A family of accessibility measures derived from spatial interaction principles

Anastasia Soukhov[1], Rafael H. M. Pereira[2], Christopher D. Higgins[3*], Antonio Páez[1]

1 McMaster University, School of Earth, Environment & Society, Hamilton, Canada,
2 Institute for Applied Economic Research - Ipea, Data Science Division, Brasília, Brazil,
3 University of Toronto Scarborough, Department of Human Geography, Toronto, Canada,

☯These authors contributed equally to this work.

¤Current Address: Dept/Program/Center, Institution Name, City, State, Country

†Deceased

¶Membership list can be found in the Acknowledgments sections

* cd.higgins@utoronto.ca

Abstract

Transportation planning has long prioritized the efficiency of movement. However, the concept of accessibility represents a more comprehensive evolution, shifting focus from movement to the potential to reach–or spatially interact with–desired destinations. Despite growing recognition of accessibility-based planning approaches, the concept remains fragmented, with inconsistent definitions and unclear interpretations. To this end, this paper offers a methodological contribution by specifying a family of accessibility measures grounded in the shared theoretical roots of gravity-based accessibility and spatial interaction models, particularly their balancing factors. From this foundation, we outlined four memebers of the family: the 'unconstrained' measure (i.e., Hansen-type accessibility), the 'total constrained' measure (i.e., a constrained version of the Hansen-type accessibility), the 'singly constrained' measure (i.e., related to the popular two step floating catchment approach - 2SFCA), and the 'doubly constrained' measure representing realized access (i.e., equal to the doubly constrained spatial interaction model). These measures can be interpreted as either the number of accessible opportunities or accessible population (i.e., market potential). A toy example illustrates how they produce interpretable, unit-based values, offering a clearer and more coherent basis for accessibility analysis.

Introduction

In the early nineteenth century, rapid urbanisation driven by industrialisation expanded cities across Great Britain, Northern Europe, and North America. This expansion led to traffic congestion and the emergence of a transportation planning paradigm focused primarily on mobility. Within this new practice, access to destinations was treated as a by-product of movement. After World War II, major investments in automobile and transportation infrastructure cemented this mobility-oriented model, fostering flight from urban congested cities, lower-density sprawl, car-dependent development and entrenching automobility in planning practice [1, 2]. Despite continued road and highway expansion, this automobility monoculture has proven ineffective at reducing travel costs or environmental burdens, and has not clearly improved people's ability to reach destinations [3, 4, 5].

In response, transportation researchers have increasingly advocated for the adoption of accessibility as a planning criterion, in contrast to traditional mobility-oriented transportation planning approaches which translate into indicators that benchmark movement (e.g., vehicle kilometres traveled, intersection through traffic, etc.) which are not necessarily linked to improved accessibility [6, 7, 8, 9]. Accessibility, by contrast, is the "potential of opportunities for [spatial] interaction" [10]. While mobility reflects movement, accessibility captures the combined influence of transport and land use, emphasizing destinations and the potential to reach them [11].

Accessibility research has expanded across diverse domains including: employment [12, 13, 14, 15, 16], healthcare [17, 5, 18, 19, 20, 21], green space [22, 23, 24], education [25, 26, 27], social contact [28, 29, 30], and regional economics [31, 32, 33, 34], among many other domains of application. Despite its popularity in scholarly works, accessibility still remains difficult to implement in planning due to definitional inconsistencies [35, 8, 36] and challenges in interpreting and communicating results [37, 35, 38].

More specifically, the wide range of accessibility definitions–with novel methods being more sophisticated but less intuitive [36]–can further hinder practical uptake [35]. Geurs and van Wee [37] classify accessibility measures into four categories: infrastructure-, place-, person-, and utility-based. Among place-based measures (this work's focus), variants include gravity-based [10], cumulative opportunity [39], the 2 Step Floating Catchment Area (FCA) approach [17], and a variety of modifications to these approaches e.g., Enhanced 2-Step FCA [40], 3-Stage FCA [18], Modified 2-Step FCA [19], inverse 2-Step FCA [41], and n-steps FCA [24]. While these methods are tailored to address specific research contexts, overall this diversity does not demystify existing questions like those raised in [35]: How should practitioners interpret difference in scores between modes, and how should results be communicated?

Rather than propose a new measure, this work argues for a return to the spatial interaction foundations of accessibility. Specifically, we show how the family of spatial interaction models [42] can be reformulated in the context of accessibility, namely as a "family of accessibility measures". This formulation results in constrained versions of gravity-based accessibility [e.g., 10], but in units of *spatially reachable opportunities*. This approach offers a direct mathematical link to existing accessibility measures while restoring tangible meaning to zonal values. Instead of abstract proportional scores, constrained accessibility expresses the number of opportunities a population may potentially spatially interact with.

In summary, this paper makes two contributions: (1) Reviews how spatial interaction modeling and accessibility share similar origins, but diverged in focus and interpretation. (2) Introduces a family of accessibility measures grounded in spatial interaction principles, including total, singly, and doubly constrained cases and variants of "accessible opportunities" and "accessible population". These cases and variants align with common measures such as Hansen-type accessibility [10], competitive accessibility measures such as the 2SFCA method [43, 17], and market potential models [44, 45].

We contend that accessibility research should re-engage with spatial interaction modeling, particularly the use of Wilson's system constraints. While spatial interaction models embraced such constraints to improve interpretability, most accessibility models have not. This lack of adoption contributes to fuzziness in current analyses, limiting interpretive clarity to simple proportional comparisons (e.g., “higher-than”, “lower-than”) [46]. Without such constraints, accessibility scores lack clear units and comparability across cities or modes. In contrast, constrained measures yield values that can be tied to tangible values without any post-hoc treatment, theoretically making them more interpretable, communicable, and actionable in planning.

Accessibility and spatial interaction modelling literature share common headwaters, and the latter has given careful attention to measurement units and their interpretability. Following this introduction, this paper revisits that lineage–from Ravenstein’s Newtonian gravitational expression of spatial interaction [47], through Hansen [10] to Wilson [42]–to propose a unified family of constrained accessibility measures. We illustrate each case of the family with a numerical toy example and conclude by discussing how these constrained measures can inform planning decisions and improve clarity in accessibility analysis.

Newtonian’s roots of human spatial interaction research

The patterns of people’s movement in space have been a subject of scientific inquiry for at least a century and a half, from as far back as Henry C. Carey’s *Principles of Social Science* [48]. It was in this work where Carey stated that “man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance” [49, pp. 37-38]. This statement shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton’s Law of Universal Gravitation, first posited in 1687’s *Principia Mathematica* and expressed as in Eq 1.

$$F_{ij} \propto \frac{M_i M_j}{D_{ij}^2} \tag{1}$$

This famous equation expresses that the attractive force F between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. As mass increases, so does force; as distance increases, force decreases. However, Eq 1 only demonstrates a *proportional* relationship. To quantify the magnitude (not the *proportional* magnitude) of the force, it must be *constrained* with an empirical constant. This constant G converts Eq 1 from an expression of proportionality to the following expression of equality:

$$F_{ij} = G\frac{M_i M_j}{D_{ij}^2} \tag{2}$$

Where G is the gravitational constant–an empirically calibrated value that ensures the model reflects observed forces. Newton’s initial estimate of G was based on a speculation that the mean density of earth was between five or six times that of water, an assumption that received support after Hutton’s experiments of 1778 [50, p. 783] and Cavendish’s 1798 experiment [51] to estimate of the proportionality constant to within 1% accuracy. In sum: it took over a century from the publication of *Principia* to refine the estimate of the proportionality constant.

The Newtonian gravitational relationship laid the conceptual groundwork for later empirical studies of human spatial interaction, such as those by Ravenstein, Stewart, and others, who began translating these principles into observed patterns of human movement. However, the majority of these attempts described a proportional relationship, or one arbitrarily set to equality.

Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Following Carey's *Principles of Social Science* [48], research into human spatial interaction continued in different contexts. Namely, since Carey to the 1940s, a number of researchers theoretically and empirically attempted to characterise human spatial interaction as a force F directly proportional to the "masses" M_i and M_j of two locations, and inversely proportional to their separation distance–conceptually parallel to Newtonian gravity, but often omitting a proportionality constant.

Beginning with Ravenstein in the late 1880s, his works proposed some "Laws of Migration" based on his empirical analysis of migration flows in various countries [52, 47]. These works posited 1) a directly proportional relationship between migration flows and the attractive size of destinations, and 2) an inversely proportional relationship between the size of flows and the separation between origins and destinations. As with Carey, these propositions echo Newton's gravitational laws.

Over time, other researchers discovered similar relationships. For example, Reilly [53] formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations that could be understood as 'potential trade territories'. Later, Zipf proposed a $\frac{P_1 P_2}{D}$ hypothesis for the case of information [54], intercity personal movement [55], and goods movement by railways [56]. The $\frac{P_1 P_2}{D}$ hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

A common feature of these early investigations of human spatial interaction is that a proportionality constant, similar to G in Eq 2 was never considered. Of the researchers cited above, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly's hypothesis was presented in the following form:

$$B_a = \frac{(P_a P_T)^N}{D_{aT}^n} \tag{3}$$

where B_a is the amount of business drawn to a from T, P_a and P_T are the populations of the two settlements, and D_{aT} is the distance between them. Quantity N was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of $n = 2$.

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

$$C^2 = \frac{P_1 P_2}{D_{12}} \tag{4}$$

where C is the volume of goods exchanged between 1 and 2, P_1 and P_2 are the populations of the two settlements, and D_{12} is the distance between them.

These early formulations (Eq 3, Eq 4) clearly reflect the influence of Newtonian gravity on human spatial interaction theory, revealing a shared mathematical structure across migration, trade, and communication models. However, none included a proportionality constant–highlighting a consistent omission of the empirical calibration necessary to convert these proportional relationships into measurable and comparable quantities.

After Carey and folowing Reilly and Zipf, it is only in Stewart [57] that we find the most explicit connection yet to Newton's Gravitational law and the use of a proportionality constant. Besides awareness of preceding research (i.e., Stewart cites both Reilly and Zipf as predecessors in the analysis of human spatial interaction), Stewart appears to have been the first author to express his theorized relationships for human spatial interaction with a proportionality constant G, enabling his formulation to be interpreted as a measurable 'demographic' force:

$$F = G\frac{(\mu_1 N_1)(\mu_2 N_2)}{d_{12}^2} = G\frac{M_1 M_2}{d_{12}^2} \tag{5}$$

Where:

- F is the *demographic force*
- N_1 and N_2 are the numbers of people of in groups 1 and 2
- μ_1 and μ_2 are so-called *molecular weights*, the attractive weight of groups 1 and 2
- $M_1 = \mu_1 N_1$ and $M_2 = \mu_2 N_2$ are the demographic masses at 1 and 2
- d_{12}^2 is the distance between 1 and 2
- And finally proportionality constant G

What is notable about Eq 5, however, is the proportionality constant G was specified but "left for future determination" [57, p. 34]. We can infer that it is crucial for ensuring F is maintained in some units of demographic force.

In addition to demographic force F, Stewart defined a measure of the "potential" of group 2 with respect to group 1. The partial sum of the demographic force experienced by group 1, or the potential number of people from location 2 that could visit location 1, as $V_1 = G\frac{M_2}{d_{12}}$. For a system with more than two population bodies, Stewart formulated the population potential at i by summing the contributions from each group j, after arbitrarily setting $G = 1$:

$$V_i = \sum_j M_j d_{ij}^{-1} \tag{6}$$

Where M_j is the demographic mass at location j and d_{ij} is the distance between i and j. A version of this discrete form is what was used in Hansen [10], going on to become a foundation of modern accessibility definitions, as will be discussed.

Although Stewart's concept of "social physics" eventually fell out of favour–due to inconsistent mathematical notation (e.g., G is used as both a proportionality constant p. 34 and then later as *demographic energy* on p. 53.) and its racist and unscientific assumptions (e.g., view of humans as particles following physical laws and assumptions of the molecular weight μ of the average American being one, but "presumably…much less than one….for an Australian aborigine" [p. 35])–his introduction of a proportionality constant G in modeling demographic force marks an important conceptual step: recognizing that moving from proportionality to equality requires empirical calibration. In other words, the addition of G shifts results from being abstract indicators of potential (i.e., $\frac{\text{people}^2}{\text{distance}^2}$) to having interpretable units grounded in consistent, albeit still abstract, quantities (i.e., units of demographic force).

As will be discussed, when Hansen [10] later adopted Stewart's formula (Eq 6) for accessibility, he omitted any mention of G, effectively setting it to 1 arbitrarily as Stewart had done. This omission has persisted in accessibility research, leaving a conceptual gap in how such measures are interpreted and compared.

Hansen's gravity-based accessibility to today

From Stewart [57], we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature [10]. In this seminal paper, Hansen defined accessibility as "the potential of opportunities for interaction… a generalization of the population-over-distance relationship or *population potential* concept developed by Stewart [57]" (p. 73). As well as being a student of city and regional planning at the Massachusetts Institute of Technology, Hansen was also an engineer with the Bureau of Roads, and preoccupied with the power of transportation to shape land uses in a very practical sense. Hansen [10] drew directly from Stewart's population potential formula (see Eq 6), but left aside the broader (and often problematic) framework of "social physics".

Hansen recast Stewart's population potential to reflect accessibility, a model of human behaviour useful to capture regularities in mobility patterns. Hansen replaced M_j in Eq 6 with *opportunities* to derive an *opportunity potential*, or more specifically, a *potential of opportunities for interaction* as $S_i = \sum_j \frac{O_j}{d_{ij}^{\beta}}$ with a contemporary rewriting to accounts for a variety of impedance functions beyond the inverse power $d^{-\beta}$ as follows:

$$S_i = \sum_j O_j f(d_{ij}) \tag{7}$$

S_i in Eq 7 is a measure of the accessibility from site i. This is a function of O_j (the mass of opportunities at j), d_{ij} (the cost, e.g., distance or travel time, incurred to reach j from i), and β (a parameter that modulates the friction of cost). Today, Hansen is frequently cited as the father of modern accessibility analysis [e.g., 58], and Hansen-type accessibility is commonly referred to as the gravity-based accessibility measure.

However, Hansen's formulation carried forward a crucial omission that continues to affect the literature: the proportionality constant G included in Stewart's original formulation (Eq 6) has vanished entirely. Although Stewart included G explicitly (with a note that "G [was] left for future determination: a suitable choice of other units can reduce it to unity" [p. 34].) Hansen made no mention of it. As a result, modern accessibility analysis has largely evolved without addressing the constant's role, leaving G effectively and implicitly fixed at 1. This omission has significant implications. Without a proportionality constant, the accessibility formula expresses only a proportional relationship: $S_i \propto \sum_j g(O_j) f(d_{ij})$, not of calibrated equality. Recognition of the nature of this relationship is not common in the literature, but is known, i.e., this proportional equation is shown in Figure 1 in Wu and Levinson [59].

Furthermore, working with a proportional relationship generates fundamental issues in comparability between and, arguably within, studies. Namely, accessibility estimates have no fixed unit, rendering them sensitive to the choice of impedance functions. For instance, if travel cost d_{ij} is measured in meters, then when the travel impedance function $f(d_{ij})$ equals $d_{ij}^{-\beta}$, the resulting S_i has units of opportunities per metres$^{\beta}$. However, when $f(d_{ij})$ is set to equal $e^{-\beta d_{ij}}$, the units become opportunities per $e^{\beta \text{metres}}$.

Such variation impairs comparability across analysis and obscures the meaning of accessibility scores, making them difficult to understand and communicate without post-hoc treatment.

Therefore in practice, Hansen-type measures are ones of proportionality and are better understood as *ordinal indicators*; they rank accessibility but lack cardinal meaning or consistent units [46]. The continued absence of a sort of proportionality constant G leaves a conceptual and practical gap in accessibility analysis: a missing link between theoretical form and empirical measurement.

Wilson's family of spatial interaction models

While accessibility research evolved in North America with Hansen [10], a parallel development was taking place across the Atlantic with Alan G. Wilson. Wilson's ground breaking paper [42], advanced a general framework for spatial interaction modeling–focused on modeling flows of interaction between places. This work was not focused on the 'potential' concept as accessibility has been. Wilson [42] formalized the general spatial interaction model through the following equation:

$$T_{ij} = kW_i^{(1)}W_j^{(2)}f(c_{ij}) \tag{8}$$

The model in Eq 8 posits a quantity T_{ij} that represents a value in a matrix of flows of size $n \times m$, that is, between $i = 1, s, n$ origins and $j = 1, s, m$ destinations. The quantities $W_i^{(1)}$ and $W_j^{(2)}$ are proxies for the masses at $i = 1, s, n$ origins and $j = 1, s, m$ destinations. The super-indices (1) and (2) indicate that these masses can be any number of different things associated with the zones, i.e., $W_i^{(1)}$ could be population at a zone as an origin, and $W_j^{(2)}$ hectares of park space at a zone as a destination. $f(c_{ij})$ is some function of travel cost c_{ij} which reflects travel impedance.

Of important note for this paper, k in Eq 8 acts as a proportionality constant, shifting the equation from a proportional to an equal relationship by incorporating known system totals. In this sense, we infer that k serves a role similar to the gravitational constant in Newton's law–it calibrates the model so that outputs match real-world quantities. However, these real-world quantities was not a set empirical constant (like Newton's G) but instead, sensitive to the system and known information about the system.

From the outset, spatial interaction models emphasized interpretability of results [60, 61, 42]. But unlike earlier approaches that borrowed heuristically from Newtonian gravity (i.e., interaction between masses over distance), Wilson's innovation was to ground the model in *entropy maximisation*. By maximizing the number of ways individual trip probabilities could be arranged under known constraints, Wilson derived models that estimate *statistical averages* of flows between zones [42, 62].

Crucially, to ensure that T_{ij} in Eq 8 is maintained in units of flow between i and j, the model moves from proportionality to calibrated equality by incorporating empirical constraints. At a minimum, this requires knowledge of the total number of flows T in the system, leading to the basic constraint:

$$\sum_i \sum_j T_{ij} = T \tag{9}$$

Additional information can be introduced. For example, when information is available about the total number of flows produced by each origin, $W_i^{(1)}$ in Eq 8 is represented as O_i and the following constraint can be used:

$$\sum_j T_{ij} = O_i \tag{10}$$

As well, if there is information available about the total number of flows attracted by each destination, $W_j^{(2)}$ is represented as D_j and the following constraint can be used:

$$\sum_i T_{ij} = D_j \tag{11}$$

It is also possible to have information about both O_i and D_j, in which case both constraints can be imposed on the model at once.

Depending on which of the three system constraints are applied a family of spatial interaction models can be derived from Eq 8. The proportionality constant k is replaced with different *balancing factors*. This change in name is more useful as these factors not only preserve proportionality but also ensure that the predicted flows T_{ij} align with the known constraints considered in the system. In other words, the different balancing factors adjust the model so that flows become statistical averages consistent with observed origin and/or destination data. In the framework introduced and inferred from Wilson [42], three types of balancing factors are specified: (1) an unconstrained model that only matches the total volume of interaction K, (2) a singly constrained model (either by origins A_i or destinations B_j), and (3) a doubly constrained model that satisfies both.

In the unconstrained model, constraints in Eq 10 and Eq 11 do not hold. In practical terms, this means that the total number of flows predicted by the model must be equal to sum of all flows from origins i to destinations j. The balancing factor K takes the place of k and is equal to the following (as specified in [63] and [64]):

$$K = \frac{T}{\sum_i \sum_j T_{ij}} \tag{12}$$

In the singly-constrained model, only constraint Eq 10 or constraint Eq 11 hold. When only Eq 10 holds, entropy maximisation leads to the production-constrained singly-constrained version of Eq 8, where the proxy for the mass at the origin $W_i^{(1)}$ is replaced with O_i. Also, k is replaced with a set of balancing factors specific to origins A_i which ensures that constraint Eq 10 is satisfied (i.e., the sum of predicted flows from one origin going to all destinations must equal the known mass at that origin O_i). Satisfying this constraint also implicitly fulfills the total constraint (Eq 9), since the sum of O_i values across all origins equals the total number of flows. A_i takes the following form:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \tag{13}$$

The singly-constrained attraction-constrained model is similar to the production-constrained version but from the perspective of the mass at the destination. For the attraction-constrained model, the proxy for the mass at the destination $W_j^{(2)}$ is replaced with D_j in Eq 8, representing the spatial interaction inbound flow. Also, k is replaced with a set of destination-specific balancing factors B_j that ensure that Eq 11 is satisfied (hence the total constraint Eq 9 is as well), meaning that the sum of predicted flows going to one destination from all origins must equal the known mass of that destination D_j. As before, destination-specific balancing factors B_j were derived by Wilson as:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \tag{14}$$

Lastly, the doubly-constrained model is the production-attraction constrained model in Wilson [42]. In this case, both constraints Eq 10 and Eq 11 hold simultaneously. These constraints ensure that the sum of predicted flows from one origin to all destination, and the predicted flows going to one destination from all origins must equal the known mass of the origin O_i and of the destination D_j. This should hold for all origins and destinations. The resulting model is, in Wilson's terms, doubly constrained, and from Eq 8, k becomes both A_i and B_j shown in Eq 15, and $W_i^{(1)}$ and $W_j^{(2)}$ is replaced with O_i and D_j. Derivation of these models is demonstrated in detail elsewhere [e.g., 65, 61].

$$\begin{aligned} A_i &= \frac{1}{\sum_j B_j D_j f(c_{ij})} \\ B_j &= \frac{1}{\sum_i A_i O_i f(c_{ij})} \end{aligned} \tag{15}$$

Wilson's work is notable for many reasons, one of which is his departure from the metaphor of Newtonian gravitational physics. Instead, he grounded his models in statistical mechanics, particularly the principle of entropy maximisation. For accessibility research, Wilson's approach is especially significant because it can address a problem that has quietly afflicted this field (as this paper has argued): the transition from proportionality to equality.

Rather than relying on a universal constant like G, Wilson's models calibrate interaction flows through known empirical constraints (e.g., total system flows, or origin-side and destination-side flow totals). This results in interpretable, balanced (given the constraints), and unit-consistent ij flows. In short, when aspects of the systems are inputs into the balancing factors, ij flows can be robustly predicted in tangible units of 'flow', units that align with the system being modelled. In this way, spatial interaction modeling tradition can be seen to have succeeded where accessibility modeling stalled. While Wilson's model produces results that are in units of flow tethered to the system of analysis, Hansen-type measures (still widely used today) yield outputs that *indicate* a proportional relationship and not one of equality nor yielding results without interpretable units.

As a preview for the work ahead: Wilson's balancing factors can be manipulated into *proportional allocation factors* and equations can be established to reflect accessibility. The resulting ij flows can be understood as accessibility flows–and aggregated at the zone level to express accessibility in units of actual opportunities or population.

In this way, these balancing factors can be used to recover what Stewart's G was meant to accomplish, but through a different conceptual path. But before demonstrating a family of accessibility measures that are sensitive to constraints, in the next section we review how conceptually intertwined the spatial interaction modelling and accessibility literatures are, and where they began to diverge. This investigation sheds light on why accessibility research may have failed to adopt a comparable approach focused, until this paper.

Accessibility and spatial interaction modelling: two divergent research streams

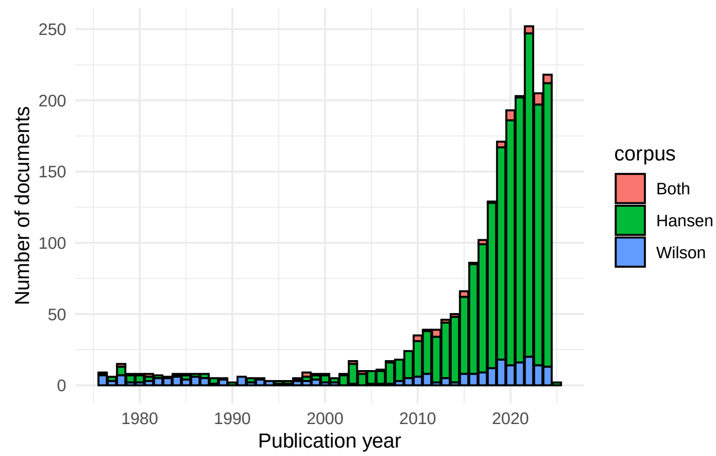

Figure 1. Historical pattern of publication: documents per year.

Despite their close conceptual ties, the accessibility and spatial interaction modeling literatures have developed along largely separate paths since the 1970s. Hansen's [10] formulation of accessibility became the method used for decades of work on transport equity, land use analysis, and urban accessibility planning. Meanwhile, Wilson's [42] entropy-maximising framework reshaped how spatial interaction models were constructed, particularly in transportation demand forecasting. We argue the framework's quiet innovation–introducing empirically grounded constraints to shift from proportionality to calibrated equality–made the framework immediately relevant for policy applications as outputs were in tangible units. Yet, this mechanism was never widely adopted in accessibility analysis.

This divergence is especially striking given the context in which both frameworks emerged. As noted in [66], large scale spatial interaction models (like Wilson's) responded to important developments at the time, a need "to meet the dictates and needs of public policy for strategic land use and transportation planning". And these needs were far from trivial: in the U.S., for instance, the Federal-Aid Highway Act of 1956 set in motion the construction of the Interstate Highway System with an eventual budget exceeding $600 billion in today's dollars [67, 68]. In this context, spatial interaction models were incorporated into institutional practices focused on "predict and provide" travel demand forecasting [69, 67]. Accessibility analysis, by contrast, remained more conceptually diffuse, focused on indicators of "potential" spatial interaction with opportunities rather than flows that could tangibly guide infrastructure decisions (e.g., roadway capacity expansion, new construction). Whereas spatial interaction modelling became a key element of transportation planning practice, accessibility remained a somewhat more academic pursuit, and the two streams of literature only rarely connected.

To explore why Wilson's approach never crossed over to accessibility modeling, we conduct a review of the literature citing Hansen [10], Wilson [42], or both (on Web of Science using the "Cited References" functionality, and the digital object identifiers of Hansen [10] and Wilson [42]) . Only 76 out of the 2,122 documents that emerged from our search cite both. The number of articles, by year and if they cite Hasen, Wilson, or both are shown in Fig 1.

Through the close analysis of *how* articles that cite both works, we identify two distinct citation patterns: one group of articles focused on accessibility, the other on spatial interaction. In examining these articles, we uncover how the relationship between the two has often been misunderstood, underexplored, or entirely overlooked.

In the first stream of literature–which cite both but are focused more so on spatial interaction models–they treat spatial interaction and accessibility as separate but related phenomenons. Four subsets of this stream emerge.

First, some of the more early works interpret the spatial interaction model's balancing factors (Eq 13 or Eq 15) as the inverse of Hansen's accessibility measure [70, 71, 72, 73], likely following Wilson's own recognition of this similarity between balancing factor A_i and Hansen-type measure S_i on p. 10 in Wilson [42]. In some ways, this relationship has been recognized as a "common sense" approach to incorporating accessibility in the spatial interaction model [74, p. 99], though acknowledgment of its further exploration has been recommended [75].

The second subset of articles within this stream use both Hansen [10] and Wilson's [42] framework in conjunction. For instance, some articles argue that spatial interaction models fail to explain certain spatial patterns on their own, for instance, as in Fotheringham [73] who demonstrates how the spatial interaction model may insufficiently explain spatial patterns, and suggests that explicitly defining destinations' accessibility (Hansen-type accessibility) as a variable within the model may remedy the issue (e.g., the *competition destination* model). Other works take a more applied approach: such as in defining location-allocation problems in operations research [71, 76], estimating trips (or some other spatial interaction flows) alongside accessibility [e.g., 77, 78, 79], or considering accessibility as a variable within spatial interaction models, in line with Fotheringham's [73] demonstration [e.g., 80].

The third subset of the spatial-interaction focused literature, depart from Hansen's [10] definition, aligning instead with microeconomic or utility-based interpretations of potential spatial interaction e.g., [74, 81]. In sum though, across these works, they recognize Hansen-type accessibility as an indicator of 'potential' but as a separate but related concept to spatial interaction.

Moving onto the group of accessibility-focused literature that cites both works, we categorise their citation of Wilson [42] within three general groups. Overall though, these works do not engage, or only superficially engage with Wilson [42].

Firstly, there is a group of articles within this stream that cite Wilson [42] exclusively as attribution for using context-dependent travel cost functions. This trend is common: for instance it is done in [82, 11, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95]. However, these works do not engage with spatial interaction beyond this attribution.

Secondly, a subset of literature explicitly associates spatial interaction, as defined in Wilson [42], with accessibility's potential for spatial interaction–but only superficially. These works acknowledge the conceptual link, but do not go beyond this recognition in the scope of their works e.g., [96, 97, 98, 13, 99, 100, 101, 102, 103, 104, 59, 105, 106, 107]. Indeed, while accessibility can be seen as the *potential* for spatial interaction–and Wilson [42] briefly touches on this–such mentions have not resulted in deeper analytical integration of these concepts. Furthermore, some of this literature also occasionally conflates or blurs the distinction entirely, for instance by co-citing Hansen and Wilson as being 'gravity models' [e.g., 108, 109, 110, 92]. This conflation reveals ongoing murkiness between the distinction of spatial interaction and the *potential for* spatial interaction in the literature.

Thirdly, there is a group of accessibility-focused works that interprets the measure used in Hansen [10] as the singly- or doubly- constrained spatial interaction model's inverse balancing factor [e.g., 45]. This group often cites the spatial interaction works that make this connection (i.e., the first subset of the first stream of literature) and is especially prominent in the investigation of competitive accessibility topics e.g., [12, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124]. Only the works of Soukhov et al. [125, 126] use Wilson's [42] balancing factors as a method for maintaining constraints on opportunities within the context of competitive accessibility.

On that note, Soukhov et al., 2023 and 2024 [125, 126] introduce the balancing factors as a mechanisms to ensure that opportunities at each destination are proportionally allocated to each zone (based on the proportion of population seeking opportunities and the relative travel impedance). This is to ensure that each zonal accessibility value is the sum of this proportional allocation from each destination, and that all zonal values ultimately sum to the total number of opportunities in the region. However, these balancing factors were deduced intuitively. These works do not explicitly state that the mathematical formulation of the equations are effectively equivalent to Wilson's singly constrained model (derived from entropy maximization). This equivalence is only discovered in hindsight, as will be demonstrated in the following section. These two works also do not discuss other constrained cases that will also be addressed.

In sum, despite the interpretative advantaged offered by the statistical logic of Wilson [42]'s framework, the spatial interaction-oriented Hansen [10] citing literature nor the accessibility-oriented literature that cite Wilson [42] have yet to meaningfully adopt Wilson's constraint-based logic to the concept of accessibility. So, in the next section, we demonstrate how applying Wilson's framework enables accessibility to move from a proportional relationship to calibrated equality–tying outputs to tangible system knowns. This re-expression of accessibility using constraints yields interpretable, unit-consistent measures of opportunity. This approach takes the same path of entropy-maximisation as in Wilson [42], and does not rely on specifying some universal constant G like initially proposed in Stewart [57] (recall, Eq 6 which Hansen [10] operationalised).

A family of accessibility measures: from proportionality to equality

Despite the close conceptual ties between accessibility and spatial interaction modeling, the former has not meaningfully absorbed the constraint-based logic of the latter. This section introduces this reframing: defining a family of accessibility measures using Wilson's approach grounded in statistical mechanics, shifting place-based accessibility (a la Hansen [10]) from a relationship describing proportionality to equality.

This shift addresses the fundamental issue of interpretability associated with the proportional nature of Hansen-type accessibility indicators we previous outlined. Rather than relying on arbitrary or uncalibrated proportional relationships (or invoking a universal constant G as proposed in Stewart [57]) we adopt Wilson's approach grounded in statistical mechanics.

To do so, we propose a revised definition of accessibility considerate of the constraint-based spatial interaction model: the potential for spatial interaction with opportunities (or population). We can specify k as a type of proportional allocation factor κ which incorporates Wilson's balancing factor(s) to define the *potential for* *spatial interaction with opportunities* V_{ij} and the *potential for spatial interaction with population* M_{ji}. In effect, κ is unitless and proportionally allocates (based on the constraints of the case) opportunities (for V_{ij}) and population (for M_{ji}). The equation is as follows:

$$\begin{aligned} V_{ij}^X &= \kappa_{ij}^X W_X^{(2)} \\ M_{ji}^X &= \hat{\kappa}_{ji}^X W_X^{(1)} \end{aligned} \tag{16}$$

Where $W_X^{(2)}$ and $W_X^{(1)}$ is the mass of the destination (i.e., opportunities) and mass of the origin (i.e., population) at either the zone or for the full region, depending on the case (hence represented by a X).

Accessibility flows can also be summarised as as a partial sum of the potential at i and at j to express accessibility at the origin zone or at the destination zone, respectively, as often done in accessibility research:

$$\begin{aligned} V_i^X &= \sum_j \kappa_{ij}^X W_X^{(2)} \\ M_j^X &= \sum_i \hat{\kappa}_{ji}^X W_X^{(1)} \end{aligned} \tag{17}$$

Figure Fig 2 illustrates our analytical framework using a simple 3-zone system. The most detailed values, X_{ij}, represent the potential for spatial interaction from origin i to destination j. Here, X stands for all cases and variants to be discussed (e.g., V_{ij}^0, M_{ji}^0, V_{ij}^T, M_{ji}^T, V_{ij}^S, M_{ji}^S, V_{ij}^D, and M_{ji}^D). Single marginals show origin and destination weights, while the total marginal sums these values.

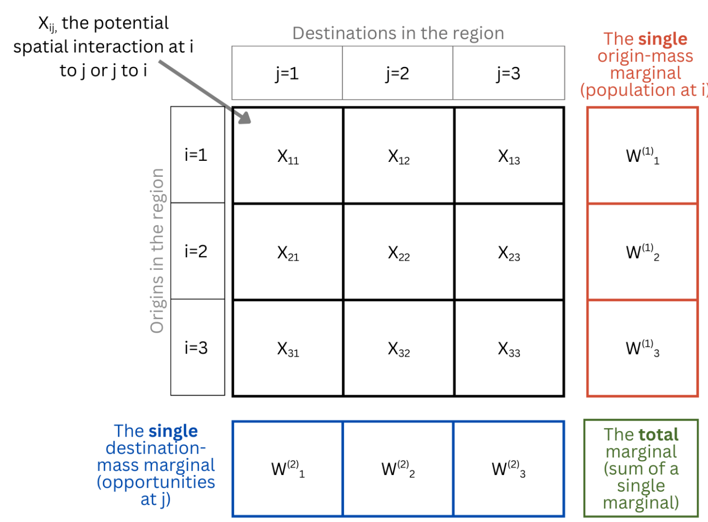

Figure 2. The family of accessibility measures analytical framework: labelling and associating ij flows, zonal weights, the single marginals, and the total marginal.

The proportional allocation constant κ takes the form of a balancing factor that varies depending on the constraints applied. Each member of the accessibility measure family is defined by the constraints used, and can be grouped into the following four cases:

1. **Unconstrained Case** (V_i^0, M_j^0)

- Equivalent to Hansen's [10] and Reilly's [53] original formulations; the status quo of accessibility modelling.
- No balancing factors applied; units are in "opportunities-by-impedance" for V_i^0 or "population-by-impedance" for M_j^0.
- No constraints are applied, so values reflect proportionality only and are not calibrated to known system totals.

2. **Total Constrained Case (V_i^T, M_j^T)**

- Applies a total proportional allocation factor (κ_{ij}^T, $\hat{\kappa}_{ji}^T$) based only on the total marginal (green box in Fig 2) i.e., total number of opportunities or population in the system. This ensures the sum of all values in the system match the total marginal.
- Units of V_i^T: accessible opportunities from i, a value that is total constrained and linearly proportion to V_i^0.
- Units of M_j^T: accessible population from j, a value that is total constrained and linearly proportion to M_j^0.

3. **Singly Constrained Case (V_i^S, M_j^S)**

- Applies singly-constrained proportional allocation factors (κ_{ij}^S, $\hat{\kappa}_{ji}^S$) based on Wilson's balancing factors (B_j, A_i) to preserve either the destination-side or origin-side marginal totals (blue and red boxes in Fig 2) i.e., the number of opportunities or population at each zone. Reflects how the literature calculates competitive accessibility.
- Units of V_i^S: accessible opportunities from i, a value that is the sum of opportunity supply flows allocated proportionally based on demand at i. Mathematically equivalent in per-capita form to 2SFCA [17].
- Units of M_j^S: accessible population from j, a value that is the sum of population demand flows allocated proportionally based on supply at j.

4. **Doubly Constrained Case (V_{ij}^D, M_{ji}^D)**

- Constrained on both origin and destination sides using both A_i and B_j simultaneously, which can also be expressed as proportional allocation factors (κ_{ij}^D, $\hat{\kappa}_{ji}^D$); equivalent in interpretation to Wilson's [42] doubly constrained spatial interaction model.
- Simultaneous application ensures both the destination-side *and* origin-side marginal totals are maintained (blue and red boxes in Fig 2).
- Interpretable only as ij and ji flows, since aggregation at i and j simply reproduces known totals. Represents 'interaction capacity' or 'realized access' serving as predictions of real interaction flows.

Toy example setup

Consider the simple 3-zone region in Figure Fig 2, where each zone serves as both origin (i) and destination (j). The system includes three inputs: zonal population and opportunities, a zonal cost (travel time) matrix, and three travel impedance functions representing different travel behaviours.

First, Table Table 1 summarizes population (in 10,000s) and physicians per zone. For context, the Provider-to-Population Ratio (PPR) is 24.5 comparable to Canada's 2022 PPR of 24.97 physicians per 10,000 [127]. Second, Table Table 2 shows travel times (minutes); it can be discerned that Zones 1 and 3 are closer to each other than to Zone 2. Zones 1 and 3 together have a population roughly equal to Zone 2 but offer more than twice the physician availability. We interpret Zone 1 as the Urban Edge, Zone 3 as part of the Urban Core, and Zone 2 as the Suburban.

Table 1. Simple system with three zones (ID 1, 2 and 3). Population is in 10,000 persons and opportunities in number of physicians.

ID (i or j)	Population[1]	Opportunities[2]
1	4	160
2	10	150
3	6	180

[1] Population is $Wi^{(1)}$ when used as a proxy for the mass at the origin, and Oi when used as a constraint.
[2] Opportunities is $Wj^{(2)}$ when used as a proxy for the mass at the destination, and Dj when used as a constraint.

Table 2. Cost matrix for system with three zones (travel time in minutes).

	Destination ID		
Origin ID	1	2	3
1	10	30	15
2	30	10	25
3	15	25	10

And third, Eq 18 presents the assumed travel impedance functions reflecting three different travel behaviours. A helpful analogy may be tying travel behaviour to the used mode's mobility potential, i.e., the most decaying travel behaviour ($f_1(c_{ij})$) would assume all travel in the region being done by foot, while calculating accessibility assuming the least decay ($f_3(c_{ij})$) would assume unfettered automobility.

$$\begin{aligned} f_1(c_{ij}) &= \frac{1}{c_{ij}^3} \\ f_2(c_{ij}) &= \frac{1}{c_{ij}^2} \\ f_3(c_{ij}) &= \frac{1}{c_{ij}^{0.1}} \end{aligned} \tag{18}$$

Any set of concepts representing population, opportunities, and their associated travel behaviour, whether representing the entire region uniformly (as will be demonstrated) or representing specific subgroups, can be substituted into our simple toy example. The purpose of the following simple example is to demonstrate the calculation and interpretation of the four accessibility measure variants.

Unconstrained accessibility

In V_i^0, no proportional allocation factor is defined, simply $f(c_{ij})$ is used to weight the number of opportunities at each j and the weighted values for each j are summed for each i, yielding the an expression identical to Hansen's accessibility S_i [10], the current standard practice in accessibility measurement:

$$V_i^0 = \sum_j V_{ij}^0 = \sum_j W_j^{(2)} f(c_{ij}) = S_i \tag{19}$$

However $\sum_i V_i^0$ generally does not equal the total opportunities O, so units here are 'opportunities weighted by travel impedance' and lack meaningful scaling or direct interpretability. Comparing values across different decay functions or contexts (i.e., different number of zones) is therefore limited to ordinal statements (more vs. less), not intervals or ratios (i.e., the magnitude of differences).

Table 3. Simple system: unconstrained accessibility.

	$V_i{}^0$		
	$f_1\ (c_{ij}) = 1/c_{ij}{}^3$	$f_2\ (c_{ij}) = 1/c_{ij}{}^2$	$f_3\ (c_{ij}) = 1/c_{ij}{}^{0.1}$
Origin	units: *physicians-minuteˆ-3*	units: *physicians-minuteˆ-2*	units: *physicians-minuteˆ-0.1*
1	0.219	2.567	371.143
2	0.167	1.966	363.479
3	0.237	2.751	373.738
Sum	0.6233422	7.283556	1108.361

For example, Table Table 3 shows V_i^0 under each decay function. Comparing across decay types is meaningless in absolute terms. For instance, the difference in zone 1 (edge of urban core)'s accessibility under f_3 vs f_1 is 370.92, but in what units? These two values are a product of different impedance functions (*physicians-minute*$^{-3}$ and *physicians-minute*$^{-0.1}$), making the comparison uninterpretable (and arguably incorrect). The fundamental uninterpretability of what is a *opportunity-weighted-travel-impedance* unit remains.

As the different impedance functions represent different travel behaviours, comparing the raw unconstrained accessibility values across groups is meaningless beyond notions of higher or lower. While one could attempt to adjust the units post-calculation (e.g., scaling, population normalization) or select impedance functions to facilitate comparison across scenarios (potentially at the expense of accurately reflecting travel behavior), such adjustments may introduce bias. The unconstrained scores are best used for ranking within a single context.

The next sections introduce constraints to calibrate these measures for better interpretability and comparability, applying each to this example in turn.

Total constrained accessibility

The total constrained accessibility case can be interpreted in a few ways. In the one that connects to the status quo: the total balancing factor proportionally adjusts unconstrained zonal accessibility values V_i^0 so their total sum of V_i^0 matches a known system total–either total opportunities or total population. Another interpretation is in reformulating the equation to use a a proportional allocation constant based on the total balancing factor. The proportional allocation constant distributes opportunities (or population) proportionally by travel impedance.

In both formulations, all zonal values become a proportion of a known system total, be it the regional opportunities or regional population depending on the variant.

We define two variants for this case:

- V_i^T: accessibility is constrained by the total number of opportunities (total constrained accessible opportunity) and which is interpreted as Hansen's accessibility with a constraining constant, and
- M_j^T: where i and j of the first variant is transposed, yielding a measure constrained by the total number of population and to be interpreted as constrained 'market potential'.

Total constrained accessible opportunities: Hansen's accessibility with a total constraint

In the total constrained case, accessibility is expressed as a share of the total number of opportunities in the region D, allocated based on travel impedance. The total constrained accessibility from i to j:

$$V_{ij}^T = \kappa_{ij}^T D \tag{20}$$

This formulation satisfies the total constraint, analogous to the one in the Wilson framework:

$$\sum_i \sum_j V_{ij}^T = D \tag{21}$$

Next, the proportional allocation factor κ_{ij}^T determines the share of total opportunities assigned to each origin–destination pair, based on the relative proportion of opportunities-weighted travel impedance:

$$\kappa_{ij}^T = \frac{W_j^{(2)} f(c_{ij})}{\sum_i \sum_j W_j^{(2)} f(c_{ij})}$$

This renders V_i^T (equal to $\sum_j V_{ij}^T$) into units of opportunities (e.g., physicians), and allows direct interpretation and comparison of results between zones and scenarios.

Alternatively, this formulation can be rewritten to be expressed using a total constrained balancing factor K^T, which scales Hansen's unconstrained accessibility V_i^0 to meet the total opportunity constraint:

$$V_i^T = \sum_j V_{ij}^T = K^T \sum_j W_j^{(2)} f(c_{ij}) = K^T V_i^0 \tag{22}$$

Where the total constrained balancing factor K^T is:

$$K^T = \frac{D}{\sum_i \sum_j W_j^{(2)} f(c_{ij})} \tag{23}$$

This expression is consistent with Wilson's entropy-maximizing framework and analogous to the total flow spatial interaction model (e.g., Equation 2.11 in [63]).

In summary, κ_{ij}^T proportionally allocates the total number of opportunities D to each origin–destination pair based on relative opportunity-weighted travel impedance. These values can be aggregated across destinations to obtain total constrained accessibility at each origin. Alternatively, the measure can be expressed using the balancing factor K^T, demonstrating that it is algebraically proportional to unconstrained accessibility V_i^0, but with interpretable units (i.e., opportunities). This allows for meaningful comparisons of differences across zones and travel behaviour scenarios.

Referring back to our simple numeric example, K^T for the highest decay t scenario $f_1(c_{ij}) = 1/c_{ij}^3$ would then be:

$$K^T = \frac{D}{\sum_i \sum_j W_j^{(2)} f(c_{ij})} = \frac{490}{0.6233} = 786.085$$

K^T for other decay scenarios are calculated similarly in code. Applying each K^T to the unconstrained values V_i^0 yields total constrained accessibility values (Table Table 4), all in units of physicians.

Table 4. Simple system: total constrained accessible opportunities.

	${V_i}^T$		
	$f_1\ (c_{ij}) = 1/{c_{ij}}^3$	$f_2\ (c_{ij}) = 1/{c_{ij}}^2$	$f_3\ (c_{ij}) = 1/{c_{ij}}^{0.1}$
Origin	units: *physicians*	units: *physicians*	units: *physicians*
1	172.065	172.672	164.080
2	131.627	132.247	160.692
3	186.308	185.081	165.228
Sum	490	490	490

Compared to the unconstrained case, values now sum to the known regional total D, allowing interpretation of absolute and relative differences across zones and travel scenarios. For example, in the highest decay case, Zone 1 (Urban Edge) captures an intermediate number of physicians (172.0652825), like in the unconstrained accessibility case. However, unlike in the unconstrained case, we can say that this value is out of the 490 physicians in the region, which allows us also to deduce that zone 1 captures 1.3072213 and 0.9235529 times more than zone 2 and 3. Values for the lesser decay ($f_2(c_{ij})$) and lowest decay ($f_3(c_{ij})$) scenarios are calculated separately, with decay scenario values also summing to equal 490 physicians accessible in the region.

One can also directly compare values at a specific zone, across travel impedance scenarios, due to the consistent units. As the decay scenario decreases, all zones become more accessible to each other and the differences between pairs diminishes (i.e., in $f_3(c_{ij})$ each zone captures close to an average amount of physicians, a third of 490 or ~163). This averaging of the total amount is how the total constrained proportional allocation factor functions. However, what's notable is how Zones change between scenarios. For instance, Zone 1's share only declines slightly–these declines are outpaced by Zone 2's relative gains. This shift reflects how κ_{ij}^T redistributes opportunities in proportion to impedance under different travel behaviours.

The total constrained accessibility measure resolves the interpretability issue of Hansen's accessibility (i.e., unconstrained accessibility) by grounding values in a meaningful total, enabling robust comparisons across zones and scenarios but also by keeping values proportional to V_i^0 so interpretation is similar.

Total constrained accessible population: Reilly's potential trade territories with a total constraint

This second total constrained variant is a transpose of the opportunity-constrained formulation, switching indices i and j to yield s measure of market potential–the number of people who can spatially interact with a destination.

Though not outlined in the "Unconstrained accessibility" section, the unconstrained form aligns with Reilly's "potential trade territories" [53] and Harris' and Vickerman's formulations of regional market potential [44, 45]. In its unconstrained form, market potential has also been more recently to estimate potentially accessible populations following infrastructure investments [e.g., 128, 129, 130]. Market potential can also be thought of as a form of *passive accessibility*, indicating the number of people that can reach each destination.

However, like V_{ij}^0, issues of unit interpretability arise in unconstrained market potential M_j^0. To address this, we introduce the total constrained accessible population measure M_{ji}^T, which allocates the total population O across all origin-destination pairs proportionally:

$$M_{ji}^T = \kappa_{ji}^T O \tag{24}$$

Subject to the total constraint::

$$\sum_i \sum_j M_{ji}^T = O \tag{25}$$

Next, $\hat{\kappa}_{ij}$ is the total constrained proportional allocation factor, a dimensionless term which distributes population based on impedance-weighted accessibility:

$$\hat{\kappa}_{ij}^T = \frac{W_i^{(1)} f(c_{ij})}{\sum_i \sum_j W_i^{(1)} f(c_{ij})}$$

This renders M_j^T (equal to $\sum_j M_{ji}^T$) into units of population. Alternatively, this formulation can be rewritten to be expressed using a total constrained balancing factor $\hat{K}^T$, which scales unconstrained market potential M_j^0 to meet the total population constraint:

$$M_j^T = \sum_i M_{ji}^T = \hat{K}^T \sum_i W_i^{(1)} f(c_{ij}) = \hat{K}^T M_j^0 \tag{26}$$

Where the total constrained balancing factor $\hat{K}^T$ is:

$$\hat{K}^T = \frac{D}{\sum_i \sum_j W_i^{(1)} f(c_{ij})} \tag{27}$$

In summary, $\hat{\kappa}_{ij}^T$ allocates the total number of population O proportionally to each origin–destination pair based on relative population-weighted travel impedance. As well, the measure can be expressed using the balancing factor $\hat{K}^T$, demonstrating that it is algebraically proportional to unconstrained market potential, but also yielding interpretable units that allow for meaningful comparison.

Returning to the numerical example, the balancing factor $\hat{K}^T$ is solved for each travel behaviour scenario, and the market potential of each zone M_j^T is expressed as units of population (e.g., the number of people accessible from each origin at that destination) in Table 5.

Table 5. Simple system: total constrained accessible population.

	M_i^S		
	$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
Destination	units: *population in 10,000s*	units: *population in 10,000s*	units: *population in 10,000s*
1	5.018	5.447	6.598
2	8.596	7.986	6.717
3	6.386	6.567	6.684
Sum	20	20	20

Readers may note the difference in trends in accessible population (Table 5) and accessible physicians (i.e., the preceding subsection, Table 4).

In Table 4, zone 1, 2, 3 represent destinations and the accessibility values reflect the number of accessible people from the vantage of physicians. Zone 1, in its role as a destination, is no longer intermediately-ranked relative to other zones; it now attracts the fewest number of people across all three travel behaviour scenarios. However, similar to the total constrained opportunity case, as travel decay reduces, the availability of population begins to converge (though Zone 1 continues as the lowest-ranked) for similar reasons. As decay reduces, the population's travel impedance to all zones become more similar, making the relative location of the zones less important and all people in the region more equally accessible.

Like in the total constrained accessible opportunities variant, the total constrained accessible population enables direct comparison of raw values, supporting both ordinal and interval interpretations across space and travel behaviour scenarios.

Singly constrained accessibility

The singly constrained accessibility case can also be expressed in two variants, each defined by the direction in which a constraint is applied:

- V_i^S: accessibility constrained by opportunities at destinations (singly constrained accessible opportunities), and
- M_j^S: its transpose, constrained by population at origins (singly constrained accessible population, or market potential).

Similar to the total constrained case, the singly constrained measures adjust unconstrained zonal accessibility values (V_i^0 or M_j^0) using a balancing factor to satisfy the known system constraint. However, unlike the total constraint (which enforces a global/total sum), the singly constrained case applies a localized constraint at one end of the interaction–either origin or destination.

In the opportunities-constrained variant, the balancing factor ensures that only a proportion of opportunities at each destination are allocated to origins, based on their relative demand (population) and travel impedance. This variant mirrors the concept of spatial availability as discussed in Soukhov et al. [125]. In the population-constrained variant, the logic is reversed: population at each origin is allocated proportionally across destinations, informed by the distribution of opportunities and impedance.

In both cases, the singly constrained formulation introduces zonal-level competition, unlike the total constrained case which distributes a fixed regional sum. Each zonal accessibility value becomes not only a fraction of the regional total (opportunities or population), but also a balanced sum of interactions, weighted by impedance and relative competition. The result remains in interpretable units–accessible opportunities or accessible population–but reflects more complex spatial dynamics.

Singly constrained accessible opportunities: spatial availability

In this singly constrained variant, accessibility is constrained at the destination side: the sum of accessible opportunities allocated from each destination must equal the known number of opportunities D_j. This is comparable to the single attraction-constraint (Eq 11) from Wilson's framework:

$$\sum_i V_{ij}^S = D_j \tag{28}$$

The underlying spatial interaction model is now the attraction-constrained model and our accessibility measure becomes:

$$V_i^S = \sum_j B_j D_j W_i^{(1)} f(c_{ij}) \tag{29}$$

where $W_i^{(1)}$ is a measure of the mass at origin i (i.e., the opportunity-seeking population). The corresponding balancing factor, as per Wilson, is:

$$B_j = \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \tag{30}$$

Introducing the balancing factor in Eq 29, we obtain:

$$V_i^S = \sum_j D_j \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \tag{31}$$

Further, we can express the formula even more simply, by defining the following proportional allocation factor:

$$\kappa_{ij}^S = \frac{W_i^{(1)} f(c_{ij})}{\sum_i W_i^{(1)} f(c_{ij})} \tag{32}$$

After this, it is possible to rewrite Eq 31 as an origin summary expression of proportionally allocated known opportunities (i.e., D_j).

$$V_i^S = \sum_j \kappa_{ij}^S D_j \tag{33}$$

This formulation has been referred to as **spatial availability** by Soukhov et al. [125], since it incorporates spatial competition by allocating opportunities based on demand (population), impedance, and the known opportunity totals D_j. The dimensionless factor κ_{ij}^S ensures that each destination's opportunities are distributed proportionally to origins. As in the total constrained case, V_i^S is expressed in the units of accessible opportunities. Finally, the **per capita** version of this measure:

Soukhov et al., 2023 [125] also showed that the following expression (accessibility per capita) is a constrained version of the popular 2SFCA approach of Shen 1998 [43] and Luo and Wang 2003 [17]:

$$v_i^S = \frac{V_i^S}{W_i^{(1)}} \tag{34}$$

Returning to the simple numeric example, as an example of the solved B_j for the highest decay travel behaviour $f_1(c_{ij})$:

$$
\begin{aligned}
B_j &= \frac{1}{\sum_i W_i^{(1)} f(c_{ij})} \\
B_1 &= \frac{1}{\frac{4}{10^3} + \frac{10}{30^3} + \frac{6}{15^3}} = 162.6506 \\
B_2 &= \frac{1}{\frac{4}{30^3} + \frac{10}{10^3} + \frac{6}{25^3}} = 94.9474 \\
B_3 &= \frac{1}{\frac{4}{10^3} + \frac{10}{25^3} + \frac{6}{10^3}} = 93.9850
\end{aligned}
$$

The balancing factors B_j for the $f_2(c_{ij})$ decay group for zones 1, 2 and 3 area 12.8571429, 8.7685113 and 10.6635071, respectively. For the $f_3(c_{ij})$ decay group, they are 0.0672461, 0.0660559 and 0.0663798. Using these these balancing constants, we can calculate the singly constrained opportunity accessibility (Table 6).

Table 6. Simple system: singly constrained accessible opportunities.

		V_i^S		
		$f_1\ (c_{ij}) = 1/{c_{ij}}^3$	$f_2\ (c_{ij}) = 1/{c_{ij}}^2$	$f_3\ (c_{ij}) = 1/{c_{ij}}^{0.1}$
Origin	Population (10k)	units: *physicians*	units: *physicians*	units: *physicians*
1	4	133.469	122.255	98.848
2	10	166.781	185.096	241.877
3	6	189.750	182.650	149.275
Sum	—	490	490	490

Imposing the single proportional allocation factor κ_{ij}^S allows for the comparison of differences and ratios of the accessibility values, like previously discussed in the total constrained accessible opportunities case. The proportional allocation factor ensures that resulting values are in units of *physicians*, with the impedance units already accounted for in the allocation process.

However, unlike κ_{ij}^T, κ_{ij}^S introduces zonal competition based on the mass of the origin (population). In the total constrained case, opportunities are distributed based on impedance alone, regardless of population at i. In contrast, the singly constrained case allocates each zone's opportunities proportionally across the region based on the relative impedance-weighted demand from all origins.

This consideration has important implications. Consider the highest decay scenario $f_1(c_{ij})$. Under this scenario, Zone 1–despite hosting a medium amount of physicans–captures the fewest physicians (133.4687282), compared to 166.7813387 at Zone 2, and 189.7499331 at Zone 3. Why? Zone 1 has the smallest population and is adjacent to Zone 3, the urban core. Its low impedance-weighted demand means κ_{ij}^S allocates it fewer opportunities. By contrast, in the total constrained case, Zone 1 fares better, capturing 35% of all physicians (compared to 27%).

As travel decay decreases (e.g., $f_3(c_{ij})$), competition becomes more diffuse. Zone 2, with the largest population, initially dominates accessibility—but under low decay, Zones 1 and 3 also begin drawing more opportunities from Zone 2. For instance, Zone 1 gains 6% more from Zone 2 in $f_3(c_{ij})$ than in $f_1(c_{ij})$. This shift reflects a drop in $\kappa_{2,2}^S$ by 14%, showing Zone 2's decreasing hold on its own opportunities as other zones gain accessibility 'parity'.

This dynamic reveals how κ_{ij}^S embeds both travel impedance and population competition. Unlike the total constraint lowest decay scenarios that allocate evenly, the singly constrained case reflects how competition evolves continues to influence allocation.

In this way, the consideration of constrained accessibility *per capita* may be clarifying. Often, accessibility values are reported as raw scores without the consideration for population. But, as we introduced constraints, these constrained accessibility values can be normalized using anything that is relevant to the zone. In Table 7, we present per capita accessibility for the numeric example, simply in units of number of physicians accessible per population at each zone. Notably, these per capita rates are equivalent to the 2SFCA values.

Table 7. Simple system: singly constrained accessible opportunities per capita.

		v_i^S		
		$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
Origin	Population (10k)	units: *physicians per capita*	units: *physicians per capita*	units: *physicians per capita*
1	4	33.367	30.564	24.712
2	10	16.678	18.510	24.188
3	6	31.625	30.442	24.879

This simple example was constructed so that the regional average equals 24.5 physicians per 10,000 people. As distance decay decreases and becomes *relatively* uniform (all zones can reach all zones), the effect of population drives the proportional allocation of opportunities. Consequently, per capita accessibility values begin to stabilise to the regional per capita average (e.g., in the lowest distance decay $f_3(c_{ij})$, per capita values are all ~24 physicians accessible per capita).

This convergence mirrors the trend in the total constrained opportunity case, where accessibility values approach a third of the 490 physicians under the unfettered mobility scenario $f_3(c_{ij})$. In both cases, the balancing factors (K^S and B_j) act as averaging mechanisms but at different scales. As distance decay becomes more *relatively* uniform, the role of remaining variables (i.e., total population or opportunities) drive the proportional allocation differences. In the total constrained case, this is the proportion of opportunities relative to the regional opportunities, and in the case of the single opportunity constrained case, this is the population at a zone relative to the regional population.

Singly constrained accessible population: market availability

Similar to Eq 27 in transposing the origins and destinations, we can define a *singly constrained* measure of market potential that preserves the known population (i.e., the mass weight at the origin $W_i^{(1)}$ is now represented by O_i). In it's per-capita expression, i.e., equivalent to 2SFCA, this constrained concept of market potential been used to express "facility crowdedness" as in Wang [131].

The underlying spatial interaction model is now the production-constrained model version of Eq 8, and our market potential measure M_j^S becomes:

$$M_j^S = \sum_i A_i O_i W_j^{(2)} f(c_{ij}) \tag{35}$$

In this variant, the measure is singly constrained by the population *by origin* (i.e., O_i), like Eq 11 from Wilson's framework:

$$\sum_j M_{ji}^S = O_i \tag{36}$$

And the corresponding balancing factor, as per Wilson, is:

$$A_i = \frac{1}{\sum_j W_j^{(2)} f(c_{ij})} \tag{37}$$

Following the same logic as in the preceding section on total constrained market potential, one arrives at the following expression of accessible population M_j^S being the product of proportionally allocated ($\hat{\kappa}_{ji}^S$) popultion:

$$M_j^S = \sum_i \hat{\kappa}_{ji}^S O_i \tag{38}$$

with:

$$\hat{\kappa}_{ji}^S = \frac{W_j^{(2)} f(c_{ij})}{\sum_i W_j^{(2)} f(c_{ij})} \tag{39}$$

As well, the single (population) constraint in Eq 36 ensures that the the total constraint (e.g., $\sum_j M_j^S = \sum_i \sum_j M_{ji}^S = O$) is maintained.

With these constraints, $\frac{M_j^S}{O}$ can be interpreted as the proportion of the total population serviced by location j.

For the sake of brevity, we'll move forward onto the doubly constrained case.

Doubly constrained accessibility

This accessibility case adopts the structure of the doubly constrained spatial interaction model, where V_{ij}^D flows are constrained by both origin populations O_i and destination opportunities D_j. That is, the resulting accessibility outflow from each origin must match the origin's population demand, and the resulting accessibility inflow to each destination must match the number of opportunities supplied:

$$\sum_j V_{ij}^D = O_i \text{ and } \sum_i V_{ij}^D = D_j \tag{40}$$

Because results are made to match both margins, the results cannot be interpreted as a traditional summary at i or j (e.g., "opportunities accessible from i")–those sums simply reproduce the constraint totals. Instead, the meaningful unit of analysis is the ij flow itself.

This distinguishes doubly constrained accessibility from the total and singly constrained cases discussed previously. In those cases, only one side of the interaction–either the total marginal or opportunity/population marginal–was constrained, while the other was treated as a demand/supply weight (e.g., D or O for total constrained and $W_j^{(2)}$ or $W_i^{(1)}$ for singly constrained).

By contrast, the doubly constrained model assumes both the demand (population) and the supply (opportunity) are known and bounded, and allocates flows accordingly. This makes it less suitable for traditional accessibility analysis, namely because origin and destination masses often differ in kind and units. For instance, the number of people accessing an opportunity such as park may be known, but the capacity of each park is not known. A doubly constrained approach only makes conceptual sense when population and opportunities are comparable, have a one-to-one correspondence or are paired somehow–e.g., job per worker, student per school-seat, or vaccine doses per person.

A doubly constrained approach to accessibility adopts the structure of the production-attraction spatial interaction model, where both population and opportunity totals are fixed. Mathematically, this model requires the simultaneous imposition of both the population- and opportunity- constraints in the preceding singly constrained variants (Eq 28 and Eq 36), namely the sum of population in all origins should match the sum of opportunities in all destinations (Eq 41):

$$\sum_i O_i = \sum_j D_j \tag{41}$$

As before, the simultaneous imposition of both constraints ensures the total system constraint is maintained i.e., $\sum_i V_i^D = \sum_i \sum_j V_{ij}^D = D$ remains equal to the total number of opportunities in the region O.

As the doubly constrained accessibility measure V_{ij}^D takes the form of the production-attraction (doubly constrained) spatial interaction model as follows:

$$V_{ij}^D = A_i B_j O_i D_j f(c_{ij}) \tag{42}$$

where the corresponding balancing factors A_i and B_j, as per Wilson, are:

$$A_i = \frac{1}{\sum_j B_j D_j f(c_{ij})}$$
$$B_j = \frac{1}{\sum_i A_i O_i f(c_{ij})}$$

Calibration of the two sets of proportionality constants is accomplished by means of iterative proportional fitting, whereby the values of A_i are initialized as 1 for all i to obtain an initial estimate of B_j. The values of B_j are used to update the underlying V_{ij}^D matrix, before calibrating A_i. This process continues to update A_i and B_j until a convergence criterion is met [see 65, p. 193-195].

The doubly constrained model completely distributes origin populations to destination opportunities according to travel impedance and supply-demand balance. This ensures that: summing V_{ij}^D across j returns O_i; summing across i returns D_j. Thus, aggregating over i or j yields only the known constraints. In this way, a per-capita form (e.g., V_i^D/O_i) is not meaningful–since the output already reflects population-normalized allocation. As such, the ij matrix V_{ij}^D is the only interpretable output.

We could define the proportional allocation factor κ_{ij}^D such that:

$$\kappa_{ij}^D = \sum_j \frac{1}{\sum_j B_j D_j f(c_{ij})} \frac{1}{\sum_i A_i O_i f(c_{ij})} O_i f(c_{ij})$$

and represent V_{ij}^D as equal to $\kappa_{ij}^D D_j$, allowing the analyst to understand the proportional allocation of D_js to each ij flow.

Following this logic, the market potential form M_{ji}^D is effectively equivalent to V_{ij}^D, but can be read with a different interpretation: i.e., the opportunities accessed from j at an i vs. the population accessed from i at a j. The inputs of ‘opportunities accessed’ and ‘accessed population’ can already be interpreted as inherently being sensitive to both opportunities and population.

Moving onto the toy example, to calculate doubly constrained accessibility, the interpretation of the population data and the counts of the opportunity data in the numeric example must be reinterpreted. Namely, a count of physician *capacity* per destination D_j is needed instead of simply the number of physicians. Also, we must be able to clearly state that the population is the *capacity* of the origin to interact with opportunities O_i, i.e., the count of people seeking opportunities.

This adjusted simple example is summarised in Table 8: with the population (in units of 10,000s of people seeking physicians) and the opportunities (in units of 10,00s of physician-capacity) per zone. For the population, we leave this unchanged numerically but we now must keep in mind that each person interacts with one physician capacity. The number of providers per destination is however revised to represent physician capacity, scaled approximately from the original number of physicians used in previous cases (Table 1). The system-wide PPR is now:1, this is compared to the unmodified example which yields system PPR of24.5.

We keep the same zonal cost matrix, and travel impedance functions for three types of travel behaviour as before (Table 2 and Eq 18).

Table 8. Modified simple system with three zones reflecting matched population and opportunities. Population is in 10,000 persons and opportunities in 10,000 of physician-capacity.

ID (i or j)	Population	Opportunities
1	4	7
2	10	5
3	6	8

And with these modifications to the example, our objective is slightly different: to predict the flows from j knowing that the amount of physician-capacity at each j must be preserved and all flows to i should match the number of people at i, under different travel behaviour scenarios. Put another way, we’re interested in the ij flows assuming we already know accessibility at each i. The highest decay travel behaviour scenario $(f_1(c_i j))$ is presented in Table 9.

Table 9. Doubly constrained accessible opportunities assuming highest travel decay in the modified simple system.

		Destination ID			
	Origin ID	1	2	3	sum
	1	3.235859	0.01032226	0.7556568	4
	2	2.132602	4.95932483	2.9044391	10
	3	1.631539	0.03035291	4.3399040	6
Sum	—	7	5	8	—

As shown in Table 9 for the highest-decay scenario $f_1(c_{ij})$, accessibility is no longer meaningfully represented as zonal summaries like V_i^D or M_j^D, since these values reproduce the original constraints–i.e., $V_i^D = O_i$ so the physician-capacity accessible for Zones 1, 2, and 3 would be 4.0018381, 9.9963656, and 6.0017963. Hence, the usefulness of the doubly constrained measure lies in the interpretation as V_{ij}^D values. V_{ij}^D values represent the number of opportunities from zone j allocated to populations in zone i, shaped by both mass and travel impedance.

To illustrate this, consider the results for Zone 2 (Suburban zone–a high-population, low-opportunity, relatively remote zone). As shown in Table 10, its intrazonal allocation (i.e., V_{22}^D) declines as travel impedance decay decreases–from 4.9593248 under $f_1(c_{ij})$ to 2.6672837 under $f_3(c_{ij})$, out of the ~10 opportunities allocated to Zone 2 (a population of 10).

Following the intuition discussed in the singly constrained opportunity case, as decay decreases (i.e., more relatively uniform for all zones), the mass effects (effect of the population and opportunities magnitudes) become more relatively dominant in the spatial allocation.

Table 10. Doubly constrained accessible opportunities at Zone 2 for all travel decay groups in the modified simple system.

			$V_{\{ij\}}^D$		
			$f_1(c_{ij}) = 1/c_{ij}^3$	$f_2(c_{ij}) = 1/c_{ij}^2$	$f_3(c_{ij}) = 1/c_{ij}^{0.1}$
Dest.	Population at 2 (units: *people in 10,000s*)	Opportunities (units: *capacity in 10,000s*)	units: *physician-capacity in 10,000s*	units: *physician-capacity in 10,000s*	units: *physician-capacity in 10,000s*
1	10.000	7.000	2.133	2.272	3.411
2	10.000	5.000	4.959	4.766	2.667
3	10.000	8.000	2.904	2.958	3.919

Recall, accessibility is traditionally presented as a summary zonal measure. However, in the doubly constrained case, since we force the allocation of zonal population demand and zonal opportunities supplied to be paired and allocation to be proportional, V_i^D is simply the number of opportunities that matches our known population at i. So following the logic of the family of accessibility measures, in the doubly constrained case, V_{ij}^D flows are the only relevant unit of analysis: spatial proportional allocations between population and opportunity capacity. Furthermore, V_{ij}^D and its transposed counterpart M_{ji}^D are structurally identical, differing only in interpretation (referring to $kappa_{ij}^D$ and $\hat{k}appa_{ij}^D$): one reflects the proportional allocation of opportunity to population flows; the other, population to opportunity flows.

Readers should recall, V_{ij}^D are also mathematically equivalent to Wilson's spatial interaction flows. And as Wilson [42] explicitly noted, origin and destination weights defined in the spatial interaction model *can* be defined using any unit. However, traditionally the focus of these models have typically on ij flows and often calibrated using trips (i.e., outbound and inbound trips, inherently in the same units).

Accessibility is often understood as a zonal summary of potential spatial interaction, often involving origin and destination masses in different units. These units also often misaligned–for instance, we may not know how much park space, grocery area, or childcare capacity is accessible per person. When they do align–such as people to physician capacity–we're essentially modeling realised access flows based on known quantities of *those that interact* and the *interacted*. In such cases, the traditional accessibility question is already answered by the known information (i.e., how many opportunities can be reached by a zone? the number of people at that zone). This is why we do not foresee the doubly constrained measure being widely used in accessibility analysis: the literature has largely focused on questions of potential access, not on predicting flows of realised access.

Conclusions

In this paper, we examined the historical and mathematical commonalities between spatial interaction models and place-based accessibility measures. As accessibility research evolved largely influenced by Hansen [10], researchers in the field neglected the proportionality constant that was originally present in gravity-based models, and is still present in spatial interaction modeling.

Firstly, we have demonstrated that by reintroducing Wilson's system constraints and defining associated balancing factors and proportional allocation factors, we can derive a unifying family of accessibility measures that reintroduces units to the resulting values. These values become more intuitive for the purpose of analysis and comparison. Secondly, the constraining constants, depending on the case (i.e., total, singly- or doubly- constrained), restrict the degree of *potential*, linking accessibility (the potential for spatial interaction) with access (spatial interaction) on the same continuum based on the constraint used. Lastly, we discussed how popular measures such as the one used in Hansen [10] and the 2SFCA link into the family of accessibility measures.

A summary of the family of accessibility measures, is detailed; First, we place the popular Hansen-type accessibility measure [10] within this family of measures as an "unconstrained" case, demonstrating that resulting values cannot be directly compared across different travel scenarios without ad-hoc adjustments. We then show how applying a total constraint balances the units and produces a statistically averaged solution that converges to the regional average for each zone as the decay effect decreases. In other words, the total-constraint model could be a more interpretable alternative for the unconstrained case if population-competition is not relevant and one is interested in capturing the maximum *potential*; specifically, if there is a fixed number of opportunities in the region, and if it makes sense to assume that people accessing proximate opportunities leave fewer for others, *without* considering the population size at the origins.

We then introduce the singly-constrained case, which *does* take into account the population size at the origin in the allocation of opportunities. It is also mathematically equivalent to the spatial availability introduced in Soukhov et al. 2023 [125]. In this case, all accessibility values are fixed to sum to a known zonal opportunity-size value (implicitly, the regional total of opportunities), but they are not required to sum to any population-based values at the zone or regional level. The singly constrained model could be useful if regional competition is a factor and if the acknowledgment that only a finite number of opportunities can be allocated from each destination (with those allocations distributed based on origin population size) is suitable. We also introduce an 'accessible' PPR (e.g., opportunities per capita), calculated by dividing each accessibility value by the zonal population. To clarify, this per capita expression of the singly constrained case is equivalent to the 2SFCA [17, 43], hence linking this literature back to spatial interaction principles.

Lastly, the doubly constrained case is introduced. In this case, the sums must equal both the regional total and ensure that no zone allocates more opportunities than it has available. Specifically, accessibility values for each i-j pair must be a proportion of he zonal opportunity and population values simultaneously. For example, the accessibility at zone 1 must equal the sum of opportunities from zones 1, 2, and 3, as well as the sum of the population at zone 1. Satisfying the double constraint means the opportunities and population data must match one-to-one, so working with the accessibility i-j pair values should be of interest. In this sense, the research question should be concerned with ‘access’ (how many opportunities accessed from j at i based on given zonal opportunities and populations) instead of potential spatial interaction (e.g., typically expressed as a zonal summary measure of how many opportunities one could reach (out of a regional total and/or zonal-allocation).

Building on Wilson’s [42] foundational work, this paper proposed a unified framework for accessibility.. By reintroducing Wilson’s proportionality constant, the proposed family of constrained accessibility measures restores measurement units to accessibility estimates. This enhancement provides a more interpretable, consistent, and theoretically grounded basis for accessibility analysis, which could help advance the adoption of accessibility-oriented planning.

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