Introducing spatial availability, a singly-constrained measure of competitive accessibility

Abstract

Accessibility indicators are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability, mainly by introducing competition. In this paper, we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

1. Introduction

The concept of accessibility in transportation studies derives its appeal from the combination of the spatial distribution of opportunities and the cost of reaching them (Handy and Niemeier, 1997; Hansen, 1959). Accessibility analysis is employed in transportation, geography, public health, and many other areas, with the number of applications growing (Shi et al., 2020), especially as mobility-based planning is de-emphasized in favor of access-oriented planning (Deboosere et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021).

Accessibility analysis stems from the foundational works of Harris (1954) and Hansen (1959). From these seminal efforts, many accessibility measures have been derived, particularly after the influential work of Wilson (1971) on spatial interaction[1]. Of these, gravity-type accessibility is arguably the most common; since its introduction in the literature, it has been widely adopted in numerous forms (Arranz-López et al., 2019; Cervero et al., 2002; Geurs and van Wee, 2004; Levinson, 1998; Paez, 2004). Hanson-type accessibility indicators are essentially weighted sums of opportunities, with the weights given by an impedance function that depends on the cost of movement, and thus measure the *intensity of the possibility of interaction* (Hansen, 1959). This type of accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure (Handy and Niemeier, 1997).

Despite their usefulness, the interpretability of Hansen-type accessibility measures can be challenging (Geurs and van Wee, 2004; Miller, 2018). Since they aggregate opportunities, the results are sensitive to the size of the region of interest (e.g., a large city has more jobs than a smaller city). As a consequence, raw outputs are not necessarily comparable across study areas (Allen and Farber, 2019). This limitation becomes evident when surveying studies that implement this type of analysis. For example, Páez et al. (2010) (in Montreal) and Campbell et al. (2019) (in Nairobi) report accessibility as the number of health care facilities that can potentially be reached from origins. But what does it mean for a zone to have accessibility to less than 100 facilities in each of these two cities, with their different populations and number of facilities? For that matter, what does it mean for a zone to have accessibility to more than 700 facilities in Montreal, besides being "accessibility rich"? As another example, Bocarejo S. and Oviedo H. (2012) (in Bogota), El-Geneidy et al. (2016) (in Montreal), and Jiang and Levinson (2016) (in Beijing) report accessibility as numbers of jobs, with accessibility values often in the hundreds of thousands, and even exceeding one million jobs for some zones in Beijing and Montreal. As indicators of urban structure, these measures are informative, but the meaning of one million accessible jobs is harder to pin down: how many jobs must any single person have access to? Clearly, the answer to this question depends on how many people demand jobs.

[1] Utility-based measures derive from a very different theoretical framework, random utility maximization

The interpretability of Hansen-type accessibility has been discussed in numerous studies, including recently by Hu and Downs (2019), Kelobonye et al. (2020), and in greater depth by Merlin and Hu (2017). As hinted above, the limitations in interpretability are frequently caused by ignoring competition - without competition, each opportunity is assumed to be equally available to every single opportunity-seeking individual that can reach it (Kelobonye et al., 2020; Paez et al., 2019; Shen, 1998). This assumption is appropriate when the opportunity of interest is non-exclusive, that is, if use by one unit of population does not preclude use by another. For instance, national parks with abundant space are seldom used to full capacity, so the presence of some population does not exclude use by others. When it comes to exclusive opportunities, or when operations may be affected by congestion, the solution has been to account for competition. Several efforts exist that do so. In our reckoning, the first such approach was proposed by Weibull (1976), whereby the distance decay of the supply of employment and the demand for employment (by workers) were formulated under so-called axiomatic assumptions. This approach was then applied by Joseph and Bantock (1984) in the context of healthcare, to quantify the availability of general practitioners in Canada. About two decades later, Shen (1998) independently re-discovered Weibull's (1976) formula (see footnote (7) in Shen, 1998) and deconstructed it to consider accessibility for different modes. These advances were subsequently popularized as the family of Two-Stage Floating Catchment area (2SFCA) methods (Luo and Wang, 2003) that have found widespread adoption in healthcare, education, and food systems (B. Y. Chen et al., 2020; Chen, 2019; Z. Chen et al., 2020; Yang et al., 2006; Ye et al., 2018).

An important development contained in Shen's work is a proof that the population-weighted sum of the accessibility measure with competition equates to the number of opportunities available (footnote (7) and Appendix A in Shen, 1998). This demonstration gives the impression that Shen-type accessibility allocates *all* opportunities to the origins, however to the authors' knowledge, it has not interpreted by literature in this way. For instance, Hu (2014), Merlin and Hu (2017), and Tao et al. (2020) all use Shen-type accessibility to calculate job access but report values as 'competitive accessibility scores' or simply 'job accessibility'. These works do not explicitly recognize that jobs that are assigned to each origin are in fact a proportion of *all* the opportunities in the system. This recognition, we argue, is critical to interpreting the meaning of the final result. Thus, in this paper we intend to revisit accessibility with competition within the context of disentangling how opportunities are allocated. We first argue that Shen's competitive accessibility misleadingly refers to the the total zonal population to equal the travel-cost discounted opportunity-seeking population. This equivocation, we believe, results in a ambigious interpretation of what Shen-type accessibility represents as the allocation of opportunities to population is masked by the results presenting as rates (i.e., opportunities per capita). We then propose an alternative formulation of accessibility that incorporates competition by adopting a proportional allocation mechanism; we name this measure *spatial availability.* The use of balancing factors for proportional allocation is akin to imposing a single constraint on the accessibility indicator, in the spirit of Wilson's (1971) spatial interaction model.

In this way, the aim of the paper is three-fold:

- First, we aim to demonstrate that Shen-type (and thus Weibull (1976) accessibility and the popular 2SFCA methods) produce equivocal estimates of opportunities allocated as the result is presented as a rate (i.e., opportunities per capita);
- Second, we introduce a new measure, *spatial availability*, which we submit is a more interpretable alternative to Shen-type accessibility, since opportunities in the system are preserved and proportionally allocated to the population; and
- Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.

Discussion is supported by the use of the small synthetic example of Shen (1998) and empirical data drawn from the 2016 Transportation Tomorrow Survey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the spirit of openness of research in the spatial sciences (Brunsdon and Comber, 2021; Páez, 2021) this paper has a companion open data product (Arribas-Bel et al., 2021), and all code is available for replicability and reproducibility purposes.

2. Accessibility measures revisited

In this section we revisit Hansen-type and Shen-type accessibility indicators. We adopt the convention of using a capital letter for absolute values (number of opportunities) and lower case for rates (opportunities per capita).

2.1. Hansen-type accessibility

Hansen-type accessibility measures follow the general formulation shown in Equation (1):

$$S_i = \sum_{j=1}^{J} O_j \cdot f(c_{ij}) \tag{1}$$

where:

- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}; it can take the form of any monotonically decreasing function chosen based on positive or normative criteria (Paez et al., 2012).
- i is a set of origin locations ($i = 1, \cdots, N$).
- j is a set of destination locations ($j = 1, \cdots, J$).

- O_j is the number of opportunities at location j; $O = \sum_{j=1}^{J} O_j$ is the total supply of opportunities in the study region.
- S is Hansen-type accessibility as weighted sum of opportunities.

As formally defined, accessibility S_i is the sum of opportunities that can be reached from location i, weighted down by an impedance function of the cost of travel c_{ij}. Summing the opportunities in the neighborhood of i provides estimates of the number of opportunities that can *potentially* be reached from i. Several variants of this method result from using a variety of impedance functions; for example, cumulative opportunities measures are obtained when $f(\cdot)$ is a binary or indicator function (e.g., El-Geneidy et al., 2016; Geurs and van Wee, 2004; Qi et al., 2018; Rosik et al., 2021). Other measures use impedance functions modeled after any monotonically decreasing function (e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, *inter alia*, Kwan, 1998; Li et al., 2020; Reggiani et al., 2011; Vale and Pereira, 2017). In practice, accessibility measures with different impedance functions tend to be highly correlated (Higgins, 2019; Kwan, 1998; Santana Palacios and El-geneidy, 2022).

Gravity-based accessibility has been shown to be an excellent indicator of the intersection between spatially distributed opportunities and transportation infrastructure (Kwan, 1998; Reggiani et al., 2011; Shi et al., 2020). However, beyond enabling comparisons of relative values they are not highly interpretable on their own (Miller, 2018). To address the issue or interpretability, previous research has aimed to index and normalize values on a per demand-population basis (e.g., Barboza et al., 2021; Pereira et al., 2019; Wang et al., 2021). However, as recent research on accessibility discusses (Allen and Farber, 2019; Kelobonye et al., 2020; Merlin and Hu, 2017; Paez et al., 2019), these steps do not adequately consider competition. In effect, when calculating S_i, every opportunity enters the weighted sum once for every origin i that can reach it. This makes interpretability opaque, and to complicate matters, can also bias the estimated landscape of opportunity.

2.2. Shen-type competitive accessibility

To account for competition, the influential works of Shen (1998) and Weibull (1976), as well as the widely used 2SFCA approach of Luo and Wang (2003), adjust Hansen-type accessibility with the population in the region of interest. The mechanics of this approach consist of calculating, for every destination j, the population that can reach it given the impedance function $f(\cdot)$; let us call this the *effective opportunity-seeking population* (Equation (2)). This value can be seen as the Hansen-type *market area* (accessibility to population) of j. The opportunities at j are then divided by the sum of the *effective opportunity-seeking population* to obtain a measure of opportunities per capita, i.e., R_j in Equation (3). This can be thought of as the *level of service* at j. Per capita values are then allocated back to the population at i, again subject to the impedance function as seen in Equation (4); this is accessibility with competition.

$$P_{ij}^{*} = P_i \cdot f(c_{ij}) \tag{2}$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^{*}} \tag{3}$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \tag{4}$$

where:

- a is Shen-type accessibility as weighted sum of opportunities per capita (or weighted level of service).
- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}.
- i is a set of origin locations ($i = 1, \cdots, N$).
- j is a set of destination locations ($j = 1, \cdots, J$).
- O_j is the number of opportunities at location j; $O = \sum_{j=1}^{J} O_j$ is the total supply of opportunities in the study region.
- P_i is the population at location i.
- P_{ij}^{*} is the population at location i that can reach destination j according to the impedance function; we call this the *effective opportunity-seeking population.*
- R_j is the ratio of opportunities at j to the sum over all origins of the *effective opportunity-seeking population* that can reach j; in other words, this is the total number of opportunities per capita found at j.

Shen (1998) describes P_i as the *"the number of people in location i seeking opportunities"*. In our view, this is somewhat equivocal and where misinterpretation of the final result may arise. Consider a population center where the population is only willing to travel if a trip is less than or equal to 60 minutes. This is identical to the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 \text{ if } c_{ij} \leq 60 \text{ min} \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

If an employment center is less than 60 minutes away, the population can seek opportunities there (i.e., $f(c_{ij}) = 1$). But are these people still part of the opportunity-seeking population for jobs located two hours away? Four hours? Ten hours? We would assume that they are not because their travel behaviour, as represented by the impedance function would yield $f(c_{ij}) = 0$, eliminating them from what we call, the *effective opportunity-seeking population* $P_i j*$. We see Shen's definition as ambiguous because, for the purpose of calculating accessibility, the impedance function defines what constitutes the population that effectively can seek opportunities at remote locations. Thus P_i should be plainly understood as the population at location i (as we define above) and not the *"the number of people in location i seeking opportunities"*. In other words, Shen confounds P_i with P_ij*.

Furthermore, the same misunderstanding can be described for O_j which Shen defines as *"the number of **relevant** opportunities in location j"* . O_j is adjusted by the same $f(c_{ij})$ in Equation (4), so the *relevancy* is determined by the travel behaviour associated with the impedance function not by O_j itself. For this reason, O_j should be understand plainly as the opportunities at location j (as we also define above).

Misunderstanding P_i and O_j may led to a misinterpretation of the final result a_i, especially as expressed in Shen's proof as shown in Equation (6).

$$\sum_{i=1}^{N} a_i P_i = \sum_{j=1}^{J} O_j \tag{6}$$

Notice, misunderstanding P_i and O_j may allow some to misunderstand the units of a_i as *"relevant opportunities"* per *"people in location i seeking opportunities"*. Instead, as mathematically expressed in the proof, a_i is a proportion of the opportunities available to the population, since multiplying a_i by the population at i and summing for all origins in the system equals to the total number of opportunities in the system. Embedded in a_i is already the travel behaviour so P_i and O_j must be plainly understand as population at i and opportunities at j to have Equation (6) hold true.

2.3. Shen's toy example

In this section we use the example in Shen (1998) to detail the importance of understanding P_i and O_j as simply the population at the origin i and the opportunities at destination j respectively. It is critical to understanding how the opportunities are allocated to the population based on the impedance function.

Table 1 contains the information needed to calculate S_i and a_i for this example. We use a negative exponential impedance function with $\beta = 0.1$ as done in Shen (1998, see footnote (5)):

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

In Table 1, we see that population centers A and B have equal Hansen-type accessibility ($S_A = S_B = 27{,}292$ jobs). On the other hand, the isolated satellite town of C has low accessibility ($S_C = 2{,}240$ jobs). But center B, despite its high accessibility, is a large population center. C, in contrast, is smaller but also relatively isolated and has a balanced ratio of jobs (10,0000 jobs) to population (10,000 people). It is difficult from these outputs to determine whether the accessibility at C is better or worse than that at A or B.

The results are easier to interpret when we consider Shen-type accessibility. The results indicate that $a_A \approx 1.337$ jobs per capita, $a_B \approx 0.888$, and $a_C \approx 0.996$. The latter value is sensible given the jobs-population balance of C. Center A is relatively close to a large number of jobs (more jobs than the population of A). The opposite is true of B. According to Shen (1998), the sum of the

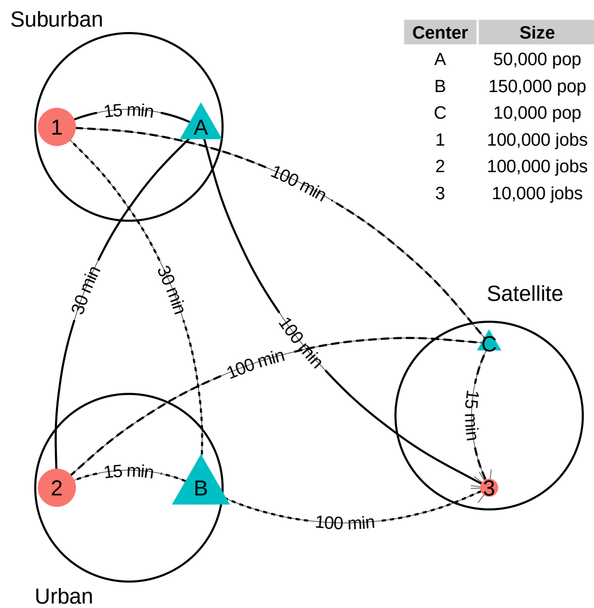


Figure 1: Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

population-weighted accessibility a_i is exactly equal to the number of jobs in the region following Shen's proof:

$$\sum_{i=1}^{N} a_i P_i = \sum_{j=1}^{J} O_j$$
$$50,000 \times 1.3366693$$
$$+150,000 \times 0.8880224$$
$$+10,000 \times 0.9963171 = 210,000$$

As mentioned earlier, this property under Shen's definition of P_i *"people in location i seeking opportunities"* , gives the impression that all sought jobs are allocated to the people located at each origin i. In other words, Shen defines P_i to mean P^*_{ij} (i.e., the *effective opportunity-seeking population* which is already adjusted by travel behaviour) while it in fact should be simply understood as the full population at i and it is adjusted by a_i which already has travel behaviour embedded in its formulation. As seen in column **Pop * f(TT)** in Table 1 (i.e., $P^*_{ij} = P_i \cdot f(c_{ij})$), the number of individuals from population center A that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2,27 respectively. Therefore, the total *effective opportunity-seeking population* in the system (i.e., $P^*_A = \sum_j P^*_{Aj}$) is 13,647.27 people, which is considerably lower than the total population of A (i.e., $P_A =$ 50,000 people). Continuing to understand P_i to equal P^*_{ij}, we would think that the proof would actually allocate 56,834.59 jobs in the region (as demonstrated as follows) instead of the nominal number

Table 1: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition with beta = 0.1

Origin	**Pop.**	**Dest.**	**Jobs**	**TT**	**f(TT)**	**Pop * f(TT)**	**Jobs * f(TT)**	**S_i**	**a_i**
A	50,000	1	100,000	15	0.223130	11,157	22,313	27,292	1.34
		2	100,000	30	0.049787	2,489	4,979		
		3	10,000	100	0.000045	2.27	0.454		
B	150,000	1	100,000	30	0.049787	7,468	4,979	27,292	0.888
		2	100,000	15	0.223130	33,470	22,313		
		3	10,000	100	0.000045	6.81	0.454		
C	10,000	1	100,000	100	0.000045	0.454	4.54	2,240	0.996
		2	100,000	100	0.000045	0.454	4.54		
		3	10,000	15	0.223130	2,231	2,231		

of jobs in the region which is over three times this number (i.e., 210,000 jobs).

$$
\begin{aligned}
&\textstyle\sum_{i=1}^{N} a_i P_{ij}^{*} = ???\\
&(11,156.51 + 2,489.35 + 2.26) \times 1.3366693\\
&+(7,468.06 + 33,469.52 + 6.81) \times 0.8880224\\
&+(4.54 + 4.54 + 2,231.20) \times 0.9963171 \approx 56,834.59
\end{aligned}
$$

Furthermore, even when Shen's P_i is understood plainly as the total population at i, the meaning of the proof may still be ambiguous. The proof can still give the impression that all jobs are allocated to the total population since total population ($\sum_{i=1}^{N} P_i$) goes into the equation and total jobs ($\sum_{j=1}^{J} O_j$) in the region is the result. However, this impression is incomplete since it does not reflect the quantity of population which is being allocated jobs and the quantity of population being considered for jobs; these magnitudes are a product of being weighted down by the impedance function (which reflects travel behaviour). Why these magnitudes may be hard to disentangle from a_i is because it is presented as a rate (i.e., opportunities per capita).

Let's consider a modification to the example in Table 2, to illustrate how the presentation of a_i as a rate obscures the magnitude of *effective opportunity-seeking population* and how that can impact the interpretation of the result. We modify the example by increasing the β to 0.5 (compared to the previous value of 0.1; see Figure 2). This modification increases the cost of travel and thus the impedance function, which is an expression of the population's relative willingness to travel to opportunities, reflects a population which is relatively

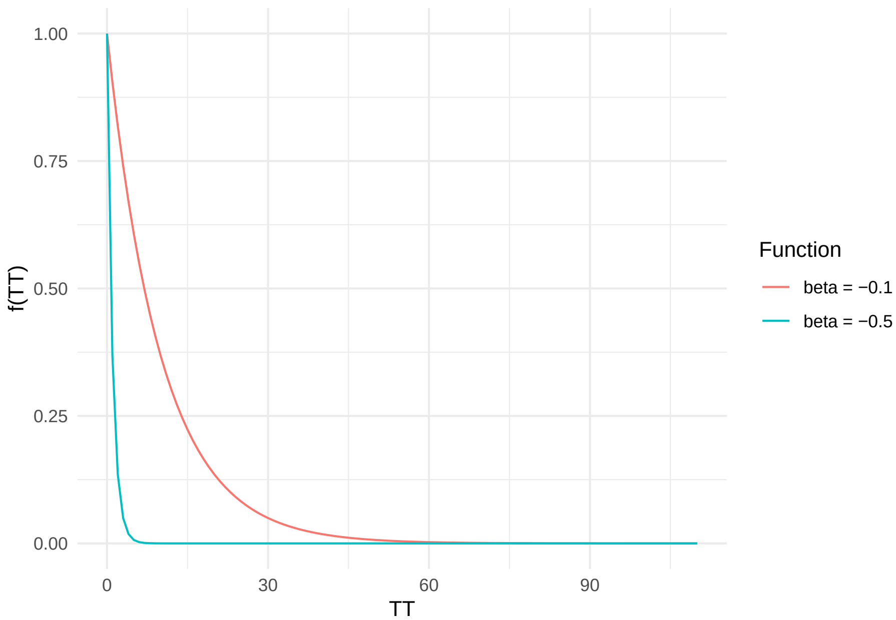


Figure 2: Comparison of two impedance functions in the example.

less willing to travel to opportunities further away compared to the previous β value.

As expected, Hansen-type accessibility drops quite dramatically after this β modification: the friction of distance is so high that few opportunities are within reach. In contrast, Shen-type accessibility converges to the jobs:population ratio (i.e., origin A is $\frac{100{,}000}{50{,}000} = 2$). This could led to some interpreting the value indicating that there are an over-supply of jobs to population; especially when knowing there are two times more jobs than population at A. However, what is the magnitude of population who effectively seek jobs and the number of jobs allocated? For origin A for jobs in job center 1, the demand is only 0.015 people ($Pop * f(TT)$), while the supply of jobs allocated from center 1 to A is only 0.031 jobs ($Jobs * f(TT)$). So yes, the jobs:population ratio is 2, but only for a tiny fraction of the population of A that effectively seeks opportunities at center 1.

In what follows, we propose an alternative derivation of Shen (1998) accessibility with competition that explicitly clarifies the opportunities allocated to the *effective opportunity-seeking population* within it's formulation. Hence, the resulting value is more interpretable.

Table 2: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition with beta = 0.5

Origin	**Pop.**	**Dest.**	**Jobs**	**TT**	**f(TT)**	**Pop * f(TT)**	**Jobs * f(TT)**	**S_i**	**a_i**
		1	100,000	15	< 0.001	0.015	0.031		
A	50,000	2	100,000	30	< 0.001	< 0.001	< 0.001	0.0306	2
		3	10,000	100	< 0.001	< 0.001	< 0.001		
		1	100,000	30	< 0.001	< 0.001	< 0.001		
B	150,000	2	100,000	15	< 0.001	0.046	0.031	0.0306	0.667
		3	10,000	100	< 0.001	< 0.001	< 0.001		
		1	100,000	100	< 0.001	< 0.001	< 0.001		
C	10,000	2	100,000	100	< 0.001	< 0.001	< 0.001	0.00306	1
		3	10,000	15	< 0.001	0.003	0.003		

3. Introducing spatial availability: a singly-constrained measure of accessibility

In brief, we define the *spatial availability* at i (V_i) as the proportion of all opportunities O that are allocated to i from all destinations j:

$$V_i = \sum_{j=1}^{K} O_j F_{ij}^t$$

where:

- F_{ij}^t is a balancing factor that depends on the population and cost of movement in the system.
- O_j is the number of opportunities at j.
- V_i is the number of spatially available opportunities from the perspective of i.

The general form of spatial availability is also as a sum, and the fundamental difference with Hansen- and Shen-type accessibility is that opportunities are allocated proportionally. Balancing factor F_{ij}^t consists of two components: the population-based balancing factor F_i^p and the impedance-based balancing factor F_{ij}^c which, respectively, allocates opportunities to i in proportion to the size of the population of the different competing centers (the mass effect of the gravity model) and the cost of reaching opportunities (the impedance effect). In the next two subsections, we explain the intuition behind the method before defining it in full.

3.1. Proportional allocation by population

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by population. Recall, the total population in the example is 210,000. The proportion of the population by population center is as follows:

$$F_A^p = \frac{50{,}000}{210{,}000}$$

$$F_B^p = \frac{150{,}000}{210{,}000}$$

$$F_C^p = \frac{10{,}000}{210{,}000}$$

Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors F_i^p. In this way, employment center 1 allocates $100,000 \cdot \frac{50{,}000}{210{,}000} = 23,809.52$ jobs to A; $100,000 \cdot \frac{150{,}000}{210{,}000} = 71,428.57$ jobs to B; and $100,000 \cdot \frac{10{,}000}{210{,}000} = 7,142.857$ jobs to C. Notice how this mechanism ensures that the total number of jobs at employment center 1 is preserved at 100,000.

We can verify that the number of jobs allocated is consistent with the total number of jobs in the system:

Employment center 1 to population centers A, B, and C:
$$100,000 \cdot \frac{50{,}000}{210{,}000} + 100,000 \cdot \frac{150{,}000}{210{,}000} + 100,000 \cdot \frac{10{,}000}{210{,}000} = 100,000$$

Employment center 2 to population centers A, B, and C:
$$100,000 \cdot \frac{50{,}000}{210{,}000} + 100,000 \cdot \frac{150{,}000}{210{,}000} + 100,000 \cdot \frac{10{,}000}{210{,}000} = 100,000$$

Employment center 3 to population centers A, B, and C:
$$10,000 \cdot \frac{50{,}000}{210{,}000} + 10,000 \cdot \frac{150{,}000}{210{,}000} + 10,000 \cdot \frac{10{,}000}{210{,}000} = 10,000$$

In the general case where there are N population centers in the region, we define the following population-based balancing factors:

$$F_i^p = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \tag{7}$$

Balancing factor F_i^p corresponds to the proportion of the population in origin i relative to the population in the region. On the right hand side of the equation, the numerator P_i^α is the population at origin i. The summation in the denominator is over $i = 1, \cdots, N$, and adds up to the total population of the region. Notice that we incorporate an empirical parameter α. The role of α is to modulate the effect of demand by population. When $\alpha < 1$, opportunities are allocated more rapidly to smaller centers relative to larger ones; $\alpha > 1$ achieves the opposite effect.

Balancing factor F_i^p can now be used to proportionally allocate a share of available jobs at j to origin i. The number of jobs available to i from j balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

In the general case where there are J employment centers, the total number of jobs available from all destinations to i is simply the sum of V_{ij}^p over $j = 1, \cdots, J$:

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

Since the factor F_i^p, when summed over $i = 1, \cdots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^p = 1$), the sum of all spatially available jobs equals O, the total number of opportunities in the region:

$$\begin{aligned}
&\textstyle\sum_{i=1}^N V_i^p = \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\
&\textstyle= \sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\
&\textstyle= \sum_{j=1}^J O_j = O
\end{aligned}$$

The terms F_i^p act here as the balancing factors of the gravity model when a single constraint is imposed (i.e., to ensure that the sums of columns are equal to the number of opportunities per destination, see Ortúzar and Willumsen, 2011, pp. 179–180 and 183-184). As a result, the sum of spatial availability for all population centers equals the total number of opportunities.

The discussion so far concerns only the mass effect (i.e., population size) of the gravity model. In addition, the potential for interaction is thought to decrease with increasing cost, so next we define similar balancing factors but based on the impedance.

3.2. Proportional allocation by cost

Clearly, using only balancing factors F_i^p to calculate spatial availability V_i^p does not account for the cost of reaching employment centers. Consider instead a set of balancing factors F_{ij}^c that account for the friction of distance for our example. Recall that the impedance function $f(c_{ij})$ equals $\exp(-\beta \cdot c_{ij})$ where $\beta = 0.1$ and travel time c_{ij} is either 15, 30 or 60 minutes. For instance, the impedance-based balancing factors F_{ij}^c would be the following for employment center 1 (employment center 2 and 3 have their own balancing factor values for each origin i as will be discussed later):

$$\begin{aligned}
F_{A1}^c &= \tfrac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\
F_{B1}^c &= \tfrac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\
F_{C1}^c &= \tfrac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581
\end{aligned}$$

Balancing factors F_{ij}^c use the impedance function to proportionally allocate more jobs to closer population centers, that is, to those with populations *more willing to reach the jobs*. Indeed, the factors F_{ij}^c can be thought of as the proportion of the population at i willing to travel to destination j, conditional on the travel behavior as given by the impedance function. For instance, 81.74398% of jobs from employment center 1 are allocated to population center A based on impedance.

So as follows from our example, of the 100,000 jobs at employment center 1 the number of jobs allocated to population center A is $100,000 \times 0.8174398 = 81,743.98$ jobs; the number allocated to population center B is $100,000 \times 0.1823954 = 18,239.54$ jobs; and the number allocated to population center C is $100,000 \times 0.0001648581 = 16.48581$ jobs. We see once more that the total number of jobs at the employment center is preserved at 100,000. In this example, the proportional allocation mechanism assigns the largest share of jobs to population center A, which is the closest to employment center 1, and the smallest to the more distant population center C.

In the general case where there are N population centers and J employment centers in the region, we define the following impedance-based balancing factors:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \tag{8}$$

The total number of jobs available to i from j according to impedance is defined as follows:

$$V_{ij}^c = O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c}$$

The total number of jobs available to i from all destinations is:

$$V_i^c = \sum_{j=1}^J O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c}$$

Like the population-based allocation factors, F_i^c summed over $i = 1, \cdots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^c = 1$). As before, the sum of all spatially available jobs equals O, the total number of opportunities in the region:

$$\begin{aligned}
&\sum_{i=1}^N V_i^c = \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c} \\
&= \sum_{i=1}^N \frac{F_i^c}{\sum_{i=1}^N F_i^c} \cdot \sum_{j=1}^J O_j \\
&= \sum_{j=1}^J O_j = O
\end{aligned}$$

We are now ready to more formally define spatial availability with due consideration to both population and travel cost effects.

3.3. Assembling mass and impedance effects

Population and the cost of travel are both part of the gravity modelling framework. Since the balancing factors defined in the preceding sections are proportions (alternatively, can be understood as probabilities), they can be combined multiplicatively to obtain their joint effect. This multiplicative relationship can alternatively be understood as the joint probability of allocating opportunities and is captured by Equation (9), where F_i^p is the population-based balancing factor that grants a larger share of the existing opportunities to larger centers and F_{ij}^c is the impedance-based balancing factor that grants a larger share of the existing opportunities to closer centers. This is in line with the tradition of gravity modeling.

$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \tag{9}$$

with F_i^p and F_{ij}^c as defined in Equations (7) and (8) respectively. The resulting combined balancing factor F_{ij}^t is used to proportionally allocate jobs from j to i. Hence, spatial availability is given by Equation (10).

$$V_i = \sum_{j=1}^J O_j \ F_{ij}^t \tag{10}$$

The terms in Equation 10 are as follows:

- F_{ij}^t is a balancing factor as defined in Equation (9).
- i is a set of origin locations in the region $i = 1, \cdots, N$.
- j is a set of destination locations in the region $j = 1, \cdots, J$.
- O_j is the number of opportunities at location j.
- V_i is the spatial availability at i.

Notice that, unlike S_i in Hansen-type accessibility (Equation (1)), the population enters the calculation of V_i through F_i^p. Returning to Shen's example in Figure 1, Table 3 contains the information needed to calculate V_i.

In Table 3, column **V_ij** are the jobs available to each origin from each employment center. In this column $V_{A1} = 59{,}901$ is the number of jobs available at A from employment center 1. Column **V_i** (i.e., $\sum_{j=1}^J V_{ij}$) gives the total number of jobs available to origin i. We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

Compare the calculated values of V_i to column **S_i** (Hansen-type accessibility) in Table 1. The spatial availability values are more intuitive. Recall that population centers A and B had identical Hansen-type accessibility to employment opportunities. According to V_i, population center A has greater job availability due to: 1) its close proximity to employment center 1; combined with 2) less competition (i.e., a majority of the population have to travel longer

Table 3: Summary description of synthetic example: spatial availability

Origin	Pop.	Dest.	Jobs	TT	f(TT)	F^p	F^c	F	V_ij	V_i
A	50,000	1	100,000	15	0.223130	0.238095	0.817438	0.599006	59,901	66,833
		2	100,000	30	0.049787	0.238095	0.182395	0.069227	6,923	
		3	10,000	100	0.000045	0.238095	0.000203	0.001013	10	
B	150,000	1	100,000	30	0.049787	0.714286	0.182395	0.400969	40,097	133,203
		2	100,000	15	0.223130	0.714286	0.817438	0.930760	93,076	
		3	10,000	100	0.000045	0.714286	0.000203	0.003040	30	
C	10,000	1	100,000	100	0.000045	0.047619	0.000166	0.000024	2.4	9,963
		2	100,000	100	0.000045	0.047619	0.000166	0.000013	1.3	
		3	10,000	15	0.223130	0.047619	0.999593	0.995947	9,959	

distances to reach employment center 1). Job availability is lower for population center B due to much higher competition (150,000 people can reach 100,000 jobs at equal cost). And center C has almost as many jobs available as it has population.

As discussed above, Hansen-type accessibility is not designed to preserve the number of jobs in the region. Shen-type accessibility ends up preserving the number of jobs in the region but the definitions of variables are internally obscured; the only way it preserves the number of jobs is if the effect of the impedance function is ignored within the definition of P_i when expanding the values of jobs per capita to obtain the total number of opportunities. The proportional allocation procedure described above, in contrast, consistently returns a number of jobs available that matches the total number of jobs in the region.

Since the jobs spatially available are consistent with the jobs in the region, it is possible to define a measure of spatial availability per capita:

$$v_i = \frac{V_i}{P_i} \tag{11}$$

And, since the jobs are preserved, it is possible to use the regional jobs per capita as a benchmark to compare the spatial availability of jobs per capita at each origin:

$$\frac{\sum_{j=1}^{J} O_j}{\sum_{i=1}^{N} P_i} \tag{12}$$

In the example, since the population is equal to the number of jobs, the regional value of jobs per capita is 1.0. To complete the illustrative example, the spatial availability of jobs per capita by origin is:

$$\begin{aligned} v_1 &= \frac{V_1}{P_1} = \frac{66{,}833.47}{50{,}000} = 1.337 \\ v_2 &= \frac{V_2}{P_2} = \frac{133{,}203.4}{150{,}000} = 0.888 \\ v_3 &= \frac{V_3}{P_3} = \frac{9{,}963.171}{10{,}000} = 0.996 \end{aligned} \tag{13}$$

We can see that population center A has fewer jobs per capita than the regional benchmark, center B has more, and center C is at parity. Remarkably, the spatial availability per capita matches the values of a_i in Table 1. Appendix A has a proof of the mathematical equivalence between the two measures. It is interesting to notice how Weibull (1976), Shen (1998), as well as this paper, all reach identical expressions starting from different assumptions; this effect is known as *equifinality* (see Ortúzar and Willumsen, 2011, p. 333; and Williams, 1981). Incidentally, this result means that Shen-type accessibility and 2SFCA can be re-conceptualized as singly-constrained accessibility measures.

3.4. Why does proportional allocation matter?

Having shown that Shen-type accessibility and spatial availability produce equifinal results, it is reasonable to ask whether the distinction between them is of any importance.

Conceptually, we would argue that the incorrect interpretation of variables leads to internal inconsistency in the calculation of total opportunities in Shen (1998): this points to a deeper issue that is only evident when we consider the intermediate values of the method. To illustrate, Table 1 shows results of a_i that are reasonable (and they match exactly the spatial availability per capita) when expressed as a rate. But when we dig deeper, these results mask potentially misleading interpretations of the jobs allocated and the number of jobs taken. For instance, a population region with a high jobs:population ratio but a prohibitive transportation network which results in a high cost of travel may yield a high a_i value. This value, however, can conceal a low *effective opportunity-seeking population* and proportionally low number of allocated jobs while additionally obscuring the number of population which does *not* take jobs and the jobs *not* taken.

In addition, the intermediate accessibility values of a_i (Shen-type measure) may also lead to impact estimates that are deceptive (see Sarlas et al., 2020). For example, the estimated region-wide cost of travel considering the jobs allocated by a_i in Table 1 (i.e., $Jobs * f(TT)$) is as follows:

$$\begin{aligned} &22,313 \times 15 \text{ min} + 4,979 \times 30 \text{ min} + 0.454 \times 100 \text{ min} \\ &4,979 \times 30 \text{ min} + 22,313 \times 15 \text{ min} + 0.454 \times 100 \text{ min} \\ &4.54 \times 100 \text{ min} + 4.54 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,594 \text{ min} \end{aligned}$$

In contrast, the estimated region-wide cost of travel according to V_i in Table

3 is as follows:

$$59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min}$$
$$40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min}$$
$$2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min}$$

Often referred to as 'the supply of jobs' (or simply Hansen-style accessibility) in the Shen-type measure: $Jobs * f(TT)$ cannot be used to understand the region-wide travel-time. Recall how we define $Pop * f(TT)$ as the *effective opportunity-seeking population* (P^*_{ij}), $Jobs * f(TT)$ similarly represents the *effective* ***opportunities allocated*** and sums to approximately 56,824 out of a total of 210,000 jobs. Like $Pop * f(TT)$, the *effective opportunities allocated* to each origin is only a reflection of the impedance function and not the *actual* number of opportunities allocated to each origin. Therefore, the resulting $1,002,594$ min is practically meaningless in measuring the region-wide travel time.

However, since spatial availability allocates the *actual* number of opportunities to each origin; the 3,859,054 min can be practically used to quantify the system-wide impacts of competitive accessibility in this region. We know spatial availability's output is the number of opportunities at each i since the combined balancing factors allocate a proportional amount of the total opportunities to each i such that the number of opportunities allocated to each i sum to equal the total opportunities in the region.

4. Empirical example of Toronto

In this section we illustrate the application of spatial availability through an empirical example. For this, we use full-time employment flows from the Greater Golden Horseshoe (GGH) area in Ontario, Canada. Contained with the GGH is the Greater Toronto and Hamilton (GTHA) which forms the most populous metropolitan regions in Canada and the core urban agglomeration in the GGH.

The GTHA contains the city of Toronto, the most populous city in Canada. The city of Toronto is the focus of this empirical example, it will be used to demonstrate the application of the proposed spatial availability measure along with how it compares to Hansen- and Shen-type measures. We begin this section by explaining the data and then detailing the calculated comparisons.

4.1. GGH Data

We obtained full-time employment flows from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 3) (Data Management Group, 2018) every five years. The data set includes origin to destination flows associated with full-time employment trips; the number of jobs (n=3,081,885) and workers (n=3,446,957) (i.e., the number of originating trips and destination trips) at each origin and destination are represented at the level of Traffic Analysis Zones (TAZ) (n=3,764). TAZ are a unit of spatial analysis which are defined as part of the TTS, however, TAZ are commonly used to ascribe production and attraction of trips in the context of transportation planning modelling. In the GGH data set, the TAZ contain on average 916 workers and jobs 819 with more detailed descriptive statistics discussed later. The TTS data is based on a representative sample of between 3% to 5% of households in the GGH and is weighted to reflect the population covering the study area as a whole (Data Management Group, 2018).

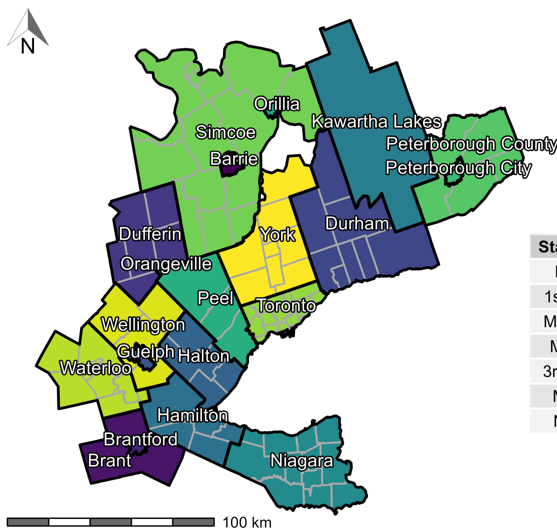
To generate the travel cost for the full-time employment trips, travel times between origins and destinations are calculated for car travel using the `R` package {r5r} (Rafael H. M. Pereira et al., 2021) with a street network retrieved from OpenStreetMap. For the calculations a 3 hr travel time threshold was selected as it captures 99% of population-employment pairs (see the travel times summarized in Figure 3). This method does not account for traffic congestion or modal split, which can be estimated through other means (e.g., Allen and Farber, 2021; Higgins et al., 2021). For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions. All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

4.2. Spatial employment characteristics in Toronto

As mentioned, the focus of this empirical example is on the city of Toronto. It is the largest city in the GGH and represents a significant subset of workers and jobs in the GGH; 22% of workers in the GGH live in Toronto and 25% of jobs are located within Toronto that these workers take. The spatial distribution of jobs (purple) and workers (orange) are captured in Figure 4. It can be seen that a large cluster of jobs can be found in the central southern part of Toronto (the downtown core). Spatial trends in the distribution of workers is less apparent relative to the distribution of jobs.

Next, the spatial distribution of the jobs to workers ratio (jobs:workers) (red) and the estimated car travel time (grey) is visualized in Figure 5. Some of the jobs:workers balance predictably clusters near major transportation networks (i.e, the north-south subway line and highways) where surrounding land is more commonly zoned for commercial use. By contrast, the estimated average travel time to work is more even throughout the city, with lower travel times within the downtown core and longer travel times for TAZ located further and further from the core. Interestingly, travel times in certain TAZ with a high jobs:worker balance occur.

Nonetheless, the point of these visualizations are to demonstrate the spatial distribution of worker and job data in the city of Toronto to contextualize spatial availability and Shen- and Hansen- type measures. Assessing contributing factors to patterns of variable clustering are outside of the paper's scope and left to further research.



Statistic	Trips	TT	Workers	Jobs
Min.	1	0	0	0
1st Qu.	14	13	25	64
Median	22	20	464	244
Mean	33	23	916	819
3rd Qu.	38	30	1378	700
Max.	1129	179	8491	41821
NA's	NA	3507	NA	NA

Figure 3: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

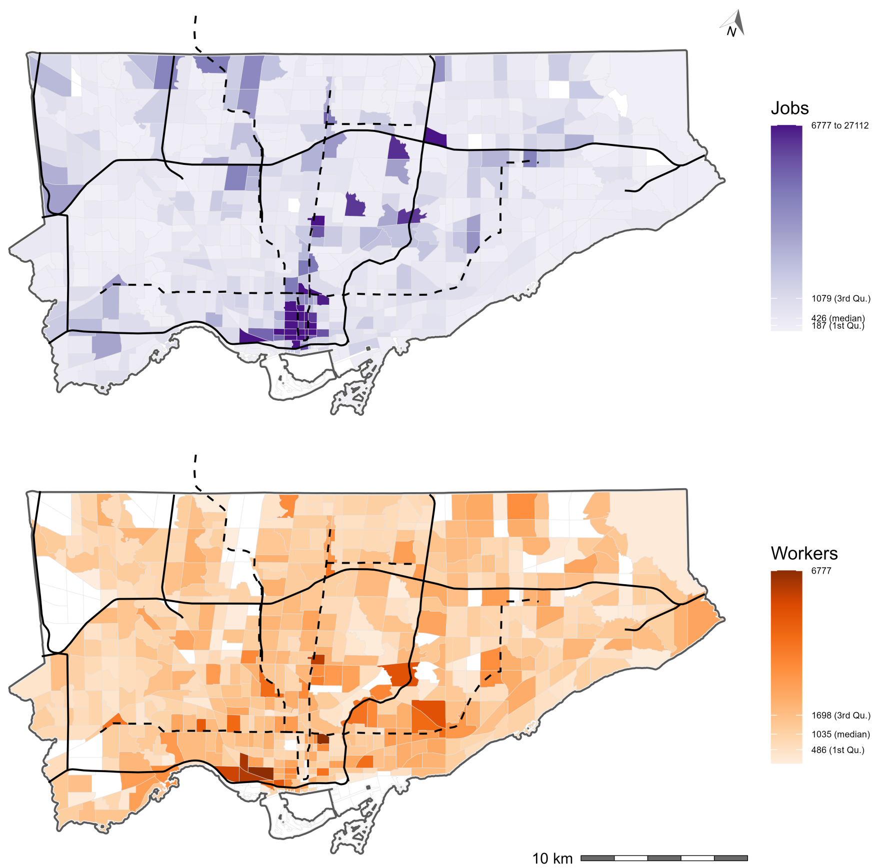


Figure 4: Spatial distribution of full-time jobs (top) and full-time working population (bottom) for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines.

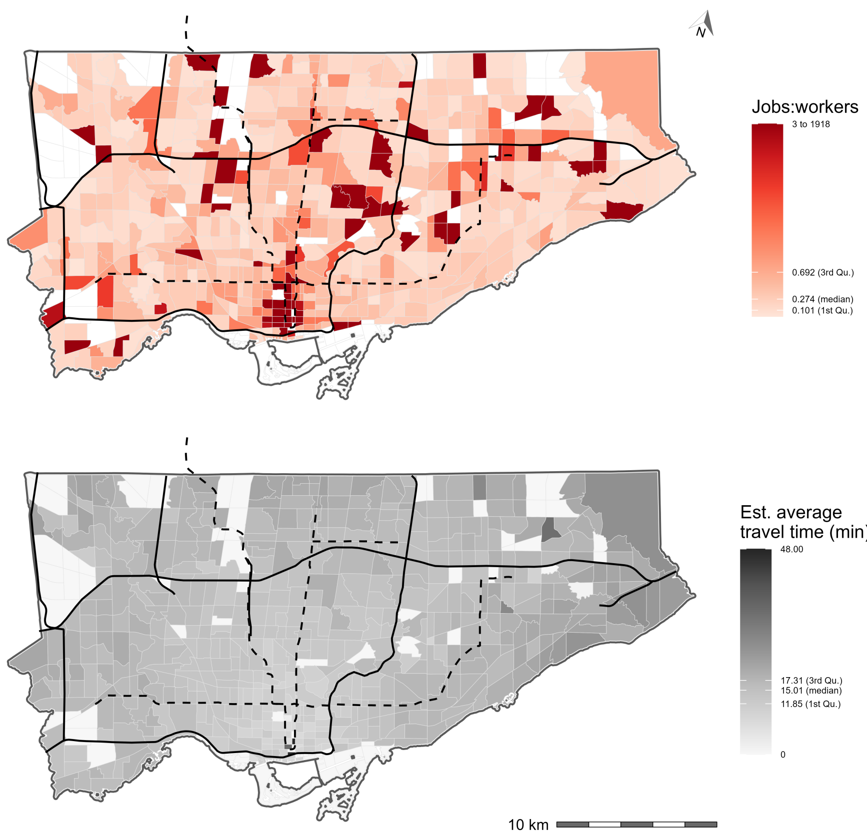


Figure 5: Spatial distribution of full-time working jobs and worker ratio (top) and car travel time to jobs estimated using R5R for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines.

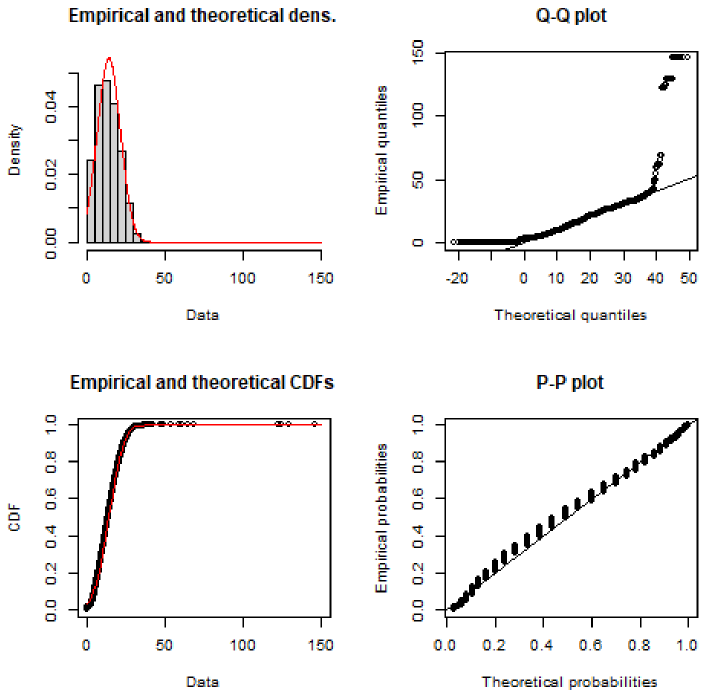


Figure 6: Car trip length distribution and calibrated normal distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

4.3. Calibration of an impedance function for Toronto

In toy example introduced in a preceding section, we used a negative exponential function with the parameter reported by Shen (1998). For the empirical Toronto data set, we calibrate an impedance function on the trip length distribution (TLD) of commute trips. Briefly, a TLD represents the proportion of trips that are taken at a specific travel cost (e.g., travel time); this distribution is commonly used to derive impedance functions in accessibility research (Batista et al., 2019; Horbachov and Svichynskyi, 2018; Lopez and Paez, 2017).

As mentioned, the calculations are undertaken for the city of Toronto to facilitate a comparison across measures. As such, only the work-related trips with origins and destinations within the boundaries of Toronto are selected. Specifically, edge trips are not included such as trips originating in Toronto but finishing outside of Toronto and trips originating outside of Toronto but finishing in Toronto. The empirical and theoretical TLD for this Toronto data set are represented in the top-left panel of Figure 6. Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the {fitdistrplus} package (Delignette-Muller and Dutang, 2015) were used. Based on goodness-of-fit criteria and diagnostics the normal distribution was selected (see Figure 6).

The normal distribution is defined in Equation (14), where we see that it depends on a mean parameter μ and a standard deviation parameter σ. The estimated values of these parameters are $\mu = 14.707$ and $\sigma = 7.697$.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{14}$$

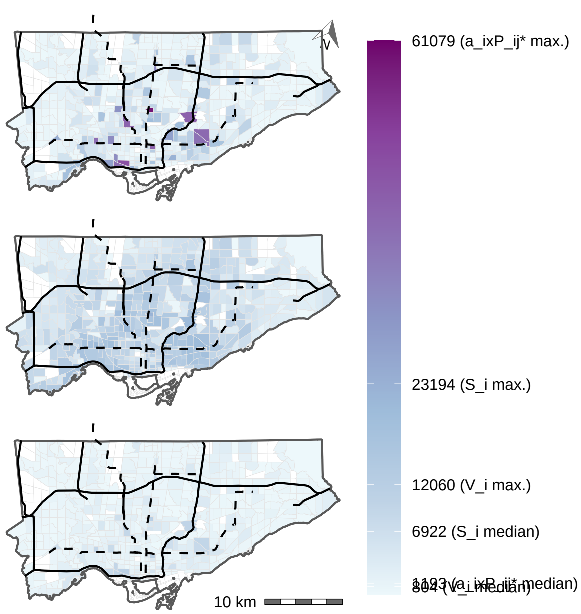
$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

4.4. Accessibility and spatial availability of jobs in Toronto

jobs
7.69e+05

jobs
7.69e+05

workers
1.14e+06



workers
1.98e+06

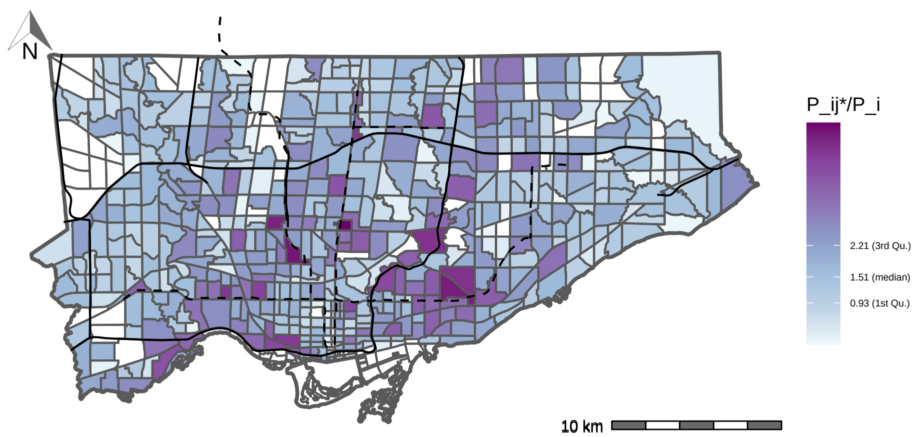
workers
1.33e+06

Figure **??** contains the absolute accessibility values in number of jobs accessible using Shen-type accessibility, Hansen-type accessibility, and the number of jobs *available* using the spatial availability measure. The Shen-type accessibility is multiplied by the *effective opportunity-seeking population* to yield a value that corresponds to absolute number of accessible jobs (considering competition) according to Shen's definition. The Hansen-type accessibility is an unconstrained case of accessibility in which all jobs which are in-reach of each origin (according to the impedance function); each value corresponds to the number of jobs which can be reach at each origin assuming no competition. Lastly, the spatial availability measure is a constrained case of accessibility which yields the number of jobs, at each origin, considering competition from the population in nearby origin and the relative travel cost (according to the impedance function).

The proportional allocation mechanism of spatial availability ensures that the job availability value for each origin all sums to the city-wide total of 769,231 jobs (i.e., the number of destination flows from Toronto origins to Toronto destinations). The regional total for Hansen-type accessibility is 4,318,005 jobs, which as a value is practically meaningless since the measure is unconstrained; it represents the sum of opportunities that have been counted anywhere from 1 to many times depending on the impedance function. As previously discussed, unconstrained counting of the same opportunity by all origins is not an issue if the opportunity itself is non-exclusive, but since one origin-to-destination full-time employment trip corresponds to 1 job and 1 worker, it is inappropriate to use unconstrained measures to capture employment characteristics. Though, using unconstrained Hansen-style measure for employment data can depicted urban form but applications to create actionable policy is lacking .

On a similar note to Hasen-type, the *absolute* Shen-type measure (as understood by Shen's definition of P_i being equal to P^*_{ij} sums to the city-wide value of 2,064,055 jobs; this value demonstrates how the Shen-type measure that is presented a rate (i.e., opportunities per capita) can be misunderstood and multiplied by the *effective opportunity-seeking population* (i.e., the denominator of the rate P^*_{ij}) to express an absolute accessibility score. The misleading interpretation of confounding P_i with P^*_{ij} yields an *incorrect* competitively accessible job values because P^*_{ij} greatly exceeds greatly exceeds the city-wide total of workers (i.e., Toronto origin trips). To the authors' knowledge, Shen-type accessibility has not been converted to the absolute value in the way demonstrated within this paper perhaps potentially because the values are incorrect since Shen's proof (Shen (1998) footnote) does not hold true. However, we also have not seen the Shen-type measure converted from the opportunities per capita score to absolute opportunities, we suspect, because of the ambiguous definition which conflates P^*_{ij} with P_i. If a_i is multiplied by P_i, it yields the same value as V_i, but, since the definition of Shen-type measure is equivocal doing so is not clear since the denominator of a_i (which is a rate) is *not* P_i.

osp	total_workers
1.73e+06	7.69e+05



It is important to understand the difference between P_i and P^*_{ij} since their impacts is not the same across space. See Figure 7 which presents the ratio of P^*_{ij} to P_i which presents how this effect is not constant across space. As a consequence of the inflated population (i.e., $P^*_{ij} > P_i$), if attempting to calculate impacts associated with Shen-type measure (absolute value) such as the city-wide travel time, you can see how the travel time is exaggerated by the Shen-type measure. Consequently, when using spatial availability (recall: this equal to a_i multiplied by P_i) which sums to the *total* number of jobs in the city, the travel time can be accuratedly calculated (as accurately as the input data).

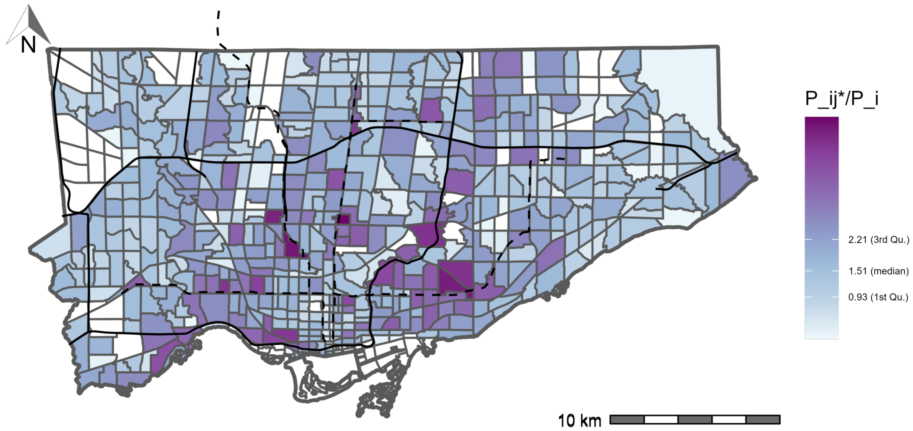
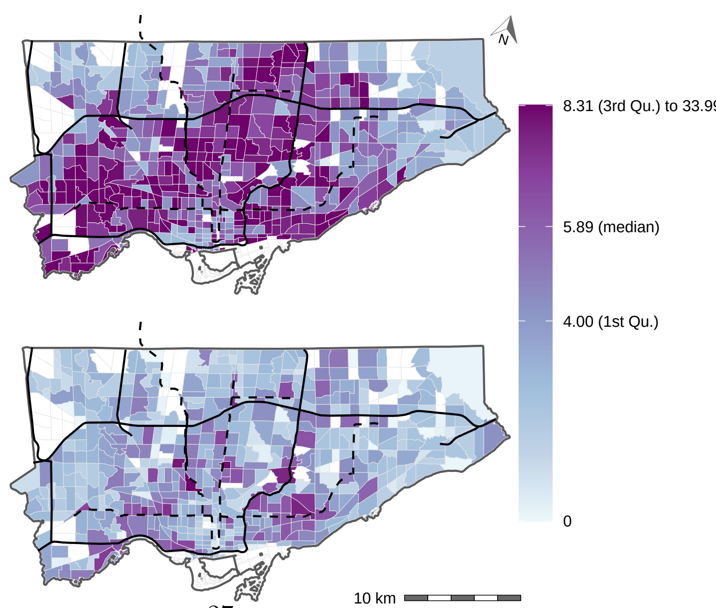


Figure 7: The ratio of the effective opportunity seeking population to the population in Toronto. Black lines represent expressways and black dashed lines represent subway lines.

total_travel_time
497258.183377296



563

total_travel_time
187393.373611765

Spatial availability untangles Shen-stype measure so an absolute value of *opportunity availability* is expressed. Because the proportional allocation mechanism makes clear that all the opportunities are being allocated proportionally to all the origins, it makes the interpretation more clear. It thus can be directly divided by origin population and expressed as an opportunities per capita score. Coincidentally, as mentioned, this score is equal to Shen-type measure, but with a renewed interpretability. When normalized spatial availability is compared to Hansen-type measure, normalizing the output by population directly is unique; spatial availability can be normalized by opportunities without the issue of multiple counting of opportunities which obscures accessibility's meaning. See Figure **??** which displays the normalized opportunity per capita spatial availability score and plainly normalizing Hansen-type accessibility by dividing each value by the origin population.

5. Conclusions

Why do differences between Hansen-style measure and the interpretation of Shen-type measure matter? Because of equity analysis and policy planning! With spatial availability, we can push accessibility analysis forward by making it more interpretable. As discussed, Hansen-style accessibility value cannot be interpreted directly in either form (though it can be interpreted relatively to speak about urban form). By contrast, spatial availability yields the number of jobs available directly.

This property and understanding spatial availability (along with Shen's accessibility with competition and 2SFCA) as a singly-constrained measure are important to understanding how accessibility measures can be broadled interepted. The following are recommendations:

1) The Hansen-style accessibilty should be used when opportunities are non-exclusive. When opportunities are perfectly exclusive (i.e., 1 spot for 1 person), spatial availability (i.e., accessibility with competition) should be used.
2) Though outside of scope, when opportunities are perfectly exclusive ad -We argue that spatial availability should be used

- some literature which uses Shen focuses on how travel cost discounts job supply (opportunities) and demand (opportunity-seeking population). This gets close to disentangling which population *effectively* seeking opportunities - but comparing it to the population which doesn't seek opportunities has not been made, to the authors opportunity. This is critical for equity analysis; i.e., which populations can't seek opportunities because of friction of distance? This can be plainly seen by seeing the proportion of population, in spatial availability, which is allocated to the origin

6. Appendix A

Equivalence of Shen-type accessibility and spatial availability

Population allocation factor:

$$F^p_{ij} = \frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}}$$

$$F^p_A = \frac{P^\alpha_A}{P^\alpha_A + P^\alpha_B + P^\alpha_C}$$

Cost allocation factor:

$$F^c_{ij} = \frac{f(c_{ij})}{\sum_{i=A}^K f(c_{ij})}$$

$$F^c_{A1} = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F^c_{B1} = \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$F^c_{C1} = \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$$

Now let's put it together with P, and see how the denominators end up cancelling out:

$$v_i = \sum_j \frac{O_j}{P^\alpha_{i\in r}} \frac{\frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}{\sum_i^K \frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}$$

$$v_A = \frac{O_1}{P^\alpha_A}(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})} + \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{B1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})} + \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{C1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}})+$$

$$\frac{O_2}{P^\alpha_A}(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})} + \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{B2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})} + \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{C2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}})+$$

$$\frac{O_3}{P^\alpha_A}(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})} + \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{B3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})} + \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{C3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}})$$

First, notice how the denominator on the denominator is the same across the summation? Let's simplify it:

$$v_A = \frac{O_1}{P^\alpha_A}(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P^\alpha_A \cdot f(c_{A1})+P^\alpha_A \cdot f(c_{B1})+P^\alpha_A \cdot f(c_{C1})}{(P^\alpha_A+P^\alpha_B+P^\alpha_C)\cdot(f(c_{A1})+f(c_{B1})+f(c_{C1}))}})+$$

$$\frac{O_2}{P^\alpha_A}(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P^\alpha_A \cdot f(c_{A2})+P^\alpha_A \cdot f(c_{B2})+P^\alpha_A \cdot f(c_{C2})}{(P^\alpha_A+P^\alpha_B+P^\alpha_C)\cdot(f(c_{A2})+f(c_{B2})+f(c_{C2}))}})+$$

$$\frac{O_3}{P^\alpha_A}(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C} \cdot \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P^\alpha_A \cdot f(c_{A3})+P^\alpha_A \cdot f(c_{B3})+P^\alpha_A \cdot f(c_{C3})}{(P^\alpha_A+P^\alpha_B+P^\alpha_C)\cdot(f(c_{A3})+f(c_{B3})+f(c_{C3}))}})$$

See how the denominator of the denominator is the same as the denominator of the numerator's denominator for each J (J=1, J=2, and J=3)? Let's cancel those out and simplify:

$$v_A = \frac{O_1}{P^\alpha_A}(\frac{P^\alpha_A \cdot f(c_{A1})}{P^\alpha_A \cdot f(c_{A1}) + P^\alpha_A \cdot f(c_{B1}) + P^\alpha_A \cdot f(c_{C1})}+$$

$$\frac{O_2}{P_A^\alpha}\frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})}+$$

$$\frac{O_3}{P_A^\alpha}\frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})})$$

Next, see how we can cancel out the P_A^α? Let's do that.

$$v_A = O_1(\frac{f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})}+O_2\frac{f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})}+O_3\frac{}{P_A^\alpha \cdot f(c_{A3})}$$

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