Multimodal spatial availability: a singly-constrained measure of accessibility considering multiple modes

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Abstract

Recent research has aimed to address the way opportunities are counted in accessibility analysis. In conventional accessibility measures, opportunities are often multiply counted, which leads to values of accessibility that are difficult to interpret. Constraining the calculations to match a known quantity ensures that the measurements sum up to a predetermined quantity (i.e., the total number of opportunities), and so each value can be meaningfully related to this total. A recent effort is spatial availability, a singly-constrained accessibility measure. In this paper we extend spatial availability for use in the case of multiple modes or, more generally, heterogeneous population segments with distinct travel behaviors. After deriving a multimodal version of spatial availability, we proceed to illustrate its features using a synthetic example. Next, we apply it to an empirical example of low emission zones in Madrid, Spain. We conclude the paper with suggestions for future research and its use in evaluating policy interventions.

Introduction

Accessibility is a key concept in the analysis of land use and transportation systems [e.g., 1,2,3], and too is coming of age from the perspective of planning [see *inter alia*, 4,5–8]. Beginning with the work of Hansen [1], accessibility measures have been widely used to evaluate the efficiency of transportation systems when combined with the distribution of opportunities in space [9]. As such, it is a holistic measure of spatial systems that measures the ease of reaching destinations [10,11].

In practice, the most common form of accessibility measure is based on the gravity model. These measures are sums of weighted opportunities around a focal point (i.e., a potential origin), based on how expensive it is to reach them. Recent research in accessibility analysis has paid attention to the way opportunities are counted in the pertinent calculations. Conventionally, the sums are not constrained, which means that the same opportunity can enter the sum for different origins. Counting the same opportunity multiple times treats it as if it was inexhaustible. But opportunities in general are not inexhaustible, and in fact some of them are by definition exclusive: for example, once a job is taken up by someone in the population, the same job is no longer available for any other person to take. More generally, opportunities are subject to

congestion: for example, multiple people can obtain services from the same family [18] doctor, but the more people who do, the more congested the service will be. [19]

The issue of congestion in accessibility measures was the motivation for the [20] development of floating catchment area approaches [12,13]. While these approaches [21] purport to account for congestion, Paez et al. [14] demonstrate that in general they do [22] not solve the issue of multiple counting of opportunities, thus leading to biases in the [23] calculation of total demand and supply, sometimes inflating them, other times deflating [24] them. In response to this, recent research has paid closer attention to the way [25] opportunities are counted in accessibility analysis. Paez et al. [14], for example, tackle [26] floating catchment area methods and introduce a normalization of the impedance [27] matrix to allocate the population and then the level of service proportionally. More [28] recently, Soukhov et al. [15] introduced a singly-constrained measure of accessibility, [29] called spatial availability, that employs a similar, but more sophisticated proportional [30] allocation mechanism. The work of these authors show that floating catchment area [31] methods can be seen as singly-constrained accessibility measures, and improve on [32] existing approaches by guaranteeing that each opportunity is counted only once - in [33] other words, treating opportunities as *finite*. The proportional allocation of spatial [34] availability constrains the calculations to match a known quantity, therefore ensuring [35] that the measurements sum up to a predetermined quantity (i.e., the total number of [36] opportunities), and so each value can be meaningfully related to this total. [37]

A limitation of spatial availability as introduced by Soukhov et al. [15] is that it was [38] developed for the case of a homogeneous population, for example for the case of a single [39] mode of transportation. However, the finite nature of opportunities makes the analysis [40] of heterogeneous populations very relevant. In the case of multiple modes of [41] transportation, people who travel by slower modes (e.g., active modes) can usually [42] reach fewer opportunities than people who travel by faster modes and whose range is [43] typically far wider (e.g., car). This implies that slower travelers will often face increased [44] competition for local opportunities from travelers who can reach said opportunities from [45] farther afield. [46]

The objective of this paper is to address this limitation of spatial availability. Our [47] primary motivation is to extend spatial availability for the case of multimodal [48] accessibility, but it is worthwhile noting that this is in fact just one case of [49] heterogeneous populations (i.e., travel by different modes). The method itself can easily [50] accommodate other forms of heterogeneity, for example variations in travel behavior [51] between older and younger adults [e.g., 16], the propensity of older adults to use [52] different modes of transportation [e.g., 17], the usually shorter trip lengths of children [53] compared to grown-ups [e.g., 18], or the more limited travel ranges of single parents [54] [e.g., 19]. [55]

The paper rest of this paper is organized as follows. In Section 2 we provide a brief [56] review of multimodal accessibility. In the following Section 3 we demonstrate the [57] derivation of the new spatial availability expression for multiple modes. In Section 4 we [58] illustrate relevant issues through a synthetic example. This is followed in Section 5 by [59] an empirical example using data from the city of Madrid after the implementation of its [60] Low Emission Zones (LEZ). Data for this example comes from the city's 2018 travel [61] survey. The example shows the differences in spatial availability within and outside the [62] LEZ for travelers using different modes, namely car, transit, cycling and walking. In [63] Section 6, we provide concluding remarks on the strengths of the use of spatial [64] availability as a multimodal accessibility measure, and discuss potential future uses in [65] policy planning scenarios as well as directions for future research. [66]

A brief review of multimodal accessibility

Location-based accessibility indicators are quantitative measures of *potential* interaction with opportunities for locations within a given region: they are summary measures of the relationship between land-use and transport systems. Arguably, the most commonly used are measures based on the gravity model [20], of which cumulative opportunity measures and weighted cumulative opportunity measures are particular forms [5]. These measures assign a weight to opportunities based on how easy it is to reach them. Given an origin (i) and a destination (j), an impedance function $f^m(c_{ij}^m)$ converts the cost of travel (e.g., time, money, generalized cost) into a score that represents the propensity for interaction. These measures originate from that proposed by Hansen [1], which can take the following form in the multimodal case: $S_i^m = \sum_j O_j f^m(c_{ij}^m)$ where m is a set of modes which have mode-specific travel costs (c_{ij}^m) and/or travel impedance functions ($f^m(\cdot)$).

Hansen-type accessibility is not constrained, which is to say it does not consider the opportunities as finite. To cite an example, Tahmasbi et al. [21] use Hansen-type accessibility to assess the potential interaction with retail locations by three modes: walking, public transit, and car (i.e., $m = w, p, c$). S_i^m is the sum of retail locations j that can potentially be reached under the travel impedance as calculated for each i and m. In other words, for each origin i three accessibility scores are calculated. In this work, Tahmasbi et al. [21] show that car travel affords the highest S_i^m values in the majority of i, i.e., travelers who use a car can potentially reach more retail opportunities than populations using other modes. However, higher S_i^m values for car do not affect the values of S_i^m for other modes: in effect, each mode is analysed as if the others did not exist. Since the measure is not constrained, each opportunity is typically counted multiple times within and between modes, and as a result the sum of accessibility is not necessarily a meaningful quantity. The accessibility scores for the modes are often values that are difficult to interpret beyond making statements about relative size. For example, Lunke [22], researching the region of Oslo, reports accessibility scores for car in the order of tens of thousands of employment opportunities. The corresponding scores for transit are lower, but still often in the thousands or tens of thousands. As reported, the ratio of the transit to the car score can be lower than 0.2 (meaning transit gives access to less than 20% of the opportunities than car). But despite the discussion about "sufficient accessibility", it is unclear what the unconstrained scores mean: is having access to but 10,000 jobs by transit insufficient? After all, 10,000 employment opportunities are still plenty of opportunities. These ratios can be found elsewhere in the literature [e.g., 23, and 9,16,19, and 8], and they are useful as relative assessment of when some members of the public are better or worse off than others, but they do not say much about how bad is "worse off".

Besides ratios of accessibility, another way to improve interpretability of scores sometimes seen in the literature is to standardize them to lie in the range [0-1]. This adjustment is only helpful insofar as it facilitates relative comparisons, but interpretation of the scores remains challenging because the values are specific to a region and convey no meaning about the magnitude of the scores. In this approach, zones always have values between 0 and 1, but how remarkable is a zone with a low score for pedestrians and a high value for car? And if remarkable, what does the difference in these standardized values mean for planners? By how much should transport systems and land-use configurations be changed to improve conditions? And in what way can these scores be used to track differences over time? Or between regions? These questions lack straightforward answers since certain values will always be relatively 'low' or 'high', but do not track to a quantity that can be intuitively understood. Presentation or discussion of Hansen-type accessibility that has been standardized in this way is not uncommon in the literature [e.g., 24,25].

Once we understand opportunities to be finite, it is possible for an accessibility measure to take on a crisper meaning. As considered in the long tradition of accessibility research, capacity of opportunities is limited and thus is subject to competition by population [12,15,26–30]. There are only so many school-seats, hospital capacity, employment opportunities, etc., in a region and if one person reaches an opportunity, it is taken: the supply of an opportunity and the demand for that opportunity are two components of accessibility. These are clear examples of opportunities that are unambiguously competitive. But we would go as far as to argue that every type of opportunity is subject to congestion or capacity constraint, even when the opportunities are conventionally seen as non-competitive.

Amenities are a good example of this. For instance, standards for providing green spaces are often stated in the form of *exclusive access*, in units of amenity per capita. A case in point is a the Ile-de-France region, a jurisdiction that suggested in a 2013 planning document that at the municipal level at least $10m^2$ of public green space should be supplied *per inhabitant* [31]. Green spaces are not evenly distributed, which means that who has access to them hinges on where they are and how easy is to reach them. This formulation of provision of amenities is not unusual. For example, Natural England, an organization that recommends an Accessible Natural Greenspace Standard such that the minimum supply of space is one ha of statutory Local Nature Reserves per thousand population[1]. Similarly, the World Health Organization [cited in 32] recommends that cities provide a minimum of $9m^2$ of green area per inhabitant. For our purposes, standards of this type translate into "how much of this resource is available to one individual that has not been claimed by anyone else?". Green spaces often have large capacities, but they still have a capacity, and it is not the same for a person to have access to $5m^2$ of *uncongested* green space as $15m^2$. This difference is in fact a matter of justice [31,33]. Constraining accessibility is in this way a useful way to evaluate the congested availability of any type of opportunity. As development of sound standards is emphasized in the planning literature, in particular in regards to fairness in transportation [see 34], spatial availability analysis is a useful way to develop and assess standards.

The relevance of the considerations above is put in sharper relief when we think about the use of multiple modes (or heterogeneous populations). If we return to Oslo for a moment [22], we notice that the places that have high accessibility by transit are also the places that have *very high* accessibility by car (in their Figure 2). Those two populations are going for the same opportunities, and those travelling by transit have fewer to choose from to begin with. More generally, people in a zone who are advantaged with relatively low cost of travel will have the ability to potentially reach more opportunities than other people. Due to this advantage, through the perspective of finite opportunities, there are fewer opportunities left for everyone else, especially for those who use modes that are slower or otherwise more expensive.

As noted in the Introduction, competitive accessibility was the rationale for developing floating catchment area methods (FCA), popularized by Luo et al. [13] who reformulated the work of Shen [12] into two steps (although similar, and earlier, developments are found in [30,35]). Shen-type accessibility is formulated as: $a_i^m = \sum_j \frac{O_j f^m(c_{ij}^m)}{\sum_m D_j^m}$ where D_j^m is the potential demand for opportunities equal to travel impedance weighted population $\sum_i P_i^m f^m(c_{ij}^m)$ and the remaining variables are repeated in the Hansen-type measure. Shen-type modal accessibility (a_i^m) can be understood as a ratio of the travel impedance-weighted supply of opportunities for m-mode in i over the travel impedance-weighted demand for opportunities. In this way, it considers competition. That said, the measure remains unconstrained, meaning both

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[1]see https://redfrogforum.org/wp-content/uploads/2019/11/67-Nature-Nearby%E2%80%99-Accessible-Natural-Greenspace-Guidance.pdf

population *and* opportunities are multiply counted [see 14]. In other words, interpretation of the Shen-type accessibility scores between modes is fraught as it is for Hansen-type measures.

To illustrate, [36] calculate a_i^m to jobs for different income-group populations in Shenzhen, China using $m =$ public transit and $m =$ car. Their results indicate that zones with low-income populations have lower a_i^m than zones with higher-income populations. Further, they show that $a_i^{m=\text{public transit}}$ is lower than $a_i^{m=\text{car}}$ at many zones, arguing that this may further place those zones with lower-income populations at a disadvantage. a_i and/or a_i^m are used to compare relative spatial differences in overall competitive accessibility and multimodal competitive accessibility, but because opportunities were doubly counted (entering the sums of both modes), this makes for uneasy interpretations of the differences in a_i^m between modes. Questions that this approach leaves unaddressed include: what is the impact of competition on the difference in a_i^m values? How does the impact vary spatially? And what is the interpretation of this difference?

Spatial availability improves on previously discussed accessibility approaches using the Hansen-type measure and the Shen-type measure by constraining the sum of opportunities, that is, by treating opportunities as finite. This is done by means of proportional allocation factors that follow well established principles of spatial interaction and the gravity model [see 37]. In Soukhov et al. [15] these factors consider: the mass effect (e.g., the size of populations at different origins); and the cost of travel from different zones (e.g., some sub-populations face relatively higher or lower costs). The following section introduces the multimodal form of spatial availability.

Multimodal spatial availability

In brief, we define the spatial availability at an origin i (V_i) as the proportion of all opportunities in the region that are allocated to location i from all destinations j. V_i is a value of how many opportunities are available to each location i out of all the opportunities in the region. The general formulation of spatial availability V_i is shown in Equation (1) [see 15]:

$$V_i = \sum_{j=1}^{J} O_j F_{ij}^t \tag{1}$$

Where:

- F_{ij}^t is a balancing factor that depends on the size of the populations at different locations that demand opportunities O_j, as well as the cost of movement in the system $f(c_{ij})$.

- V_i is the number of spatially available opportunities at i; the sum of V_i is identical to the total number of opportunities in the region (i.e., $\sum_j O_j = \sum_i V_i$); in other words, opportunities are dealt with as finite resources.

Compared to Hansen-type accessibility:

$$A_i = \sum_{j=1}^{J} O_j f(c_{ij}) \tag{2}$$

we see that spatial availability is, like the Hansen-type measure, a weighted sum of the opportunities. What makes spatial availability stand apart from other approaches is how the weight used in the sum, balancing factor F_{ij}^t, implements a proportional

allocation mechanisms to ensure that the sum of V_i is constrained to match the total number of opportunities in the region. As such, spatial availability is singly-constrained and naturally implements competition or congestion. F_{ij}^t consists of two parts. The first part is a population-based proportional allocation factor to model the mass effect of the gravity model:

$$F_i^p = \frac{P_i}{\sum_i P_i}$$

This factor makes opportunities available based on demand. Secondly, there is an impedance-based proportional allocation factor that models the cost effect:

$$F_{ij}^c = \frac{F_{ij}^c}{\sum_j F_{ij}^c}$$

This factor makes opportunities available preferentially to those who can reach them at a lower cost. F_i^p and F_{ij}^c are designed so that they both equal 1 when summed across all i in the region (e.g., $\sum_i F_i^p = 1$ and $\sum_i F_{ij}^c = 1$). These factors are combined multiplicatively to yield F_{ij}^t which ensures that a proportion of the opportunities O_j are allocated to each i accordingly. In other words, assuming a finite number of opportunities in the region, F_{ij}^t proportionally allocates O_j to each i such that the resulting V_i value represents the number of opportunities *available* to the population at i. Each zonal value is a proportion of the opportunities in the region (i.e., $\sum_j O_j = \sum_i V_i$).

The focus of this paper is to extend V_i for the measurement of multimodal applications (or more generally heterogeneous populations). To do so, the balancing factors need to be reformulated so that 1) the mass effect now accounts not only for the size of the population at i, but also the size of sub-populations within i; and 2) the cost of travel is not only for different zones, but by sub-populations within each zone (e.g., the cost of travel from i by car, transit, walking, etc.) When we introduce modes (or sub-populations) m, the proportional allocation factors need to satisfy the condition that F_i^{pm} and F_{ij}^{cm} can be summed across each m at each i and then across all i to equal to 1. They are also similarly combined multiplicatively to obtain their joint effect, represented as the combined balancing factor F_{ij}^{tm} similar to that detailed in Equation (3). This factor is given by:

$$F_{ij}^{tm} = \frac{F_i^{pm} \cdot F_{ij}^{cm}}{\sum_{m=1}^{M} \sum_{i=1}^{N} F_i^{pm} \cdot F_{ij}^{cm}} \tag{3}$$

where:

- The factor for allocation by population for each m at each i is $F_i^{pm} = \frac{P_i^m}{\sum_m \sum_i P_i^m}$; and
- The factor for allocation by cost of travel for each m at i is $F_{ij}^{cm} = \frac{f^m(c_{ij}^m)}{\sum_m \sum_i f^m(c_{ij}^m)}$

Implementing F_{ij}^{tm}, the following Equation (4) gives the multimodal version of spatial availability V_i^m:

$$V_i^m = \sum_{j=1}^{J} O_j \, F_{ij}^{tm} \tag{4}$$

Where:

- $m = 1, 2, \cdots, M$ is a set of M modes (or sub-populations) of interest.
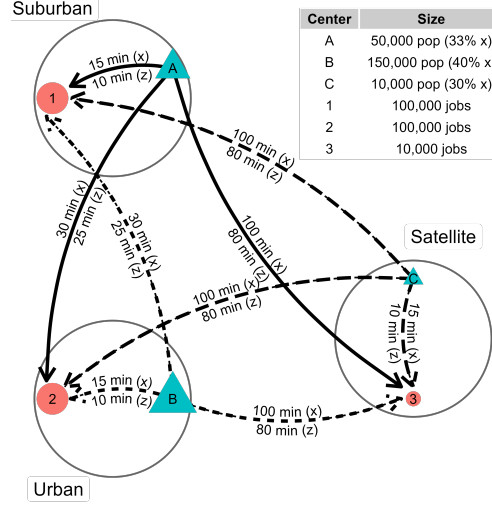- F_{ij}^{tm} is a balancing factor F_{ij}^t for each m at each i.

Fig 1. Multimodal synthetic example: locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times for two modes (slower mode x and faster mode z).

- V_i^m is the spatial availability V_i for each m at each i; the sum of V_i^m for all m at each i is equivalent to the total sum of opportunities in the region (i.e., $\sum_j O_j = \sum_i V_i = \sum_m \sum_i V_i^m$).

Next we use a synthetic example to contrast multimodal accessibility and spatial availability.

An illustrative synthetic example

Consider the simple system shown in Figure 1. The figure shows a region with population at three population centers (A, B, C) and jobs at three employment centers (1, 2, 3). The population at each origin i is consists of two sub-populations, one using a faster mode z and another using a slower mode x, to travel to employment centers. Population center A is Suburban: it is closest to its own relatively large employment center at 1, close to the Urban's equally large employment center 2, and has a population that is smaller than the Urban B and larger than the Satellite C. B has the largest x-using population, followed by then A, then C. This synthetic example is inspired by the single-mode example used in [12] and reconfigured in [15].

From the perspective of access to a *finite* amount of opportunities in the region (210,000 jobs), the sub-population that is most proximate to jobs (low cost to reach), furthest from large populations (less competition), and uses the fastest mode z (greater range) can potentially reach the largest number of opportunities. This appears to be the sub-population at A using mode z. Sub-populations located in opposite conditions (i.e., more distant from jobs, close to large populations, and using slow mode x) are at a relative disadvantage. The competition for opportunities between different mode-using populations matters as it reflects how well the land-use and transport system serves (or does not serve) certain populations.

Table 1. Accessibility values at each origin (i) per mode (m) (columns three to five) and aggregated per i (columns six and seven) for the synthetic example.

i	m	S_i^m	a_i^m	V_i^m	a_i	V_i
A	x	27,292.18	0.95	15,696.89	1.36	67,482.61
	z	44,999.80	1.57	51,785.72		
B	x	27,292.18	0.64	38,170.03	0.88	132,638.94
	z	44,999.80	1.05	94,468.91		
C	x	2,240.38	0.68	2,035.86	0.99	9,878.45
	z	3,745.89	1.12	7,842.59		
TOTALS		150,570.22	N/A	210,000.00	N/A	210,000.00

The values calculated for S_i^m (Hansen-type accessibility), a_i^m (Shen-type accessibility), and V_i^m (spatial availability) for each i and m are shown in the middle three columns and are aggregated for each i in the final two columns in Table 1 . As in the example in Shen [12], we use a negative exponential impedance function $f^m(c_{ij}^m) = \exp(-\beta \cdot c_{ij})$ with $\beta = 0.1$ for both x and z modes for all accessibility measures calculations. Notice that in this example we use the same impedance function but the travel times are different for the two modes. More generally, it is possible to use different impedance functions for the modes, as demonstrated in the empirical example in the following section.

Hansen-type accessibility S_i^m is presented for each origin and mode in the third column of Table 1 . For all i, the travel by z results in higher values of S_i^m than travel by x. Lack of competition, or alternatively the assumption of an inexhaustible resource in the calculation of S_i^m, lead to a curious result. Since the populations in A and B have the same travel impedance to employment centers 1, 2 and 3 (either 15, 30, or 100 minutes using x or 10, 25, or 80 minutes using z), their values of S_i^m are the same for both A and B. Furthermore, the total sum of S_i^m in the region is equal to 150,570.2. This value lacks an intuitive interpretation: it represents the weighted sum of opportunities that may be reached within the region according to the travel impedance (i.e., the travel behavior and the characteristics of the modes) and does not usefully translate into any sort of benchmark. To connect this example to the aforementioned literature, S_i^m is calculated in the work of Tahmasbi et al. [21]; they contrast differences in S_i^m values between modes in a relative and comparative sense, but make no further interpretation of the S_i^m values. More densely populated metropolitan regions will tend to have more opportunities and hence large S_i^m values and less densely regions, smaller values; how much of these differences may simply an artifact of region density?

In the fourth and sixth columns in Table 1 the results for Shen-type accessibility are reported: first for both origin and mode a_i^m as well as aggregated by the weighted mean mode-population ($\sum_m \frac{P_i^m}{P_i} * a_i^m$) to represent a value for each origin a_i. Unlike S_i^m, this measure does considers *competition*. For instance, the population travelling by x from A and B do not have the same values of a_i^m as those travelling by z. In fact, A has the highest values a_i^m and a_i values since this center has the lowest travel impedance to opportunities (lower than at C, A and B are equal) and faces relatively low competition, not being close to a relatively large population (lower than at B).

However, the calculations of a_i^m are not constrained: the total sum of a_i^m or a_i is practically meaningless since it represents a sum of ratios. For instance, the population travelling by z from A has a value of 1.57 jobs per job-seeking population compared to 0.95 for users of mode x. What is the meaning of these values? The difference between

these modes is equal to 0.62, but 0.62 of what? How many more job opportunities can users of z reach compared to user of x? When a_i^m is aggregated to a_i as shown in the sixth column, the values face similar interpretability issues. The Shen-type measure is implemented in aforementioned work of Tao et al. [36] to calculate modal a_i^m values and the aggregated a_i is implemented in the work of Carpentier et al. [38]. However, similar to Hansen-type accessibility, these works discuss relative and spatially comparative differences in values, but veer from interpreting the values of a_i^m or a_i themselves. In fairness, interpretation is complicated by the multiple counting of opportunities between zones and modes.

In contrast, spatial availability V_i considers competition and is constrained such that the total sum of values is equal to the total number of opportunities in the region (i.e., $210,000$ jobs). Seen in fifth column of Table 1 , the values of V_i^m in A and B are not the same within each mode (as this measure considers competition). In fact, at A, users of mode z capture 36,088.84 more spatially available jobs (of the $210,000$ jobs in the region) than the sub-population travelling by x. The numerical difference is clear since it refers to opportunities out of the total.

Furthermore, the proportional allocation mechanism also means that the values of V_i^m for any origin i can be aggregated across m and compared between zones ($V_i = \sum_m \sum_i V_i^m$). This aggregation, V_i, is shown in the seventh column in Table 1 . Again looking at center A, A is allocated 67,482.61 spatially available opportunities for both modes. 77% of this spatial availability allocated to A is assigned to users of mode z despite representing 66% of A's population.

Spatial availability can be further aggregated to better interpret competition between modes. Across the entire region, 130,000 people use z (62% of the region population). However, users of z account for 73% of the region's total spatial availability - while the remaining 27% is allocated to users of mode x who are 38%of the total population. Notably, the population who uses x have 11% fewer spatially available opportunities than its share in the population. This realization leads us to ask normative questions such as, how unequal should availability of opportunities be by mode? What intervention could help to redistribute spatial availability to sub-populations commensurate with their proportion of the total?

Since spatial availability is constrained and has an interpretable meaning as a proportion of the total opportunities in the region, the values at i have a straighforward interpretation. Inequality in V_i^m values can be explored through a variety of approaches. For instance, consider travel times. The population of travelers who use z accounts for 67% of the potential travel time traveled in the region: this is 7% less travel time than the proportion of spatial available opportunities that is allocated to them. In other words, the population of users of z travels fewer minutes overall and has more spatial availability of opportunities than users of the slower mode x.

Alternatively, inequities in spatial availability between modes can be explored through proportional benchmarks. A spatial availability per capita v_i^m is presented in Equation (5):

$$v_i^m = \frac{V_i^m}{P_i^m} \tag{5}$$

The values of v_i^m for A, B, and C for users of x are 0.95, 0.64 and 0.68 spatially available jobs per capita, respectively. The values of v_i^m for users of z are much higher, with values of: 1.57, 1.05 and 1.12 respectively. Users of x, especially those at B and C, are directly impacted by the jobs that are spatially available to users of z *in addition to* the mass effect (occurring at B, high population density) and high travel impedance (occurring at the Satellite C).

If, let us say, the planning goal was to have one spatially available job per

mode-using population, a policy intervention could be devised, to reduce the values of v_i^z (making it slower or more expensive) and increase he values of v_i^x (making it faster or les expensive). The purpose of this simple demonstration is to show how spatial availability can be used to quantify the competitive (dis)advantage in a multimodal application. In what follows, we demonstrate the use of multimodal spatial availability through an empirical example.

Empirical example

Context

The context for the empirical example is Madrid, Spain. This city implemented a Low Emission Zone (LEZ) in 2017 to: pursue goals set out in the national climate change agenda, cut nitrogen dioxide levels, and to prioritize people's movement in the city. LEZs elsewhere have similarly been implemented as interventions to reduce GHG emissions, improve air quality, and support sustainable mobility [39,40]. Though the rules of exclusion vary by city, LEZs aim to deter/reduce traffic in designated zones under threat of penalty (e.g., fines, seizure of vehicle). In other words, LEZs implement a form of *geographic discrimination* as they change how people can reach opportunities by making it more costly for some forms of travel, typically cars, to circulate in predetermined zones. When considering opportunities as finite in a region, this discrimination reduces the competition of one mode and opens up opportunities for other modes to better thrive. At their core, LEZs operate by changing the accessibility landscape of a city from the perspective of multiple modes.

In geographic scope, the 2017 boundaries of the LEZ in Madrid were relatively modest, covering only approximately 4.72 km² of the central business district of the city (the so-called LEZ Centro). As of this writing, there are plans to expand these boundaries to the area inside the M-30, an orbital highway in proximity to the city center (i.e., LEZ M-30). Within the 2017 LEZ Centro implementation, all cars, motorcycles and freight vehicles with environmental labels A or B (older makes and models of fossil fuel internal combustion engine vehicles), were disallowed from entering the zone unless they are used by residents or meet other exemptions. This restriction impacted approximately half of all car trips that used to travel into what is now the LEZ Centro [41].

For this case study, we use spatial availability to quantify access to opportunities by different modes in Madrid. Particularly, we demonstrate how V_i^m can be used to derive insights into how the restriction of car mobility in areas around/within the LEZ Centro may have allowed more sustainable (but often slower or more costly modes) to become more competitive.

Data

The source of data for our empirical example is the 2018 Travel Survey of the Community of Madrid [42]. This is a representative survey that offers a snap-shot of travel patterns for a typical weekday in 2018. The survey collected 222,744 trips from a representative sample of 85,064 households across the traffic analysis zones (TAZ) in the Community of Madrid. For context, the population older than 3 years in the Community is 6,507,184.

In this example, we use all home-to-full-time-work trips, by all modes. The trips are expanded using population weights. Figures 2 and 3 show the number of workers and the distribution of full-time jobs in the City of Madrid by TAZ. The light grey boundary represents the LEZ Centro in effect in 2017. The survey was conducted after the
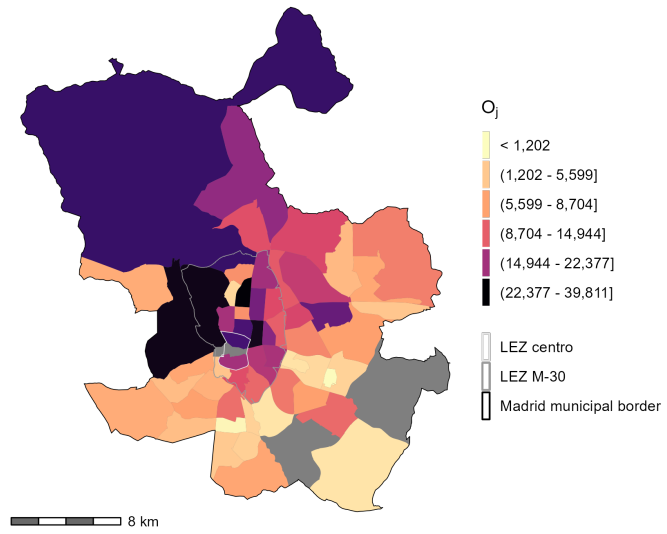
Fig 2. Distribution of jobs taken by people living and working in Madrid as reported in the 2018 travel survey. Grey TAZs has no jobs. Ranges of values in the legend are quintiles.
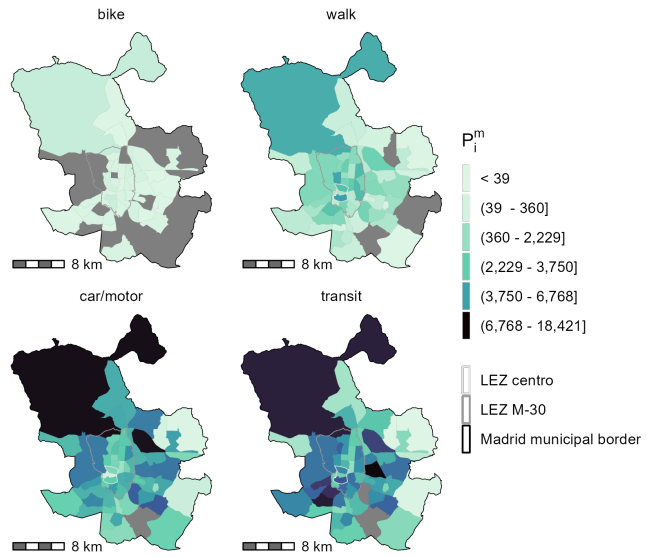

Fig 3. Population living and working in Madrid by mode of transportation as reported in the 2018 travel survey. Grey TAZs have no population. Ranges of values in the legend are quintiles.

introduction of LEZ Centro. The dark grey boundary represents the LEZ planned for the boundaries of the M-30 highway and is present in the plots as a spatial reference for areas in proximity to the LEZ Centro.

The total sum of jobs O_j are shown in Figure 2 and the populations that go to a work destination by four modal categories P_i^m, is displayed in Figure 3. The modal shares in Figure 3 are calculated based on those measured in the survey. The modal categories and the mode types within each category are reported as follows:

- Car/motor: all cars and operating modes (e.g., cab, private driver, company, rental car, main driver of a private car, passenger in a private car) and all public, private or company motorcycle/mopeds.
- Transit: all bus, trams, and trains.
- Bike: all bicycle trips (e.g., private, public, or company bike trips) and "other" types of micromobility options.
- Walk: walking or by foot.

Some aggregation of modes is necessary to calculate the travel impedance functions by mode. From Figure 2, it can be seen that the largest concentration of jobs is within, near, and to the north of LEZ Centro. The populations with access to those jobs by mode (Figure 3) are spatially distinct. Travel by car and transit represent 37% and 47% of the modal share respectively. The population that travels by transit is more spatially distributed than those using cars - particularly near and within LEZ Centro. This distribution is likely caused by a variety of factors including: transit coverage and service within with city, effective car infrastructure outside of the M-30, and/or the impact of the LEZ Centro itself. From Figure 3, it can be seen that active travel is less common than motorised trips at 1% and 15% for cycling and walking respectively. Noticeably, there is a positive trend between the walking and cycling in zones where transit is also present. This positive trend is higher than for car trip populations.

Travel times are provided within the travel survey by mode. This information is used to calibrate mode-specific travel impedance functions $f^m(c_{ij}^m)$. To illustrate the modal differences in travel times, the following descriptive statistics per mode are presented:

- Car/motor: mean 36 minutes (min: 0 minutes, Q2: 15 minutes, Q3: 55 minutes, max: 120 minutes)
- Transit: mean 55 minutes (min: 1 minutes, Q2: 30 minutes, Q3: 80 minutes, max: 120 minutes)
- Bike: mean 34 minutes (min: 5 minutes, Q2: 15 minutes, Q3: 40 minutes, max: 115 minutes)
- Walk: mean 27 minutes (min: 1 minutes, Q2: 10 minutes, Q3: 45 minutes, max: 119 minutes)

Impedance functions $f^m(c_{ij}^m)$ are calibrated from the travel times in the survey via the empirical trip length distribution (TLD). An empirical TLD is given by the proportion of trips at various travel cost bins. This distribution is then used to estimate the parameters of a function for the travel impedance [as done in 43,44,45]. To fit the impedance functions, we use the Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the R {fitdistrplus} package [46]. Based on goodness-of-fit criteria and associated diagnostics, the gamma and log-normal probability density functions are selected as best fitting curves for the motorised and non-motorised modes respectively. The selection of functional forms aligns with empirical examples in other regions [15,47,48]. The shape and rate parameters for the gamma functions (motorised modes) are 1.8651852 and 0.051468 for car/motor and 2.7566235 and 0.0499193 for transit; for the log-normal functions (non-motorised
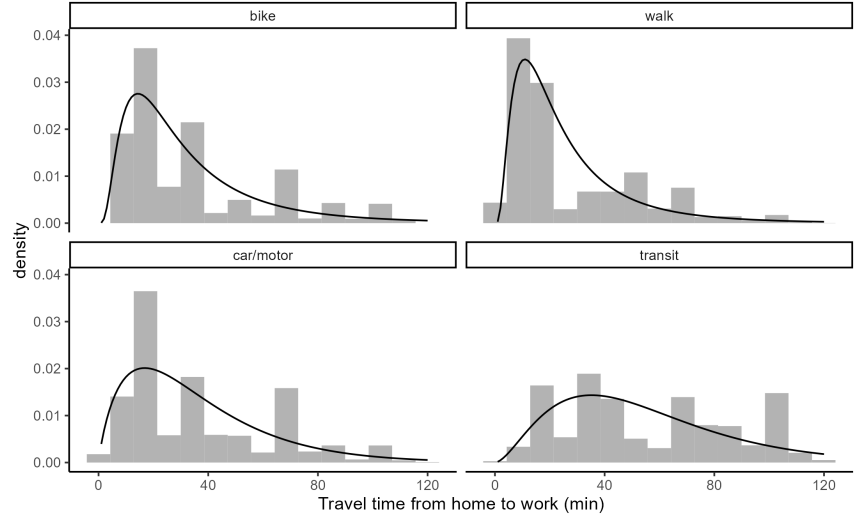
Fig 4. Fitted impedance function against empirical TLD (bars) corresponding to the home to full-time work origin destination flows for the City of Madrid from the 2018 travel survey.

modes), the mean and standard deviation parameters are 3.2372212 and 0.7575986 for bike and 2.9918042 and 0.7575986 for walk.

Figure 4 includes four plots to visualize the calibrated impedance functions (represented as black lines) superimposed on the empirical TLD. The impedance functions can be interpreted as the propensity to travel (y-axis) given a trip travel time (x-axis). The functions reflect a combination of possibilities and preferences: the travel behavior given the transportation technologies available. For example, trips shorter than 5 minutes do not occur frequently for any mode; this reflects the spatial separation between places of residence and places of work commonly seen in many cities. In terms of the non-motorised modes, there is a preference towards walking trips around 15 minutes in duration, as seen from the highest value of $f^{walk}(c_{ij}^{walk})$. With respect to travel by bicycle, longer travel times are more common; although the highest value of the impedance also corresponds to approximately 15 minutes, the curve has a longer tail and values decrease less rapidly at longer travel times than is the case of $f^{walk}(c_{ij}^{walk})$. A similar trend can be observed for the motorised modal options where transit mode is more spread out than car/motor mode. All in all, these functions represent the propensity of travel by mode by duration of trip, and are used to calculate the proportional allocation factors F_{ij}^m for V_i^m.

Results

At this point, it is worthwhile reiterating that the empirical example is a snap-shot of spatial availability by mode using data from the 2018 travel survey. Our purpose in this empirical example is to investigate the trends in availability of employment opportunities by mode, and illustrate how spatial availability can be used in discussions about the competitive advantage of various modes within Madrid Centro. The spatial availability of jobs V_i^m is calculated for each of the four modes m at the level of traffic analysis zones i in Madrid and displayed in Figure 5.

In the figure, V_i^m is a proportion of the total number of 847,574 jobs in the region. Since V_i^m is calculated based on the population of workers and the distribution of jobs, the values can be understood as the number of full-time jobs that are spatially available
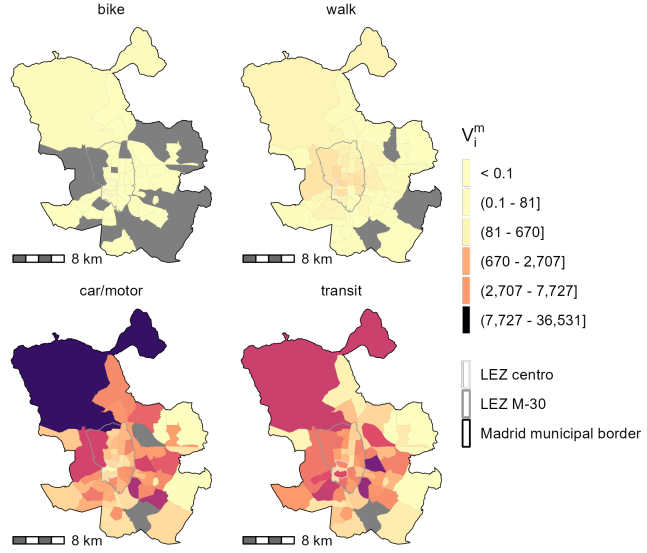
Fig 5. Spatial availability of jobs per origin and mode V_i^m in Madrid. Grey TAZs have no population. Ranges of values in the legend are quintiles.

to full-time workers at that i traveling by mode m, relative to all the jobs in the city. There are noticeable differences in the magnitude of V_i^m between modes as seen in Figure 5. The majority of V_i^m (which is to say of spatially available jobs) are allocated to workers travelling by car and transit. In a way, this is to be expected since users of these modes represent 84.1% of the total population. However, the ability to travel at greater speeds also impacts these results. Furthermore, differences in V_i^m values within modes also exist in space: car users outside of the M-30 region appear to enjoy greater spatial availability, while some zones inside the M-30 to have greater spatial availability for transit. Overall, the magnitude of V_i^m values for cyclists and pedestrians are lower than for car and transit but the highest values of V_i^{bike} and V_i^{walk} tend to be found in zones within the M-30 and origins with higher spatial availability by transit.

The differences between the shares of modes and their shares of spatially available opportunities highlights the competitive advantage of certain modes, although this effect is not geographically uniform. As seen in the left-most columns of Figure 6, users of 'car/motor' and 'transit' together can avail 95.3% of all jobs in the city (Spatial Availability by Mode). However, car/motor users have a disproportionate share of V_i^m relative to the population of users of this mode (Population by Mode), compared to opportunities that are spatially available to transit users. The combined population of car and transit users is 36.6% and 47.5% respectively, but these populations are allocated 48.0% and 47.3% respectively, of the city's jobs. When treating the number of opportunities that can be reached as a finite value (total: 847,574 opportunities), fewer opportunities are spatially available to slower modes (i.e., walking and cycling), even taking into account that their share is smaller overall. These modes are at a disadvantage as a result of: the travel impedance for longer trips (see Figure 4; their low population values values overall; and larger populations in origins with high shares of travel by motorised modes. These factors all contribute to the the car/motor mode being most advantaged in capturing spatially available job opportunities overall.

The big picture demonstrates how car is the most advantageous mode, however it is interesting to notice that this advantage appears to be blunted by the LEZ. Unlike the results for the whole city and even within the M-30 ring, the proportion of car users in Centro is larger than the proportion of opportunities spatially available to them. The
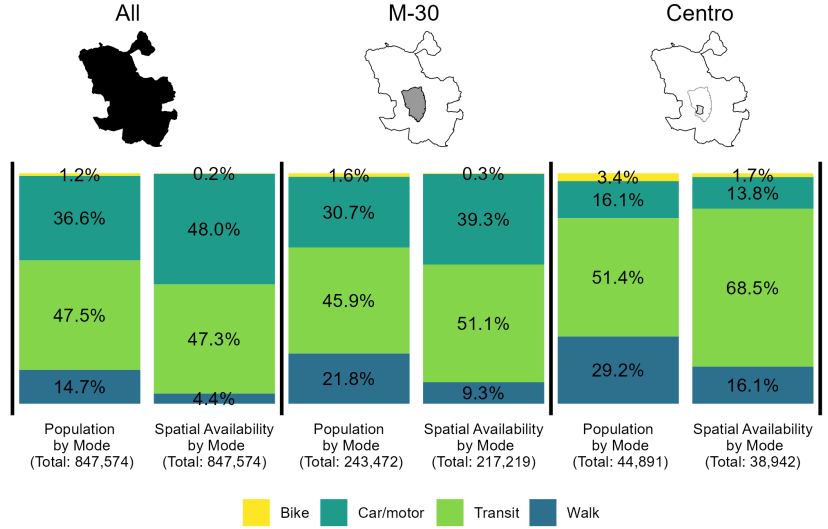
Fig 6. Proportion of population by mode and spatial availability of jobs by mode aggregated for three areas. From left to right, the city of Madrid (All), the area within the M-30 highway (M-30), the area within the Centro region (Centro).

restriction on cars in effect reduces competition by this mode, and leads to a relative increment of the mass effect of the modes allowed within the LEZ, which also contains a large number of jobs (see Figure 2). As summarized in the two right-most columns in Figure 6, the proportion of jobs spatially available to car in Centro is (13.8% or 5,373 opportunities). For reference, this is less than the proportion of the car users in Centro (16.1%), evidently less then the proportion of car users in the city, and is the opposite of the overall trend (left-most columns) and within the M-30 (middle columns).

It is also clear that more opportunities are spatially available to non-car users within Centro. In the case of active travel, the proportions of cyclists and walkers within LEZ Centro still exceeds the proportions of jobs spatially available to them — however, the disparity is drastically reduced compared to the rest of the city. As seen in Figure 6, there is a higher proportion of opportunities that are spatially available to pedestrians and cyclists in Centro than in the City overall and in areas within the M-30. Notably, within Centro, 1.7% and 16.1% of opportunities are spatially available to bike and walk modes respectively, while their populations represent 1.2% and 14.7% of the population. By restricting the ability of cars to enter Centro, the LEZ seems to contribute to leveling the playing field for slower modes, in particular cycling and walking, but also transit. As seen in the Figure 6, transit users are generally close to parity across the region, with nearly as many spatially available jobs as transit users. Still, this mode has the greatest advantage in LEZ Centro with 68.5% of spatially available jobs in Centro for 51.4% of transit users in Centro. This result makes intuitive sense: after car, it is the mode with the greatest range, and unlike car it is unrestricted in the LEZ Centro.

The spatial differences in the competitive dis/advantage of spatial availability between modes can also be visualized at a finer level of granularity. Figure 7 shows v_i^m, the spatial availability V_i^m divided by the population of users of m. Values of v_i^m below one are shown in shades of orange, and indicate TAZs with less than one spatially available opportunity per capita for the mode. Values above one are shown in shades of green, and indicate TAZs with more than one spatially available opportunity per capita for the mode. The highest spatial availability per capita (shown in blue) is for car users in a zone northeast just beyond the M-30. These plots illustrate in unambiguous
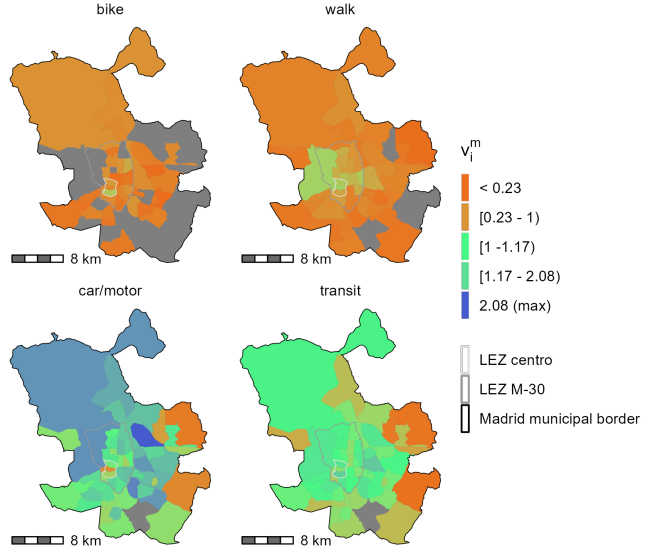
Fig 7. Distribution of spatially available jobs per capita by mode of transportatin (v_i^m). Grey TAZs have no population that use the mode. Ranges of values in the legend are quintiles.

fashion, and in a quantity that is comparable over space and time, the advantage in terms of spatial availability of car for most of the city (bottom left plot, areas denoted with green v_i^m values above one). It can also be observed that spatial availability of jobs is relatively well balanced for transit users over most of the regions (i.e., many zones are light orange or light green). Spatial availability of jobs for non-motorised modes, in contrast, is low (under one) overall, although less so within LEZ Centro.

Incidentally, v_i^m values for car within and near LEZ Centro is close to or below one in Figure 7, while all non-car modes have relatively higher v_i^m values. Since these values are comparable across regions and over time, Figure 7 potentially provides a benchmark for quantifying changes in LEZ policies in the future. As Figure 7 also shows, many areas within the M-30 have high (white/green) v_i^m values for car, but the results for LEZ Centro give reasonable grounds to speculate that a spatial expansion of the LEZ to include all areas within the M-30 would likely increase the spatial availability of jobs for transit users, cyclists and pedestrians.

Discussion and conclusions

Accessibility measures are an important tool in transportation research [9] and are increasingly seen as valuable for planning purposes [4–8]. They boast a long history of development, beginning with Hansen-type S_i^m measures, with other developments like Shen's a_i^m, to account for competition/congestion. The more recent spatial availability measure V_i^m has in common with these accessibility indicators that it is a weighted sum of the opportunities in a region from the perspective of a determined origin i. Aggregations of opportunities embody principles of gravitational/spatial interaction modelling that date back to at least H.C. Carey [49], and are part of a line of research that includes the work of Ravenstein [50], Reilly [51], Stewart [52–54], Zipf [55,56], Wilson [37], and many others. In this way, S_i^m, a_i^m, and yes, V_i^m, can be interpreted as scores of the potential for interaction with opportunities in space.

Different accessibility indicators are characterized by how they weight and aggregate opportunities. Spatial availability's contribution to the literature is to incorporate a

proportional allocation mechanism that essentially constrains the sums to match the number of opportunities in the region; in this way it is a singly-constrained accessibility measure that naturally accommodates congestion and competition. The effort with spatial availability is in line with previous research on proportional allocation by Paez et al. [14]. As initially introduced by Soukhov et al. [15], spatial availability was designed for a homogeneous population traveling by a single mode of transportation. In this paper, we extended spatial availability for the case of heterogeneous populations. We discussed this in terms of multiple modes of transportation, but the framework can accommodate equally well variations in travel behavior by population segments.

An empirical example using data from Madrid helped to illustrate the potential of multimodal spatial availability analysis, including its ability to account for competition for opportunities within and between modes. Particularly relevant is the fact that spatial availability scores relate directly to the total number of opportunities in the region. This makes it possible to compare the results to intuitive benchmarks, such as opportunities per population, in ways that other accessibility measures cannot or tend to obfuscate. This comparability is preserved between regions and over time. The example suggests that once that opportunities are treated as being finite, restrictions to travel by car leave more spatially available opportunities for non-car-users. This difference for car travel in locations within and immediately around the LEZ Centro seems to increase the number of opportunities spatially available to transit users (transit being the second most competitive mode), as well as the spatial availability from the perspective of non-motorised modes. In effect, a policy such as Low Emission Zones help to improve the accessibility situation of active travel and transit in the parts of the city where it is implemented.

The purpose of the empirical example is to illustrate the kind of insights that can be derived from the application of multimodal spatial availability. But there are some intriguing opportunities for future research. Accessibility indicators are not designed to work as modal split models, and yet, in the case of policies that alter the relative cost of various forms of transportation, one can reasonably expect to see some shifts between modes. In our empirical example we used data collected *after* the introduction of LEZ Centro. However, given a modal split model to project model shares, accessibility indicators, including spatial availability, can be used to investigate changes to the accessibility landscape. Ditto for destination choice. Our empirical example presented but a snapshot of this, and in future research it will be interesting to investigate changes *between* policy interventions. The expansion of Madrid's LEZ to the ring contained by the M-30 orbital presents an excellent opportunity to do so. Given the intuitive and straightforward interpretation of spatial availability scores as fractions of opportunities from the total, relative and absolute changes in the accessibility landscape can be assessed, thus helping to evaluate the implications of policy interventions.

Finally, our example dealt with differences in travel by mode only, but it is possible to think of the intersection between mode of travel and different types of travelers. This would expand the number of sub-populations in the analysis from, say, $m = M$ (modes) to $m = M \cdot Q$ (modes times population segments), each with their own characteristic impedance function. Evaluations of this kind will be especially relevant as LEZ are implemented in cities globally, and the question of their impact on disadvantaged populations who have become mobility-restricted increasingly come to the fore [40,57,58].

Acknowledgements

Author contributions

References

1. Hansen WG. How accessibility shapes land use. Journal of the American Institute of Planners. 1959;25: 73–76. doi:10.1080/01944365908978307

2. Geurs KT, Wee B van. Accessibility evaluation of land-use and transport strategies: Review and research directions. Journal of Transport Geography. 2004;12: 127–140. Available: http://www.sciencedirect.com/science/article/B6VG8-4B28VY7-1/2/61478339c14cab4fa58438ad7b1f4610 C:/Papers/Journal of Transport Geography/Journal of Transport Geography (2004) 12 (2) 127-140.pdf

3. Paez A, Scott DM, Morency C. Measuring accessibility: Positive and normative implementations of various accessibility indicators. Journal of Transport Geography. 2012;25: 141–153. doi:10.1016/j.jtrangeo.2012.03.016

4. Handy S. Is accessibility an idea whose time has finally come? Transportation Research Part D: Transport and Environment. 2020;83: 102319. doi:https://doi.org/10.1016/j.trd.2020.102319

5. Levinson D, King D. Transport access manual: A guide for measuring connection between people and places. University of Sydney; 2020. Available: https://ses.library.usyd.edu.au/handle/2123/23733

6. Siddiq F, D. Taylor B. Tools of the trade?: Assessing the progress of accessibility measures for planning practice. Journal of the American Planning Association. 2021;87: 497–511. doi:10.1080/01944363.2021.1899036

7. Yan X. Toward accessibility-based planning addressing the myth of travel cost savings. JOURNAL OF THE AMERICAN PLANNING ASSOCIATION. 2021;87: 409–423. doi:10.1080/01944363.2020.1850321

8. El-Geneidy A, Levinson D. Making accessibility work in practice. Transport Reviews. 2022;42: 129–133. doi:10.1080/01441647.2021.1975954

9. Shi Y, Blainey S, Sun C, Jing P. A literature review on accessibility using bibliometric analysis techniques. Journal of Transport Geography. 2020;87: 102810. doi:10.1016/j.jtrangeo.2020.102810

10. Handy S, Niemeier D. Measuring accessibility: An exploration of issues and alternatives. Environment and Planning A. 1997;29: 1175–1194.

11. Kwan MP. Space-time and integral measures of individual accessibility: A comparative analysis using a point-based framework. Geographical Analysis. 1998;30: 191–216. Available: ISI:000074579200001 C:/Papers/Geographical Analysis/Geographical Analysis (1998) 30 (3) 191-216.pdf

12. Shen Q. Location characteristics of inner-city neighborhoods and employment accessibility of low-wage workers. Environ Plann B. 1998;25: 345–365. doi:10.1068/b250345

13. Luo W, Wang F. Measures of spatial accessibility to health care in a GIS environment: Synthesis and a case study in the chicago region. Environ Plann B Plann Des. 2003;30: 865–884. doi:10.1068/b29120

14. Paez A, Higgins CD, Vivona SF. Demand and level of service inflation in floating catchment area (FCA) methods. PloS one. 2019;14: e0218773. doi:10.1371/journal.pone.0218773

15. Soukhov A, Paez A, Higgins CD, Mohamed M. Introducing spatial availability, a singly-constrained measure of competitive accessibility | PLOS ONE. PLOS ONE. 2023; 1–30. doi:https:// doi.org/10.1371/journal.pone.0278468

16. Páez A, Mercado R, Farber S, Morency C, Roorda M. Accessibility to health care facilities in montreal island: An application of relative accessibility indicators from the perspective of senior and non-senior residents. International Journal of Health Geographics. 2010;9: 1–9.

17. Moniruzzaman M, Páez A, Nurul Habib KM, Morency C. Mode use and trip length of seniors in montreal. Journal of Transport Geography. 2013;30: 89–99. doi:http://dx.doi.org/10.1016/j.jtrangeo.2013.03.007

18. Reyes M, Paez A, Morency C. Walking accessibility to urban parks by children: A case study of montreal. Landscape and Urban Planning. 2014;125: 38–47. doi:10.1016/j.landurbplan.2014.02.002

19. Páez A, Farber S, Mercado R, Roorda M, Morency C. Jobs and the single parent: An analysis of accessibility to employment in toronto. Urban Geography. 2013;34: 815–842. doi:10.1080/02723638.2013.778600

20. Wu H, Levinson D. Unifying access. Transportation Research Part D: Transport and Environment. 2020;83: 102355. doi:10.1016/j.trd.2020.102355

21. Tahmasbi B, Mansourianfar MH, Haghshenas H, Kim I. Multimodal accessibility-based equity assessment of urban public facilities distribution. Sustainable Cities and Society. 2019;49: 101633. doi:10.1016/j.scs.2019.101633

22. Lunke EB. Modal accessibility disparities and transport poverty in the oslo region. Transportation Research Part D: Transport and Environment. 2022;103: 103171. doi:10.1016/j.trd.2022.103171

23. Paez A, Mercado RG, Farber S, Morency C, Roorda M. Relative accessibility deprivation indicators for urban settings: Definitions and application to food deserts in montreal. Urban Studies. 2010;47: 1415–1438. doi:10.1177/0042098009353626

24. Campbell KB, Rising JA, Klopp JM, Mbilo JM. Accessibility across transport modes and residential developments in nairobi. Journal of Transport Geography. 2019;74: 77–90.

25. Maharjan S, Tilahun N, Ermagun A. Spatial equity of modal access gap to multiple destination types across chicago. Journal of Transport Geography. 2022;104: 103437. doi:10.1016/j.jtrangeo.2022.103437

26. Grengs J. Job accessibility and the modal mismatch in detroit. Journal of Transport Geography. 2010;18: 42–54. doi:10.1016/j.jtrangeo.2009.01.012

27. Kawabata M, Shen Q. Job accessibility as an indicator of auto-oriented urban structure: A comparison of boston and los angeles with tokyo. Environ Plann B Plann Des. 2006;33: 115–130. doi:10.1068/b31144

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28. Kwok RCW, Yeh AGO. The use of modal accessibility gap as an indicator for sustainable transport development. Environ Plan A. 2004;36: 921–936. doi:10.1068/a3673

29. Morris JM, Dumble PL, Wigan MR. Accessibility indicators for transport planning. Transportation Research Part A: General. 1979;13: 91–109. doi:10.1016/0191-2607(79)90012-8

30. Weibull JW. An axiomatic approach to the measurement of accessibility. Regional Science and Urban Economics. 1976;6: 357–379. doi:10.1016/0166-0462(76)90031-4

31. Liotta C, Kervinio Y, Levrel H, Tardieu L. Planning for environmental justice - reducing well-being inequalities through urban greening. Environmental Science & Policy. 2020;112: 47–60. doi:https://doi.org/10.1016/j.envsci.2020.03.017

32. OECD. Frameworks and sector policies for urban development in chile. OECD urban policy reviews, chile 2013. 2013. doi:http://dx.doi.org/10.1787/9789264191808-en

33. Lara-Valencia F, García-Pérez H. Space for equity: Socioeconomic variations in the provision of public parks in hermosillo, mexico. Local Environment. 2015;20: 350–368. doi:10.1080/13549839.2013.857647

34. Martens K, Golub A. A fair distribution of accessibility: Interpreting civil rights regulations for regional transportation plans. Journal of Planning Education and Research. 2021;41: 425–444. doi:10.1177/0739456x18791014

35. Joseph AE, Bantock PR. Measuring potential physical accessibility to general practitioners in rural areas: A method and case study. Social Science & Medicine. 1982;16: 85–90. doi:10.1016/0277-9536(82)90428-2

36. Tao Z, Zhou J, Lin X, Chao H, Li G. Investigating the impacts of public transport on job accessibility in Shenzhen, China: A multi-modal approach. Land Use Policy. 2020;99: 105025. doi:10.1016/j.landusepol.2020.105025

37. Wilson AG. A family of spatial interaction models, and associated developments. Environment and Planning A. 1971;3: 1–32.

38. Carpentieri G, Guida C, Masoumi HE. Multimodal Accessibility to Primary Health Services for the Elderly: A Case Study of Naples, Italy. Sustainability. 2020;12: 781. doi:10.3390/su12030781

39. Margaryan S. Low emission zones and population health. Journal of Health Economics. 2021;76: 102402. doi:10.1016/j.jhealeco.2020.102402

40. Verbeek T, Hincks S. The "just" management of urban air pollution? A geospatial analysis of low emission zones in brussels and london. Applied Geography. 2022;140: 102642. doi:10.1016/j.apgeog.2022.102642

41. Tarriño-Ortiz J, Gómez J, Soria-Lara JA, Vassallo JM. Analyzing the impact of low emission zones on modal shift. Sustainable Cities and Society. 2022;77: 103562. doi:10.1016/j.scs.2021.103562

42. Comunidad de Madrid. Resultados de la EDM 2018 - Datos Abiertos. 2020 [cited 31 Jul 2023]. Available: https://datos.comunidad.madrid/catalogo/dataset/resultados-edm2018

43. Lopez FA, Paez A. Spatial clustering of high-tech manufacturing and knowledge-intensive service firms in the greater toronto area. Canadian Geographer-Geographe Canadien. 2017;61: 240–252. doi:10.1111/cag.12326

44. Horbachov P, Svichynskyi S. Theoretical substantiation of trip length distribution for home-based work trips in urban transit systems. Journal of Transport and Land Use. 2018;11: 593–632. Available: `https://www.jstor.org/stable/26622420`

45. Batista SFA, Leclercq L, Geroliminis N. Estimation of regional trip length distributions for the calibration of the aggregated network traffic models. Transportation Research Part B: Methodological. 2019;122: 192–217. doi:10.1016/j.trb.2019.02.009

46. Delignette-Muller ML, Dutang C. fitdistrplus: An R package for fitting distributions. Journal of Statistical Software. 2015;64: 1–34. Available: `https://www.jstatsoft.org/article/view/v064i04`

47. Reggiani A, Bucci P, Russo G. Accessibility and impedance forms: Empirical applications to the german commuting network. International Regional Science Review. 2011;34: 230–252. doi:10.1177/0160017610387296

48. Soukhov A, Páez A. TTS2016R: A data set to study population and employment patterns from the 2016 transportation tomorrow survey in the greater golden horseshoe area, ontario, canada. Environment and Planning B: Urban Analytics and City Science. 2023; 23998083221146781. doi:10.1177/23998083221146781

49. Carey HC. Principles of social science. Philadelphia: J.B. Lippincot; Co.; 1858.

50. Ravenstein EG. The laws of migration. Journal of the Royal Statistical Society. 1889;52: 241–305. doi:10.2307/2979333

51. Reilly WJ. Methods for the study of retail relationships. 1929.

52. Stewart JQ. An inverse distance variation for certain social influences. Science. 1941;93: 89–90. Available: `http://www.jstor.org/stable/1668130`

53. Stewart JQ. Suggested principles of "social physics". Science. 1947;106: 179–180. Available: `http://www.jstor.org/stable/1675368`

54. Stewart JQ. Demographic gravitation: Evidence and applications. Sociometry. 1948;11: 31–58. doi:10.2307/2785468

55. Zipf GK. The p 1 p 2/d hypothesis: The case of railway express. The Journal of Psychology. 1946;22: 3–8. doi:10.1080/00223980.1946.9917292

56. Zipf GK. The P1P2/d hypothesis: On the intercity movement of persons. American Sociological Review. 1946;11: 677–686. doi:10.2307/2087063

57. De Vrij E, Vanoutrive T. "No-one visits me anymore": Low emission zones and social exclusion via sustainable transport policy. 2022 [cited 27 Jul 2023]. Available: `https://www.tandfonline.com/doi/epdf/10.1080/1523908X.2021.2022465?needAccess=true&role=button`

58. Liotta C. What drives inequalities in low emission zones'impacts on job accessibility? Virtual; 2023. doi:10.17605/OSF.IO/9VQU7