A family of accessibility measures (bringing balance back?)

Introduction

Accessibility, a central concept in transport geography, planning, and engineering, is conceptually related to human mobility and the opportunity landscape. In simple terms, it is defined as the "potential of opportunities for interaction" (Hansen 1959). The range of opportunities considered in accessibility analysis has grown to include employment [REFS], health care [REFS], green spaces [REFS], schools [REFS], social contacts [REFS], emergency services [e.g., shelters, REFS], and many more. In other words, accessibility analysis is today used to understand the potential to reach opportunities that are important to people (Ferreira and Papa 2020). Compared to earlier measures of performance used in transportation (e.g., VKT, PKT, etc.) that benchmark the amount of movement, accessibility brings a more holistic understanding of transportation and land use systems combined (S. L. Handy and Niemeier 1997). However, the focus of transportation planning has historically prioritized mobility and treated accessibility as a by-product of movement, leading to problematic consequences (S. Handy 2020). With the rise of an automobility mono-culture (H. J. Miller 2011; Lavery, Páez, and Kanaroglou 2013), recent decades of mobility-based practice (e.g., road and highway expansion) have been marked by increased travel cost (time, effort, energy, financial cost), often with limited impact on accessibility levels (e.g., S. Handy 2002; Páez et al. 2010). In response, transportation planning practice has begin to adopt accessibility planning approaches, via accessibility planning tools (Silva et al. 2017; Paez et al. 2013) or through the traditional four-step transportation demand model (Ortúzar and Willumsen 2011). However, institutional and conceptual barriers remain, many of which relate to the diversity of accessibility definitions (S. Handy 2020) and interpretability and communicability of the outputs (Karst T. Geurs and Wee 2004; Ferreira and Papa 2020).

In adopting the perspective that accessibility is an evolution of mobility conceptually, this paper aims to re-unify accessibility measures with its spatial interaction-based roots. It proposes to do so by mapping accessibility onto Wilson's family of spatial interactions models (Wilson 1971). In this, we aim not only to clarify accessibility's interpretation as a measure of performance, but also to encourage future developments that draw from its gravity modelling siblings, and spatial interaction history. To this end, we begin by tracing the development of spatial interaction from Ravenstein (1889) to its split into accessibility research from Hansen (1959) to today. After this, we derive different types of accessibility measures, following the family of gravity models namely: unconstrained, total constrained, origin- and destination- constrained, composite destination-constrained, and doubly constrained. Then, we will provide empirical examples of the types of measures. Finally, we will discuss the use of each measure and broadly the implications of ignoring the role of constrains in accessibility analysis.

Background

The inquiry into spatial interaction traces to Newton's Universal Gravitation

The patterns of people's movement in space have been a subject of scientific inquiry for centuries. From as far back as Henry C. Carey's *Principles of Social Science* (Carey 1858) there was already a concern with the scientific study of human interaction over space. It is in this work where Carey states that "man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance" (McKean 1883, 37–38), which shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton's Law of Universal Gravitation posited in *Principia Mathematica* of 1687 and expressed in 1.

$$F_{ij} \propto \frac{M_i M_j}{D_{ij}^2} \tag{1}$$

To be certain, the expression above, a proportionality, is one of the most famous in all of science. In brief, it states that the force of attraction F between a pair of bodies i and j is directly *proportional* to the product of their masses M_i and M_j, and inversely *proportional* to the square of the distance between them D_{ij}. Direct proportionality only means that as the product of the masses increases, so does the force. Likewise, inverse proportionality only means that as the distance increases, the force decreases. To quantify the magnitude of the force, an empirical factor, say G, is required to convert the proportionality into an equality, which ensures that the force F matches empirical observations of the force of attraction between masses. Ultimately, the equation for the force is as seen in Equation 2, where G is a proportionality constant obtained empirically:

$$F_{ij} = G\frac{M_i M_j}{D_{ij}^2} \tag{2}$$

Newton's initial estimate of G was based on a speculation that the mean density of earth was between five or six times that of water, an assumption that Hutton's experiments of 1778 validated (Hutton 1778, 783). Still, it took over a century from the publication of *Principia* to refine the estimate of this constant to within 1% accuracy, with Cavendish's 1798 experiment (Cavendish 1798).

Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Following Carey's *Principles* of 1858, research into human spatial interaction continued in different contexts. In the late 1880s Ravenstein proposed his "Laws of Migration", based on his empirical analysis of migration flows in various countries (Ravenstein 1885, 1889). In these works, Ravenstein posited a directly proportional relationship of migration flows with the size of the destination (i.e., centres of commerce and industry), and an inversely proportional relationship between the size of flows and separation between origin and destination. These propositions are similar to Newton's gravitational laws. Over time, other researchers discovered similar relationships. For example, Reilly et al. (1929) formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations. Later, Zipf proposed a $\frac{P_1 P_2}{D}$ hypothesis for the case of goods movement by railways Zipf (1946c), intercity personal movement Zipf (1946b),and information Zipf (1946a). The $\frac{P_1 P_2}{D}$ hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

A common theme among these early researchers of human spatial interaction is that a proportionality constant similar to G in Equation 2 is missing. Of the researchers cited, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly's hypothesis was expressed as follows:

$$B_a = \frac{(P_a \cdot P_T)^N}{D_{aT}^n} \tag{3}$$

Where B_a is the amount of business drawn to a from T, P_a and P_T are the populations of the two settlements, and D_{aT} is the distance between them. Quantity N was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of $n = 2$.

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

$$C^2 = \frac{P_1 \cdot P_2}{D} \tag{4}$$

Where C is the volume of goods exchanged between 1 and 2, P_1 and P_2 are the populations of the two settlements, and D_{12} is the distance between them.

After Carey, it is in Stewart's work on the principles of demographic gravitation that we find the strongest connection yet to Newton's law (Stewart 1948). We suspect that this may relate to academic backgrounds; Stewart was a physicist, whereas Ravenstein, Reilly, and Zipf were social scientists. Besides awareness of preceding research, as he cites Reilly and Zipf as predecessors in the analysis of human spatial interaction, Stewart appears to have been the first author to express his theorized relationships for human spatial interaction with a proportionality constant G, as follows:

$$F = G\frac{(N_1\mu_1)(N_2\mu_2)}{d_{12}^2} \tag{5}$$

where:

- F is the *demographic force*
- N_1 and N_2 are the numbers of people of in groups 1 and 2
- μ_1 and μ_2 are so-called *molecular weights*
- d_{12}^2 is the distance between 1 and 2
- And finally G, a constant that Stewart "left for future determination" (1948, 34)

In addition to demographic force, Stewart defined a measure of "population potential" of 2 with respect to 1 as follows:

$$V_1 = G\frac{M_2}{d} \tag{6}$$

For a system with more than 2 population bodies, Stewart formulated the population potential at i as:

$$V_i = \int \frac{D}{r} ds \tag{7}$$

where D is population density over an infinitesimal area ds, and r is the distance to i. In more contemporary terms, this can be rewritten as:

$$V_i = \sum_j \frac{M_j}{d_{ij}} \tag{8}$$

which astute readers will identify as our modern definition of accessibility.

Stewart's formulation of demographic force, developed in the context of what he called "social physics" (Stewart 1947), was problematic. It had issues with inconsistent mathematical notation. For example, e.g., in Stewart (1948), G is used as a proportionality constant for both *demographic force* and *demographic energy* on p. 34. Perhaps more seriously, Stewart's work was permeated by a view of humans as particles following physical laws, but with ideas that were afflicted by unscientific racism. For instance, the molecular weight μ was presumed to be one for the average American, but "presumably…much less than one….for an Australian aborigine" [p. 35]. Stewart's ideas about "social physics" fell out of favour among social scientists, but not before influencing the emerging field of accessibility research.

In summary: from the 1850s to the 1940s, researchers attempted to theoretically and empirically characterise human spatial interaction as some force of attraction F that is directly proportion to the masses M of a pair of locations (i and j) and inversely proportional by some sort of separation distance D_{ij}. Those that captured the relationship empirically, approached the concept with different expressions, some including a proportionality constant and some not, but all tying back to the Newtonian gravity analogy.

Hansen's gravity-based accessibility to today

From Stewart (1948), we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature (Hansen 1959). In his seminal paper, Hansen defined accessibility, i.e., "the potential of opportunities for interaction" [p.73], as "a generalization of the population-over-distance relationship or 'population potential' concept developed by Stewart (Stewart 1948)" [p.73]. As well as being a student of city and regional planning at Massachusetts Institute of Technology, Hansen was also an engineer with the the Bureau of Roads, and likely preoccupied with applied research into the power of transportation to shape land uses. Hansen (1959) focused on Stewart (1948)'s *population potential* (expressed in Equation 8), leaving Stewart's other formulaic contributions and objectionable aspects of "social physics" behind. Hansen (1959) recast Equation 8 to reflect accessibility, a model of human behaviour useful to capture regularities in mobility patterns . In this equation, Hansen (1959) replaced M_j with *opportunities* to derive an *opportunity potential*, or more accurately, a *potential of opportunities for interaction* as follows:

$$S_i = \sum_j O_j \cdot d_{ij}^{\beta} \tag{9}$$

Alternatively, a more general form of this expression to account for different impedance functions:

$$S_i = \sum_j O_j \cdot f(d_{ij}) \tag{10}$$

S_i in Equation 9 is a measure of the accessibility at i, whereas O_j is the size of the activity at j and d_{ij} is the distance or travel time between i and j with β describing the effect size. Today, Hansen is is frequently cited as the father of modern accessibility analysis (e.g., Reggiani and Martín 2011), and Hansen-type accessibility is commonly referred to today as the gravity-based accessibility measure.

Of note, however is that between Stewart (1948) and Hansen (1959) the proportionality constant G in Equation 6 vanished. There is some evidence that Hansen (1959) was aware of the importance of this constant as he wrote about *direct* and *inversely proportional* relationships between opportunities in one area, opportunities at another, and their separation distance. Furthermore, those reading Hansen (1959) should remember that Stewart (1948) set the proportionality constant G to 1, with a note that"G [was] left for future determination: a suitable choice of other units can reduce it to unity".

Since Hansen (1959), accessibility analysis has been widely employed across planning disciplines (Koenig 1980; Karst T. Geurs and Wee 2004; E. J. Miller 2018; Wu and Levinson 2020). However, to our knowledge, the proportionality constant has remained forgotten, with no notable developments to explicitly determining it. In this way, G continues to be implicitly set to 1 even when the fundamental relationship in accessibility is proportionality (e.g., $ S_{i} G *j O_j f(d*{ij})$ and not equality (for instance, see the formula for accessibility at the top of Figure 1 in Wu and Levinson 2020). Without a proportionality constant, however, the units of S_i remain unclear: the unit of 'potential of opportunity for interaction' is left free to change as β is calibrated for the population center with no discussion of a constant to balance the units.

However, it has been raised that there is a difficulty in interpreting gravity-based accessibility's meaning as anything other than a relative index of potential interaction (higher accessibility/lower accessibility) (E. J. Miller 2018). Interpretability issues plague this type of analysis, as implications beyond high-or-low access hotspots are often desired. What does the accessibility value mean? Difficultly in interpreting the output's tangible meaning contributes to resistance to its application in practice (Ferreira and Papa 2020). Furthermore, The majority of accessibility studies within the past few decades develop on the breath and geographic case study selection of the opportunity O_j studied and potentially impacted population []; the accuracy of parameters that capture distance-decay $f(c_{ij})$ []; and additions to the opportunity O_j attributes such as competitive demand considerations (Stouffer 1960; Jörgen W. Weibull 1976; Q. Shen 1998a; Allen and Farber 2020; Soukhov et al. 2024). Overall, due to the diverse range of questions accessibility measures have been applied to help clarify, a plethora of variations in Equation 10 are present in the literature. These variations also compounds the issue of interpretability across studies.

A testament to this inconsistent interpretation are the unsettled classifications generated by influential review works of the accessibility literature; for instance, S. L. Handy and Niemeier (1997) classifies studies into gravity- and utility-based measures, Karst T. Geurs and Wee (2004) introduces further classification by expanding categories into potential- (or gravity), utility-, location-, and person- based measures with different components. Curtis and Scheurer (2010) further classifies accessibility measures into spatial separation, contour-, gravity-, competition-, time-space, utility-, and network- based measures (Curtis and Scheurer 2010), though methodologies may clearly overlap. Broader approaches have also been taken: Paez, Scott, and Morency (2012) categories indicators on how they have been normatively and positively applied, and Levine (2020) reviews accessibility's evolution along definition-based and application-based dimensions throughout the years.

Among these reviews, we find Wu and Levinson (2020)'s contribution compelling for unifying accessibility measures: they demonstrate that most or all of these concepts can be unified into its main features: the relationship between travel-cost and corresponding reachable opportunities. Indeed, these authors demonstrate how previous classifications are special cases of travel-cost or opportunities constraints. From the initial Newtonian analogy, we interpret Wu and Levinson (2020)'s work as offering a positive contribution by circling back to re-unify mass M and separation distance D; however, an important component is still missing, the proportionality constant G.

As such, this paper's focus is on the proportionality constant. We wish to demonstrate the importance of transiting from proportionality to equality in Equation 10, in order to balance the units of accessibility into more interpretable values with clear meanings. To address this matter, we need not reinvent the wheel: instead we refer to the spatial interaction model as developed by Wilson (1971). In essence, in what follows, we reunite the accessibility indicators literature with the spatial interaction literature as developed by Wilson (1971) based on entropy maximization concepts. Although Wilson's approach is based on a different conceptual foundation than the old reference to Newtonian gravity, the work succeeded at identifying the steps from proportionality to equality to yield variations of proportionality constants: including the one that eluded Stewart (1948) and has been ignored in almost all subsequent accessibility research.

Introducing a family of accessibility measures

Gravity-based accessibility's cousin: Wilson's spatial interaction model

Today's gravity-based accessibility measures (Equation 10) share the same early 20th century Newtonian-based origins as the model's of Reilly, Zipf, and Stewart, versions of which continue to be extensively used (Ortúzar and Willumsen 2011; Wilson 1971). However, in some ways the gravity model and spatial interaction continued to evolve independently from Equation 10, the former being developed into formalized transportation demand modelling practice to 'predict-and-provide', and the later taken up to describe land-use and transportation interactions by geography and social science-adjacent researchers and planners. From the perspective of gravity-based accessibility measure, this paper's focus is on the proportionality constant of the spatial interaction model, which takes the following form after Wilson (1971):

$$T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij}) \tag{11}$$

Where T_{ij} represents a n x m matrix of flows between an n number of origins i and an m number of destinations j, $W_i^{(1)}$ and $W_j^{(2)}$ is a vector of mass attributes representing origin i and destination j respectively, and $f(c_{ij})$ is a n x m matrix representing some function of travel cost c_{ij} which reflects travel impedance. In other words, T_{ij} explicitly measures *interaction* as a unit of trips and the single proportionality constant k ensures the total sum of T_{ij} represents the total flows in the data. It should be noted that the early analogy with the gravitational law proved too simplistic, and improvements to the gravity model (Equation 11) are the use of total trip ends (number of originating trips from zone i are O_i and ending trips at zone j are D_j) instead of total populations and separation distance calibrated by a power n (such as P_i, P_j, and n in Equation 4, Equation 3 and Equation 5). Finally, and of most important note in this paper, k is a scale parameter that makes the overall equation proportional to the rate characteristic of the modeled phenomenon.

Traditionally, the development of the gravity model has put an emphasis on the interpretability of the resulting units (Kirby 1970; Wilson 1967, 1971). But instead of relying on the heuristic Newtonian gravity analogy, entropy maximisation has emerged as a popular model building approach taken in the spatial interaction studies to generate the gravity model (Ortúzar and Willumsen 2011). Namely, it is mathematically rigorousness and offers a sensible interpretation of the resulting model as a statistical averaging procedure (Wilson 1971; Senior 1979). As such, Equation 11 can be derived to account for additional information to include constraints that reflect known information about the system and is mathematically rigorous. The interaction variable T_ij is often in the unit of trips, so additional knowledge can be the total trip flows originating at each zone $W_i^{(1)}$ and the total trip flows terminating at zone $W_j^{(2)}$. These pieces of information can constrain T_{ij} as follows.

The total flow constraint: Equation 12, where the total number of flows in the region equals T:

$$\sum_i \sum_j T_{ij} = T \tag{12}$$

The production constraint: the sum of trips ending at each j is equal to the total flow originating from each i:

$$\sum_j T_{ij} = W_i^{(1)} \tag{13}$$

The attraction constraint: the sum of trips starting at each i is equal to the total flow ending at each j:

$$\sum_i T_{ij} = W_j^{(2)} \tag{14}$$

The entropy maximization derivation is demonstrated in detail elsewhere (e.g., Ortúzar and Willumsen (2011), Wilson (1967)). The basis of the approach is to accept that there are macro, meso, and micro states. In the case of trips being the interaction variable, a micro state is an individual trip maker, the meso state is the statistically representative zone representing a collection of individual trip makers, and the macro state is the entire study region. Using these assumptions, a set of T_{ij} that maximizes the number of micro states in each meso state subject to some sensible macro-state constraints (Equation 12, Equation 13, and Equation 14). From this approach, the gravity model can be interpreted as the given the total number of individual trips (micro level) statistically represented at origin and destination zones (meso level), the cost of travelling (micro level, represented at the meso level), and the fixed total cost of travelling in the region (number of trips and cost of travel at the macro level). There is then a most probable distribution of trips between zones (meso-states) that stasis all the macro-state constraints.

Using these constraints, the gravity model Equation 11 is distinguished into four general cases. These cases are presented in increasing order of constraint restrictiveness and continued to be useful in classifying the gravity model in trip distribution modelling to this day (Ortúzar and Willumsen 2011):

- The unconstrained case: neither Equation 12, Equation 13, or Equation 14 hold;
- The total constrained case: only Equation 12 hold
- The singly-constrained case: either Equation 13 holds (origin-constrained) or Equation 14 holds (destination-constrained);
- The doubly-constrained case: both Equation 13 and Equation 14 hold simultaneously

In each of these four cases, the single proportionality constant k is replaced by balancing factors A_i, B_j, and K as follows to ensure the case-specific constraints are satisfied:

$$A_i = \frac{1}{\sum_j B_j W_j^{(2)} f(c_{ij})} \tag{15}$$

$$B_j = \frac{1}{\sum_i A_i W_i^{(1)} f(c_{ij})} \tag{16}$$

$$K = \frac{T}{\sum_i \sum_j W_i^{(1)} W_j^{(2)} f(c_{ij})} \tag{17}$$

For the doubly-constrained case, A_i and B_j must be solved together along with K. For the singly-constrained cases, origin-constrained case replaces B_j with 1 for all j and A_i is Equation 15, and the the destination-constrained case replaces A_i with 1 for all i and B_j is Equation 16. For both singly-constrained cases, K also holds. For the total constrained case, A_i and B_j are set to 1 and K is the defining constraint. And for the unconstrained case, all other cases (the total constrained, singly-constrained and doubly-constrained cases) are simply constrained variations.

Accessibilities' development seperate from the family of gravity models

Upon detailing the gravity model, we can now discuss how accessibility has been developed within that field and how it has not. Gravity modellers (interaction variable being trips, a model of spatial interaction - mobility) continued to use accessibility within transportation demand modelling. As succinctly summarized in Morris, Dumble, and Wigan (1979), "Mobility and accessibility together influence an individual's capacity to travel in daily life". However, the gravity model literature that focuses on Hansen's accessibility is relatively scarce. To capture this sentiment, see Figure 1 which demonstrates the bibliometric coupling of papers that cites Hansen (1959), Wilson (1971), and both Hansen (1959) and Wilson (1971). Figure 1 is based on a SCOPUS search of papers that cite Hansen (1959)'s DOI (2,134 documents) and another SCOPUS search of papers that cite Wilson (1971)'s DOI (368 documents) conducted on November 9, 2024 and a bibliometric coupling matrix created using the {bibliometrix} R package (Aria and Cuccurullo 2017).

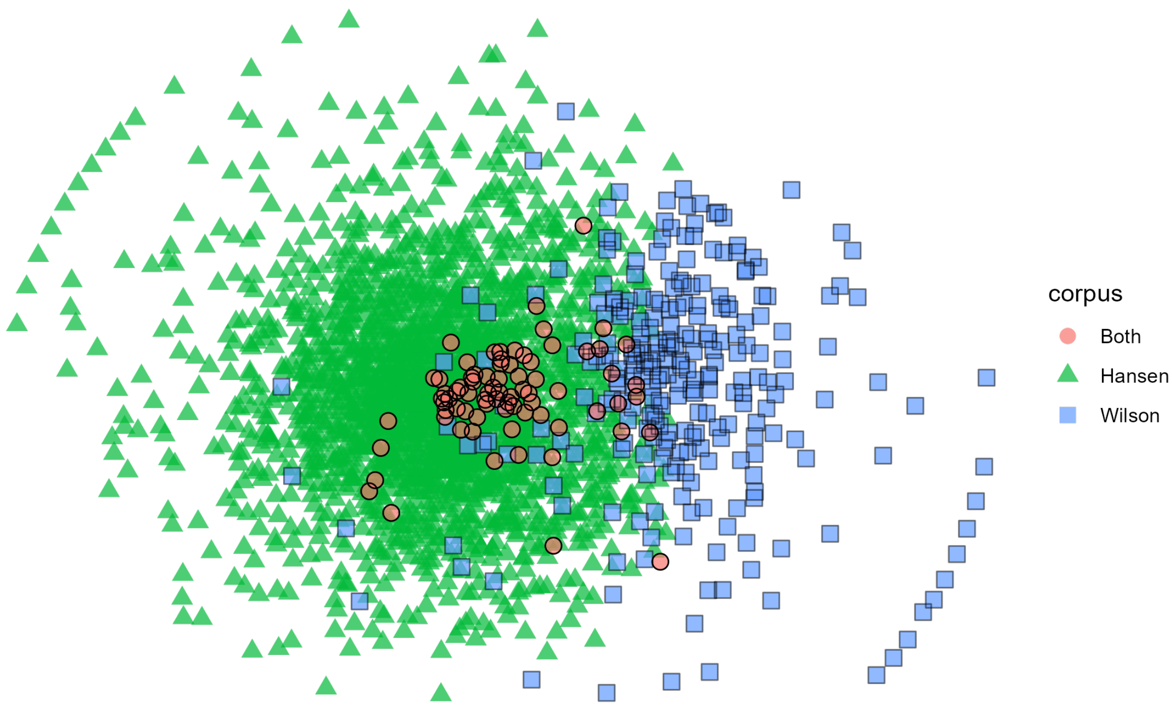

Figure 1: The bibliometric coupling of papers that cite Hansen (1959), Wilson (1971) and both Hansen (1959) and Wilson (1971).

Many of the papers that cite both Hansen (1959) and Wilson (1971) (77 journal articles) are transportation modelling papers working on integrating accessibility within the gravity model (mobility).

For instance, Batty and March (1976) states that the relationship between the gravity model and Hansen's original accessibility-potential model may be worth exploring… There is a consistent acknowledgement that these two models are related. Specifically, that the Hansen-type accessibility model is embedded within the balancing factors (Harris and Wilson 1978; G. Leonardi 1978; A. Stewart Fotheringham 1981; A. S. Fotheringham 1985). This is a "common sense" approach at integrating accessibility (Morris, Dumble, and Wigan 1979).

Though there are other approaches to consider accessibility with spatial interaction modelling beyond Hansen founded on micro-economic consumer behaviour (Morris, Dumble, and Wigan 1979). For instance, others explore entropy maximising solutions (Wilson's framework and hence the balancing factors defined as Hansen's accessibility) when considering demand behavior described by random utility choice models (Giorgio Leonardi and Tadei 1984).

Some work on location-allocation models also used both Hansen and Wilson. For instance, G. Leonardi (1978) and Beaumont (1981) define a location-allocation problem using the singly-constrained gravity model, and then defines one using Hansen-type accessibility as the objective function subject.

Then, A. S. Fotheringham (1985) demonstrated that destination's accessibility should be explicitly specified within Wilson's framework, citing Hansen measure as convenient formulation. Without the consideration for accessibility explicitly, unexplained spatial pattern remains. This later came to be the 'competition destination' model, and is one of a few techniques to address spatial variation in gravity models. (Oshan 2021). Implicitly, we gather that at this point: it was understood balancing factors in the gravity model could not be understood as capturing accessibility. We also gather that in this moment of growing complexity of spatial interaction modelling, accessibility branched off and sprouted in the medium of budding GIS technology and the emergence of space-time dimensions (H. J. Miller 1999; Occelli 2000). With the possibility for showcasing complex GIS methodologies for varied case studies, accessibility became mature enough to branch off on its own.

In papers after A. S. Fotheringham (1985), Hansen is typically cited as providing a definition of accessibility but how Wilson is cited changes through the years. Wilson is attributed for using different travel cost functions in depending on the context (J. W. Weibull 1980; S. L. Handy and Niemeier 1997; Kwan 1998; Q. Shen 1998b; Ashiru, Polak, and Noland 2003; Rau and Vega 2012; Pan 2013; Margarida Condeço Melhorado et al. 2016; Caschili, De Montis, and Trogu 2015; Grengs 2015; Pan, Jin, and Liu 2020; Chia and Lee 2020; Roblot et al. 2021; Sharifiasl, Kharel, and Pan 2023; Kharel, Sharifiasl, and Pan 2024).

Othertimes, Wilson and Hansen measures were progressively incorrectly conflated throughout the years (Giuliano et al. 2010; Grengs et al. 2010; Grengs 2010, 2012; Levine et al. 2012; Levinson and Huang 2012; Tong, Zhou, and Miller 2015; X. Liu and Zhou 2015; He et al. 2017; Wu and Levinson 2020; Ng et al. 2022; Naqavi et al. 2023; Suel et al. 2024) - indeed they both measure spatial interaction so are somewhat related, but they are not the same. They are also co-cited to refer to the concept of 'gravity' in models (S. Liu and Zhu 2004; Dai, Wan, and Gai 2017; Y. Shen 2019; Chia and Lee 2020)

And finally, Wilson's balancing factors are interpreted as inverse Hansen-type accessibility, especially in the context of competition (Vickerman 1974; Karst and Van Eck 2003; Karst T. Geurs, Wee, and Rietveld 2006; Willigers, Floor, and Wee 2007; El-Geneidy and Levinson 2011; Curtis and Scheurer 2010; Manaugh and El-Geneidy 2012; Chen and Silva 2013; Alonso et al. 2014; Albacete et al. 2017; Sahebgharani, Mohammadi, and Haghshenas 2019; Mayaud et al. 2019; Allen and Farber 2020; Levinson and Wu 2020; Marwal and Silva 2022; Su and Goulias 2023). This interpretation is fraut, upon considering A. S. Fotheringham (1985)'s demonstration, which many of these works do not contend with.

THE SPATIAL STRUCTURE DEBATE: Oshan (2021)

Some papers also use gravity models to estimate trip or some other interaction flows alongside accessibility indicators (Clarke, Eyre, and Guy 2002; Grengs 2004; Türk 2019) or integrate accessibility within the gravity model, in line with A. S. Fotheringham (1985)'s demonstration and most often in context with retailing (Beckers et al. 2022).

Hansen is a relatively straightforward simple application, while Wilon's contribution is methodological and dense - must be carefully read. Many takeaways can be drawn, so with accessibilities changing applications throughout the years – different things about Wilon's contribution becomes relevant. The most contemprary connection between the two is in the relam of competitive accessibility – in the interpretation of Wilson's balancing factors as a way to consider competition. We fit this contribution in this literature. Competition fits on a spectrum - by introducing balancing factors trips are *constrained* to equal regional or zonal totals. This same application can be applied to accessibility.

Some Additionally, trending research topics of the time such as space-time and more []

previously mentioned in accessibility development: the breadth of opportunity O_j that can be studied [] as well as the accuracy of parameters that capture distance-decay $f(c_{ij})$ []. Outside of these factors, accessibility literature that delves into competition, does not cite Wilson, with some exception… Soukhov for the balancing factor.

Overall, work that cities Wilson is closer related to transportation demand modelling than accessibility. However there are a few works of the 77 documents that are in the realm of accessibility. …

However there are a few papers that are accessibility focused that focus on

There are plenty of insights to be shared across these sibling branches; they are both spatial interaction models.

A family of accessibility measures

Instead of the interaction variable being in the unit of trips, it is in the unit of potential interaction with opportunities. This focus is the preoccupation of accessibility measures. The same analytical framework to define the balancing factors k can be used for the family of accessibility measures as in the family of gravity models, however with different interpretation. In this section, we explicitly define these cases.

In this context, accessibility is our *interaction* variable of choice, so T_{ij} in Equation 11 is replaced with V_{ij}. As the interaction variable represents *potential*, i.e., the number of potential reachable opportunities, as opposed to trips:

$$V_{ij} = kW_j^{(2)} f(c_{ij}) = kO_j f(c_{ij}) \tag{18}$$

Where V_{ij} consists of the same variables, however V_{ij} explicitly measures *potential interaction* as a unit of number of potentially reachable opportunities and the single proportionality constant k ensures the total sum of V_{ij} represents the total potential reachable opportunities in the data. Notably, the mass vectors $W_i^{(1)}$ and $W_j^{(2)}$ are replaced by P_i and O_j, representing the weight of the origin (population) and destination (opportunities), the weights common in accessibility literature. As accessibility is the sum of opportunities for a j from i, when the mass of the origin (P_i) is relevant, it enters Equation 18 through the k constant. Accessibility is also most often presented as a locational measure, so the value of V_i is of most interest, e.g., the number of potentially reachable opportunities at a zone:

$$V_i = k \sum_j O_j f(c_{ij}) \tag{19}$$

Like with Equation 11, we distinguish Equation 18 into four cases based on versions of the gravity-model building constraints (Equation 20, Equation 21, and Equation 22) to reflect macro-level knowledge, i.e., the sum of the number of potential reachable opportunities.

$$\sum_i \sum_j V_{ij} = V \tag{20}$$

Where the sum of all potentially reached opportunities in the region is equal to a total opportunities in the region V.

$$\sum_j V_{ij} = W_i^{(1)} = P_i \tag{21}$$

Where sum of all potentially reachable opportunities for all j zones in the region is equal to the production weight at each i in the region.

$$\sum_i V_{ij} = W_j^{(2)} = O_j \tag{22}$$

Where sum of all potentially reachable opportunities for all i zones in the region is equal to the attraction weight at each j in the region.

All four cases are differentiated like: - The unconstrained case: neither Equation 20, Equation 21, or Equation 22 hold; - The total constrained case: only Equation 20 hold - The singly-constrained case: either Equation 21 holds (origin-constrained) or Equation 22 holds (destination-constrained); - The doubly-constrained case: both Equation 21 and Equation 22 hold simultaneously

Like gravity model problem, we can solve these cases using spatial weights matrix and travel cost matrix, but with alternative meanings. P_i (our $W_i^{(1)}$) is population at i who seek opportunities. O_j (our $W_j^{(2)}$) are the opportunities being sought. For this simple example, we can say that the opportunities are the population's jobs. In our region, each person has a job - but we only know at an aggregate how many people are at each i, the jobs that are located at

j and the travel cost c_{ij}. We assume the the travel impedance ($f(c_{ij})$) is the associated travel propensity of $f(c_{ij}) = 1/C$.

The following is the known accessibility matrix. The V_{ij}, the number of jobs spatially accessible at i from destination j. The TOTALs are the V_i and V_j, the total number of jobs spatially available at i and the number of jobs spatially available from j.

```
        1   2   3 TOTAL
1      95  23  42   160
2      27 378  45   450
3      38  36 106   180
TOTAL 160 437 193     1
```

And the matrix of cost of travel from i (rows) to j (columns) is as follows:

```
   1  2  3
1  2 15  5
2 15  2 10
3  5 10  2
```

Unconstrained accessibility

This accessibility is standard practice. There is no explicit consideration of a proportionality constant (k). We can presume it is still set to 1 as Stewart (1947). No constraints hold, making the interpretation of the units of V_{ij} simply some magnitude of O_j weighted by some $f(c_ij)$, and V_i the sum of these values for each j. V_i reflects the magnitude of opportunity accessibility but it is not scaled to equal the number opportunities O and/or population P.

$$V_{ij} = O_j f(c_{ij}) V_i = \sum_j O_j f(c_{ij})$$

Each origin (row) all the opportunities at each destination are available. Likewise, each destination (column) offers all opportunities to each origin:

```
    1   2   3
1 160 437 193
2 160 437 193
3 160 437 193
```

The resulting matrix of V_{ij} along with the V_i and V_j totals for this example:

```
            1      2     3  TOTAL
1       80.00  29.13  38.6 147.73
2       10.67 218.50  19.3 248.47
3       32.00  43.70  96.5 172.20
TOTAL 122.67 291.33 154.4 568.40
```

We'd report origin 1 as having 148 accessibility (i.e., $V_{i=1} = 148$), origin 2 as 248 accessibility, and origin 3 as 172 accessibility. The total accessibility in the region is 568 ($\sum_i V_i = 568$), not equal to any meaningful value. This value is not often reported in the accessibility literature for this reason. One could also report the accessibility values that destinations offers (V_j), namely destination 1 offers 123, destination 2 offer 291, and destination 3 offers 154. However, these values are also not reported in the literature currently… as they aren't constrained to match up to the number of opportunities at each j (i.e., 160, 437 and 193). Assuming $k = 1$, V_i and V_j represent magnitudes of accessibility concepts, but in an unconstrained way. The population matrix is no incorporated in the V_{ij} calculation, so V_i cannot be directly compared to population. From a different perspective, V_j values also cannot be compared to known O_j values as the equation unconstrained.

OF NOTE: in the context of spatial interaction modelling, the unconstrained model is not often useful. While the resulting T_{ij} values are in units of 'trips' - the assumption of k equaling 1 is practically meaningless.

Accessibility, or the *potential* for interaction is different than interaction (V_{ij} vs. T_{ij}), namely since the number of opportunities that are accessible at an origin from a destination cannot be known in the same way as a trip from an origin to a destination. However, we would argue that getting closer to knowning this number and planning for it - is what accessibility research should strive to achieve.

Total flow constrained accessibility

In the total constrained case, k is replaced with K (Equation 18 satisfying only constraint Equation 20) as follows for V_{ij} and V_i:

$$V'_{ij} = KO_j f(c_{ij}) V'_i = K \sum_j O_j f(c_{ij})$$

Solved for our example, K is equal to the total accessibility V, divided by the sum of the unconstrained accessibility:

$$K = \frac{V}{\sum_i \sum_j O_j f(c_{ij})} K = \frac{V}{\frac{O_1}{c_{1,1}} + \frac{O_1}{c_{2,1}} + \frac{O_1}{c_{3,1}} + \frac{O_2}{c_{1,2}} + \ldots \frac{O_3}{c_{3,3}}} K = \frac{790}{568.4} K = 1.389866$$

```
        1   2   3 TOTAL
1     111  40  54   205
2      15 304  27   345
3      44  61 134   239
TOTAL 170 405 215   790
```

While we can see that the 'number of potentially reachable opportunities' 790 results, the total access at a each i or j is not equal to the known known total access ends (because these constraints are not specified and thus not satisfied by the model). This approach is not yet seen in accessibility literature, but could be an intelligible way to compare accessibility values (V_i or V_j) without considering the population matrix.

Singly-constrained case

Similarly, singly-constrained can be either origin-constrained $V_i''^{(1)}$ or destination-constrained $V_i''^{(2)}$. For the origin- and destination- constrained versions, k is replaced with A_{ij} and B_{ij} respectively (Equation 18 enforcing constraint Equation 21 or Equation 22, and implicitly Equation 20):

$$V_{ij}''^{(1)} = A_{ij} O_j f(c_{ij}) V_{ij}''^{(2)} = B_{ij} O_j f(c_{ij}) \tag{23}$$

Where A_{ij} and B_{ij} are similar to the balancing factors of the gravity model for the singly-constrained cases, though with an addition to the numerator to maintain the associated enforcing constraints:

$$A_{ij} = \frac{O_j}{\sum_j B_{ij} O_j f(c_{ij})} B_{ij} = \frac{P_i}{\sum_i A_{ij} P_i f(c_{ij})}$$

In solving the origin-constrained $V_{ij}^{''(1)}$ case (e.g., Equation 21 of $\sum_j V_{ij} = P_i$ is enforced), $B_{ij} = 1$. In solving the destination-constrained $V_{ij}^{''(2)}$ case (e.g., Equation 22 of $\sum_i V_{ij} = O_j$ is enforced), $A_{ij} = 1$.

Solving for the destination-constrained case $V_{ij}^{''(2)}$ may be helpful in an example where the number of people seeking opportunities is known (P_i), along with the number of opportunities O_j offered by a destination as in unconstrained and total accessibility. An example could be the number of people at each residence seeking hospital-beds (origin) and hospital-beds (opportunities) at each healthcare location (destination) to calculate the opportunity access at each origin. Another case could be calculating the opportunity access at each origin using the quantity of goods available to purchase (opportunities) at each commercial location (destination).

From our review, related versions of this defined destination-constrained accessibility are formulaically similar to retail accessibility models e.g., decision model for shops to optimally spatially locate to maximize their potential consumer interaction. Starting with reilly's law of gravitational retail (Reilly et al. 1929), formalized later in Huff (1964) and Lakshmanan and Hansen (1965). Other literature also demonstrates similar *competition* considerations through its analytical formulation; for instance, in the Intervening Opportunities model (Stouffer 1940, 1960) "axiomatically" Jörgen W. Weibull (1976), and competition consideration expressed as a per population rate particularly in healthcare and job opportunity accessibility analysis (Q. Shen 1998a), and 2FCA methods in general (Soukhov et al. 2023). Most of these works do not explicitly focus on balancing accessibility units; rather, they center on *competition* considerations. We suspect this may be why they have been used in case studies examining access to opportunities that are inherently competitive or where the concept of supply and demand resonates. It was only until the recent work of (Soukhov et al. 2023), where the units of accessibility given competition, e.g., the spatial availability, was introduced with the focus on both competition and balanced units. NOTE: See Appendix where I demonstrate how this destination-constrained case is equivalent to spatial availability

Returning to our simple example, the destination-constrained singly-constrained case would yield the following B_{ij} values:

$$B_{ij} = \frac{P_i}{\sum_i P_i f(c_{ij})} B_{1,1} = \frac{P_i}{P_1 f(c_{1,1}) + P_2 f(c_{1,2}) + P_3 f(c_{1,3})} ... B_{1,1} = 160 * \frac{1}{160/2 + 450/15 + 180/5} = 1.09589 B_{2,}$$

	1	2	3
1	1.095890	0.6307490	0.9580838
2	3.082192	1.7739816	2.6946108
3	1.232877	0.7095926	1.0778443

V_{ij} values:

	1	2	3	TOTAL
1	88	18	37	143
2	33	388	52	472
3	39	31	104	174
TOTAL	160	437	193	790

The O_j values are equal to the known opportunities, whereas the P_i values are not. The total accessibility in the region is maintained at 790. Each V_{ij} can be interpreted as the number of opportunities accessible from each destination by anyone in the region all weighted by travel cost of course.

Doubly constrained case (origin-destination constrained)

(only when the units of the marginals are the same, therefore of more limited application) ex. Horner (2004) and Allen and Farber (2020)

```
Checking population, opportunity balancing:
Population:  790
Opportunities:  790
Population, opportunities balancing OK.

Iteration: 1
Ai: 0.007 0.004 0.006
Bj: 1.148 0.924 1.087

Iteration: 2
Ai: 0.006 0.004 0.005
Bj: 1.215 0.891 1.13

Iteration: 3
Ai: 0.006 0.004 0.005
Bj: 1.245 0.876 1.15

Iteration: 4
Ai: 0.006 0.004 0.005
Bj: 1.259 0.87 1.159

Iteration: 5
Ai: 0.006 0.004 0.005
Bj: 1.265 0.867 1.163

Iteration: 6
Ai: 0.006 0.004 0.005
Bj: 1.267 0.866 1.165

Iteration: 7
Ai: 0.006 0.004 0.005
Bj: 1.269 0.865 1.165

Iteration: 8
Ai: 0.006 0.004 0.005
Bj: 1.269 0.865 1.166
```

```
Stopping Condition:  Slow improvement
Final Error: 199.248%
```

```
          [,1]       [,2]      [,3]
[1,] 0.5914286 0.07885714 0.2365714
[2,] 0.1119605 0.83970381 0.1679408
[3,] 0.2357480 0.11787398 0.5893699
```

The A_i and B_j matrices:

```
            [,1]
[1,] 0.005824592
[2,] 0.004442850
[3,] 0.005239296
```

```
         [,1]      [,2]     [,3]
[1,] 1.268701 0.8652486 1.165342
```

The trip matrix (notice, all summed up opportunities and populations are equivalent to our knowns)

```
      1 2 3 TOTAL
1     1 0 0   143
2     0 1 0   472
3     0 0 1   174
TOTAL 1 1 1     3
```

Hybrid singly-constrained case:

Balanced floating catchment areas (not truly doubly constrained)

Empirical examples

Use the TTS2016R package to illustrate every measure.

Discussion

When to use each of these measures?

Conclusions

APPENDIX (some musings...)

Singly-constrained case is spatial availability

The destination-constrained case is also spatial availability, see below! As follows then, this case is also related to 2SFCA - if the value is divided by per capita (OR P_i from the numerator in B_{ij} is removed).

Spatial availability is formulated like so:

$$V_{ij}''^{(2)} = O_j \frac{F_{ij}^p \cdot F_{ij}^c}{\sum_{i=1}^K F_{ij}^p \cdot F_{ij}^c}$$

It turns out that the denominators of the Population balancing factor $F_{ij}^p = \frac{P_{i\in r}^\alpha}{\sum_{i=1}^K P_{i\in r}^\alpha}$ and Travel cost balancing factor $F_{ij}^c = F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^K f(c_{ij})}$ cancel out, so spatial availability can be expressed as:

$$V_{ij}''^{(2)} = O_j \frac{P_i f(c_{ij})}{\sum_i P_i f(c_{ij})}$$

Reformatted in the general family of accessibility measure form, it looks like:

$$V_{ij}''^{(2)} = kO_j f(c_{ij}) V_{ij}''^{(2)} = \frac{P_i}{\sum_i P_i f(c_{ij})} O_j f(c_{ij}) V_{ij}''^{(2)} = O_j \frac{F_{ij}^p \cdot F_{ij}^c}{\sum_{i=1}^K F_{ij}^p \cdot F_{ij}^c} = O_j \frac{P_i f(c_{ij})}{\sum_i P_i f(c_{ij})}$$

Checking singly-constrained case against spatial availability

```
# A tibble: 3 x 2
      j   V_j
  <dbl> <dbl>
1     1   160
2     2   437
3     3   193
```

Destination- (or ATTRACTION) constrained is the only singly-constrained case... population- (or PRODUCTION) constrained doesn't make sense!

Then the origin-constrained (singly-constrained) case $V_{ij}^{(1)}$, no P_i values are needed only O_j values. Using this method, V_i values equal P_i values...? while O_j values are unconstrained.

This case may be helpful in an example where the number of opportunities available to an origin is known and fixed. This could be a normative minimum threshold, such as 100 opportunities at each origin.

Returning to our simple example, the origin-constrained singly-constrained case would yield the following A_{ij} values:

$$A_{ij} = \frac{O_j}{\sum_j O_j f(c_{ij})} A_{1,1} = \frac{O_j}{O_1 f(c_{1,1}) + O_2 f(c_{1,2}) + O_3 f(c_{1,3})} ... A_{1,1} = 160 * \frac{1}{160/2 + 437/15 + 193/5} = 1.083032A$$

```
          1        2         3
1 1.0830325 2.958032 1.3064079
2 0.6439496 1.758787 0.7767642
3 0.9291521 2.537747 1.1207898
```

Starting off with this opportunity matrix, namely - at i=1, only a fixed number of opportunities are accessible to this origin. same for i=2, i=3.

```
     [,1] [,2] [,3]
[1,]  160  160  160
[2,]  437  437  437
[3,]  193  193  193
```

V_{ij} values:

```
              1         2         3 TOTAL
1      86.64260  31.55235  41.80505   160
2      18.76040 384.29501  33.94459   437
3      35.86527  48.97851 108.15621   193
TOTAL 141.26827 464.82587 183.90586   790
```

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