A family of accessibility measures derived from spatial interaction principles

Introduction

Historically, the focus of modern transportation planning has been to prioritize mobility while treating access to destinations as a by-product of movement. This has had problematic consequences: with the car seen as the ultimate mobility tool, this approach has led to the emergence and continued dominance of an automobility mono-culture (H. J. Miller 2011; Lavery, Páez, and Kanaroglou 2013). Decades of planning for this mono-culture (often characterized by road and highway expansion) have been marked by increased travel cost and environmental burdens, but often having only limited impact on the ease with which people can reach destinations (Steven Farber and Páez 2011; S. Handy 2002; Páez et al. 2010). In response to this situation, transportation researchers have increasingly advocated for the adoption of accessibility as a planning criterion (Silva et al. 2017; Paez et al. 2013; S. Handy 2020) using both positive and normative approaches (Paez, Scott, and Morency 2012; Levine 2020). Accessibility, a central concept in transport geography, planning, and engineering, is conceptually related to human mobility and the opportunity landscape. In simple terms, it is defined as the "potential of opportunities for interaction" (Hansen 1959). Compared to other measures of performance used in transportation that benchmark movement (e.g., VKT, PKT, etc.), accessibility brings a more holistic understanding of transportation and land use systems combined (S. L. Handy and Niemeier 1997).

An ascendant interest in accessibility has been accompanied by a remarkable boom in scholarly research, which has grown to include studies of access to employment (e.g., Karst and Van Eck 2003; Grengs 2010; Páez et al. 2013; Merlin and Hu 2017; Tao et al. 2020), health care (e.g., Luo and Wang 2003; Páez et al. 2010; Wan, Zou, and Sternberg 2012; Delamater 2013; Boisjoly, Moreno-Monroy, and El-Geneidy 2017; Pereira et al. 2021), green spaces (Reyes, Paez, and Morency 2014; Rojas et al. 2016; Liang, Yan, and Yan 2024), schools (e.g., Williams and Wang 2014; Romanillos and Garcia-Palomares 2018; Marques, Wolf, and Feitosa 2021), social contacts (e.g., Neutens et al. 2007; S. Farber, Páez, and Morency 2012; S. Farber et al. 2013), and regional economic analysis (e.g., R. Vickerman, Spiekermann, and Wegener 1999; Lopez, Gutierrez, and Gomez 2008; Ribeiro, Antunes, and Páez 2010; Gutierrez et al. 2011) among many other domains of application. In other words, accessibility analysis is used today to broadly understand the potential to reach opportunities that are important to people (Ferreira and Papa 2020). However, despite its growth in popularity in scholarly works, challenges remain with respect to the more widespread adoption of accessibility in planning practice. Several barriers to bridging the scholarly-practice accessibility gap have been identified. For example, the diversity of accessibility definitions has been flagged by van Wee (2016), S. Handy (2020), and Kapatsila et al. (2023). Further, difficulties in the interpretability and communicability of outputs has also been noticed by Karst T. Geurs and van Wee (2004), van Wee (2016), and Ferreira and Papa (2020).

The adoption of accessibility in planning practice is not necessarily made easier when potential adopters have to contend with a plethora of definitions, each seemingly more sophisticated but less intuitive than the last (Kapatsila et al. 2023). At a high level, Karst T. Geurs and van Wee (2004) identify four families of accessibility measures: infrastructure-, place-, person-, and utility-based. Of the place-based family, which is the focus of this paper, the menu has grown to include gravity-based accessibility (e.g., Hansen 1959; Pirie 1979), cumulative opportunities (e.g., Wachs and Kumagai 1973; Pirie 1979; Ye et al. 2018), modified gravity (e.g., Schuurman, Berube, and Crooks 2010), 2-Step Floating Catchment Areas (e.g., Luo and Wang 2003), Enhanced 2-Step Floating Catchment Areas (e.g., Luo and Qi 2009), 3-Stage Floating Catchment Areas (e.g., Wan, Zou, and Sternberg 2012), Modified 2-Step Floating Catchment Areas (e.g., Delamater 2013), inverse 2-Step Floating Catchment Areas (e.g., F. Wang 2021), and n-steps Floating Catchment Areas (Liang, Yan, and Yan 2024). How is a practitioner to choose among this myriad options? What differences in accessibility scores should matter, and how should they be communicated? [see van Wee (2016); p. 14].

In this respect, Wu and Levinson (2020)'s contribution provides a unifying framework to think about accessibility measures, the most general of which they describe as S_i in Equation 1. By considering the key features of accessibility, namely travel-cost $f(c_{ij})$ and the distribution of opportunities $g(O_i)$, Wu and Levinson (2020) demonstrate that a majority of the concepts found scattered throughout the accessibility literature can be seen as particular cases of a general accessibility formula (Equation 1). One needs only to judiciously change the way travel costs and opportunities are formulated to derive almost any known place-based accessibility measure. In this way, Wu and Levinson (2020) have taken a considerable step towards clarifying the differences between various accessibility indicators.

$$S_i \propto \sum_j g(O_j) f(c_{ij}) \tag{1}$$

However, there is room to further clarify another relevant aspect: competition. Wu and Levinson (2020)'s unifying framework does not appear to accommodate many (or any) of the floating catchment area methods, a popular approach within the competitive accessibility literature. There have also been more recent developments in the literature: for instance, Soukhov et al. (2023) demonstrate that the introduction of a single constraint is sufficient to turn the general accessibility measure (i.e, Equation 1) into a competitive measure of accessibility in accordance with the 2-Step Floating Catchment Area approach. Moreover, the proportional allocation balancing factors in Soukhov et al. (2023) apply a singly-constraint to the accessibility measure, akin to the constraints introduced within the spatial interaction modelling framework outlined in Wilson (1971).

Motivated by the challenge of incorporating competition into a unified framework for accessibility, this paper contends that accessibility research must reconnect with its spatial interaction origins. Particularly, we argue that an important aspect of spatial interaction modelling–namely, constraining the results to match empirical observations–was never effectively reincorporated into accessibility analysis. Empirical constraints were embraced by early spatial interaction literature following the work of Wilson (1971), but this stream of literature tended to flow separately from research inspired by Hansen (1959)' accessibility. The application of Wilson (1971)'s empirical constraints supported the development of various spatial interaction models that remain relevant in research and practice today (Ortúzar and Willumsen 2011). However, the same cannot be said of the contemporary accessibility literature, where empirical constraints were not explicitly adopted. We argue that the absence of empirical constraints (and their attendant proportionality constants) has contributed to some of accessibility analysis' interpretability issues; for instance, the fuzziness of insights beyond simple proportional statements like 'higher-than' or 'lower-than' (E. J. Miller 2018).

These streams of literature share common headwaters. It is by looking to the past that we believe accessibility analysis can newly wade into the future. Hence, this work's primary focus is to show that the same empirical constraints used in Wilson (1971)'s family of spatial interaction models can be mapped onto accessibility (REITERATE WHY). We begin by tracing the development of accessibility from its origins in spatial interaction, from the Newtonian gravitational expression in Ravenstein (1889) through to the seminal accessibility work of Hansen (1959). We then present evidence for a narrative highlighting the marked divergence between accessibility and spatial interaction modelling research after the work of Wilson (1971). Next, we hark back to Wilson (1971)'s spatial interaction models, and use it to derive a family of accessibility measures based on different types of constraints. We illustrate various members of this family with a simple numerical example and a real world data set. We then conclude by discussing the uses of these measure and their interpretation.

Newtonian's roots of human spatial interaction research

The patterns of people's movement in space have been a subject of scientific inquiry for at least a century and a half. From as far back as Henry C. Carey's *Principles of Social Science* (Carey 1858), a concern with the scientific study of human spatial interaction can be observed. It was in this work where Carey stated that "man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance" (McKean 1883, 37–38). This statement shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton's Law of Universal Gravitation, first posited in 1687's *Principia Mathematica* and expressed as in Equation 2.

$$F_{ij} \propto \frac{M_i M_j}{D_{ij}^2} \tag{2}$$

To be certain, the expression above, a proportionality, is one of the most famous in all of science. In brief, it states that the force of attraction F between a pair of bodies i and j is directly *proportional* to the product of their masses M_i and M_j, and inversely *proportional* to the square of the distance between them D_{ij}. Direct proportionality means that as the product of the masses increases, so does the force. Likewise, inverse proportionality means that as the distance increases, the force decreases. Equation 2, however, does not quantify the magnitude of the force. To do so, an empirical constant is required to convert the proportionality into an equality, ensuring that values of the force F in Equation 2 match the observed force of attraction between masses. In other words, Equation 2 needs to be *constrained* using empirical data. Ultimately, the equation for the force is as seen in Equation 3, where G is an empirically calibrated proportionality constant:

$$F_{ij} = G\frac{M_i M_j}{D_{ij}^2} \tag{3}$$

Newton's initial estimate of G was based on a speculation that the mean density of earth was between five or six times that of water, an assumption that received support after Hutton's experiments of 1778 (Hutton 1778, 783). Still, it took over a century from the publication of *Principia* to refine the estimate of the proportionality constant to within 1% accuracy, with Cavendish's 1798 experiment (Cavendish 1798).

Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Following Carey's *Principles* of 1858, research into human spatial interaction continued in different contexts. In the late 1880s, Ravenstein proposed some "Laws of Migration" based on his empirical analysis of migration flows in various countries (Ravenstein 1885, 1889). In these works, Ravenstein posited 1) a directly proportional relationship between migration flows and the size of destinations (i.e., centres of commerce and industry), and 2) an inversely proportional relationship between the size of flows and the separation between origins and destinations. As with Carey, these propositions echo Newton's gravitational laws. Over time, other researchers discovered similar relationships. For example, Reilly (1929) formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations. Later, Zipf proposed a $\frac{P_1 P_2}{D}$ hypothesis for the case of information (Zipf 1946a), intercity personal movement (Zipf 1946b), and goods movement by railways (Zipf 1946c). The $\frac{P_1 P_2}{D}$ hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

A common feature of these early investigations of human spatial interaction is that a proportionality constant similar to G in Equation 3 was never considered. Of the researchers cited above, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly's hypothesis was presented in the following form:

$$B_a = \frac{(P_a \cdot P_T)^N}{D_{aT}^n} \tag{4}$$

where B_a is the amount of business drawn to a from T, P_a and P_T are the populations of the two settlements, and D_{aT} is the distance between them. Quantity N was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of $n = 2$.

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

$$C^2 = \frac{P_1 \cdot P_2}{D_{12}} \tag{5}$$

where C is the volume of goods exchanged between 1 and 2, P_1 and P_2 are the populations of the two settlements, and D_{12} is the distance between them.

After Carey, it is in Stewart's work on the principles of demographic gravitation that we find the strongest connection yet to Newton's law (Stewart 1948). This may relate to academic backgrounds; where Ravenstein, Reilly, and Zipf were social scientists, Stewart was a physicist. Besides awareness of preceding research (he cites both Reilly and Zipf as predecessors in the analysis of human spatial interaction), Stewart appears to have been the first author to express his theorized relationships for human spatial interaction with a proportionality constant G, as follows:

$$F = G\frac{(\mu_1 N_1)(\mu_2 N_2)}{d_{12}^2} = G\frac{M_1 \cdot M_2}{d_{12}^2} \tag{6}$$

Where:

- F is the *demographic force*
- N_1 and N_2 are the numbers of people of in groups 1 and 2
- μ_1 and μ_2 are so-called *molecular weights*
- $M_1 = \mu_1 N_1$ and $M_2 = \mu_2 N_2$ are the demographic masses at 1 and 2
- d_{12}^2 is the distance between 1 and 2
- And finally G, a constant that Stewart "left for future determination" (1948, 34)

In addition to demographic force, Stewart defined a measure of the "population potential" of 2 with respect to 1 as follows:

$$V_1 = G\frac{M_2}{d_{12}} \tag{7}$$

For a system with more than two population bodies, Stewart formulated the population potential at i as follows (after arbitrarily assuming that $G = 1$):

$$V_i = \int \frac{D}{r} ds \tag{8}$$

where D is the population density over an infinitesimal area ds and r is the distance to i. In Equation 8, $D \cdot ds$ gives an infinitesimal count of the population, say dm, and so, after discretizing space, Equation 8 can be rewritten as:

$$V_i = \sum_j \frac{M_j}{d_{ij}} \tag{9}$$

Alerted readers will notice that Equation 9, with some re-organization of terms, is formally equivalent to our modern definition of accessibility:

$$V_i = \sum_j M_j d_{ij}^{-1} \tag{10}$$

Stewart's formulation of demographic force, developed in the context of what he called "social physics" (Stewart 1947), was problematic. It had issues with inconsistent mathematical notation. More seriously though, Stewart's work was permeated by a view of humans as particles following physical laws, but tinted by unscientific ideas that were unadultered racism. For instance, he assumed that the molecular weight μ of the average American was one, but "presumably…much less than one….for an Australian aborigine" [p. 35]. Stewart's ideas about "social physics" soon fell out of favour among social scientists, but not before influencing the nascent field of accessibility research, as detailed next.

Hansen's gravity-based accessibility to today

From Stewart (1948), we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature (Hansen 1959). In his seminal paper, Hansen defined accessibility as "the potential of opportunities for interaction… a generalization of the population-over-distance relationship or *population potential* concept developed by Stewart (1948)" (p. 73). As well as being a student of city and regional planning at Massachusetts Institute of Technology, Hansen was also an engineer with the Bureau of Roads, and preoccupied with the power of transportation to shape land uses in a very practical sense.

Hansen (1959) focused on Stewart (1948)'s *population potential* (expressed in Equation 9), leaving Stewart's other formulaic contributions and objectionable aspects of "social physics" behind. Hansen (1959) recast Stewart's population potential to reflect accessibility, a model of human behaviour useful to capture regularities in mobility patterns. Hansen (1959) replaced M_j in Equation 9 with *opportunities* to derive an *opportunity potential*, or more accurately, a *potential of opportunities for interaction* as follows:

$$S_i = \sum_j \frac{O_j}{d_{ij}^{\beta}} \tag{11}$$

A contemporary rewriting of Equation 11 accounts for a variety of impedance functions beyond the inverse power $d^{-\beta}$:

$$S_i = \sum_j O_j \cdot f(d_{ij}) \tag{12}$$

S_i in Equation 11 is a measure of the accessibility of site i. This is a function of O_j (the mass of opportunities at j), d_{ij} (the cost, e.g., distance or travel time, incurred to reach j from i), and β (a parameter that modulates the friction of cost). Today, Hansen is frequently cited as the father of modern accessibility analysis (e.g., Reggiani and Martín 2011), and Hansen-type accessibility is commonly referred to as the gravity-based accessibility measure.

Of note, however is that between Stewart (1948) and Hansen (1959) the proportionality constant G in Equation 7 vanished. There is some evidence that Hansen (1959) was aware of the importance of this constant as he wrote about *directly* and *inversely proportional* relationships when discussing population, opportunities, and their separation in space. At any rate, those reading Hansen (1959) must recall that Stewart (1948) had set the proportionality constant G to 1, with a note that "G [was] left for future determination: a suitable choice of other units can reduce it to unity".

After Hansen (1959), accessibility analysis has been widely used in numerous disciplines but, to our knowledge, the proportionality constant has remained implicit or forgotten, with no notable developments to explicitly determine it. In this way, G continues to be implicitly set to 1, even when the fundamental relationship in accessibility is proportionality (e.g., $S_i \propto \sum_j g(O_j)f(d_{ij})$) and not equality (for instance, see the formula for accessibility at the top of Figure 1 in Wu and Levinson 2020). Alas, without a proportionality constant, the units of S_i remain unclear: the unit of "potential of opportunity for interaction" is left free to change as β is calibrated. For example, if c_{ij} is distance in meters, it will be number of opportunities per m^{β} when $f(c_{ij}) = d^{-\beta}$ but number of opportunities per $e^{-\beta \cdot m}$ when $f(c_{ij}) = e^{\beta \cdot m}$. Hansen-style accessibility, therefore, is better thought of as an ordinal measure of potential that can only be interpreted in terms of higher and lower accessibility (E. J. Miller 2018).

Wilson's family of spatial interaction models

In his groundbreaking work, Wilson (1971) defined a general spatial interaction model as follows:

$$T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij}) \tag{13}$$

The model in Equation 13 posits a quantity T_{ij} that represents a value in a matrix of flows of size $n \times m$, that is, between $i = 1, \cdots, n$ origins and $j = 1, \cdots, m$ destinations. The quantities $W_i^{(1)}$ and $W_j^{(2)}$ are proxies for the masses at $i = 1, \cdots, n$ origins and $j = 1, \cdots, m$ destinations. Finally, $f(c_{ij})$ is the $i-j$ element of an $n \times m$ matrix representing some function of travel cost c_{ij} which reflects travel impedance. In this way, T_{ij} explicitly measures *interaction* in the unit of trips, and the role of k is to ensure that the system-wide sum of T_{ij} represents the total flows in the data. In other words, k is a scale parameter that makes the overall amount of flows identical to the magnitude of the phenomenon being modeled: it balances the units.

Traditionally, development of the spatial interaction model put an emphasis on the interpretability of the results (Kirby 1970; Wilson 1967, 1971). But instead of relying on the heuristic of Newtonian gravity (e.g., some interaction between a mass at i and a mass at j separated by some distance), Wilson's approach was to maximise the entropy of the system. Entropy maximisation in this case achieves stable results as a statistical average that represents the population. The approach works by assuming undifferentiated individual interactions, and assessing their probabilities of making a particular journey. The result of Equation 13 then is a statistical average (Wilson 1971; Senior 1979).

To ensure that T_{ij} in Equation 13 is in the unit of trips, additional knowledge about the system is required. At the very least, this framework assumes that the total number of trips in the system T is known, and therefore:

$$\sum_i \sum_j T_{ij} = T \tag{14}$$

Additional information can be introduced. For example, when information is available about the total number of trips produced by each origin (O_i), the following constraint can be used:

$$\sum_j T_{ij} = O_i \tag{15}$$

Alternatively, if there is information available about the total number of trips attracted by each destination (D_j), the following constraint can be used:

$$\sum_i T_{ij} = D_j \tag{16}$$

It is also possible to have information about both O_i and D_j, in which case both constraints could be imposed on the model.

Building on this, a family of spatial interaction models can be derived. In the framework introduced in Wilson (1971), this includes the "unconstrained" (which we refer to as the total flow constrained) model in Equation 13. The balancing constant K in this case is (see Cliff, Martin, and Ord 1974; A. S. Fotheringham 1984):

$$K = \frac{T}{\sum_i \sum_j T_{ij}} \tag{17}$$

When only one of Equation 15 or Equation 16 holds, the resulting models are, in Wilson's terms, singly-constrained. Entropy maximisation leads to the following production-constrained model:

$$T_{ij} = A_i O_i W_j^{(2)} f(c_{ij}) \tag{18}$$

Notice how, in this model, the proxy for the mass at the origin $W_i^{(1)}$ is replaced by the actual mass, as measured by the trips produced O_i. Also, there is no longer a single system-wide proportionality constant, but rather a set of proportionality constants (i.e., balancing factors) specific to origins. According to Wilson, these constants, namely A_i, are:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \tag{19}$$

The attraction-constrained model, in turn, takes the following form:

$$T_{ij} = B_j D_j W_i^{(1)} f(c_{ij}) \tag{20}$$

Notice how now the proxy for the mass at the destination $W_j^{(2)}$ is replaced by the actual mass, as measured by the trips attracted D_j. As before, destination-specific proportionality constants (i.e., balancing factors) B_j were derived by Wilson as:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \tag{21}$$

When both Equation 15 and Equation 16 hold, the resulting models is, in Wilson's terms, doubly-constrained, and takes the following form:

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \tag{22}$$

In this model, both proxies for the masses are replaced with the known masses, that is, the trips produced by origin and the trips attracted by destination. And, now there are two sets of mutually dependent proportionality constants:

$$
\begin{aligned}
A_i &= \frac{1}{\sum_j B_j D_j f(c_{ij})} \\
B_j &= \frac{1}{\sum_i A_i O_i f(c_{ij})}
\end{aligned}
\tag{23}
$$

Derivation of these models is demonstrated in detail elsewhere (e.g., Ortúzar and Willumsen 2011; Wilson 1967). It is worth noting, however, that although Wilson's approach is built on a different conceptual foundation than the old reference to Newtonian gravity, the work succeeded at identifying the steps from proportionality to equality to yield variations of proportionality constants, including the one that eluded Stewart (1948) and that has been ignored in almost all subsequent accessibility research. Why was this key element of spatial interaction models ignored in accessibility research? In the next section we aim to address this question.

Accessibility and spatial interaction modelling: two divergent research streams

The work of Hansen (1959) and Wilson (1971) responded to important developments in the Anglo-American world at the time, in particular a need "to meet the dictates and needs of public policy for strategic land use and transportation planning" (Michael Batty 1994). Said dictates and needs were far from trivial. In the United States alone, the Federal-Aid Highway Act of 1956 authorized the creation of the U.S. Interstate Highway System, with a budget that ultimately exceeded one hundred billion dollars (Weiner 2016; MDOT 2007). Spatial interaction modelling, with its ability to quantify trips, was incorporated into institutional modelling practices meant to "predict and provide", i.e., predict travel demand and supply transportation infrastructure (Kovatch, Zames, et al. 1971; Weiner 2016). Accessibility at the time did not quite have that power, as it did not quantify trips, but rather something somewhat more elusive: the less tangible "potential for interaction". In this way, where spatial interaction modelling became a key element of transportation planning practice, accessibility remained a somewhat more academic pursuit, and the two streams of literature only rarely connected.

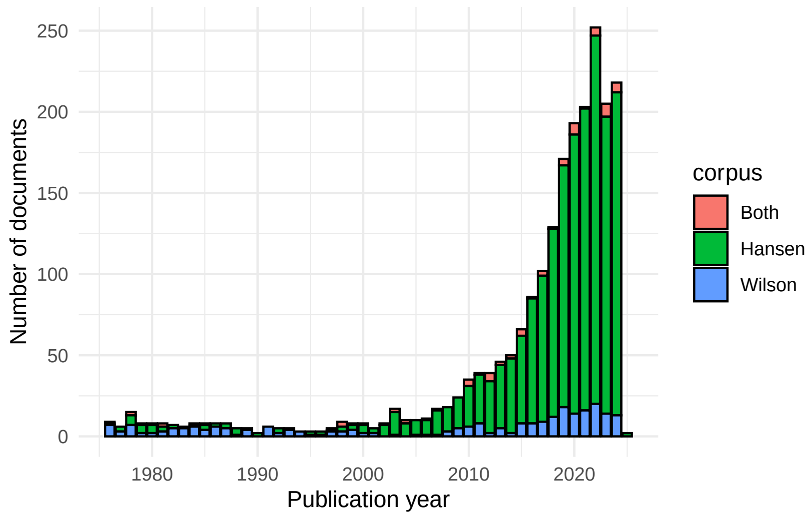

Figure 1: Historical pattern of publication: documents per year.

To illustrate this point, we conducted a bibliographic analysis of the literature that cites Hansen (1959), Wilson (1971), or both. We retrieved all relevant documents using the Web of Science "Cited References" functionality, and the digital object identifiers of Hansen (1959) and Wilson (1971). As a result of this search we identified 1,788 documents that cite Hansen (1959), 258 documents that cite Wilson (1971), and 76 that cite both. The earliest document in this corpus dates to 1976 and the most recent is from 2025. The number of documents per year appears in Figure 1, where we see the frequency of documents over a span of almost fifty years. In particular, we notice the remarkable growth in the number of papers that cite Hansen (1959) compared to those that cite Wilson (1971) or both.

After compiling this corpus of documents, we used the {bibliometrix} package (Aria and Cuccurullo 2017) to create a bibliographical coupling matrix. In this matrix, two documents are coupled if they share at least one common reference, and the strength of the coupling increases with the number of references that the documents have in common. Figure 2 presents the results, where each symbol represents a document. The distance between symbols on the plot indicates the strength of bibliographical coupling: documents that are more strongly coupled (i.e., those sharing more references) are plotted closer to each other.

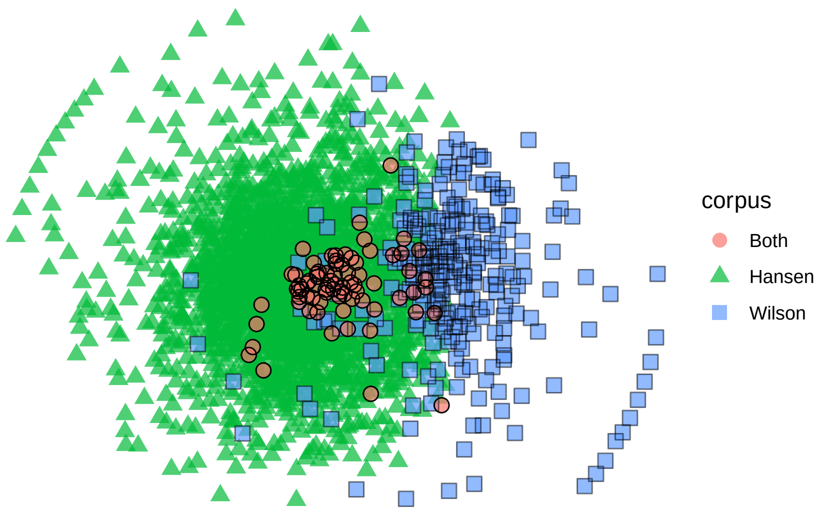

Figure 2: Bibliometric coupling of papers that cite Hansen (1959), Wilson (1971), or both.

Further examination of the bibliographical coupling matrix allows us to check the coupling strength within and between subgroups of documents (i.e., those that cite only Hansen, only Wilson, or both papers). First, we note that when we look at the three subgroups of documents pooled together, the average number of references shared by a pair of documents is 1.18. Table 1 presents the average number of items cited in common within each subgroup, as well as between subgroups. As seen in the table, the coupling within subgroup tends to be higher than when we consider the full corpus with all three groups together. The tightest coupling is seen for the group of documents that cite both Hansen and Wilson, with an average number of references in common of 3.36. The weakest coupling is between the Hansen and Wilson subgroups, with only 0.09 references in common on average, which indicates a large degree of decoupling (independence) between these bodies of work.

Table 1: Average coupling strength within and between groups of documents. The average coupling strength is the number of references shared on average by a pair of documents.

corpus	Hansen	Wilson	Both
Hansen (n = 1788)	1.45	-	-
Wilson (n = 258)	0.09	1.31	-
Both (n = 76)	1.65	1.23	3.36

Average coupling for the pooled set of documents (n = 2122) is 1.18

As noted, literature that cites both Wilson (1971) and Hansen (1959) are sparse (only 3.6%

of the corpus) but highly coupled. However, we can discern from a manual reading of these works that they too are divergent, with one stream focused on developing accessibility (i.e., *potential for* spatial interaction) and another on spatial interaction. The focuses of these divergent streams contribute this paper's broader hypothesis that the concepts of accessibility and spatial interaction have remained largely disconnected.

The spatial-interaction-focused stream of literature inspired by Wilson (1971) that cites both Hansen (1959) and Wilson (1971), tends to contribute to understanding how accessibility is interpreted and incorporated in spatial interaction models, treating it as a separate (but related) phenomenon or variable. Specifically, works interpret the spatial interaction model's balancing factors (Equation 19 or Equation 23) as the inverse of Hansen (1959)'s model (B. Harris and Wilson 1978; G. Leonardi 1978; A. Stewart Fotheringham 1981; A. S. Fotheringham 1985), recognizing it as a "common sense" approach (Morris, Dumble, and Wigan 1979, 99) to including accessibility in the spatial interaction model, though further exploration of its relationship is warranted (M. Batty and March 1976). Some authors have explored this relationship, for instance as in A. S. Fotheringham (1985) who demonstrates how the spatial interaction model may insufficiently explain spatial patterns and suggest that explicitly defining destinations' accessibility as a variable within the model may remedy the issue (e.g., the *competition destination* model). Other works used both Hansen (1959) and Wilson (1971)'s framework in conjunction, such as in defining location-allocation problems in Operations research (G. Leonardi 1978; Beaumont 1981), estimating trip flows (or some other interaction flows) alongside accessibility (e.g., Clarke, Eyre, and Guy 2002; Grengs 2004; Türk 2019), or considering accessibility within spatial interaction models, in line with A. S. Fotheringham (1985)'s demonstration (e.g., Beckers et al. 2022). Other works departed from Hansen (1959)'s definition and aligned with spatial interaction in different ways, such as using micro-economic consumer behaviour concepts to express potential for spatial interaction (Morris, Dumble, and Wigan 1979; Giorgio Leonardi and Tadei 1984).

On the other hand, we discern that the potential-for-spatial-interaction-focused stream of literature inspired by Hansen (1959) that cite both Hansen (1959) and Wilson (1971), cite Wilson (1971) for three general reasons: - Firstly, as attribution for using using different context-dependent travel cost functions (Weibull 1980; S. L. Handy and Niemeier 1997; Kwan 1998; Q. Shen 1998a; Ashiru, Polak, and Noland 2003; Rau and Vega 2012; Pan 2013; Margarida Condeço Melhorado et al. 2016; Caschili, De Montis, and Trogu 2015; Grengs 2015; Pan, Jin, and Liu 2020; Chia and Lee 2020; Roblot et al. 2021; Sharifiasl, Kharel, and Pan 2023; Kharel, Sharifiasl, and Pan 2024). - Secondly, associating spatial interaction with accessibility's potential for spatial interaction (e.g., Giuliano et al. 2010; Grengs et al. 2010; Grengs 2010, 2012; Levine et al. 2012; Levinson and Huang 2012; Tong, Zhou, and Miller 2015; X. Liu and Zhou 2015; He et al. 2017; Wu and Levinson 2020; Ng et al. 2022; Naqavi et al. 2023; Suel et al. 2024). While accessibility is an expression of *potential for* spatial interaction and Wilson (1971) touches on the concept H. J. Miller (1999), it is distinct concept from spatial interaction. This distinction, however, is often unclear, as Hansen (1959) and Wilson (1971) are frequently co-cited as both being 'gravity models' (e.g., S. Liu and Zhu 2004; Dai, Wan, and Gai 2017; Y. Shen 2019; Chia and Lee 2020). - Thirdly, interpretation of Hansen (1959)'s model as an inverse balancing factor (R. W. Vickerman 1974), especially in competitive opportunity or population contexts (Karst and Van Eck 2003; Karst T. Geurs, van Wee, and Rietveld 2006; Willigers, Floor, and van Wee 2007; El-Geneidy and Levinson 2011; Curtis and Scheurer 2010; Manaugh and El-Geneidy 2012; Chen and Silva 2013; Alonso et al. 2014; Albacete et al. 2017; Sahebgharani, Mohammadi, and Haghshenas 2019; Mayaud et al. 2019; Allen and Farber 2020; Levinson and Wu 2020; Marwal and Silva 2022; Su and Goulias 2023). However, as outlined in preceding sections, we argue the resulting indicator values are plagued by similar interpretability issues. Of the literature in this stream, only the work of Soukhov et al. (2023) and Soukhov et al. (2024) cite Wilson (1971)'s use of balancing factors as ways for maintaining constraints in opportunities in the context of competitive accessibility.

A family of accessibility measures

As argued in the preceding section, the accessibility and spatial interaction modelling literature streams tended to evolve with little contact after Hansen (1959) and Wilson (1971). This may explain the failure of the constraints/proportionality constant(s) of spatial interaction models to cross over to accessibility analysis. This is intriguing since Wilson himself made an effort to connect his developments in spatial interaction modelling to accessibility, noting the denominator of Equation 19 and is the inverse of A_i, offering a potential interpretation for the proportionality constant (Wilson 1971, 10):

$$S_i = \frac{1}{A_i} = \sum_j W_j^{(2)} f(c_{ij}) \tag{24}$$

This, however, only helps to illustrate the ease with which confusion may arise in this context. The relationship in Equation 24 appears to be misaligned. In the preceding section on early research, we noted that Hansen, and Stewart before him, defined accessibility as a partial sum of the demographic force F, essentially going from the demographic force ($F = G\frac{M_1 \cdot M_2}{d_{12}^2}$ to the pairwise population potential $V_1 = G\frac{M_2}{d_{12}}$ and from there to the system-wide population potential (after losing G) $V_i = \sum_j \frac{M_j}{d_{ij}}$. While the inverse of A_i may appear to be accessibility, its functional role in Wilson's general model $T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij})$ is that of a balancing factor k (maintaining proportionality).

Hence, we propose a more useful definition: accessibility as a partial sum of the spatial interaction. A way to define the *potential for spatial interaction* (i.e., accessibility) is as follows:

$$V_{ij} = kW_j^{(2)} f(c_{ij}) \tag{25}$$

where V_{ij} is the potential for interaction, and therefore:

$$V_i = k \sum_j W_j^{(2)} f(c_{ij}) \tag{26}$$

Similar to Equation 13, $W_j^{(2)}$ above is the mass at the destination, and the sub-indices are for $i = 1, \cdots, n$ origins and $j = 1, \cdots, m$ destinations.

In the following subsections, we detail how various accessibility measures can be defined depending on selected constraints: how k is specified and depending on the combination of population and opportunity data is available and measurement focus. Together, these constraints delineate different members of the family of accessibility measures. Along with a simple numerical example, we detail the unconstrained case (Hansen (1959)'s formulation), then total constrained cases (opportunity-constrained and population-constrained), then singly-constrained cases (opportunity-constrained and population-constrained), and lastly the doubly-constrained case. These cases are presented in increasing order of constraint restrictiveness and information availability.

Simple numeric example setup

Consider a simple region with three zones, IDs 1, 2 and 3. Each zone is both an origin i and a destination j. The following three pieces of information are defined: zonal population and opportunities, zonal cost matrix, and travel impedance functions for three types of travel behaviour. In this example, the population are people, the opportunities are physicians, and the region can be described by three possible travel behaviours.

Firstly, Table 2 summarises the population (in units of 10,000s of people) and the opportunities (the number of physicians) per zone. [1]. Considering Table 2's values, the Provider-to-Population-Ratio (PPR) in this system is 24.5. The population is $W_i^{(1)}$ when used as a proxy for the mass at the origin, and O_i when used as a constraint. Similarly, the opportunities are represented by $W_j^{(2)}$ when used as a proxy for the mass at the destination, and D_j when used as a constraint.

[1]For reference, the number of physicians per 10,000 in Canada in 2022 was 24.97 (WHO 2025)

Table 2: Simple system with three zones. Population is in 10,000 persons and opportunities in number of physicians.

ID (i or j)	Population[1]	Opportunities[2]
1	4	160
2	10	150
3	6	180

[1]Population is $Wi^{(1)}$ when used as a proxy for the mass at the origin, and Oi when used as a constraint.
[2]Opportunities is $Wj^{(2)}$ when used as a proxy for the mass at the destination, and Dj when used as a constraint.

Secondly, to pair with the zonal population and opportunity information, the assumed cost of movement (in minutes of travel time) between origins and destinations is as shown in Table 3. From both Table 2 and Table 3, it can be interpreted that Zones 1 and 3 are more proximate to each other than to Zone 2. Combined, Zone 1 and 3 have the same amount of population as Zone 2 but more than double the opportunities.

Table 3: Cost matrix for system with three zones (travel time in minutes).

	Destination ID		
Origin ID	1	2	3
1	10	30	15
2	30	10	25
3	15	25	10

And lastly, we distinguish accessibility measure values for the following three impedance functions that represent the potential for spatial interaction travel behaviour of the population to opportunities in the region are in Equation 27:

$$\begin{aligned} f_1(c_{ij}) &= \tfrac{1}{c_{ij}^3} \\ f_2(c_{ij}) &= \tfrac{1}{c_{ij}^2} \\ f_3(c_{ij}) &= \tfrac{1}{c_{ij}^{0.1}} \end{aligned} \tag{27}$$

Accessibility will be calculated three times for each case, one assuming the most decay ($f_1(c_{ij})$), another assuming medium decay ($f_2(c_{ij})$), and a third assuming the least decaying travel behaviour ($f_3(c_{ij})$) for the entire region. A helpful analogy may be tying travel behaviour to the used mode's mobility potential, i.e., the most decaying travel behaviour ($f_1(c_{ij})$) would assume all travel in the region being done by foot, while calculating accessibility assuming the least decay ($f_3(c_{ij})$) would assume unfettered automobility. Or alternatively, these functions could represent travel behaviour on snowstorm-affected day ($f_1(c_{ij})$) for the entire region versus a clear, ideal travel day ($f_3(c_{ij})$). As an example of a discussion on how travel behaviour has been considered in accessibility measures cost of travel see Paez, Scott, and Morency (2012).

Any set of concepts representing population, opportunities, and their associated travel behaviour -whether representing the entire region uniformly (as we will demonstrate) or representing specific subgroups- can be substituted into our simple example, depending on the research question. The purpose of the following simple example is to demonstrate the calculation of each member of the accessibility measure family, interpret the values, and compare them both within and across travel behavior groups and members of the accessibility family.

Unconstrained accessibility

Setting the balancing factor k to one in Equation 25 results in the unconstrained accessibility case:

$$V_{ij}^0 = W_j^{(2)} f(c_{ij}) \tag{28}$$

Then, the partial sum becomes the following: simply Hansen's accessibility S_i (Hansen 1959), standard practice in accessibility measurement:

$$V_i^0 = \sum_j V_{ij}^0 = \sum_j W_j^{(2)} f(c_{ij}) = S_i \tag{29}$$

Where the sum of V_i^0 in the region generally does not equal the total number of opportunities O (e.g., $\sum_i V_i^0 \neq O$), as setting k to 1 to arbitrarily eliminates the term and imposes no meaningful interpretation to the resulting values. To complicate matters, when comparing V_i^0 values between travel behaviour contexts, their difference depends on the choice of impedance function $f(c_{ij})$, associated parameters, and the number of zones for which an accessibility score is calculated. For this reason, the unconstrained accessibility should more appropriately be used as an ordinal variable to make comparisons of size (i.e., greater than, less than, equal to), not to calculate ratios or intervals (i.e., the magnitude of differences).

Returning to the simple example, consider the calculated unconstrained accessibility V_i^0 for each origin, a sum of all the travel impedance weighted opportunities at each destination ($\sum_i V_i^0$), in Table 4.

Table 4: Simple system: unconstrained accessibility.

Origin	V_i^0		
	f_1 $(c_{ij}) = 1/{c_{ij}}^3$	f_2 $(c_{ij}) = 1/{c_{ij}}^2$	f_3 $(c_{ij}) = 1/{c_{ij}}^{0.1}$
	units: *f1(cij)-weighted physicians*	units: *f2(cij)-weighted physicians*	units: *f3(cij)-weighted physicians*
1	0.219	2.567	371.143
2	0.167	1.966	363.479
3	0.237	2.751	373.738
Sum	0.6233422	7.283556	1108.361

As the different impedance functions represent different travel behaviours, comparing unconstrained accessibility values across groups is meaningless beyond notions of higher or lower. For instance, at zone 1 the difference between the least decay ($f_3(c_{ij})$) and most decay ($f_1(c_{ij})$) groups is 370.92, but in what units? These two values have different units so this difference value cannot be meaningfully interpreted.

However, we can compare values within the same travel behavior scenario across different zones but values are not inherently meaningful due to the units. For example, considering the most decaying scenario at zone 1, it captures 0.051443 more $f_1(c_{ij})$-weighted-physicians than zone 2, and 0.0181185 fewer than zone 3. In the least decaying scenario, zone 1 captures 7.6638289 more $f_3(c_{ij})$-weighted-physicians physicians than zone 2, and 2.5954449 fewer than zone 3. Both scenarios at zone 1 captures an intermediate V_i^0 value between the two other zones, but at different ratios: the least decaying scenario captures 1.3072213 and 0.9235529 times the $f_1(c_{ij})$-weighted-physicians than zone 2 and 3 respectively, while the most decaying scenario captures 1.0210846 and 0.9930555 times the $f_3(c_{ij})$-weighted-physicians. The ratios between the zones in the two scenarios are different. These differences are tied up in the travel impedance functions, and hence the unconstrained accessibility values themselves should not be compared. This simple example illustrates that the differences and ratios of unconstrained accessibility values are not inherently meaningful. While one could attempt to adjust the units post-calculation (e.g., scaling, population normalization) or select comparable impedance functions (at the cost of accurately reflecting travel behavior), such adjustments may introduce bias.

Totally-constrained accessibility

A total constraint proportionally adjusts zonal accessibility values based on the total population and/or opportunities in the region. The introduction of this constraint introduces a form of competition, as all zonal values are a proportion of the system total, be it the regional opportunities or regional population depending on the case. We define two cases: a *totally-constrained opportunity* accessibility case, interpreted as Hansen's accessibility with a constraint, and a *totally-constrained population* accessibility case, interpreted as constrained market potential.

Totally-constrained opportunity: Hansen's accessibility with a constraint

Unlike in Equation 25, the proportionality constant k is retained, not set arbitrarily to 1, and represented as K^T:

$$V_{ij}^T = K^T \cdot W_j^{(2)} \cdot f(c_{ij}) \tag{30}$$

Where the total opportunity constrained accessibility measure now becomes Hansen's accessibility with a proportionality constant:

$$V_i^T = \sum_j V_{ij}^T = K^T \sum_j W_j^{(2)} f(c_{ij}) = K^T \cdot V_i^0 \tag{31}$$

The constraint that we impose in this case is the total number of opportunities D in the region.

$$\sum_i V_i^T = \sum_i \sum_j V_{ij}^T = D \tag{32}$$

This constraint is analogous to the total constraint of Equation 14 which is congruent with Wilson's framework. We can then substitute Equation 31 in Equation 32 and solve for K^T to yield:

$$K^T = \frac{D}{\sum_i \sum_j V_{ij}^0} = \frac{D}{\sum_i \sum_j W_j^{(2)} f(c_{ij})} \tag{33}$$

Which is also congruent with Wilson's framework as it comparable to the total flow spatial interaction model (e.g., Equation 2.11 in Cliff, Martin, and Ord (1974)). Hence, rearranging the equation to have opportunities and the proportional constant distinctly represented, our totally-constrained opportunity accessibility model is:

$$V_i^T = K^T \sum_j W_j^{(2)} f(c_{ij}) = \sum_j W_j^{(2)} \frac{D \cdot f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

Further, we can see that, since D and $W_j^{(2)}$ are both in units of opportunities, the following term, the proportional allocation factor for the totally-constrained opportunity case κ^T is dimensionless:

$$\kappa^T = \frac{D \cdot f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

and therefore V_i^T is now in the units of $W_j^{(2)}$, that is, the mass at the destination ($V_i^T = \kappa^T \sum_j W_j^{(2)}$). The role of κ^T in this reformulation of accessibility is to adjust the number of opportunities accessible from i so that they represent a proportion of the total number of opportunities in the region. Constant κ^T then assigns opportunities in proportion to the impedance between i and j. For this reason, we refer to it as a proportional allocation factor.

Referring to our simple numeric example, proportionality constant for the $f_1(c_{ij}) = 1/c_{ij}^3$ travel behaviour scenario would then be:

$$K^T = \frac{D}{\sum_i \sum_j O_j f(c_{ij})}$$
$$K^T = \frac{D}{\frac{D_1}{c_{11}^3} + \frac{D_1}{c_{21}^3} + \frac{D_1}{c_{31}^3} + \cdots + \frac{D_3}{c_{31}^3} + \frac{D_3}{c_{32}^3} + \frac{D_3}{c_{33}^3}}$$
$$K^T = \frac{490}{0.6233422}$$
$$K^T = 786.085$$

Using the calculated proportionality constants for all zones, the total opportunity constrained accessibility values for all zones and different travel behaviour scenarios is presented in Table 5.

Table 5: Simple system: total opportunity constrained accessibility.

	V_i^T		
	$f_1\ (c_{ij}) = 1/c_{ij}{}^3$	$f_2\ (c_{ij}) = 1/c_{ij}{}^2$	$f_3\ (c_{ij}) = 1/c_{ij}{}^{0.1}$
Origin	units: *physicians*	units: *physicians*	units: *physicians*
1	172.065	172.672	164.080
2	131.627	132.247	160.692
3	186.308	185.081	165.228
Sum	490	490	490

In contrast to unconstrained accessibility, imposing a constraint -in this case a total opportunity constraint- allows for the comparison of differences and ratios. Each value is effectively in units of physicians, with the impedance units already accounted for by κ^T.

Consider the highest decay scenario ($f_1(c_{ij})$), zone 1 again captures an intermediate amount of physicians (172.0652825). However, we can say that this is out of the 490 in the region, which allows us also to deduce that zone 1 captures 1.3072213 and 0.9235529 times more than zone 2 and 3. Values for the lesser decay ($f_2(c_{ij})$) and lowest decay ($f_3(c_{ij})$) scenarios are calculated separately, with decay scenario values also summing to equal 490 physicians accessible in the region. Between zonal comparisons can be more interpretably made for these travel behaviour scenarios as well.

One can also directly compare values at a specific zone due to the consistent units. For instance, zone 1 remains intermediate in capturing accessible physicians relative to zones 2 and 3 across scenarios, similar to the unconstrained case. However, the difference between travel behaviour scenarios differ in direction. Specifically, Zone 1 captures 0.6067946 more and 7.9850478 fewer opportunities than the lesser decay scenarios $f_2(c_{ij})$ and $f_3(c_{ij})$ respectively. Why? κ^T ensures proportional allocation for each travel behaviour scenario. Meaning, while the unconstrained accessibility increases, κ^T adjusts the values to remain proportional to the total number of opportunities (490 physicians accessible in the region). As the decay behaviour decreases, more opportunities are accessible for all zones. In the medium decay scenario $f_2(c_{ij})$, zone 1 sees a slight increase in values (relative to the highest decay scenario) as the zone can accessible more opportunities relative to increases seen in other zones. However, in the lowest decay scenario, zone 1 sees a decrease, as it is outpaced by increases in other zones - namely zone 2 (recall: zone 2 has the lowest number of opportunities, hence the increases in opportunity gains is much higher in a low decay scenario).

Using the total opportunity constrained formulation of accessibility offers a solution to the unit interpretability issue of Hansen (1959)'s accessibility measure. Intuitively, the use of the constraint illustrates how the differences and ratios of values between zones and decay groups can be compared.

Totally-constrained population: market potential

Effectively transposing i and j, this case of the totally-constrained accessibility measure (Equation 34) is an expression of the concept of market potential (i.e., potential users) as proposed in C. D. Harris (1954) and R. W. Vickerman (1974). The unconstrained form of market potential, effectively the i j transpose of V_{ij}^0, has been used in recent research to express the potentially accessible population (i.e., users) as a result of regional transportation infrastructure investment projects (e.g., Gutiérrez 2001; Holl 2007; Condeço-Melhorado and Christidis 2018). To formulate this case, the total constraint may instead be applied to the mass of the *population* at i instead of the opportunities at j.

$$M_{ij}^T = \hat{K}^T \cdot W_i^{(1)} f(c_{ij}) = \hat{K}^T \cdot M_{ij}^0 \tag{34}$$

with M_j^0 being the i j transpose of V_{ij}^0:

$$M_j^0 = W_i^{(1)} f(c_{ij})$$

The totally-constrained population accessibility (market potential) becomes:

$$M_j^T = \sum_i M_{ij}^T = \hat{K}^T \sum_i W_i^{(1)} f(c_{ij})$$

The constraint that we impose in this case is the total market potential equals the total population O in the region :

$$\sum_j M_j^T = \sum_i \sum_j M_{ij}^T = O \tag{35}$$

Substituting Equation 34 in Equation 35, and solving for $\hat{K}^T$, we obtain:

$$\hat{K}^T = \frac{O}{\sum_i \sum_i M_{ij}^T} = \frac{O}{\sum_i \sum_j W_i^{(1)} f(c_{ij})} \tag{36}$$

The constrained market potential then takes the following form:

$$M_i^T = \hat{K}^T \sum_i W_i^{(1)} f(c_{ij}) = \sum_i W_i^{(2)} \frac{O \cdot f(c_{ij})}{\sum_i \sum_j W_i^{(2)} f(c_{ij})}$$

Where the following $\hat{\kappa}^T$ proportional allocation factor is dimensionless:

$$\hat{\kappa}^T = \frac{O \cdot f(c_{ij})}{\sum_i \sum_j W_i^{(2)} f(c_{ij})}$$

Returning back to the numerical example, the proportionality constant would be solved for each travel behaviour scenario, and the market potential of each zone is expressed as units of population (e.g., the number of people accessible from each zone) in Table 6. Readers may note the difference in trends in accessible population (Table 6) and accessible physicians (i.e., the preceding subsection, Table 5). In Table 5, zone 1, 2, 3 represent destinations and the accessibility values reflect the number of accessible people from the vantage of physicians. Zone 1, in its role as a destination, is no longer intermediately-ranked relative to other zones; it now attracts the fewest number of people across all three travel behaviour scenarios. However, similar to the totally-constrained opportunity case, as travel decay reduces, the availability of population begins to converge (though Zone 1 continues as the lowest-ranked) for similar reasons. As decay reduces, the population's travel impedance to all zones become more similar, making the relative location of the zones less important and all people in the region more equally accessible.

Table 6: Simple system: total opportunity constrained accessibility.

Destination	M_i^S		
	$f_1\ (c_{ij}) = 1/{c_{ij}}^3$	$f_2\ (c_{ij}) = 1/{c_{ij}}^2$	$f_3\ (c_{ij}) = 1/{c_{ij}}^{0.1}$
	units: *population in 10,000s*	units: *population in 10,000s*	units: *population in 10,000s*
1	5.018	5.447	6.598
2	8.596	7.986	6.717
3	6.386	6.567	6.684
Sum	20	20	20

Singly-constrained accessibility

Increasing in restrictiveness relative to the totally-constrained accessibility measure, a single constraint proportionally adjusts the origin-destination zonal accessibility values based on the population or opportunities in the zone for all zones, while still maintaining the total system population or opportunity constraint, depending on the case. We define two cases: a *singly-constrained opportunity* accessibility case and a *singly-constrained population* accessibility case.

Singly-constrained opportunity: A.K.A spatial availability

If additional information is introduced into our analysis, say, the number of opportunities *by destination* (i.e., D_j), we can impose the following constraint (Equation 37)- comparable to the single attraction-constraint (Equation 16) from Wilson's framework.

$$\sum_i V_{ij}^S = D_j \tag{37}$$

The underlying spatial interaction model is now the attraction-constrained model in Equation 20, and our accessibility measure becomes:

$$V_i^S = \sum_j B_j D_j W_i^{(1)} f(c_{ij}) \tag{38}$$

where $W_i^{(1)}$ is a measure of the mass at origin i (i.e., the opportunity-seeking population). The corresponding balancing factor, as per Wilson, is:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \tag{39}$$

Introducing the balancing factor in Equation 38, we obtain:

$$V_i^S = \sum_j D_j \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \tag{40}$$

Further, we define the following proportional allocation factor:

$$\kappa_i^S = \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \tag{41}$$

After this, it is possible to rewrite Equation 40 as an origin summary expression of proportionally allocated known opportunities (i.e., D_j):

$$V_i^S = \kappa_i^S \sum_j D_j \tag{42}$$

Soukhov et al. (2023) have shown that the role of κ_i^S is to allocate opportunities D_j proportionally to the mass at each origin i and the impedance between i and j. As in the totally-constrained opportunity case, κ_i^S is dimensionless and V_i^S is in the units of opportunities D_j. The singly-constrained accessibility measure in Equation 42 is called spatial availability by Soukhov et al. (2023), because it represents the number of opportunities that can be reached *and* are available, in the sense that accessible opportunities have been proportionally allocated based on relative demand, travel impedance and the regional total number of opportunities, i.e., spatial competition for them has been considered. These authors also show that the following expression (accessibility per capita) is a constrained version of the popular two-stage floating catchment area measure of Q. Shen (1998b) and Luo and Wang (2003):

$$v_i^S = \frac{V_i^S}{W_i^{(1)}}$$

To reinforce this point, Equation 37 also implies that the total system constraint (e.g., $\sum_i V_i^S = \sum_i \sum_j V_{ij}^S = D$ in Equation 32) is maintained.

Due to the constraints, $\frac{V_i^S}{O}$ can be interpreted as the proportion of opportunities accessible *and* available to location i out of the total number of opportunities in the system.

Returning to the simple numeric example, the opportunity-constrained case would yield the following B_j for $f_1(c_{ij})$:

$$
\begin{aligned}
B_j &= \frac{1}{\sum_i O_i f(c_{ij})} \\
B_1 &= \frac{1}{\frac{4}{10^3}+\frac{10}{30^3}+\frac{6}{15^3}} = 162.6506 \\
B_2 &= \frac{1}{\frac{4}{30^3}+\frac{10}{10^3}+\frac{6}{25^3}} = 94.9474 \\
B_3 &= \frac{1}{\frac{4}{10^3}+\frac{10}{25^3}+\frac{6}{10^3}} = 93.9850
\end{aligned}
$$

The balancing factors B_j for the $f_2(c_{ij})$ decay group for zones 1, 2 and 3 is 12.8571429, 8.7685113 and 10.6635071, and for $f_3(c_{ij})$ decay group is 0.0672461, 0.0660559 and 0.0663798. Using these these proportionality constants, we can calculate the singly-constrained opportunity accessibility:

Table 7: Simple system: singly-constrained opportunity accessibility.

		V_i^S		
		$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
Origin	Population (units: *people in 10,000s*)	units: *physicians*	units: *physicians*	units: *physicians*
1	4	133.469	122.255	98.848
2	10	166.781	185.096	241.877
3	6	189.750	182.650	149.275
Sum	—	490	490	490

Imposing the single constraint κ_i^S allows for the comparison of differences and ratios of the accessibility values like previously discussed in the case of the totally-constrained opportunity case. Namely, when the single constraint is applied to the opportunities at j, each resulting accessibility value at i is in units of physicians, with the impedance units already accounted for.

However, unlike the totally-constrained opportunity case, κ_i^S captures competition from the population. Again, let's consider the highest decay scenario $f_1(c_{ij})$. In this case, zone 1 no longer captures an intermediate amount of physicians as in the totally-constrained opportunity case: it now captures the fewest in the region i.e., 133.4687282 at zone 1, 166.7813387 at zone 2, and 189.7499331 at zone 3. Why may zone 1 capture 27% of the physicians regionally while this same zone captures 35% in the total opportunity constrained case? This difference is due to the single-opportunity constraint κ_i^S's role in V_i^S. The only inputs needed in the total-opportunity constraint κ^T is the total number of opportunities in the region D as well as the associated opportunities at each j and travel impedance. In other words, opportunities are proportionally allocated to each i based on the proportion of impedance-weighted opportunities at all j for that i to the total sum of impedance-weighted opportunities in the entire region. Opportunities are allocated to is, regardless of the mass weights of origin. Whereas the single opportunity constraint κ_i^S requires the population at i as an input. In fact, κ_i^S is the proportion of the sum of impedance-weighted population at an i to the sum of impedance-weighted population for the entire region. Hence, since zone 1 has the lowest amount of people in the region, and is not located -travel cost effectively- to other opportunities, V_1^S is lowest.

From a different perspective: V_2^S is intermediate in rank when considering the highest decay scenario ($f_1(c_{ij})$) but is the highest ranked zone under $f_3(c_{ij})$, from 166.7813387 to 241.8772095 accessible physicians out of 490 in the region; a 1.4502654 times increase. Zone 2 has the highest amount of population and the least travel-impedance proximate to other zones. So when the the decay is high - it can most effectively capture opportunities at Zone 2, in fact it captures (κ_2^T) is 0.363834, or 36% of the opportunities in the region, with the vast majority coming from its own zone (87% out of the total opportunities that are allocated to Zone 2). This is unlike other zones, that capture opportunities from their own zones at lower rates (i.e., (κ_1^S corresponds to a capture of 80% of its allocation from Zone 1 and κ_3^S corresponds to 70% of its allocation from Zone 3)).

In scenarios with low decay (e.g., $f_3(c_{ij})$), the connection between opportunities and population changes: spatial impedance among zones is more equal - zonal populations can get to zonal opportunities at relatively more similar travel costs. In this sense, spatial impedance has less relative impact on the effect of zonal population. For this reason, under the low decay scenario of $f_3(c_{ij})$, Zone 2 's competitive population advantage is more evident. Zone 1 captures more opportunities from Zones 1 and 3 than Zone 1 and 3 capture themselves (i.e., Zone 2 captures 16% from both Zone 1 and 3, while Zone 1 and Zone 3 only captures 7% and 11% from itself respectively.

Readers may also notice the change in the proportion of opportunities drawn from different zones depending on the travel scenarios. Again, returning to Zone 2 as an example, while its competitive population advantage is more evident in the $f_3(c_{ij})$ scenario than in higher decay scenarios: this zone does not have an exceptionally large population for the region - Zone 2 only represents 50% of the population in the three zone region. In this sense, other zones are not *that* disadvantaged, and in this scenario with unfettered travel cost, Zones 1 and 3 also take opportunities from Zone 2 (i.e., Zone 1 and Zone 3 takes 6% and 8% more from Zone 2 between $f_3(c_{ij})$ and $f_1(c_{ij})$ scenarios hence $\kappa_{2,2}^S$ decreases by 14%). Zones 1 and 3 are allocated opportunities at relates similar to their relative population size.

In this way, the consideration of accessibility *per capita* may be clarifying. Often, accessibility values are reported as raw scores without the consideration for population. But, as we introduced constraints, these constrained accessibility values can be normalized using anything that is relevant to the zone. In Table 8, we present per capita accessibility for the numeric example, simply in units of number of physicians accessible per population at each zone. These per capita rates are equivalent to the 2SFCA values.

Table 8: Simple system: singly-constrained opportunity accessibility per capita.

Origin	Population (units: *people in 10,000s*)	v_i^S		
		$f_1(c_{ij}) = 1/{c_{ij}}^3$	$f_2(c_{ij}) = 1/{c_{ij}}^2$	$f_3(c_{ij}) = 1/{c_{ij}}^{0.1}$
		units: *physicians per capita*	units: *physicians per capita*	units: *physicians per capita*
1	4	33.367	30.564	24.712
2	10	16.678	18.510	24.188
3	6	31.625	30.442	24.879

Our simple example was designed such that the regional average equals 24.5 physicians per 10,000 people. As the decay decreases and the effect of population is more independent: per capita values begin to stabilise to this regional per capita average, similar to how the accessibility values in the totally-constrained opportunity case stabilizes to the V_i^T regional average. These responses make intuitive sense: the use of constraints are an regional and/or zonal averaging methodology. By equalising the impact of travel impedance, the impact of the remaining constraint variable remains. In the case of the total opportunity constraint, this is the proportion of opportunities relative to the regional opportunities, and in the case of the single opportunity constraint, this is the population at a zone relative to the regional population.

Like the totally-constrained opportunity case, this case offers a solution to the unit interpretability issue of Hansen (1959)'s accessibility measure. Building on this, the singly-constrained opportunity case also offers a way to consider origin-side competition while constraining the opportunities to match the input number of regional opportunities and zonal opportunity totals. This case builds on the previous intuition of totally-constrained opportunity case by providing additional insights from the proportional allocation factor and per capita considerations (that are mathematically equivalent to the 2SFCA, but with additional intuition).

Singly-constrained population: market potential

Similar to Equation 36 in transposing the origins and destinations, we can define a *singly-constrained* measure of market potential that preserves the known population (i.e., the mass weight at the origin $W_i^(1)$ is now represented by O_i). In it's per-capita expression, i.e., equivalent to 2SFCA, this constrained concept of market potential been used to express "facility crowdedness" as in F. H. Wang (2018).

The underlying spatial interaction model is now the production-constrained model in Equation 18, and our market potential measure M_i^S becomes:

$$M_j^S = \sum_i A_i O_i W_j^{(2)} f(c_{ij}) \tag{43}$$

In this case, the measure is singly-constrained by the population *by origin* (i.e., O_i), like Equation 16 from Wilson's framework:

$$\sum_j M_{ij}^S = O_i \tag{44}$$

And the corresponding balancing factor, as per Wilson, is:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \tag{45}$$

Following the same logic as in the preceding section on totally-constrained population accessibility (market potential), one arrives at the following expression:

$$M_j^S = \sum_i \hat{\kappa}_{ij}^S O_i \tag{46}$$

with:

$$\kappa_{ij}^S = \frac{W_j^{(2)} f(c_{ij})}{\sum_i W_j^{(2)} f(c_{ij})} \tag{47}$$

As well, the constraint in Equation 44 ensures that the the total constraint (e.g., $\sum_j M_j^S = \sum_i \sum_j M_{ij}^S = O$) is maintained.

With these constraints, $\frac{M_j^S}{O}$ can be interpreted as the proportion of the total population serviced by location j.

For the sake of brevity, we'll move forward onto the doubly-constrained case.

Doubly-constrained accessibility

The accessibility measures above have used either O_i, D_j, or the regional sums of either, but not both. When D_j, the opportunities, was used to constrain Equation 38, the mass of the population at the origin was given by $W_i^{(1)}$ (also in Equation 39). When O_i, the population, was used as a constraint in Equation 43, the mass at the destination was given by $W_j^{(2)}$ (also in Equation 45). The reason is that the population and the opportunities are typically different things without a common metric. For example, on the side of the population we usually have people, but on the side of the opportunities we can have physicians, clinics, grocery stores, green space, schools, libraries, and so on. There are a few cases where there might be a one-to-one relationship between the population and the opportunities, with the most commonly studied opportunity in the competitive accessibility literature being employment, e.g., one person with one job. Another possible opportunity type is healthcare, e.g., one person and one unit of capacity.

In these latter cases, a doubly-constrained approach can be implemented when one-to-one relationships between population and opportunities are present. This is required because the production- and attraction- constraints in Wilson's doubly-constrained case (Equation 15 and Equation 16) are imposed simultaneously. The accessibility-specific forms are used in the singly-constrained opportunities and population cases in the preceding section (Equation 37 and Equation 44). To simultaneously impose both population- and destination- constraints, they must match (Equation 48):

$$\sum_i O_i = \sum_j D_j \tag{48}$$

Like before, the imposition of these two constraints implies that the total system constraint is maintained e.g., the sum of all doubly-constrained accessibility $\sum_i V_i^D = \sum_i \sum_j V_{ij}^D = D$ in Equation 32 is also maintained, and also equal to the total opportunities in the region O.

To explicitly define the underlying accessibility measure in this case, it follows that of the production-attraction constrained (doubly-constrained) spatial interaction model in Equation 22, hence our accessibility measure becomes:

$$V_i^D = \sum_j A_i B_j O_i D_j f(c_{ij}) \tag{49}$$

where the corresponding balancing factors, as per Wilson, are:

$$A_i = \frac{1}{\sum_j B_j D_j f(c_{ij})}$$
$$B_j = \frac{1}{\sum_i A_i O_i f(c_{ij})}$$

Calibration of the two sets of proportionality constants is accomplished by means of iterative proportional fitting, whereby the values of A_i are initialized as one for all i to obtain an initial estimate of B_j. The values of B_j are used to update the underlying V_{ij}^D matrix, before calibrating A_i. This process continues to update A_i and B_j until a convergence criterion is met (see Ortúzar and Willumsen 2011, 193–95).

The proportional allocation factor κ_{ij}^D would then be:

$$\kappa_{ij}^D = \sum_j \frac{1}{\sum_j B_j D_j f(c_{ij})} \frac{1}{\sum_i A_i O_i f(c_{ij})} O_i f(c_{ij})$$

{eq-doubly-constrained-proportional-allocation-factor-forpop}

Rewriting Equation 49 as an origin summary expression of proportionally allocated opportunities:

$$V_i^D = \kappa_{ij}^D \sum_j D_j \tag{50}$$

Unlike in the totally- and singly-constrained cases, representing V_i^D per capita is not as meaningful, as the the value *already* matches population to opportunities, e.g., the number of opportunities accessed by the population. Similarly, the market potential form M_j^D would be the transposed way of summarizing V_{ij}^D. In other words, the inputs of 'opportunities accessed' and 'accessed population' can already be interpreted as inherently being sensitive to both opportunities and population. As opportunities match population and both marginals are constrained, M_j^D would simply be a summary of V_{ij}^D values at each j, and would represent the number of people accessible to opportunities. The formulation is as follows:

$$M_j^D = \sum_i A_i B_j O_i D_j f(c_{ij}) \tag{51}$$

Let's return to the numeric example with a new objective. Say, we're interested in measuring the zonal values of accessed physician-capacity. And we have new information: given the same zonal populations, a region-wide survey indicates the number of people that live in each zone, and the number of physician capacity per zone. The survey does not tell use what person is matched with what physician (assume that's confidential), we only have the numbers of population at i and the number of physician-capacity at j. Lucky for us, this survey keeps our example simple: we find out that the physician-capacity at each zone is a approximately scaled version of the number of destination-side physicians at each zone. With this new information, we are interested in understanding what does the zonal access to physician-capacity for the population looks under different travel behaviour scenarios. Table 9 summarises this adjusted simple example: with the population (in units of 10,000s of people) and the opportunities (in units of 10,00s of physician-capacity) per zone . Considering this new definition of 'provider', the new (realized) Provider-to-Population-Ratio is simply 1. The zonal cost matrix, and travel impedance functions for three types of travel behaviour are the same as before (Table 3 and Equation 27).

Table 9: Modified simple system with three zones reflecting matched population and opportunities. Population is in 10,000 persons and opportunities in 10,000 of physician-capacity.

ID (i or j)	Population	Opportunities
1	4	7
2	10	5
3	6	8

In this case, V_{ij}^D for the most decay travel behaviour scenario is presented in Table 10.

Table 10: Adjusted simple system: doubly-constrained opportunity accessibility for the most decay travel behaviour scenario.

		Destination ID			
	Origin ID	1	2	3	sum
	1	3.235859	0.01032226	0.7556568	4
	2	2.132602	4.95932483	2.9044391	10
	3	1.631539	0.03035291	4.3399040	6
Sum	—	7	5	8	—

As mentioned, accessibility can be interpreted as a summary of the proportionally allocated opportunities at each i. In interpreting the doubly-constrained accessibility from Table 10, V_i^D, these values would be 4.0018381, 9.9963656, and 6.0017963 physician-capacity for Zones 1, 2 and 3 respectively, approximately equal to the number of population at each of these zones. Conversely, M_j^D values would be 7, 5, and 8 people accessible from Zones 1, 2 and 3 respectively, equal to the number of *opportunities* (physician-capacities) at each of these zones. And that's precisely the function of the double constraint - to have the mass weight at the origin equal the mass weight of the destination. In other words, V_i^D is simply accessed opportunities and M_j^D population accessed. And following the same logic, the same V_i^D and M_j^D values are calculated for the other two travel behaviour scenarios.

What may be interesting are the differences in V_{ij}^D values between travel behaviour scenarios, these values can be directly compared to discuss their mass and distance decay impacts. Returning to Zone 2, Table 11 demonstrates these i to j access values for our more relatively remote, higher-populated and lower-opportunity rich zone. It can be observed that the number of intrazonal opportunities proportionally allocated decreases as the assumed distance decay decreases e.g., from 4.9593248 to 2.6672837 out of the ~10 opportunities allocated to Zone 2 (a population of 10). Following the intuition discussed in the singly-constrained opportunity case, as decay decreases, the mass effect of the population (at origin) and opportunity (at destination) is more evident: zonal opportunities are supplied and zonal populations demand at the weights assigned to these zones, with minimal decay adjustment, reflected by the proportional allocation factors κ_{ij}^D.

Table 11: Adjusted simple system: doubly-constrained opportunity accessibility for all travel behaviour scenarios for Zone 2.

			$V_{\{ij\}}^D$		
			$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
Dest.	Population at 2 (units: *people in 10,000s*)	Opportunities (units: *capacity in 10,000s*)	units: *physician-capacity in 10,000s*	units: *physician-capacity in 10,000s*	units: *physician-capacity in 10,000s*
1	10.000	7.000	2.133	2.272	3.411
2	10.000	5.000	4.959	4.766	2.667
3	10.000	8.000	2.904	2.958	3.919

Recall that accessibility is defined as "the potential for interaction" and is traditionally presented as a summary zonal measure. In the doubly-constrained case, we force the zonal population and zonal opportunities to match one-to-one, hence providing a zonal summary is no longer relevant: the sum of V_{ij}^D for all is ends up being equal to the population at the zone. However, if what interests readers is the "potential for interaction" in the case population and opportunities do match one-to-one, perhaps reframing the investigation is needed. In this sense, it would be examining "interaction" (much less room for potential) through the values of V_{ij}^D e.g., how many opportunities are being allocated to an origin from a destination (or in a transposed sense for M_{ij}^D). In this sense, the doubly-constrained case it better thought of as an estimate of "access" instead of "accessibility". It is also formulaically identical to the doubly-constrained spatial interaction model, but with specific interpretations of the origin and destination weights as 'population' and 'opportunity' respectively.

As Wilson explicitly noted, origin and destination weights defined in the spatial interaction model *can* be defined using any unit. Accessibility, however, is a summary of *potential* for interaction: it's typically been used and understood as a zonal summary. Arriving at the doubly-constrained case through the unconstrained, totally, and singly-constrained case make the connection between the *potential* for spatial interaction (accessibility) and spatial interaction (access) more clear(?).

Discussion

SUMMARY

By introducing constraints, we fold in competition considerations into a family of accessibility measures. While previous methodologically clarifying work, such as Wu and Levinson (2020), has largely overlooked competition, this paper demonstrates how applying constraints consistent with the spatial interaction model allows for incorporating competition into accessibility. The constraints used depends on the research question, the suitability of assumptions, the granularity of data, but ultimately: the addition of constraints re-introduces unit into accessibility, make comparisons between scenarios and regions more intuitive.

This work builds on spatial interaction modeling principles, following a review of earlier spatial interaction literature. We show that, while recent accessibility research has diverged from its spatial interaction foundations, it can be explicitly linked back. We demonstrate that accessibility can be consistently calculated for an i-j pair by stacking constraints that preserve the units of "accessibility" while maintaining zonal or regional constraints.

We first place the popular Hansen accessibility measure within this family of measures as an "unconstrained" case, demonstrating that resulting values cannot be directly compared across different travel scenarios without ad-hoc adjustments. We then show how applying a total constraint balances the units and produces a statistically averaged solution that converges to the regional average for each zone as the decay effect decreases. In other words, the total-constraint model could be a more interpretable alternative for the unconstrained case if regional competition is relevant; specifically, if there is a fixed number of opportunities in the region, and if it makes sense to assume that people accessing proximate opportunities leave fewer for others, *without* considering the population size at the origins.

We then introduce the singly-constrained case, which *does* takes into account the population size at the origin in the allocation of opportunities (unlike the total constraint). In this case, all accessibility values are fixed to sum to a known zonal opportunity-size value (implicitly, the regional total of opportunities), but they are not required to sum to any population-based values at the zone or regional level. The singly-constrained model could be useful if regional competition is a factor and if the acknowledgment that only a finite number of opportunities can be allocated from each destination (with those allocations distributed based on origin population size) is suitable. We also introduce an 'accessible' PPR (e.g., opportunities per capita), calculated by dividing each accessibility value by the zonal population. To clarify, this per capita expression of the singly-constrained case is equivalent to the 2SFCA, hence linking this literature back to spatial interaction principles.

Lastly, the doubly-constrained case is introduced. In this case, the sums must equal both the regional total and ensure that no zone allocates more opportunities than it has available. Specifically, accessibility values for each i-j pair must be a proportion of he zonal opportunity and population values simultaneously. For example, the accessibility at zone 1 must equal the sum of opportunities from zones 1, 2, and 3, as well as the sum of the population at zone 1. Satisfying the double constraint means the opportunities and population data must match one-to-one, so working with the accessibility i-j pair values should be of interest. In this sense, the research question should be concerned with realized access (how many opportunities accessed from j at i based on given zonal opportunities and populations) instead of potential 'access' (e.g., typically expressed as a zonal summary measure of how many opportunities one could reach (out of a regional total and/or zonal-allocation).

A note on interpretation and possible applications

When to use each of these measures? Embedded assumptions? and dangers of using one over the other?

Conclusions

Accessibility development has a rich history. There have been developments in the cases/breath of travel impedance functions, competition considerations, and in incorporating individual choice as part of activity-based accessibility. However, no work yet has examined accessibilities explicit connecting to SIM. We examine this connection, and provide a origin-destination (i to j) formulation of accessibility as part of the SIM framework. Novel stuff.

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