Multimodal spatial availability: a singly-constrained measure of accessibility considering multiple modes

Anastasia Soukhov [1] [*], Javier Tarriño-Ortiz [2], Julio A. Soria-Lara [2], Antonio Páez [1]

1 Department of Earth, Environment and Society, McMaster University, Hamilton, Canada
2 Centro de Investigación del Transporte (TRANSyT), Universidad Politécnica de Madrid, Madrid, Spain

* Corresponding author: soukhoa@mcmaster.ca (AS)

Abstract

~~An increasing number of studies within the domain of transportation planning are concerned with the inequities in accessibility to opportunities. A dimension of these inequities arises from differences in access by mode type (e.g., commuting using a car as opposed to transit). However, methods implemented in current accessibility literature are lacking within the context of multimodal analysis. This paper presents an extension of~~ Recent research has aimed to address the way opportunities are counted in accessibility analysis. In conventional accessibility measures, opportunities are often multiply counted, which leads to values of accessibility that are difficult to interpret. Constraining the calculations to match a known quantity ensures that the measurements sum up to a predetermined quantity (i.e., the total number of opportunities), and so each value can be meaningfully related to this total. A recent effort is spatial availability, a singly-constrained ~~competitive accessibility measure, for the context of multimodal accessibility analysis. We first illustrate the features of spatial availability that lend itself to multimodal analysis. We then demonstrate its use on the case study of Low Emission Zones in Madrid (Spain) and highlight how this policy intervention changes the accessibility of populations using different modes . In summary, spatial availability can be used to create and interpret multimodal policy intervention scenarios unlike previous methods: this creation and interpretation can help regions envision a more sustainable and equitable access-to-opportunity landscape.~~ accessibility measure. In this paper we extend spatial availability for use in the case of multiple modes or, more generally, heterogeneous population segments with distinct travel behaviors. After deriving a multimodal version of spatial availability, we proceed to illustrate its features using a synthetic example. Next, we apply it to an empirical example in Madrid, Spain. We conclude the paper with suggestions for future research.

Introduction

~~Implementing urban policies that re-shape cities through accessibility gains (i.e., the *potential to interact* with opportunities as a result of land-use mix and transport systems as originally defined by~~ Accessibility is a key concept in the analysis of land use and transportation systems [e.g., 1,2,3]~~) have been widely applied within the~~

~~transportation literature and is increasingly discussed by planners~~ , and too is coming of age from the perspective of planning [~~2–5~~see *inter alia*, 4,5–8]. ~~An important challenge in the identification of interventions that equitably transform cities is the effective evaluation of *trade-offs*: cities are complex and dynamic ecologies, and advantaging one component of the city can disadvantage another area, population, or sub-component In this way, policy evaluation should take a *systems* approach~~ Beginning with the work of Hansen [~~6~~1]~~. One way of considering systems is from the perspective of the *finite*. As an illustration, consider the amount of transport space within a city: the amount is typically finite so re-allocating road space away from one mode directly impacts the performance of the others (see the literature on road space reallocation e.g.,~~ , accessibility measures have been widely used to evaluate the efficiency of transportation systems when combined with the distribution of opportunities in space [~~7~~9]~~). Evaluating policy impacts in the context of *finity* provides a way to contextualize the balance of trade-offs that the citizens of a city should tolerate.~~ . As such, it is a holistic measure of spatial systems that measures the ease of reaching destinations [10,11].

~~From the perspective of urban transport systems~~ In practice, the most common form of accessibility measure is based on the gravity model. These measures are sums of weighted opportunities around a focal point (i.e., a potential origin), based on how expensive it is to reach them. Recent research in accessibility analysis has paid attention to the way opportunities are counted in the pertinent calculations. Conventionally, the sums are not constrained, which means that the same opportunity can enter the sum for different origins. Counting the same opportunity multiple times treats it as if it was inexhaustible. But opportunities in general are not inexhaustible, and in fact some of them are by definition exclusive: for example, once a job is taken up by someone in the population, the same job is no longer available for any other person to take. More generally, opportunities are subject to congestion: for example, multiple people can obtain services from the same family doctor, but the more people who do, ~~location-based accessibility measures have been used in the context of policy evaluation. For instance,~~ the more congested the service will be.

The issue of congestion in accessibility measures was the motivation for the development of floating catchment area approaches [~~8~~12,13]~~assesses the transit accessibility gains to healthcare and employment opportunities for disadvantaged neighbourhood in Columbus, Ohio, USA after the transit system's re-design and introduction of a rapid bus system. However, a limitation of this study, like others that implement accessibility measures, is they donot calculate results under a *constrained* framework i.e., one of *finity*. The citizens of Columbus should experience quantitative accessibility gains - but is it at the expense of access using other modes? As another example,~~. While these approaches purport to account for congestion, Paez et al. [~~9~~14] ~~implements a modified cumulative opportunity measure to assess differences between private vehicle and transit system accessibility to jobs in Melbourne, but a similar question remains: does the accessibility afforded to the private vehicle using population come at the expense of accessibility losses to transit users?~~

demonstrate that in general they do not solve the issue of multiple counting of opportunities, thus leading to biases in the calculation of total demand and supply, sometimes inflating them, other times deflating them. In response to this, recent research has paid closer attention to the way opportunities are counted in accessibility analysis. Paez et al. [~~8~~14]~~and~~ , for example, tackle floating catchment area methods and introduce a normalization of the impedance matrix to allocate the population and then the level of service proportionally. More recently, Soukhov et al. [~~9~~15] ~~both use *non-competitive* accessibility measures. There is a branch of location-based accessibilitymeasures that do incorporate the effect of competition for opportunities by~~

~~the population in the region. However, we argue that these existing methods fall short in acknowledging the *finity* of opportunities. For instance,~~ introduced a singly-constrained measure of accessibility, called spatial availability, that employs a similar, but more sophisticated proportional allocation mechanism. The work of these authors show that floating catchment area methods can be seen as singly-constrained accessibility measures, and improve on existing approaches by guaranteeing that each opportunity is counted only once - in other words, treating opportunities as *finite*. The proportional allocation of spatial availability constrains the calculations to match a known quantity, therefore ensuring that the measurements sum up to a predetermined quantity (i.e., the total number of opportunities), and so each value can be meaningfully related to this total.

A limitation of spatial availability as introduced by Soukhov et al. [~~10~~15] ~~applies a competitive measure, two-step floating catchment approach (2SFCA), for~~ is that it was developed for the case of a homogeneous population, for example for the case of ~~access to healthcare services in Florida for both a multimodal network and a single modal network. While the differences in modal access are discussed, the question of how the advantage in access afforded by one mode over another impacts access for different mode users is unanswered.~~

~~This question of how much one mode-using population can access at the expense of another mode-using population is a pertinent equity question in the evaluation of policy scenarios that are multimodal . For instance, consider the impact of a low emission zone (LEZ). LEZ is a policy of spatial and modal discrimination: the circulation of vehicles that are excessively polluting are restricted in specific regions. In the recognition that opportunities are finite, the implementation of a LEZ explicitly reduces the access that the population using polluting vehicles has to opportunities. This restriction allows the population using other more sustainable modes to potentially have a higher level of access than before the LEZ implementation. This evaluation is especially urgent as LEZ are currently in effect in cities globally; their reception has been mixed~~ a single mode of transportation. However, the finite nature of opportunities makes the analysis of heterogeneous populations very relevant. In the case of multiple modes of transportation, people who travel by slower modes (e.g., active modes) can usually reach fewer opportunities than people who travel by faster modes and whose range is typically far wider (e.g., car). This implies that slower travelers will often face increased competition for local opportunities from travelers who can reach said opportunities from farther afield.

The objective of this paper is to address this limitation of spatial availability. Our primary motivation is to extend spatial availability for the case of multimodal accessibility, but it is worthwhile noting that this is in fact just one case of heterogeneous populations (i.e., travel by different modes). The method itself can easily accommodate other forms of heterogeneity, for example variations in travel behavior between older and younger adults [~~11~~e.g., 16]~~and may be having negative impacts on disadvantaged populations who have become mobility-restricted~~ , the propensity of older adults to use different modes of transportation [~~12, 13~~e.g., 17]~~.~~ ~~Measures that evaluate the accessibility of modes given both *constrained* and *competitive* considerations are lacking in the literature, but are needed, to evaluate such policy interventions impact on accessibility.~~

~~In~~ , the usually shorter trip lengths of children compared to grown-ups [~~14~~e.g., 18]~~, we introduce spatial availability, a type of location-based accessibilitymeasure that is both *constrained* and *competitive*. In this paper, we extend the spatial availability measure into a multimodal framework and explore its use in answering the question outlined: *"given opportunities are finite, how many are available to a given location depending on the mode used?"*. The answer to this question quantifies how many~~

~~opportunities can be accessed, considering competition, for different modes. To foreground this exploration, in Section 2, we discuss short falls of a few existing location-based measures in comparison to spatial availability~~ , or the more limited travel ranges of single parents [e.g., 19].

The paper rest of this paper is organized as follows. In Section 2 we provide a brief review of multimodal accessibility. In the following Section 3 we demonstrate the derivation of the new spatial availability expression for multiple modes. In Section 4 we illustrate relevant issues through a synthetic example. ~~In Section 3, the spatial availability of~~ This is followed in Section 5 by an empirical example ~~of the LEZ in the~~ using data from the city of Madrid ~~, Spain is calculated. We demonstrate how the restriction of car circulation could have impacted the spatial availability of opportunities for each sub-population using~~ after the implementation of its Low Emission Zones (LEZ). Data for this example comes from the city's 2018 travel survey. The example shows the differences in spatial availability within and outside the LEZ for travelers using different modes, namely car, transit, cycling and walking~~modes~~. In Section ~~4~~6, we provide concluding remarks on the strengths of the use of spatial availability as a multimodal accessibility measure~~and~~ , and discuss potential future uses in policy planning scenarios as well as directions for future research.

A brief review of multimodal accessibility

Location-based accessibility indicators are quantitative measures of *potential* interaction with opportunities for locations within a given region: they are ~~a product~~ summary measures of the relationship between land-use and transport systems. Arguably, the most commonly used ~~location-based measured are~~ are measures based on the gravity model [20], of which cumulative opportunity measures and weighted cumulative opportunity measures are particular forms [~~2~~5]. These measures ~~weight the opportunities that can be potentially interacted with from origin~~ assign a weight to opportunities based on how easy it is to reach them. Given an origin (i~~to destination~~) and a destination (j~~based on some sort of travel cost function~~), an impedance function $f^m(c^m_{ij})$ converts the cost of travel (e.g., ~~travel time, fare, travel distance) otherwise known as a travel impedance function $f^m(c^m_{ij})$. Many weighted cumulative opportunities (often referred to as the gravity-based measure) originate from the measure proposed by~~ time, money, generalized cost) into a score that represents the propensity for interaction. These measures originate from that proposed by Hansen [1], which can take the following ~~multimodal form~~ form in the multimodal case: $S_i^m = \sum_j O_j f^m(c_{ij}^m)$ where m is a set of modes which have mode-specific travel costs ~~(c_{ij}^m) and~~(c_{ij}^m) and/or travel impedance functions $(f^m(\cdot))$.

~~The~~ Hansen-type ~~measure does not consider competition between modes nor is it constrained. As~~ accessibility is not constrained, which is to say it does not consider the opportunities as finite. To cite an example, ~~the work of~~ Tahmasbi et al. [~~15~~21] ~~uses the~~ use Hansen-type ~~measure to measure~~ accessibility to assess the potential interaction with retail locations ~~using~~ by three modes: walking, public transit, and car ~~modes m~~(i.e., $m = w, p, c$). S_i^m is the sum of retail locations j that can potentially be ~~interacted with~~ reached under the travel impedance as calculated for each i and m. In other words, ~~each~~ for each origin i ~~has three S_i values, one per m~~three accessibility scores are calculated. In this work, ~~they demonstrate that the car mode has the highest $S_i^{m=car}$~~ Tahmasbi et al. [21] show that car travel affords the highest S_i^m values in the majority of i, i.e., ~~populations using~~ travelers who use a car can potentially ~~interact with the most~~ reach more retail opportunities than populations using other modes. However, ~~the higher $S_i^{m=car}$ values are not a result of lower~~ higher S_i^m values for car do not affect the values of S_i^m ~~values~~ for other modes: ~~it is not assumed that~~

~~car-using populations potentially accessing more opportunitiestake away potential opportunities for other populations within the measure (no consideration for competition). This measure is also not constrained: there is no global maximum for S_i or S_i^m values,they are presented as a population normalized accessibilityindex. This makes the~~ in effect, each mode is analyzed as if the others did not exist. Since the measure is not constrained, each opportunity is typically counted multiple times within and between modes, and as a result the sum of accessibility is not necessarily a meaningful quantity. The accessibility scores for the modes are often values that are difficult to interpret beyond making statements about relative size. For example, Lunke [22], researching the region of Oslo, reports accessibility scores for car in the order of tens of thousands of employment opportunities. The corresponding scores for transit are lower, but still often in the thousands or tens of thousands. As reported, the ratio of the transit to the car score can be lower than 0.2 (meaning transit gives access to less than 20% of the opportunities than car). But despite the discussion about "sufficient accessibility", it is unclear what the unconstrained scores mean: is having access to but 10,000 jobs by transit insufficient? After all, 10,000 employment opportunities are still plenty of opportunities. These ratios can be found elsewhere in the literature [e.g., 23, and 9,16,19, and 8], and they are useful as relative assessment of when some members of the public are better or worse off than others, but they do not say much about how bad is "worse off".

Besides ratios of accessibility, another way to improve interpretability of scores sometimes seen in the literature is to standardize them to lie in the range [0-1]. This adjustment is only helpful insofar as it facilitates relative comparisons, but interpretation of the ~~'potentially interacted opportunities' relative to the region, making comparisons of the results across different regions challenging.~~ scores remains challenging because the values are specific to a region and convey no meaning about the magnitude of the scores. In this approach, zones always have values between 0 and 1, but how remarkable is a zone with a low score for pedestrians and a high value for car? And if remarkable, what does the difference in these standardized values mean for planners? By how much should transport systems and land-use configurations be changed to improve conditions? And in what way can these scores be used to track differences over time? Or between regions? These questions lack straightforward answers since certain values will always be relatively 'low' or 'high', but do not track to a quantity that can be intuitively understood. Presentation or discussion of Hansen-type accessibility that has been standardized in this way is not uncommon in the literature [e.g., 24,25].

~~However, opportunities in a region can be considered finite~~Once we understand opportunities to be finite, it is possible for an accessibility measure to take on a crisper meaning. As considered in the long tradition of accessibility research, capacity of opportunities is limited and thus is subject to competition by population [12,15,26–30]. There are only so many school-seats, hospital capacity, ~~jobs~~employment opportunities, etc., in a region and if one person ~~interacts with an opportunityat a given time~~reaches an opportunity, it is taken: the supply of an opportunity and the demand for that opportunity are two components of accessibility. These are clear examples of opportunities that are unambiguously competitive. But we would go as far as to argue that every type of opportunity is subject to congestion or capacity constraint, even when the opportunities are conventionally seen as non-competitive. ~~As such~~

Amenities are a good example of this. For instance, standards for providing green spaces are often stated in the form of *exclusive access*, in units of amenity per capita. A case in point is a the Ile-de-France region, a jurisdiction that suggested in a 2013 planning document that at the municipal level at least $10m^2$ of public green space should be supplied *per inhabitant* [31]. Green spaces are not evenly distributed, which means that who has access to them hinges on where they are and how easy is to reach them. This formulation of provision of amenities is not unusual. For example, Natural England, an organization that recommends an Accessible Natural Greenspace Standard such that the minimum supply of space is one ha of statutory Local Nature Reserves per thousand population[1]. Similarly, the World Health Organization [cited in 32] recommends that cities provide a minimum of $9m^2$ of green area per inhabitant. For our purposes, standards of this type translate into "how much of this resource is available to one individual that has not been claimed by anyone else?". Green spaces often have large capacities, but they still have a capacity, and it is not the same for a person to have access to $5m^2$ of *uncongested* green space as $15m^2$. This difference is in fact a matter of justice [31,~~if one person is advantaged and has the ability to reach more opportunities through a lower travel-cost mode, than they have more opportunitiesto potentially interact with~~33]. Constraining accessibility is in this way a useful way to evaluate the congested availability of any type of opportunity. As development of sound standards is emphasized in the planning literature, in particular in regards to fairness in transportation [see 34], spatial availability analysis is a useful way to develop and assess standards.

The relevance of the considerations above is put in sharper relief when we think about the use of multiple modes (or heterogeneous populations). If we return to Oslo for a moment [22], we notice that the places that have high accessibility by transit are also the places that have *very high* accessibility by car (in their Figure 2). Those two populations are going for the same opportunities, and those travelling by transit have fewer to choose from to begin with. More generally, people in a zone who are advantaged with relatively low cost of travel will have the ability to potentially reach more opportunities than other people. ~~From the other perspective , their~~ Due to this advantage, through the perspective of finite opportunities, there are fewer opportunities left ~~to be potentially interacted with for populations using higher travel-cost modes. In this way, populations using modes with a higher travel impedance are at a higher access disadvantage than populations using lower travel impedance modes. This recognition is the motivation behind integrating *competition* for opportunities within multimodal accessibility measures. Arguably one of the most popular competitive location-based accessibility measures is the two-step~~ for everyone else, especially for those who use modes that are slower or otherwise more expensive.

As noted in the Introduction, competitive accessibility was the rationale for developing floating catchment area ~~(2SFCA)approach popularized by~~ methods (FCA), popularized by Luo et al. [~~16~~13] who ~~simplified the approach proposed by~~ reformulated the work of Shen [~~17~~12] ~~(with similarconsiderations for competition~~ into two steps (although similar, and earlier, developments are found in [~~18~~30,35]~~and 19).~~

~~The~~). Shen-type accessibility ~~measure's formulation is~~ is formulated as: $a_i^m = \sum_j \frac{O_j f^m(c_{ij}^m)}{\sum_m D_j^m}$ where D_j^m is the potential demand for opportunities equal to travel impedance weighted population $\sum_i P_i^m f^m(c_{ij}^m)$ ~~. In this way, the Shen-type measure~~and the remaining variables are repeated in the Hansen-type measure. Shen-type modal accessibility (a_i^m) can be understood as a ratio of the ~~potential opportunity supply over the potential~~ travel impedance-weighted supply of opportunities for m-mode in i over the travel impedance-weighted demand for opportunities. ~~The measure considers competition, but it is *non-constrained*. A score of competitive potential accessibility associated is associated with each location i for each mode m, but there are no global maximums~~In this way, it considers competition. That said, the measure remains unconstrained, meaning both population *and* opportunities are multiply counted [see 14]. In other words, ~~it is difficult to interpret~~

[1] see https://redfrogforum.org/wp-content/uploads/2019/11/67-Nature-Nearby%E2%80%99-Accessible-Natur

~~the meaning of differences in~~ interpretation of the Shen-type accessibility scores between modes is fraught as it is for Hansen-type measures.

To illustrate, [~~20~~36] ~~calculates~~ calculate a_i^m to jobs for different income-group populations in Shenzhen~~(China)~~, China using $m =$ public transit and ~~$m = car$. They demonstrate that i~~ $m =$ car. Their results indicate that zones with low-income populations have lower a_i^m than ~~i~~ zones with higher-income populations. Further, they ~~demonstrate~~ show that $a_i^{m=\text{public transit}}$ is lower than $a_i^{m=\text{car}}$ at many ~~i~~zones, arguing that this may ~~put i~~ further place those zones with lower-income populations ~~in a further~~ at a disadvantage. a_i and/or a_i^m are used to compare relative spatial differences in overall competitive accessibility and ~~modal~~ multimodal competitive accessibility, but because ~~there is no global maximum, making it is difficult to interpret the significance between~~ opportunities were doubly counted (entering the sums of both modes), this makes for uneasy interpretations of the differences in a_i^m ~~values. Questions such as~~between modes. Questions that this approach leaves unaddressed include: what is the impact ~~that competition has~~ of competition on the difference in a_i^m values? How does the impact vary spatially? And what is the interpretation of this difference? ~~are left unanswered.~~

Spatial availability improves on ~~previous multimodal accessibility approaches as it considers *competition* in the potential interaction with opportunitiesin a *constrained* framework (e.g., finite opportunities)~~previously discussed accessibility approaches using the Hansen-type measure and the Shen-type measure by constraining the sum of opportunities, that is, by treating opportunities as finite. This is done by ~~considering: 1) competition between~~ means of proportional allocation factors that follow well established principles of spatial interaction and the gravity model [see 37]. In Soukhov et al. [15] these factors consider: the mass effect (e.g., the ~~advantage of sub-populations residing in relatively low population-density and high opportunity-proximate areas)and 2) competition between travel impedance~~ size of populations at different origins); and the cost of travel from different zones (e.g., some sub-populations ~~with relatively low travel-impedance)through a proportional allocation mechanism~~face relatively higher or lower costs). The following ~~sub-section demonstrates how spatial availabilitycompares to the Hansen-type and Shen-type measures through a synthetic example~~section introduces the multimodal form of spatial availability.

Multimodal spatial availability

In brief, we define the ~~*spatial availability* at~~ spatial availability at an origin i (V_i) as the proportion of all opportunities in the region ~~O~~ that are allocated to location i from all ~~opportunity~~ destinations j. V_i is a value of how many opportunities are available to each location i out of all the opportunities in the region. The general formulation of spatial availability V_i is shown in Equation (1) [see 15]:

$$V_i = \sum_{j=1}^{J} O_j F_{ij}^t \tag{1}$$

Where:

- F_{ij}^t is a balancing factor that depends on the ~~demand for~~ size of the populations at different locations that demand opportunities O_j~~and~~, as well as the cost of movement in the system $f(c_{ij})$.
- V_i is the number of spatially available opportunities at i; the sum of V_i is ~~equivalent~~ identical to the total ~~sum~~ number of opportunities in the region (i.e., $\sum_j O_j = \sum_i V_i$); in other words, opportunities are dealt with as finite resources.

~~The spatial availability measure isintroduced in 14. Spatial availability 's unique feature is the~~ Compared to Hansen-type accessibility:

$$A_i = \sum_{j=1}^{J} O_j f(c_{ij}) \tag{2}$$

we see that spatial availability is, like the Hansen-type measure, a weighted sum of the opportunities. What makes spatial availability stand apart from other approaches is how the weight used in the sum, balancing factor F^t_{ij}, implements a proportional allocation mechanisms ~~, that ensures the~~ to ensure that the sum of V_i ~~calculated for each i sums, across all i in the region, to equal~~ is constrained to match the total number of opportunities in the region. As such, spatial availability is ~~a *competitive* and *constrained* accessibility measure as F^t_{ij} handles the number of opportunities in the region in a finite way (proportional allocation)~~singly-constrained and naturally implements competition or congestion. F^t_{ij} consists of two ~~components:~~ parts. The first part is a population-based ~~balancing factor $F^p_i = \frac{P_i}{\sum_i P_i}$ and an~~ proportional allocation factor to model the mass effect of the gravity model:

$$F^p_i = \frac{P_i}{\sum_i P_i}$$

This factor makes opportunities available based on demand. Secondly, there is an impedance-based ~~balancing factor $F^c_{ij} = \frac{F^c_{ij}}{\sum_j F^c_{ij}}$ that, respectively, allocate opportunities to i in proportion to the size of the population at i (the mass effect) and the cost of reaching opportunities at j (the impedance effect).~~ proportional allocation factor that models the cost effect:

$$F^c_{ij} = \frac{F^c_{ij}}{\sum_j F^c_{ij}}$$

This factor makes opportunities available preferentially to those who can reach them at a lower cost. F^p_i and ~~$ F^c_{ij}$ are calculated for each i such~~ F^c_{ij} are designed so that they both equal 1 when summed across all i in the region (e.g., $\sum_i F^p_i = 1$ and $\sum_i F^c_{ij} = 1$). These ~~balancing~~ factors are combined multiplicatively to yield F^t_{ij} which ensures that a proportion of the opportunities O_j are allocated to each i accordingly. In other words, assuming a finite number of opportunities in the region, F^t_{ij} proportionally allocates O_j to each i such that the resulting V_i value represents the number of opportunities ~~*spatially*~~ *available* to the population at i. ~~This value can be seen to represent spatial availability as it~~ Each zonal value is a proportion of the opportunities in the region (i.e., $\sum_j O_j = \sum_i V_i$).

The focus of this paper is to extend V_i for ~~multimodal applications~~the measurement of multimodal applications (or more generally heterogeneous populations). To do so, the balancing factors ~~are reformulated to yield a proportional value for the set of modes m used by populations at each~~ need to be reformulated so that 1) the mass effect now accounts not only for the size of the population at i~~. As these factors are proportional,~~ , but also the size of sub-populations within i; and 2) the cost of travel is not only for different zones, but by sub-populations within each zone (e.g., the cost of travel from i by car, transit, walking, etc.) When we introduce modes (or sub-populations) m, the proportional allocation factors need to satisfy the condition that F^{pm}_i and F^{cm}_{ij} can be summed ~~up~~ across each m at each i and then across all i to equal to 1. They are also similarly combined multiplicatively to obtain their joint effect, represented as the combined balancing factor F_{ij}^{tm} similar to that detailed in Equation (3). This factor is given by:

$$F_{ij}^{tm} = \frac{F_i^{pm} \cdot F_{ij}^{cm}}{\sum_{m=1}^{M} \sum_{i=1}^{N} F_i^{pm} \cdot F_{ij}^{cm}} \tag{3}$$

~~Where~~where:

- The ~~population balancing factor for~~ factor for allocation by population for each m at each i is $F_i^{pm} = \frac{P_i^m}{\sum_m \sum_i P_i^m}$; and
- The factor for allocation by cost of travel ~~balancing factor~~ for each m at i is ~~$F_{ij}^{cm} = \frac{f(c_{ij}^m)}{\sum_m \sum_i f(c_{ij}^m)}$~~ $F_{ij}^{cm} = \frac{f^m(c_{ij}^m)}{\sum_m \sum_i f^m(c_{ij}^m)}$

Implementing F_{ij}^{tm}, the following Equation (4) ~~demonstrates the multimodal configuration~~ gives the multimodal version of spatial availability V_i^m:

$$V_i^m = \sum_{j=1}^{J} O_j \; F_{ij}^{tm} \tag{4}$$

Where:

- ~~m~~ $m = 1, 2, \cdots, M$ is a set of ~~modes used by populations in the region~~ M modes (or sub-populations) of interest.
- F_{ij}^{tm} is a balancing factor F_{ij}^{t} for each m at each i.
- V_i^m is the spatial availability V_i for ~~mode~~ each m at each i; the sum of V_i^m for all m at each i is equivalent to the total sum of opportunities in the region (i.e., $\sum_j O_j = \sum_i V_i = \sum_m \sum_i V_i^m$).

Next we use a synthetic example to contrast multimodal accessibility and spatial availability.

~~Consider the following: Figure 1depicts~~

An illustrative synthetic example

Consider the simple system shown in Figure 1. The figure shows a region with population ~~and jobs~~ at three population centers (A, B, C) and jobs at three employment centers (1, 2, 3). The population at each ~~population center is divided into~~ origin i is consists of two sub-populations, one using a faster mode z and another using a slower mode x, to travel to employment centers. Population center A is Suburban: it is closest to its own relatively large employment center at 1, close to the Urban's equally large employment center 2, and has a population that is smaller than the Urban B and larger than the Satellite C. B has the largest x-using population, followed by then A, then C. This synthetic example ~~was~~ is inspired by the single-mode example used in [~~17~~12] and reconfigured in [~~14~~15].

From the perspective of access to a *finite* amount of opportunities in the region (210,000 jobs), the sub-population that is most proximate to jobs (low cost to reach), furthest from ~~densely populated centers, and is using the lowest travel-cost~~ large populations (less competition), and uses the fastest mode z ~~can potentially access the most job~~ (greater range) can potentially reach the largest number of opportunities. This appears to be the sub-population at A using mode z. ~~From the other perspective, sub-populations~~ Sub-populations located in opposite conditions (i.e., ~~further away~~ more distant from jobs, close to ~~dense~~ large populations, and using slow mode x) are at

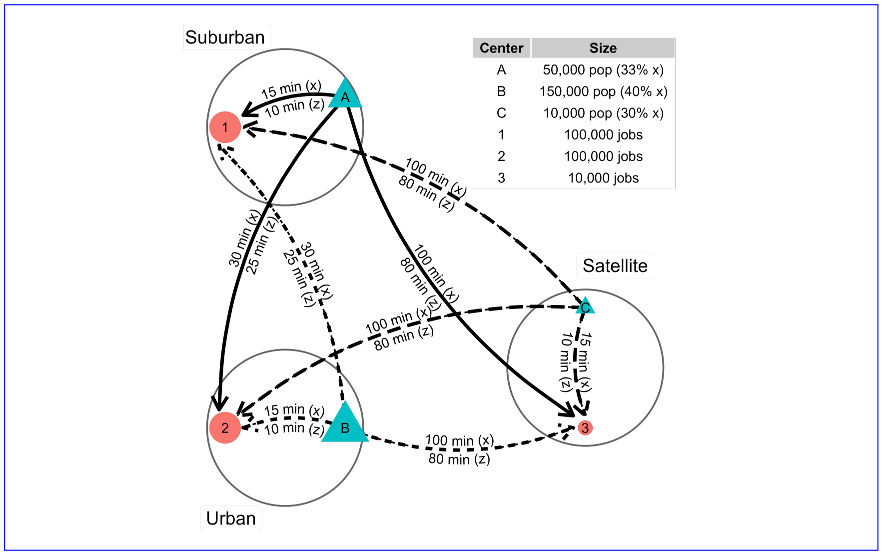

Fig 1. Multimodal synthetic example: locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times for two modes (slower mode x and faster mode z).

a relative ~~job opportunity access *disadvantage*. From the perspective of inequities, the~~ disadvantage. The competition for opportunities between different mode-using populations matters as it reflects how well the land-use and transport system serves (or ~~doesn't serve) them~~does not serve) certain populations.

Table 1. Accessibility values at each origin (i) per mode (m) ~~at~~(columns ~~each~~three ~~origin~~to ~~i~~five) and aggregated ~~between~~per ~~modes~~i ~~for~~(columns ~~each~~six ~~i~~and seven) for the synthetic example.

i	m	S_i^m	a_i^m	V_i^m	a_i	V_i
A	x	27,292.18	0.95	15,696.89	1.36	67,482.61
	z	44,999.80	1.57	51,785.72		
B	x	27,292.18	0.64	38,170.03	0.88	132,638.94
	z	44,999.80	1.05	94,468.91		
C	x	2,240.38	0.68	2,035.86	0.99	9,878.45
	z	3,745.89	1.12	7,842.59		
TOTALS		150,570.22	N/A	210,000.00	N/A	210,000.00

The ~~calculated~~ values calculated for S_i^m (Hansen-type accessibility), a_i^m (Shen-type accessibility), and V_i^m ~~accessibility values~~ (spatial availability) for each i and m are shown in the middle three columns and are aggregated for each i in the final two columns in Table 1 . ~~We~~ As in the example in Shen [12], we use a negative exponential impedance function ~~$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$~~ $f^m(c_{ij}^m) = \exp(-\beta \cdot c_{ij})$ with $\beta = 0.1$ for both x and z modes for all accessibility measures calculations. Notice that in this example we use the same impedance function but the travel times are different for the two modes. More generally, it is possible to use different impedance functions for the modes, as demonstrated in the empirical example in the following section.

~~The~~ Hansen-type ~~measure~~ accessibility S_i^m is presented for each origin and mode in the third column of Table 1 . For all i, the travel by z ~~-using sub-population has higher S_i^m values than the~~ results in higher values of S_i^m than travel by x~~-using sub-populations. Additionally,~~ . Lack of competition, or alternatively the assumption of an inexhaustible resource in the calculation of S_i^m~~is equal for both mode-using populations in A and B. This is the case because S_i^m does not consider *competition*, it only relies on reflecting the count of opportunities that may be interacted with as a product of $f^m(c_{ij}^m)$. Recall,~~ , lead to a curious result. Since the populations in A and B have the same travel impedance to employment centers 1, 2 and 3 (either 15, 30, or 100 minutes using x or 10, 25, or 80 minutes using z)~~. As such, these the calculated S_i^m values~~ , their values of S_i^m are the same for both A and B. Furthermore, the total sum of S_i^m in the region is equal to 150,570.2. This value ~~is difficult to interpret~~lacks an intuitive interpretation: it represents the weighted sum of opportunities that may be ~~interacted with~~ reached within the region ~~based on travel impedance . It cannot be interpreted as~~ according to the travel impedance (i.e., the travel behavior and the characteristics of the modes) and does not usefully translate into any sort of benchmark~~since the measure is *non-constrained*~~. To connect this example to the aforementioned literature, S_i^m is calculated in the work of Tahmasbi et al. [~~15~~21]; they ~~compare~~ contrast differences in S_i^m values between modes in a relative and comparative sense, but make no further interpretation of the S_i^m values. More densely populated metropolitan regions will tend to have more opportunities and hence large S_i^m values and less densely regions, smaller values; how much of these differences may simply an artifact of region density?

In the fourth and sixth ~~column~~ columns in Table 1 the results for Shen-type ~~measure is calculated~~accessibility are reported: first for both origin and mode a_i^m as well as aggregated by the weighted mean mode-population ($\sum_m \frac{P_i^m}{P_i} * a_i^m$) to represent a value for each origin a_i. Unlike S_i^m, this measure does considers *competition*. For instance, the population travelling by x ~~-using populations in~~ from A and B ~~centers~~ do not have the same ~~a_i^m values as the~~ values of a_i^m as those travelling by z~~-using~~. In fact, A has the highest values a_i^m and a_i values since this center has the ~~smallest~~ lowest travel impedance to opportunities (lower than at C, A and B are equal) and ~~has one of these lowest proximity~~ faces relatively low competition, not being close to a relatively ~~high amount of~~ large population (lower than at B).

However, the ~~Shen-type measure is *non-constrained*~~calculations of a_i^m are not constrained: the total sum of a_i^m or a_i is practically meaningless since it represents a sum of ratios. For instance, the population travelling by z ~~-using sub-population at~~ from A has a value of 1.57 ~~potential jobs per potential~~ jobs per job-seeking population compared to 0.95 for users of mode x~~-using sub-population~~. What is the ~~significance~~ meaning of these values? The difference between these modes is equal to 0.62, but 0.62 of what? How many more job opportunities ~~are z users interacting with than~~ can users of z reach compared to user of x~~users~~? When a_i^m is aggregated to a_i as shown in the sixth column, the values face similar interpretability issues. The Shen-type measure is implemented in ~~the previously discussed work of~~ aforementioned work of Tao et al. [~~20~~36] to calculate modal a_i^m values and the aggregated a_i is implemented in the work of Carpentier et al. [~~21~~38]. However, similar to ~~the~~ Hansen-type ~~measure~~accessibility, these works discuss relative and spatially comparative differences in values, ~~they do not make further interpretation of the~~ but veer from interpreting the values of a_i^m or a_i themselves. ~~This may be because the Shen-type measure is *non-constrained*, this is no benchmark or global maximum to which comparisons can be drawn from~~In fairness, interpretation is complicated by the multiple counting of opportunities between zones and modes.

~~By~~ In contrast, spatial availability V_i considers competition and is constrained such that the total sum of values is equal to the total number of opportunities in the region (i.e., 210,000 jobs). Seen in fifth column of Table 1 , the values of V_i^m ~~for the same mode-using populations~~ in A and B are not the same within each mode (as this measure considers competition). In fact, at A, ~~the~~ users of mode z ~~-using sub-population captures~~ capture 36,088.84 more spatially available jobs (of the 210,000 jobs in the region) than the sub-population ~~using mode~~ travelling by x. The numerical difference ~~has a practical interpretation~~is clear since it refers to opportunities out of the total.

Furthermore, ~~V_i^m values for an~~ the proportional allocation mechanism also means that the values of V_i^m for any origin i can be aggregated across m and compared ~~across i~~ between zones ($V_i = \sum_m \sum_i V_i^m$)~~as a result of the proportional allocation mechanism~~. This aggregation, V_i, is shown in the seventh column in Table 1 . Again looking at center A, A is allocated 67,482.61 spatially available opportunities for both modes. 77% of this spatial availability allocated to A is assigned to ~~the~~ users of mode z ~~-using population~~ despite representing 66% of A's population.

Spatial availability can be further aggregated to better interpret competition between modes. Across the entire region, 130,000 people use z (62% of the region population). However, ~~the~~ users of z ~~-using population accounts~~ account for 73% of the region's total spatial availability - ~~the rest~~ while the remaining 27% is allocated to ~~the~~ users of mode x ~~-using population (~~who are 38%of the total population~~)~~. Notably, the population who uses x ~~-using population captures~~ have 11% ~~less spatial availability to~~ fewer spatially available opportunities than its ~~population proportion. This understanding can lead~~ share in the population. This realization leads us to ask normative questions such as, how unequal should ~~opportunity access for the two mode-using populations be~~ availability of opportunities be by mode? What intervention could help to redistribute spatial availability to sub-populations commensurate with their proportion of the total? ~~Can the lower-travel-cost populations spare some spatial availability if a policy of modal-restriction (like a LEZ) was introduced?~~

Since spatial availability is constrained and has an interpretable meaning as a proportion of the total opportunities in the region, the values at i have a ~~new significance~~straighforward interpretation. Inequality in V_i^m values can be explored through a variety of approaches. For instance, consider travel times. The population of travelers who use z ~~-using population~~ accounts for 67% of the potential travel time traveled in the region: this is 7% less travel time than the proportion of spatial available opportunities that is allocated to them. In other words, the ~~z-using population travels less~~ population of users of z travels fewer minutes overall and has more spatial availability of opportunities than ~~the x-using population using the~~ users of the slower mode x.

Alternatively, inequities in spatial availability between ~~mode-using populations~~ modes can be explored through proportional benchmarks. A spatial availability per capita v_i^m ~~as~~ is presented in Equation (5):

$$v_i^m = \frac{V_i^m}{P_i^m} \tag{5}$$

The ~~v_i^m values~~ values of v_i^m for A, B, and C for ~~the~~ users of x ~~-using sub-populations~~ are 0.95, 0.64 and 0.68 spatially available jobs per capita, respectively. The values of v_i^m for ~~the~~ users of z ~~-using sub-populations~~ are much higher, with values of: 1.57, 1.05 and 1.12 respectively. ~~The~~ Users of x~~-using population, especially~~ , especially those at B and C, are directly impacted by the jobs that are spatially available to ~~the~~ users of z ~~-using population~~ *in addition to* the mass effect (occurring at B, high population density) and high travel impedance (occurring at the Satellite C).

If, ~~lets~~ let us say, the planning goal ~~is~~ was to have one spatially available job per mode-using population, a policy intervention ~~can be put in place~~could be devised, to reduce the ~~v_i^z values and increase v_i^x values . This~~ values of v_i^z (making it slower or more expensive) and increase he values of v_i^x (making it faster or les expensive). The purpose of this simple demonstration is to show how ~~simply the V_i^m framework can be manipulated~~ spatial availability can be used to quantify the competitive (dis)advantage in a multimodal application. In what follows, we ~~further explore competition between multiple modes~~ demonstrate the use of multimodal spatial availability through an empirical example.

Empirical example

Context

~~Low emission zones~~ The context for the empirical example is Madrid, Spain. This city implemented a Low Emission Zone (LEZ) ~~have~~ in 2017 to: pursue goals set out in the national climate change agenda, cut nitrogen dioxide levels, and to prioritize people's movement in the city. LEZs elsewhere have similarly been implemented as ~~a climate change policy intervention~~ interventions to reduce GHG emissions, improve air quality, and support sustainable mobility ~~in many countries. Though rules vary~~[39,~~LEZ~~ 40]. Though the rules of exclusion vary by city, LEZs aim to deter/reduce traffic in designated zones under threat of penalty (e.g., fines, seizure of vehicle). ~~From the perspective of restriction for passenger transport, LEZ are a policy~~ In other words, LEZs implement a form of *geographic discrimination* as they change how people ~~access~~ can reach opportunities by making ~~the travel impedance~~ it more costly for ~~car-mode users. If seeing~~ some forms of travel, typically cars, to circulate in predetermined zones. When considering opportunities as finite in a region, this discrimination ~~allows populations to access opportunities by other modes more readily than before. In this way, LEZ change the multimodal competitive~~ reduces the competition of one mode and opens up opportunities for other modes to better thrive. At their core, LEZs operate by changing the accessibility landscape of a city ~~.~~ from the perspective of multiple modes.

~~Spain is one of a few countries with active LEZ and plans to expand their implementation as specified in their climate-change-related plans: *Plan Nacional Integrado de Energía y Clima 2021-2030* 22and *Plan Nacional de Control de la Contaminación Atmosférica* 23. Specifically, the national Spanish law 7/2021 (*Ley de Cambio Climático y Transición Energética*) will require all municipalities to implement LEZ by 2023 if they meet at least one of the following requirements: (i) municipalities >50,000 inhab. ; (ii) islands; and (iii) municipalities > 20,000 inhab. when air quality exceeds limits specified in *RD 102/2011 de Mejora de Calidad del Aire* 24.~~

In ~~2017, LEZs were implemented in the Spanish capital city of Madrid following the goals set out in the national agenda . In~~ geographic scope, the 2017 boundaries of the LEZ ~~were relatively small (covering~~ in Madrid were relatively modest, covering only approximately 4.72 km~~^{2}) and within the center (i.e.,~~ ^{2} of the central business district of the city (the so-called LEZ Centro). ~~These boundaries were expanded in 2023 to inside of~~ As of this writing, there are plans to expand these boundaries to the area inside the M-30, ~~a~~ an orbital highway in proximity to the city center (i.e., LEZ M-30)~~and the city has plans to further spatially expand the LEZ~~. Within the 2017 LEZ Centro implementation, all cars, motorcycles and freight ~~with environmental label~~ vehicles with environmental labels A or B (~~higher polluting classification, associated with older make and model~~ older makes and models of fossil fuel internal combustion engine vehicles), ~~are not permitted to enter the area~~ were disallowed from entering the zone unless they are used by residents or meet other exemptions. This restriction

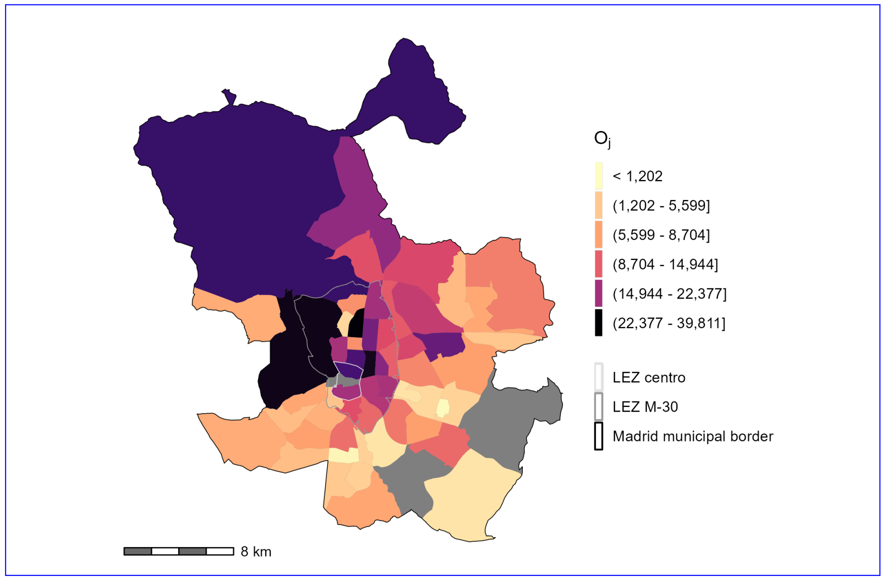

Fig 2. ~~Jobs O_j~~ Distribution of jobs taken by people living and working in Madrid as reported ~~by~~ in the 2018 travel survey. Grey TAZs has no jobs. Ranges of values in the legend are quintiles.

impacted approximately half of all car trips that ~~were typically made into~~ used to travel into what is now the LEZ Centro [~~25~~41].

For this case study, we use ~~V_i^m to quantify the competition of spatially available opportunities between modes after the LEZ Centro implementation~~spatial availability to quantify access to opportunities by different modes in Madrid. Particularly, we demonstrate how V_i^m can be used to ~~spectate on~~ derive insights into how the restriction of car mobility in areas around/within the LEZ Centro ~~allowed the other, more sustainable but often with higher travel impedance modes,~~ may have allowed more sustainable (but often slower or more costly modes) to become more competitive.

~~The~~

Data

The source of data for our empirical example is the 2018 Travel Survey of the Community of Madrid ~~travel survey (~~[~~26~~42]~~) is the source of data for this empirical example: it is~~. This is a representative survey that ~~reflects~~ offers a snap-shot of ~~the~~ travel patterns for ~~one typical day of the working week (e.g., n=~~a typical weekday in 2018. The survey collected 222,744 trips ~~with representative population elevation factors)~~ from a representative sample of 85,064 households across the traffic analysis zones (TAZ) in the Community of Madrid. For context, the population older than 3 years in the Community is 6,507,184.

In this example, we use all home-to-full-time-work trips, by all modes. ~~In this paper, a sample of the travel survey is used,namely the residential home origin to work destination tripsof all modesand those that originate and end in the city of Madrid. These totals are displayed in Figure 2 and Figure 3 . Both figures are displayed at the level of traffic analysis zones (i and j) that correspond to the survey. The red~~ The trips are expanded using population weights. Figures 2 and 3 show the number of workers and the distribution of full-time jobs in the City of Madrid by TAZ. The light grey boundary represents the LEZ Centro in effect in ~~2017 and thus those travel patterns of car-restriction reflected in the survey . The cyan~~ 2017. The survey was conducted

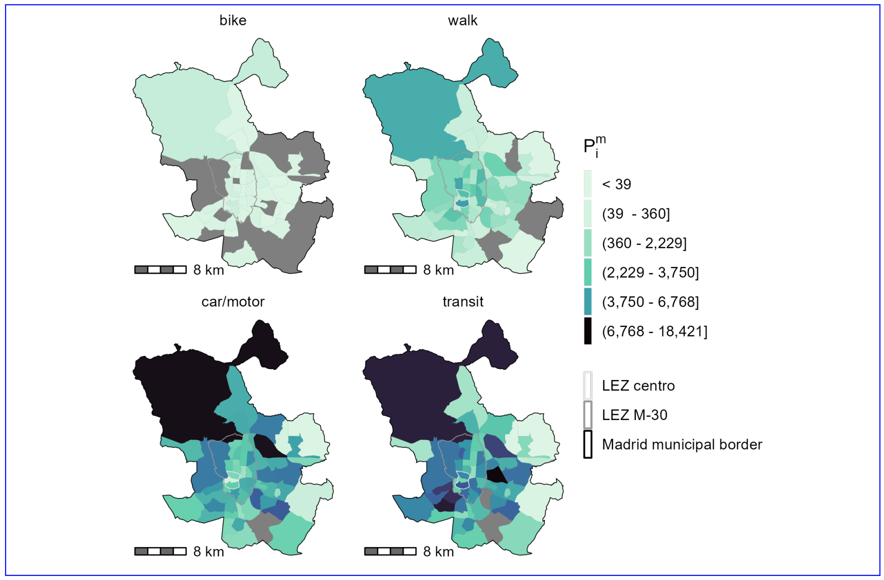

Fig 3. Population living and working in Madrid ~~,~~ by ~~four summarized modal categories, P_i^m~~ mode of transportation as reported ~~by~~ in the 2018 travel survey. Grey TAZs have no population. Ranges of values in the legend are quintiles.

after the introduction of LEZ Centro. The dark grey boundary represents the LEZ ~~that will be within~~ planned for the boundaries of the M-30 highway ~~in 2023~~ and is present in the plots as a spatial reference for areas in proximity to the LEZ Centro.

The total sum of jobs O_j ~~that are held are~~ are shown in Figure 2 and the populations that go to a work destination by four modal categories P_i^m, is ~~reflected~~ displayed in Figure 3. The modal ~~categories represented~~ shares in Figure 3 are ~~summarized for the following trip mode types~~ calculated based on those measured in the survey. The modal categories and the mode types within each category are reported as follows:

- Car/motor: all cars and operating modes (e.g., cab, private driver, company, rental ~~care~~car, main driver ~~, passenger, etc.~~of a private car, passenger in a private car) and all public, private or company motorcycle/mopeds.
- Transit: all bus, trams, and trains~~,~~.
- Bike: all bicycle trips (e.g., private, public, or company bike trips) and "other" types of micromobility options~~,~~.
- Walk: walking or by foot~~,~~.

Some aggregation of modes is necessary to calculate the travel impedance functions by mode. From Figure 2, it can be seen that the largest concentration of jobs ~~are~~ is within, near, and to the north of ~~the~~ LEZ Centro. The ~~population that is accessing~~ populations with access to those jobs by mode (Figure 3) ~~, appear~~ are spatially distinct. ~~Car and transit trips~~ Travel by car and transit represent 37% and 47% of the modal share respectively. The population that travels ~~using~~ by transit is more spatially distributed than those using cars - particularly near and within LEZ Centro. This distribution ~~could be a result of a~~ is likely caused by a variety of factors including: transit coverage and service within with city, effective car infrastructure outside of the M-30, and/or the impact of the ~~Central LEZ itself.~~

LEZ Centro itself. From Figure 3, it can ~~also~~ be seen that ~~biking and walking trips are~~ active travel is less common than motorized trips at 1% and 15% ~~respectively. Noteably~~for cycling and walking respectively. Noticeably, there is a positive trend between the ~~populations of walking and biking trips in zones and populations of~~

~~transit trips~~walking and cycling in zones where transit is also present. This positive trend is higher than for car trip populations.

~~The travel time for each trip is~~ Travel times are provided within the ~~survey. These travel times, per modal category, are~~ travel survey by mode. This information is used to calibrate mode-specific travel impedance functions $f^m(c_{ij}^m)$. To illustrate the modal differences in travel ~~lengths, summary descriptive~~ times, the following descriptive statistics per mode are ~~detailed~~presented:

- Car/motor: mean 36 ~~min~~ minutes (min: 0 ~~min.~~minutes, Q2: 15 ~~min.~~minutes, Q3: 55 ~~min.~~minutes, max: 120 ~~min.~~minutes)
- Transit: mean 55 ~~min.~~ minutes (min: 1 ~~min.~~minutes, Q2: 30 ~~min.~~minutes, Q3: 80 ~~min.~~minutes, max: 120 ~~min.~~minutes)
- Bike: mean 34 ~~min.~~ minutes (min: 5 ~~min.~~minutes, Q2: 15 ~~min.~~minutes, Q3: 40 ~~min.~~minutes, max: 115 ~~min.~~minutes)
- Walk: mean 27 ~~min.~~ minutes (min: 1 ~~min.~~minutes, Q2: 10 ~~min.~~minutes, Q3: 45 ~~min.~~minutes, max: 119 ~~min.~~minutes)

~~To calculate~~ Impedance functions $f^m(c_{ij}^m)$ ~~from the survey travel times , a concept known as the~~ are calibrated from the travel times in the survey via the empirical trip length distribution (TLD)~~was used. A TLD represents~~. An empirical TLD is given by the proportion of trips ~~that are taken at a specific travel cost such as travel time (i.e., probability density distribution of trips taken by travel cost)~~at various travel cost bins. This distribution is then used to ~~derive impedance functions (e.g., done in the accessibility works of~~ estimate the parameters of a function for the travel impedance [~~27~~as done in 43,44,45]~~,28,and 29).~~. To fit the impedance functions, we use the Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the R {fitdistrplus} package [~~30~~46]~~is used to fit the impedance functions. As shown as shown in Figure 4, based~~. Based on goodness-of-fit criteria and associated diagnostics, the gamma and log-normal probability density ~~function (line curves)~~ functions are selected as best fitting curves for the motorized and non-motorized modes respectively. The selection of functional ~~form~~ forms aligns with empirical examples in other regions [~~14,31,32~~15,47,48]. ~~Overall, the plots in Figure 4 display the probability of travel~~ The shape and rate parameters for the gamma functions (motorized modes) are 1.8651852 and 0.051468 for car/motor and 2.7566235 and 0.0499193 for transit; for the log-normal functions (non-motorized modes), the mean and standard deviation parameters are 3.2372212 and 0.7575986 for bike and 2.9918042 and 0.7575986 for walk.

Figure 4 includes four plots to visualize the calibrated impedance functions (represented as black lines) superimposed on the empirical TLD. The impedance functions can be interpreted as the propensity to travel (y-axis) given a trip travel time ~~, based on trip flows from the survey. These 'probability of travel' at each travel time for each mode are realized observations that reflect the land-use,~~ (x-axis). The functions reflect a combination of possibilities and preferences: the travel behavior given the transportation technologies available. For example, trips shorter than 5 minutes do not occur frequently for any mode; this reflects the spatial separation between places of residence and places of work commonly seen in many cities. In terms of the ~~transport system, and~~ non-motorized modes, there is a preference towards walking trips around 15 minutes in duration, as seen from the highest value of $f^{walk}(c_{ij}^{walk})$. With respect to travel by bicycle, longer travel times are more common; although the highest value of the impedance also corresponds to approximately 15 minutes, the curve has a longer tail and values decrease less rapidly at longer travel times than is the case of $f^{walk}(c_{ij}^{walk})$. A similar trend can be observed for the ~~population travel behaviour in Madrid~~motorized modal options where transit mode is

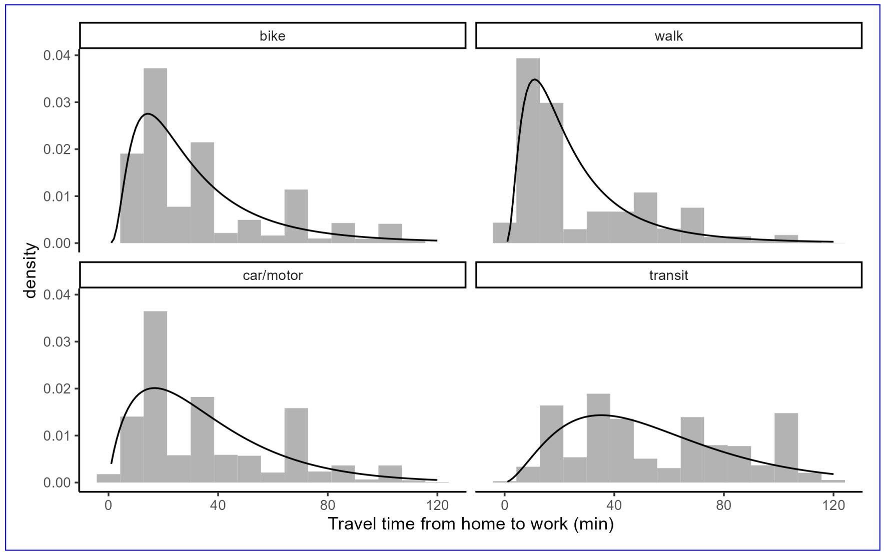

Fig 4. Fitted impedance function ~~curve (line)~~ against empirical TLD (bars) corresponding to the ~~home-to-work origin-destination~~ home to full-time work origin destination flows ~~from~~ for the City of Madrid from the 2018 travel survey.

more spread out than car/motor mode. All in all, these functions represent the propensity of travel by mode by duration of trip, and are used to calculate the proportional allocation factors F_{ij}^m for V_i^m.

Results

At this point, it is worthwhile reiterating that the empirical example is a snap-shot of spatial availability by mode using data from the 2018 travel survey. Our purpose in this empirical example is to investigate the trends in availability of employment opportunities by mode, and illustrate how spatial availability can be used in discussions about the competitive advantage of various modes within Madrid Centro. The spatial availability of jobs V_i^m is calculated for each of the four ~~modal categories~~ modes m at the level of traffic analysis zones i in Madrid and ~~demonstrated~~ displayed in Figure 5.

In the figure, V_i^m is a proportion of the total number of ~~the~~ 847,574 jobs in the region~~and is visualized in Figure 5~~. Since V_i^m is calculated based on the ~~likelihood of travel from observed home-to-work journeys~~population of workers and the distribution of jobs, the values can be understood as the number of full-time jobs that are spatially available to ~~the~~ full-time ~~working population~~ workers at that i ~~and their associated~~ traveling by mode m, relative to all the jobs in the city. ~~V_i^m is the number of jobs that are *spatially available* to a m-using population located at i, relative to the travel impedance and size of *all* populations in the region.~~

~~Notable are the~~ There are noticeable differences in the magnitude of V_i^m between modes as seen in Figure 5. The majority of V_i^m (which is to say of spatially available jobs) are allocated to workers travelling by car and transit. In a way, this is ~~allocated~~ to ~~car- and transit- using populations. This is to~~ be expected ~~, as the population that commutes using these modes represents~~ since users of these modes represent 84.1% of the total population. ~~Differences~~ However, the ability to travel at greater speeds also impacts these results. Furthermore, differences in V_i^m values within ~~mode-using populations also exist : car-using populations~~ modes also exist in space: car users outside of the M-30 region appear to ~~have greater V_i^m values~~enjoy greater spatial

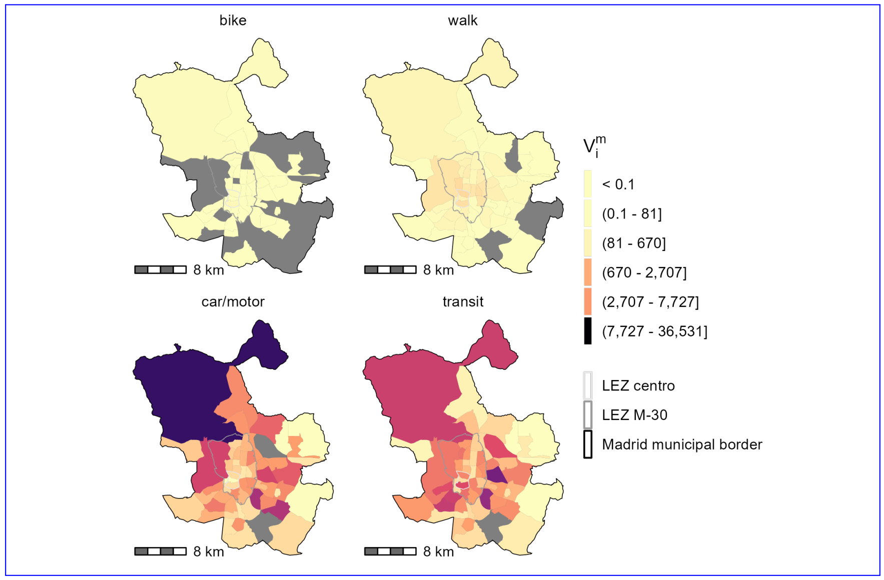

Fig 5. Spatial availability of ~~job opportunities~~ jobs per origin and mode V_i^m in Madrid. ~~Calculated using~~ Grey TAZs have no population. Ranges of values in the ~~home-to-work origin-destination flows from the 2018 travel survey~~legend are quintiles.

availability, while some ~~i areas~~ zones inside the M-30 ~~appear to have higher V_i^m values for the transit-using populations~~to have greater spatial availability for transit. Overall, the magnitude of V_i^m values for ~~the bikers and walkers~~ cyclists and pedestrians are lower than for car and transit but the highest values of V_i^{bike} and V_i^{walk} ~~values~~ tend to be ~~allotted to~~ ~~is~~ found in zones within the M-30 and ~~is that have higher $V_i^{transit}$ values~~origins with higher spatial availability by transit.

The differences between the ~~mode-using population and their mode-specific spatial availability~~ shares of modes and their shares of spatially available opportunities highlights the competitive advantage ~~offered to certain modesin certain spatial extents. As summarized~~ of certain modes, although this effect is not geographically uniform. As seen in the left-most columns ~~in~~ of Figure 6, ~~the~~ users of 'car/motor' and 'transit' ~~populations represent a combined~~ together can avail 95.3% of ~~the total spatial availability~~ all jobs in the city (Spatial Availability by Mode). However, ~~the '~~car/motor ~~' using population is allocated disproportionately more~~ users have a disproportionate share of V_i^m ~~than its size compared to the transit-using population . The car-using and transit-using population~~ relative to the population of users of this mode (Population by Mode), compared to opportunities that are spatially available to transit users. The combined population of car and transit users is 36.6% and 47.5% respectively, but ~~is~~ these populations are allocated 48.0% and 47.3% respectively, of the city's ~~spatial availability~~jobs. When treating the number of opportunities that can be reached as a finite value (total: 847,574 opportunities), fewer opportunities are ~~spatial availability to the lesser competitive modes-using populations, in this case~~ spatially available to slower modes (i.e., walking and cycling), even taking into account that their share is smaller overall. These modes are ~~less competitive~~ at a disadvantage as a result of: ~~their lower travel impedance values at longer travel times~~ the travel impedance for longer trips (see Figure 4~~at travel times beyond ~30 minutes~~); their low population values values overall; and ~~higher populations present~~ larger populations in origins with high ~~motorized mode commuting~~shares of travel by motorized modes. These factors all contribute to the the car/motor mode being most advantaged in capturing spatially available job opportunities overall.

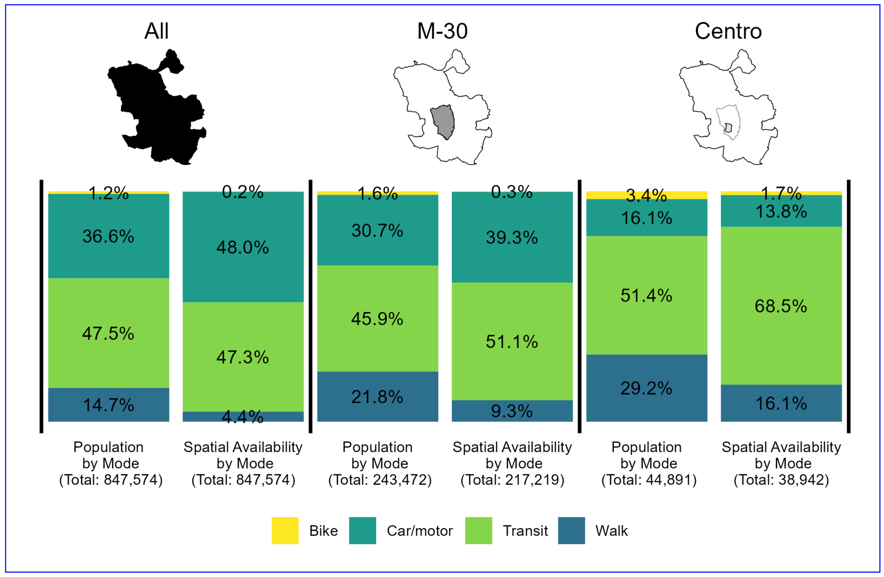

Fig 6. ~~Displays the proportion~~ Proportion of ~~the working~~ population by mode and spatial availability of ~~job opportunities~~ jobs by mode aggregated for three ~~spatial~~ areas. From left to right, the city of Madrid (All), the area within the M-30 highway (M-30), the area within the Centro region (Centro).

~~There are spatial variations in the competitive advantage of the car-using populations. The proportion of car-using population in the Centro is smaller and has higher travel impedance values relative to the inputs in other areas and mode-using populations. The LEZ Centro implementation further restricts the car-advantage as it shifted more than half of all car trips into the LEZ to another mode 25. This restriction decreased the number of car-using population from *is* going into the LEZ Centro (an area with~~ The big picture demonstrates how car is the most advantageous mode, however it is interesting to notice that this advantage appears to be blunted by the LEZ. Unlike the results for the whole city and even within the M-30 ring, the proportion of car users in Centro is larger than the proportion of opportunities spatially available to them. The restriction on cars in effect reduces competition by this mode, and leads to a relative increment of the mass effect of the modes allowed within the LEZ, which also contains a large number of jobs ~~overall,~~ (see Figure 2)~~, thus increasing the mass effect for non-car modes and resulting in proportionally higher~~ ~~v_i^m~~ ~~values for non-car modes. As such, the lower amount of access to opportunities by car-mode allows more opportunities in the LEZ to be available by populations using other modes.~~

. As summarized in the two right-most columns in Figure 6, the proportion of ~~spatial availability allocated to the car-using population in the Centro~~ jobs spatially available to car in Centro is (13.8% or 5,373 opportunities). ~~As a comparative~~ For reference, this is less than the proportion of the ~~car-using population in the~~ car users in Centro (16.1%), evidently less then the proportion of ~~car-using population~~ car users in the city, and is the opposite of the ~~trend overall~~ overall trend (left-most columns) and within the M-30 (middle columns). ~~More opportunities are spatial availability~~

It is also clear that more opportunities are spatially available to non-car ~~using populations within the Centro, particularly transit-using populations (68.5% of spatially available jobs in the Centro despite representing 51.4% of the population in the Centro and 47.5% in the cityoverall).~~

~~From~~ users within Centro. In the case of active travel, the proportions of cyclists and walkers within LEZ Centro still exceeds the proportions of jobs spatially available to them — however, the disparity is drastically reduced compared to the rest of the city. As seen in Figure 6, ~~it is also summarized that~~ there is a higher proportion of opportunities that are spatially available to ~~walking and cycling populations in the~~ pedestrians and cyclists in Centro than in the City overall and in ~~all~~ areas within the M-30. Notably, within ~~the~~ Centro, 1.7% and 16.1% of opportunities are spatially available to bike and walk modes respectively, while their populations represent ~~smaller proportions of~~ 1.2% and 14.7% of the population~~overall. Though the proportion of spatial availability for these mode-using populations is still lower than the proportion of mode-using population located in the Centro, these modes are more competitive within the Centro than outside of the Centro~~. ~~By restricting the more competitive carmode through the LEZ, the advantage in the spatial availability of opportunities afforded to the otherwise lesser competitive modes is made apparent~~By restricting the ability of cars to enter Centro, the LEZ seems to contribute to leveling the playing field for slower modes, in particular cycling and walking, but also transit. As seen in the Figure 6, transit users are generally close to parity across the region, with nearly as many spatially available jobs as transit users. Still, this mode has the greatest advantage in LEZ Centro with 68.5% of spatially available jobs in Centro for 51.4% of transit users in Centro. This result makes intuitive sense: after car, it is the mode with the greatest range, and unlike car it is unrestricted in the LEZ Centro.

The spatial differences in the competitive dis/advantage of spatial availability between modes can also be visualized ~~per origin~~at a finer level of granularity. Figure 7 ~~visualizes~~ shows v_i^m, the spatial availability V_i^m divided by the ~~mode-population.~~ population of users of m. Values of v_i^m ~~values above 1 are represented in increasing red shades , values below 1 are represented in increasingly green shades , and values equal to 1 are white. These plots illustrates the discussion of the disproportionately high over-allocation of spatial availability relative to the mode-using population in many of the origins for the car/motor mode~~ below one are shown in shades of orange, and indicate TAZs with less than one spatially available opportunity per capita for the mode. Values above one are shown in shades of green, and indicate TAZs with more than one spatially available opportunity per capita for the mode. The highest spatial availability per capita (shown in blue) is for car users in a zone northeast just beyond the M-30. These plots illustrate in unambiguous fashion, and in a quantity that is comparable over space and time, the advantage in terms of spatial availability of car for most of the city (bottom left plot, areas denoted with green v_i^m values above ~~1). These plots also visualize areas that disproportionately capture lower spatial availability (under 1), represented in shades of red. It can~~ one). It can also be observed that ~~the transit-using population's spatial availability to~~ spatial availability of jobs is relatively ~~balanced~~ well balanced for transit users over most of the regions (i.e., many zones are ~~white), while the~~ light orange or light green). Spatial availability of jobs for non-motorized modes~~v_i^m values are~~, in contrast, is low (under ~~1) overall~~one) overall, although less so within LEZ Centro.

~~Interestingly, as also represented in Figure 6,~~ Incidentally, v_i^m ~~for car/motor~~ values for car within and near ~~the~~ LEZ Centro is ~~near or below 1 (white/red)~~ close to or below one in Figure 7, while all non-car modes have relatively higher v_i^m values. ~~Though the spatial availability from before the LEZ Centro implementation is unknown~~Since these values are comparable across regions and over time, Figure 7 potentially provides a benchmark for quantifying ~~potential LEZ implementations~~ changes in LEZ policies in the future~~(given 2018 travel conditions).~~. As Figure 7 also shows~~that~~, many areas within the M-30 have high (white/green) v_i^m values for ~~car-mode, signaling that the~~ car, but the results for LEZ Centro give reasonable grounds to speculate that a spatial expansion of the LEZ ~~Centro stands to~~ to include

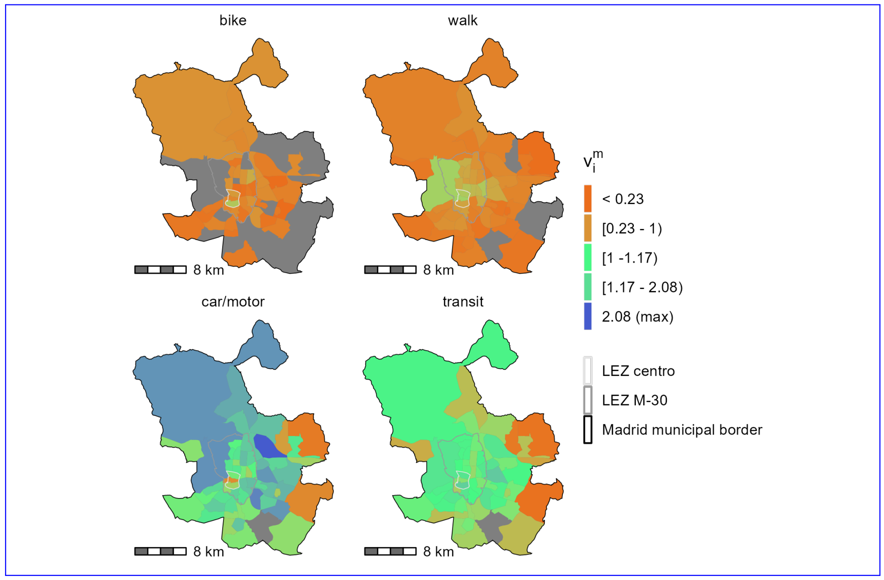

Fig 7. ~~Spatial availability~~ Distribution of ~~job opportunities~~ spatially available jobs per ~~mode-using~~ capita by mode of transportatin (v_i^m ~~per origin in Madrid~~). ~~Calculated using~~ Grey TAZs have no population that use the ~~home-to-work origin destination flows from~~ mode. Ranges of values in the ~~2018 travel survey~~legend are quintiles.

all areas within the M-30 would likely increase the spatial availability of jobs for ~~non-car mode using populations~~transit users, cyclists and pedestrians.

Discussion and conclusions

~~Location-based accessibility measures like the~~ Accessibility measures are an important tool in transportation research [9] and are increasingly seen as valuable for planning purposes [4–8]. They boast a long history of development, beginning with Hansen-type S_i^m ~~, Shen-type~~ measures, with other developments like Shen's a_i^m, ~~and spatial availability~~ to account for competition/congestion. The more recent spatial availability measure V_i^m ~~measures share a commonality; they are a~~ has in common with these accessibility indicators that it is a weighted sum of ~~opportunities assigned to each spatial unit~~ ~~i~~ the opportunities in a region ~~.~~ from the perspective of a determined origin i. Aggregations of opportunities embody principles of gravitational/spatial interaction modelling that date back to at least H.C. Carey [49], and are part of a line of research that includes the work of Ravenstein [50], Reilly [51], Stewart [52–54], Zipf [55,56], Wilson [37], and many others. In this way, ~~they all~~ S_i^m, a_i^m, and yes, V_i^m, can be interpreted as ~~a score of how many opportunities can be potentially interacted with by the population at~~ ~~i~~~~. How the weight and sum of the potentially-interacted-with opportunitiesis considered is what defines the type of accessibility measure.~~ scores of the potential for interaction with opportunities in space.

~~Within this paper, the location-based singly-~~ ~~*constrained*~~ ~~and~~ ~~*competitive*~~ ~~accessibility measure , known as spatial availability~~ ~~V_i~~ Different accessibility indicators are characterized by how they weight and aggregate opportunities. Spatial availability's contribution to the literature is to incorporate a proportional allocation mechanism that essentially constrains the sums to match the number of opportunities in the region; in this way it is a singly-constrained accessibility measure that naturally accommodates congestion and competition. The effort with spatial availability is in line with previous research on proportional allocation by Paez et al. [14]~~, is extended~~. As initially introduced by Soukhov et al. [15], spatial availability was designed for a homogeneous population traveling by a single mode of transportation. In this paper, we extended spatial availability for the case of ~~capturing multimodal accessibility to opportunities V_i^m. A synthetic example and then an empirical case of LEZ in Madrid are detailed to demonstrate this multimodal extension.~~ heterogeneous populations. We discussed this in terms of multiple modes of transportation, but the framework can accommodate equally well variations in travel behavior by population segments.

~~The spatial availability measure is capable of capturing a new interpretation of multimodal competition that previous accessibility measures have not yet done. Competitive measures hypothesis that populations using modes with lower travel impedance, when competing for a finite set of opportunities , will capture more opportunities. With spatial availability , the~~ An empirical example using data from Madrid helped to illustrate the potential of multimodal spatial availability analysis, including its ability to account for competition for opportunities within and between modes. Particularly relevant is the fact that spatial availability scores relate directly to the total number of opportunities ~~that are captured (of the total opportunities~~ in the region~~) by each mode can be individually calculated. From there, the difference between how many spatially available opportunities one mode captures versus another can be investigated. This is the advantage of the spatial availability measure, particularly its multimodal extension.~~

~~The flexibility and need for an accessibility measure such as spatial availability is pertinent in policy scenario evaluation. As showcased in the empirical example of the LEZ in Madrid, competition for job opportunity availability varies spatially *as well as* between modes. The car and transit modes have the highest spatial availability, with the car-mode having highest availability with exception to the areas within the LEZ Centro. Since car travel has been highly restricted within the LEZ Centro, fewer car-using people potentially interact with jobs within the LEZ Centro, leaving more *spatially available* jobs for non-car-using populations~~. This makes it possible to compare the results to intuitive benchmarks, such as opportunities per population, in ways that other accessibility measures cannot or tend to obfuscate. This comparability is preserved between regions and over time. The example suggests that once that opportunities are treated as being finite, restrictions to travel by car leave more spatially available opportunities for non-car-users. This difference ~~in car-using populations in locations accessing jobs~~ for car travel in locations within and immediately ~~outside~~ around the LEZ Centro ~~increases the competitiveness of~~ seems to increase the number of opportunities spatially available to transit users (transit being the ~~transit-using population (the~~ second most competitive mode), as well as the spatial availability from the perspective of non-motorized modes. In effect, a policy such as Low Emission Zones help to improve the accessibility situation of active travel and transit in the parts of the city where it is implemented.

~~Spatial availability V_i^m can also be divided by the mode-using population at each i to yield mode-population normalized values. These values, reflected in Figure 7, can be used as a benchmark to investigate existing conditions and plan future LEZ implementation (i.e., target areas with exceptionally high car spatial availability such that more opportunities are available to other mode-users).~~ The purpose of the empirical example is to illustrate the kind of insights that can be derived from the application of multimodal spatial availability. But there are some intriguing opportunities for future research. Accessibility indicators are not designed to work as modal split models, and yet, in the case of policies that alter the relative cost of various forms of transportation, one can reasonably expect to see some shifts between modes. In our empirical example we used data collected *after* the introduction of LEZ Centro.

However, given a modal split model to project model shares, accessibility indicators, including spatial availability, can be used to investigate changes to the accessibility landscape. Ditto for destination choice. Our empirical example presented but a snapshot of this, and in future research it will be interesting to investigate changes *between* policy interventions. The expansion of Madrid's LEZ to the ring contained by the M-30 orbital presents an excellent opportunity to do so. Given the intuitive and straightforward interpretation of spatial availability scores as fractions of opportunities from the total, relative and absolute changes in the accessibility landscape can be assessed, thus helping to evaluate the implications of policy interventions.

~~In summary, conventional *non-constrained* accessibility measures are difficult for planners to operationalize for a variety of reasons including issues of computation and interpretability~~ Finally, our example dealt with differences in travel by mode only, but it is possible to think of the intersection between mode of travel and different types of travelers. This would expand the number of sub-populations in the analysis from, say, $m = M$ (modes) to $m = M \cdot Q$ (modes times population segments), each with their own characteristic impedance function. Evaluations of this kind will be especially relevant as LEZ are implemented in cities globally, and the question of their impact on disadvantaged populations who have become mobility-restricted increasingly come to the fore [~~2~~40,57,58]~~. With spatial availability, the magnitude of opportunities that are available as a proportion of all the opportunities in the region is equal to V_i. As a result of its proportional allocation mechanism, V_i can be naturally extended into multimodal applications. This flexibility is helpful to modelling policy scenarios in our cities that are increasingly multimodal. The interpretation of V_i allows for manipulation of V_i^m values to investigate differences of availability between neighbourhoods, modes, and regions, generate per capita benchmarks, and/or generate average values per population-group~~.

~~From a spatial equity perspective, spatial availability measure can provide researchers, policy makers, and citizens a new-found interpretation of accessibility measures. With a plot of spatial availability values, one can begin asking, how much is enough and what level may be too much. These interpretations were difficult to be made with accessibility measures in the past.~~

Acknowledgements

This research was funded by the Canada Graduate Scholarship - Doctoral Program (CGS D) provided by the Social Sciences and Humanities Research Council (SSHRC) and Project Mobilizing Justice, also supported by SSHRC. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

All work is fully-reproducible and available within this GitHub repository.

Author contributions

The authors confirm contribution to the paper as follows: study conception and design: AS, JTO, JSL, AP.; data collection: AS, JTO, JSL.; analysis and interpretation of results: AS, JTO, JSL, AP.; draft manuscript preparation: AS, JSL, AP. All authors reviewed the results and approved the final version of the manuscript.

References

1. Hansen WG. How accessibility shapes land use. Journal of the American Institute of Planners. 1959;25: 73–76. doi:10.1080/01944365908978307
2. Geurs KT, Wee B van. Accessibility evaluation of land-use and transport strategies: Review and research directions. Journal of Transport Geography. 2004;12: 127–140. Available: http://www.sciencedirect.com/science/article/B6VG8-4B28VY7-1/2/61478339c14cab4fa58438ad7b1f4610 C:/Papers/Journal of Transport Geography/Journal of Transport Geography (2004) 12 (2) 127-140.pdf
3. Paez A, Scott DM, Morency C. Measuring accessibility: Positive and normative implementations of various accessibility indicators. Journal of Transport Geography. 2012;25: 141–153. doi:10.1016/j.jtrangeo.2012.03.016
4. Handy S. Is accessibility an idea whose time has finally come? Transportation Research Part D: Transport and Environment. 2020;83: 102319. doi:`https://doi.org/10.1016/j.trd.2020.102319`
5. Levinson D, King D. Transport access manual: A guide for measuring connection between people and places. University of Sydney; 2020. Available: `https://ses.library.usyd.edu.au/handle/2123/23733`
6. Siddiq F, D. Taylor B. Tools of the trade?: Assessing the progress of accessibility measures for planning practice. Journal of the American Planning Association. 2021;87: 497–511. doi:10.1080/01944363.2021.1899036
7. Yan X. Toward accessibility-based planning addressing the myth of travel cost savings. JOURNAL OF THE AMERICAN PLANNING ASSOCIATION. 2021;87: 409–423. doi:10.1080/01944363.2020.1850321
8. El-Geneidy A, Levinson D. Making accessibility work in practice. Transport Reviews. 2022;42: 129–133. doi:10.1080/01441647.2021.1975954
9. Shi Y, Blainey S, Sun C, Jing P. A literature review on accessibility using bibliometric analysis techniques. Journal of Transport Geography. 2020;87: 102810. doi:10.1016/j.jtrangeo.2020.102810
10. Handy S, Niemeier D. Measuring accessibility: An exploration of issues and alternatives. Environment and Planning A. 1997;29: 1175–1194.
11. Kwan MP. Space-time and integral measures of individual accessibility: A comparative analysis using a point-based framework. Geographical Analysis. 1998;30: 191–216. Available: ISI:000074579200001 C:/Papers/Geographical Analysis/Geographical Analysis (1998) 30 (3) 191-216.pdf
12. Shen Q. Location characteristics of inner-city neighborhoods and employment accessibility of low-wage workers. Environ Plann B. 1998;25: 345–365. doi:10.1068/b250345
13. Luo W, Wang F. Measures of spatial accessibility to health care in a GIS environment: Synthesis and a case study in the chicago region. Environ Plann B Plann Des. 2003;30: 865–884. doi:10.1068/b29120
14. Paez A, Higgins CD, Vivona SF. Demand and level of service inflation in floating catchment area (FCA) methods. PloS one. 2019;14: e0218773. doi:10.1371/journal.pone.0218773
15. Soukhov A, Paez A, Higgins CD, Mohamed M. Introducing spatial availability, a singly-constrained measure of competitive accessibility | PLOS ONE. PLOS ONE. 2023; 1–30. doi:https:// doi.org/10.1371/journal.pone.0278468

16. Páez A, Mercado R, Farber S, Morency C, Roorda M. Accessibility to health care facilities in montreal island: An application of relative accessibility indicators from the perspective of senior and non-senior residents. International Journal of Health Geographics. 2010;9: 1–9.
17. Moniruzzaman M, Páez A, Nurul Habib KM, Morency C. Mode use and trip length of seniors in montreal. Journal of Transport Geography. 2013;30: 89–99. doi:`http://dx.doi.org/10.1016/j.jtrangeo.2013.03.007`
18. Reyes M, Paez A, Morency C. Walking accessibility to urban parks by children: A case study of montreal. Landscape and Urban Planning. 2014;125: 38–47. doi:10.1016/j.landurbplan.2014.02.002
19. Páez A, Farber S, Mercado R, Roorda M, Morency C. Jobs and the single parent: An analysis of accessibility to employment in toronto. Urban Geography. 2013;34: 815–842. doi:10.1080/02723638.2013.778600
20. Wu H, Levinson D. Unifying access. Transportation Research Part D: Transport and Environment. 2020;83: 102355. doi:10.1016/j.trd.2020.102355
21. Tahmasbi B, Mansourianfar MH, Haghshenas H, Kim I. Multimodal accessibility-based equity assessment of urban public facilities distribution. Sustainable Cities and Society. 2019;49: 101633. doi:10.1016/j.scs.2019.101633
22. Lunke EB. Modal accessibility disparities and transport poverty in the oslo region. Transportation Research Part D: Transport and Environment. 2022;103: 103171. doi:10.1016/j.trd.2022.103171
23. Paez A, Mercado RG, Farber S, Morency C, Roorda M. Relative accessibility deprivation indicators for urban settings: Definitions and application to food deserts in montreal. Urban Studies. 2010;47: 1415–1438. doi:10.1177/0042098009353626
24. Campbell KB, Rising JA, Klopp JM, Mbilo JM. Accessibility across transport modes and residential developments in nairobi. Journal of Transport Geography. 2019;74: 77–90.
25. Maharjan S, Tilahun N, Ermagun A. Spatial equity of modal access gap to multiple destination types across chicago. Journal of Transport Geography. 2022;104: 103437. doi:10.1016/j.jtrangeo.2022.103437
26. Grengs J. Job accessibility and the modal mismatch in detroit. Journal of Transport Geography. 2010;18: 42–54. doi:10.1016/j.jtrangeo.2009.01.012

pre

27. Kawabata M, Shen Q. Job accessibility as an indicator of auto-oriented urban structure: A comparison of boston and los angeles with tokyo. Environ Plann B Plann Des. 2006;33: 115–130. doi:10.1068/b31144

pre

28. Kwok RCW, Yeh AGO. The use of modal accessibility gap as an indicator for sustainable transport development. Environ Plan A. 2004;36: 921–936. doi:10.1068/a3673

pre

29. Morris JM, Dumble PL, Wigan MR. Accessibility indicators for transport planning. Transportation Research Part A: General. 1979;13: 91–109. doi:10.1016/0191-2607(79)90012-8

pre

30. Weibull JW. An axiomatic approach to the measurement of accessibility. Regional Science and Urban Economics. 1976;6: 357–379. doi:10.1016/0166-0462(76)90031-4

pre

31. Liotta C, Kervinio Y, Levrel H, Tardieu L. Planning for environmental justice - reducing well-being inequalities through urban greening. Environmental Science & Policy. 2020;112: 47–60. doi:https://doi.org/10.1016/j.envsci.2020.03.017
pre

32. OECD. Frameworks and sector policies for urban development in chile. OECD urban policy reviews, chile 2013. 2013. doi:http://dx.doi.org/10.1787/9789264191808-en
pre

33. Lara-Valencia F, García-Pérez H. Space for equity: Socioeconomic variations in the provision of public parks in hermosillo, mexico. Local Environment. 2015;20: 350–368. doi:10.1080/13549839.2013.857647
pre

34. Martens K, Golub A. A fair distribution of accessibility: Interpreting civil rights regulations for regional transportation plans. Journal of Planning Education and Research. 2021;41: 425–444. doi:10.1177/0739456x18791014
pre

35. Joseph AE, Bantock PR. Measuring potential physical accessibility to general practitioners in rural areas: A method and case study. Social Science & Medicine. 1982;16: 85–90. doi:10.1016/0277-9536(82)90428-2
pre

36. Tao Z, Zhou J, Lin X, Chao H, Li G. Investigating the impacts of public transport on job accessibility in Shenzhen, China: A multi-modal approach. Land Use Policy. 2020;99: 105025. doi:10.1016/j.landusepol.2020.105025
pre

37. Wilson AG. A family of spatial interaction models, and associated developments. Environment and Planning A. 1971;3: 1–32.
pre

38. Carpentieri G, Guida C, Masoumi HE. Multimodal Accessibility to Primary Health Services for the Elderly: A Case Study of Naples, Italy. Sustainability. 2020;12: 781. doi:10.3390/su12030781
pre

39. Margaryan S. Low emission zones and population health. Journal of Health Economics. 2021;76: 102402. doi:10.1016/j.jhealeco.2020.102402
pre

40. Verbeek T, Hincks S. The “just” management of urban air pollution? A geospatial analysis of low emission zones in brussels and london. Applied Geography. 2022;140: 102642. doi:10.1016/j.apgeog.2022.102642
pre

41. Tarriño-Ortiz J, Gómez J, Soria-Lara JA, Vassallo JM. Analyzing the impact of low emission zones on modal shift. Sustainable Cities and Society. 2022;77: 103562. doi:10.1016/j.scs.2021.103562
pre

42. Comunidad de Madrid. Resultados de la EDM 2018 - Datos Abiertos. 2020 [cited 31 Jul 2023]. Available: https://datos.comunidad.madrid/catalogo/dataset/resultados-edm2018

43. Lopez FA, Paez A. Spatial clustering of high-tech manufacturing and knowledge-intensive service firms in the greater toronto area. Canadian Geographer-Geographe Canadien. 2017;61: 240–252. doi:10.1111/cag.12326

44. Horbachov P, Svichynskyi S. Theoretical substantiation of trip length distribution for home-based work trips in urban transit systems. Journal of Transport and Land Use. 2018;11: 593–632. Available: https://www.jstor.org/stable/26622420

45. Batista SFA, Leclercq L, Geroliminis N. Estimation of regional trip length distributions for the calibration of the aggregated network traffic models. Transportation Research Part B: Methodological. 2019;122: 192–217. doi:10.1016/j.trb.2019.02.009

46. Delignette-Muller ML, Dutang C. fitdistrplus: An R package for fitting distributions. Journal of Statistical Software. 2015;64: 1–34. Available: `https://www.jstatsoft.org/article/view/v064i04`

47. Reggiani A, Bucci P, Russo G. Accessibility and impedance forms: Empirical applications to the german commuting network. International Regional Science Review. 2011;34: 230–252. doi:10.1177/0160017610387296

48. Soukhov A, Páez A. TTS2016R: A data set to study population and employment patterns from the 2016 transportation tomorrow survey in the greater golden horseshoe area, ontario, canada. Environment and Planning B: Urban Analytics and City Science. 2023; 23998083221146781. doi:10.1177/23998083221146781

49. Carey HC. Principles of social science. Philadelphia: J.B. Lippincot; Co.; 1858.

50. Ravenstein EG. The laws of migration. Journal of the Royal Statistical Society. 1889;52: 241–305. doi:10.2307/2979333

51. Reilly WJ. Methods for the study of retail relationships. 1929.

52. Stewart JQ. An inverse distance variation for certain social influences. Science. 1941;93: 89–90. Available: `http://www.jstor.org/stable/1668130`

53. Stewart JQ. Suggested principles of "social physics". Science. 1947;106: 179–180. Available: `http://www.jstor.org/stable/1675368`

54. Stewart JQ. Demographic gravitation: Evidence and applications. Sociometry. 1948;11: 31–58. doi:10.2307/2785468

55. Zipf GK. The p 1 p 2/d hypothesis: The case of railway express. The Journal of Psychology. 1946;22: 3–8. doi:10.1080/00223980.1946.9917292

56. Zipf GK. The P1P2/d hypothesis: On the intercity movement of persons. American Sociological Review. 1946;11: 677–686. doi:10.2307/2087063

57. De Vrij E, Vanoutrive T. "No-one visits me anymore": Low emission zones and social exclusion via sustainable transport policy. 2022 [cited 27 Jul 2023]. Available: `https://www.tandfonline.com/doi/epdf/10.1080/1523908X.2021.2022465?needAccess=true&role=button`

58. Liotta C. What drives inequalities in low emission zones'impacts on job accessibility? Virtual; 2023. doi:10.17605/OSF.IO/9VQU7