

¹ Introducing spatial availability, a singly-constrained
² measure of competitive accessibility

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⁴ **Abstract**

Accessibility indicators are widely used in transportation, urban, and health-care planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability, mainly by introducing competition. In this paper, we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

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5 **1. Introduction**

6 The concept of accessibility in transportation studies derives its appeal from
7 the combination of the spatial distribution of opportunities and the cost of
8 reaching them (Handy and Niemeier, 1997; Hansen, 1959). Accessibility analysis
9 is employed in transportation, geography, public health, and many other areas,
10 with the number of applications growing (Shi et al., 2020), especially as mobility-
11 based planning is de-emphasized in favor of access-oriented planning (Deboosere
12 et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021).

13 Accessibility analysis stems from the foundational works of Harris (1954)
14 and Hansen (1959). From these seminal efforts, many accessibility measures
15 have been derived, particularly after the influential work of Wilson (1971) on
16 spatial interaction¹. Of these, gravity-type accessibility is arguably the most
17 common; since its introduction in the literature, it has been widely adopted in
18 numerous forms (Arranz-López et al., 2019; Cervero et al., 2002; Geurs and van
19 Wee, 2004; Levinson, 1998; Paez, 2004). Hansen-type accessibility indicators
20 are essentially weighted sums of opportunities, with the weights given by an
21 impedance function that depends on the cost of movement, and thus measure
22 the *intensity of the possibility of interaction* (Hansen, 1959). This type of acces-
23 sibility analysis offers a powerful tool to study the intersection between urban
24 structure and transportation infrastructure (Handy and Niemeier, 1997).

25 Despite their usefulness, the interpretability of Hansen-type accessibility
26 measures can be challenging (Geurs and van Wee, 2004; Miller, 2018). Since
27 they aggregate opportunities, the results are sensitive to the size of the region
28 of interest (e.g., a large city has more jobs than a smaller city). As a conse-
29 quence, raw outputs are not necessarily comparable across study areas (Allen
30 and Farber, 2019). This limitation becomes evident when surveying studies that
31 implement this type of analysis. For example, Páez et al. (2010) (in Montreal)
32 and Campbell et al. (2019) (in Nairobi) report accessibility as the number of
33 health care facilities that can potentially be reached from origins. But what
34 does it mean for a zone to have accessibility to less than 100 facilities in each of
35 these two cities, with their different populations and number of facilities? For
36 that matter, what does it mean for a zone to have accessibility to more than 700
37 facilities in Montreal, besides being “accessibility rich”? As another example,
38 Bocarejo S. and Oviedo H. (2012) (in Bogota), El-Geneidy et al. (2016) (in
39 Montreal), and Jiang and Levinson (2016) (in Beijing) report accessibility as
40 numbers of jobs, with accessibility values often in the hundreds of thousands,
41 and even exceeding one million jobs for some zones in Beijng and Montreal. As
42 indicators of urban structure, these measures are informative, but the meaning
43 of one million accessible jobs is harder to pin down: how many jobs must any
44 single person have access to? Clearly, the answer to this question depends on
45 how many people demand jobs.

¹Utility-based measures derive from a very different theoretical framework, random utility maximization

46 The interpretability of Hansen-type accessibility has been discussed in nu-
47 merous studies, including recently by Hu and Downs (2019), Kelobonye et al.
48 (2020), and in greater depth by Merlin and Hu (2017). As hinted above, the
49 limitations in interpretability are frequently caused by ignoring competition -
50 without competition, each opportunity is assumed to be equally available to
51 every single opportunity-seeking individual that can reach it (Kelobonye et al.,
52 2020; Paez et al., 2019; Shen, 1998). This assumption is appropriate when the
53 opportunity of interest is non-exclusive, that is, if use by one unit of population
54 does not preclude use by another. For instance, national parks with abundant
55 space are seldom used to full capacity, so the presence of some population does
56 not exclude use by others. When it comes to exclusive opportunities, or when
57 operations may be affected by congestion, the solution has been to account
58 for competition. Several efforts exist that do so. In our reckoning, the first
59 such approach was proposed by Weibull (1976), whereby the distance decay of
60 the supply of employment and the demand for employment (by workers) were
61 formulated under so-called axiomatic assumptions. This approach was then ap-
62 plied by Joseph and Bantock (1984) in the context of healthcare, to quantify
63 the availability of general practitioners in Canada. About two decades later,
64 Shen (1998) independently re-discovered Weibull's (1976) formula (see footnote
65 (7) in Shen, 1998) and deconstructed it to consider accessibility for different
66 modes. These advances were subsequently popularized as the family of Two-
67 Stage Floating Catchment area (2SFCA) methods (Luo and Wang, 2003) that
68 have found widespread adoption in healthcare, education, and food systems (B.
69 Y. Chen et al., 2020; Chen, 2019; Z. Chen et al., 2020; Yang et al., 2006; Ye et
70 al., 2018).

71 An important development contained in Shen's work is a proof that the
72 population-weighted sum of the accessibility measure with competition equates
73 to the number of opportunities available (footnote (7) and Appendix A in Shen,
74 1998). This demonstration gives the impression that Shen-type accessibility al-
75 locates *all* opportunities to the origins, however to the authors' knowledge, it
76 has not interpreted by literature in this way. For instance, Hu (2014), Merlin
77 and Hu (2017), and Tao et al. (2020) all use Shen-type accessibility to calcu-
78 late job access but report values as 'competitive accessibility scores' or simply
79 'job accessibility'. These works do not explicitly recognize that jobs that are
80 assigned to each origin are in fact a proportion of *all* the opportunities in the
81 system. This recognition, we argue, is critical to interpreting the meaning of the
82 final result. Thus, in this paper we intend to revisit accessibility with compe-
83 tition within the context of disentangling how opportunities are allocated. We
84 first argue that Shen's competitive accessibility misleadingly refers to the the
85 total zonal population to equal the travel-cost discounted opportunity-seeking
86 population. This equivocation, we believe, results in a ambiguous interpretation
87 of what Shen-type accessibility represents as the allocation of opportunities to
88 population is masked by the results presenting as rates (i.e., opportunities per
89 capita). We then propose an alternative formulation of accessibility that incor-
90 porates competition by adopting a proportional allocation mechanism; we name
91 this measure *spatial availability*. The use of balancing factors for proportional

92 allocation is akin to imposing a single constraint on the accessibility indicator,
93 in the spirit of Wilson's (1971) spatial interaction model.

94 In this way, the aim of the paper is three-fold:

- 95 • First, we aim to demonstrate that Shen-type (and thus Weibull (1976)
96 accessibility and the popular 2SFCA methods) produce equivocal esti-
97 mates of opportunities allocated as the result is presented as a rate (i.e.,
98 opportunities per capita);
- 99 • Second, we introduce a new measure, *spatial availability*, which we submit
100 is a more interpretable alternative to Shen-type accessibility, since oppor-
101 tunities in the system are preserved and proportionally allocated to the
102 population; and
- 103 • Third, we show how Shen-type accessibility (and 2SFCA methods) can be
104 seen as measures of singly-constrained accessibility.

105 Discussion is supported by the use of the small synthetic example of Shen
106 (1998) and empirical data drawn from the 2016 Transportation Tomorrow Sur-
107vey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the
108 spirit of openness of research in the spatial sciences (Brunsdon and Comber,
109 2021; Páez, 2021) this paper has a companion open data product (Arribas-
110 Bel et al., 2021), and all code is available for replicability and reproducibility
111 purposes at <https://github.com/soukhova/Spatial-Availability-Measure>.

112 2. Accessibility measures revisited

113 In this section we revisit Hansen-type and Shen-type accessibility indicators.
114 We adopt the convention of using a capital letter for absolute values (number
115 of opportunities) and lower case for rates (opportunities per capita).

116 2.1. Hansen-type accessibility

117 Hansen-type accessibility measures follow the general formulation shown in
118 Equation (1):

$$S_i = \sum_{j=1}^J O_j \cdot f(c_{ij}) \quad (1)$$

119 where:

- 120 • c_{ij} is a measure of the cost of moving between i and j .
- 121 • $f(\cdot)$ is an impedance function of c_{ij} ; it can take the form of any monoton-
122 ically decreasing function chosen based on positive or normative criteria
123 (Páez et al., 2012).
- 124 • i is a set of origin locations ($i = 1, \dots, N$).
- 125 • j is a set of destination locations ($j = 1, \dots, J$).

- 126 • O_j is the number of opportunities at location j ; $O = \sum_{j=1}^J O_j$ is the total
 127 supply of opportunities in the study region.
 128 • S is Hansen-type accessibility as weighted sum of opportunities.

129 As formally defined, accessibility S_i is the sum of opportunities that can be
 130 reached from location i , weighted down by an impedance function of the cost
 131 of travel c_{ij} . Summing the opportunities in the neighborhood of i provides es-
 132 timates of the number of opportunities that can *potentially* be reached from i .
 133 Several measures result from using a variety of impedance functions; for exam-
 134 ple, cumulative opportunities measures are obtained when $f(\cdot)$ is a binary or
 135 indicator function (e.g., El-Geneidy et al., 2016; Geurs and van Wee, 2004; Qi et
 136 al., 2018; Rosik et al., 2021). Other measures use impedance functions modeled
 137 after any monotonically decreasing function (e.g., Gaussian, inverse power, neg-
 138 ative exponential, or log-normal, among others, see, *inter alia*, Kwan, 1998; Li
 139 et al., 2020; Reggiani et al., 2011; Vale and Pereira, 2017). In practice, accessi-
 140 bility measures with different impedance functions tend to be highly correlated
 141 (Higgins, 2019; Kwan, 1998; Santana Palacios and El-geneidy, 2022).

142 Gravity-based accessibility has been shown to be an excellent indicator of
 143 the intersection between spatially distributed opportunities and transportation
 144 infrastructure (Kwan, 1998; Reggiani et al., 2011; Shi et al., 2020). However,
 145 beyond enabling comparisons of relative values they are not highly interpretable
 146 on their own (Miller, 2018). To address the issue of interpretability, previous re-
 147 search has aimed to index and normalize values on a per demand-population ba-
 148 sis (e.g., Barboza et al., 2021; Pereira et al., 2019; Wang et al., 2021). However,
 149 as recent research on accessibility discusses (Allen and Farber, 2019; Kelobonye
 150 et al., 2020; e.g., Merlin and Hu, 2017; Paez et al., 2019), these steps do not ad-
 151 equately consider competition. In effect, when calculating S_i , every opportunity
 152 enters the weighted sum once for every origin i that can reach it. This makes
 153 interpretability opaque, and to complicate matters, can also bias the estimated
 154 landscape of opportunity.

155 2.2. *Shen-type competitive accessibility*

156 To account for competition, the influential works of Shen (1998) and Weibull
 157 (1976), as well as the widely used 2SFCA approach of Luo and Wang (2003), ad-
 158 just Hansen-type accessibility with the population in the region of interest. The
 159 mechanics of this approach consist of calculating, for every destination j , the
 160 population that can reach it given the impedance function $f(\cdot)$; let us call this
 161 the *effective opportunity-seeking population* (Equation (2)). This value can be
 162 seen as the Hansen-type *market area* (accessibility to population) of j . The op-
 163 portunities at j are then divided by the sum of the effective opportunity-seeking
 164 population to obtain a measure of opportunities per capita, i.e., R_j in Equa-
 165 tion (3). This can be thought of as the *level of service* at j . Per capita values
 166 are then allocated back to the population at i , again subject to the impedance
 167 function as seen in Equation (4); this is accessibility with competition.

$$P_{ij}^* = P_i \cdot f(c_{ij}) \quad (2)$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^*} \quad (3)$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \quad (4)$$

168 where:

- 169 • a is Shen-type accessibility as weighted sum of opportunities per capita
(or weighted level of service).
- 170 • c_{ij} is a measure of the cost of moving between i and j .
- 171 • $f(\cdot)$ is an impedance function of c_{ij} .
- 172 • i is a set of origin locations ($i = 1, \dots, N$).
- 173 • j is a set of destination locations ($j = 1, \dots, J$).
- 174 • O_j is the number of opportunities at location j ; $O = \sum_{j=1}^J O_j$ is the total
175 supply of opportunities in the study region.
- 176 • P_i is the population at location i .
- 177 • P_{ij}^* is the population at location i that can reach destination j according
178 to the impedance function; we call this the *effective opportunity-seeking
population*.
- 179 • R_j is the ratio of opportunities at j to the sum over all origins of the
180 *effective opportunity-seeking population* that can reach j ; in other words,
181 this is the total number of opportunities per capita found at j .
- 182
- 183

184 Shen (1998) describes P_i as the “*the number of people in location i seeking
185 opportunities*”. In our view, this is somewhat equivocal and where misinterpre-
186 tation of the final results may arise. Consider a population center where the
187 population is only willing to take an opportunity if the trip required is less than
188 or equal to 60 minutes. This is identical to the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 & \text{if } c_{ij} \leq 60 \text{ min} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

189 If an employment center is less than 60 minutes away, the population can
190 seek opportunities there (i.e., $f(c_{ij}) = 1$). But are these people still part of the
191 opportunity-seeking population for jobs located two hours away? Four hours?
192 Ten hours? We assume that they are not because their travel behaviour, as rep-
193 resented by the impedance function would yield $f(c_{ij}) = 0$, eliminating them
194 from the effective opportunity-seeking population P_{ij}^* . We see Shen’s defi-
195 nition as ambiguous because, for the purpose of calculating accessibility, the
196 impedance function defines what constitutes the population that effectively can
197 seek opportunities at remote locations. Thus P_i should be plainly understood
198 as the population at location i (as defined above) and not the “*the number of
199 people in location i seeking opportunities*”. In other words, P_i and P_{ij}^* are
200 confounded.

201 Furthermore, an identical misunderstanding can be described for O_j which is
202 defined as “*the number of relevant opportunities in location j* ” in Shen (1998)

(our emphasis). O_j is adjusted by the same $f(c_{ij})$ in Equation (4), so the *relevancy* is determined by the travel behaviour associated with the impedance function not purely by O_j itself. For this reason, O_j should be understand plainly as the opportunities at location j (as we also defined them above).

Misunderstanding P_i and O_j may lead to a misleading interpretation of the final result a_i , especially as expressed in Shen's proof (see Equation (6)).

$$\sum_{i=1}^N a_i P_i = \sum_{j=1}^J O_j \quad (6)$$

Notice, confounding P_i with the effective opportunity-seeking population and O_j with the jobs taken may cause us to misunderstand a_i as "*relevant opportunities*" per "*people in location i seeking opportunities*". Instead, as mathematically expressed in the proof, a_i is a proportion of the opportunities available to the population, since multiplying a_i by the population at i and summing for all origins in the system equals to the total number of opportunities in the system. Embedded in a_i is already the travel behaviour so P_i and O_j must be plainly understand as population at i and opportunities at j to have Equation (6) hold true.

2.3. Shen's synthetic example

In this section we use the example in Shen (1998) to detail the importance of understanding P_i and O_j as simply the population at the origin i and the opportunities at destination j respectively. This is critical to understanding how the opportunities are allocated to the population based on the impedance function.

comebackhere

Table 1 contains the information needed to calculate S_i and a_i for this example. We use a negative exponential impedance function with $\beta = 0.1$ as done in Shen (1998, see footnote (5)):

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

In Table 1, we see that population centers A and B have equal Hansen-type accessibility ($S_A = S_B = 27,292$ jobs). On the other hand, the isolated satellite town of C has low accessibility ($S_C = 2,240$ jobs). But center B , despite its high accessibility, is a large population center. C , in contrast, is smaller but also relatively isolated and has a balanced ratio of jobs (10,0000 jobs) to population (10,000 people). It is difficult from these outputs to determine whether the accessibility at C is better or worse than that at A or B .

The results are easier to interpret when we consider Shen-type accessibility. The results indicate that $a_A \approx 1.337$ jobs per capita, $a_B \approx 0.888$, and $a_C \approx 0.996$. The latter value is sensible given the jobs-population balance of C . Center A is relatively close to a large number of jobs (more jobs than the population of A). The opposite is true of B . According to Shen (1998), the sum of the

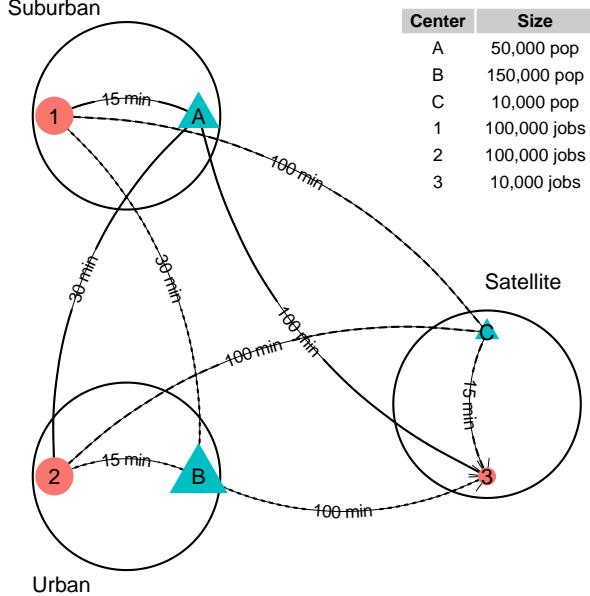


Figure 1: Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

population-weighted accessibility a_i is exactly equal to the number of jobs in the region following Shen's proof:

$$\begin{aligned}
 \sum_{i=1}^N a_i P_i &= \sum_{j=1}^J O_j \\
 50,000 \times 1.3366693 & \\
 + 150,000 \times 0.8880224 & \\
 + 10,000 \times 0.9963171 &= 210,000
 \end{aligned}$$

As mentioned earlier, this property under Shen's definition of P_i “*people in location i seeking opportunities*”, gives the impression that all jobs sought are allocated to the people located at each origin i . In other words, Shen defines P_i to mean P_{ij}^* (i.e., the *effective opportunity-seeking population* which is already adjusted by travel behaviour) instead of defining it to simply be the full population at i (i.e., P_i). As seen in column **Pop * f(TT)** in Table 1 (i.e., $P_{ij}^* = P_i \cdot f(c_{ij})$), the number of individuals from population center A that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2.27 respectively. Therefore, the total effective opportunity-seeking population at A is $P_A^* = \sum_j P_{Aj}^*$, that is, 13,647.27 people, which is considerably lower than the total population of A (i.e., $P_A = 50,000$ people). Demonstrated as follows, using P_{ij}^* in the calculations associated with this proof results in only 56,834.59 jobs being allocated to the population, instead of the nominal number of jobs

Table 1: Summary description of synthetic example: Hansen-type accessibility S_i , Shen-type accessibility a_i , and spatial availability V_i with beta = 0.1 (light yellow) and beta = 0.6 (light grey).

Origin	A			B			C		
Destination	1	2	3	1	2	3	1	2	3
Pop.	50000	50000	50000	150000	150000	150000	10000	10000	10000
Jobs	100000	100000	10000	100000	100000	10000	100000	100000	10000
TT	15	30	100	30	15	100	100	100	15
f(TT)	0.223	0.050	< 0.001	0.050	0.223	< 0.001	< 0.001	< 0.001	0.223
Pop * f(TT)	11156.5	2489.4	2.3	7468.1	33469.5	6.8	0.5	0.5	2231.3
Jobs * f(TT)	22313.0	4978.7	0.5	4978.7	22313.0	0.5	4.5	4.5	2231.3
S_i	27292.2	27292.2	27292.2	27292.2	27292.2	27292.2	2240.4	2240.4	2240.4
a_i	1.337	1.337	1.337	0.888	0.888	0.888	0.996	0.996	0.996
f(TT)	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
Pop * f(TT)	6.170	< 0.001	< 0.001	0.002	18.511	< 0.001	< 0.001	< 0.001	1.234
Jobs * f(TT)	12.341	0.002	< 0.001	0.002	12.341	< 0.001	< 0.001	< 0.001	1.234
S_i	12.343	12.343	12.343	12.343	12.343	12.343	1.234	1.234	1.234
a_i	1.999	1.999	1.999	0.667	0.667	0.667	1.000	1.000	1.000
F^c	0.238	0.238	0.238	0.714	0.714	0.714	0.048	0.048	0.048
F^p	0.817	0.182	< 0.001	0.182	0.817	< 0.001	< 0.001	< 0.001	1.000
F	0.599	0.069	0.001	0.401	0.931	0.003	< 0.001	< 0.001	0.996
V_ij	59900.6	6922.7	10.1	40096.9	93076.0	30.4	2.4	1.3	9959.5
V_i	66833.5	66833.5	66833.5	133203.4	133203.4	133203.4	9963.2	9963.2	9963.2

in the region that is over three times this number (i.e., 210,000 jobs).

$$\begin{aligned}
& \sum_{i=1}^N a_i P_{ij}^* = \\
& (11,156.51 + 2,489.35 + 2.26) \times 1.3366693 \\
& +(7,468.06 + 33,469.52 + 6.81) \times 0.8880224 \\
& +(4.54 + 4.54 + 2,231.20) \times 0.9963171 \approx 56,834.59
\end{aligned}$$

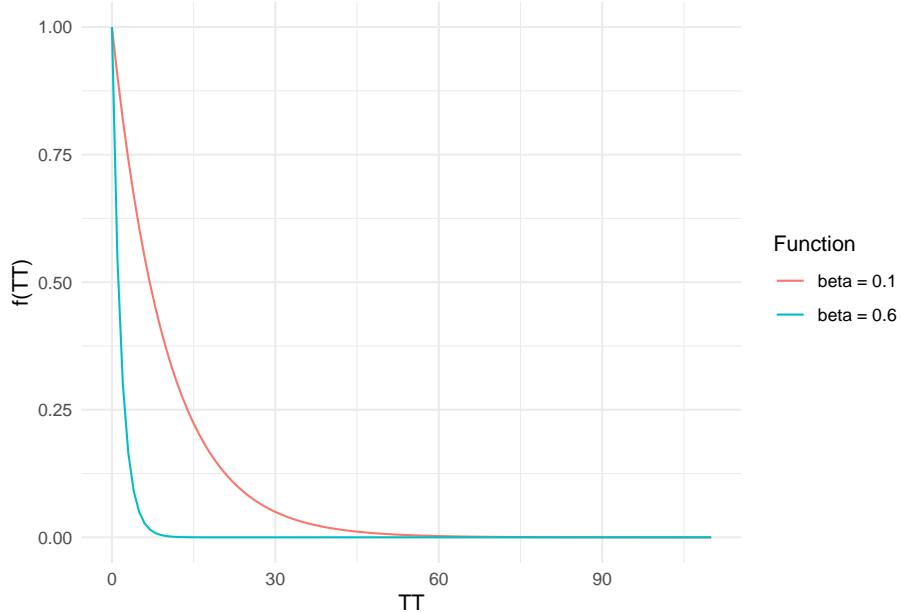


Figure 2: Comparison of two negative exponential impedance functions used in the synthetic example. The x-axis represents the travel time (in mins) and the y-axis represents the impedance function at each travel time.

Furthermore, even when Shen's P_i is understood plainly as the total population at i , the meaning of the proof may still be ambiguous. The proof can still give the impression that all jobs are allocated to the total population since total population ($\sum_{i=1}^N P_i$) goes into the equation and total jobs ($\sum_{j=1}^J O_j$) in the region is the result. However, this impression is incomplete since it does not reflect the amount of population which takes jobs and the number of people being considered for jobs; these magnitudes are a product of being weighted down by the impedance function. These magnitudes are not obvious from a_i is because the result is presented as a rate (i.e., opportunities per capita).

Let us consider a modification to the travel behaviour of the example discussed to illustrate how the presentation of a_i as a rate obscures the magnitude of the effective opportunity-seeking population. We modify the example by increasing the β to 0.6 (compared to the previous value of 0.1; see Figure 2). This modification increases the cost of travel and thus the impedance function, which is an expression of the population's relative willingness to travel to opportunities, reflects a population which is relatively less willing to travel to opportunities further away compared to the previous β value. The Hansen-type and Shen-type values are presented in the yellow rows of Table 1.

As expected, Hansen-type accessibility drops quite dramatically after this β modification: the friction of distance is so high that few opportunities are within reach. In contrast, Shen-type accessibility converges to the jobs:population ratio

(i.e., origin A is $\frac{100,000}{50,000} = 2$). This is explained by the way the impedance function excludes the population in droves, thus reducing the competition for jobs: as seen in Table 1, the effective opportunity-seeking population from A is only about equal to 6.17; likewise, the number of jobs at 1 weighted by the impedance is only 12.341. In other words, competition is low because jobs are expensive to reach, but those willing to reach jobs enjoy relatively high accessibility (in the limit, the jobs/population ratio). On the other hand, the accessibility is effectively zero for those in the population prevented by the impedance from reaching any jobs.

In what follows, we propose an alternative derivation of Shen (1998) accessibility with competition that explicitly clarifies the opportunities allocated to the *effective opportunity-seeking population* within its formulation. Hence, the results are not only more interpretable, but also extend the potential of accessibility analysis.

3. Introducing spatial availability: a singly-constrained measure of accessibility

In brief, we define the *spatial availability* at i (V_i) as the proportion of all opportunities O that are allocated to i from all destinations j :

$$V_i = \sum_{j=1}^N O_j F_{ij}^t$$

where:

- F_{ij}^t is a balancing factor that depends on the population and cost of movement in the system.
- O_j is the number of opportunities at j .
- V_i is the number of spatially available opportunities from the perspective of i .

The general form of spatial availability is also as a sum, and the fundamental difference with Hansen- and Shen-type accessibility is that opportunities are allocated proportionally. Balancing factor F_{ij}^t consists of two components: a population-based balancing factor F_i^p and an impedance-based balancing factor F_{ij}^c which, respectively, allocate opportunities to i in proportion to the size of the population of the different competing centers (the mass effect of the gravity model) and the cost of reaching opportunities (the impedance effect). In the next two subsections, we explain the intuition behind the method before defining it in full.

3.1. Proportional allocation by population

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the

impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by population. Recall, the total population in the example is 210,000. The proportion of the population by population center is as follows:

$$F_A^p = \frac{50,000}{210,000}$$

$$F_B^p = \frac{150,000}{210,000}$$

$$F_C^p = \frac{10,000}{210,000}$$

Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors F_i^p . In this way, employment center 1 allocates $100,000 \cdot \frac{50,000}{210,000} = 23,809.52$ jobs to A ; $100,000 \cdot \frac{150,000}{210,000} = 71,428.57$ jobs to B ; and $100,000 \cdot \frac{10,000}{210,000} = 7,142.857$ jobs to C . Notice how this mechanism ensures that the total number of jobs at employment center 1 is preserved at 100,000.

We can verify that the number of jobs allocated is consistent with the total number of jobs in the system:

Employment center 1 to population centers A, B, and C:

$$100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$$

Employment center 2 to population centers A, B, and C:

$$100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$$

Employment center 3 to population centers A, B, and C:

$$10,000 \cdot \frac{50,000}{210,000} + 10,000 \cdot \frac{150,000}{210,000} + 10,000 \cdot \frac{10,000}{210,000} = 10,000$$

In the general case where there are N population centers in the region, we define the following population-based balancing factors in Equation (7):

$$F_i^p = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \quad (7)$$

Balancing factor F_i^p corresponds to the proportion of the population in origin i relative to the population in the region. On the right hand side of the equation, the numerator P_i^α is the population at origin i . The summation in the denominator is over $i = 1, \dots, N$, and adds up to the total population of the region. Notice that we incorporate an empirical parameter α . The role of α is to modulate the effect of demand by population. When $\alpha < 1$, opportunities are allocated more rapidly to smaller centers relative to larger ones; $\alpha > 1$ achieves the opposite effect.

Balancing factor F_i^p can now be used to proportionally allocate a share of available jobs at j to origin i . The number of jobs available to i from j balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

In the general case where there are J employment centers, the total number of jobs available from all destinations to i is simply the sum of V_{ij}^p over $j = 1, \dots, J$:

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

Since the factor F_i^p , when summed over $i = 1, \dots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^p = 1$), the sum of all spatially available jobs equals O , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^p &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\ &= \sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

301 The terms F_i^p act here as the balancing factors of the gravity model when a
302 single constraint is imposed (i.e., to ensure that the sums of columns are equal
303 to the number of opportunities per destination, see Ortúzar and Willumsen,
304 2011, pp. 179–180 and 183–184). As a result, the sum of spatial availability for
305 all population centers equals the total number of opportunities.

306 The discussion so far concerns only the mass effect (i.e., population size)
307 of the gravity model. In addition, the potential for interaction is thought to
308 decrease with increasing cost, so next we define similar balancing factors but
309 based on the impedance.

310 3.2. Proportional allocation by cost

Clearly, using only balancing factors F_i^p to calculate spatial availability V_i^p does not account for the cost of reaching employment centers. Consider instead a set of balancing factors F_{ij}^c that account for the friction of distance for our example. Recall that the impedance function $f(c_{ij})$ equals $\exp(-\beta \cdot c_{ij})$ where $\beta = 0.1$ and travel time c_{ij} is either 15, 30 or 60 minutes. For instance, the impedance-based balancing factors F_{ij}^c would be the following for employment center 1 (employment center 2 and 3 have their own balancing factor values for each origin i as will be discussed later):

$$\begin{aligned} F_{A1}^c &= \frac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\ F_{B1}^c &= \frac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\ F_{C1}^c &= \frac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581 \end{aligned}$$

311 Balancing factors F_{ij}^c use the impedance function to proportionally allocate
312 more jobs to closer population centers, that is, to those with populations *more*
313 *willing to reach the jobs*. Indeed, the factors F_{ij}^c can be thought of as the
314 proportion of the population at i willing to travel to destination j , conditional on
315 the travel behavior as given by the impedance function. For instance, 81.74398%
316 of jobs from employment center 1 are allocated to population center A based
317 on impedance.

318 So as follows from our example, of the 100,000 jobs at employment center 1
 319 the number of jobs allocated to population center A is $100,000 \times 0.8174398 =$
 320 81,743.98 jobs; the number allocated to population center B is $100,000 \times$
 321 $0.1823954 = 18,239.54$ jobs; and the number allocated to population center
 322 C is $100,000 \times 0.0001648581 = 16.48581$ jobs. We see once more that the total
 323 number of jobs at the employment center is preserved at 100,000. In this ex-
 324 ample, the proportional allocation mechanism assigns the largest share of jobs
 325 to population center A , which is the closest to employment center 1, and the
 326 smallest to the more distant population center C .

327 In the general case where there are N population centers and J employ-
 328 ment centers in the region, we define the following impedance-based balancing factors:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \quad (8)$$

The total number of jobs available to i from j according to impedance is defined as follows:

$$V_{ij}^c = O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

The total number of jobs available to i from all destinations is:

$$V_i^c = \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

Like the population-based allocation factors, F_i^c summed over $i = 1, \dots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_{ij}^c = 1$). As before, the sum of all spatially available jobs equals O , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^c &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c} \\ &= \sum_{i=1}^N \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

329 We are now ready to more formally define spatial availability with due con-
 330 sideration to both population and travel cost effects.

331 3.3. Assembling mass and impedance effects

332 Population and the cost of travel are both part of the gravity modelling
 333 framework. Since the balancing factors defined in the preceding sections are
 334 proportions (alternatively, can be understood as probabilities), they can be
 335 combined multiplicatively to obtain their joint effect. This multiplicative rela-
 336 tionship can alternatively be understood as the joint probability of allocating
 337 opportunities and is captured by Equation (9), where F_i^p is the population-
 338 based balancing factor that grants a larger share of the existing opportunities
 339 to larger centers and F_{ij}^c is the impedance-based balancing factor that grants a

³⁴⁰ larger share of the existing opportunities to closer centers. This is in line with
³⁴¹ the tradition of gravity modeling.

$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \quad (9)$$

³⁴² with F_i^p and F_{ij}^c as defined in Equations (7) and (8) respectively. The combined
³⁴³ balancing factor F_{ij}^t is used to proportionally allocate jobs from j to i . Hence,
³⁴⁴ spatial availability is given by Equation (10).

$$V_i = \sum_{j=1}^J O_j F_{ij}^t \quad (10)$$

³⁴⁵ The terms in Equation 10 are as follows:

- ³⁴⁶ • F_{ij}^t is a balancing factor as defined in Equation (9).
- ³⁴⁷ • i is a set of origin locations in the region $i = 1, \dots, N$.
- ³⁴⁸ • j is a set of destination locations in the region $j = 1, \dots, J$.
- ³⁴⁹ • O_j is the number of opportunities at location j .
- ³⁵⁰ • V_i is the spatial availability at i .

Notice that, unlike S_i in Hansen-type accessibility (Equation (1)), the population enters the calculation of V_i through F_i^p . Returning to the example in Figure 1, Table 1 also contains the information needed to calculate V_i , with β set again to 0.1. Column **V_ij** are the jobs available to each origin from each employment center. In this column $V_{A1} = 59,901$ is the number of jobs available at A from employment center 1. Column **V_i** (i.e., $\sum_{j=1}^J V_{ij}$) gives the total number of jobs available to origin i . We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

³⁵¹ Compare the calculated values of V_i to column **S_i** (Hansen-type accessibility)
³⁵² in Table 1. The spatial availability values are more intuitive. Recall
³⁵³ that population centers A and B had identical Hansen-type accessibility to em-
³⁵⁴ployment opportunities. According to V_i , population center A has greater job
³⁵⁵availability due to: 1) its close proximity to employment center 1; combined
³⁵⁶ with 2) less competition (i.e., a majority of the population have to travel longer
³⁵⁷distances to reach employment center 1). Job availability is lower for popula-
³⁵⁸tion center B due to much higher competition (150,000 people can reach 100,000
³⁵⁹jobs at equal cost). And center C has almost as many jobs available as it has
³⁶⁰population.

³⁶¹ As discussed above, Hansen-type accessibility is not designed to preserve
³⁶² the number of jobs in the region. Shen-type accessibility ends up preserving

363 the number of jobs in the region but the definitions of variables are internally
 364 obscured; the only way it preserves the number of jobs is if the effect of the
 365 impedance function is ignored when expanding the values of jobs per capita to
 366 obtain the total number of opportunities. The proportional allocation procedure
 367 described above, in contrast, consistently returns a number of jobs available that
 368 matches the total number of jobs in the region.

369 Since the jobs spatially available are consistent with the jobs in the region,
 370 it is possible to define a measure of spatial availability per capita as presented
 371 in Equation (11):

$$v_i = \frac{V_i}{P_i} \quad (11)$$

372 And, since the jobs are preserved, it is possible to use the regional jobs per
 373 capita ($\frac{\sum_{j=1}^J O_j}{\sum_{i=1}^N P_i}$) as a benchmark to compare the spatial availability of jobs per
 374 capita at each origin.

375 In the example, since the population is equal to the number of jobs, the
 376 regional value of jobs per capita is 1.0. To complete the illustrative example,
 377 the spatial availability of jobs per capita by origin is:

$$\begin{aligned} v_1 &= \frac{V_1}{P_1} = \frac{66,833.47}{50,000} = 1.337 \\ v_2 &= \frac{V_2}{P_2} = \frac{133,203.4}{150,000} = 0.888 \\ v_3 &= \frac{V_3}{P_3} = \frac{9,963.171}{10,000} = 0.996 \end{aligned} \quad (12)$$

378 We can see that population center A has fewer jobs per capita than the
 379 regional benchmark, center B has more, and center C is at parity. Remarkably,
 380 the spatial availability per capita matches the values of a_i in Table 1. Appendix
 381 A has a proof of the mathematical equivalence between the two measures. It
 382 is interesting to notice how Weibull (1976), Shen (1998), as well as this paper,
 383 all reach identical expressions starting from different assumptions; this effect is
 384 known as *equifinality* (see Ortúzar and Willumsen, 2011, p. 333; and Williams,
 385 1981). This result means that Shen-type accessibility and 2SFCA can be re-
 386 conceptualized as singly-constrained accessibility measures.

387 3.4. Why does proportional allocation matter?

388 We have shown that Shen-type accessibility and spatial availability produce
 389 equifinal results when accessibility per-capita is computed. At this point it is
 390 reasonable to ask whether the distinction between these two measures is of any
 391 importance.

392 Conceptually, we would argue that the confounded populations in Shen-type
 393 accessibility leads to internal inconsistency in the calculation of total opportu-
 394 nities in Shen (1998): this points to a deeper issue that is only evident when
 395 we consider the intermediate values of the method. To illustrate, Table 1 shows
 396 results of a_i that are reasonable (and they match exactly the spatial availability
 397 per capita). But when we dig deeper, these results mask potentially misleading
 398 values for the jobs allocated and the number of jobs taken. For instance, a

399 region with a high jobs:population ratio but a prohibitive transportation net-
 400 work that results in a high cost of travel may yield a high a_i value. This value,
 401 however, can conceal a low *effective opportunity-seeking population* and propor-
 402 tionally low number of allocated jobs while additionally obscuring the number
 403 of population which does *not* take jobs and the jobs *not* taken.

404 In addition, the intermediate accessibility values of a_i (Shen-type measure)
 405 may also lead to impact estimates that are deceptive (see Sarlas et al., 2020). For
 406 example, the estimated region-wide cost of travel considering the jobs allocated
 407 by a_i in Table 1 (i.e., $Jobs * f(TT)$) is as follows:

$$\begin{aligned}
 & 22,313 \times 15 \text{ min} + 4,979 \times 30 \text{ min} + 0.454 \times 100 \text{ min} \\
 & 4,979 \times 30 \text{ min} + 22,313 \times 15 \text{ min} + 0.454 \times 100 \text{ min} \\
 & 4.54 \times 100 \text{ min} + 4.54 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,594 \text{ min}
 \end{aligned}$$

In contrast, the estimated region-wide cost of travel according to V_i in Table 1 is as follows:

$$\begin{aligned}
 & 59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min} \\
 & 40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min} \\
 & 2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min}
 \end{aligned}$$

408 Often referred to as ‘the supply of jobs’ (or simply Hansen-style accessibil-
 409 ity) in the Shen-type measure: $Jobs * f(TT)$ cannot be used to understand the
 410 region-wide cost of travel. Recall how we define $Pop * f(TT)$ as the *effective*
 411 *opportunity-seeking population* (P_{ij}^*), $Jobs * f(TT)$ similarly represents the *ef-*
 412 *fective opportunities allocated* and sums to approximately 56,824 out of a
 413 total of 210,000 jobs. Like $Pop * f(TT)$, the *effective opportunities allocated*
 414 to each origin is only a reflection of the impedance function and not the *ac-*
 415 *tual* number of opportunities allocated to each origin. Therefore, the resulting
 416 1,002,594 min is not a meaningful measure of the cost of travel in the system.

417 However, since spatial availability allocates the *actual* number of opportuni-
 418 ties to each origin; the 3,859,054 min can be used to quantify the system-wide
 419 impacts of competitive accessibility in this region. We know spatial availability’s
 420 output is the number of opportunities at each i since the combined balancing
 421 factors allocate a proportional amount of the total opportunities to each i such
 422 that the number of opportunities allocated to each i sum to equal the total
 423 opportunities in the region.

424 4. Empirical example of Toronto

425 In this section we illustrate the application of spatial availability through
 426 an empirical example. For this, we use full-time employment flows from the
 427 Greater Golden Horseshoe (GGH) area in Ontario, Canada. Contained with
 428 the GGH is the Greater Toronto and Hamilton (GTHA) which forms the most
 429 populous metropolitan regions in Canada and the core urban agglomeration in
 430 the GGH.

431 The GTHA contains the city of Toronto, the most populous city in Canada.
432 The city of Toronto is the focus of this empirical example, it will be used to
433 demonstrate the application of the proposed spatial availability measure along
434 with how it compares to Hansen- and Shen-type measures. We begin this section
435 by explaining the data and then detailing the calculated comparisons.

436 *4.1. GGH Data*

437 We obtained full-time employment flows from the 2016 Transportation To-
438 morrow Survey (TTS). This survey collects representative urban travel infor-
439 mation from 20 municipalities contained within the GGH area in the southern
440 part of Ontario, Canada (see Figure 3) (Data Management Group, 2018) ev-
441 ery five years. The data set includes origin to destination flows associated
442 with full-time employment trips; the number of jobs ($n=3,081,885$) and work-
443 ers ($n=3,446,957$) (i.e., the number of originating trips and destination trips)
444 at each origin and destination are represented at the level of Traffic Analysis
445 Zones (TAZ) ($n=3,764$). TAZ are a unit of spatial analysis which are defined
446 as part of the TTS, however, TAZ are commonly used to ascribe production
447 and attraction of trips in the context of transportation planning modelling. In
448 the GGH data set, the TAZ contain on average 916 workers and jobs 819 with
449 more detailed descriptive statistics discussed later. The TTS data is based on
450 a representative sample of between 3% to 5% of households in the GGH and
451 is weighted to reflect the population covering the study area as a whole (Data
452 Management Group, 2018).

453 To generate the travel cost for the full-time employment trips, travel times
454 between origins and destinations (i.e., centroids of the TAZ) are calculated for
455 car travel using the R package {r5r} (Rafael H. M. Pereira et al., 2021) with
456 a street network retrieved from OpenStreetMap. It is also assumed that intra-
457 TAZ trips are equal to 0.1 minutes. For inter-TAZ trips, a 3 hr travel time
458 threshold was selected as it captures 99% of population-employment pairs (see
459 the travel times summarized in Figure 3). This method does not account for
460 traffic congestion or modal split, which can be estimated through other means
461 (e.g., Allen and Farber, 2021; Higgins et al., 2021). For simplicity, we carry
462 on with the assumption that all trips are taken by car in uncongested travel
463 conditions. All data and data preparation steps are documented and can be
464 freely explored in the companion open data product {TTS2016R}.

465 *4.2. Spatial employment characteristics in Toronto*

466 As mentioned, the focus of this empirical example is on the city of Toronto.
467 It is the largest city in the GGH and represents a significant subset of workers
468 and jobs in the GGH; 22% of workers in the GGH live in Toronto and 25% of
469 jobs that these workers take are located within Toronto. The spatial distribution
470 of jobs and workers is shown in Figure 4. It can be seen that a large cluster
471 of jobs can be found in the central southern part of Toronto (the downtown
472 core). Spatial trends in the distribution of workers is more even relative to the
473 distribution of jobs.

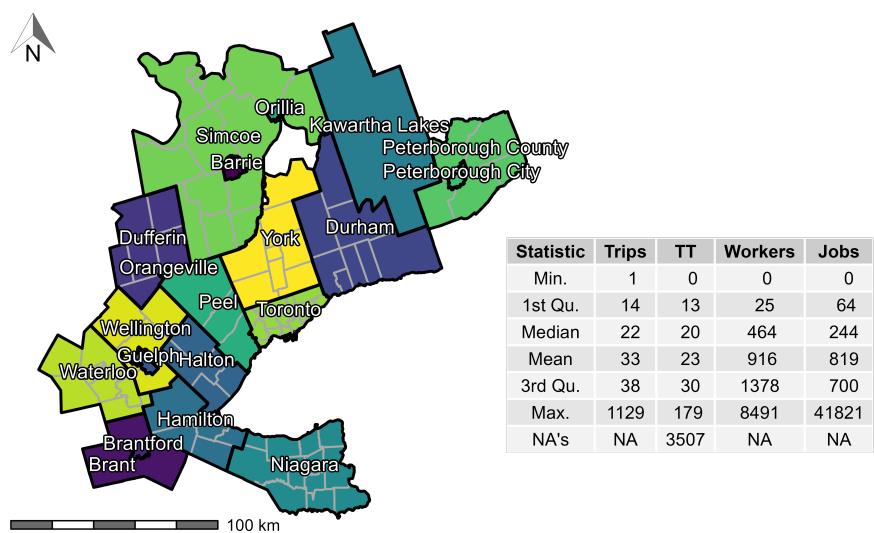


Figure 3: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

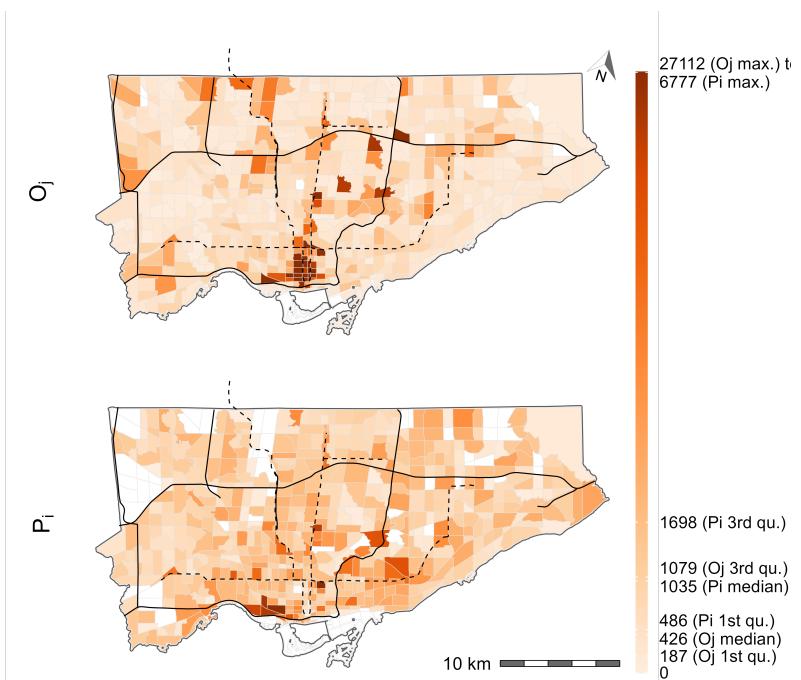


Figure 4: Spatial distribution of full-time jobs (top) and full-time working population (bottom) at each TAZ for Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs.

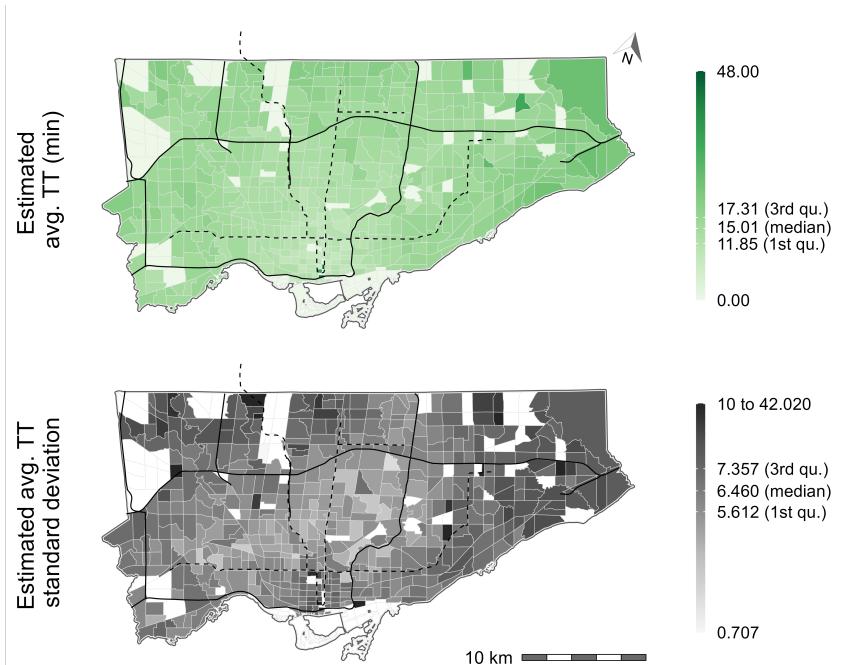


Figure 5: Spatial distribution of full-time working population to jobs ratio (top) and car travel time to jobs estimated using R5R (bottom) for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. White TAZ represent a TAZ with no workers thus no travel time for the top plot, for the bottom plot they represent no travel time TAZ and TAZ with only 1 travel time.

474 Next, the spatial distribution of the estimated car travel time (green) and the
 475 associated standard deviation (grey) is visualized in Figure 5. It can be seen that
 476 the car travel time is lower within the downtown core and, unexpectedly higher
 477 as the TAZ is further from the downtown core. These travel time estimations
 478 are to be expected, as these car travel time are calculated using an uncongested
 479 OpenStreetMaps road network from the centroid of origin TAZ to destination
 480 TAZ. Since within Toronto trips are only considered, trips which originate from
 481 the center of Toronto, an area with high job density, relatively closer proximity
 482 to all other Toronto TAZ, and high road connectivity, travel times are lower than
 483 outside in an areas further from the downtown core. In terms of the variability
 484 of the travel times, the center TAZ of Toronto have lower variability than TAZ
 485 closer to the borders of Toronto. Trends from both plots indicate that trips
 486 originating from within the center of Toronto are shorter and more similar in
 487 length than traips originating from closer to the border of Toronto.

488 Nonetheless, the point of these visualizations is to demonstrate the spatial
 489 distribution of worker and job data in the city of Toronto to contextualize spatial
 490 availability and Shen- and Hansen- type measures.

491 *4.3. Calibration of an impedance function for Toronto*

492 In the synthetic example introduced before, we used a negative exponential
 493 function with the parameter reported by Shen (1998). For the empirical Toronto
 494 data set, we calibrate an impedance function on the trip length distribution
 495 (TLD) of commute trips. Briefly, a TLD represents the proportion of trips
 496 that are taken at a specific travel cost (e.g., travel time); this distribution is
 497 commonly used to derive impedance functions in accessibility research (Batista
 498 et al., 2019; Horbachov and Svichynskyi, 2018; Lopez and Paez, 2017).

499 As mentioned, the calculations are undertaken for the city of Toronto using
 500 only the employed population in the city and jobs taken by residents of Toronto.
 501 Specifically, edge trips are not included such as trips originating in Toronto but
 502 finishing outside of Toronto and trips originating outside of Toronto but finish-
 503 ing in Toronto. The empirical and theoretical TLD for this Toronto data set
 504 are represented in the top-left panel of Figure 6. Maximum likelihood estima-
 505 tion and the Nelder-Mead method for direct optimization available within the
 506 `{fitdistrplus}` package (Delignette-Muller and Dutang, 2015) were used. Based
 507 on goodness-of-fit criteria and diagnostics the normal distribution was selected
 508 (see Figure 6).

509 The normal distribution is defined in Equation (13), where we see that it
 510 depends on a mean parameter μ and a standard deviation parameter σ . The
 511 estimated values of these parameters are $\mu = 14.169$ and $\sigma = 7.369$.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad (13)$$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

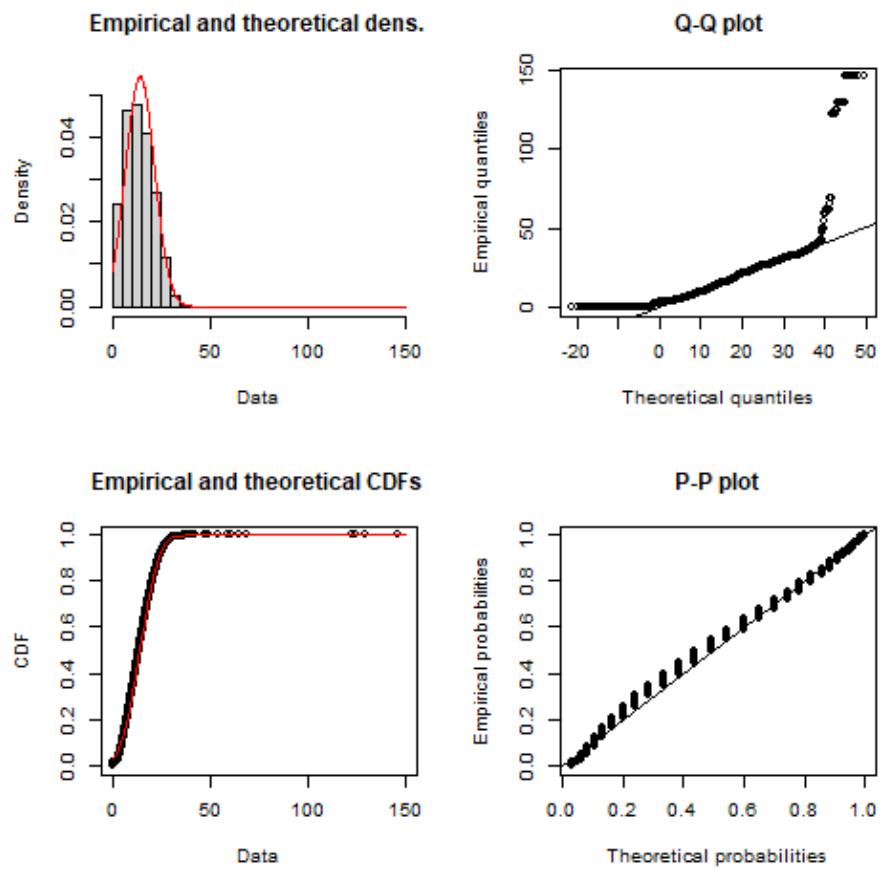


Figure 6: Car trip length distribution and calibrated normal distribution impedance function (red line) with associated Q-Q and P-P plots. Based on the estimated car travel times for full-time employment and workers in Toronto from the TTS 2016.

512 4.4. Accessibility and spatial availability of jobs in Toronto

513 4.4.1. Absolute opportunity values

514 Figure 7 contains the number of jobs accessible using Shen-type accessibility,
515 Hansen-type accessibility, and the number of jobs *available* using the spatial
516 availability measure. The values from all these measures are represented on
517 the same axis as they are comparable as they measure the absolute value of
518 *jobs* accessible to the workers in the origin. In the top plot, the Shen-type
519 accessibility is multiplied by the *effective opportunity-seeking population* to yield
520 a value that corresponds to absolute number of accessible jobs (considering
521 competition) according to Shen's definition. In the middle plot, the Hansen-
522 type accessibility is an unconstrained case of accessibility in which all jobs which
523 are in-reach of each origin (according to the impedance function); each value
524 corresponds to the number of jobs which can be reach at each origin assuming
525 no competition. Lastly, in the bottom plot, the spatial availability measure is a
526 constrained case of accessibility which yields the number of jobs, at each origin,
527 considering competition from the population in nearby origin and the relative
528 travel cost (according to the impedance function).

529 What is notable about the bottom plot is that the proportional allocation
530 mechanism of spatial availability ensures that the job availability value for each
531 origin all sums to the city-wide total of 769,231 jobs (i.e., the number of des-
532 tination flows from Toronto origins to Toronto destinations). The number of
533 accessible jobs at each origin can therefore be interpreted as the number of
534 *available* jobs to each origin based on the relative travel behaviour and density
535 of competition for jobs (i.e., worker population). A proportion of each of the
536 769,231 jobs in Toronto are only allocated once to each origin. In terms fo
537 the middle plot, the city-wide total for Hansen-type accessibility is 4,366,743
538 jobs, which as a value is meaningless since the measure is unconstrained; it
539 represents the sum of opportunities that have been counted anywhere from 1
540 to many times depending on the impedance function. As previously discussed,
541 unconstrained counting of the same opportunity by all origins is not an issue
542 if the opportunity itself is non-exclusive, but since one job can only be given
543 to one worker (especially since the worker and job data is derived from origin-
544 destination flows), it is inappropriate to use unconstrained measures to capture
545 employment characteristics. Comparing the middle and bottom plots, it is evi-
546 dent that the unconstrained counting of opportunities (Hansen-style) results in
547 absolute values that are higher throughout the city, particularly in TAZ that
548 are in proximity to high job density (recall Figure 4). These same trends are
549 not present in the spatial availability bottom plot, as the absolute value is lower
550 than Hansen-style accessibility as the proximity to high job density and com-
551 petition from worker density is proportionally metered; the resulting values are
552 thus lower than the middle plot and reflect the spatial distribution trends of
553 both the workers and job density (recall Figure 4).

554 Lastly, the top plot that visualizes the *absolute* Shen-type measure (as un-
555 derstood by Shen's definition of P_i being equal to P_{ij}^* sums to the city-wide
556 value of 2,117,774 by multiplying a_i by the *effective opportunity-seeking popu-*

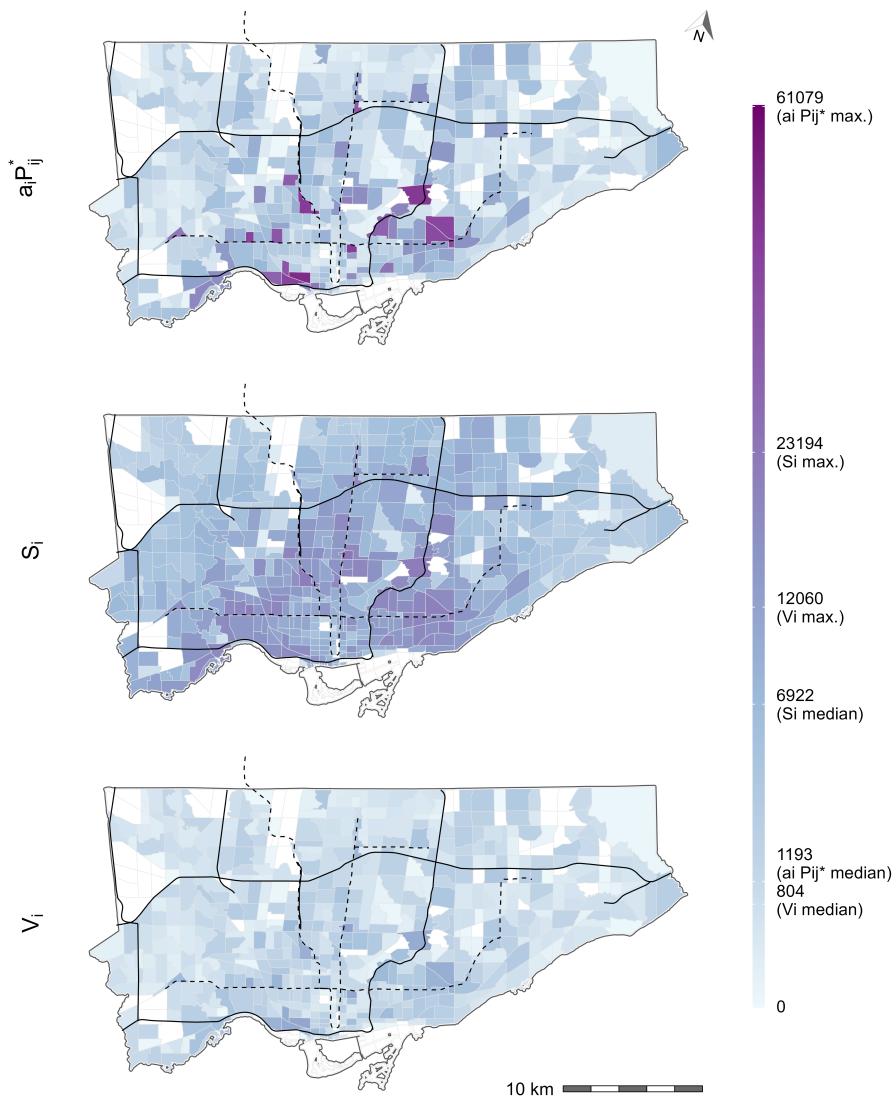


Figure 7: Estimated accessibility to jobs (# of jobs) in Toronto according to Shen-type measure times effective opportunity-seeking population (top), Hansen-type measure (middle), and spatial availability (bottom). Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values. Legend scale is square root transformed to effectively visualize the spread range.

557 *lation* P_{ij}^* (i.e., the denominator of the rate). This plot thus demonstrates how
 558 confounding P_i with P_{ij}^* yields an *incorrect* number of competitively accessible
 559 jobs: it is evidently incorrect because the sum of $a_i P_{ij}^*$ greatly exceeds the city-
 560 wide total of workers (i.e., $2,117,774 > 769,231$). To the authors' knowledge,
 561 literature has not attempted to convert Shen-type accessibility to the absolute
 562 value of accessible jobs in the way demonstrated in the top plot: we suspect
 563 this is the case because of the ambiguous definition that conflates P_{ij}^* with P_i .
 564 If a_i is multiplied by P_i , it yields the same value as V_i , but since the definition
 565 of Shen-type measure is equivocal doing so is not clear since the denominator of
 566 a_i (which is a rate) is *not* P_i . The resulting plot, spatially, is similar to spatial
 567 availability (bottom plot) but certain TAZ have exceptionally high values in
 568 an inconsistent way. This is because a_i uses the impedance function values for
 569 both access to jobs (numerator) and the competition from neighboring workers
 570 (denominator P_{ij}^*) to adjust their impact: using P_{ij}^* does not *consistently* isolate
 571 the absolute value of accessible jobs. However, if a_i is multiplied by P_i it yields
 572 the same values at V_i (bottom plot) (the proof for mathematically equivalency is
 573 in Appendix A). As also mentioned earlier, the formulation of the denominator
 574 and numerator of a_i is ambiguous so to presume that multiplying it by P_i would
 575 disintangle the rate and yield the absolute value of accessible *and available* (i.e.,
 576 considering competition) jobs is unclear.

577 4.4.2. Internal values

578 Carrying on the discussion on how to retrieve the absolute value of *available*
 579 jobs using the Shen-type measure (a_i), Figure 8 highlights how the differences
 580 between P_i and P_{ij}^* are not uniform across space; the values at each origin are
 581 equivalent to $\sum_j f(c_{ij})$. Recall, P_i is the number of workers at each TAZ (city-
 582 wide sum of 769,231) while P_{ij}^* is the number of workers who *seek* jobs (city-wide
 583 sum of 1,770,609) in that TAZ based on their travel behaviour. P_{ij}^* is an internal
 584 value of a_i and the top plot presents the ratio of P_{ij}^* to P_i which reflects how the
 585 effective opportunity-seeking population is sometimes inflated (i.e., impedance
 586 values is greater than 1) and others deflated (i.e., impedance value is less than
 587 1) by the Shen-type measure (a_i). As such, using P_{ij}^* to untangle the absolute
 588 job availability from a_i instead of P_{ij}^* can lead to exaggerating the total travel
 589 time in the city since it does not represent the *actual* number of workers but
 590 the *effective* number of workers. For instance, when trying to calculate the city-
 591 wide travel time using $a_i P_{ij}^*$, Shen-type accessibility yields 499,740.1 [h] instead
 592 of the city-wide travel time of 183,736.8 [h] that corresponds to the *absolute*
 593 (i.e., the total number of jobs in the city is preserved) number of available jobs
 594 from V_i . The absolute number of opportunities cannot be easily disentangled
 595 from a_i .

596 By contrast, not only are the absolute values a direct result of V_i , the inter-
 597 nal combined balancing factor F_{ij}^t (Equation (9)) can be used for analysis. The
 598 bottom plot shows the average F_{ij}^t for each TAZ which is the proportional allo-
 599 cation mechanism of opportunities to origins in the V_i calculation. Practically,
 600 the visualized values corresponds to the average of *proportion* of opportunities
 601 available that are claimed by the zone based on travel behaviour and population

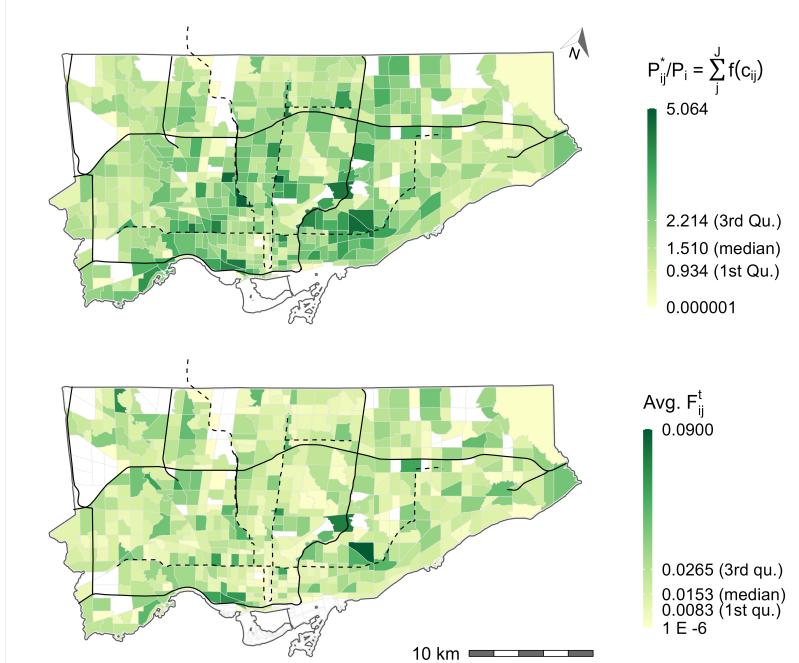


Figure 8: The ratio of the effective opportunity seeking population to the population (top) and the average spatial availability's balancing factor (Equation (9)) (bottom) for Toronto TAZ. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

competition for opportunities. These values can allow the analyst to understand the magnitude of the *proportion of opportunities* that the origin TAZ is assigned based on the opportunities located at reachable destination TAZ. For instance, the TAZ with the maximum value of 0.090 has many origin to destination trips (112 trips, upper 3rd quantile), many workers (5538 workers, upper 3rd quantile), and located centrally within Toronto. Averaging F_{ij}^t demonstrates that this TAZ claims on average a high proportion of jobs from reachable TAZs. This does not necessarily mean TAZ with a high V_i have an exceptionally high average F_{ij}^t ; for instance, many TAZ around the downtown core have high V_i values but do not have exceptionally high average F_{ij}^t . The average F_{ij}^t can thus be used to identify relatively “greedy” areas that could possibly withstand reductions in availability, if that meant increasing spatial availability in areas with a deficit of jobs available. The balancing factor is an interesting feature of spatial availability which opens up avenues for future analysis; alas, there does not seem to be an equivalent for the Shen-type measure.

4.4.3. Benchmarking opportunity availability

Figure 9 presents the number of jobs per capita for Hansen-type accessibility (top plot), the raw number of jobs per capita (middle plot), and the spatially available jobs per capita (bottom plot). In addition to clarifying the meaning of internal values, spatial availability can also be divided by population at each origin and expressed as a rate: this rate can be used as a benchmark for equity analysis and compared directly to the raw number of jobs per capita.

The bottom plot features a value which is mathematically equivalent to Shen-type measure, but with stronger interpretability thanks to the proportional allocation mechanism. This mechanism makes clear that all the opportunities are allocated proportionally to origins, which improves interpretability since the V_i values are the absolute value of *opportunity availability*. The value can thus be directly divided by the population at the origin and expressed as opportunities per capita. When spatial availability is compared to Hansen-type measure (top plot), dividing the output by population directly yield's a more difficult to interpret number of *unconstrained* accessible jobs per capita. For instance, the median light-pink shaded TAZ corresponds to approximately 5.89 unconstrained accessible jobs per capita; this value is difficult to intercept, because as discussed in the introduction, jobs are *exclusive* opportunity types so their accessibility value should take into consideration competition.

The bottom plot displays the spatially available jobs per capita. It can be interpreted as a benchmark its values can be compared directly to the raw number of jobs per capita (middle plot) since the total number of opportunities are preserved (and the population, in this case, is equivalent to the number of opportunities). For instance, a TAZ with a $v_i > 1$ have more *available jobs* (based on travel behaviour and competition) than their working population. This TA has sufficient employment opportunities (under the assumptions of the input data), while TAZ with a $v_i < 1$ do not have sufficient employment opportunities. From an equity perspective, v_i can be used to target where residential housing, job opportunities, and/or transportation system improvements should be created.

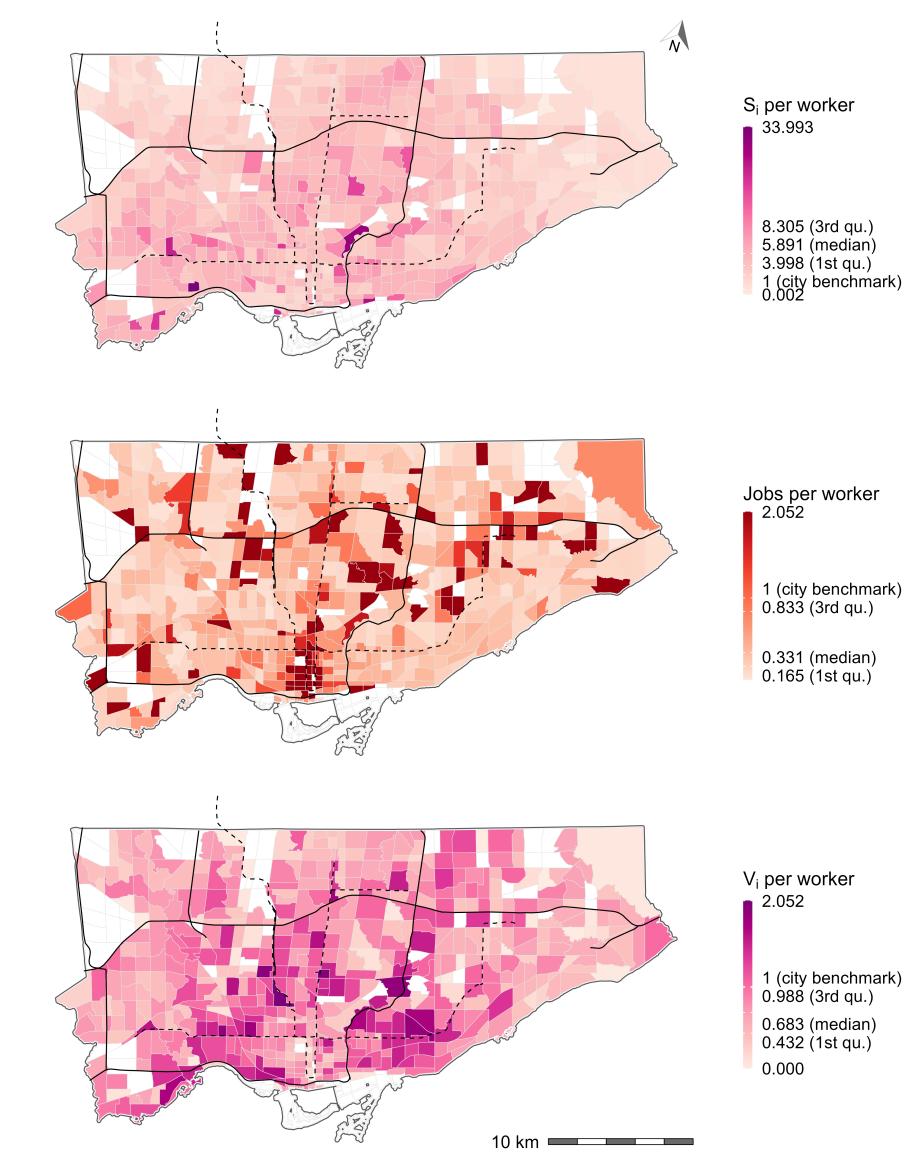


Figure 9: Hansen-type accessible jobs per capita (top), number of jobs to population ratio (middle), and spatially available jobs per capita (bottom) for Toronto. The city benchmark corresponds to total number of jobs in Toronto divided by the number of workers in Toronto, since this is equal the value is 1. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

647 For TAZ with v_i values significantly greater than 1 (dark pinks), constructing
648 more residential housing for the type of workers who occupy the *available jobs*
649 in the proximate TAZ should be considered. Assuming the input data is cor-
650 rect, increasing the competition in the area will decrease the v_i score but if can
651 be decreased up to threshold of $v_i = 1$. For TAZ with v_i values significantly
652 less than 1 (light pinks), constructing more employment opportunities for the
653 type of workers who live in proximate TAZ and/or prioritizing transportation
654 network improvements to create more favourable travel time conditions. De-
655 pending on the raw jobs per worker ratio, different approaches are appropriate.
656 For instance, adding more residential locations near the downtown core (bottom
657 center on the bottom plot) could be a good approach to increasing v_i as there
658 is already a high jobs per worker ratio (middle plot). However, doing so will
659 decrease the v_i availability in areas near the border of the city, so in addition
660 to doing so, adding more employment opportunities to areas with low raw jobs
661 per worker ratio and low v_i is needed. In addition to these changes, the travel
662 time landscape would also influence the resulting v_i score, so transportation
663 network improves to areas with low v_i could also be considered. This is to say,
664 v_i is dependent on the magnitude and spatial distribution of residential housing,
665 job opportunities, and transportation system so the region could be optimized
666 to achieve thresholds of specific v_i values and thus the difference in residential
667 housing, job opportunities, and transportation system can become policy tar-
668 gets. It should also be kept in mind, that though $v_i = 1$ and the comparison
669 to the raw jobs per worker values can be used for policy planning, v_i can easily
670 be transformed back to V_i to understand the magnitude of the job availability
671 within that origin.

672 **5. Conclusion**

673 In this paper we show how a widely used measure of accessibility with com-
674 petition obscures some important internal values of opportunities taken. This
675 is caused by confounding the population of zones with the *effective opportunity-*
676 *seeking population*. We then propose an alternative derivation of accessibility
677 with competition that we call spatial availability. This measure ensures that
678 opportunities are allocated in a proportional way and preserved in the regional
679 total. We also show that spatial availability and Shen-type accessibility are
680 equifinal: formally the equations are the same (along with 2SFCA) and can be
681 consider as singly-constrained measures.

682 Why do differences between Hansen-style measure and the interpretation of
683 Shen-type measure matter? In equity analysis and policy planning, an analyst
684 might be interested in the internal values of their accessibility analysis, for ex-
685 ample travel times, and who pays how much for accessibility. The increased
686 interpretability and internal consistency of spatial availability can help to push
687 accessibility analysis forward. Hansen-type measure tend to result in values
688 which are very extreme as a result of multiple-counting opportunities as shown
689 in empirical example. Multiple-counting may not be an issue if the opportunity-
690 type is non-exclusive, but with the case of employment where one worker can
691 only take one job, the resulting values are difficult to interpret (though it can be
692 interpreted relatively to speak about urban form). In this paper, we also demon-
693 strated how attempting to disentangle the absolute values of opportunities from
694 the Shen-type measure is difficult as a result of Shen's definition which confounds
695 the population with the effective-opportunity seeking population. As demon-
696 strated in this paper, spatial availability increases interpretability by presenting
697 first, the absolute value of *available* jobs and then by dividing the available jobs
698 value by the number of working population. This rate is equivalent to Shen-
699 type measure but contains internal values, such as the proportional allocation
700 mechanism, that yield more realistic estimates of opportunities taken, as well
701 as a set of balancing factors that can be used to better understand the absolute
702 and rate values obtained.

703 Based on this research we suggest the following guidelines for the application
704 of spatial availability and the topic of future work:

- 705 1) The Hansen-style accessibility should be used when opportunities are non-
706 exclusive. When opportunities are perfectly exclusive (i.e., 1 spot for 1
707 person), spatial availability (i.e., accessibility with competition) should be
708 used.
- 709 2) Shen-type accessibility can be used to compute the availability of jobs (the
710 rate and the absolute values if the original definition is corrected), however,
711 if the analyst is interested in internal values and secondary analysis of the
712 results, spatial availability should be considered.
- 713 3) With the renewed interpretability of what the absolute *opportunity avail-*
714 *ability* is at each origin, the spatial availability per capita v_i value of 1 can
715 be used as a policy goal. For areas with a value below 1, targeted increases

716 to the quantity of opportunities, residential housing, and transportation
717 system improvements can be considered such that the number of *available*
718 *jobs* per capita in the zone is at least equal to 1. Since spatial availability
719 per capita implicitly preserves the number of opportunities in the region,
720 it can be directly compared to the the region's raw jobs to population ratio
721 to inform policy. Additionally, the absolute values of spatial availability
722 can be used to understand the magnitude of the opportunity availability
723 deficit (or surplus).

- 724 4) Spatial availability per capita can also be compared directly to other re-
725 gions as done by literature using Shen-type measure/2SFCA (e.g., Gian-
726 notti et al. (2021)). However, as a result of the renewed interpretation,
727 the magnitude of *spatially available* opportunities can be quantified.
- 728 5) Lastly, since opportunities are preserved, many new avenues of analysis
729 can be pursued. This is especially important in light of emerging concerns
730 with equity. For instance, the population and opportunities can be seg-
731 mented (i.e., transit users, active transportation users, low income, low
732 education, new comers, children) and their spatial availability to opportu-
733 nities can be assessed, benchmarked, and corresponding policy to target
734 inequities can be theorized. As another example, the combined balancing
735 factor can be analysed to identify which populations currently do not seek
736 opportunities because of friction of distance. This is a topic for future
737 research.

⁷³⁸ **6. Appendix A**

⁷³⁹ The mathematical equivalence of Shen-type accessibility measure and spatial availability is provided in 4 steps in this
⁷⁴⁰ appendix. Spatial availability per population v_i is solved for population center A (Shen's synthetic example as discussed in
⁷⁴¹ Section 2.3): this value is represented by v_A and is equivalent to a_A as follows.

First step: the population-based balancing factor F_i^p used in V_i is defined as:

$$F_i^p = \frac{P_i^\alpha}{\sum_i^N P_i^\alpha}$$

For population center A , F_i^p is equal to:

$$F_A^p = \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha}$$

Second step: the impedance-based balancing factor F_{ij}^c in V_i is defined as:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=A}^N f(c_{ij})}$$

In this synthetic example, combinations of workers from population center A are permitted to go to all employment centers (1, 2, 3), so their relative impedance value is experienced in all of the nine origin-destination trip combinations. Therefore, all nine F_{ij}^c are computed as follows, since they all consider the impact of population center A trip combinations (i.e., either $A1$, $A2$, $A3$).

$$F_{A1}^c = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{B1}^c = \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{C1}^c = \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$\begin{aligned}
F_{A2}^c &= \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} \\
F_{B2}^c &= \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} \\
F_{C2}^c &= \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} \\
F_{A3}^c &= \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} \\
F_{B3}^c &= \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} \\
F_{C3}^c &= \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}
\end{aligned}$$

Third step: when these balancing factors (F_i^p and F_{ij}^c) concerning population center A are assembled and divided by P_i allow the denominators of the denominators to cancel out. The following equation is the assigned general form, with the strike-through indicating which values cancel out:

$$v_i = \sum_j \frac{O_j}{P_i^\alpha} \frac{\cancel{\sum_i^N P_i^\alpha} \cdot \cancel{\sum_i^N f(c_{ij})}}{\cancel{\sum_i^N P_i^\alpha} \cdot \cancel{\sum_i^N f(c_{ij})}}$$

To demonstrate that those denominator terms cancel out, the following following terms for v_A are subbed into the general form as follows:

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} \right) +$$

$$\frac{O_2}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} \right) + \\ \frac{O_3}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} \right)$$

v_A simplifies to the following:

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A1}) + f(c_{B1}) + f(c_{C1}))}} \right) + \frac{O_2}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A2}) + f(c_{B2}) + f(c_{C2}))}} \right) + \frac{O_3}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A3}) + f(c_{B3}) + f(c_{C3}))}} \right)$$

35

Now, notice how the denominator of the denominator is the same as the denominator of the numerator for each j ($j=1$, $j=2$, and $j=3$)? We remove those cancelled out terms (as indicated at the beginning of this step) and re-write v_a as follows:

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} \right) + \frac{O_2}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} \right) + \frac{O_3}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

Fourth step: We can now cancel out one more term, P_A^α , from the denominator and numerator as follows:

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} \right) + \frac{O_2}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} \right) + \frac{O_3}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

Which can be expressed as:

$$v_A = \left(\frac{O_1 \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})} + \frac{O_2 \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})} + \frac{O_3 \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_B^\alpha \cdot f(c_{B3}) + P_C^\alpha \cdot f(c_{C3})} \right)$$

And generalized to be formally identical to the Shen-type accessibility measure with competition as follows:

$$v_i = a_i = \sum_j \frac{O_j \cdot f(c_{ij})}{\sum_i P_i \cdot f(c_{ij})}$$

742 \end{landscape}

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