Introducing spatial availability, a singly-constrained measure of competitive accessibility

Abstract

Accessibility indicators are widely used in transportation, urban, and health-care planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability, mainly by introducing competition. In this paper, we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

1. Introduction

The concept of accessibility in transportation studies derives its appeal from the combination of the spatial distribution of opportunities and the cost of reaching them (Handy and Niemeier, 1997; Hansen, 1959). Accessibility analysis is employed in transportation, geography, public health, and many other areas, with the number of applications growing (Shi et al., 2020), especially as mobility-based planning is de-emphasized in favor of access-oriented planning (Deboosere et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021).

Accessibility analysis stems from the foundational works of Harris (1954) and Hansen (1959). From these seminal efforts, many accessibility measures have been derived, particularly after the influential work of Wilson (1971) on spatial interaction[1]. Of these, gravity-type accessibility is arguably the most common; since its introduction in the literature, it has been widely adopted in numerous forms (Arranz-López et al., 2019; Cervero et al., 2002; Geurs and van Wee, 2004; Levinson, 1998; Paez, 2004). Hanson-type accessibility indicators are essentially weighted sums of opportunities, with the weights given by an impedance function that depends on the cost of movement, and thus measure the *intensity of the possibility of interaction* (Hansen, 1959). This type of accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure (Handy and Niemeier, 1997).

Despite their usefulness, the interpretability of Hansen-type accessibility measures can be challenging (Geurs and van Wee, 2004; Miller, 2018). Since they aggregate opportunities, the results are sensitive to the size of the region of interest (e.g., a large city has more jobs than a smaller city). As a consequence, raw outputs are not necessarily comparable across study areas (Allen and Farber, 2019). This limitation becomes evident when surveying studies that implement this type of analysis. For example, Páez et al. (2010) (in Montreal) and Campbell et al. (2019) (in Nairobi) report accessibility as the number of health care facilities that can potentially be reached from origins. But what does it mean for a zone to have accessibility to less than 100 facilities in each of these two cities, with their different populations and number of facilities? For that matter, what does it mean for a zone to have accessibility to more than 700 facilities in Montreal, besides being "accessibility rich"? As another example, Bocarejo S. and Oviedo H. (2012) (in Bogota), El-Geneidy et al. (2016) (in Montreal), and Jiang and Levinson (2016) (in Beijing) report accessibility as numbers of jobs, with accessibility values often in the hundreds of thousands, and even exceeding one million jobs for some zones in Beijing and Montreal. As indicators of urban structure, these measures are informative, but the meaning of one million accessible jobs is harder to pin down: how many jobs must any single person have access to? Clearly, the answer to this question depends on how many people demand jobs.

[1] Utility-based measures derive from a very different theoretical framework, random utility maximization

The interpretability of Hansen-type accessibility has been discussed in numerous studies, including recently by Hu and Downs (2019), Kelobonye et al. (2020), and in greater depth by Merlin and Hu (2017). As hinted above, the limitations in interpretability are frequently caused by ignoring competition - without competition, each opportunity is assumed to be equally available to every single opportunity-seeking individual that can reach it (Kelobonye et al., 2020; Paez et al., 2019; Shen, 1998). This assumption is appropriate when the opportunity of interest is non-exclusive, that is, if use by one unit of population does not preclude use by another. For instance, national parks with abundant space are seldom used to full capacity, so the presence of some population does not exclude use by others. When it comes to exclusive opportunities, or when operations may be affected by congestion, the solution has been to account for competition. Several efforts exist that do so. In our reckoning, the first such approach was proposed by Weibull (1976), whereby the distance decay of the supply of employment and the demand for employment (by workers) were formulated under so-called axiomatic assumptions. This approach was then applied by Joseph and Bantock (1984) in the context of healthcare, to quantify the availability of general practitioners in Canada. About two decades later, Shen (1998) independently re-discovered Weibull's (1976) formula (see footnote (7) in Shen, 1998) and deconstructed it to consider accessibility for different modes. These advances were subsequently popularized as the family of Two-Stage Floating Catchment area (2SFCA) methods (Luo and Wang, 2003) that have found widespread adoption in healthcare, education, and food systems (B. Y. Chen et al., 2020; Chen, 2019; Z. Chen et al., 2020; Yang et al., 2006; Ye et al., 2018).

An important development contained in Shen's work is a proof that the population-weighted sum of the accessibility measure with competition equates to the number of opportunities available (footnote (7) and Appendix A in Shen, 1998). This demonstration gives the impression that Shen-type accessibility allocates *all* opportunities to the origins, however to the authors' knowledge, it has not interpreted in this way in the literature. For instance, Hu (2014), Merlin and Hu (2017), , and Tao et al. (2020) all use Shen-type accessibility to calculate job access but report values as 'competitive accessibility scores' or simply 'job accessibility' and focus on the way in which job supply (opportunities) and demand (job-seeking population) are both discounted by travel cost. These works do not explicitly recognize that jobs that are assigned to each origin are in fact *all* the opportunities in the system. This recognition, we argue, is critical to interpreting the meaning of the final result. Thus, in this paper we intend to revisit accessibility with competition within the context of disentangling how opportunities are allocated. We first argue that Shen's competitive accessibility misleadingly equates the travel-cost discounted opportunity-seeking population to the total zonal population provided in Shen's proof. This equivocation, we believe, results in a misleading interpretation of what Shen-type accessibility actually represents since the allocation of opportunities to population are masked by the presentation of results as rates (i.e., opportunities per capita). We then propose an alternative formulation of accessibility that incorporates competition by adopting a proportional allocation mechanism; we name this measure *spatial availability*. The use of balancing factors for proportional allocation is akin to imposing a single constraint on the accessibility indicator, in the spirit of Wilson's (1971) spatial interaction model.

In this way, the aim of the paper is three-fold:

- First, we aim to demonstrate that Shen-type (and thus Weibull (1976) accessibility and the popular 2SFCA methods) produce misleading estimates of the opportunities allocated;

- Second, we introduce a new measure, *spatial availability*, which we submit is a more interpretable alternative to Shen-type accessibility, since opportunities in the system are preserved and proportionally allocated to the population; and

- Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.

Discussion is supported by the use of the small synthetic example of Shen (1998) and empirical data drawn from the 2016 Transportation Tomorrow Survey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the spirit of openness of research in the spatial sciences (Brunsdon and Comber, 2021; Páez, 2021) this paper has a companion open data product (Arribas-Bel et al., 2021), and all code is available for replicability and reproducibility purposes.

2. Accessibility measures revisited

In this section we revisit Hansen-type and Shen-type accessibility indicators. We adopt the convention of using a capital letter for absolute values (number of opportunities) and lower case for rates (opportunities per capita).

2.1. Hansen-type accessibility

Hansen-type accessibility measures follow the general formulation shown in Equation (1):

$$S_i = \sum_{j=1}^{J} O_j \cdot f(c_{ij}) \tag{1}$$

where:

- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}; it can take the form of any monotonically decreasing function chosen based on positive or normative criteria (Paez et al., 2012).
- i is a set of origin locations ($i = 1, \cdots, N$).
- j is a set of destination locations ($j = 1, \cdots, J$).

- O_j is the number of opportunities at location j; $O = \sum_{j=1}^{J} O_j$ is the total supply of opportunities in the study region.
- S is Hansen-type accessibility as weighted sum of opportunities.

As formally defined, accessibility S_i is the sum of opportunities that can be reached from location i, weighted down by an impedance function of the cost of travel c_{ij}. Summing the opportunities in the neighborhood of i provides estimates of the number of opportunities that can *potentially* be reached from i. Several variants of this method result from using a variety of impedance functions; for example, cumulative opportunities measures are obtained when $f(\cdot)$ is a binary or indicator function (e.g., El-Geneidy et al., 2016; Geurs and van Wee, 2004; Qi et al., 2018; Rosik et al., 2021). Other measures use impedance functions modeled after any monotonically decreasing function (e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, *inter alia*, Kwan, 1998; Li et al., 2020; Reggiani et al., 2011; Vale and Pereira, 2017). In practice, accessibility measures with different impedance functions tend to be highly correlated (Higgins, 2019; Kwan, 1998; Santana Palacios and El-geneidy, 2022).

Gravity-based accessibility has been shown to be an excellent indicator of the intersection between spatially distributed opportunities and transportation infrastructure (Kwan, 1998; Reggiani et al., 2011; Shi et al., 2020). However, beyond enabling comparisons of relative values they are not highly interpretable on their own (Miller, 2018). To address the issue or interpretability, previous research has aimed to index and normalize values on a per demand-population basis (e.g., Barboza et al., 2021; Pereira et al., 2019; Wang et al., 2021). However, as recent research on accessibility discusses (Allen and Farber, 2019; Kelobonye et al., 2020; Merlin and Hu, 2017; Paez et al., 2019), these steps do not adequately consider competition. In effect, when calculating S_i, every opportunity enters the weighted sum once for every origin i that can reach it. This makes interpretability opaque, and to complicate matters, can also bias the estimated landscape of opportunity.

2.2. Shen-type competitive accessibility

To account for competition, the influential works of Shen (1998) and Weibull (1976), as well as the widely used 2SFCA approach of Luo and Wang (2003), adjust Hansen-type accessibility with the population in the region of interest. The mechanics of this approach consist of calculating, for every destination j, the population that can reach it given the impedance function $f(\cdot)$; let us call this the *effective opportunity-seeking population* (Equation (2)). This value can be seen as the Hansen-type *market area* (accessibility to population) of j. The opportunities at j are then divided by the sum of the *effective opportunity-seeking population* to obtain a measure of opportunities per capita, i.e., R_j in Equation (3). This can be thought of as the *level of service* at j. Per capita values are then allocated back to the population at i, again subject to the impedance function as seen in Equation (4); this is accessibility with competition.

$$P_{ij}^* = P_i \cdot f(c_{ij}) \tag{2}$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^*} \tag{3}$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \tag{4}$$

where:

- a is Shen-type accessibility as weighted sum of opportunities per capita (or weighted level of service).
- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}.
- i is a set of origin locations ($i = 1, \cdots, N$).
- j is a set of destination locations ($j = 1, \cdots, J$).
- O_j is the number of opportunities at location j; $O = \sum_{j=1}^{J} O_j$ is the total supply of opportunities in the study region.
- P_i is the population at location i.
- P_{ij}^* is the population at location i that can reach destination j according to the impedance function; we call this the *effective opportunity-seeking population*.
- R_j is the ratio of opportunities at j to the sum over all origins of the *effective opportunity-seeking population* that can reach j; in other words, this is the total number of opportunities per capita found at j.

Shen (1998) describes P_i as the *"the number of people in location i seeking opportunities"*. In our view, this is somewhat equivocal. Consider a population center where the population are willing to travel at most 60 minutes. This is identical to the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 \text{ if } c_{ij} \leq 60 \text{ min} \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

If an employment center is less than 60 minutes away, the population can seek opportunities there. But are these people still part of the opportunity-seeking population for jobs located two hours away? Four? Ten? We would submit that they are not, according to the travel behavior represented by the impedance function. For the purpose of calculating accessibility, the impedance function defines what constitutes the population that effectively can seek opportunities at remote locations.

According to Shen (1998), the sum of $A_i = a_i P_i$ over i equates the total number of opportunities in the full study region.

$$\sum_{i=1}^{N} a_i P_i = \sum_{i=1}^{N} A_i = \sum_{j=1}^{J} O_j = O \tag{6}$$

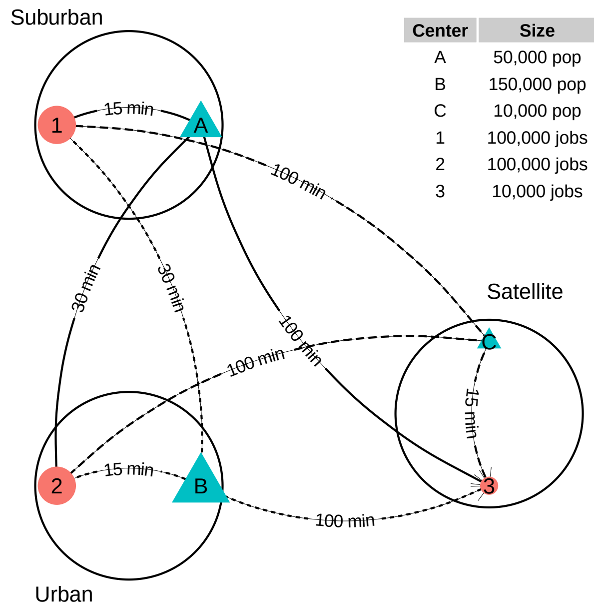


Figure 1: Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

Notice, however, that the opportunities per capita are multiplied by the total zonal population, which is not necessarily the same as the effective opportunity-seeking population. Thus, Equation (6) holds only if we choose to ignore the travel behavior of the population.

2.3. Example

In this section we use the example in Shen (1998) to flesh out with concrete detail the arguments above. The example is the simple system shown in Figure 1.

Table 1 contains the information needed to calculate S_i and a_i for this example. We use a negative exponential impedance function with $\beta = 0.1$ (see Shen, 1998, footnote (5)):

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

In the table we see that population centers A and B have equal Hansen-type accessibility ($S_A = S_B =$ 27,292 jobs). On the other hand, the isolated satellite town of C has low accessibility ($S_C =$ 2,240 jobs). But center B, despite its high accessibility, is a large population center. C, in contrast, is smaller but also relatively isolated and has a balanced ratio of jobs (10,0000) to population (10,000). It is difficult from these outputs to determine whether the accessibility at C is better or worse than that at A or B.

Table 1: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition with beta = 0.1

Origin	**Pop.**	**Dest.**	**Jobs**	**TT**	**f(TT)**	**Pop * f(TT)**	**Jobs * f(TT)**	**S_i**	**a_i**
		1	100,000	15	0.223130	11,157	22,313		
A	50,000	2	100,000	30	0.049787	2,489	4,979	27,292	1.34
		3	10,000	100	0.000045	2.27	0.454		
		1	100,000	30	0.049787	7,468	4,979		
B	150,000	2	100,000	15	0.223130	33,470	22,313	27,292	0.888
		3	10,000	100	0.000045	6.81	0.454		
		1	100,000	100	0.000045	0.454	4.54		
C	10,000	2	100,000	100	0.000045	0.454	4.54	2,240	0.996
		3	10,000	15	0.223130	2,231	2,231		

The results are easier to interpret when we consider Shen-type accessibility. The results indicate that $a_A \approx 1.337$ jobs per capita, $a_B \approx 0.888$, and $a_C \approx 0.996$. The latter value is sensible given the jobs-population balance of C. Center A is relatively close to a large number of jobs (more jobs than the population of A). The opposite is true of B. According to Shen (1998), the sum of the population-weighted accessibility a_i is exactly equal to the number of jobs in the region:

$$\begin{aligned}&50,000 \times 1.3366693\\&+150,000 \times 0.8880224\\&+10,000 \times 0.9963171 = 210,000\end{aligned}$$

As mentioned earlier, this property gives the impression that jobs are allocated in their totality. However, for this property to work, the accessibility values need to be multiplied by the total population of their corresponding zones. Alas, there is a logical inconsistency in this calculation, since the travel behavior (i.e., the impedance function), means that the effective opportunity-seeking population $P_i^* = \sum_j P_{ij}^*$ is not necessarily equal to the total population P_i. In other words, the effective opportunity-seeking population and the total population are confounded. As seen in column **Pop * f(TT)** in Table 1 (i.e., $P_{ij}^* = P_i \cdot f(c_{ij})$), the number of individuals from population center A that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2.27 respectively. Therefore, the effective opportunity-seeking population is $P_A^* = \sum_j P_{Aj}^*$ 13,647.27, which is considerably lower than the total population of A (i.e., $P_A =$ 50,000).

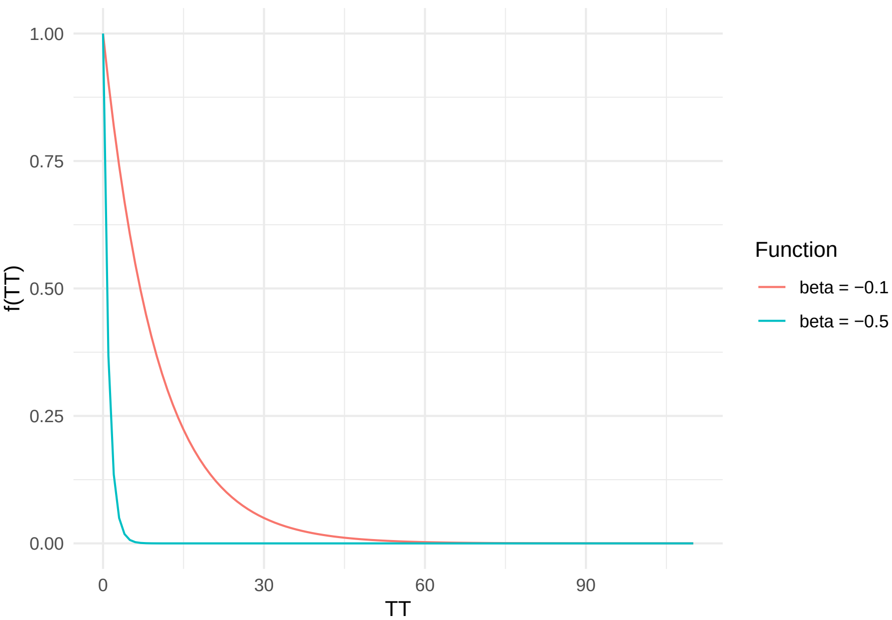


Figure 2: Comparison of two impedance functions in the example.

To ensure that the calculations are consistent with the travel behavior given by the impedance function, the number of accessible jobs per capita should be multiplied by the population who are willing to travel to the employment centers; hence, instead of the nominal number of jobs in the region, the number of jobs the method actually allocates is:

$$
\begin{aligned}
&(11,156.51 + 2,489.35 + 2.26) \times 1.3366693\\
&+(7,468.06 + 33,469.52 + 6.81) \times 0.8880224\\
&+(4.54 + 4.54 + 2,231.20) \times 0.9963171 \approx 56,834.59
\end{aligned}
$$

which is less than one-third of the total number of jobs in the region.

Use of the total zonal population in the calculation, instead of the effective opportunity-seeking population, gives the impression that all jobs are allocated - however the result is inconsistent with the travel behavior in the model. When the impedance-weighted opportunity-seeking population is used, it becomes apparent that the number of jobs allocated is not equal to the total number of jobs in the region. This feature of the method is not immediately apparent because the results are given in terms of opportunities per capita.

Consider the example in Table 2, where we increase the friction of distance by changing β to 0.5 (compared to the previous value of 0.1; see Figure 2).

As expected, Hansen-type accessibility drops, quite dramatically in this case: the friction of distance is so high that few opportunities are within reach. In contrast, Shen-type accessibility converges to the jobs/population ratio. Notice however that the population from center A that effectively seeks opportunities

Table 2: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition with beta = 0.5

Origin	**Pop.**	**Dest.**	**Jobs**	**TT**	**f(TT)**	**Pop * f(TT)**	**Jobs * f(TT)**	**S_i**	**a_i**
		1	100,000	15	< 0.001	0.015	0.031		
A	50,000	2	100,000	30	< 0.001	< 0.001	< 0.001	0.0306	2
		3	10,000	100	< 0.001	< 0.001	< 0.001		
		1	100,000	30	< 0.001	< 0.001	< 0.001		
B	150,000	2	100,000	15	< 0.001	0.046	0.031	0.0306	0.667
		3	10,000	100	< 0.001	< 0.001	< 0.001		
		1	100,000	100	< 0.001	< 0.001	< 0.001		
C	10,000	2	100,000	100	< 0.001	< 0.001	< 0.001	0.00306	1
		3	10,000	15	< 0.001	0.003	0.003		

at center 1 has collapsed to 0.015, while the number of jobs allocated from center 1 to A given the friction of distance is only 0.031. So yes, the jobs/population ratio is 2, but only for a tiny fraction of the population of A that effectively seeks opportunities at center 1.

In what follows, we propose an alternative derivation of competitive accessibility that resolves the inconsistency described above.

3. Introducing spatial availability: a singly-constrained measure of accessibility

In brief, we define the *spatial availability* at i (V_i) as the proportion of all opportunities O that are allocated to i from all destinations j:

$$V_i = \sum_{j=1}^{K} O_j F_{ij}^t$$

where:

- F_{ij}^t is a balancing factor that depends on the population and cost of movement in the system.
- O_j is the number of opportunities at j.
- V_i is the number of spatially available opportunities from the perspective of i.

The general form of spatial availability is also as a sum, and the fundamental difference with Hansen- and Shen-type accessibility is that opportunities are allocated proportionally. Balancing factors F_{ij} allocate opportunities to i in proportion to the size of the population of the different competing centers (the mass effect of the gravity model), and the cost of reaching opportunities (the impedance effect). In the next two subsections, we explain the intuition behind the method before defining it in full.

3.1. Proportional allocation by population

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by population. The total population in the example is 210,000. The proportion of the population by population center is:

$$F^p_A = \frac{50{,}000}{210{,}000}$$

$$F^p_B = \frac{150{,}000}{210{,}000}$$

$$F^p_C = \frac{10{,}000}{210{,}000}$$

Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors F^p_i. In this way, employment center 1 allocates $100,000 \cdot \frac{50{,}000}{210{,}000} = 23,809.52$ jobs to A; $100,000 \cdot \frac{150{,}000}{210{,}000} = 71,428.57$ jobs to B; and $100,000 \cdot \frac{10{,}000}{210{,}000} = 7,142.857$ jobs to C. Notice how this mechanism ensures that the total number of jobs at employment center 1 is preserved at 100,000.

We can verify that the number of jobs allocated is consistent with the total number of jobs in the region:

Employment center 1 to population centers A, B, and C:
$100,000 \cdot \frac{50{,}000}{210{,}000} + 100,000 \cdot \frac{150{,}000}{210{,}000} + 100,000 \cdot \frac{10{,}000}{210{,}000} = 100,000$

Employment center 2 to population centers A, B, and C:
$100,000 \cdot \frac{50{,}000}{210{,}000} + 100,000 \cdot \frac{150{,}000}{210{,}000} + 100,000 \cdot \frac{10{,}000}{210{,}000} = 100,000$

Employment center 3 to population centers A, B, and C:
$10,000 \cdot \frac{50{,}000}{210{,}000} + 10,000 \cdot \frac{150{,}000}{210{,}000} + 10,000 \cdot \frac{10{,}000}{210{,}000} = 10,000$

In the general case where there are N population centers in the region, we define the following population-based balancing factors:

$$F^p_i = \frac{P^\alpha_i}{\sum_{i=1}^N P^\alpha_i} \tag{7}$$

Balancing factor F^p_i corresponds to the proportion of the population in origin i relative to the population in the region. On the right hand side of the equation, the numerator P_i^α is the population at origin i. The summation in the denominator is over $i = 1, \cdots, N$, and adds up to the total population of the region. Notice that we incorporate an empirical parameter α. The role of α is to modulate the effect of demand by population. When $\alpha < 1$, opportunities are allocated more rapidly to smaller centers relative to larger ones; $\alpha > 1$ achieves the opposite effect.

Balancing factor F_i^p can now be used to proportionally allocate a share of available jobs at j to origin i. The number of jobs available to i from j balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

In the general case where there are J employment centers, the total number of jobs available from all destinations to i is simply the sum of V_{ij}^p over $j = 1, \cdots, J$:

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

Since the factor F_i^p, when summed over $i = 1, \cdots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^p = 1$), the sum of all spatially available jobs equals O, the total number of opportunities in the region:

$$\begin{aligned}
&\textstyle\sum_{i=1}^N V_i^p = \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\
&= \textstyle\sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\
&= \textstyle\sum_{j=1}^J O_j = O
\end{aligned}$$

The terms F_i^p act here as the balancing factors of the gravity model when a single constraint is imposed (i.e., to ensure that the sums of columns are equal to the number of opportunities per destination, see Ortúzar and Willumsen, 2011, pp. 179–180 and 183-184). As a result, the sum of spatial availability for all population centers equals the total number of opportunities.

The discussion so far concerns only the mass effect (i.e., population size) of the gravity model. In addition, the potential for interaction is thought to decrease with increasing cost, so next we define similar balancing factors but based on the impedance.

3.2. Proportional allocation by cost

Clearly, using only balancing factors F_i^p to calculate spatial availability V_i^p does not account for the cost of reaching employment centers. Consider instead a set of balancing factors F_{ij}^c that account for the friction of distance:

$$\begin{aligned}
F_{A1}^c &= \tfrac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\
F_{B1}^c &= \tfrac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\
F_{C1}^c &= \tfrac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581
\end{aligned}$$

Balancing factors F_{ij}^c use the impedance function to proportionally allocate more jobs to closer population centers, that is, to those with populations *more willing to reach the jobs*. Indeed, the factors F_{ij}^c can be thought of as the proportion of the population at i willing to travel to destination j, conditional on the travel behavior as given by the impedance function.

In our example, the number of jobs allocated from employment center 1 to population center A is $100,000 \times 0.8174398 = 81,743.98$; to population center B is $100,000 \times 0.1823954 = 18,239.54$; and to population center C is $100,000 \times 0.0001648581 = 16.48581$. We see once more that the total number of jobs at the employment center is preserved at 100,000. In this example, the proportional allocation mechanism assigns the largest share of jobs to population center A, which is the closest to employment center 1, and the smallest to the more distant population center C.

In the general case where there are N population centers and J employment centers in the region, we define the following impedance-based balancing factors:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \tag{8}$$

The total number of jobs available to i from j according to impedance is defined as follows:

$$V_{ij}^c = O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c}$$

The total number of jobs available to i from all destinations is:

$$V_i^c = \sum_{j=1}^J O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c}$$

Like the population-based allocation factors, F_i^c summed over $i = 1, \cdots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^c = 1$). As before, the sum of all spatially available jobs equals O, the total number of opportunities in the region:

$$\begin{aligned}
&\sum_{i=1}^N V_i^c = \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c} \\
&= \sum_{i=1}^N \frac{F_i^c}{\sum_{i=1}^N F_i^c} \cdot \sum_{j=1}^J O_j \\
&= \sum_{j=1}^J O_j = O
\end{aligned}$$

We are now ready to more formally define spatial availability with due consideration to both mass and cost effects.

3.3. Assembling mass and impedance effects

Population and the cost of travel are both part of the gravity modelling framework. Since the balancing factors defined in the preceding sections are proportions (alternatively probabilities), they can be combined multiplicatively to obtain their joint effect (alternatively, the joint probability of allocating opportunities). This idea is captured by Equation (9), where F_i^p is the population-based balancing factor that grants a larger share of the existing opportunities to larger centers and F_{ij}^c is the impedance-based balancing factor that grants a larger share of the existing opportunities to closer centers. This is in line with the tradition of gravity modeling.

$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \tag{9}$$

with F_i^p and F_i^c as defined in Equations (7) and (8) respectively.

Balancing factors F_{ij}^t are used to proportionally allocate jobs from j to i. The spatial availability is given by Equation (10).

$$V_i = \sum_{j=1}^J O_j \ F_{ij}^t \tag{10}$$

The terms in Equation 10 are as follows:

- F_{ij}^t is a balancing factor as defined in Equation (9).
- i is a set of origin locations in the region $i = 1, \cdots, N$.
- j is a set of destination locations in the region $j = 1, \cdots, J$.
- O_j is the number of opportunities at location j.
- V_i is the spatial availability at i.

Notice that, unlike S_i in Equation (1), the population enters the calculation of V_i through F_i^p. Returning to the example in Figure 1, Table 3 contains the information needed to calculate V_i.

In the table, column **V_ij** are the jobs available to each origin from each employment center. In this column $V_{A1} = 59{,}901$ is the number of jobs available at A from employment center 1. Column **V_i** (i.e., $\sum_{j=1}^J V_{ij}$) gives the total number of jobs available to origin i. We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

Compare the calculated values of V_i to column **S_i** (Hansen-type accessibility) in Table 1. The spatial availability values are more intuitive. Recall that population centers A and B had identical Hansen-type accessibility to employment opportunities. According to V_i, population center A has greater job availability due to: 1) its close proximity to employment center 1; combined with 2) less competition (i.e., a majority of the population have to travel longer distances to reach employment center 1). Job availability is lower for population center B due to much higher competition (150,000 people can reach 100,000

Table 3: Summary description of synthetic example: spatial availabilityy

Origin	**Pop.**	**Dest.**	**Jobs**	**TT**	**f(TT)**	**F^p**	**F^c**	**F**	**V__ij**	**V__i**
A	50,000	1	100,000	15	0.223130	0.238095	0.817438	0.599006	59,901	66,833
		2	100,000	30	0.049787	0.238095	0.182395	0.069227	6,923	
		3	10,000	100	0.000045	0.238095	0.000203	0.001013	10	
B	150,000	1	100,000	30	0.049787	0.714286	0.182395	0.400969	40,097	133,203
		2	100,000	15	0.223130	0.714286	0.817438	0.930760	93,076	
		3	10,000	100	0.000045	0.714286	0.000203	0.003040	30	
C	10,000	1	100,000	100	0.000045	0.047619	0.000166	0.000024	2.4	9,963
		2	100,000	100	0.000045	0.047619	0.000166	0.000013	1.3	
		3	10,000	15	0.223130	0.047619	0.999593	0.995947	9,959	

jobs at equal cost). And center C has almost as many jobs available as it has population.

As discussed above, Hansen-type accessibility is not designed to preserve the number of jobs in the region. Shen-type accessibility is internally inconsistent: the only way it preserves the number of jobs is if the effect of the impedance function is ignored when expanding the values of jobs per capita to obtain the total number of opportunities. The proportional allocation procedure described above, in contrast, consistently returns a number of jobs available that matches the total number of jobs in the region.

Since the jobs spatially available are consistent with the jobs in the region, it is possible to define a measure of spatial availability per capita:

$$v_i = \frac{V_i}{P_i} \tag{11}$$

And, since the jobs are preserved, it is possible to use the regional jobs per capita as a benchmark to compare the spatial availability of jobs per capita at each origin:

$$\frac{\sum_{j=1}^{J} O_j}{\sum_{i=1}^{N} P_i} \tag{12}$$

In the example, since the population is equal to the number of jobs, the regional value of jobs per capita is 1.0. To complete the illustrative example, the spatial availability of jobs per capita by origin is:

$$
\begin{array}{l}
v_1 = \frac{V_1}{P_1} = \frac{66{,}833.47}{50{,}000} = 1.337 \\
v_2 = \frac{V_2}{P_2} = \frac{133{,}203.4}{150{,}000} = 0.888 \\
v_3 = \frac{V_3}{P_3} = \frac{9{,}963.171}{10{,}000} = 0.996
\end{array} \tag{13}
$$

We can see that population center A has fewer jobs per capita than the regional benchmark, center B has more, and center C is at parity. Remarkably, the spatial availability per capita matches the values of a_i in Table 1. Appendix A has a proof of the mathematical equivalence between the two measures. It is interesting to notice how Weibull (1976), Shen (1998), as well as this paper, all reach identical expressions starting from different assumptions; this effect is known as *equifinality* (see Ortúzar and Willumsen, 2011, p. 333; and Williams, 1981). Interestingly, this result means that Shen-type accessibility and 2SFCA can be re-conceptualized as singly-constrained accessibility measures.

3.4. Why does proportional allocation matter?

Having shown that Shen-type accessibility and spatial availability produce equifinal results, it is reasonable to ask whether the distinction between them is of any import.

Conceptually, we would argue that the internal inconsistency in the calculation of total opportunities in Shen (1998) points to a deeper issue that is only evident when we consider the internal values of the method. To illustrate, Table 1 shows results of a_i that are reasonable (and they match exactly the spatial availability per capita). But when we dig deeper, these results mask potentially misleading values of jobs allocated and taken. In addition, the internal values also lead to estimates of the impact of accessibility that are deceptive (see Sarlas et al., 2020). For example, the estimated system-wide cost of travel considering the jobs allocated by a_i in Table 1 is as follows:

$$
\begin{array}{l}
11,157 \times 15 \text{ min} + 2,489 \times 30 \text{ min} + 2.27 \times 100 \text{ min} \\
7,468 \times 30 \text{ min} + 33,470 \times 15 \text{ min} + 6.81 \times 100 \text{ min} \\
0.454 \times 100 \text{ min} + 0.454 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,581 \text{ min}
\end{array}
$$

In contrast, the estimated system-wide cost of travel according to V_i in Table 3 is as follows:

$$
\begin{array}{l}
59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min} \\
40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min} \\
2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min}
\end{array}
$$

Therefore, not only does the Shen-type measure effectively allocate fewer than 56,835 out of a total of 210,000 jobs in the example, it also gives a biased estimate of the potential cost of travel in the system by obscuring the number of jobs not allocated.

4. Empirical example of Toronto

In this section we illustrate the application of spatial availability through an empirical example. For this, we use population and employment data from the Greater Toronto and Hamilton Area (GTHA) in Ontario, Canada. This is the largest metropolitan region in Canada. For comparison, we calculate Hansen- and Shen-type accessibility, as well as the proposed spatial availability measure.

4.1. Data

We obtained population and employment data from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GTHA area in the southern part of Ontario, Canada (see Figure 3) (Data Management Group, 2018). The data set includes Traffic Analysis Zones (TAZ) (n=3,764), the number of jobs (n=3,081,885) and workers (n=3,446,957) at each origin and destination. The TTS data is based on a representative sample of between 3% to 5% of households in the GTHA and is weighted to reflect the population covering the study area has a whole (Data Management Group, 2018).

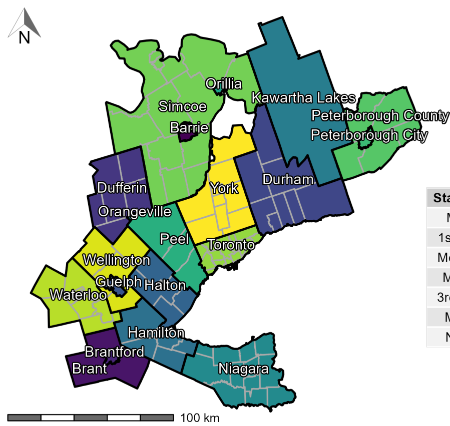
To generate the travel cost for these trips, travel times between origins and destinations are calculated for car travel using the `R` package {r5r} (Rafael H. M. Pereira et al., 2021) with a street network retrieved from OpenStreetMap. For the calculations a 3 hr travel time threshold was selected as it captures 99% of population-employment pairs (see the travel times summarized in Figure 3). This method does not account for traffic congestion or modal split, which can be estimated through other means (e.g., Allen and Farber, 2021; Higgins et al., 2021). For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions. All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

4.2. Calibration of an impedance function

In the synthetic example introduced in a preceding section, we used a negative exponential function with the parameter reported by Shen (1998). For the empirical example, we calibrate an impedance function on the trip length distribution (TLD) of commute trips. Briefly, a TLD represents the proportion of trips that are taken at a specific travel cost (e.g., travel time); this distribution is commonly used to derive impedance functions in accessibility research (Batista et al., 2019; Horbachov and Svichynskyi, 2018; Lopez and Paez, 2017).

The empirical and theoretical TLD for this data set are represented in the top-left panel of Figure 4. Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the {fitdistrplus} package (Delignette-Muller and Dutang, 2015) were used. Based on goodness-of-fit criteria and diagnostics the gamma distribution was selected (see Figure 4).

The gamma distribution is defined in Equation (14), where we see that it depends on a shape parameter α and a rate parameter β. The estimated values of these paramters are $\alpha = 2.019$ and $\beta = 0.094$.



Statistic	Trips	TT	Workers	Jobs
Min.	1	0	0	0
1st Qu.	14	13	25	64
Median	22	20	464	244
Mean	33	23	916	819
3rd Qu.	38	30	1378	700
Max.	1129	179	8491	41821
NA's	NA	3507	NA	NA

Figure 3: TTS 2016 study area (GTHA, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

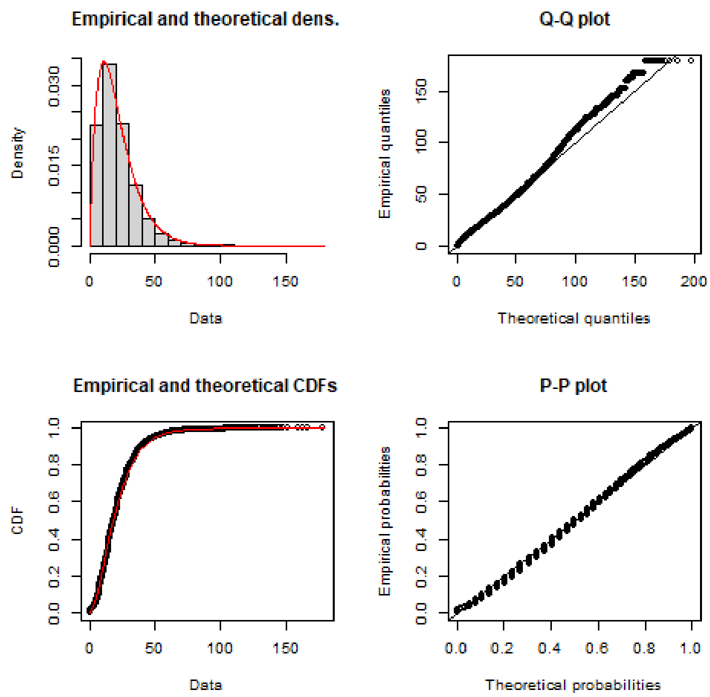


Figure 4: Car trip length distribution and calibrated gamma distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

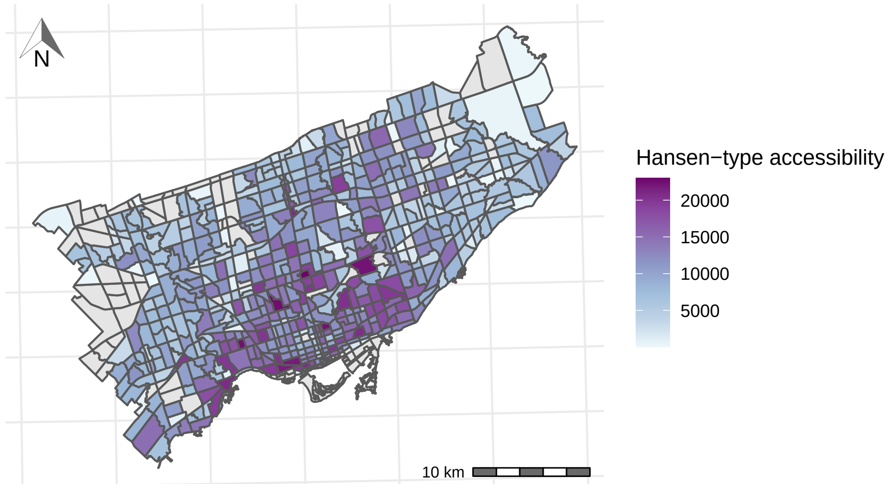


Figure 5: Estimated accessibility to employment in Toronto according to Hansen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.

$$\begin{aligned} &f(x,\alpha,\beta) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} \quad \text{for } 0 \leq x \leq \infty \\ &\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}\,dx \end{aligned} \tag{14}$$

Figures 8, 9, and 10 are the absolute accessibility values in number of jobs accessible/available.

How do Shen-type internal values perform? The opportunity seeking population according to Shen-type measure greatly exceeds the population:

osp	population
1.98e+06	1.14e+06

The ratio of effective opportunity-seeking population to population is shown next:

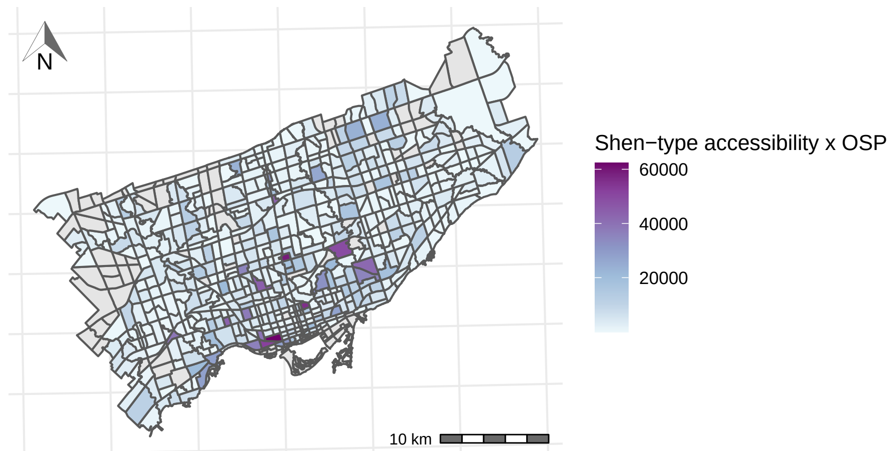


Figure 6: Estimated accessibility to employment in Toronto according to Shen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null accessibility values.

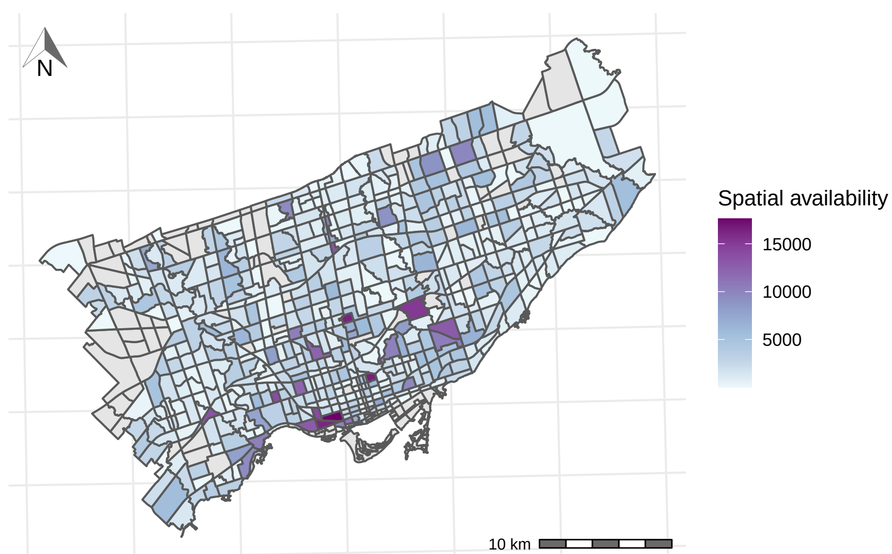


Figure 7: Estimated spatial availability of employment in Toronto. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.

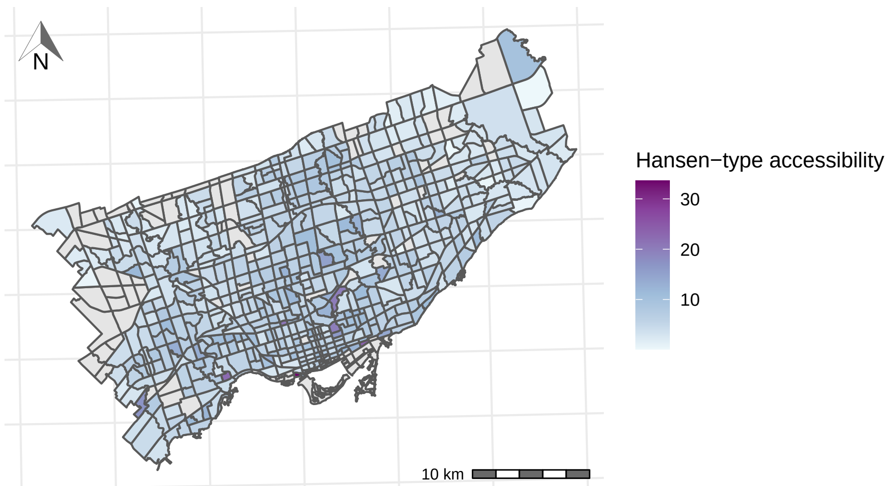
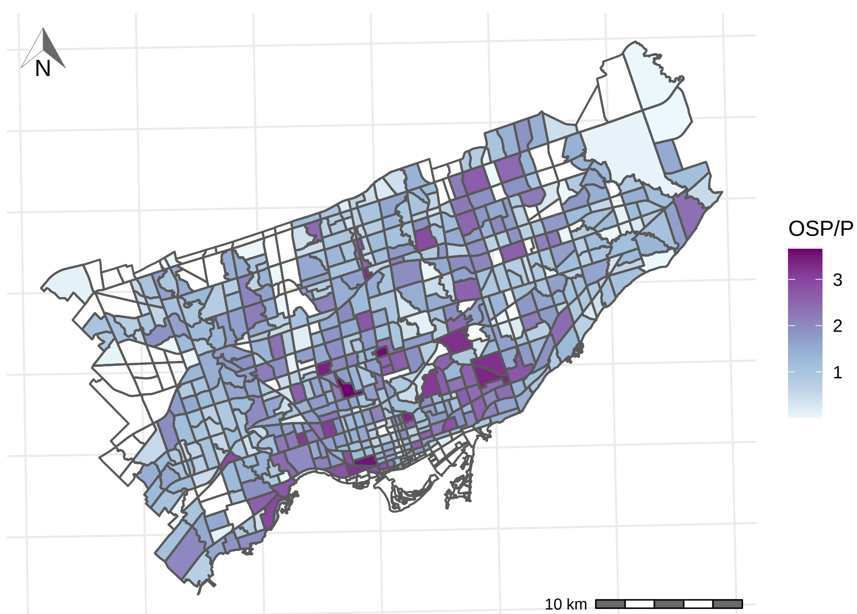


Figure 8: Estimated accessibility per capita to employment in Toronto according to Hansen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.



The effect is not constant in space.

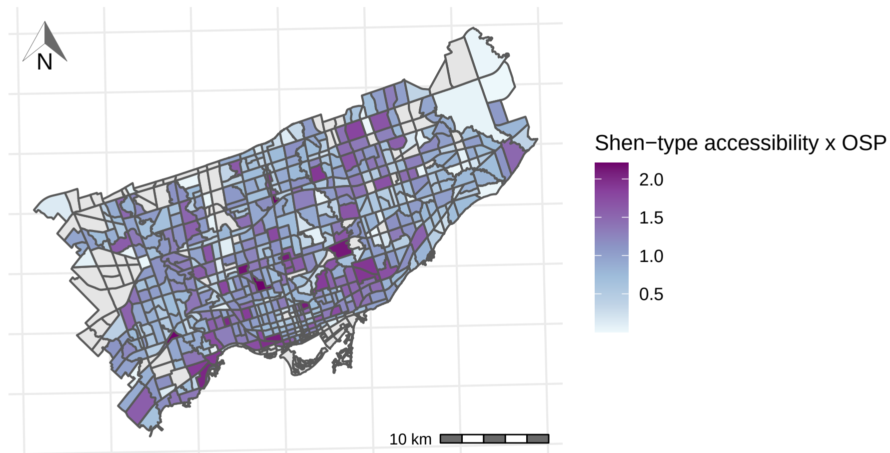


Figure 9: Estimated accessibility to employment in Toronto according to Shen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null accessibility values.

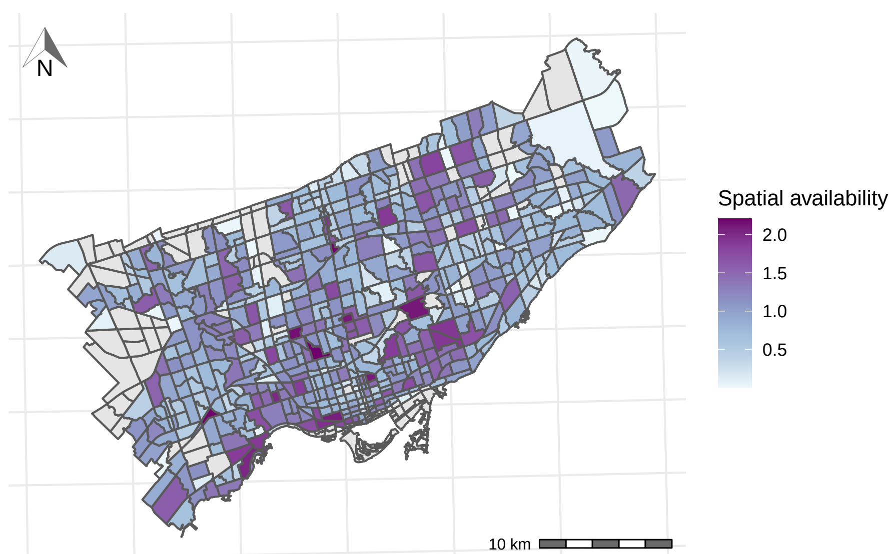


Figure 10: Estimated spatial availability of employment in Toronto. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.

As a consequence of the inflated population (osp > population), the travel time is exaggerated by Shen-type measure:

total_travel_time
668586.666173916

total_travel_time
344376.94962808

Why do these differences matter? Think about equity analysis!

4.3. Accessibility and spatial availability of jobs in Toronto

Toronto is the largest city in the GTHA and represents a significant subset of workers and jobs in the GTHA; 31% of workers in the GTHA travel to jobs in Toronto and 40% of jobs are located within Toronto.

5. Discussion and Conclusions

Words go here.

6. Appendix A

Equivalence of Shen-type accessibility and spatial availability

Population allocation factor:

$$F_{ij}^{p} = \frac{P_{i\in r}^{\alpha}}{\sum_{i}^{K} P_{i\in r}^{\alpha}}$$

$$F_{A}^{p} = \frac{P_{A}^{\alpha}}{P_{A}^{\alpha} + P_{B}^{\alpha} + P_{C}^{\alpha}}$$

Cost allocation factor:

$$F_{ij}^{c} = \frac{f(c_{ij})}{\sum_{i=A}^{K} f(c_{ij})}$$

$$F_{A1}^{c} = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{B1}^{c} = \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$F_{C1}^{c} = \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$$

Now let's put it together with P, and see how the denominators end up cancelling out:

$$v_i = \sum_j \frac{O_j}{P_{i\in r}^{\alpha}} \frac{\frac{P_{i\in r}^{\alpha}}{\sum_i^K P_{i\in r}^{\alpha}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}{\sum_i^K \frac{P_{i\in r}^{\alpha}}{\sum_i^K P_{i\in r}^{\alpha}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}$$

$$v_A = \frac{O_1}{P_A^{\alpha}}\left(\frac{\frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})} + \frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{B1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})} + \frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{C1})}{f(c_{A1})+f(c_{B1})+f(c}}\right.$$

$$\frac{O_2}{P_A^{\alpha}}\left(\frac{\frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})} + \frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{B2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})} + \frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{C2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}\right)+$$

$$\frac{O_3}{P_A^{\alpha}}\left(\frac{\frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})} + \frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{B3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})} + \frac{P_A^{\alpha}}{P_A^{\alpha}+P_B^{\alpha}+P_C^{\alpha}} \cdot \frac{f(c_{C3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}\right)$$

First, notice how the denominator on the denominator is the same across the summation? Let's simplify it:

$$v_A = \frac{O_1}{P_A^\alpha}(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P_A^\alpha \cdot f(c_{A1})+P_A^\alpha \cdot f(c_{B1})+P_A^\alpha \cdot f(c_{C1})}{(P_A^\alpha+P_B^\alpha+P_C^\alpha)\cdot(f(c_{A1})+f(c_{B1})+f(c_{C1}))}})+$$

$$\frac{O_2}{P_A^\alpha}(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P_A^\alpha \cdot f(c_{A2})+P_A^\alpha \cdot f(c_{B2})+P_A^\alpha \cdot f(c_{C2})}{(P_A^\alpha+P_B^\alpha+P_C^\alpha)\cdot(f(c_{A2})+f(c_{B2})+f(c_{C2}))}})+$$

$$\frac{O_3}{P_A^\alpha}(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P_A^\alpha \cdot f(c_{A3})+P_A^\alpha \cdot f(c_{B3})+P_A^\alpha \cdot f(c_{C3})}{(P_A^\alpha+P_B^\alpha+P_C^\alpha)\cdot(f(c_{A3})+f(c_{B3})+f(c_{C3}))}})$$

See how the denominator of the denominator is the same as the denominator of the numerator's denominator for each J (J=1, J=2, and J=3)? Let's cancel those out and simplify:

$$v_A = \frac{O_1}{P_A^\alpha}(\frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})}+$$

$$\frac{O_2}{P_A^\alpha}\frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})}+$$

$$\frac{O_3}{P_A^\alpha}\frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})})$$

Next, see how we can cancel out the P_A^α? Let's do that.

$$v_A = O_1(\frac{f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})}+O_2\frac{f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})}+O_3\frac{}{P_A^\alpha \cdot f(c_{A3})}$$

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