

<sup>1</sup> Introducing spatial availability, a singly-constrained  
<sup>2</sup> measure of competitive accessibility

<sup>3</sup> **Abstract**

Accessibility indicators are widely used in transportation, urban, and health-care planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability, mainly by introducing competition. In this paper, we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

4      **1. Introduction**

5      The concept of accessibility in transportation studies derives its appeal from  
6      the combination of the spatial distribution of opportunities and the cost of  
7      reaching them (Handy and Niemeier, 1997; Hansen, 1959). Accessibility analysis  
8      is employed in transportation, geography, public health, and many other areas,  
9      with the number of applications growing (Shi et al., 2020), especially as mobility-  
10     based planning is de-emphasized in favor of access-oriented planning (Deboosere  
11     et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021).

12     Accessibility analysis stems from the foundational works of Harris (1954)  
13     and Hansen (1959). From these seminal efforts, many accessibility measures  
14     have been derived, particularly after the influential work of Wilson (1971) on  
15     spatial interaction<sup>1</sup>. Of these, gravity-type accessibility is arguably the most  
16     common; since its introduction in the literature, it has been widely adopted in  
17     numerous forms (Arranz-López et al., 2019; Cervero et al., 2002; Geurs and van  
18     Wee, 2004; Levinson, 1998; Paez, 2004). Hansen-type accessibility indicators  
19     are essentially weighted sums of opportunities, with the weights given by an  
20     impedance function that depends on the cost of movement, and thus measure  
21     the *intensity of the possibility of interaction* (Hansen, 1959). This type of acces-  
22     sibility analysis offers a powerful tool to study the intersection between urban  
23     structure and transportation infrastructure (Handy and Niemeier, 1997).

24     Despite their usefulness, the interpretability of Hansen-type accessibility  
25     measures can be challenging (Geurs and van Wee, 2004; Miller, 2018). Since  
26     they aggregate opportunities, the results are sensitive to the size of the region  
27     of interest (e.g., a large city has more jobs than a smaller city). As a conse-  
28     quence, raw outputs are not necessarily comparable across study areas (Allen  
29     and Farber, 2019). This limitation becomes evident when surveying studies that  
30     implement this type of analysis. For example, Páez et al. (2010) (in Montreal)  
31     and Campbell et al. (2019) (in Nairobi) report accessibility as the number of  
32     health care facilities that can potentially be reached from origins. But what  
33     does it mean for a zone to have accessibility to less than 100 facilities in each of  
34     these two cities, with their different populations and number of facilities? For  
35     that matter, what does it mean for a zone to have accessibility to more than 700  
36     facilities in Montreal, besides being “accessibility rich”? As another example,  
37     Bocarejo S. and Oviedo H. (2012) (in Bogota), El-Geneidy et al. (2016) (in  
38     Montreal), and Jiang and Levinson (2016) (in Beijing) report accessibility as  
39     numbers of jobs, with accessibility values often in the hundreds of thousands,  
40     and even exceeding one million jobs for some zones in Beijng and Montreal. As  
41     indicators of urban structure, these measures are informative, but the meaning  
42     of one million accessible jobs is harder to pin down: how many jobs must any  
43     single person have access to? Clearly, the answer to this question depends on  
44     how many people demand jobs.

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<sup>1</sup>Utility-based measures derive from a very different theoretical framework, random utility maximization

45 The interpretability of Hansen-type accessibility has been discussed in nu-  
46 merous studies, including recently by Hu and Downs (2019), Kelobonye et al.  
47 (2020), and in greater depth by Merlin and Hu (2017). As hinted above, the  
48 limitations in interpretability are frequently caused by ignoring competition -  
49 without competition, each opportunity is assumed to be equally available to  
50 every single opportunity-seeking individual that can reach it (Kelobonye et al.,  
51 2020; Paez et al., 2019; Shen, 1998). This assumption is appropriate when the  
52 opportunity of interest is non-exclusive, that is, if use by one unit of population  
53 does not preclude use by another. For instance, national parks with abundant  
54 space are seldom used to full capacity, so the presence of some population does  
55 not exclude use by others. When it comes to exclusive opportunities, or when  
56 operations may be affected by congestion, the solution has been to account  
57 for competition. Several efforts exist that do so. In our reckoning, the first  
58 such approach was proposed by Weibull (1976), whereby the distance decay of  
59 the supply of employment and the demand for employment (by workers) were  
60 formulated under so-called axiomatic assumptions. This approach was then ap-  
61 plied by Joseph and Bantock (1984) in the context of healthcare, to quantify  
62 the availability of general practitioners in Canada. About two decades later,  
63 Shen (1998) independently re-discovered Weibull's (1976) formula (see footnote  
64 (7) in Shen, 1998) and deconstructed it to consider accessibility for different  
65 modes. These advances were subsequently popularized as the family of Two-  
66 Stage Floating Catchment area (2SFCA) methods (Luo and Wang, 2003) that  
67 have found widespread adoption in healthcare, education, and food systems (B.  
68 Y. Chen et al., 2020; Chen, 2019; Z. Chen et al., 2020; Yang et al., 2006; Ye et  
69 al., 2018).

70 An important development contained in Shen's work is a proof that the  
71 population-weighted sum of the accessibility measure with competition equates  
72 to the number of opportunities available (footnote (7) and Appendix A in Shen,  
73 1998). This demonstration gives the impression that Shen-type accessibility al-  
74 locates *all* opportunities to the origins, however to the authors' knowledge, it  
75 has not interpreted by literature in this way. For instance, Hu (2014), Merlin  
76 and Hu (2017), and Tao et al. (2020) all use Shen-type accessibility to calcu-  
77 late job access but report values as 'competitive accessibility scores' or simply  
78 'job accessibility'. These works do not explicitly recognize that jobs that are  
79 assigned to each origin are in fact a proportion of *all* the opportunities in the  
80 system. This recognition, we argue, is critical to interpreting the meaning of the  
81 final result. Thus, in this paper we intend to revisit accessibility with compe-  
82 tition within the context of disentangling how opportunities are allocated. We  
83 first argue that Shen's competitive accessibility misleadingly refers to the the  
84 total zonal population to equal the travel-cost discounted opportunity-seeking  
85 population. This equivocation, we believe, results in a ambiguous interpretation  
86 of what Shen-type accessibility represents as the allocation of opportunities to  
87 population is masked by the results presenting as rates (i.e., opportunities per  
88 capita). We then propose an alternative formulation of accessibility that incor-  
89 porates competition by adopting a proportional allocation mechanism; we name  
90 this measure *spatial availability*. The use of balancing factors for proportional

91 allocation is akin to imposing a single constraint on the accessibility indicator,  
92 in the spirit of Wilson's (1971) spatial interaction model.

93 In this way, the aim of the paper is three-fold:

- 94 • First, we aim to demonstrate that Shen-type (and thus Weibull (1976)  
95 accessibility and the popular 2SFCA methods) produce equivocal esti-  
96 mates of opportunities allocated as the result is presented as a rate (i.e.,  
97 opportunities per capita);
- 98 • Second, we introduce a new measure, *spatial availability*, which we submit  
99 is a more interpretable alternative to Shen-type accessibility, since oppor-  
100 tunities in the system are preserved and proportionally allocated to the  
101 population; and
- 102 • Third, we show how Shen-type accessibility (and 2SFCA methods) can be  
103 seen as measures of singly-constrained accessibility.

104 Discussion is supported by the use of the small synthetic example of Shen  
105 (1998) and empirical data drawn from the 2016 Transportation Tomorrow Sur-  
106vey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the  
107 spirit of openness of research in the spatial sciences (Brunsdon and Comber,  
108 2021; Páez, 2021) this paper has a companion open data product (Arribas-  
109 Bel et al., 2021), and all code is available for replicability and reproducibility  
110 purposes at <https://github.com/soukhova/Spatial-Availability-Measure>.

## 111 2. Accessibility measures revisited

112 In this section we revisit Hansen-type and Shen-type accessibility indicators.  
113 We adopt the convention of using a capital letter for absolute values (number  
114 of opportunities) and lower case for rates (opportunities per capita).

### 115 2.1. Hansen-type accessibility

116 Hansen-type accessibility measures follow the general formulation shown in  
117 Equation (1):

$$S_i = \sum_{j=1}^J O_j \cdot f(c_{ij}) \quad (1)$$

118 where:

- 119 •  $c_{ij}$  is a measure of the cost of moving between  $i$  and  $j$ .
- 120 •  $f(\cdot)$  is an impedance function of  $c_{ij}$ ; it can take the form of any monoton-  
121 ically decreasing function chosen based on positive or normative criteria  
122 (Páez et al., 2012).
- 123 •  $i$  is a set of origin locations ( $i = 1, \dots, N$ ).
- 124 •  $j$  is a set of destination locations ( $j = 1, \dots, J$ ).

- 125     •  $O_j$  is the number of opportunities at location  $j$ ;  $O = \sum_{j=1}^J O_j$  is the total  
 126       supply of opportunities in the study region.  
 127     •  $S$  is Hansen-type accessibility as weighted sum of opportunities.

128     As formally defined, accessibility  $S_i$  is the sum of opportunities that can be  
 129       reached from location  $i$ , weighted down by an impedance function of the cost  
 130       of travel  $c_{ij}$ . Summing the opportunities in the neighborhood of  $i$  provides es-  
 131       timates of the number of opportunities that can *potentially* be reached from  $i$ .  
 132       Several measures result from using a variety of impedance functions; for exam-  
 133       ple, cumulative opportunities measures are obtained when  $f(\cdot)$  is a binary or  
 134       indicator function (e.g., El-Geneidy et al., 2016; Geurs and van Wee, 2004; Qi et  
 135       al., 2018; Rosik et al., 2021). Other measures use impedance functions modeled  
 136       after any monotonically decreasing function (e.g., Gaussian, inverse power, neg-  
 137       ative exponential, or log-normal, among others, see, *inter alia*, Kwan, 1998; Li  
 138       et al., 2020; Reggiani et al., 2011; Vale and Pereira, 2017). In practice, accessi-  
 139       bility measures with different impedance functions tend to be highly correlated  
 140       (Higgins, 2019; Kwan, 1998; Santana Palacios and El-geneidy, 2022).

141     Gravity-based accessibility has been shown to be an excellent indicator of  
 142       the intersection between spatially distributed opportunities and transportation  
 143       infrastructure (Kwan, 1998; Reggiani et al., 2011; Shi et al., 2020). However,  
 144       beyond enabling comparisons of relative values they are not highly interpretable  
 145       on their own (Miller, 2018). To address the issue of interpretability, previous re-  
 146       search has aimed to index and normalize values on a per demand-population ba-  
 147       sis (e.g., Barboza et al., 2021; Pereira et al., 2019; Wang et al., 2021). However,  
 148       as recent research on accessibility discusses (Allen and Farber, 2019; Kelobonye  
 149       et al., 2020; e.g., Merlin and Hu, 2017; Paez et al., 2019), these steps do not ad-  
 150       equately consider competition. In effect, when calculating  $S_i$ , every opportunity  
 151       enters the weighted sum once for every origin  $i$  that can reach it. This makes  
 152       interpretability opaque, and to complicate matters, can also bias the estimated  
 153       landscape of opportunity.

154     2.2. *Shen-type competitive accessibility*

155     To account for competition, the influential works of Shen (1998) and Weibull  
 156       (1976), as well as the widely used 2SFCA approach of Luo and Wang (2003), ad-  
 157       just Hansen-type accessibility with the population in the region of interest. The  
 158       mechanics of this approach consist of calculating, for every destination  $j$ , the  
 159       population that can reach it given the impedance function  $f(\cdot)$ ; let us call this  
 160       the *effective opportunity-seeking population* (Equation (2)). This value can be  
 161       seen as the Hansen-type *market area* (accessibility to population) of  $j$ . The op-  
 162       portunities at  $j$  are then divided by the sum of the effective opportunity-seeking  
 163       population to obtain a measure of opportunities per capita, i.e.,  $R_j$  in Equa-  
 164       tion (3). This can be thought of as the *level of service* at  $j$ . Per capita values  
 165       are then allocated back to the population at  $i$ , again subject to the impedance  
 166       function as seen in Equation (4); this is accessibility with competition.

$$P_{ij}^* = P_i \cdot f(c_{ij}) \quad (2)$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^*} \quad (3)$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \quad (4)$$

167 where:

- 168 •  $a$  is Shen-type accessibility as weighted sum of opportunities per capita  
(or weighted level of service).
- 169 •  $c_{ij}$  is a measure of the cost of moving between  $i$  and  $j$ .
- 170 •  $f(\cdot)$  is an impedance function of  $c_{ij}$ .
- 171 •  $i$  is a set of origin locations ( $i = 1, \dots, N$ ).
- 172 •  $j$  is a set of destination locations ( $j = 1, \dots, J$ ).
- 173 •  $O_j$  is the number of opportunities at location  $j$ ;  $O = \sum_{j=1}^J O_j$  is the total  
174 supply of opportunities in the study region.
- 175 •  $P_i$  is the population at location  $i$ .
- 176 •  $P_{ij}^*$  is the population at location  $i$  that can reach destination  $j$  according  
177 to the impedance function; we call this the *effective opportunity-seeking  
population*.
- 178 •  $R_j$  is the ratio of opportunities at  $j$  to the sum over all origins of the  
179 *effective opportunity-seeking population* that can reach  $j$ ; in other words,  
180 this is the total number of opportunities per capita found at  $j$ .
- 181
- 182

183 Shen (1998) describes  $P_i$  as the “*the number of people in location  $i$  seeking  
184 opportunities*”. In our view, this is somewhat equivocal and where misinterpre-  
185 tation of the final results may arise. Consider a population center where the  
186 population is only willing to take an opportunity if the trip required is less than  
187 or equal to 60 minutes. This is identical to the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 & \text{if } c_{ij} \leq 60 \text{ min} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

188 If an employment center is less than 60 minutes away, the population can  
189 seek opportunities there (i.e.,  $f(c_{ij}) = 1$ ). But are these people still part of the  
190 opportunity-seeking population for jobs located two hours away? Four hours?  
191 Ten hours? We assume that they are not because their travel behaviour, as rep-  
192 resented by the impedance function would yield  $f(c_{ij}) = 0$ , eliminating them  
193 from the effective opportunity-seeking population  $P_{ij}^*$ . We see Shen’s defi-  
194 nition as ambiguous because, for the purpose of calculating accessibility, the  
195 impedance function defines what constitutes the population that effectively can  
196 seek opportunities at remote locations. Thus  $P_i$  should be plainly understood  
197 as the population at location  $i$  (as defined above) and not the “*the number of  
198 people in location  $i$  seeking opportunities*”. In other words,  $P_i$  and  $P_{ij}^*$  are  
199 confounded.

200 Furthermore, an identical misunderstanding can be described for  $O_j$  which is  
201 defined as “*the number of relevant opportunities in location  $j$* ” in Shen (1998)

(our emphasis).  $O_j$  is adjusted by the same  $f(c_{ij})$  in Equation (4), so the *relevancy* is determined by the travel behaviour associated with the impedance function not purely by  $O_j$  itself. For this reason,  $O_j$  should be understand plainly as the opportunities at location  $j$  (as we also defined them above).

Misunderstanding  $P_i$  and  $O_j$  may lead to a misleading interpretation of the final result  $a_i$ , especially as expressed in Shen's proof (see Equation (6)).

$$\sum_{i=1}^N a_i P_i = \sum_{j=1}^J O_j \quad (6)$$

Notice, confounding  $P_i$  with the effective opportunity-seeking population and  $O_j$  with the jobs taken may cause us to misunderstand  $a_i$  as "*relevant opportunities*" per "*people in location  $i$  seeking opportunities*". Instead, as mathematically expressed in the proof,  $a_i$  is a proportion of the opportunities available to the population, since multiplying  $a_i$  by the population at  $i$  and summing for all origins in the system equals to the total number of opportunities in the system. Embedded in  $a_i$  is already the travel behaviour so  $P_i$  and  $O_j$  must be plainly understand as population at  $i$  and opportunities at  $j$  to have Equation (6) hold true.

### 2.3. Shen's synthetic example

In this section we use the example in Shen (1998) to detail the importance of understanding  $P_i$  and  $O_j$  as simply the population at the origin  $i$  and the opportunities at destination  $j$  respectively. This is critical to understanding how the opportunities are allocated to the population based on the impedance function.

Table 1 contains the information needed to calculate  $S_i$  and  $a_i$  for this example. We use a negative exponential impedance function with  $\beta = 0.1$  as done in Shen (1998, see footnote (5)):

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

In Table 1, we see that population centers  $A$  and  $B$  have equal Hansen-type accessibility ( $S_A = S_B = 27,292$  jobs). On the other hand, the isolated satellite town of  $C$  has low accessibility ( $S_C = 2,240$  jobs). But center  $B$ , despite its high accessibility, is a large population center.  $C$ , in contrast, is smaller but also relatively isolated and has a balanced ratio of jobs (10,0000 jobs) to population (10,000 people). It is difficult from these outputs to determine whether the accessibility at  $C$  is better or worse than that at  $A$  or  $B$ .

The results are easier to interpret when we consider Shen-type accessibility. The results indicate that  $a_A \approx 1.337$  jobs per capita,  $a_B \approx 0.888$ , and  $a_C \approx 0.996$ . The latter value is sensible given the jobs-population balance of  $C$ . Center  $A$  is relatively close to a large number of jobs (more jobs than the population of  $A$ ). The opposite is true of  $B$ . According to Shen (1998), the sum of the population-weighted accessibility  $a_i$  is exactly equal to the number of jobs in

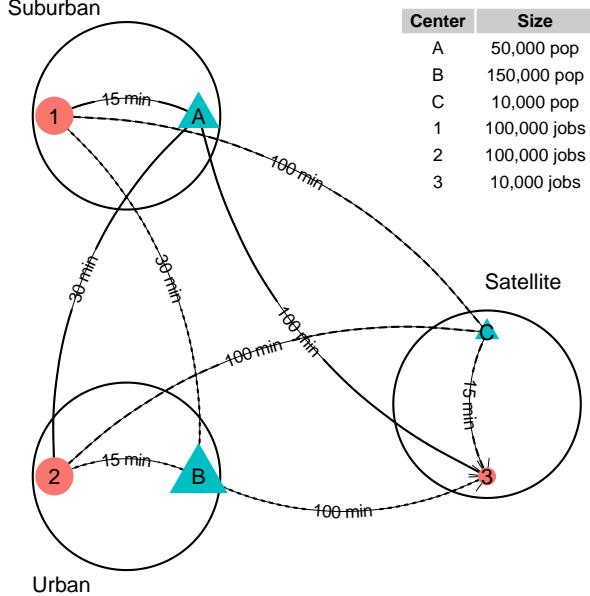


Figure 1: Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

the region following Shen's proof:

$$\begin{aligned}
 \sum_{i=1}^N a_i P_i &= \sum_{j=1}^J O_j \\
 50,000 \times 1.3366693 & \\
 + 150,000 \times 0.8880224 & \\
 + 10,000 \times 0.9963171 &= 210,000
 \end{aligned}$$

As mentioned earlier, this property under Shen's definition of  $P_i$  “people in location  $i$  seeking opportunities”, gives the impression that all jobs sought are allocated to the people located at each origin  $i$ . In other words, Shen defines  $P_i$  to mean  $P_{ij}^*$  (i.e., the *effective opportunity-seeking population* which is already adjusted by travel behaviour) instead of defining it to simply be the full population at  $i$  (i.e.,  $P_i$ ). As seen in column **Pop \* f(TT)** in Table 1 (i.e.,  $P_{ij}^* = P_i \cdot f(c_{ij})$ ), the number of individuals from population center  $A$  that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2.27 respectively. Therefore, the total effective opportunity-seeking population at  $A$  is  $P_A^* = \sum_j P_{Aj}^*$ , that is, 13,647.27 people, which is considerably lower than the total population of  $A$  (i.e.,  $P_A = 50,000$  people). Demonstrated as follows, using  $P_{ij}^*$  in the calculations associated with this proof results in only 56,834.59 jobs being allocated to the population, instead of the nominal number of jobs

in the region that is over three times this number (i.e., 210,000 jobs).

$$\begin{aligned} \sum_{i=1}^N a_i P_{ij}^* = & \\ (11,156.51 + 2,489.35 + 2.26) \times 1.3366693 & \\ +(7,468.06 + 33,469.52 + 6.81) \times 0.8880224 & \\ +(4.54 + 4.54 + 2,231.20) \times 0.9963171 \approx 56,834.59 & \end{aligned}$$

Furthermore, even when Shen's  $P_i$  is understood plainly as the total population at  $i$ , the meaning of the proof may still be ambiguous. The proof can still give the impression that all jobs are allocated to the total population since total population ( $\sum_{i=1}^N P_i$ ) goes into the equation and total jobs ( $\sum_{j=1}^J O_j$ ) in the region is the result. However, this impression is incomplete since it does not reflect the amount of population which takes jobs and the number of people being considered for jobs; these magnitudes are a product of being weighted down by the impedance function. These magnitudes are not obvious from  $a_i$  is because the result is presented as a rate (i.e., opportunities per capita).

Let us consider a modification to the travel behaviour of the example discussed to illustrate how the presentation of  $a_i$  as a rate obscures the magnitude of the effective opportunity-seeking population. We modify the example by increasing the  $\beta$  to 0.6 (compared to the previous value of 0.1; see Figure 2). This modification increases the cost of travel and thus the impedance function, which is an expression of the population's relative willingness to travel to opportunities, reflects a population which is relatively less willing to travel to opportunities further away compared to the previous  $\beta$  value. The Hansen-type and Shen-type values are presented in the yellow rows of Table 1.

As expected, Hansen-type accessibility drops quite dramatically after this  $\beta$  modification: the friction of distance is so high that few opportunities are within reach. In contrast, Shen-type accessibility converges to the jobs:population ratio (i.e., origin  $A$  is  $\frac{100,000}{50,000} = 2$ ). This is explained by the way the impedance function excludes the population in droves, thus reducing the competition for jobs: as seen in Table 1, the effective opportunity-seeking population from  $A$  is only about equal to 6.17; likewise, the number of jobs at 1 weighted by the impedance is only 12.341. In other words, competition is low because jobs are expensive to reach, but those willing to reach jobs enjoy relatively high accessibility (in the limit, the jobs/population ratio). On the other hand, the accessibility is effectively zero for those in the population prevented by the impedance from reaching any jobs.

In what follows, we propose an alternative derivation of Shen (1998) accessibility with competition that explicitly clarifies the opportunities allocated to the *effective opportunity-seeking population* within its formulation. Hence, the results are not only more interpretable, but also extend the potential of accessibility analysis.

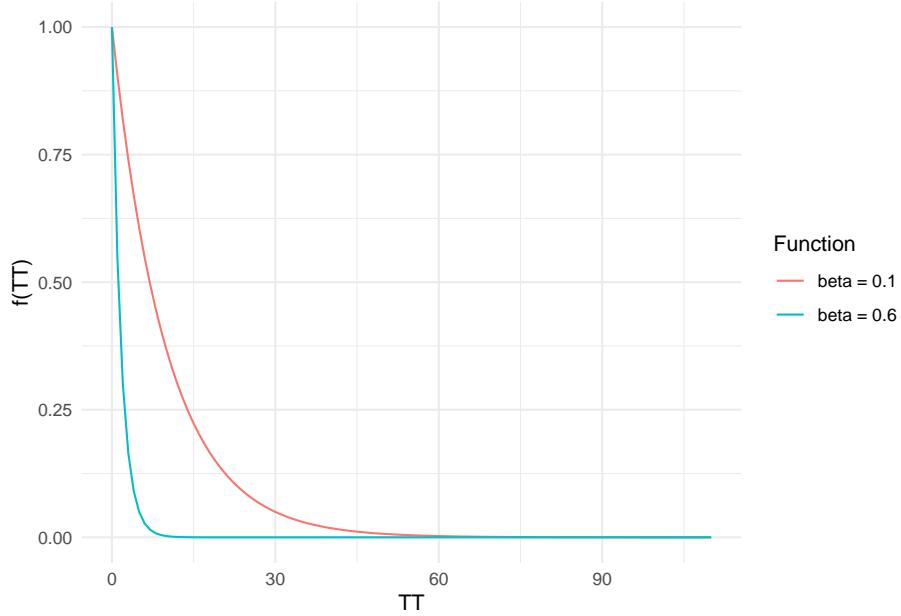


Figure 2: Comparison of two negative exponential impedance functions used in the synthetic example. The x-axis represents the travel time (in mins) and the y-axis represents the impedance function at each travel time.

265     **3. Introducing spatial availability: a singly-constrained measure of  
266       accessibility**

In brief, we define the *spatial availability* at  $i$  ( $V_i$ ) as the proportion of all opportunities  $O$  that are allocated to  $i$  from all destinations  $j$ :

$$V_i = \sum_{j=1}^N O_j F_{ij}^t$$

267     where:

- 268     •  $F_{ij}^t$  is a balancing factor that depends on the population and cost of move-  
269       ment in the system.
- 270     •  $O_j$  is the number of opportunities at  $j$ .
- 271     •  $V_i$  is the number of spatially available opportunities from the perspective  
272       of  $i$ .

273     The general form of spatial availability is also as a sum, and the fundamental  
274     difference with Hansen- and Shen-type accessibility is that opportunities are  
275     allocated proportionally. Balancing factor  $F_{ij}^t$  consists of two components: a  
276     population-based balancing factor  $F_i^p$  and an impedance-based balancing factor  
277      $F_{ij}^c$  which, respectively, allocate opportunities to  $i$  in proportion to the size of

<sup>278</sup> the population of the different competing centers (the mass effect of the gravity  
<sup>279</sup> model) and the cost of reaching opportunities (the impedance effect). In the  
<sup>280</sup> next two subsections, we explain the intuition behind the method before defining  
<sup>281</sup> it in full.

<sup>282</sup> *3.1. Proportional allocation by population*

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by population. Recall, the total population in the example is 210,000. The proportion of the population by population center is as follows:

$$F_A^p = \frac{50,000}{210,000}$$

$$F_B^p = \frac{150,000}{210,000}$$

$$F_C^p = \frac{10,000}{210,000}$$

<sup>283</sup> Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors  
<sup>284</sup>  $F_i^p$ . In this way, employment center 1 allocates  $100,000 \cdot \frac{50,000}{210,000} = 23,809.52$  jobs  
<sup>285</sup> to  $A$ ;  $100,000 \cdot \frac{150,000}{210,000} = 71,428.57$  jobs to  $B$ ; and  $100,000 \cdot \frac{10,000}{210,000} = 7,142.857$   
<sup>286</sup> jobs to  $C$ . Notice how this mechanism ensures that the total number of jobs at  
<sup>287</sup> employment center 1 is preserved at 100,000.  
<sup>288</sup>

We can verify that the number of jobs allocated is consistent with the total number of jobs in the system:

Employment center 1 to population centers A, B, and C:

$$100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$$

Employment center 2 to population centers A, B, and C:

$$100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$$

Employment center 3 to population centers A, B, and C:

$$10,000 \cdot \frac{50,000}{210,000} + 10,000 \cdot \frac{150,000}{210,000} + 10,000 \cdot \frac{10,000}{210,000} = 10,000$$

<sup>289</sup> In the general case where there are  $N$  population centers in the region, we  
<sup>290</sup> define the following population-based balancing factors in Equation (7):

$$F_i^p = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \tag{7}$$

<sup>291</sup> Balancing factor  $F_i^p$  corresponds to the proportion of the population in origin  $i$  relative to the population in the region. On the right hand side of the  
<sup>292</sup> equation, the numerator  $P_i^\alpha$  is the population at origin  $i$ . The summation in  
<sup>293</sup> the denominator is over  $i = 1, \dots, N$ , and adds up to the total population of the  
<sup>294</sup>

<sup>295</sup> region. Notice that we incorporate an empirical parameter  $\alpha$ . The role of  $\alpha$  is to  
<sup>296</sup> modulate the effect of demand by population. When  $\alpha < 1$ , opportunities are  
<sup>297</sup> allocated more rapidly to smaller centers relative to larger ones;  $\alpha > 1$  achieves  
<sup>298</sup> the opposite effect.

Balancing factor  $F_i^p$  can now be used to proportionally allocate a share of available jobs at  $j$  to origin  $i$ . The number of jobs available to  $i$  from  $j$  balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

In the general case where there are  $J$  employment centers, the total number of jobs available from all destinations to  $i$  is simply the sum of  $V_{ij}^p$  over  $j = 1, \dots, J$ :

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

Since the factor  $F_i^p$ , when summed over  $i = 1, \dots, N$  always equals to 1 (i.e.,  $\sum_{i=1}^N F_i^p = 1$ ), the sum of all spatially available jobs equals  $O$ , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^p &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\ &= \sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

<sup>299</sup> The terms  $F_i^p$  act here as the balancing factors of the gravity model when a  
<sup>300</sup> single constraint is imposed (i.e., to ensure that the sums of columns are equal  
<sup>301</sup> to the number of opportunities per destination, see Ortúzar and Willumsen,  
<sup>302</sup> 2011, pp. 179–180 and 183–184). As a result, the sum of spatial availability for  
<sup>303</sup> all population centers equals the total number of opportunities.

<sup>304</sup> The discussion so far concerns only the mass effect (i.e., population size)  
<sup>305</sup> of the gravity model. In addition, the potential for interaction is thought to  
<sup>306</sup> decrease with increasing cost, so next we define similar balancing factors but  
<sup>307</sup> based on the impedance.

### <sup>308</sup> 3.2. Proportional allocation by cost

Clearly, using only balancing factors  $F_i^p$  to calculate spatial availability  $V_i^p$  does not account for the cost of reaching employment centers. Consider instead a set of balancing factors  $F_{ij}^c$  that account for the friction of distance for our example. Recall that the impedance function  $f(c_{ij})$  equals  $\exp(-\beta \cdot c_{ij})$  where  $\beta = 0.1$  and travel time  $c_{ij}$  is either 15, 30 or 60 minutes. For instance, the impedance-based balancing factors  $F_{ij}^c$  would be the following for employment center 1 (employment center 2 and 3 have their own balancing factor values for

each origin  $i$  as will be discussed later):

$$\begin{aligned} F_{A1}^c &= \frac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\ F_{B1}^c &= \frac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\ F_{C1}^c &= \frac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581 \end{aligned}$$

309     Balancing factors  $F_{ij}^c$  use the impedance function to proportionally allocate  
 310   more jobs to closer population centers, that is, to those with populations *more*  
 311   *willing to reach the jobs*. Indeed, the factors  $F_{ij}^c$  can be thought of as the  
 312   proportion of the population at  $i$  willing to travel to destination  $j$ , conditional on  
 313   the travel behavior as given by the impedance function. For instance, 81.74398%  
 314   of jobs from employment center 1 are allocated to population center  $A$  based  
 315   on impedance.

316     So as follows from our example, of the 100,000 jobs at employment center 1  
 317   the number of jobs allocated to population center  $A$  is  $100,000 \times 0.8174398 =$   
 318   81,743.98 jobs; the number allocated to population center  $B$  is  $100,000 \times$   
 319    $0.1823954 = 18,239.54$  jobs; and the number allocated to population center  
 320    $C$  is  $100,000 \times 0.0001648581 = 16.48581$  jobs. We see once more that the total  
 321   number of jobs at the employment center is preserved at 100,000. In this ex-  
 322   ample, the proportional allocation mechanism assigns the largest share of jobs  
 323   to population center  $A$ , which is the closest to employment center 1, and the  
 324   smallest to the more distant population center  $C$ .

325     In the general case where there are  $N$  population centers and  $J$  employment  
 326   centers in the region, we define the following impedance-based balancing factors:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \quad (8)$$

The total number of jobs available to  $i$  from  $j$  according to impedance is defined as follows:

$$V_{ij}^c = O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

The total number of jobs available to  $i$  from all destinations is:

$$V_i^c = \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

Like the population-based allocation factors,  $F_i^c$  summed over  $i = 1, \dots, N$  always equals to 1 (i.e.,  $\sum_{i=1}^N F_{ij}^c = 1$ ). As before, the sum of all spatially available jobs equals  $O$ , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^c &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c} \\ &= \sum_{i=1}^N \frac{\sum_{j=1}^J O_j F_{ij}^c}{\sum_{i=1}^N \sum_{j=1}^J O_j F_{ij}^c} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

327     We are now ready to more formally define spatial availability with due con-  
 328   sideration to both population and travel cost effects.

329     3.3. Assembling mass and impedance effects

330     Population and the cost of travel are both part of the gravity modelling  
 331     framework. Since the balancing factors defined in the preceding sections are  
 332     proportions (alternatively, can be understood as probabilities), they can be  
 333     combined multiplicatively to obtain their joint effect. This multiplicative rela-  
 334     tionship can alternatively be understood as the joint probability of allocating  
 335     opportunities and is captured by Equation (9), where  $F_i^p$  is the population-  
 336     based balancing factor that grants a larger share of the existing opportunities  
 337     to larger centers and  $F_{ij}^c$  is the impedance-based balancing factor that grants a  
 338     larger share of the existing opportunities to closer centers. This is in line with  
 339     the tradition of gravity modeling.

$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \quad (9)$$

340     with  $F_i^p$  and  $F_{ij}^c$  as defined in Equations (7) and (8) respectively. The combined  
 341     balancing factor  $F_{ij}^t$  is used to proportionally allocate jobs from  $j$  to  $i$ . Hence,  
 342     spatial availability is given by Equation (10).

$$V_i = \sum_{j=1}^J O_j F_{ij}^t \quad (10)$$

343     The terms in Equation 10 are as follows:

- 344     •  $F_{ij}^t$  is a balancing factor as defined in Equation (9).
- 345     •  $i$  is a set of origin locations in the region  $i = 1, \dots, N$ .
- 346     •  $j$  is a set of destination locations in the region  $j = 1, \dots, J$ .
- 347     •  $O_j$  is the number of opportunities at location  $j$ .
- 348     •  $V_i$  is the spatial availability at  $i$ .

349     Notice that, unlike  $S_i$  in Hansen-type accessibility (Equation (1)), the pop-  
 350     ulation enters the calculation of  $V_i$  through  $F_i^p$ . Returning to Shen's example  
 351     in Figure 1, Table ?? contains the information needed to calculate  $V_i$ , with  $\beta$   
 352     set again to 0.1 as in Table 1.

In Table ??, column **V\_ij** are the jobs available to each origin from each employment center. In this column  $V_{A1} = 59,901$  is the number of jobs available at  $A$  from employment center 1. Column **V\_i** (i.e.,  $\sum_{j=1}^J V_{ij}$ ) gives the total number of jobs available to origin  $i$ . We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

353     Compare the calculated values of  $V_i$  to column **S\_i** (Hansen-type accessi-  
 354     bility) in Table 1. The spatial availability values are more intuitive. Recall

355 that population centers  $A$  and  $B$  had identical Hansen-type accessibility to em-  
 356 ployment opportunities. According to  $V_i$ , population center  $A$  has greater job  
 357 availability due to: 1) its close proximity to employment center 1; combined  
 358 with 2) less competition (i.e., a majority of the population have to travel longer  
 359 distances to reach employment center 1). Job availability is lower for popula-  
 360 tion center  $B$  due to much higher competition (150,000 people can reach 100,000  
 361 jobs at equal cost). And center  $C$  has almost as many jobs available as it has  
 362 population.

363 As discussed above, Hansen-type accessibility is not designed to preserve  
 364 the number of jobs in the region. Shen-type accessibility ends up preserving  
 365 the number of jobs in the region but the definitions of variables are internally  
 366 obscured; the only way it preserves the number of jobs is if the effect of the  
 367 impedance function is ignored when expanding the values of jobs per capita to  
 368 obtain the total number of opportunities. The proportional allocation procedure  
 369 described above, in contrast, consistently returns a number of jobs available that  
 370 matches the total number of jobs in the region.

371 Since the jobs spatially available are consistent with the jobs in the region,  
 372 it is possible to define a measure of spatial availability per capita as presented  
 373 in Equation (11):

$$v_i = \frac{V_i}{P_i} \quad (11)$$

374 And, since the jobs are preserved, it is possible to use the regional jobs per  
 375 capita ( $\frac{\sum_{j=1}^J O_j}{\sum_{i=1}^N P_i}$ ) as a benchmark to compare the spatial availability of jobs per  
 376 capita at each origin.

377 In the example, since the population is equal to the number of jobs, the  
 378 regional value of jobs per capita is 1.0. To complete the illustrative example,  
 379 the spatial availability of jobs per capita by origin is:

$$\begin{aligned} v_1 &= \frac{V_1}{P_1} = \frac{66,833.47}{50,000} = 1.337 \\ v_2 &= \frac{V_2}{P_2} = \frac{133,203.4}{150,000} = 0.888 \\ v_3 &= \frac{V_3}{P_3} = \frac{9,963.171}{10,000} = 0.996 \end{aligned} \quad (12)$$

380 We can see that population center  $A$  has fewer jobs per capita than the  
 381 regional benchmark, center  $B$  has more, and center  $C$  is at parity. Remarkably,  
 382 the spatial availability per capita matches the values of  $a_i$  in Table 1. Appendix  
 383 A has a proof of the mathematical equivalence between the two measures. It  
 384 is interesting to notice how Weibull (1976), Shen (1998), as well as this paper,  
 385 all reach identical expressions starting from different assumptions; this effect is  
 386 known as *equifinality* (see Ortúzar and Willumsen, 2011, p. 333; and Williams,  
 387 1981). This result means that Shen-type accessibility and 2SFCA can be re-  
 388 conceptualized as singly-constrained accessibility measures.

### 389 3.4. Why does proportional allocation matter?

390 We have shown that Shen-type accessibility and spatial availability produce  
 391 equifinal results when accessibility per-capita is computed. At this point it is

392 reasonable to ask whether the distinction between these two measures is of any  
393 importance.

394 Conceptually, we would argue that the confounded populations in Shen-type  
395 accessibility leads to internal inconsistency in the calculation of total opportu-  
396 nities in Shen (1998): this points to a deeper issue that is only evident when  
397 we consider the intermediate values of the method. To illustrate, Table 1 shows  
398 results of  $a_i$  that are reasonable (and they match exactly the spatial availability  
399 per capita). But when we dig deeper, these results mask potentially misleading  
400 values for the jobs allocated and the number of jobs taken. For instance, a re-  
401 gion with a high jobs:population ratio but a prohibitive transportation network  
402 which results in a high cost of travel may yield a high  $a_i$  value. This value,  
403 however, can conceal a low *effective opportunity-seeking population* and propor-  
404 tionally low number of allocated jobs while additionally obscuring the number  
405 of population which does *not* take jobs and the jobs *not* taken.

406 In addition, the intermediate accessibility values of  $a_i$  (Shen-type measure)  
407 may also lead to impact estimates that are deceptive (see Sarlas et al., 2020). For  
408 example, the estimated region-wide cost of travel considering the jobs allocated  
409 by  $a_i$  in Table 1 (i.e.,  $Jobs * f(TT)$ ) is as follows:

$$\begin{aligned} & 22,313 \times 15 \text{ min} + 4,979 \times 30 \text{ min} + 0.454 \times 100 \text{ min} \\ & 4,979 \times 30 \text{ min} + 22,313 \times 15 \text{ min} + 0.454 \times 100 \text{ min} \\ & 4.54 \times 100 \text{ min} + 4.54 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,594 \text{ min} \end{aligned}$$

In contrast, the estimated region-wide cost of travel according to  $V_i$  in Table ?? is as follows:

$$\begin{aligned} & 59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min} \\ & 40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min} \\ & 2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min} \end{aligned}$$

410 Often referred to as ‘the supply of jobs’ (or simply Hansen-style accessibil-  
411 ity) in the Shen-type measure:  $Jobs * f(TT)$  cannot be used to understand the  
412 region-wide cost of travel. Recall how we define  $Pop * f(TT)$  as the *effective*  
413 *opportunity-seeking population* ( $P_{ij}^*$ ),  $Jobs * f(TT)$  similarly represents the *ef-*  
414 *fective opportunities allocated* and sums to approximately 56,824 out of a  
415 total of 210,000 jobs. Like  $Pop * f(TT)$ , the *effective opportunities allocated*  
416 to each origin is only a reflection of the impedance function and not the *act-*  
417 *ual* number of opportunities allocated to each origin. Therefore, the resulting  
418 1,002,594 min is not a meaningful measure of the cost of travel in the system.

419 However, since spatial availability allocates the *actual* number of opportuni-  
420 ties to each origin; the 3,859,054 min can be used to quantify the system-wide  
421 impacts of competitive accessibility in this region. We know spatial availability’s  
422 output is the number of opportunities at each  $i$  since the combined balancing  
423 factors allocate a proportional amount of the total opportunities to each  $i$  such  
424 that the number of opportunities allocated to each  $i$  sum to equal the total  
425 opportunities in the region.

426     **4. Empirical example of Toronto**

427     In this section we illustrate the application of spatial availability through  
428     an empirical example. For this, we use full-time employment flows from the  
429     Greater Golden Horseshoe (GGH) area in Ontario, Canada. Contained with  
430     the GGH is the Greater Toronto and Hamilton (GTHA) which forms the most  
431     populous metropolitan regions in Canada and the core urban agglomeration in  
432     the GGH.

433     The GTHA contains the city of Toronto, the most populous city in Canada.  
434     The city of Toronto is the focus of this empirical example, it will be used to  
435     demonstrate the application of the proposed spatial availability measure along  
436     with how it compares to Hansen- and Shen-type measures. We begin this section  
437     by explaining the data and then detailing the calculated comparisons.

438     **4.1. GGH Data**

439     We obtained full-time employment flows from the 2016 Transportation To-  
440     morrow Survey (TTS). This survey collects representative urban travel infor-  
441     mation from 20 municipalities contained within the GGH area in the southern  
442     part of Ontario, Canada (see Figure 3) (Data Management Group, 2018) ev-  
443     ery five years. The data set includes origin to destination flows associated  
444     with full-time employment trips; the number of jobs ( $n=3,081,885$ ) and work-  
445     ers ( $n=3,446,957$ ) (i.e., the number of originating trips and destination trips)  
446     at each origin and destination are represented at the level of Traffic Analysis  
447     Zones (TAZ) ( $n=3,764$ ). TAZ are a unit of spatial analysis which are defined  
448     as part of the TTS, however, TAZ are commonly used to ascribe production  
449     and attraction of trips in the context of transportation planning modelling. In  
450     the GGH data set, the TAZ contain on average 916 workers and jobs 819 with  
451     more detailed descriptive statistics discussed later. The TTS data is based on  
452     a representative sample of between 3% to 5% of households in the GGH and  
453     is weighted to reflect the population covering the study area as a whole (Data  
454     Management Group, 2018).

455     To generate the travel cost for the full-time employment trips, travel times  
456     between origins and destinations (i.e., centroids of the TAZ) are calculated for  
457     car travel using the R package {r5r} (Rafael H. M. Pereira et al., 2021) with  
458     a street network retrieved from OpenStreetMap. It is also assumed that intra-  
459     TAZ trips are equal to 0.1 minutes. For inter-TAZ trips, a 3 hr travel time  
460     threshold was selected as it captures 99% of population-employment pairs (see  
461     the travel times summarized in Figure 3). This method does not account for  
462     traffic congestion or modal split, which can be estimated through other means  
463     (e.g., Allen and Farber, 2021; Higgins et al., 2021). For simplicity, we carry  
464     on with the assumption that all trips are taken by car in uncongested travel  
465     conditions. All data and data preparation steps are documented and can be  
466     freely explored in the companion open data product {TTS2016R}.

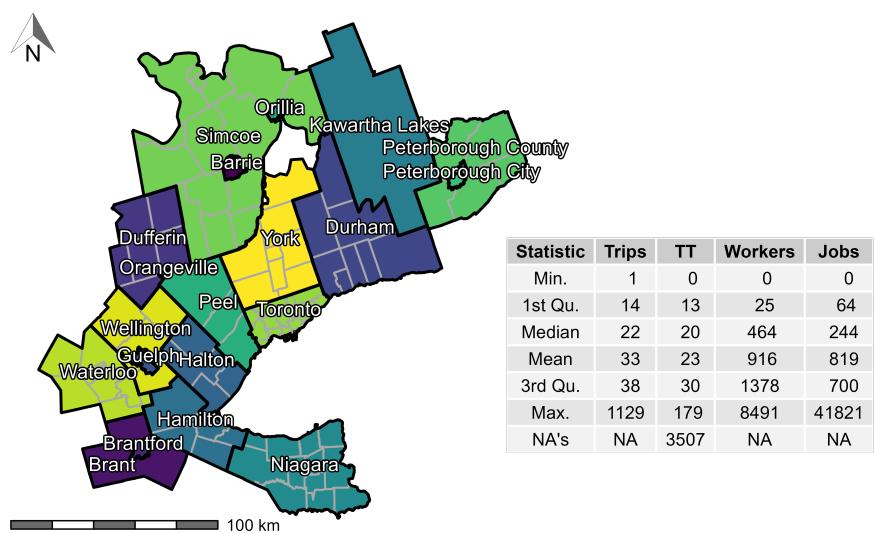


Figure 3: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

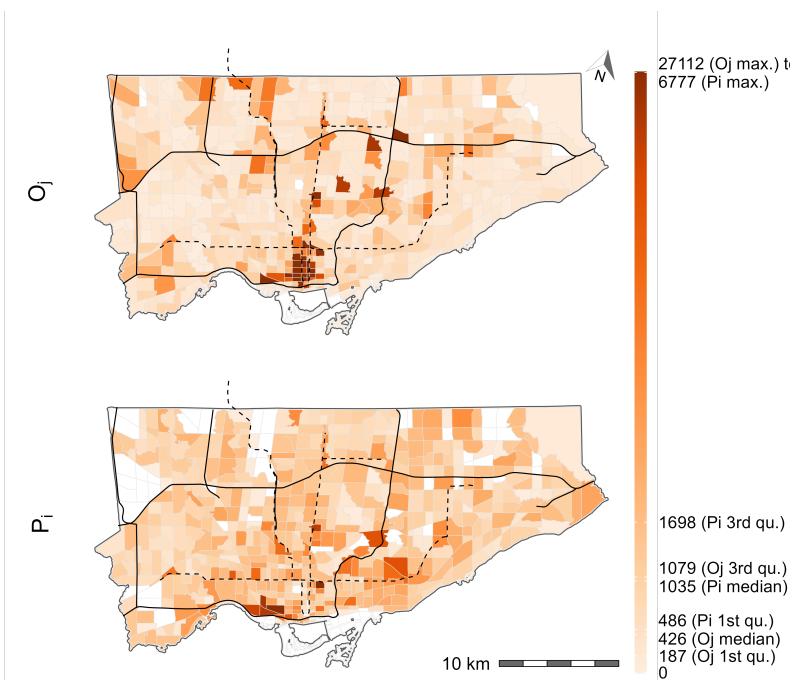


Figure 4: Spatial distribution of full-time jobs (top) and full-time working population (bottom) at each TAZ for Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs.

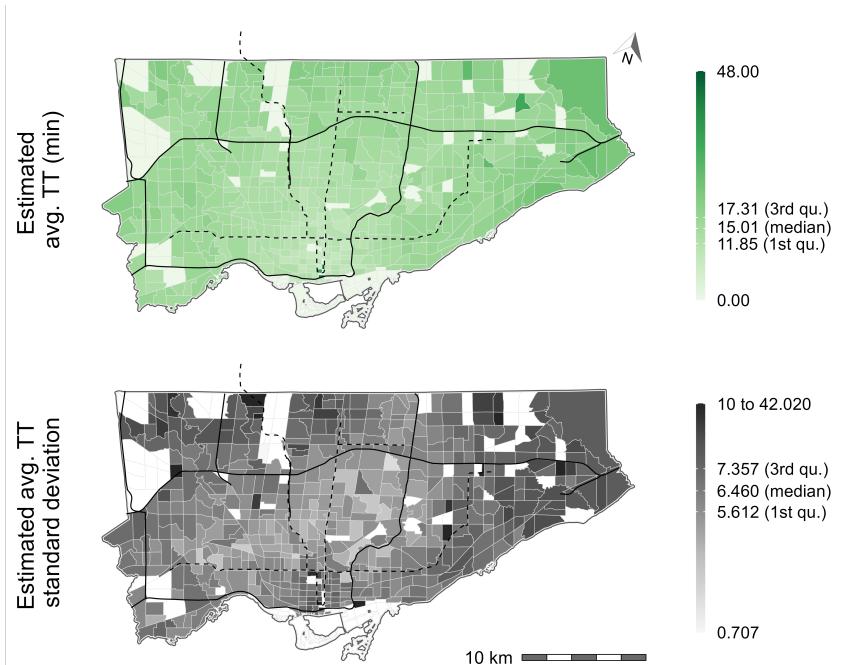


Figure 5: Spatial distribution of full-time working population to jobs ratio (top) and car travel time to jobs estimated using R5R (bottom) for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. White TAZ represent a TAZ with no workers thus no travel time for the top plot, for the bottom plot they represent no travel time TAZ and TAZ with only 1 travel time.

467     *4.2. Spatial employment characteristics in Toronto*

468     As mentioned, the focus of this empirical example is on the city of Toronto.  
469     It is the largest city in the GGH and represents a significant subset of workers  
470     and jobs in the GGH; 22% of workers in the GGH live in Toronto and 25% of  
471     jobs that these workers take are located within Toronto. The spatial distribution  
472     of jobs and workers is shown in Figure 4. It can be seen that a large cluster  
473     of jobs can be found in the central southern part of Toronto (the downtown  
474     core). Spatial trends in the distribution of workers is more even relative to the  
475     distribution of jobs.

476     Next, the spatial distribution of the estimated car travel time (green) and the  
477     associated standard deviation (grey) is visualized in Figure 5. It can be seen that  
478     the car travel time is lower within the downtown core and, unexpectedly higher  
479     as the TAZ is further from the downtown core. These travel time estimations  
480     are to be expected, as these car travel time are calculated using an uncongested  
481     OpenStreetMaps road network from the centroid of origin TAZ to destination  
482     TAZ. Since within Toronto trips are only considered, trips which originate from  
483     the center of Toronto, an area with high job density, relatively closer proximity  
484     to all other Toronto TAZ, and high road connectivity, travel times are lower than  
485     outside in an areas further from the downtown core. In terms of the variability  
486     of the travel times, the center TAZ of Toronto have lower variability than TAZ  
487     closer to the borders of Toronto. Trends from both plots indicate that trips  
488     originating from within the center of Toronto are shorter and more similar in  
489     length than trips originating from closer to the border of Toronto.

490     Nonetheless, the point of these visualizations is to demonstrate the spatial  
491     distribution of worker and job data in the city of Toronto to contextualize spatial  
492     availability and Shen- and Hansen- type measures.

493     *4.3. Calibration of an impedance function for Toronto*

494     In the synthetic example introduced before, we used a negative exponential  
495     function with the parameter reported by Shen (1998). For the empirical Toronto  
496     data set, we calibrate an impedance function on the trip length distribution  
497     (TLD) of commute trips. Briefly, a TLD represents the proportion of trips  
498     that are taken at a specific travel cost (e.g., travel time); this distribution is  
499     commonly used to derive impedance functions in accessibility research (Batista  
500     et al., 2019; Horbachov and Svichynskyi, 2018; Lopez and Paez, 2017).

501     As mentioned, the calculations are undertaken for the city of Toronto using  
502     only the employed population in the city and jobs taken by residents of Toronto.  
503     Specifically, edge trips are not included such as trips originating in Toronto but  
504     finishing outside of Toronto and trips originating outside of Toronto but finish-  
505     ing in Toronto. The empirical and theoretical TLD for this Toronto data set  
506     are represented in the top-left panel of Figure 6. Maximum likelihood estima-  
507     tion and the Nelder-Mead method for direct optimization available within the  
508     {fitdistrplus} package (Delignette-Muller and Dutang, 2015) were used. Based  
509     on goodness-of-fit criteria and diagnostics the normal distribution was selected  
510     (see Figure 6).

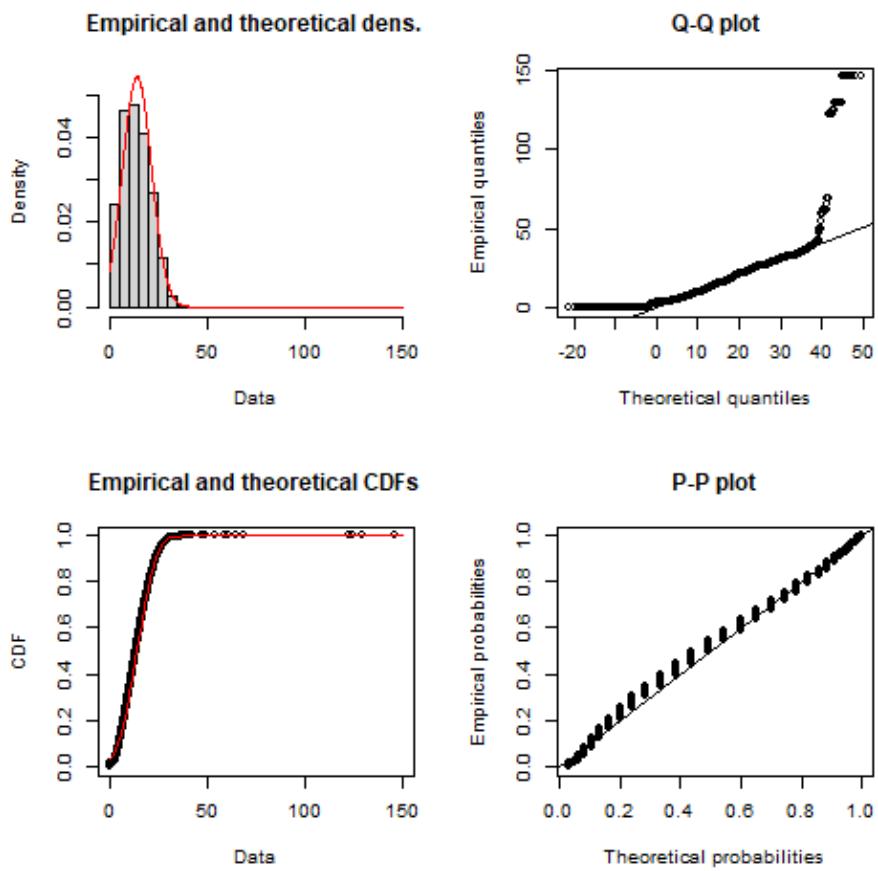


Figure 6: Car trip length distribution and calibrated normal distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

511     The normal distribution is defined in Equation (13), where we see that it  
 512 depends on a mean parameter  $\mu$  and a standard deviation parameter  $\sigma$ . The  
 513 estimated values of these parameters are  $\mu = 14.169$  and  $\sigma = 7.369$ .

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad (13)$$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

514     *4.4. Accessibility and spatial availability of jobs in Toronto*

515     *4.4.1. Absolute opportunity values*

516     Figure 7 contains the number of jobs accessible using Shen-type accessibility,  
 517 Hansen-type accessibility, and the number of jobs *available* using the spatial  
 518 availability measure. The values from all these measures are represented on  
 519 the same axis as they are comparable as they measure the absolute value of  
 520 *jobs* accessible to the workers in the origin. In the top plot, the Shen-type  
 521 accessibility is multiplied by the *effective opportunity-seeking population* to yield  
 522 a value that corresponds to absolute number of accessible jobs (considering  
 523 competition) according to Shen's definition. In the middle plot, the Hansen-  
 524 type accessibility is an unconstrained case of accessibility in which all jobs which  
 525 are in-reach of each origin (according to the impedance function); each value  
 526 corresponds to the number of jobs which can be reach at each origin assuming  
 527 no competition. Lastly, in the bottom plot, the spatial availability measure is a  
 528 constrained case of accessibility which yields the number of jobs, at each origin,  
 529 considering competition from the population in nearby origin and the relative  
 530 travel cost (according to the impedance function).

531     What is notable about the bottom plot is that the proportional allocation  
 532 mechanism of spatial availability ensures that the job availability value for each  
 533 origin all sums to the city-wide total of 769,231 jobs (i.e., the number of des-  
 534 tination flows from Toronto origins to Toronto destinations). The number of  
 535 accessible jobs at each origin can therefore be interpreted as the number of  
 536 *available* jobs to each origin based on the relative travel behaviour and density  
 537 of competition for jobs (i.e., worker population). A proportion of each of the  
 538 769,231 jobs in Toronto are only allocated once to each origin. In terms fo  
 539 the middle plot, the city-wide total for Hansen-type accessibility is 4,366,743  
 540 jobs, which as a value is meaningless since the measure is unconstrained; it  
 541 represents the sum of opportunities that have been counted anywhere from 1  
 542 to many times depending on the impedance function. As previously discussed,  
 543 unconstrained counting of the same opportunity by all origins is not an issue  
 544 if the opportunity itself is non-exclusive, but since one job can only be given  
 545 to one worker (especially since the worker and job data is derived from origin-  
 546 destination flows), it is inappropriate to use unconstrained measures to capture  
 547 employment characteristics. Comparing the middle and bottom plots, it is evi-  
 548 dent that the unconstrained counting of opportunities (Hansen-style) results in  
 549 absolute values that are higher throughout the city, particularly in TAZ that

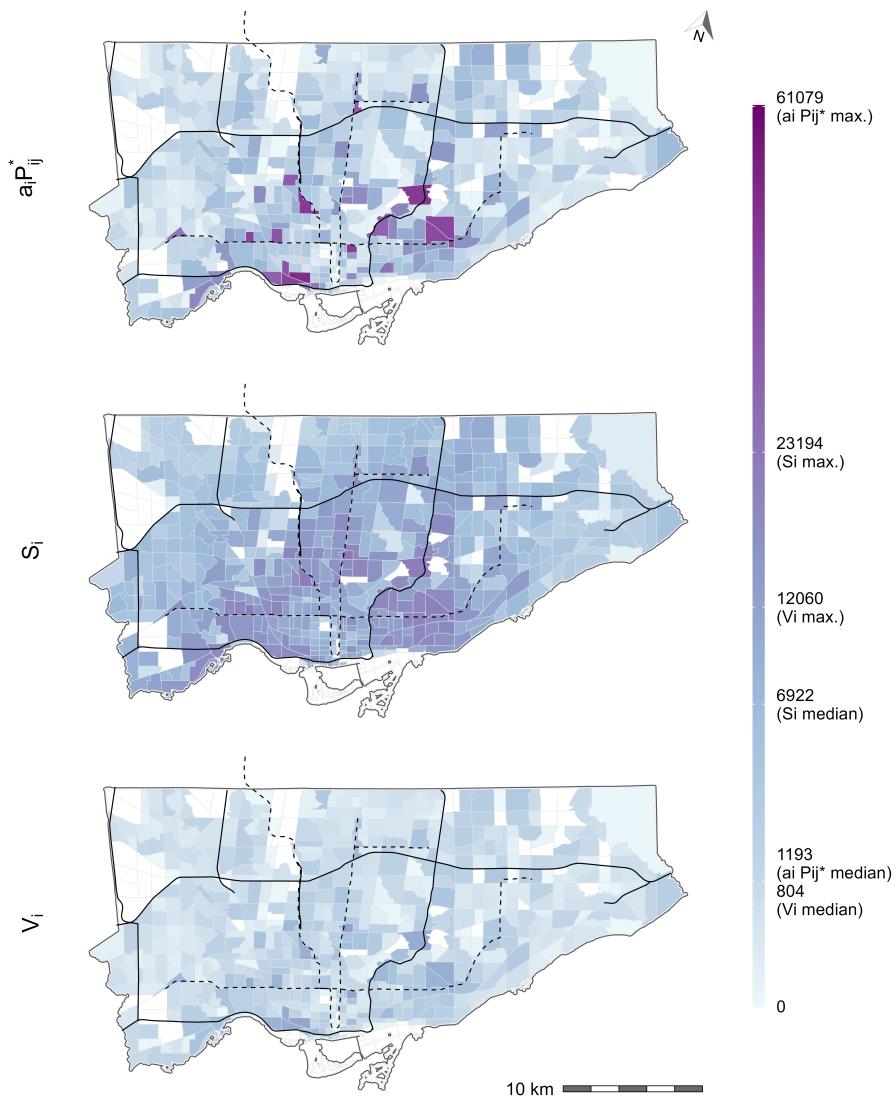


Figure 7: Estimated accessibility to jobs (# of jobs) in Toronto according to Shen-type measure times effective opportunity-seeking population (top), Hansen-type measure (middle), and spatial availability (bottom). Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values. Legend scale is square root transformed to effectively visualize the spread range.

550 are in proximity to high job density (recall Figure 4). These same trends are  
551 not present in the spatial availability bottom plot, as the absolute value is lower  
552 than Hansen-style accessibility as the proximity to high job density and com-  
553 petition from worker density is proportionally metered; the resulting values are  
554 thus lower than the middle plot and reflect the spatial distribution trends of  
555 both the workers and job density (recall Figure 4).

556 Lastly, the top plot that visualizes the *absolute* Shen-type measure (as under-  
557 stood by Shen's definition of  $P_i$  being equal to  $P_{ij}^*$  sums to the city-wide  
558 value of 2,117,774 by multiplying  $a_i$  by the *effective opportunity-seeking popu-*  
559  $P_{ij}^*$  (i.e., the denominator of the rate ). This plot thus demonstrates how  
560 confounding  $P_i$  with  $P_{ij}^*$  yields an *incorrect* number of competitively accessible  
561 jobs: it is evidently incorrect because the sum of  $a_i P_{ij}^*$  greatly exceeds the city-  
562 wide total of workers (i.e.,  $2,117,774 > 769,231$ ). To the authors' knowledge,  
563 literature has not attempted to convert Shen-type accessibility to the absolute  
564 value of accessible jobs in the way demonstrated in the top plot: we suspect  
565 this is the case because of the ambiguous definition that conflates  $P_{ij}^*$  with  $P_i$ .  
566 If  $a_i$  is multiplied by  $P_i$ , it yields the same value as  $V_i$ , but since the definition  
567 of Shen-type measure is equivocal doing so is not clear since the denominator of  
568  $a_i$  (which is a rate) is *not*  $P_i$ . The resulting plot, spatially, is similar to spatial  
569 availability (bottom plot) but certain TAZ have exceptionally high values in  
570 an inconsistent way. This is because  $a_i$  uses the impedance function values for  
571 both access to jobs (numerator) and the competition from neighboring workers  
572 (denominator  $P_{ij}^*$ ) to adjust their impact: using  $P_{ij}^*$  does not *consistently* isolate  
573 the absolute value of accessible jobs. However, if  $a_i$  is multiplied by  $P_i$  it yields  
574 the same values at  $V_i$  (bottom plot) (the proof for mathematically equivalency is  
575 in Appendix A). As also mentioned earlier, the formulation of the denominator  
576 and numerator of  $a_i$  is ambiguous so to presume that multiplying it by  $P_i$  would  
577 disintangle the rate and yield the absolute value of accessible *and available* (i.e.,  
578 considering competition) jobs is unclear.

579 *4.4.2. Internal values*

580 Carrying on the discussion on how to retrieve the absolute value of *available*  
581 jobs using the Shen-type measure ( $a_i$ ), Figure 8 highlights how the differences  
582 between  $P_i$  and  $P_{ij}^*$  are not uniform across space; the values at each origin are  
583 equivalent to  $\sum_j f(c_{ij})$ . Recall,  $P_i$  is the number of workers at each TAZ (city-  
584 wide sum of 769,231) while  $P_{ij}^*$  is the number of workers who *seek* jobs (city-wide  
585 sum of 1,770,609) in that TAZ based on their travel behaviour.  $P_{ij}^*$  is an internal  
586 value of  $a_i$  and the top plot presents the ratio of  $P_{ij}^*$  to  $P_i$  which reflects how the  
587 effective opportunity-seeking population is sometimes inflated (i.e., impedance  
588 values is greater than 1) and others deflated (i.e., impedance value is less than  
589 1) by the Shen-type measure ( $a_i$ ). As such, using  $P_{ij}^*$  to untangle the absolute  
590 job availability from  $a_i$  instead of  $P_{ij}^*$  can lead to exaggerating the total travel  
591 time in the city since it does not represent the *actual* number of workers but  
592 the *effective* number of workers. For instance, if attempting to calculate the  
593 city-wide travel time using  $a_i P_{ij}^*$ , you will yield 499740.136324239 instead of the  
594 city-wide travel time of 183736.816926917 that corresponds to the *absolute* (i.e.,

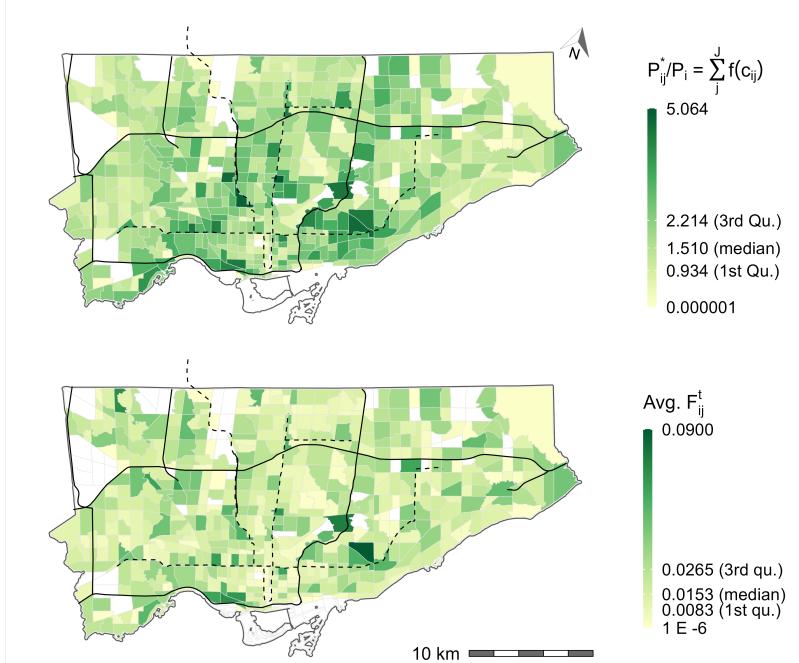


Figure 8: The ratio of the effective opportunity seeking population to the population (top) and the average spatial availability's balancing factor (Equation (9)) (bottom) for Toronto TAZ. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

595 the total number of jobs in the city is preserved) number of available jobs from  
596  $V_i$ . The absolute number of opportunities cannot be easily distangled from  $a_i$ .

597 By contrast, not only are the absolute values a direct result of  $V_i$ , the internal  
598 combined balancing factor  $F_{ij}^t$  (Equation (9)) can be used for analysis.  
599 The bottom plot showcases the average  $F_{ij}^t$  for each TAZ which is the proportional  
600 allocation mechanism of opportunities to origins in the  $V_i$  calculation.  
601 Practically, the visualized values corresponds to the average of the opportunity  
602 *proportions* that a TAZ is allocated based on travel behaviour and population  
603 competition to work destinations. These values can allow the spatial availability  
604 analyst to understand the magnitude of the *proportion of opportunities*  
605 that the origin TAZ is assigned based on the opportunities located at reachable  
606 destination TAZ. For instance, the TAZ with the maximum value of 0.090 has  
607 many origin to destination trips (112 trips, upper 3rd quantile), many workers  
608 (5538 workers, upper 3rd quantile), and located centrally within Toronto. Av-  
609 eraging  $F_{ij}^t$  demonstrates that this TAZ is allocated many proportions of jobs  
610 from reachable TAZ. This doesn't necessarily mean TAZ with a high  $V_i$  have  
611 an exceptionally high average  $F_{ij}^t$ ; for instance, many TAZ around the down-  
612 town core have high  $V_i$  values but don't have exceptionally high average  $F_{ij}^t$ .  
613 The average  $F_{ij}^t$  can thus be used to identify which areas are being relatively  
614 *high* opportunity attractors and can potentially be areas which can withstand a  
615 lower spatial availability value at the expense of increasing spatial availability  
616 elsewhere. The balancing factor is an interesting feauture of spatial availability  
617 which can open up avenues for future analysis. While this balancing factor  
618 from spatial availability can be used for analysis: there is no equivalent for the  
619 Shen-type measure; this is another advantage for spatial availability.

620 *4.4.3. Benchmarking opportunity availability*

621 Figure 9 presents the number of jobs per capita for Hansen-type accessibility  
622 (top plot), the raw number of jobs per capita (middle plot), and the spatially  
623 available jobs per capita (bottom plot). In addition to clarifying the meaning  
624 of internal values, spatial availability can also be divided by population at each  
625 origin and expressed as a rate: this rate can be used as a benchmark for equity  
626 analysis and compared directly to the raw number of jobs per capita.

627 The bottom plot features a value which is mathematically equivalent to  
628 Shen-type measure, but with a renewed interpretability. Because the proportional  
629 allocation mechanism makes clear that all the opportunities are being  
630 allocated proportionally to origins, it clarifies the interpretation since the  $V_i$   
631 values are the absolute value of *opportunity availability*. The value can thus can  
632 be directly divided by origin population and expressed as an opportunities per  
633 capita value. When spatial availability is compared to Hansen-type measure  
634 (top plot), dividing the output by population directly yield's a more difficult  
635 to interpret number of *unconstrained* accessible jobs per capita. For instance,  
636 the median light-pink shaded TAZ corresponds to approximately 5.89 uncon-  
637 strained accessible jobs per capita; this value is difficult to intercept, because  
638 as discussed in the introduction, jobs are *exclusive* opportunity types so their  
639 accessibility value should take into consideration competition.

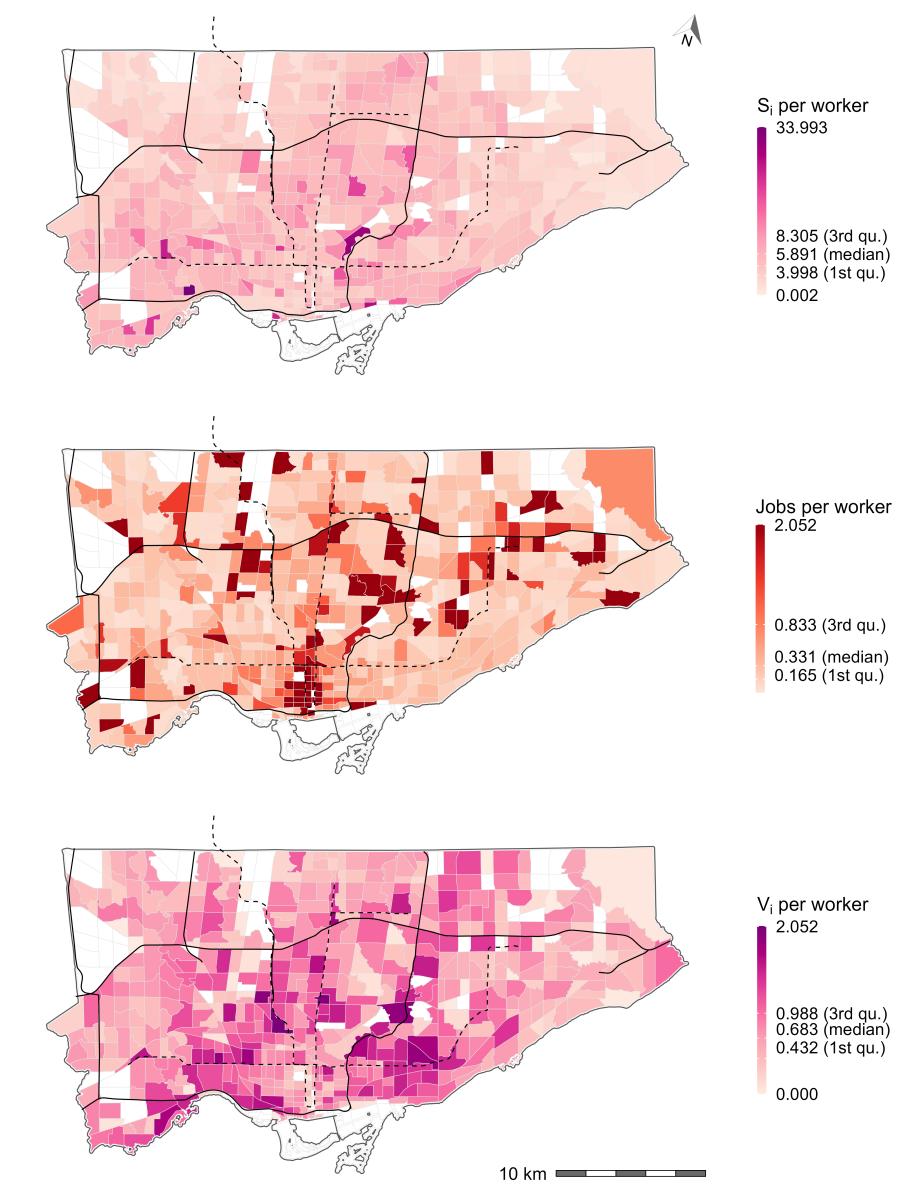


Figure 9: Hansen-type accessible jobs per capita (top), number of jobs to population ratio (middle), and spatially available jobs per capita (bottom) for Toronto. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

640     The bottom plot displays the spatially available jobs per capita. It can be  
641     interpreted as a benchmark its values can be compared directly to the raw num-  
642     ber of jobs per capita (middle plot) since the total number of opportunities are  
643     preserved (and the population, in this case, is equivalent to the number of oppor-  
644     tunities). For instance, a TAZ with a  $v_i > 1$  have more *available jobs* (based on  
645     travel behaviour and competition) than their working population. This TA has  
646     sufficient employment opportunities (under the assumptions of the input data),  
647     while TAZ with a  $v_i < 1$  do not have sufficient employment opportunities. From  
648     an equity perspective,  $v_i$  can be used to target where residential housing, job  
649     opportunities, and/or transportation system improvements should be created.  
650     For TAZ with  $v_i$  values significantly greater than 1 (dark pinks), constructing  
651     more residential housing for the type of workers who occupy the *available jobs*  
652     in the proximate TAZ should be considered. Assuming the input data is cor-  
653     rect, increasing the competition in the area will decrease the  $v_i$  score but if can  
654     be decreased up to threshold of  $v_i = 1$ . For TAZ with  $v_i$  values significantly  
655     less than 1 (light pinks), constructing more employment opportunities for the  
656     type of workers who live in proximate TAZ and/or prioritizing transportation  
657     network improvements to create more favourable travel time conditions. De-  
658     pending on the raw jobs per worker ratio, different approaches are appropriate.  
659     For instance, adding more residential locations near the downtown core (bottom  
660     center on the bottom plot) could be a good approach to increasing  $v_i$  as there  
661     is already a high jobs per worker ratio (middle plot). However, doing so will  
662     decrease the  $v_i$  availability in areas near the border of the city, so in addition  
663     to doing so, adding more employment opportunities to areas with low raw jobs  
664     per worker ratio and low  $v_i$  is needed. In addition to these changes, the travel  
665     time landscape would also influence the resulting  $v_i$  score, so transportation  
666     network improves to areas with low  $v_i$  could also be considered. This is to say,  
667      $v_i$  is dependent on the magnitude and spatial distribution of residential housing,  
668     job opportunities, and transportation system so the region could be optimized  
669     to achieve thresholds of specific  $v_i$  values and thus the difference in residential  
670     housing, job opportunities, and transportation system can become policy tar-  
671     gets. It should also be kept in mind, that though  $v_i = 1$  and the comparison  
672     to the raw jobs per worker values can be used for policy planning,  $v_i$  can easily  
673     be transformed back to  $V_i$  to understand the magnitude of the job availability  
674     within that origin.

675     **5. Conclusion**

676     In this paper we show how a widely used accessibility measure with com-  
677     petition obscures internal values of opportunities taken due to the confounding  
678     of population of zones with the *effective opportunity-seeking population*. We  
679     then propose an alternative derivation of accessibility with competition that  
680     we call spatial availability. This measure makes sure that the opportunities  
681     are allocated in a proportional way and preserved in the regional total. We  
682     also show that spatial availability and Shen-type accessibility are equifinal: for-  
683     mally the equations are the same (along with 2SFCA) and can be considered as  
684     singly-constrained measures.

685     Why do differences between Hansen-style measure and the interpretation of  
686     Shen-type measure matter? Because of equity analysis and policy planning.  
687     With spatial availability, we can push accessibility analysis forward by making  
688     it competitive accessibility more interpretable. Hansen-type measure tends  
689     to result in values which are very extreme as a result of multiple-counting op-  
690     portunities as shown in empirical example. Multiple-counting may not be an  
691     issue if the opportunity-type is non-exclusive, but with the case of employment  
692     where one worker can only take one job, the resulting values are difficult to  
693     interpret (though it can be interpreted relatively to speak about urban form).  
694     In this paper, we also demonstrate that attempting to disentangle the absolute  
695     values of opportunities from the Shen-type accessibility measure is difficult as  
696     a result of Shen's definition which confounds the population with the effective-  
697     opportunity seeking population. As demonstrated within this paper, spatial  
698     availability overcomes the interpretability issue associated with Shen's measure  
699     by presenting first, the absolute value of *available* jobs and then by dividing the  
700     available jobs value by the number of working population. This rate is equiv-  
701     alent to Shen-type measure but contains internal values, such as the proportional  
702     allocation mechanism that yields the combined balancing factor, which can be  
703     used to interpret the absolute and rate values.

704     Based on this research we suggest the following guidelines for the application  
705     of spatial availability and the topic of future work:

- 706       1) The Hansen-style accessibility should be used when opportunities are non-  
707       exclusive. When opportunities are perfectly exclusive (i.e., 1 spot for 1  
708       person), spatial availability (i.e., accessibility with competition) should be  
709       used.
- 710       2) Shen-type accessibility can be used to compute the availability of jobs  
711       (the rate and the absolute values if the original definition is corrected),  
712       however, it should not be used if the analyst is not interested in internal  
713       values. If willing to push the analysis further, the use of spatial availability  
714       should be considered.
- 715       3) With the renewed interpretability of what the absolute *opportunity avail-*  
716       *ability* is at each origin, the spatial availability per capita (Equation X)  
717       value of 1 can be used as a policy goal. For areas with a value below  
718       1, targeted increases to the quantity of opportunities, residential housing,

719 and transportation system improvements can be theorized such that the  
 720 number of *available jobs* per capita in the region is at least equal to 1.  
 721 Since spatial availability per capita implicitly preserves the number of op-  
 722 portunities in the region, it can be directly compared to the the region's  
 723 raw jobs to population ratio to inform policy. Additionally, the absolute  
 724 values of spatial availability can be used to understand the magnitude of  
 725 the opportunity availability deficit (or surplus).

- 726 4) Spatial availability per capita can also be compared directly to other re-  
 727 gions as done by literature using Shen-type measure/2SFCA (e.g., Gian-  
 728 notti et al. (2021)). However, as a result of the renewed interpretation,  
 729 the magnitude of *spatially available* opportunities can be quantified.  
 730 5) Lastly, since opportunities are preserved, many new avenues of analysis  
 731 can be pursued which have not been conceptualized yet. This is espe-  
 732 cially important in light of emerging concerns with equity. For instance,  
 733 the population and opportunities can be segmented (i.e., transit users,  
 734 active transportation users, low income, low education, new comers, chil-  
 735 dren) and their spatial availability to opportunities can be assessed, bench-  
 736 marked, and corresponding policy to target inequities can be theorized.  
 737 As another example, the combined balancing factor can be analysed to  
 738 identify which populations currently do not seek opportunities because of  
 739 friction of distance.

740 **6. Appendix A**

741 The mathematical equivalence of Shen-type accessibility measure and spatial  
 742 availability is provided in this appendix.

The population-based balancing factor used in  $V_i$  is defined as:

$$F_{ij}^p = \frac{P_{i \in r}^\alpha}{\sum_i^N P_{i \in r}^\alpha}$$

$$F_A^p = \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha}$$

743 The impedance-based balancing factor in  $V_i$  is:

$$\begin{aligned} F_{ij}^c &= \frac{f(c_{ij})}{\sum_{i=A}^N f(c_{ij})} \\ F_{A1}^c &= \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} \\ F_{B1}^c &= \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} \\ F_{C1}^c &= \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} \end{aligned}$$

<sup>744</sup> These factors when assembled together with  $P$  makes the denominators can-  
<sup>745</sup> cel out:

$$v_i = \sum_j \frac{O_j}{P_A^\alpha} \frac{\frac{P_{i \in r}^\alpha}{\sum_i^N P_{i \in r}^\alpha} \cdot \frac{f(c_{ij})}{\sum_i^N f(c_{ij})}}{\sum_i^N \frac{P_{i \in r}^\alpha}{\sum_i^N P_{i \in r}^\alpha} \cdot \frac{f(c_{ij})}{\sum_i^N f(c_{ij})}}$$

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} \right) +$$

$$\frac{O_2}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} \right) +$$

$$\frac{O_3}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} \right)$$

<sup>746</sup> First, notice how the denominator on the denominator is the same across  
<sup>747</sup> the summation. Simplifying:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A1}) + f(c_{B1}) + f(c_{C1}))}} \right)$$

$$\frac{O_2}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A2}) + f(c_{B2}) + f(c_{C2}))}} \right)$$

$$\frac{O_3}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A3}) + f(c_{B3}) + f(c_{C3}))}} \right)$$

<sup>748</sup> Notice how the denominator of the denominator is the same as the denomi-  
<sup>749</sup> nator of the numerator for each  $J$  ( $J=1$ ,  $J=2$ , and  $J=3$ )? Cancel those out and  
<sup>750</sup> simplify:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} \right)$$

$$\frac{O_2}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} \right)$$

$$\frac{O_3}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

<sup>751</sup> Next, we can cancel out the  $P_A^\alpha$ :

$$v_A = O_1 \left( \frac{f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})} + O_2 \frac{f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})} + O_3 \frac{f(c_{A3})}{P_A^\alpha \cdot f(c_{A3})} \right)$$

<sup>752</sup> which shows how  $v_A$  if formally identical to the Shen-type accessibility measure  
<sup>753</sup> with competition.

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Table 1: Summary description of synthetic example: Hansen-type accessibility  $S_i$ , Shen-type accessibility  $a_i$ , and spatial availability  $V_i$  with beta = 0.1 (light yellow) and beta = 0.6 (light grey).

Origin	A			B			C		
	1	2	3	1	2	3	1	2	3
Dest.									
Pop.	50000	50000	50000	150000	150000	150000	10000	10000	10000
Jobs	100000	100000	10000	100000	100000	10000	100000	100000	10000
TT	15	30	100	30	15	100	100	100	15
f(TT)	0.223	0.050	< 0.001	0.050	0.223	< 0.001	< 0.001	< 0.001	0.223
Pop * f(TT)	11156.5	2489.4	2.3	7468.1	33469.5	6.8	0.5	0.5	2231.3
Jobs * f(TT)	22313.0	4978.7	0.5	4978.7	22313.0	0.5	4.5	4.5	2231.3
S_i	27292.2	27292.2	27292.2	27292.2	27292.2	27292.2	2240.4	2240.4	2240.4
a_i	1.337	1.337	1.337	0.888	0.888	0.888	0.996	0.996	0.996
f(TT)	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
Pop * f(TT)	6.170	< 0.001	< 0.001	0.002	18.511	< 0.001	< 0.001	< 0.001	1.234
Jobs * f(TT)	12.341	0.002	< 0.001	0.002	12.341	< 0.001	< 0.001	< 0.001	1.234
S_i	12.343	12.343	12.343	12.343	12.343	12.343	1.234	1.234	1.234
a_i	1.999	1.999	1.999	0.667	0.667	0.667	1.000	1.000	1.000
F^c	0.238	0.238	0.238	0.714	0.714	0.714	0.048	0.048	0.048
F^p	0.817	0.182	< 0.001	0.182	0.817	< 0.001	< 0.001	< 0.001	1.000
F	0.599	0.069	0.001	0.401	0.931	0.003	< 0.001	< 0.001	0.996
V_ij	59900.6	6922.7	10.1	40096.9	93076.0	30.4	2.4	1.3	9959.5
V_i	66833.5	66833.5	66833.5	133203.4	133203.4	133203.4	9963.2	9963.2	9963.2

jobs
7.69e+05

<b>jobs</b>
7.69e+05

<b>workers</b>
7.69e+05

<b>workers</b>
1.77e+06

<b>workers</b>
7.69e+05