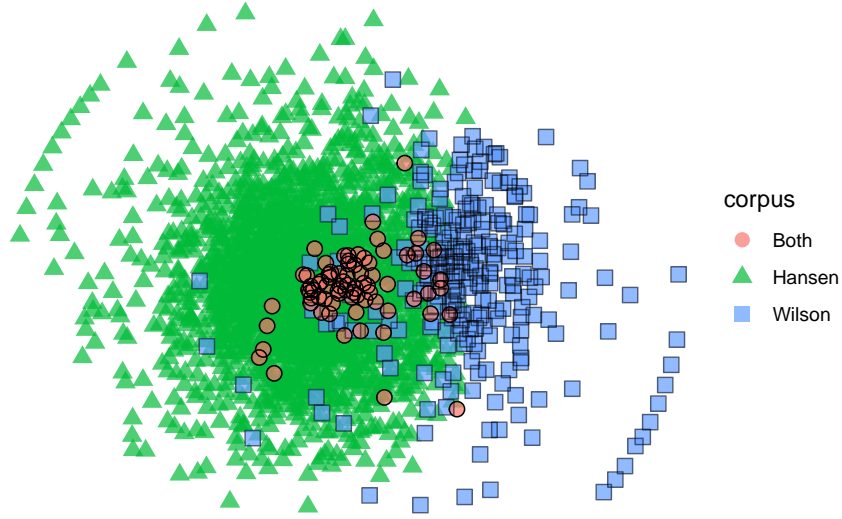


Figure 2: Bibliometric coupling of papers that cite Hansen (1959), Wilson (1971), or both.

Further examination of the bibliographical coupling matrix allows us to check the coupling strength within and between subgroups of documents (i.e., those that cite only Hansen, only Wilson, or both papers). First, we note that when we look at the three subgroups of documents pooled together, the average number of references shared by a pair of documents is 1.18. Table 1 presents the average number of items cited in common within each subgroup, as well as between subgroups. As seen in the table, the coupling within subgroup tends to be higher than when we consider the full corpus with all three groups together. The tightest coupling is seen for the group of documents that cite both Hansen and Wilson, with an average number of references in common of 3.36. The weakest coupling is between the Hansen and Wilson subgroups, with only 0.09 references in common on average, which indicates a large degree of decoupling (independence) between these bodies of work.

As noted, literature that cites both Wilson (1971) and Hansen (1959) are sparse (only 3.6% of the corpus) but highly coupled. However, we can discern from a manual reading of these

works that they too are divergent, with one stream focused on developing accessibility (i.e., *potential for* spatial interaction) and another on spatial interaction. The focuses of these divergent streams contribute this paper’s broader hypothesis that the concepts of accessibility and spatial interaction have remained largely disconnected.

The spatial-interaction-focused stream of literature inspired by Wilson (1971) that cites both Hansen (1959) and Wilson (1971), tends to contribute to understanding how accessibility is interpreted and incorporated in spatial interaction models, treating it as a separate (but related) phenomenon or variable. Specifically, works interpret the spatial interaction model’s balancing factors (Equation 18 or Equation 22) as the inverse of Hansen (1959)’s model (B. Harris and Wilson 1978; G. Leonardi 1978; A. Stewart Fotheringham 1981; A. S. Fotheringham 1985), recognizing it as a “common sense” approach (Morris, Dumble, and Wigan 1979, 99) to including accessibility in the spatial interaction model, though further exploration of its relationship is warranted (M. Batty and March 1976). Some authors have explored this relationship, for instance as in A. S. Fotheringham (1985) who demonstrates how the spatial interaction model may insufficiently explain spatial patterns and suggest that explicitly defining destinations’ accessibility as a variable within the model may remedy the issue (e.g., the *competition destination* model). Other works used both Hansen (1959) and Wilson (1971)’s framework in conjunction, such as in defining location-allocation problems in Operations research d(G. Leonardi 1978; Beaumont 1981), estimating trip flows (or some other interaction flows) alongside accessibility (e.g., Clarke, Eyre, and Guy 2002; Grengs 2004; Türk 2019), or considering accessibility within spatial interaction models, in line with A. S. Fotheringham (1985)’s demonstration (e.g., Beckers et al. 2022). Other works departed from Hansen (1959)’s definition and aligned with spatial interaction in different ways, such as using micro-economic consumer behaviour concepts to express potential for spatial interaction (Morris, Dumble, and Wigan 1979; Giorgio Leonardi and Tadei 1984).

On the other hand, we discern that the potential-for-spatial-interaction-focused stream of literature inspired by Hansen (1959) that cite both Hansen (1959) and Wilson (1971), cite Wilson (1971) for three general reasons: - Firstly, as attribution for using using different context-dependent travel cost functions (Weibull 1980; S. L. Handy and Niemeier 1997; Kwan 1998; Q. Shen 1998a; Ashiru, Polak, and Noland 2003; Rau and Vega 2012; Pan 2013; Margarida Condeço Melhorado et al. 2016; Caschili, De Montis, and Trogu 2015; Grengs 2015; Pan, Jin, and Liu 2020; Chia and Lee 2020; Roblot et al. 2021; Sharifiasl, Kharel, and Pan 2023; Kharel, Sharifiasl, and Pan 2024). - Secondly, associating spatial interaction with accessibility’s potential for spatial interaction (e.g., Giuliano et al. 2010; Grengs et al. 2010; Grengs 2010, 2012; Levine et al. 2012; Levinson and Huang 2012; Tong, Zhou, and Miller 2015; X. Liu and Zhou 2015; He et al. 2017; Wu and Levinson 2020; Ng et al. 2022; Naqavi et al. 2023; Suel et al. 2024). While accessibility is an expression of *potential for* spatial interaction and Wilson (1971) touches on the concept H. J. Miller (1999), it is distinct concept from spatial interaction. This distinction, however, is often unclear, as Hansen (1959) and Wilson (1971) are frequently co-cited as both being ‘gravity models’ (e.g., S. Liu and Zhu 2004; Dai, Wan, and Gai 2017; Y. Shen 2019; Chia and Lee 2020). - Thirdly, interpretation of Hansen (1959)’s model as an inverse balancing factor (R. W. Vickerman 1974), especially in competitive opportunity or

population contexts (Karst and Van Eck 2003; Karst T. Geurs, van Wee, and Rietveld 2006; Willigers, Floor, and van Wee 2007; El-Geneidy and Levinson 2011; Curtis and Scheurer 2010; Manaugh and El-Geneidy 2012; Chen and Silva 2013; Alonso et al. 2014; Albacete et al. 2017; Sahebgharani, Mohammadi, and Haghshenas 2019; Mayaud et al. 2019; Allen and Farber 2020; Levinson and Wu 2020; Marwal and Silva 2022; Su and Goulias 2023). However, as outlined in preceding sections, we argue the resulting indicator values are plagued by similar interpretability issues. Of the literature in this stream, only the work of Soukhov et al. (2023) and Soukhov et al. (2024) cite Wilson (1971)’s use of balancing factors as ways to maintaining constraints in opportunities in the context of competitive accessibility.

A family of accessibility measures

As argued in the preceding section, the accessibility and spatial interaction modelling literature streams tended to evolve with little contact after Hansen (1959) and Wilson (1971). This may explain the failure of the constraints/proportionality constant(s) of spatial interaction models to cross over to accessibility analysis. This is intriguing since Wilson himself made an effort to connect his developments in spatial interaction modelling to accessibility, noting the denominator of Equation 18 and is the inverse of A_i , offering a potential interpretation for the proportionality constant (Wilson 1971, 10):

$$S_i = \frac{1}{A_i} = \sum_j W_j^{(2)} f(c_{ij}) \quad (23)$$

This, however, only helps to illustrate the ease with which confusion may arise in this context. The relationship in Equation 23 appears to be misaligned. In the preceding section on early research, we noted that Hansen, and Stewart before him, defined accessibility as a partial sum of the demographic force F , essentially going from the demographic force ($F = G \frac{M_1 \cdot M_2}{d_{12}^2}$ to the pairwise population potential $V_1 = G \frac{M_2}{d_{12}}$ and from there to the system-wide population potential (after losing G) $V_i = \sum_j \frac{M_j}{d_{ij}}$. While the inverse of A_i may appear to be accessibility, its functional role in Wilson’s general model $T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij})$ is that of a balancing factor k (maintaining proportionality).

Hence, we propose a more useful definition: accessibility as a partial sum of the spatial interaction. A way to define the *potential for spatial interaction* (i.e., accessibility) is as follows:

$$V_{ij} = kW_j^{(2)}f(c_{ij}) \quad (24)$$

where V_{ij} is the potential for interaction, and therefore:

$$V_i = k \sum_j W_j^{(2)} f(c_{ij}) \quad (25)$$

Similar to Equation 12, $W_j^{(2)}$ above is the mass at the destination, and the sub-indices are for $i = 1, \dots, n$ origins and $j = 1, \dots, m$ destinations.

In the following subsections, we detail how various accessibility measures can be defined depending on selected constraints: how k is specified and depending on the combination of population and opportunity data is available and measurement focus. Together, these constraints delineate different members of the family of accessibility measures. Along with a simple numerical example, we detail the unconstrained case (Hansen (1959)'s formulation), then total constrained cases (opportunity-constrained and population-constrained), then singly-constrained cases (opportunity-constrained and population-constrained), and lastly the doubly-constrained case. These cases are presented in increasing order of constraint restrictiveness and information availability.

Simple numeric example setup

Consider a simple region with three zones, IDs 1, 2 and 3. Each zone is both an origin i and a destination j . The following three pieces of information are defined: zonal population and opportunities, zonal cost matrix, and travel impedance functions for three types of travel behaviour. In this example, the population are people, the opportunities are physicians, and the region can be described by three possible travel behaviours.

Firstly, Table 2 summarises the population (in units of 10,000s of people) and the opportunities (the number of physicians) per zone.¹ Considering Table 2's values, the Provider-to-Population-Ratio (PPR) in this system is 24.5. The population is $W_i^{(1)}$ when used as a proxy for the mass at the origin, and O_i when used as a constraint. Similarly, the opportunities are represented by $W_j^{(2)}$ when used as a proxy for the mass at the destination, and D_j when used as a constraint.

Secondly, to pair with the zonal population and opportunity information, the assumed cost of movement (in minutes of travel time) between origins and destinations is as shown in Table 3. From both Table 2 and Table 3, it can be interpreted that Zones 1 and 3 are more proximate to each other than to Zone 2. Combined, Zone 1 and 3 have the same amount of population as Zone 2 but more than double the opportunities.

And lastly, we distinguish accessibility measure values for the following three impedance functions that represent the potential for spatial interaction travel behaviour of the population to

¹For reference, the number of physicians per 10,000 in Canada in 2022 was 24.97 (WHO 2025)

Table 2: Simple system with three zones. Population is in 10,000 persons and opportunities in number of physicians.

ID (i or j)	Population ¹	Opportunities ²
1	4	160
2	10	150
3	6	180

¹Population is $W_i^{(1)}$ when used as a proxy for the mass at the origin, and O_i when used as a constraint.

²Opportunities is $W_j^{(2)}$ when used as a proxy for the mass at the destination, and D_j when used as a constraint.

Table 3: Cost matrix for system with three zones (travel time in minutes).

Origin ID	Destination ID		
	1	2	3
1	10	30	15
2	30	10	25
3	15	25	10

opportunities in the region:

$$\begin{aligned} f_1(c_{ij}) &= \frac{1}{c_{ij}^3} \\ f_2(c_{ij}) &= \frac{1}{c_{ij}^2} \\ f_3(c_{ij}) &= \frac{1}{c_{ij}^{0.1}} \end{aligned}$$

Accessibility will be calculated three times for each case, one assuming the most decay ($f_1(c_{ij})$), another assuming medium decay ($f_2(c_{ij})$), and a third assuming the least decaying travel behaviour ($f_3(c_{ij})$) for the entire region. A helpful analogy may be tying travel behaviour to the used mode's mobility potential, i.e., the most decaying travel behaviour ($f_1(c_{ij})$) would assume all travel in the region being done by foot, while calculating accessibility assuming the least decay ($f_3(c_{ij})$) would assume unfettered automobility. Or alternatively, these functions could represent travel behaviour on snowstorm-affected day ($f_1(c_{ij})$) for the entire region versus a clear, ideal travel day ($f_3(c_{ij})$). As an example of a discussion on how travel behaviour has been considered in accessibility measures cost of travel see Paez, Scott, and Morency (2012).

Any set of concepts representing population, opportunities, and their associated travel behaviour -whether representing the entire region uniformly (as we will demonstrate) or representing specific subgroups- can be substituted into our simple example, depending on the research question. The purpose of the following simple example is to demonstrate the calculation of each member of the accessibility measure family, interpret the values, and compare them both within and across travel behavior groups and members of the accessibility family.

Unconstrained accessibility

Setting the balancing factor k to one in Equation 24 results in the unconstrained accessibility case:

$$V_{ij}^0 = W_j^{(2)} f(c_{ij}) \quad (26)$$

Then, the partial sum becomes the following: simply Hansen's accessibility S_i (Hansen 1959), standard practice in accessibility measurement:

$$V_i^0 = \sum_j V_{ij}^0 = \sum_j W_j^{(2)} f(c_{ij}) = S_i \quad (27)$$

Where the sum of V_i^0 in the region generally does not equal the total number of opportunities O (e.g., $\sum_i V_i^0 \neq O$), as setting k to 1 to arbitrarily eliminates the term and imposes no meaningful interpretation to the resulting values. To complicate matters, when comparing V_i^0 values between travel behaviour contexts, their difference depends on the choice of impedance function $f(c_{ij})$, associated parameters, and the number of zones for which an accessibility score is calculated. For this reason, the unconstrained accessibility should more appropriately be

Table 4: Simple system: unconstrained accessibility.

Zone	V_i^0		
	$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
	units: *f_1(c_ij)-weighted physicians	units: *f_2(c_ij)-weighted physicians	units: *f_3(c_ij)-weighted physicians
1	0.219	2.567	371.143
2	0.167	1.966	363.479
3	0.237	2.751	373.738

used as an ordinal variable to make comparisons of size (i.e., greater than, less than, equal to), not to calculate ratios or intervals (i.e., the magnitude of differences).

Returning to the simple example, consider the calculated unconstrained accessibility V_i^0 for each origin, a sum of all the travel impedance weighted opportunities at each destination ($\sum_i V_i^0$), in Table 4.

As the different impedance functions represent different travel behaviours, comparing unconstrained accessibility values across groups meaningless. For instance, at zone 1 the difference between the least decay ($f_3(c_{ij})$) and most decay ($f_1(c_{ij})$) groups is 370.924131, but in what units? These two values, though representing the same zone but different groups in that zone, have different units so this difference value cannot be meaningfully interpreted.

However, we can compare values within the same travel behavior scenario across different zones but values are not inherently meaningful due to the units. For example, considering the most decaying scenario at zone 1, it captures 0.051443 more $f_1(c_{ij})$ -weighted-physicians than zone 2, and 0.0181185 fewer than zone 3. In the least decaying scenario, zone 1 captures 7.6638289 more $f_3(c_{ij})$ -weighted-physicians than zone 2, and 2.5954449 fewer than zone 3. Both scenarios at zone 1 captures an intermediate V_i^0 value between the two other zones, but at different ratios: the least decaying scenario captures 1.3072213 and 0.9235529 times the $f_1(c_{ij})$ -weighted-physicians than zone 2 and 3 respectively, while the most decaying scenario captures 1.0210846 and 0.9930555 times the $f_3(c_{ij})$ -weighted-physicians. The ratios between the zones in the two scenarios different. These differences are tied up in the travel impedance functions, and hence the unconstrained accessibility values themselves should not be compared. This simple example illustrates that the differences and ratios of unconstrained accessibility values are not inherently meaningful. While one could attempt to adjust the units post-calculation (e.g., scaling, population normalization) or select comparable impedance functions (at the cost of accurately reflecting travel behavior), such adjustments may introduce bias.

Totally-constrained accessibility

A total constraint proportionally adjusts zonal accessibility values based on the total population and/or opportunities in the region. The introduction of this constraint introduces a form of competition, as all zonal values are a proportion of the system total, be it the regional opportunities or regional population depending on the case. We define two cases: a *totally-constrained opportunity* accessibility case, interpreted as Hansen's accessibility with a constraint, and a *totally-constrained population* accessibility case, interpreted as constrained market potential.

Totally-constrained opportunity: Hansen's accessibility with a constraint

Unlike in Equation 24, the proportionality constant k is retained and not set arbitrarily to 1:

$$V_{ij}^T = k^D W_j^{(2)} f(c_{ij}) \quad (28)$$

Where the total opportunity constrained accessibility measure now becomes Hansen's accessibility with a proportionality constant:

$$V_i^T = \sum_j V_{ij}^T = k^D \sum_j W_j^{(2)} f(c_{ij}) = k^D V_i^0 \quad (29)$$

The constraint that we impose in this case is the total number of opportunities D in the region.

$$\sum_i V_i^T = \sum_i \sum_j V_{ij}^T = D \quad (30)$$

This constraint is analogous to the total constraint of Equation 13 which is congruent with Wilson's framework. We can then substitute Equation 29 in Equation 30 and solve for k to yield:

$$k^D = \frac{D}{\sum_i \sum_j V_{ij}^0} = \frac{D}{\sum_i \sum_j W_j^{(2)} f(c_{ij})} \quad (31)$$

Which is also congruent with Wilson's framework as it comparable to the total flow spatial interaction model (e.g., Equation 2.11 in Cliff, Martin, and Ord (1974)). Hence, our totally-constrained opportunity accessibility model is:

$$V_i^T = k^D \sum_j W_j^{(2)} f(c_{ij}) = \sum_j W_j^{(2)} \frac{D \cdot f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

Table 5: Simple system: total opportunity constrained accessibility.

Origin	V_i^T		
	$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
	units: <i>physicians</i>	units: <i>physicians</i>	units: <i>physicians</i>
1	172.065	172.672	164.080
2	131.627	132.247	160.692
3	186.308	185.081	165.228

Further, we can see that, since D and $W_j^{(2)}$ are both in units of opportunities, the following term is dimensionless:

$$\kappa_{ij}^D = \frac{D \cdot f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

and therefore V_i^T is now in the units of $W_j^{(2)}$, that is, the mass at the destination ($V_i^T = \sum_j W_j^{(2)} \kappa_{ij}^D$). The role of κ_{ij}^D in this reformulation of accessibility is to adjust the number of opportunities accessible from i so that they represent a proportion of the total number of opportunities in the region. Constant κ_{ij}^D then assigns opportunities in proportion to the impedance between i and j . For this reason, we refer to it as a proportional allocation factor.

Referring to our simple numeric example, proportionality constant for $f_1(c_{ij}) = 1/c_{ij}^3$ would then be:

$$\begin{aligned} k^D &= \frac{D}{\sum_i \sum_j O_j f(c_{ij})} \\ k^D &= \frac{D}{\frac{D_1}{c_{11}^3} + \frac{D_1}{c_{21}^3} + \frac{D_1}{c_{31}^3} + \dots + \frac{D_3}{c_{31}^3} + \frac{D_3}{c_{32}^3} + \frac{D_3}{c_{33}^3}} \\ k^D &= \frac{490}{0.75} \\ k^D &= 653.3256 \end{aligned}$$

Using the calculated proportionality constants for all zones, the total opportunity constrained accessibility values for all zones and different travel behaviour scenarios is presented in Table 5.

In contrast to unconstrained accessibility, imposing a constraint -in this case a total opportunity constraint- allows for the comparison of differences and ratios. Each value is effectively in units of physicians, with the impedance units already accounted for by κ_{ij}^D .

Consider the highest decay scenario ($f_1(c_{ij})$), zone 1 again captures an intermediate amount of physicians (172.0652825). However, we can say that this is out of the 490 in the region, which allows us also to deduce that zone 1 captures 1.3072213 and 0.9235529 times more than zone 2 and 3. Values for the lesser decay ($f_2(c_{ij})$) and lowest decay ($f_3(c_{ij})$) scenarios are calculated

separately, with decay scenario values also summing to equal 490 physicians accessible in the region. Between zonal comparisons can be more interpretably made for these travel behaviour scenarios as well.

One can also directly compare values at a specific zone due to the consistent units. For instance, zone 1 remains intermediate in capturing accessible physicians relative to zones 2 and 3 across scenarios, similar to the unconstrained case. However, the difference between travel behaviour scenarios differ in direction. Specifically, Zone 1 captures 0.6067946 more and 7.9850478 fewer opportunities than the lesser decay scenarios $f_2(c_{ij})$ and $f_3(c_{ij})$ respectively. Why? k^D ensures proportional allocation for each travel behaviour scenario. Meaning, while the unconstrained accessibility increases, k^D adjusts the values to remain proportional to the total number of opportunities (490 physicians accessible in the region). As the decay behaviour decreases, more opportunities are accessible for all zones. In the medium decay scenario $f_2(c_{ij})$, zone 1 sees a slight increase in values (relative to the highest decay scenario) as the zone can access more opportunities relative to increases seen in other zones. However, in the lowest decay scenario, zone 1 sees a decrease, as it is outpaced by increases in other zones - namely zone 2 (recall: zone 2 has the lowest number of opportunities, hence the increases in opportunity gains is much higher in a low decay scenario).

Using the total opportunity constrained formulation of accessibility offers a solution to the unit interpretability issue of (han1959?)'s accessibility measure. Intuitively, the use of the constraint illustrates how the differences and ratios of values between zones and decay groups can be compared.

Totally-constrained population: market potential

Effectively transposing i and j , this case of the totally-constrained accessibility measure (Equation 33) is an expression of the concept of market potential (i.e., potential users) as proposed in C. D. Harris (1954; R. W. Vickerman 1974). The unconstrained form of market potential has been used in recent research (e.g., Gutiérrez 2001; Holl 2007; Condeço-Melhorado and Christidis 2018) to express the potentially accessible population (i.e., users) as a result of regional transportation infrastructure investment projects. To formulate this case, the total constraint may instead be applied to the mass of the *population* at i instead of the opportunities at j .

$$M_{ij}^T = k^O W_i^{(1)} f(c_{ij}) = k^O M_{ij}^0 \quad (32)$$

with:

$$M_j^0 = W_i^{(1)} f(c_{ij})$$

The population-constrained accessibility (market potential) becomes:

Table 6: Simple system: total opportunity constrained accessibility.

Destination	M_i^D		
	$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
	units: <i>population</i> <i>in 10,000s</i>	units: <i>population</i> <i>in 10,000s</i>	units: <i>population</i> <i>in 10,000s</i>
1	5.018	5.447	6.598
2	8.596	7.986	6.717
3	6.386	6.567	6.684

$$M_j^T = \sum_i M_{ij}^T = k^O \sum_i W_i^{(1)} f(c_{ij}) \quad (33)$$

The constraint that we impose in this case is the total market potential equals the total population O in the region :

$$\sum_j M_j^T = \sum_i \sum_j M_{ij}^T = O \quad (34)$$

Substituting Equation 33 in Equation 34, and solving for k , we obtain:

$$k^O = \frac{O}{\sum_i \sum_j M_{ij}^T} = \frac{O}{\sum_i \sum_j W_i^{(1)} f(c_{ij})} \quad (35)$$

The constrained market potential then takes the following form:

$$M_i^T = k \sum_j W_j^{(1)} f(c_{ij}) = \frac{O}{\sum_i \sum_j M_{ij}^T} \sum_j W_j^{(1)} f(c_{ij})$$

Returning back to the numerical example, the proportionality constant would be solved for each travel behaviour scenario, and the market potential of each zone is expressed as units of population (e.g., the number of people accessible from each zone) in Table 6. Readers should not, zonals trends in accessible population and accessible physicians. in the preceding subsection (i.e., calculated using totally-constrained opportunity) are not the same: zone 1, 2, 3 represent destinations and the accessibility values represent the number of accessible people from the vantage of physicians.

Singly-constrained accessibility

Increasing in restrictiveness relative to the totally-constrained accessibility measure, a single constraint proportionally adjusts the origin-destination zonal accessibility values based on the population or opportunities in the zone for all zones, while still maintaining the total system population or opportunity constraint, depending on the case. We define two cases: a *singly-constrained opportunity* accessibility case and a *singly-constrained population* accessibility case.

Singly-constrained opportunity: spatial availability(?)

If additional information is introduced into our analysis, say, the number of opportunities *by destination* (i.e., D_j), we can impose the following constraint (Equation 36)- comparable to the single constraint (Equation 15) from Wilson's framework. This constraint implicitly maintains the total system constraint (Equation 30).

$$\sum_i V_{ij}^D = D_j \quad (36)$$

The underlying spatial interaction model is now the attraction-constrained model in Equation 19, and our accessibility measure becomes:

$$V_i^D = \sum_j B_j D_j W_i^{(1)} f(c_{ij}) \quad (37)$$

where $W_i^{(1)}$ is a measure of the mass at origin i (i.e., the opportunity-seeking population). The corresponding balancing factor, as per Wilson, is:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \quad (38)$$

If we introduce this balancing factor in Equation 37 we obtain:

$$V_i^D = \sum_j D_j \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \quad (39)$$

Further, we define the following proportional allocation factor:

$$\kappa_{ij}^D = \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \quad (40)$$

After this, it is possible to rewrite Equation 39 as:

$$V_i^D = \sum_j \kappa_{ij}^D D_j \quad (41)$$

Soukhov et al. (2023) have shown that the role of κ_{ij}^D is to allocate opportunities D_j proportionally to the mass at each origin i and the impedance between i and j . As before, κ_{ij}^D is dimensionless and V_i^D is in the units of opportunities D_j . The singly-constrained accessibility measure in Equation 41 is called spatial availability by Soukhov et al. (2023), because it represents the number of opportunities that can be reached *and* are available, in the sense that there is no competition for them. These authors also show that the following expression (accessibility per capita) is a constrained version of the popular two-stage floating catchment area measure of Q. Shen (1998b) and Luo and Wang (2003):

$$v_i^D = \frac{V_i^D}{W_i^{(1)}}$$

As an additional point, Equation 36 also implies that the total constraint (e.g., $\sum_i V_i^D = \sum_i \sum_j V_{ij}^D = D$) is maintained.

Due to the constraints, $\frac{V_i^D}{O}$ can be interpreted as the proportion of opportunities accessible *and* available to location i out of the total number of opportunities in the system.

Returning to the simple numeric example, the opportunity-constrained case would yield the following B_j for $f_1(c_{ij})$:

$$\begin{aligned} B_j &= \frac{1}{\sum_i O_i f(c_{ij})} \\ B_1 &= \frac{1}{\frac{4}{10^3} + \frac{10}{30^3} + \frac{6}{15^3}} = 162.6506 \\ B_2 &= \frac{1}{\frac{4}{30^3} + \frac{10}{10^3} + \frac{6}{25^3}} = 94.9474 \\ B_3 &= \frac{1}{\frac{4}{10^3} + \frac{10}{25^3} + \frac{6}{10^3}} = 93.9850 \end{aligned}$$

The balancing factors B_j for the $f_2(c_{ij})$ decay group for zones 1, 2 and 3 is 12.8571429, 8.7685113 and 10.6635071, and for $f_3(c_{ij})$ decay group is 0.0672461, 0.0660559 and 0.0663798. Using these these proportionality constants, we can calculate the singly-constrained opportunity accessibility:

Similar to the accessibility differences and between pairs of zone using the two different impedance functions:

$$[1] \quad 33.31261$$

Table 7: Simple system: singly-constrained opportunity accessibility.

Origin	V_i^D		
	$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
	units: <i>physicians</i>	units: <i>physicians</i>	units: <i>physicians</i>
1	133.469	122.255	98.848
2	166.781	185.096	241.877
3	189.750	182.650	149.275

[1] 62.8411

[1] 143.0296

[1] 1.249591

[1] 1.514018

[1] 2.44697

This illustrates how the differences and ratios are now meaningful.

The total accessibility compared to the number of opportunities:

```
# A tibble: 1 x 3
  V_opp_1 V_opp_2 V_opp_3
  <dbl>   <dbl>   <dbl>
1     490     490     490
```

```
0
1 490
```

This shows how the number of opportunities is preserved.

And the ratio of accessibility to population compared to the ratio of opportunities to population:

```

# A tibble: 3 x 8
  oid   V_opp_i_1 V_opp_i_2 V_opp_i_3   O_i v_opp_1 v_opp_2 v_opp_3
<chr>   <dbl>   <dbl>   <dbl> <dbl>   <dbl>   <dbl>   <dbl>
1 1         133.    122.    98.8     4    33.4    30.6    24.7
2 2         167.    185.   242.    10    16.7    18.5    24.2
3 3         190.    183.   149.     6    31.6    30.4    24.9

# A tibble: 1 x 3
  v_opp_1 v_opp_2 v_opp_3
  <dbl>   <dbl>   <dbl>
1   24.5    24.5    24.5

PPR
1 24.5

```

Singly-constrained population: market potential

Similar to Equation 35, if the origins and destinations are transposed, we can define a *singly-constrained* measure of market potential that preserves the population. In its per-capita expression, i.e., equivalent to 2SFCA, has been used to express “facility crowdedness” as in F. H. Wang (2018).

The underlying spatial interaction model is now the production-constrained model in Equation 17, and our market potential measure M_i^P becomes:

$$M_j^P = \sum_i A_i O_i W_j^{(2)} f(c_{ij}) \quad (42)$$

In this case, the measure is constrained by the population *by origin* (i.e., O_i), like Equation 15 from Wilson’s framework:

$$\sum_j M_{ij}^P = O_i \quad (43)$$

And the corresponding balancing factor, as per Wilson, is:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \quad (44)$$

Following the same logic as in the preceding section on totally-constrained population accessibility (market potential), one arrives at the following expression:

$$M_j^P = \sum_i \kappa_{ij}^O O_i \quad (45)$$

with:

$$\kappa_{ij}^O = \frac{W_j^{(2)} f(c_{ij})}{\sum_i W_j^{(2)} f(c_{ij})} \quad (46)$$

As well, the constraint in Equation 43 ensures that the the total constraint (e.g., $\sum_j M_j^P = \sum_i \sum_j M_{ij}^P = O$) is maintained.

With these constraints, $\frac{M_j^O}{O}$ can be interpreted as the proportion of the total population serviced by location j .

Doubly-constrained accessibility

Readers will have noticed that the accessibility measures above have used either O_i or D_j but not both. When D_j , the opportunities, were used to constrain Equation 37, the mass of the population at the origin was given by $W_i^{(1)}$ (also in Equation 38). Contrariwise, When O_i , the population, was used as a constraint in Equation 42, the mass at the destination was given by $W_j^{(2)}$ (also in Equation 44). The reason is that the population and the opportunities are typically different things without a common metric. For example, on the side of the population we usually have people, but on the side of the opportunities we can have physicians, clinics, grocery stores, green space, schools, libraries, and so on. There are a few cases where there might be a one-to-one relationship between the population and the opportunities, with the most common one being employment, i.e., one person, one job.

A doubly-constrained approach can be implemented when such one-to-one relationship between population and opportunities. In this case, the two constraints in Equation 14 and Equation 15 are imposed. Furthermore, the two constraints must match:

$$\sum_i O_i = \sum_j D_j$$

This is because now the proportionality constants are mutually dependent and cannot be estimated if the constraints diverge. For this reason, the applicability seems to be relatively limited, because even in the case of jobs, there might be imbalances between O and D , say, where there are more jobs than people or vice-versa.

Table 8: Simple system with three zones. Population is in 10,000 persons and opportunities in 10,000s physicians.

ID	Population ¹	Opportunities ²
1	4	7
2	10	6
3	6	7

¹Population is $W_i^{(1)}$ when used as a proxy for the mass at the origin, and O_i when used as a constraint.

²Opportunities is $W_j^{(2)}$ when used as a proxy for the mass at the destination, and D_j when used as a constraint.

Another important point to note about the doubly-constrained approach is as follows. Recall that accessibility is defined as “the potential for interaction”. Additional constraints leave less room for that potential. In fact, a doubly constrained approach essentially converges on the spatial interaction model, where one person represents one trip to one opportunity, and it is probably better thought of as an estimate of “access” instead of “accessibility”.

Calibration of the two sets of proportionality constants is accomplished by means of iterative proportional fitting, whereby the values of A_i are initialized as one for all i to obtain an initial estimate of B_j . The values of B_j are used to update the underlying trip matrix, before calibrating A_i . This process continues to update A_i and B_j until a convergence criterion is met (see Ortúzar and Willumsen 2011, 193–95).

Returning to the simple numeric example

NOTE: The example needs to be revised in this case to have the population and opportunities match, as in, one person-one opportunity

In this case, V_{ij} would be:

Checking production, attraction balancing:

Production: 20

Attraction: 20

Production, attraction balancing OK.

Iteration: 1

Ai: 107.57 149.092 105.73

Bj: 1.485 0.646 1.166

Iteration: 2

Ai: 77.178 208.992 87.008

Bj: 1.849 0.468 1.338

Iteration: 3

Ai: 63.197 257.101 74.725

Bj: 2.08 0.383 1.454

Iteration: 4

Ai: 56.631 286.517 68.316

Bj: 2.202 0.345 1.514

Iteration: 5

Ai: 53.669 301.346 65.36

Bj: 2.26 0.328 1.542

Iteration: 6

Ai: 52.382 308.064 64.08

Bj: 2.285 0.321 1.553

Iteration: 7

Ai: 51.836 310.958 63.541

Bj: 2.296 0.318 1.558

Iteration: 8

Ai: 51.607 312.176 63.317

Bj: 2.301 0.317 1.56

Iteration: 9

Ai: 51.511 312.685 63.224

Bj: 2.303 0.317 1.561

Stopping Condition: Error threshold met

Final Error: 0.068%

Checking production, attraction balancing:

Production: 20

Attraction: 20

Production, attraction balancing OK.

Iteration: 1

Ai: 9.278 12.662 9.033

Bj: 1.329 0.717 1.1

Iteration: 2
Ai: 7.576 15.224 7.988
Bj: 1.459 0.612 1.166

Iteration: 3
Ai: 7.017 16.351 7.524
Bj: 1.508 0.575 1.194

Iteration: 4
Ai: 6.823 16.775 7.353
Bj: 1.526 0.562 1.204

Iteration: 5
Ai: 6.757 16.925 7.293
Bj: 1.532 0.558 1.207

Iteration: 6
Ai: 6.734 16.977 7.273
Bj: 1.534 0.556 1.208

Iteration: 7
Ai: 6.726 16.994 7.266
Bj: 1.535 0.556 1.209

Stopping Condition: Error threshold met
Final Error: 0.036%

Checking production, attraction balancing:
Production: 20
Attraction: 20
Production, attraction balancing OK.

Iteration: 1
Ai: 0.066 0.067 0.066
Bj: 1.01 0.991 0.997

Stopping Condition: Error threshold met
Final Error: 0.025%

The A_i and B_j matrices:

[,1]

```
[1,] 51.51130
[2,] 312.68463
[3,] 63.22359
```

```
      [,1]      [,2]      [,3]
[1,] 1.5339 0.5563556 1.20816
```

The trip matrix (notice, all summed up opportunities and populations are equivalent to our knowns)

Nice. But notice how the accessibility is simply the number of opportunities at the destination! In other words, it is simply the access.

Discussion

when to use what, their embedded assumptions, and dangers of using one over the other

Wu and Lev say comp. can be considered in the impeded function, but no details. we argue its' more. unconstrained - no comp, – constraints consider competition - total - regionally; not fixing it on what zones will be matched, it's just regional, - singly- with zone and regional, - doubly- per origin and destination zone.

A sort of 'accessible' provider to population ratio, or accessible opportunities per capita, can also be easily calculated by dividing each constrained value by the number of opportunities.
!!!

A note on interpretation and possible applications

When to use each of these measures?

Totally-constrained

Singly-constrained

Doubly-constrained

Conclusions

Accessibility development has a rich history. There have been developments in the cases/breath of travel impedance functions, competition considerations, and in incorporating individual choice as part of activity-based accessibility. However, no work yet has examined accessibilities explicit connecting to SIM. We examine this connection, and provide a origin-destination (i to j) formulation of accessibility as part of the SIM framework. Novel stuff.

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