A family of accessibility measures derived from spatial interaction principles

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Abstract

Transportation planning has long prioritized the efficiency of movement. However, the concept of accessibility represents a more comprehensive evolution, shifting focus from movement to the potential to reach–or spatially interact with–desired destinations. Despite growing recognition of accessibility-based planning approaches, the concept remains fragmented, with inconsistent definitions and unclear interpretations. To this end, this paper offers a methodological contribution by specifying a family of accessibility measures grounded in the shared theoretical roots of gravity-based accessibility and spatial interaction models, particularly their balancing factors. From this foundation, we outline four memebers of the family: the ‘unconstrained’ measure (i.e., Hansen-type accessibility), the ‘total constrained’ measure (i.e., a constrained version of the Hansen-type accessibility), the ‘singly constrained’ measure (i.e., related to the popular two step floating catchment approach - 2SFCA), and the ‘doubly constrained’ measure representing realized access (i.e., equal to the doubly constrained spatial interaction model). These measures can be interpreted as either the number of accessible opportunities or accessible population (i.e., market potential). A toy example illustrates how they produce interpretable, unit-based values, offering a clearer and more coherent basis for accessibility analysis.

# Introduction

In the early twentieth century, the emergence of a transportation planning paradigm focused primarily on mobility cemented major investments in automobile and transportation infrastructure, fostering lower-density sprawl, car-dependent development and entrenching automobility in planning practice (H. J. Miller 2011; Lavery, Páez, and Kanaroglou 2013). Within this new practice, access to destinations was treated as a by-product of movement. Despite continued road and highway expansion, this automobility monoculture has proven ineffective at reducing travel costs or environmental burdens, and has not clearly improved people’s ability to reach destinations (Steven Farber and Páez 2011; S. Handy 2002; Páez et al. 2010).

In response, transportation researchers have increasingly advocated for the adoption of accessibility as a planning criterion, in contrast to traditional mobility-oriented transportation planning approaches which translate into indicators that benchmark movement (e.g., vehicle kilometres traveled, intersection through traffic) which are not necessarily linked to improved accessibility (Silva et al. 2017; Paez et al. 2013; S. Handy 2020; El-Geneidy and Levinson 2022). Accessibility, by contrast, is the “potential of opportunities for [spatial] interaction” (Hansen 1959). While mobility reflects movement, accessibility captures the combined influence of transport and land use, emphasizing destinations and the potential to reach them (S. L. Handy and Niemeier 1997).

Accessibility research has expanded across diverse domains including: employment (Karst and Van Eck 2003; Grengs 2010; Páez et al. 2013; Merlin and Hu 2017; Tao et al. 2020), healthcare (Luo and Wang 2003; Páez et al. 2010; Wan, Zou, and Sternberg 2012; Delamater 2013; Boisjoly, Moreno-Monroy, and El-Geneidy 2017; Pereira et al. 2021; Yang et al. 2024), green space (Reyes, Paez, and Morency 2014; Rojas et al. 2016; Liang, Yan, and Yan 2024), education (Williams and Wang 2014; Romanillos and Garcia-Palomares 2018; Marques, Wolf, and Feitosa 2021), social contact (Neutens et al. 2007; S. Farber, Páez, and Morency 2012; S. Farber et al. 2013), and regional economics (R. Vickerman, Spiekermann, and Wegener 1999; Lopez, Gutierrez, and Gomez 2008; Ribeiro, Antunes, and Páez 2010; Gutierrez et al. 2011), among many other domains of application. Despite its popularity in scholarly works, accessibility still remains difficult to implement in planning due to definitional inconsistencies (van Wee 2016; S. Handy 2020; Kapatsila et al. 2023) and challenges in interpreting and communicating results (Karst T. Geurs and van Wee 2004; van Wee 2016; Ferreira and Papa 2020).

More specifically, the wide range of accessibility definitions, with novel methods being more sophisticated but less intuitive (Kapatsila et al. 2023), can further hinder practical uptake (van Wee 2016). Geurs and van Wee (Karst T. Geurs and van Wee 2004) classify accessibility measures into four categories: infrastructure-, place-, person-, and utility-based. Among place-based measures (this work’s focus), variants include gravity-based (Hansen 1959), cumulative opportunity (Pirie 1979), the 2 Step Floating Catchment Area (FCA) approach (Luo and Wang 2003), and a variety of modifications to these approaches e.g., Enhanced 2-Step FCA (Luo and Qi 2009), 3-Stage FCA (Wan, Zou, and Sternberg 2012), Modified 2-Step FCA (Delamater 2013), inverse 2-Step FCA (F. Wang 2021), and n-steps FCA (Liang, Yan, and Yan 2024). While these methods are tailored to address specific research contexts, overall this diversity does not demystify existing questions like those raised by van Wee (van Wee 2016): How should practitioners interpret differences in accessibility scores between modes, and how should results be communicated?

Rather than propose a new measure, this work argues for a return to the spatial interaction foundations of accessibility. Specifically, we show how the family of spatial interaction models (Wilson 1971) can be reformulated in the context of accessibility, namely as a “family of accessibility measures”. This formulation results in constrained versions of gravity-based accessibility (e.g., Hansen 1959), but in units of *spatially reachable opportunities*. This approach offers a direct mathematical link to existing accessibility measures while restoring tangible meaning to zonal values. Instead of abstract proportional scores, constrained accessibility expresses the number of opportunities a population may potentially spatially interact with.

In summary, this paper makes two contributions: (1) we review how spatial interaction modeling and accessibility share similar origins but have diverged in focus and interpretation. (2) we introduce a family of accessibility measures grounded in spatial interaction principles, including total, singly, and doubly constrained cases and variants of “accessible opportunities” and “accessible population”. These cases and variants align with common measures such as Hansen-type accessibility (Hansen 1959), competitive accessibility measures such as the 2SFCA method (Q. Shen 1998a; Luo and Wang 2003), and market potential models (C. D. Harris 1954; R. W. Vickerman 1974).

We contend that accessibility research should re-engage with spatial interaction modeling, particularly the use of Wilson’s (Wilson 1971) system constraints. While spatial interaction models embraced such constraints to improve interpretability, most accessibility models have not. This lack of adoption contributes to fuzziness in current analyses, limiting interpretive clarity to simple proportional comparisons (e.g., “higher-than”, “lower-than”) (E. J. Miller 2018). Without such constraints, accessibility scores lack clear units and comparability across cities or modes. In contrast, constrained measures yield values that can be tied to tangible values without any post-hoc treatment, theoretically making them more interpretable, communicable, and actionable in planning.

Accessibility and spatial interaction modelling literature share common headwaters, and the latter has given careful attention to measurement units and their interpretability. Following this introduction, this paper revisits that lineage–from Ravenstein’s Newtonian gravitational expression of spatial interaction (Ravenstein 1889), through Hansen (Hansen 1959) to Wilson (Wilson 1971)–to propose a unified family of constrained accessibility measures. We illustrate each case of the family with a numerical toy example and conclude by discussing how these constrained measures can inform planning decisions and improve clarity in accessibility analysis.

# Newtonian’s roots of human spatial interaction research

The patterns of people’s movement in space have been a subject of scientific inquiry for at least a century and a half, from as far back as Henry C. Carey’s *Principles of Social Science* (Carey 1858). It was in this work where Carey stated that “man [is] the molecule of society [and their interaction is subject to] the direct ratio of the mass and the inverse one of distance” (McKean 1883, 37–38). This statement shows how investigations into human spatial interaction have often been explicitly coloured by the features of Newton’s Law of Universal Gravitation, first posited in 1687’s *Principia Mathematica* and expressed as in [Equation 1](#eq-phys-grav-prop).

This famous equation expresses that the attractive force between two bodies and is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. As mass increases, so does force; as distance increases, force decreases. However, [Equation 1](#eq-phys-grav-prop) only demonstrates a *proportional* relationship. To quantify the magnitude (not the *proportional* magnitude) of the force, it must be *constrained* with an empirical constant. This constant converts [Equation 1](#eq-phys-grav-prop) from an expression of proportionality to the following expression of equality:

Where is the gravitational constant–an empirically calibrated value that ensures the model reflects observed forces. Newton’s initial estimate of was based on a speculation but received empirical support after Hutton’s and Cavendish’ experiments in the late 1700s (Hutton 1778; Cavendish 1798) which estimated it to within 1% accuracy. That is, it took over a century from the publication of Newton’s *Principia Mathematica* to refine the estimate of the proportionality constant.

While the Newtonian gravitational relationship laid the conceptual groundwork for later empirical studies of human spatial interaction, the majority of these early attempts described a proportional relationship or one arbitrarily set to equality. They did not establish an empirical as in the Newtownian tradition, as will be discussed in the next subsection.

# Early research on human spatial interaction: from Ravenstein (1889) to Stewart (1948)

Henry C. Carey’s *Principles of Social Science* (Carey 1858) inspired empirical spatial intereaction research in different contexts. Namely, a number of researchers theoretically and empirically attempted to characterise human spatial interaction as a force directly proportional to the “masses” and of two locations, and inversely proportional to their separation distance–conceptually parallel to Newtonian gravity, but often omitting a proportionality constant.

Beginning with Ravenstein in the late 1880s, his works proposed some “Laws of Migration” based on his empirical analysis of migration flows in various countries (Ravenstein 1885, 1889). These works posited 1) a directly proportional relationship between migration flows and the attractive size of destinations, and 2) an inversely proportional relationship between the size of flows and the separation between origins and destinations. As with Carey, these propositions echo Newton’s gravitational laws.

Over time, other researchers discovered similar relationships. For example, Reilly (Reilly 1929) formulated a law of retail gravitation, expressed in terms of equal attraction to competing retail destinations that could be understood as ‘potential trade territories’. Later, Zipf proposed a hypothesis for the case of information (Zipf 1946a), intercity personal movement (Zipf 1946b), and goods movement by railways (Zipf 1946c). The hypothesis stated that the magnitude of flows was proportional to the product of the populations of settlements, and inversely proportional to the distance between them.

Of the researchers cited above, only Reilly and Zipf expressed their hypotheses in mathematical terms. Reilly’s hypothesis was presented in the following form:

where is the amount of business drawn to from , and are the populations of the two settlements, and is the distance between them. Quantity was chosen by Reilly in a somewhat *ad hoc* fashion as 1, and he used empirical observations of shoppers to choose a value of .

Zipf, on the other hand, wrote his hypothesis in mathematical form as:

where is the volume of goods exchanged between and , and are the populations of the two settlements, and is the distance between them.

These early formulations ([Equation 3](#eq-reilly), [Equation 4](#eq-zipf)) clearly reflect the influence of Newtonian gravity on human spatial interaction theory, revealing a shared mathematical structure across migration, trade, and communication models.

However, a common feature of these early investigations is that none of them included a proportionality constant (similar to in [Equation 2](#eq-phys-grav)), a consistent omission of the empirical calibration necessary to convert these proportional relationships into measurable and comparable quantities. It is only in Stewart (1948) (Stewart 1948) that we find the most explicit connection yet to Newton’s Gravitational law and the use of a proportionality constant. While acknowledging predecessors like Reilly and Zipf, the physicist Stewart was likely the first author to formalize human spatial interaction using an explicit proportionality constant , enabling his formulation to be interpreted as a measurable ‘demographic’ force:

Where:

* is the *demographic force*
* and are the numbers of people of in groups 1 and 2
* and are so-called *molecular weights*, the attractive weight of groups 1 and 2
* and are the demographic masses at 1 and 2
* is the distance between and
* And finally proportionality constant

What is notable about [Equation 5](#eq-stewart-force), however, is the proportionality constant was specified but “left for future determination” (Stewart 1948, 34). We can infer that it is crucial for ensuring is maintained in some units of demographic force.

In addition to demographic force , Stewart defined a measure of the “potential” of group with respect to group . The partial sum of the demographic force experienced by group , or the potential number of people from location that could visit location , as . For a system with more than two population bodies, Stewart formulated the population potential at by summing the contributions from each group , after arbitrarily setting :

Where is the demographic mass at location and is the distance between and . A version of this discrete form is what was used in Hansen (Hansen 1959), going on to become a foundation of modern accessibility definitions, as will be discussed.

Although Stewart’s concept of “social physics” eventually fell out of favour–due to inconsistent mathematical notation (e.g., is used as both a proportionality constant p. 34 and then later as *demographic energy* on p. 53.) and its racist and unscientific assumptions (e.g., view of humans as particles following physical laws and assumptions of the molecular weight of the average American being one, but “presumably…much less than one….for an Australian aborigine” [p. 35])–his introduction of a proportionality constant in modeling demographic force marks an important conceptual step: recognizing that moving from proportionality to equality requires empirical calibration. In other words, the addition of shifts results from being abstract indicators of potential (i.e., ) to having interpretable units grounded in consistent, albeit still abstract, quantities (i.e., units of demographic force).

As will be discussed, when Hansen (1959) (Hansen 1959) later adopted Stewart’s formula ([Equation 6](#eq-stewart-population-potential-sum)) for accessibility, he omitted any mention of , effectively setting it to 1 arbitrarily as Stewart had done. This omission has persisted in accessibility research, leaving a conceptual gap in how such measures are interpreted and compared.

# Hansen’s gravity-based accessibility to today

From Stewart (Stewart 1948), we arrive to 1959 and Walter G. Hansen, whose work proved to be exceptionally influential in the accessibility literature (Hansen 1959). In this seminal paper, Hansen defined accessibility as “the potential of opportunities for interaction… a generalization of the population-over-distance relationship or *population potential* concept developed by Stewart (Stewart 1948)” (p. 73). As well as being a student of city and regional planning at the Massachusetts Institute of Technology, Hansen was also an engineer with the Bureau of Roads, and preoccupied with the power of transportation to shape land uses in a very practical sense. Hansen (Hansen 1959) drew directly from Stewart’s population potential formula (see [Equation 6](#eq-stewart-population-potential-sum)), but left aside the broader (and often problematic) framework of “social physics”.

Hansen recast Stewart’s population potential to reflect accessibility, a model of human behaviour useful to capture regularities in mobility patterns. Hansen replaced in [Equation 6](#eq-stewart-population-potential-sum) with *opportunities* to derive an *opportunity potential*, or more specifically, a *potential of opportunities for interaction* as . A contemporary rewriting of to accounts for the variety of impedance functions beyond the inverse power used in the applied literature:

in [Equation 7](#eq-accessibility-general) is a measure of the accessibility from site . This is a function of (the mass of opportunities at ), (the cost, e.g., distance or travel time, incurred to reach from ), and (a function that modulates the friction of cost). Today, Hansen is frequently cited as the father of modern accessibility analysis (e.g., Reggiani and Martín 2011), and Hansen-type accessibility is commonly referred to as the gravity-based accessibility measure.

However, Hansen’s formulation carried forward a crucial omission that continues to affect the literature: the proportionality constant included in Stewart’s original formulation ([Equation 6](#eq-stewart-population-potential-sum)) has vanished entirely. Although Stewart included explicitly (with a note that “ [was] left for future determination: a suitable choice of other units can reduce it to unity” [p. 34].) Hansen made no mention of it. As a result, modern accessibility analysis has largely evolved without addressing the constant’s role, leaving effectively and implicitly fixed at 1. This omission has significant implications. Without a proportionality constant, the accessibility formula expresses only a proportional relationship: , not one of calibrated equality. Recognition of the nature of this relationship is not common in the literature, but is known, i.e., this proportional equation is shown in Figure 1 in Wu and Levinson (Wu and Levinson 2020).

Furthermore, working with a proportional relationship generates fundamental issues in comparability between and, arguably within, studies. Namely, accessibility estimates have no fixed unit, rendering them sensitive to the choice of impedance functions. For instance, if travel cost is measured in meters, then when the travel impedance function equals , the resulting has units of opportunities per . However, when is set to equal , the units become opportunities per . Such variation impairs comparability across analysis and obscures the meaning of accessibility scores, making them difficult to understand and communicate without post-hoc treatment.

Therefore in practice, Hansen-type measures are ones of proportionality and are better understood as *ordinal indicators*; they rank accessibility but lack cardinal meaning or consistent units (E. J. Miller 2018). The continued absence of a sort of proportionality constant leaves a conceptual and practical gap in accessibility analysis: a missing link between theoretical form and empirical measurement that hinders the interpretability of accessibility measures.

# Wilson’s family of spatial interaction models

While accessibility research evolved in North America with Hansen (Hansen 1959), a parallel development was taking place across the Atlantic with Alan G. Wilson. Wilson’s ground breaking paper (Wilson 1971) advanced a general framework for spatial interaction modeling focused on flows of interaction between places and derived from a non-Newtonian analogy. Additionally, this work was not focused on the ‘potential’ concept, as accessibility has been. Wilson (Wilson 1971) formalized the general spatial interaction model through the following equation:

The model in [Equation 8](#eq-phys-gravity-model) posits a quantity that represents a value in a matrix of flows of size , that is, between origins and destinations. The quantities and are proxies for the masses at origins and destinations. The super-indices and indicate that these masses can be any number of different things associated with the zones, i.e., could be population at a zone as an origin, and hectares of park space at a zone as a destination. is some function of travel cost which reflects travel impedance.

Of important note for this paper, in [Equation 8](#eq-phys-gravity-model) acts as a proportionality constant, shifting the equation from a proportional to an equal relationship by incorporating known system totals. In some sense, serves a role similar to the gravitational constant in Newton’s law–it calibrates the model so that outputs match real-world quantities. However, these real-world quantities are not a set empirical constant (like Newton’s ) but instead, sensitive to the system and known information about the system.

From the outset, spatial interaction models emphasized interpretability of results (Kirby 1970; Wilson 1967, 1971). But unlike earlier approaches that borrowed heuristically from Newtonian gravity (i.e., interaction between masses over distance), Wilson’s innovation was to ground and generalise the model using *entropy maximisation*. By maximizing the number of ways individual trip probabilities could be arranged under known constraints, Wilson derived models that estimate *statistical averages* of flows between zones (Wilson 1971; Senior 1979).

Crucially, to ensure that in [Equation 8](#eq-phys-gravity-model) is maintained in units of flow between and , the model moves from proportionality to calibrated equality by incorporating empirical constraints. At a minimum, this requires knowledge of the total number of flows in the system, leading to the basic constraint:

Additional information can be introduced. For example, when information is available about the total number of flows produced by each origin, in [Equation 8](#eq-phys-gravity-model), represented as , then following constraint can be used:

As well, if there is information available about the total number of flows attracted by each destination, is represented as and the following constraint can be used:

It is also possible to have information about both and , in which case both constraints can be imposed on the model at once.

Depending on which of the three system constraints are applied, a family of spatial interaction models can be derived from [Equation 8](#eq-phys-gravity-model). The proportionality constant is replaced with different *balancing factors*. This change in name is more useful as these factors not only preserve proportionality but also ensure that the predicted flows align with the known constraints considered in the system. In other words, the different balancing factors adjust the model so that flows become statistical averages consistent with observed origin and/or destination data.

In the framework introduced and inferred from Wilson (Wilson 1971), three types of balancing factors are specified: (1) an unconstrained model that only matches the total volume of interaction , (2) a singly constrained model (either by origins or destinations - see below), and (3) a doubly constrained model that satisfies both.

In the unconstrained model, constraints in [Equation 10](#eq-constraint1-gravitymodel) and [Equation 11](#eq-constraint2-gravitymodel) do not hold. In practical terms, this means that the total number of flows predicted by the model must be equal to sum of all flows from origins to destinations . The balancing factor takes the place of and is equal to the following (as specified in (Cliff, Martin, and Ord 1974) and (A. S. Fotheringham 1984)):

In the singly-constrained model, only constraint [Equation 10](#eq-constraint1-gravitymodel) or constraint [Equation 11](#eq-constraint2-gravitymodel) hold. When only [Equation 10](#eq-constraint1-gravitymodel) holds, entropy maximisation leads to the production-constrained singly-constrained version of [Equation 8](#eq-phys-gravity-model), where the proxy for the mass at the origin is replaced with . Also, is replaced with a set of balancing factors specific to origins which ensures that constraint [Equation 10](#eq-constraint1-gravitymodel) is satisfied (i.e., the sum of predicted flows from one origin going to all destinations must equal the known mass at that origin ). Satisfying this constraint also implicitly fulfills the total constraint ([Equation 9](#eq-constraint0-gravitymodel)), since the sum of values across all origins equals the total number of flows. takes the following form:

The singly-constrained attraction-constrained model is similar to the production-constrained version but from the perspective of the mass at the destination. For the attraction-constrained model, the proxy for the mass at the destination is replaced with in [Equation 8](#eq-phys-gravity-model), representing the spatial interaction inbound flow. Also, is replaced with a set of destination-specific balancing factors that ensure that [Equation 11](#eq-constraint2-gravitymodel) is satisfied (hence the total constraint [Equation 9](#eq-constraint0-gravitymodel) is as well), meaning that the sum of predicted flows going to one destination from all origins must equal the known mass of that destination . As before, destination-specific balancing factors were derived by Wilson as:

Lastly, the doubly-constrained model is the production-attraction constrained model in Wilson (Wilson 1971). In this case, both constraints [Equation 10](#eq-constraint1-gravitymodel) and [Equation 11](#eq-constraint2-gravitymodel) hold simultaneously. These constraints ensure that the sum of predicted flows from one origin to all destination, and the predicted flows going to one destination from all origins must equal the known mass of the origin and of the destination . This should hold for all origins and destinations. The resulting model is, in Wilson’s terms, doubly constrained, and from [Equation 8](#eq-phys-gravity-model), becomes both and shown in [Equation 15](#eq-doubly-constrained-balancing-factors), and and is replaced with and . Derivation of these models is demonstrated in detail elsewhere (e.g., Ortúzar and Willumsen 2011; Wilson 1967).

Wilson’s work is notable for many reasons. Rather than relying on a universal constant like or a scaling factor to balance units, Wilson’s models calibrate interaction flows through known empirical constraints using principles of entropy maximisation. This results in interpretable, balanced (given the constraints), and unit-consistent flows. In this way, spatial interaction modeling tradition can be seen to have succeeded where accessibility modeling stalled. While Wilson’s model produces results that are in units of flow tethered to the system of analysis, Hansen-type measures (still widely used today in accessibility work) yield outputs that reflect proportional – not equal – relationships and typically lack interpretable units.

As a preview for the work ahead: Wilson’s balancing factors can be manipulated into *proportional allocation factors* and equations can be established to reflect accessibility. The resulting flows can be understood as accessibility flows–and aggregated at the zone level to express accessibility in units of actual opportunities or population.

In this way, these balancing factors can be used to recover what Stewart’s was meant to accomplish, but through a different conceptual path. But before demonstrating a family of accessibility measures that are sensitive to constraints, in the next section we review how conceptually intertwined the spatial interaction modelling and accessibility literatures are, and where they began to diverge. This investigation sheds light on why accessibility research may have failed to adopt a comparable approach focused, until this paper.

# Accessibility and spatial interaction modelling: two divergent research streams

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| Figure 1: Historical pattern of publication: documents per year. |

Despite their close conceptual ties, the accessibility and spatial interaction modeling literatures have developed along largely separate paths since the 1970s. Hansen’s (Hansen 1959) formulation of accessibility became the method used for decades of work on transport equity, land use analysis, and urban accessibility planning. Meanwhile, Wilson’s (Wilson 1971) entropy-maximising framework reshaped how spatial interaction models were constructed, particularly in transportation demand forecasting. We argue the framework’s quiet innovation–introducing empirically grounded constraints to shift from proportionality to calibrated equality–made the framework immediately relevant for policy applications as outputs were in tangible units. Yet, this mechanism was never widely adopted in accessibility analysis.

This divergence is especially striking given the context in which both frameworks emerged. As noted in (Michael Batty 1994), large scale spatial interaction models (like Wilson’s) responded to important developments at the time, a need “to meet the dictates and needs of public policy for strategic land use and transportation planning”. And these needs were far from trivial: for instance in the U.S., the Federal-Aid Highway Act of 1956 set in motion the construction of the Interstate Highway System with an eventual budget exceeding $600 billion in today’s dollars (Weiner 2016; MDOT 2007). In this context, spatial interaction models were incorporated into institutional practices focused on “predict and provide” travel demand forecasting (Kovatch, Zames, et al. 1971; Weiner 2016). Accessibility analysis, by contrast, remained more conceptually diffuse, focused on indicators of “potential” spatial interaction with opportunities rather than flows that could tangibly guide infrastructure decisions (e.g., roadway capacity expansion, new construction). Whereas spatial interaction modelling became a key element of transportation planning practice, accessibility remained a somewhat more academic pursuit, and the two streams of literature only rarely connected.

To explore why Wilson’s approach never crossed over to accessibility modeling, we conduct a review of the literature citing Hansen (Hansen 1959), Wilson (Wilson 1971), or both (on Web of Science using the “Cited References” functionality, and the digital object identifiers of Hansen (Hansen 1959) and Wilson (Wilson 1971)) . Only 76 out of the 2,122 documents that emerged from our search cite both. The number of articles, by year and if they cite Hasen, Wilson, or both are shown in [Figure 1](#fig-docs-per-year).

Through the close analysis of *how* articles that cite both works, we identify two distinct citation patterns: one group of articles focused on accessibility, the other on spatial interaction. In examining these articles, we uncover how the relationship between the two has often been misunderstood, underexplored, or entirely overlooked.

In the first stream of literature–which cite both but are focused more so on spatial interaction models–they treat spatial interaction and accessibility as separate but related phenomenons. Four subsets of this stream emerge.

First, some of the more early works interpret the spatial interaction model’s balancing factors ([Equation 13](#X6907b9d2cdd8b915698444ffab45e6902558aa6) or [Equation 15](#eq-doubly-constrained-balancing-factors)) as the inverse of Hansen’s accessibility measure (B. Harris and Wilson 1978; G. Leonardi 1978; A. Stewart Fotheringham 1981; A. S. Fotheringham 1985), likely following Wilson’s own recognition of this similarity between balancing factor and Hansen-type measure on p. 10 in Wilson (Wilson 1971). In some ways, this relationship has been recognized as a “common sense” approach to incorporating accessibility in the spatial interaction model (Morris, Dumble, and Wigan 1979, 99), though acknowledgment of its further exploration has been recommended (M. Batty and March 1976).

The second subset of articles within this stream use both Hansen (Hansen 1959) and Wilson’s (Wilson 1971) framework in conjunction. For instance, some articles argue that spatial interaction models fail to explain certain spatial patterns on their own, for instance, as in Fotheringham (A. S. Fotheringham 1985) who demonstrates how the spatial interaction model may insufficiently explain spatial patterns, and suggests that explicitly defining destinations’ accessibility (Hansen-type accessibility) as a variable within the model may remedy the issue (e.g., the *competition destination* model). Other works take a more applied approach: such as in defining location-allocation problems in operations research (G. Leonardi 1978; Beaumont 1981), estimating trips (or some other spatial interaction flows) alongside accessibility (e.g., Clarke, Eyre, and Guy 2002; Grengs 2004; Türk 2019), or considering accessibility as a variable within spatial interaction models, in line with Fotheringham’s (A. S. Fotheringham 1985) demonstration (e.g., Beckers et al. 2022).

The third subset of the spatial-interaction focused literature, depart from Hansen’s (Hansen 1959) definition, aligning instead with microeconomic or utility-based interpretations of potential spatial interaction e.g., (Morris, Dumble, and Wigan 1979; Giorgio Leonardi and Tadei 1984). In sum though, across these works, they recognize Hansen-type accessibility as an indicator of ‘potential’ but as a separate but related concept to spatial interaction.

Moving onto the group of accessibility-focused literature that cites both works, we categorise their citation of Wilson (Wilson 1971) within three general groups. Overall though, these works do not engage, or only superficially engage with Wilson (Wilson 1971).

Firstly, there is a group of articles within this stream that cite Wilson (Wilson 1971) exclusively as attribution for using context-dependent travel cost functions. This trend is common: for instance it is done in (S. L. Handy and Niemeier 1997; Weibull 1980; Kwan 1998; Q. Shen 1998b; Ashiru, Polak, and Noland 2003; Rau and Vega 2012; Pan 2013; Margarida Condeço Melhorado et al. 2016; Caschili, De Montis, and Trogu 2015; Grengs 2015; Pan, Jin, and Liu 2020; Chia and Lee 2020; Roblot et al. 2021; Sharifiasl, Kharel, and Pan 2023; Kharel, Sharifiasl, and Pan 2024). However, these works do not engage with spatial interaction beyond this attribution.

Secondly, a subset of literature explicitly associates spatial interaction, as defined in Wilson (Wilson 1971), with accessibility’s potential for spatial interaction–but only superficially. These works acknowledge the conceptual link, but do not go beyond this recognition in the scope of their works e.g., (Grengs 2010, 2012; H. J. Miller 1999; Giuliano et al. 2010; Grengs et al. 2010; Levine et al. 2012; Levinson and Huang 2012; Tong, Zhou, and Miller 2015; X. Liu and Zhou 2015; He et al. 2017; Wu and Levinson 2020; Ng et al. 2022; Naqavi et al. 2023; Suel et al. 2024). Indeed, while accessibility can be seen as the *potential* for spatial interaction–and Wilson (Wilson 1971) briefly touches on this–such mentions have not resulted in deeper analytical integration of these concepts. Furthermore, some of this literature also occasionally conflates or blurs the distinction entirely, for instance by co-citing Hansen and Wilson as being ‘gravity models’ (e.g., S. Liu and Zhu 2004; Dai, Wan, and Gai 2017; Y. Shen 2019; Chia and Lee 2020). This conflation reveals ongoing murkiness between the distinction of spatial interaction and the *potential for* spatial interaction in the literature.

Thirdly, there is a group of accessibility-focused works that interprets the measure used in Hansen (Hansen 1959) as the singly- or doubly- constrained spatial interaction model’s inverse balancing factor (e.g., R. W. Vickerman 1974). This group often cites the spatial interaction works that make this connection (i.e., the first subset of the first stream of literature) and is especially prominent in the investigation of competitive accessibility topics e.g., (Karst and Van Eck 2003; Karst T. Geurs, van Wee, and Rietveld 2006; Willigers, Floor, and van Wee 2007; El-Geneidy and Levinson 2011; Curtis and Scheurer 2010; Manaugh and El-Geneidy 2012; Chen and Silva 2013; Alonso et al. 2014; Albacete et al. 2017; Sahebgharani, Mohammadi, and Haghshenas 2019; Mayaud et al. 2019; Allen and Farber 2020; Levinson and Wu 2020; Marwal and Silva 2022; Su and Goulias 2023). Only the works of Soukhov et al. (Soukhov et al. 2023; Soukhov et al. 2024) use Wilson’s (Wilson 1971) balancing factors as a method for maintaining constraints on opportunities within the context of competitive accessibility.

On that note, Soukhov et al., 2023 and 2024 (Soukhov et al. 2023; Soukhov et al. 2024) introduce the balancing factors as a mechanisms to ensure that opportunities at each destination are proportionally allocated to each zone (based on the proportion of population seeking opportunities and the relative travel impedance). This is to ensure that each zonal accessibility value is the sum of this proportional allocation from each destination, and that all zonal values ultimately sum to the total number of opportunities in the region. However, these balancing factors were deduced intuitively. These works did not explicitly state that the mathematical formulation of the equations are effectively equivalent to Wilson’s singly constrained model (derived from entropy maximization). This equivalence is only discovered in hindsight, as will be demonstrated in the following section. These two works also do not discuss other constrained cases that will also be addressed.

In sum, despite the interpretative advantaged offered by the statistical logic of Wilson (Wilson 1971)’s framework, neither the spatial interaction literature that cites Hansen (Hansen 1959) nor the accessibility literature citing Wilson (Wilson 1971) has meaningfully applied Wilson’s constraint-based logic to the concept of accessibility. So, in the next section, we demonstrate how applying Wilson’s framework enables accessibility to move from a proportional relationship to calibrated equality–tying outputs to tangible system knowns. This re-expression of accessibility using constraints yields interpretable, unit-consistent measures of opportunity. This approach takes the same path of entropy-maximisation as in Wilson (Wilson 1971), and does not rely on specifying some universal constant like initially proposed in Stewart (Stewart 1948) (recall, [Equation 6](#eq-stewart-population-potential-sum) which Hansen (Hansen 1959) operationalised).

# A family of accessibility measures: from proportionality to equality

Despite the close conceptual ties between accessibility and spatial interaction modeling, the former has not meaningfully absorbed the constraint-based logic of the latter. This section introduces this reframing: defining a family of accessibility measures using Wilson’s approach grounded in statistical mechanics, shifting place-based accessibility (a la Hansen (Hansen 1959)) from a relationship describing proportionality to equality. This shift addresses the fundamental issue of interpretability associated with the proportional nature of Hansen-type accessibility indicators we previous outlined.

To do so, we propose a revised definition of accessibility considerate of the constraint-based spatial interaction model: the potential for spatial interaction with opportunities (or population). We can specify as a type of proportional allocation factor which incorporates Wilson’s balancing factor(s) to define the *potential for spatial interaction with opportunities* and the *potential for spatial interaction with population* . In effect, is unitless and proportionally allocates (based on the constraints of the case) opportunities (for ) and population (for ). The equation is as follows:

Where and reflect the mass of the destination (i.e., opportunities) and mass of the origin (i.e., population) at either the zone or for the full region, depending on the case (hence represented by a ).

Accessibility flows can also be summarised as as a partial sum of the potential at and at to express accessibility at the origin zone or at the destination zone, respectively, as often done in accessibility research:

Figure [Figure 2](#fig-analytical-device-conc-accessibility) illustrates our analytical framework using a simple 3-zone system. The most detailed values, , represent the potential for spatial interaction from origin to destination . Here, stands for all cases and variants to be discussed (e.g., , , , , , , , and ). Single marginals show origin and destination weights, while the total marginal sums these values.

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| Figure 2: The family of accessibility measures analytical framework: labelling and associating ij flows, zonal weights, the single marginals, and the total marginal. |

The proportional allocation constant takes the form of a balancing factor that varies depending on the constraints applied. Each member of the accessibility measure family is defined by the constraints used, and can be grouped into the following four cases:

1. **Unconstrained Case (, )**

* Equivalent to Hansen’s (Hansen 1959) and Reilly’s (Reilly 1929) original formulations; the status quo of accessibility modelling.
* No balancing factors applied; units are in “opportunities-by-impedance” for or “population-by-impedance” for .
* No constraints are applied, so values reflect proportionality only and are not calibrated to known system totals.

1. **Total Constrained Case (, )**

* Applies a total proportional allocation factor (, ) based only on the total marginal (green box in [Figure 2](#fig-analytical-device-conc-accessibility)) i.e., total number of opportunities or population in the system. This ensures the sum of all values in the system match the total marginal.
* Units of : accessible opportunities from , a value that is total constrained and linearly proportion to .
* Units of : accessible population from , a value that is total constrained and linearly proportion to .

1. **Singly Constrained Case (, )**

* Applies singly-constrained proportional allocation factors (, ) based on Wilson’s balancing factors (, ) to preserve either the destination-side or origin-side marginal totals (blue and red boxes in [Figure 2](#fig-analytical-device-conc-accessibility)) i.e., the number of opportunities or population at each zone. Reflects how the literature calculates competitive accessibility.
* Units of : accessible opportunities from , a value that is the sum of opportunity supply flows allocated proportionally based on demand at . Mathematically equivalent in per-capita form to 2SFCA (Luo and Wang 2003).
* Units of : accessible population from , a value that is the sum of population demand flows allocated proportionally based on supply at .

1. **Doubly Constrained Case (, )**

* Constrained on both origin and destination sides using both and simultaneously, which can also be expressed as proportional allocation factors (, ); equivalent in interpretation to Wilson’s (Wilson 1971) doubly constrained spatial interaction model.
* Simultaneous application ensures both the destination-side *and* origin-side marginal totals are maintained (blue and red boxes in [Figure 2](#fig-analytical-device-conc-accessibility)).
* Interpretable only as and flows, since aggregation at and simply reproduces known totals. Represents ‘interaction capacity’ or ‘realized access’ serving as predictions of real interaction flows.

As a summary, each member of the family of accessibility measure is named, explained in plain language, alongside their balancing factor(s), proportional allocation factor(s), and mathematical equation and value interpretations in Table 1.

Table 1: Summary of constrained accessibility measure types and interpretations

## Toy example setup

Consider the simple 3-zone region in [Figure 2](#fig-analytical-device-conc-accessibility), where each zone serves as both origin () and destination (). The system includes three inputs: zonal population and opportunities, a zonal cost (travel time) matrix, and three travel impedance functions representing different travel behaviours.

First, [Table 1](#tbl-small-system-land-use) summarizes population (in 10,000s) and physicians per zone. For context, the Provider-to-Population Ratio (PPR) is 24.5 comparable to Canada’s 2022 PPR of 24.97 physicians per 10,000 (WHO 2025). Second, [Table 2](#tbl-small-system-cost) shows travel times (minutes); it can be discerned that Zones 1 and 3 are closer to each other than to Zone 2. Zones 1 and 3 together have a population roughly equal to Zone 2 but offer more than twice the physician availability. We interpret Zone 1 as the Urban Edge, Zone 3 as part of the Urban Core, and Zone 2 as Suburban.

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| Table 1: Simple system with three zones (ID 1, 2 and 3). Population is in 10,000 persons and opportunities in number of physicians. |

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| Table 2: Cost matrix for system with three zones (travel time in minutes). |

And third, [Equation 18](#eq-travel-behaviour-scenarios) presents the assumed travel impedance functions reflecting three different travel behaviours. A helpful analogy may be tying travel behaviour to the used mode’s mobility potential, i.e., the most decaying travel behaviour () would assume all travel in the region being done by foot, while calculating accessibility assuming the least decay () would assume unfettered automobility.

Any set of concepts representing population, opportunities, and their associated travel behaviour, whether representing the entire region uniformly (as will be demonstrated) or representing specific subgroups, can be substituted into our simple toy example. The purpose of the following simple example is to demonstrate the calculation and interpretation of the four accessibility measure variants.

## Unconstrained accessibility

In , no proportional allocation factor is defined, simply is used to weight the number of opportunities at each and the weighted values for each are summed for each , yielding the an expression identical to Hansen’s accessibility (Hansen 1959), the current standard practice in accessibility measurement:

However generally does not equal the total opportunities , so units here are ‘opportunities weighted by travel impedance’ and lack meaningful scaling or direct interpretability. Comparing values across different decay functions or contexts (i.e., different number of zones) is therefore limited to ordinal statements (more vs. less), not intervals or ratios (i.e., the magnitude of differences).

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| Table 3: Simple system: unconstrained accessibility. |

For example, [Table 3](#X9233cab131a3b76d92e40634b9c9d5bef5451ce) shows under each decay function. Comparing across decay types is meaningless in absolute terms. For instance, the difference in zone 1 (edge of urban core)’s accessibility under vs is 370.92, but in what units? These two values are a product of different impedance functions ( *physicians-minute* and *physicians-minute*), making the comparison uninterpretable (and arguably incorrect). The fundamental uninterpretability of what is an *opportunity-weighted-travel-impedance* unit remains.

As the different impedance functions represent different travel behaviours, comparing the raw unconstrained accessibility values across groups is meaningless beyond notions of higher or lower. While one could attempt to adjust the units post-calculation (e.g., scaling, population normalization) or select impedance functions to facilitate comparison across scenarios (potentially at the expense of accurately reflecting travel behavior), such adjustments may introduce bias. The unconstrained scores are best used for ranking within a single context.

The next sections introduce constraints to calibrate these measures for better interpretability and comparability, applying each to this example in turn.

## Total constrained accessibility

The total constrained accessibility case can be interpreted in a few ways. In the one that connects to the status quo: the total balancing factor proportionally adjusts unconstrained zonal accessibility values so their total sum of matches a known system total–either total opportunities or total population. Another interpretation is in reformulating the equation to use a a proportional allocation constant based on the total balancing factor. The proportional allocation constant distributes opportunities (or population) proportionally by travel impedance.

In both formulations, all zonal values become a proportion of a known system total, be it the regional opportunities or regional population depending on the variant.

We define two variants for this case:

* : accessibility is constrained by the total number of opportunities (total constrained accessible opportunity) and which is interpreted as Hansen’s accessibility with a constraining constant, and
* : where and of the first variant is transposed, yielding a measure constrained by the total population and to be interpreted as constrained ‘market potential’.

### Total constrained accessible opportunities: Hansen’s accessibility with a total constraint

In the total constrained case, accessibility is expressed as a share of the total number of opportunities in the region , allocated based on travel impedance. The total constrained accessibility from to takes the form:

This formulation satisfies the total constraint, analogous to the one in the Wilson framework:

Next, the proportional allocation factor determines the share of total opportunities assigned to each origin–destination pair, based on the relative proportion of opportunities-weighted travel impedance:

This renders (equal to ) into units of opportunities (e.g., physicians), and allows direct interpretation and comparison of results between zones and scenarios.

Alternatively, this formulation can be rewritten to be expressed using a total constrained balancing factor , which scales Hansen’s unconstrained accessibility to meet the total opportunity constraint:

Where the total constrained balancing factor is:

This expression is consistent with Wilson’s entropy-maximizing framework and analogous to the total flow spatial interaction model (e.g., Equation 2.11 in (Cliff, Martin, and Ord 1974)).

In summary, proportionally allocates the total number of opportunities to each origin–destination pair based on relative opportunity-weighted travel impedance. These values can be aggregated across destinations to obtain total constrained accessibility at each origin. Alternatively, the measure can be expressed using the balancing factor , demonstrating that it is algebraically proportional to unconstrained accessibility , but with interpretable units (i.e., opportunities). This allows for meaningful comparisons of differences across zones and travel behaviour scenarios.

Referring back to our simple numeric example, for the highest decay travel scenario would then be:

for other decay scenarios are calculated similarly in code. Applying each to the unconstrained values yields total constrained accessibility values ([Table 4](#X950d72cddc3b41b7ffe636d17bd6f557417d467)), all in units of physicians.

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| Table 4: Simple system: total constrained accessible opportunities. |

Compared to the unconstrained case, values now sum to the known regional total , allowing interpretation of absolute and relative differences across zones and travel scenarios. For example, in the highest decay case, Zone 1 (Urban Edge) captures an intermediate number of physicians (172.065), like in the unconstrained accessibility case. However, unlike in the unconstrained case, we can say that this value is out of the 490 physicians in the region, which allows us also to deduce that zone 1 captures 1.307 and 0.924 times more than zone 2 and 3. Values for the lesser decay () and lowest decay () scenarios are calculated separately, with decay scenario values also summing to equal 490 physicians accessible in the region.

One can also directly compare values at a specific zone, across travel impedance scenarios, due to the consistent units. As the decay scenario decreases, all zones become more accessible to each other and the differences between pairs diminishes (i.e., in each zone captures close to an average amount of physicians, a third of 490 or ~163). In terms of proportional magnitude, this can also be observed in the unconstrained measure for this scenario. However, for the total constrained measure, this plateauing of results has meaning. In fact, each zone is allocated an average of the total amount in the region, as a result of the total constrained proportional allocation factor.

However, what’s notable is how zones change between scenarios. For instance, Zone 1’s share only declines slightly–these declines are outpaced by Zone 2’s relative gains. This shift reflects how redistributes opportunities in proportion to impedance under different travel behaviours.

The total constrained accessibility measure resolves the interpretability issue of Hansen’s accessibility (i.e., unconstrained accessibility) by grounding values in a meaningful total, enabling robust comparisons across zones and scenarios but also by keeping values proportional to so interpretation is similar.

### Total constrained accessible population: Reilly’s potential trade territories with a total constraint

This second total constrained variant is a transpose of the opportunity-constrained formulation, switching indices and to yield a measure of market potential–the number of people who can spatially interact with a destination.

Though not outlined in the “Unconstrained accessibility” section, the unconstrained form aligns with Reilly’s “potential trade territories” (Reilly 1929) and Harris’ and Vickerman’s formulations of regional market potential (C. D. Harris 1954; R. W. Vickerman 1974). In its unconstrained form, market potential has also been more recently to estimate potentially accessible populations following infrastructure investments (e.g., Gutiérrez 2001; Holl 2007; Condeço-Melhorado and Christidis 2018). Market potential can also be thought of as a form of *passive accessibility*, indicating the number of people that can reach each destination.

However, like , issues of unit interpretability arise in unconstrained market potential . To address this, we introduce the total constrained accessible population measure , which allocates the total population across all origin-destination pairs proportionally:

Subject to the total constraint::

Next, is the total constrained proportional allocation factor, a dimensionless term which distributes population based on impedance-weighted accessibility:

This renders (equal to ) into units of population. Alternatively, this formulation can be rewritten to be expressed using a total constrained balancing factor , which scales unconstrained market potential to meet the total population constraint:

Where the total constrained balancing factor is:

In summary, allocates the total number of population proportionally to each origin–destination pair based on relative population-weighted travel impedance. As well, the measure can be expressed using the balancing factor , demonstrating that it is algebraically proportional to unconstrained market potential, but also yielding interpretable units that allow for meaningful comparison.

Returning to the numerical example, the balancing factor is solved for each travel behaviour scenario, and the market potential of each zone is expressed as units of population (e.g., the number of people accessible from each origin at that destination) in [Table 5](#Xeff7ffaf35371dd5f7484ab3799490899a7ea15).

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| Table 5: Simple system: total constrained accessible population. |

Readers may note the difference in trends in accessible population ([Table 5](#Xeff7ffaf35371dd5f7484ab3799490899a7ea15)) and accessible physicians (i.e., the preceding subsection, [Table 4](#X950d72cddc3b41b7ffe636d17bd6f557417d467)).

In [Table 4](#X950d72cddc3b41b7ffe636d17bd6f557417d467), zone 1, 2, 3 represent destinations and the accessibility values reflect the number of accessible people from the vantage of physicians. Zone 1, in its role as a destination, is no longer intermediately-ranked relative to other zones; it now attracts the fewest number of people across all three travel behaviour scenarios. However, similar to the total constrained opportunity case, as travel decay reduces, the availability of population begins to converge (though Zone 1 continues as the lowest-ranked) for similar reasons. As decay reduces, the population’s travel impedance to all zones become more similar, making the relative location of the zones less important and all people in the region more equally accessible.

Like in the total constrained accessible opportunities variant, the total constrained accessible population enables direct comparison of raw values, supporting both ordinal and interval interpretations across space and travel behaviour scenarios.

## Singly constrained accessibility

The singly constrained accessibility case can also be expressed in two variants, each defined by the direction in which a constraint is applied:

* : accessibility constrained by opportunities at destinations (singly constrained accessible opportunities), and
* : its transpose, constrained by population at origins (singly constrained accessible population, or market potential).

Similar to the total constrained case, the singly constrained measures adjust unconstrained zonal accessibility values ( or ) using a balancing factor to satisfy the known system constraint. However, unlike the total constraint (which enforces a global/total sum), the singly constrained case applies a localized constraint at one end of the interaction–either origin or destination.

In the opportunities-constrained variant, the balancing factor ensures that only a proportion of opportunities at each destination are allocated to origins, based on their relative demand (population) and travel impedance. This variant mirrors the concept of spatial availability as discussed in Soukhov et al. (Soukhov et al. 2023). In the population-constrained variant, the logic is reversed: population at each origin is allocated proportionally across destinations, informed by the distribution of opportunities and impedance.

In both cases, the singly constrained formulation introduces zonal-level competition, unlike the total constrained case which distributes a fixed regional sum. Each zonal accessibility value becomes not only a fraction of the regional total (opportunities or population), but also a balanced sum of interactions, weighted by impedance and relative competition. The result remains in interpretable units–accessible opportunities or accessible population–but reflects more complex spatial dynamics.

### Singly constrained accessible opportunities: spatial availability

In this singly constrained variant, accessibility is constrained at the destination side: the sum of accessible opportunities allocated from each destination must equal the known number of opportunities . This is comparable to the single attraction-constraint ([Equation 11](#eq-constraint2-gravitymodel)) from Wilson’s framework:

The underlying spatial interaction model is now the attraction-constrained model and our accessibility measure becomes:

where is a measure of the mass at origin (i.e., the opportunity-seeking population). The corresponding balancing factor, as per Wilson, is:

Introducing the balancing factor in [Equation 29](#eq-opportunity-constrained-accessibility), we obtain:

Further, we can express the formula even more simply, by defining the following proportional allocation factor:

After this, it is possible to rewrite [Equation 31](#X93029e29ec311e3458a306fd52c8c33c3a750ce) as an origin summary expression of proportionally allocated known opportunities (i.e., ).

This formulation has been referred to as **spatial availability** by Soukhov et al. (Soukhov et al. 2023), since it incorporates spatial competition by allocating opportunities based on demand (population), impedance, and the known opportunity totals . The dimensionless factor ensures that each destination’s opportunities are distributed proportionally to origins. As in the total constrained case, is expressed in the units of accessible opportunities.

Soukhov et al. (Soukhov et al. 2023) also showed that the following expression (accessibility per capita) is a constrained version of the popular 2SFCA approach of Shen (Q. Shen 1998a) and Luo and Wang (Luo and Wang 2003):

Returning to the simple numeric example, as an example of the solved for the highest decay travel behaviour :

The balancing factors for the decay group for zones 1, 2 and 3 are 12.857, 8.769 and 10.664, respectively. For the decay group, they are 0.067, 0.066 and 0.066. Using these these balancing constants, we can calculate the singly constrained opportunity accessibility ([Table 6](#Xf7ca115d12f90c5dcdf6bbd2adcccca7e25c387)).

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| Table 6: Simple system: singly constrained accessible opportunities. |

Imposing the single proportional allocation factor allows for the comparison of differences and ratios of the accessibility values, like previously discussed in the total constrained accessible opportunities case. The proportional allocation factor ensures that resulting values are in units of *physicians*, with the impedance units already accounted for in the allocation process.

However, unlike , introduces zonal competition based on the mass of the origin (population). In the total constrained case, opportunities are distributed based on impedance alone, regardless of population at . In contrast, the singly constrained case allocates each zone’s opportunities proportionally across the region based on the relative impedance-weighted demand from all origins.

This consideration has important implications. Consider the highest decay scenario . Under this scenario, Zone 1–despite hosting a medium amount of physicians–captures the fewest physicians (133.469), compared to 166.781 at Zone 2, and 189.75 at Zone 3. Why? Zone 1 has the smallest population and is adjacent to Zone 3, the urban core. Its low impedance-weighted demand means allocates it fewer opportunities. By contrast, in the total constrained case, Zone 1 fares better, capturing 35% of all physicians (compared to 27%).

As travel decay decreases (e.g., ), competition becomes more diffuse. Zone 2, with the largest population, initially dominates accessibility–but under low decay, Zones 1 and 3 also begin drawing more opportunities from Zone 2. For instance, Zone 1 gains 6% more from Zone 2 in than in . This shift reflects a drop in by 14%, showing Zone 2’s decreasing hold on its own opportunities as other zones gain accessibility ‘parity’.

This dynamic reveals how embeds both travel impedance and population competition. Unlike the total constraint lowest decay scenarios that allocate evenly, the singly constrained case reflects how competition evolves continues to influence allocation.

In this way, the consideration of constrained accessibility *per capita* may be clarifying. Often, accessibility values are reported as raw scores without considering the population potentially accessing them. But, as we introduced constraints, these constrained accessibility values can be normalized using anything that is relevant to the zone. In [Table 7](#Xfaec589ab2da6a3e7265208f409fd0ed974a36a), we present per capita accessibility for the numeric example simply in units of number of physicians accessible per population at each zone. Notably, these per capita rates are equivalent to the 2SFCA values.

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| Table 7: Simple system: singly constrained accessible opportunities per capita. |

This simple example was constructed so that the regional average equals 24.5 physicians per 10,000 people. As distance decay decreases and becomes *relatively* uniform (all zones can reach all zones), the effect of population drives the proportional allocation of opportunities. Consequently, per capita accessibility values begin to stabilise to the regional per capita average (e.g., in the lowest distance decay , per capita values are all ~24 physicians accessible per capita).

This convergence mirrors the trend in the total constrained opportunity case, where accessibility values approach a third of the 490 physicians under the unfettered mobility scenario . In both cases, the balancing factors ( and ) act as averaging mechanisms but at different scales. As distance decay becomes more *relatively* uniform, the role of remaining variables (i.e., total population or opportunities) drive the proportional allocation differences. In the total constrained case, this is the proportion of opportunities relative to the regional opportunities, and in the case of the single opportunity constrained case, this is the population at a zone relative to the regional population.

### Singly constrained accessible population: market availability

Similar to [Equation 27](#eq-total-population-balancing-factor) in transposing the origins and destinations, we can define a *singly constrained* measure of market potential that preserves the known population (i.e., the mass weight at the origin is now represented by ). In it’s per-capita expression, i.e., equivalent to 2SFCA, this constrained concept of market potential has been used to express “facility crowdedness” as in Wang (F. H. Wang 2018).

The underlying spatial interaction model is now the production-constrained model version of [Equation 8](#eq-phys-gravity-model), and our market potential measure becomes:

In this variant, the measure is singly constrained by the population *by origin* (i.e., ), like [Equation 11](#eq-constraint2-gravitymodel) from Wilson’s framework:

And the corresponding balancing factor, as per Wilson, is:

Following the same logic as in the preceding section on total constrained market potential, one arrives at the following expression of accessible population being the product of proportionally allocated () popultion:

with:

As well, the single (population) constraint in [Equation 36](#eq-population-constraint) ensures that the the total constraint (e.g., ) is maintained.

With these constraints, can be interpreted as the proportion of the total population serviced by location .

For the sake of brevity, we’ll move forward on to the doubly constrained case.

## Doubly constrained accessibility

This accessibility case adopts the structure of the doubly constrained spatial interaction model, where flows are constrained by both origin populations and destination opportunities . That is, the resulting accessibility outflow from each origin must match the origin’s population demand, and the resulting accessibility inflow to each destination must match the number of opportunities supplied:

Because results are made to match both margins, the results cannot be interpreted as a traditional summary at or (e.g., “opportunities accessible from ”)–those sums simply reproduce the constraint totals. Instead, the meaningful unit of analysis is the flow itself.

This distinguishes doubly constrained accessibility from the total and singly constrained cases discussed previously. In those cases, only one side of the interaction–either the total marginal or opportunity/population marginal–was constrained, while the other was treated as a demand/supply weight (e.g., or for total constrained and or for singly constrained).

By contrast, the doubly constrained model assumes both the demand (population) and the supply (opportunity) are known and bounded, and allocates flows accordingly. This makes it less suitable for traditional accessibility analysis, namely because origin and destination masses often differ in kind and units. For instance, the number of people accessing an opportunity such as park may be known, but the capacity of each park is not known. A doubly constrained approach only makes conceptual sense when population and opportunities are comparable, have a one-to-one correspondence or are paired somehow–e.g., job per worker, student per school-seat, or vaccine doses per person.

A doubly constrained approach to accessibility adopts the structure of the production-attraction spatial interaction model, where both population and opportunity totals are fixed. Mathematically, this model requires the simultaneous imposition of both the population- and opportunity- constraints in the preceding singly constrained variants ([Equation 28](#eq-opportunity-constraint) and [Equation 36](#eq-population-constraint)), namely the sum of population in all origins should match the sum of opportunities in all destinations ([Equation 41](#eq-opportunity-population-equality-2)):

As before, the simultaneous imposition of both constraints ensures the total system constraint is maintained i.e., remains equal to the total number of opportunities in the region .

The doubly constrained accessibility measure takes the form of the production-attraction (doubly constrained) spatial interaction model as follows:

where the corresponding balancing factors and , as per Wilson, are:

Calibration of the two sets of proportionality constants is accomplished by means of iterative proportional fitting, whereby the values of are initialized as 1 for all i to obtain an initial estimate of . The values of are used to update the underlying matrix, before calibrating . This process continues to update and until a convergence criterion is met (see Ortúzar and Willumsen 2011, 193–95).

The doubly constrained model completely distributes origin populations to destination opportunities according to travel impedance and supply-demand balance. This ensures that: summing across returns ; summing across returns . Thus, aggregating over or yields only the known constraints. In this way, a per-capita form (e.g., ) is not meaningful–since the output already reflects population-normalized allocation. As such, the matrix is the only interpretable output.

We could define the proportional allocation factor such that:

and represent as equal to , allowing the analyst to understand the proportional allocation of s to each flow.

Following this logic, the market potential form is effectively equivalent to , but can be read with a different interpretation: i.e., the opportunities accessed from at an vs. the population accessed from at a . The inputs of ‘opportunities accessed’ and ‘accessed population’ can already be interpreted as inherently being sensitive to both opportunities and population.

Moving on to the toy example, to calculate doubly constrained accessibility, the interpretation of the population data and the counts of the opportunity data in the numeric example must be reinterpreted. Namely, a count of physician *capacity* per destination is needed instead of simply the number of physicians. Also, we must be able to clearly state that the population is the *capacity* of the origin to interact with opportunities , i.e., the count of people seeking opportunities.

This adjusted simple example is summarised in [Table 8](#X10590a3b0aa2aff5c62dd49c60325d397a799fa). With the population (in units of 10,000s of people seeking physicians) and the opportunities (in units of 10,000s of physician-capacity) per zone. For the population, we leave this unchanged numerically but we now must keep in mind that each person interacts with one physician capacity. The number of providers per destination is however revised to represent physician capacity, scaled approximately from the original number of physicians used in previous cases ([Table 1](#tbl-small-system-land-use)). The system-wide PPR is now 1, this is compared to the unmodified example which yields system PPR of 24.5.

We keep the same zonal cost matrix, and travel impedance functions for three types of travel behaviour as before ([Table 2](#tbl-small-system-cost) and [Equation 18](#eq-travel-behaviour-scenarios)).

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| Table 8: Modified simple system with three zones reflecting matched population and opportunities. Population is in 10,000 persons and opportunities in 10,000 of physician-capacity. |

And with these modifications to the example, our objective is slightly different: to predict the flows from knowing that the amount of physician-capacity at each must be preserved and all flows to should match the number of people at , under different travel behaviour scenarios. Put another way, we’re interested in the flows assuming we already know accessibility at each . The highest decay travel behaviour scenario () is presented in [Table 9](#Xb54e41c670fd9ff3c40fa505703612ce287036c).

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| Table 9: Doubly constrained accessible opportunities assuming highest travel decay in the modified simple system. |

As shown in [Table 9](#Xb54e41c670fd9ff3c40fa505703612ce287036c) for the highest-decay scenario , accessibility is no longer meaningfully represented as zonal summaries like or , since these values reproduce the original constraints–i.e., so the physician-capacity accessible for Zones 1, 2, and 3 would be 4.002, 9.996, and 6.002. Hence, the usefulness of the doubly constrained measure lies in the interpretation as values. values represent the number of opportunities from zone allocated to populations in zone , shaped by both mass and travel impedance.

To illustrate this, consider the results for Zone 2 (Suburban zone–a high-population, low-opportunity, relatively remote zone). As shown in [Table 10](#X37ca94bc4108af4c217e5f657ca40e28e85a6fa), its intrazonal allocation (i.e., ) declines as travel impedance decay decreases–from 4.959 under to 2.667 under , out of the ~10 opportunities allocated to Zone 2 (a population of 10).

Following the intuition discussed in the singly constrained opportunity case, as decay decreases (i.e., more relatively uniform for all zones), the mass effects (effect of the population and opportunities magnitudes) become more relatively dominant in the spatial allocation.

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| --- |
| Table 10: Doubly constrained accessible opportunities at Zone 2 for all travel decay groups in the modified simple system. |

Recall, accessibility is traditionally presented as a summary zonal measure. However, in the doubly constrained case, since we force the allocation of zonal population demand and zonal opportunities supplied to be paired and allocation to be proportional, is simply the number of opportunities that matches our known population at . So following the logic of the family of accessibility measures, in the doubly constrained case, flows are the only relevant unit of analysis: spatial proportional allocations between population and opportunity capacity. Furthermore, and its transposed counterpart are structurally identical, differing only in interpretation (referring to and ): one reflects the proportional allocation of opportunity to population flows; the other, population to opportunity flows.

are also mathematically equivalent to Wilson’s spatial interaction flows. And as Wilson (Wilson 1971) explicitly noted, origin and destination weights defined in the spatial interaction model *can* be defined using any unit. However, the focus of these models has typically been on flows, often calibrated using trips (i.e., outbound and inbound trips, inherently in the same units).

Accessibility is often understood as a zonal summary of potential spatial interaction, often involving origin and destination masses in different units. These units also often misaligned–for instance, we may not know how much park space, grocery area, or childcare capacity is accessible per person. When they do align–such as people to physician capacity–we’re essentially modeling realised access flows based on known quantities of *those that interact* and the *interacted*. In such cases, the traditional accessibility question is already answered by the known information (i.e., how many opportunities can be reached by a zone? the number of people at that zone). This is why we do not foresee the doubly constrained measure being widely used in accessibility analysis: the literature has largely focused on questions of potential access, not on predicting flows of realised access.

# Conclusions

In this paper, we examined the historical and mathematical commonalities between spatial interaction models and place-based accessibility measures. As accessibility research evolved largely influenced by Hansen (Hansen 1959), researchers in the field neglected the proportionality constant that was originally present in gravity-based models, and is still present in spatial interaction modeling. This work has demonstrated theoretically, and through a simple numeric example, that by reintroducing Wilson’s system constraints and defining associated balancing factors and proportional allocation factors, we can derive a unifying family of accessibility measures that reintroduces tangible units to the resulting values. These values may be more intuitive for the purpose of analysis and comparison.

To summarize the contributions of this work, first, we place the popular Hansen-type accessibility measure (Hansen 1959) within this family of measures as an “unconstrained” case, demonstrating that resulting values cannot be directly compared across different travel scenarios without ad-hoc adjustments. We then show how applying a total constraint balances the units and produces a statistically averaged solution that converges to the regional average for each zone as the decay effect decreases. In other words, the total-constraint model could be a more interpretable alternative for the unconstrained case if population-competition is not relevant and one is interested in capturing the maximum *potential*; specifically, if there is a fixed number of opportunities in the region, and if it makes sense to assume that people accessing proximate opportunities leave fewer for others, *without* considering the population size at the origins.

We then introduced the singly-constrained case, which *does* take into account the population size at the origin in the allocation of opportunities. It is also mathematically equivalent to the spatial availability introduced in Soukhov et al. 2023 (Soukhov et al. 2023). In this case, all accessibility values are fixed to sum to a known zonal opportunity-size value (implicitly, the regional total of opportunities), but they are not required to sum to any population-based values at the zone or regional level. The singly constrained model could be useful if regional competition is a factor and if the acknowledgment that only a finite number of opportunities can be allocated from each destination (with those allocations distributed based on origin population size) is suitable. We also introduce an ‘accessible’ PPR (e.g., opportunities per capita), calculated by dividing each accessibility value by the zonal population. To clarify, this per capita expression of the singly constrained case is equivalent to the 2SFCA (Luo and Wang 2003; Q. Shen 1998a), hence linking this literature back to spatial interaction principles.

Lastly, the doubly constrained case is introduced. In this case, the sums must equal both the regional total and ensure that no zone allocates more opportunities than it has available. Specifically, accessibility values for each - pair must be a proportion of the zonal opportunity and population values simultaneously. For example, the accessibility at Zone 1 must equal the sum of opportunities from zones 1, 2, and 3, as well as the sum of the population at Zone 1. Satisfying the double constraint means the opportunities and population data must match one-to-one, so working with the accessibility - pair values should be of interest. In this sense, the research question should be concerned with ‘access’ (how many opportunities accessed from at based on given zonal opportunities and populations) instead of potential spatial interaction (e.g., typically expressed as a zonal summary measure of how many opportunities one could reach out of a regional total and/or zonal-allocation).

Overall, and building on Wilson’s (Wilson 1971) foundational work, this paper proposes a unified framework for analyzing gravity-based accessibility. By reintroducing Wilson’s proportionality constants, the family of constrained accessibility measures restores measurement units to accessibility estimates. This enhancement provides a more interpretable, consistent, and theoretically grounded basis for accessibility analysis, which could help advance the adoption of accessibility-oriented planning.

That said, there have been more recent developments in the accessibility literature, including novel person-based approaches that are time-sensitive (Yang et al. 2024; Braga, Loureiro, and Pereira 2023), behavioural (Kar et al. 2024; Lu, Chorus, and Wee 2014) or utility-based (Guzman et al. 2023; Ben-Akiva and Lerman 1985), that are not based on top-down approaches such as Wilson’s model formalized through the entropy-maximisation procedure. While gravity-based measures of accessibility dominate the applied literature, future work could further explore how these ideas of proportionality constants and balanced units could also help inform these and other new modeling approaches.

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