

¹ Introducing spatial availability, a singly-constrained
² measure of competitive accessibility

³ **Abstract**

Accessibility indicators are widely used in transportation, urban, and health-care planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability, mainly by introducing competition. In this paper, we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

4 **1. Introduction**

5 The concept of accessibility in transportation studies derives its appeal from
6 the combination of the spatial distribution of opportunities and the cost of
7 reaching them (Handy and Niemeier, 1997; Hansen, 1959). Accessibility analysis
8 is employed in transportation, geography, public health, and many other areas,
9 with the number of applications growing (Shi et al., 2020), especially as mobility-
10 based planning is de-emphasized in favor of access-oriented planning (Deboosere
11 et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021).

12 Accessibility analysis stems from the foundational works of Harris (1954)
13 and Hansen (1959). From these seminal efforts, many accessibility measures
14 have been derived, particularly after the influential work of Wilson (1971) on
15 spatial interaction¹. Of these, gravity-type accessibility is arguably the most
16 common; since its introduction in the literature, it has been widely adopted in
17 numerous forms (Arranz-López et al., 2019; Cervero et al., 2002; Geurs and van
18 Wee, 2004; Levinson, 1998; Paez, 2004). Hansen-type accessibility indicators
19 are essentially weighted sums of opportunities, with the weights given by an
20 impedance function that depends on the cost of movement, and thus measure
21 the *intensity of the possibility of interaction* (Hansen, 1959). This type of acces-
22 sibility analysis offers a powerful tool to study the intersection between urban
23 structure and transportation infrastructure (Handy and Niemeier, 1997).

24 Despite their usefulness, the interpretability of Hansen-type accessibility
25 measures can be challenging (Geurs and van Wee, 2004; Miller, 2018). Since
26 they aggregate opportunities, the results are sensitive to the size of the region
27 of interest (e.g., a large city has more jobs than a smaller city). As a conse-
28 quence, raw outputs are not necessarily comparable across study areas (Allen
29 and Farber, 2019). This limitation becomes evident when surveying studies that
30 implement this type of analysis. For example, Páez et al. (2010) (in Montreal)
31 and Campbell et al. (2019) (in Nairobi) report accessibility as the number of
32 health care facilities that can potentially be reached from origins. But what
33 does it mean for a zone to have accessibility to less than 100 facilities in each of
34 these two cities, with their different populations and number of facilities? For
35 that matter, what does it mean for a zone to have accessibility to more than 700
36 facilities in Montreal, besides being “accessibility rich”? As another example,
37 Bocarejo S. and Oviedo H. (2012) (in Bogota), El-Geneidy et al. (2016) (in
38 Montreal), and Jiang and Levinson (2016) (in Beijing) report accessibility as
39 numbers of jobs, with accessibility values often in the hundreds of thousands,
40 and even exceeding one million jobs for some zones in Beijng and Montreal. As
41 indicators of urban structure, these measures are informative, but the meaning
42 of one million accessible jobs is harder to pin down: how many jobs must any
43 single person have access to? Clearly, the answer to this question depends on
44 how many people demand jobs.

¹Utility-based measures derive from a very different theoretical framework, random utility maximization

45 The interpretability of Hansen-type accessibility has been discussed in nu-
46 merous studies, including recently by Hu and Downs (2019), Kelobonye et al.
47 (2020), and in greater depth by Merlin and Hu (2017). As hinted above, the
48 limitations in interpretability are frequently caused by ignoring competition -
49 without competition, each opportunity is assumed to be equally available to
50 every single opportunity-seeking individual that can reach it (Kelobonye et al.,
51 2020; Paez et al., 2019; Shen, 1998). This assumption is appropriate when the
52 opportunity of interest is non-exclusive, that is, if use by one unit of population
53 does not preclude use by another. For instance, national parks with abundant
54 space are seldom used to full capacity, so the presence of some population does
55 not exclude use by others. When it comes to exclusive opportunities, or when
56 operations may be affected by congestion, the solution has been to account
57 for competition. Several efforts exist that do so. In our reckoning, the first
58 such approach was proposed by Weibull (1976), whereby the distance decay of
59 the supply of employment and the demand for employment (by workers) were
60 formulated under so-called axiomatic assumptions. This approach was then ap-
61 plied by Joseph and Bantock (1984) in the context of healthcare, to quantify
62 the availability of general practitioners in Canada. About two decades later,
63 Shen (1998) independently re-discovered Weibull's (1976) formula (see footnote
64 (7) in Shen, 1998) and deconstructed it to consider accessibility for different
65 modes. These advances were subsequently popularized as the family of Two-
66 Stage Floating Catchment area (2SFCA) methods (Luo and Wang, 2003) that
67 have found widespread adoption in healthcare, education, and food systems (B.
68 Y. Chen et al., 2020; Chen, 2019; Z. Chen et al., 2020; Yang et al., 2006; Ye et
69 al., 2018).

70 An important development contained in Shen's work is a proof that the
71 population-weighted sum of the accessibility measure with competition equates
72 to the number of opportunities available (footnote (7) and Appendix A in Shen,
73 1998). This demonstration gives the impression that Shen-type accessibility al-
74 locates *all* opportunities to the origins, however to the authors' knowledge, it
75 has not interpreted by literature in this way. For instance, Hu (2014), Merlin
76 and Hu (2017), and Tao et al. (2020) all use Shen-type accessibility to calcu-
77 late job access but report values as 'competitive accessibility scores' or simply
78 'job accessibility'. These works do not explicitly recognize that jobs that are
79 assigned to each origin are in fact a proportion of *all* the opportunities in the
80 system. This recognition, we argue, is critical to interpreting the meaning of the
81 final result. Thus, in this paper we intend to revisit accessibility with compe-
82 tition within the context of disentangling how opportunities are allocated. We
83 first argue that Shen's competitive accessibility misleadingly refers to the the
84 total zonal population to equal the travel-cost discounted opportunity-seeking
85 population. This equivocation, we believe, results in a ambiguous interpretation
86 of what Shen-type accessibility represents as the allocation of opportunities to
87 population is masked by the results presenting as rates (i.e., opportunities per
88 capita). We then propose an alternative formulation of accessibility that incor-
89 porates competition by adopting a proportional allocation mechanism; we name
90 this measure *spatial availability*. The use of balancing factors for proportional

91 allocation is akin to imposing a single constraint on the accessibility indicator,
92 in the spirit of Wilson's (1971) spatial interaction model.

93 In this way, the aim of the paper is three-fold:

- 94 • First, we aim to demonstrate that Shen-type (and thus Weibull (1976)
95 accessibility and the popular 2SFCA methods) produce equivocal esti-
96 mates of opportunities allocated as the result is presented as a rate (i.e.,
97 opportunities per capita);
- 98 • Second, we introduce a new measure, *spatial availability*, which we submit
99 is a more interpretable alternative to Shen-type accessibility, since oppor-
100 tunities in the system are preserved and proportionally allocated to the
101 population; and
- 102 • Third, we show how Shen-type accessibility (and 2SFCA methods) can be
103 seen as measures of singly-constrained accessibility.

104 Discussion is supported by the use of the small synthetic example of Shen
105 (1998) and empirical data drawn from the 2016 Transportation Tomorrow Sur-
106vey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the
107 spirit of openness of research in the spatial sciences (Brunsdon and Comber,
108 2021; Páez, 2021) this paper has a companion open data product (Arribas-
109 Bel et al., 2021), and all code is available for replicability and reproducibility
110 purposes.

111 2. Accessibility measures revisited

112 In this section we revisit Hansen-type and Shen-type accessibility indicators.
113 We adopt the convention of using a capital letter for absolute values (number
114 of opportunities) and lower case for rates (opportunities per capita).

115 2.1. Hansen-type accessibility

116 Hansen-type accessibility measures follow the general formulation shown in
117 Equation (1):

$$S_i = \sum_{j=1}^J O_j \cdot f(c_{ij}) \quad (1)$$

118 where:

- 119 • c_{ij} is a measure of the cost of moving between i and j .
- 120 • $f(\cdot)$ is an impedance function of c_{ij} ; it can take the form of any monoton-
121 ically decreasing function chosen based on positive or normative criteria
122 (Páez et al., 2012).
- 123 • i is a set of origin locations ($i = 1, \dots, N$).
- 124 • j is a set of destination locations ($j = 1, \dots, J$).

- 125 • O_j is the number of opportunities at location j ; $O = \sum_{j=1}^J O_j$ is the total
 126 supply of opportunities in the study region.
 127 • S is Hansen-type accessibility as weighted sum of opportunities.

128 As formally defined, accessibility S_i is the sum of opportunities that can
 129 be reached from location i , weighted down by an impedance function of the
 130 cost of travel c_{ij} . Summing the opportunities in the neighborhood of i provides
 131 estimates of the number of opportunities that can *potentially* be reached from
 132 i . Several variants of this method result from using a variety of impedance
 133 functions; for example, cumulative opportunities measures are obtained when
 134 $f(\cdot)$ is a binary or indicator function (e.g., El-Geneidy et al., 2016; Geurs and van
 135 Wee, 2004; Qi et al., 2018; Rosik et al., 2021). Other measures use impedance
 136 functions modeled after any monotonically decreasing function (e.g., Gaussian,
 137 inverse power, negative exponential, or log-normal, among others, see, *inter alia*,
 138 Kwan, 1998; Li et al., 2020; Reggiani et al., 2011; Vale and Pereira, 2017). In
 139 practice, accessibility measures with different impedance functions tend to be
 140 highly correlated (Higgins, 2019; Kwan, 1998; Santana Palacios and El-geneidy,
 141 2022).

142 Gravity-based accessibility has been shown to be an excellent indicator of
 143 the intersection between spatially distributed opportunities and transportation
 144 infrastructure (Kwan, 1998; Reggiani et al., 2011; Shi et al., 2020). However,
 145 beyond enabling comparisons of relative values they are not highly interpretable
 146 on their own (Miller, 2018). To address the issue of interpretability, previous re-
 147 search has aimed to index and normalize values on a per demand-population ba-
 148 sis (e.g., Barboza et al., 2021; Pereira et al., 2019; Wang et al., 2021). However,
 149 as recent research on accessibility discusses (Allen and Farber, 2019; Kelobonye
 150 et al., 2020; Merlin and Hu, 2017; Paez et al., 2019), these steps do not ade-
 151 quately consider competition. In effect, when calculating S_i , every opportunity
 152 enters the weighted sum once for every origin i that can reach it. This makes
 153 interpretability opaque, and to complicate matters, can also bias the estimated
 154 landscape of opportunity.

155 2.2. *Shen-type competitive accessibility*

156 To account for competition, the influential works of Shen (1998) and Weibull
 157 (1976), as well as the widely used 2SFCA approach of Luo and Wang (2003), ad-
 158 just Hansen-type accessibility with the population in the region of interest. The
 159 mechanics of this approach consist of calculating, for every destination j , the
 160 population that can reach it given the impedance function $f(\cdot)$; let us call this
 161 the *effective opportunity-seeking population* (Equation (2)). This value can be
 162 seen as the Hansen-type *market area* (accessibility to population) of j . The op-
 163 portunities at j are then divided by the sum of the *effective opportunity-seeking*
 164 *population* to obtain a measure of opportunities per capita, i.e., R_j in Equa-
 165 tion (3). This can be thought of as the *level of service* at j . Per capita values
 166 are then allocated back to the population at i , again subject to the impedance
 167 function as seen in Equation (4); this is accessibility with competition.

$$P_{ij}^* = P_i \cdot f(c_{ij}) \quad (2)$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^*} \quad (3)$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \quad (4)$$

168 where:

- 169 • a is Shen-type accessibility as weighted sum of opportunities per capita
170 (or weighted level of service).
- 171 • c_{ij} is a measure of the cost of moving between i and j .
- 172 • $f(\cdot)$ is an impedance function of c_{ij} .
- 173 • i is a set of origin locations ($i = 1, \dots, N$).
- 174 • j is a set of destination locations ($j = 1, \dots, J$).
- 175 • O_j is the number of opportunities at location j ; $O = \sum_{j=1}^J O_j$ is the total
176 supply of opportunities in the study region.
- 177 • P_i is the population at location i .
- 178 • P_{ij}^* is the population at location i that can reach destination j according
179 to the impedance function; we call this the *effective opportunity-seeking
180 population*.
- 181 • R_j is the ratio of opportunities at j to the sum over all origins of the
182 *effective opportunity-seeking population* that can reach j ; in other words,
183 this is the total number of opportunities per capita found at j .

184 Shen (1998) describes P_i as the “*the number of people in location i seeking
185 opportunities*”. In our view, this is somewhat equivocal and where misinterpre-
186 tation of the final result may arise. Consider a population center where the
187 population is only willing to travel if a trip is less than or equal to 60 minutes.
188 This is identical to the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 & \text{if } c_{ij} \leq 60 \text{ min} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

189 If an employment center is less than 60 minutes away, the population can
190 seek opportunities there (i.e., $f(c_{ij}) = 1$). But are these people still part of the
191 opportunity-seeking population for jobs located two hours away? Four hours?
192 Ten hours? We would assume that they are not because their travel behaviour,
193 as represented by the impedance function would yield $f(c_{ij}) = 0$, eliminat-
194 ing them from what we call, the *effective opportunity-seeking population* P_{ij}^* .
195 We see Shen’s definition as ambiguous because, for the purpose of calculating
196 accessibility, the impedance function defines what constitutes the population
197 that effectively can seek opportunities at remote locations. Thus P_i should be
198 plainly understood as the population at location i (as we define above) and not

199 the “*the number of people in location i seeking opportunities*”. In other words,
200 Shen finds P_i with P_{ij*} .

201 Furthermore, the same misunderstanding can be described for O_j which Shen
202 defines as “*the number of relevant opportunities in location j* ”. O_j is weighted
203 down by the same $f(c_{ij})$ in Equation (4), so the *relevancy* is determined by the
204 travel behaviour associated with the impedance function not by O_j itself. For
205 this reason, O_j should be understand plainly as the opportunities at location j
206 (as we also define above).

207 Misunderstanding P_i and O_j may led to a misinterpretation of the final result
208 a_i , especially as expressed in Shen’s proof as shown in Equation (6).

$$\sum_{i=1}^N a_i P_i = \sum_{j=1}^J O_j \quad (6)$$

209 Notice, misunderstanding P_i and O_j may allow some to misunderstand the
210 units of a_i as “*relevant opportunities*” per “*people in location i seeking opportunities*”. Instead, as mathematically expressed in the proof, a_i is a proportion of
211 the opportunities available to the population, since multiplying a_i by the popu-
212 lation at i and summing for all origins in the system equals to the total number
213 of opportunities in the system. Embedded in a_i is already the travel behaviour
214 so P_i and O_j must be plainly understand as population at i and opportunities
215 at j to have Equation (6) hold true.

217 2.3. *Shen’s toy example*

218 In this section we use the example in Shen (1998) to detail the importance
219 of understanding P_i and O_j as simply the population at the origin i and the op-
220 portunities at destination j respectively. It is critical to understanding how the
221 opportunities are allocated to the population based on the impedance function.

Table 1 contains the information needed to calculate S_i and a_i for this ex-
ample. We use a negative exponential impedance function with $\beta = 0.1$ as done
in Shen (1998, see footnote (5)):

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

222 In Table 1, we see that population centers A and B have equal Hansen-type
223 accessibility ($S_A = S_B = 27,292$ jobs). On the other hand, the isolated satellite
224 town of C has low accessibility ($S_C = 2,240$ jobs). But center B , despite its
225 high accessibility, is a large population center. C , in contrast, is smaller but also
226 relatively isolated and has a balanced ratio of jobs (10,0000 jobs) to population
227 (10,000 people). It is difficult from these outputs to determine whether the
228 accessibility at C is better or worse than that at A or B .

The results are easier to interpret when we consider Shen-type accessibility.
The results indicate that $a_A \approx 1.337$ jobs per capita, $a_B \approx 0.888$, and $a_C \approx 0.996$. The latter value is sensible given the jobs-population balance of C . Center
 A is relatively close to a large number of jobs (more jobs than the population
of A). The opposite is true of B . According to Shen (1998), the sum of the

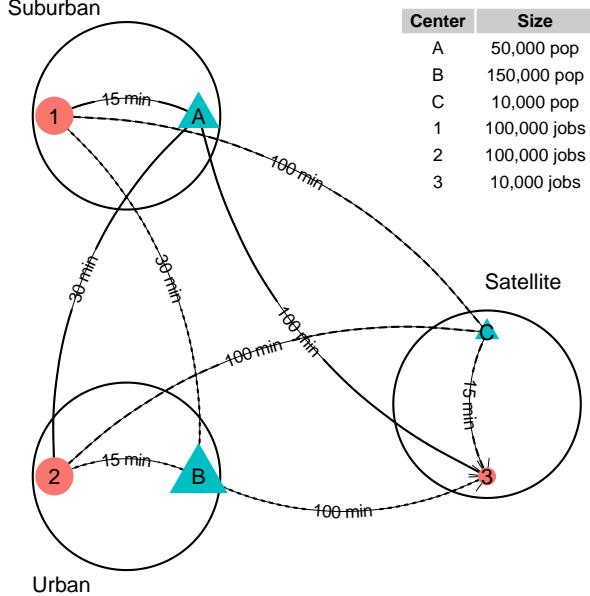


Figure 1: Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

population-weighted accessibility a_i is exactly equal to the number of jobs in the region following Shen's proof:

$$\begin{aligned} \sum_{i=1}^N a_i P_i &= \sum_{j=1}^J O_j \\ 50,000 \times 1.3366693 &+ 150,000 \times 0.8880224 \\ + 10,000 \times 0.9963171 &= 210,000 \end{aligned}$$

As mentioned earlier, this property under Shen's definition of P_i “people in location i seeking opportunities”, gives the impression that all sought jobs are allocated to the people located at each origin i . In other words, Shen defines P_i to mean P_{ij}^* (i.e., the *effective opportunity-seeking population* which is already discounted by travel behaviour) while it in fact should be simply understood as the full population at i and it is discounted by a_i which already has travel behaviour embedded in its formulation. As seen in column **Pop * f(TT)** in Table 1 (i.e., $P_{ij}^* = P_i \cdot f(c_{ij})$), the number of individuals from population center A that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2,27 respectively. Therefore, the total *effective opportunity-seeking population* in the system (i.e., $P_A^* = \sum_j P_{Aj}^*$) is 13,647.27 people, which is considerably lower than the total population of A (i.e., $P_A = 50,000$ people). Continuing to understand P_i to equal P_{ij}^* , we would think that the proof would actually allocate 56,834.59 jobs in the region (as demonstrated as follows) instead of the nominal number of jobs in the region which is over three times this number (i.e.,

Table 1: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition with beta = 0.1

Origin	Pop.	Dest.	Jobs	TT	f(TT)	Pop * f(TT)	Jobs * f(TT)	S_i	a_i
A	50,000	1	100,000	15	0.223130	11,157	22,313		
		2	100,000	30	0.049787	2,489	4,979	27,292	1.34
		3	10,000	100	0.000045	2.27	0.454		
B	150,000	1	100,000	30	0.049787	7,468	4,979		
		2	100,000	15	0.223130	33,470	22,313	27,292	0.888
		3	10,000	100	0.000045	6.81	0.454		
C	10,000	1	100,000	100	0.000045	0.454	4.54		
		2	100,000	100	0.000045	0.454	4.54	2,240	0.996
		3	10,000	15	0.223130	2,231	2,231		

210,000 jobs).

$$\begin{aligned} \sum_{i=1}^N a_i P_{ij}^* &= ??? \\ (11,156.51 + 2,489.35 + 2.26) \times 1.3366693 \\ + (7,468.06 + 33,469.52 + 6.81) \times 0.8880224 \\ + (4.54 + 4.54 + 2,231.20) \times 0.9963171 &\approx 56,834.59 \end{aligned}$$

Furthermore, even when Shen's P_i is understood plainly as the total population at i , the meaning of the proof may still be ambiguous. The proof can still give the impression that all jobs are allocated to the total population since total population ($\sum_{i=1}^N P_i$) goes into the equation and total jobs ($\sum_{j=1}^J O_j$) in the region is the result. However, this impression is incomplete since it does not reflect the quantity of population which is being allocated jobs and the quantity of population being considered for jobs; these magnitudes are a product of being weighted down by the impedance function (which reflects travel behaviour). Why these magnitudes may be hard to disentangle from a_i is because it is presented as a rate (i.e., opportunities per capita).

Let's consider a modification to the example in Table 2, to illustrate how the presentation of a_i as a rate obscures the magnitude of *effective opportunity-seeking population* and how that can impact the interpretation of the result. We modify the example by increasing the β to 0.5 (compared to the previous value of 0.1; see Figure 2). This modification increases the cost of travel and thus the impedance function, which is an expression of the population's relative willingness to travel to opportunities, reflects a population which is relatively

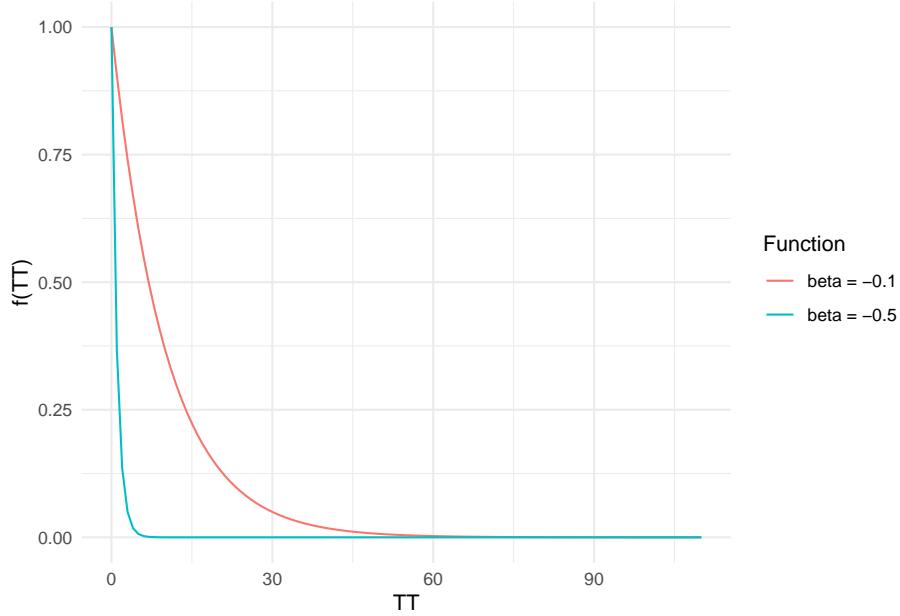


Figure 2: Comparison of two impedance functions in the example.

246 less willing to travel to opportunities further away compared to the previous β
 247 value.

248 As expected, Hansen-type accessibility drops quite dramatically after this β
 249 modification: the friction of distance is so high that few opportunities are within
 250 reach. In contrast, Shen-type accessibility converges to the jobs:population ratio
 251 (i.e., origin A is $\frac{100,000}{50,000} = 2$). This could lead to some interpreting the value
 252 indicating that there are an over-supply of jobs to population; especially when
 253 knowing there are two times more jobs than population at A. However, what is
 254 the magnitude of population who effectively seek jobs and the number of jobs
 255 allocated? For origin A for jobs in job center 1, the demand is only 0.015 people
 256 ($Pop * f(TT)$), while the supply of jobs allocated from center 1 to A is only
 257 0.031 jobs ($Jobs * f(TT)$). So yes, the jobs:population ratio is 2, but only for a
 258 tiny fraction of the population of A that effectively seeks opportunities at center
 259 1.

260 In what follows, we propose an alternative derivation of Shen (1998) acces-
 261 sibility with competition that explicitly clarifies the opportunities allocated to
 262 the *effective opportunity-seeking population* within it's formulation. Hence, the
 263 resulting value is more interpretable.

Table 2: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition with beta = 0.5

Origin	Pop.	Dest.	Jobs	TT	f(TT)	Pop * f(TT)	Jobs * f(TT)	S_i	a_i
A	50,000	1	100,000	15	< 0.001	0.015	0.031		
		2	100,000	30	< 0.001	< 0.001	< 0.001	0.0306	2
		3	10,000	100	< 0.001	< 0.001	< 0.001		
B	150,000	1	100,000	30	< 0.001	< 0.001	< 0.001		
		2	100,000	15	< 0.001	0.046	0.031	0.0306	0.667
		3	10,000	100	< 0.001	< 0.001	< 0.001		
C	10,000	1	100,000	100	< 0.001	< 0.001	< 0.001		
		2	100,000	100	< 0.001	< 0.001	< 0.001	0.00306	1
		3	10,000	15	< 0.001	0.003	0.003		

264 **3. Introducing spatial availability: a singly-constrained measure of
265 accessibility**

In brief, we define the *spatial availability* at i (V_i) as the proportion of all opportunities O that are allocated to i from all destinations j :

$$V_i = \sum_{j=1}^K O_j F_{ij}^t$$

266 where:

- 267 • F_{ij}^t is a balancing factor that depends on the population and cost of move-
268 ment in the system.
- 269 • O_j is the number of opportunities at j .
- 270 • V_i is the number of spatially available opportunities from the perspective
271 of i .

272 The general form of spatial availability is also as a sum, and the fundamental
273 difference with Hansen- and Shen-type accessibility is that opportunities are
274 allocated proportionally. Balancing factor F_{ij}^t consists of two components: the
275 population-based balancing factor F_i^p and the impedance-based balancing factor
276 F_{ij}^c which, respectively, allocates opportunities to i in proportion to the size of
277 the population of the different competing centers (the mass effect of the gravity
278 model) and the cost of reaching opportunities (the impedance effect). In the

²⁷⁹ next two subsections, we explain the intuition behind the method before defining
²⁸⁰ it in full.

²⁸¹ *3.1. Proportional allocation by population*

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by population. Recall, the total population in the example is 210,000. The proportion of the population by population center is as follows:

$$F_A^p = \frac{50,000}{210,000}$$

$$F_B^p = \frac{150,000}{210,000}$$

$$F_C^p = \frac{10,000}{210,000}$$

²⁸² Jobs are allocated proportionally from each employment center to each pop-
²⁸³ ulation center depending on their population sizes as per the balancing factors
²⁸⁴ F_i^p . In this way, employment center 1 allocates $100,000 \cdot \frac{50,000}{210,000} = 23,809.52$ jobs
²⁸⁵ to A; $100,000 \cdot \frac{150,000}{210,000} = 71,428.57$ jobs to B; and $100,000 \cdot \frac{10,000}{210,000} = 7,142.857$
²⁸⁶ jobs to C. Notice how this mechanism ensures that the total number of jobs at
²⁸⁷ employment center 1 is preserved at 100,000.

We can verify that the number of jobs allocated is consistent with the total number of jobs in the system:

Employment center 1 to population centers A, B, and C:

$$100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$$

Employment center 2 to population centers A, B, and C:

$$100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$$

Employment center 3 to population centers A, B, and C:

$$10,000 \cdot \frac{50,000}{210,000} + 10,000 \cdot \frac{150,000}{210,000} + 10,000 \cdot \frac{10,000}{210,000} = 10,000$$

²⁸⁸ In the general case where there are N population centers in the region, we
²⁸⁹ define the following population-based balancing factors:

$$F_i^p = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \tag{7}$$

²⁹⁰ Balancing factor F_i^p corresponds to the proportion of the population in ori-
²⁹¹ gin i relative to the population in the region. On the right hand side of the
²⁹² equation, the numerator P_i^α is the population at origin i . The summation in
²⁹³ the denominator is over $i = 1, \dots, N$, and adds up to the total population of the
²⁹⁴ region. Notice that we incorporate an empirical parameter α . The role of α is to
²⁹⁵ modulate the effect of demand by population. When $\alpha < 1$, opportunities are

²⁹⁶ allocated more rapidly to smaller centers relative to larger ones; $\alpha > 1$ achieves
²⁹⁷ the opposite effect.

Balancing factor F_i^p can now be used to proportionally allocate a share of available jobs at j to origin i . The number of jobs available to i from j balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

In the general case where there are J employment centers, the total number of jobs available from all destinations to i is simply the sum of V_{ij}^p over $j = 1, \dots, J$:

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

Since the factor F_i^p , when summed over $i = 1, \dots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^p = 1$), the sum of all spatially available jobs equals O , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^p &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\ &= \sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

²⁹⁸ The terms F_i^p act here as the balancing factors of the gravity model when a
²⁹⁹ single constraint is imposed (i.e., to ensure that the sums of columns are equal
³⁰⁰ to the number of opportunities per destination, see Ortúzar and Willumsen,
³⁰¹ 2011, pp. 179–180 and 183–184). As a result, the sum of spatial availability for
³⁰² all population centers equals the total number of opportunities.

³⁰³ The discussion so far concerns only the mass effect (i.e., population size)
³⁰⁴ of the gravity model. In addition, the potential for interaction is thought to
³⁰⁵ decrease with increasing cost, so next we define similar balancing factors but
³⁰⁶ based on the impedance.

³⁰⁷ 3.2. Proportional allocation by cost

Clearly, using only balancing factors F_i^p to calculate spatial availability V_i^p does not account for the cost of reaching employment centers. Consider instead a set of balancing factors F_{ij}^c that account for the friction of distance for our example. Recall that the impedance function $f(c_{ij})$ equals $\exp(-\beta \cdot c_{ij})$ where $\beta = 0.1$ and travel time c_{ij} is either 15, 30 or 60 minutes. For instance, the impedance-based balancing factors F_{ij}^c would be the following for employment center 1 (employment center 2 and 3 have their own balancing factor values for each origin i as will be discussed later):

$$\begin{aligned} F_{A1}^c &= \frac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\ F_{B1}^c &= \frac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\ F_{C1}^c &= \frac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581 \end{aligned}$$

308 Balancing factors F_{ij}^c use the impedance function to proportionally allocate
 309 more jobs to closer population centers, that is, to those with populations *more*
 310 *willing to reach the jobs*. Indeed, the factors F_{ij}^c can be thought of as the
 311 proportion of the population at i willing to travel to destination j , conditional on
 312 the travel behavior as given by the impedance function. For instance, 81.74398%
 313 of jobs from employment center 1 are allocated to population center A based
 314 on impedance.

315 So as follows from our example, of the 100,000 jobs at employment center 1
 316 the number of jobs allocated to population center A is $100,000 \times 0.8174398 =$
 317 81,743.98 jobs; the number allocated to population center B is $100,000 \times$
 318 $0.1823954 = 18,239.54$ jobs; and the number allocated to population center
 319 C is $100,000 \times 0.0001648581 = 16.48581$ jobs. We see once more that the total
 320 number of jobs at the employment center is preserved at 100,000. In this ex-
 321 ample, the proportional allocation mechanism assigns the largest share of jobs
 322 to population center A, which is the closest to employment center 1, and the
 323 smallest to the more distant population center C.

324 In the general case where there are N population centers and J employment
 325 centers in the region, we define the following impedance-based balancing factors:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \quad (8)$$

The total number of jobs available to i from j according to impedance is
 defined as follows:

$$V_{ij}^c = O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c}$$

The total number of jobs available to i from all destinations is:

$$V_i^c = \sum_{j=1}^J O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c}$$

Like the population-based allocation factors, F_i^c summed over $i = 1, \dots, N$
 always equals to 1 (i.e., $\sum_{i=1}^N F_i^c = 1$). As before, the sum of all spatially
 available jobs equals O , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^c &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^c}{\sum_{i=1}^N F_i^c} \\ &= \sum_{i=1}^N \frac{F_i^c}{\sum_{i=1}^N F_i^c} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

326 We are now ready to more formally define spatial availability with due con-
 327 sideration to both population and travel cost effects.

328 3.3. Assembling mass and impedance effects

329 Population and the cost of travel are both part of the gravity modelling
 330 framework. Since the balancing factors defined in the preceding sections are

proportions (alternatively, can be understood as probabilities), they can be combined multiplicatively to obtain their joint effect. This multiplicative relationship can alternatively be understood as the joint probability of allocating opportunities and is captured by Equation (9), where F_i^p is the population-based balancing factor that grants a larger share of the existing opportunities to larger centers and F_{ij}^c is the impedance-based balancing factor that grants a larger share of the existing opportunities to closer centers. This is in line with the tradition of gravity modeling.

$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \quad (9)$$

with F_i^p and F_{ij}^c as defined in Equations (7) and (8) respectively. The resulting combined balancing factor F_{ij}^t is used to proportionally allocate jobs from j to i . Hence, spatial availability is given by Equation (10).

$$V_i = \sum_{j=1}^J O_j F_{ij}^t \quad (10)$$

The terms in Equation 10 are as follows:

- F_{ij}^t is a balancing factor as defined in Equation (9).
- i is a set of origin locations in the region $i = 1, \dots, N$.
- j is a set of destination locations in the region $j = 1, \dots, J$.
- O_j is the number of opportunities at location j .
- V_i is the spatial availability at i .

Notice that, unlike S_i in Hansen-type accessibility (Equation (1)), the population enters the calculation of V_i through F_i^p . Returning to Shen's example in Figure 1, Table 3 contains the information needed to calculate V_i .

In Table 3, column **V_ij** are the jobs available to each origin from each employment center. In this column $V_{A1} = 59,901$ is the number of jobs available at A from employment center 1. Column **V_i** (i.e., $\sum_{j=1}^J V_{ij}$) gives the total number of jobs available to origin i . We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

Compare the calculated values of V_i to column **S_i** (Hansen-type accessibility) in Table 1. The spatial availability values are more intuitive. Recall that population centers A and B had identical Hansen-type accessibility to employment opportunities. According to V_i , population center A has greater job availability due to: 1) its close proximity to employment center 1; combined with 2) less competition (i.e., a majority of the population have to travel longer

Table 3: Summary description of synthetic example: spatial availability

Origin	Pop.	Dest.	Jobs	TT	f(TT)	F^p	F^c	F	V_ij	V_i
A	50,000	1	100,000	15	0.223130	0.238095	0.817438	0.599006	59,901	
		2	100,000	30	0.049787	0.238095	0.182395	0.069227	6,923	66,833
		3	10,000	100	0.000045	0.238095	0.000203	0.001013	10	
B	150,000	1	100,000	30	0.049787	0.714286	0.182395	0.400969	40,097	
		2	100,000	15	0.223130	0.714286	0.817438	0.930760	93,076	133.203
		3	10,000	100	0.000045	0.714286	0.000203	0.003040	30	
C	10,000	1	100,000	100	0.000045	0.047619	0.000166	0.000024	2.4	
		2	100,000	100	0.000045	0.047619	0.000166	0.000013	1.3	9,963
		3	10,000	15	0.223130	0.047619	0.999593	0.995947	9,959	

357 distances to reach employment center 1). Job availability is lower for population
 358 center B due to much higher competition (150,000 people can reach 100,000
 359 jobs at equal cost). And center C has almost as many jobs available as it has
 360 population.

361 As discussed above, Hansen-type accessibility is not designed to preserve
 362 the number of jobs in the region. Shen-type accessibility ends up preserving
 363 the number of jobs in the region but the definitions of variables are internally
 364 obscured; the only way it preserves the number of jobs is if the effect of the
 365 impedance function is ignored within the definition of P_i when expanding the
 366 values of jobs per capita to obtain the total number of opportunities. The pro-
 367 portional allocation procedure described above, in contrast, consistently returns
 368 a number of jobs available that matches the total number of jobs in the region.

369 Since the jobs spatially available are consistent with the jobs in the region,
 370 it is possible to define a measure of spatial availability per capita:

$$v_i = \frac{V_i}{P_i} \quad (11)$$

371 And, since the jobs are preserved, it is possible to use the regional jobs per
 372 capita as a benchmark to compare the spatial availability of jobs per capita at
 373 each origin:

$$\frac{\sum_{j=1}^J O_j}{\sum_{i=1}^N P_i} \quad (12)$$

374 In the example, since the population is equal to the number of jobs, the

³⁷⁵ regional value of jobs per capita is 1.0. To complete the illustrative example,
³⁷⁶ the spatial availability of jobs per capita by origin is:

$$\begin{aligned} v_1 &= \frac{V_1}{P_1} = \frac{66,833.47}{50,000} = 1.337 \\ v_2 &= \frac{V_2}{P_2} = \frac{133,203.4}{150,000} = 0.888 \\ v_3 &= \frac{V_3}{P_3} = \frac{9,963.171}{10,000} = 0.996 \end{aligned} \quad (13)$$

³⁷⁷ We can see that population center A has fewer jobs per capita than the
³⁷⁸ regional benchmark, center B has more, and center C is at parity. Remarkably,
³⁷⁹ the spatial availability per capita matches the values of a_i in Table 1. Appendix
³⁸⁰ A has a proof of the mathematical equivalence between the two measures. It
³⁸¹ is interesting to notice how Weibull (1976), Shen (1998), as well as this paper,
³⁸² all reach identical expressions starting from different assumptions; this effect is
³⁸³ known as *equifinality* (see Ortúzar and Willumsen, 2011, p. 333; and Williams,
³⁸⁴ 1981). Incidentally, this result means that Shen-type accessibility and 2SFCA
³⁸⁵ can be re-conceptualized as singly-constrained accessibility measures.

³⁸⁶ 3.4. Why does proportional allocation matter?

³⁸⁷ Having shown that Shen-type accessibility and spatial availability produce
³⁸⁸ equifinal results, it is reasonable to ask whether the distinction between them
³⁸⁹ is of any importance.

³⁹⁰ Conceptually, we would argue that the incorrect interpretation of variables
³⁹¹ leads to internal inconsistency in the calculation of total opportunities in Shen
³⁹² (1998): this points to a deeper issue that is only evident when we consider
³⁹³ the intermediate values of the method. To illustrate, Table 1 shows results
³⁹⁴ of a_i that are reasonable (and they match exactly the spatial availability per
³⁹⁵ capita) when expressed as a rate. But when we dig deeper, these results mask
³⁹⁶ potentially misleading interpretations of the jobs allocated and the number of
³⁹⁷ jobs taken. For instance, a population region with a high jobs:population ratio
³⁹⁸ but a prohibitive transportation network which results in a high cost of travel
³⁹⁹ may yield a high a_i value. This value, however, can conceal a low *effective*
⁴⁰⁰ *opportunity-seeking population* and proportionally low number of allocated jobs
⁴⁰¹ while additionally obscuring the number of population which does *not* take jobs
⁴⁰² and the jobs *not* taken.

⁴⁰³ In addition, the intermediate accessibility values of a_i (Shen-type measure)
⁴⁰⁴ may also lead to impact estimates that are deceptive (see Sarlas et al., 2020). For
⁴⁰⁵ example, the estimated region-wide cost of travel considering the jobs allocated
⁴⁰⁶ by a_i in Table 1 (i.e., $Jobs * f(TT)$) is as follows:

$$\begin{aligned} 22,313 \times 15 \text{ min} + 4,979 \times 30 \text{ min} + 0.454 \times 100 \text{ min} \\ 4,979 \times 30 \text{ min} + 22,313 \times 15 \text{ min} + 0.454 \times 100 \text{ min} \\ 4.54 \times 100 \text{ min} + 4.54 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,594 \text{ min} \end{aligned}$$

In contrast, the estimated region-wide cost of travel according to V_i in Table

3 is as follows:

$$\begin{aligned} & 59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min} \\ & 40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min} \\ & 2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min} \end{aligned}$$

Often referred to as ‘the supply of jobs’ (or simply Hansen-style accessibility) in the Shen-type measure: $\text{Jobs} * f(\text{TT})$ cannot be used to understand the region-wide travel-time. Recall how we define $\text{Pop} * f(\text{TT})$ as the *effective opportunity-seeking population* (P_{ij}^*), $\text{Jobs} * f(\text{TT})$ similarly represents the *effective opportunities allocated* and sums to approximately 56,824 out of a total of 210,000 jobs. Like $\text{Pop} * f(\text{TT})$, the *effective opportunities allocated* to each origin is only a reflection of the impedance function and not the *actual* number of opportunities allocated to each origin. Therefore, the resulting 1,002,594 min is practically meaningless in measuring the region-wide travel time.

However, since spatial availability allocates the *actual* number of opportunities to each origin; the 3,859,054 min can be practically used to quantify the system-wide impacts of competitive accessibility in this region. We know spatial availability’s output is the number of opportunities at each i since the combined balancing factors allocate a proportional amount of the total opportunities to each i such that the number of opportunities allocated to each i sum to equal the total opportunities in the region.

4. Empirical example of Toronto

In this section we illustrate the application of spatial availability through an empirical example. For this, we use population and employment data from the Greater Golden Horseshoe (GGH) area in Ontario, Canada. Contained with the GGH is the Greater Toronto and Hamilton (GTHA) which form the most populous metropolitan regions in Canada and the core urban agglomeration in the GGH.

The GTHA contains the city of Toronto, the most populous city in Canada. The city of Toronto is the focus of this empirical example, it will be used to demonstrate the application of the proposed spatial availability measure along with how it compares to Hansen- and Shen-type measures. We begin this section by explaining the data and then detailing the calculated comparisons.

4.1. GGH Data

We obtained population and employment data from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 3) (Data Management Group, 2018) every five years. The data set includes the number of jobs (n=3,081,885) and workers (n=3,446,957) at each origin and destination represented at the level of Traffic Analysis Zones (TAZ) (n=3,764). TAZ are a unit of spatial analysis which are

444 defined as part of the TTS, however, TAZ are commonly used to ascribe production
445 and attraction of trips in the context of transportation planning modelling.
446 In the GGH data set, the TAZ contain on average 916 workers and jobs 819
447 with more detailed descriptive statistics discussed later. The TTS data is based
448 on a representative sample of between 3% to 5% of households in the GGH and
449 is weighted to reflect the population covering the study area as a whole (Data
450 Management Group, 2018).

451 To generate the travel cost for these to-full-time employment trips, travel
452 times between origins and destinations are calculated for car travel using the
453 R package `{r5r}` (Rafael H. M. Pereira et al., 2021) with a street network re-
454 trieval from OpenStreetMap. For the calculations a 3 hr travel time threshold
455 was selected as it captures 99% of population-employment pairs (see the travel
456 times summarized in Figure 3). This method does not account for traffic con-
457 gestion or modal split, which can be estimated through other means (e.g., Allen
458 and Farber, 2021; Higgins et al., 2021). For simplicity, we carry on with the
459 assumption that all trips are taken by car in uncongested travel conditions. All
460 data and data preparation steps are documented and can be freely explored in
461 the companion open data product `{TTS2016R}`.

462 *4.2. Spatial employment characteristics in Toronto*

463 As mentioned, the focus of this empirical example is on the city of Toronto.
464 It is the largest city in the GGH and represents a significant subset of workers
465 and jobs in the GGH; 31% of workers in the GGH travel live in Toronto and 39%
466 of jobs are located within Toronto. The spatial distribution of jobs (purple) and
467 workers (orange) are captured in Figure 4. It can be seen that a large cluster
468 of jobs can be found in the central southern part of Toronto (the downtown
469 core). Spatial trends in the distribution of workers is less apparent relative to
470 the distribution of jobs.

471 Next, the spatial distribution of the jobs to workers ratio (jobs:workers) (red)
472 and the estimated car travel time (grey) is visualized in Figure 5. Some of the
473 jobs:workers balance predictably clusters near major transportation networks
474 (i.e, the north-south subway line and highways) where surrounding land is more
475 commonly zoned for commercial use. By contrast, the estimated average travel
476 time to work is more even throughout the city, with lower travel times within
477 the downtown core and longer travel times for TAZ located further and further
478 from the core. Interestingly, travel times in certain TAZ with a high jobs:worker
479 balance occur.

480 Nonetheless, the point of these visualizations are to demonstrate the spatial
481 distribution of worker and job data in the city of Toronto to contextualize spa-
482 tial availability and Shen- and Hansen- type measures. Assessing contributing
483 factors to patterns of variable clustering are outside of the paper's scope.

484 *4.3. Calibration of an impedance function for Toronto*

485 In toy example introduced in a preceding section, we used a negative expo-
486 nential function with the parameter reported by Shen (1998). For the empirical

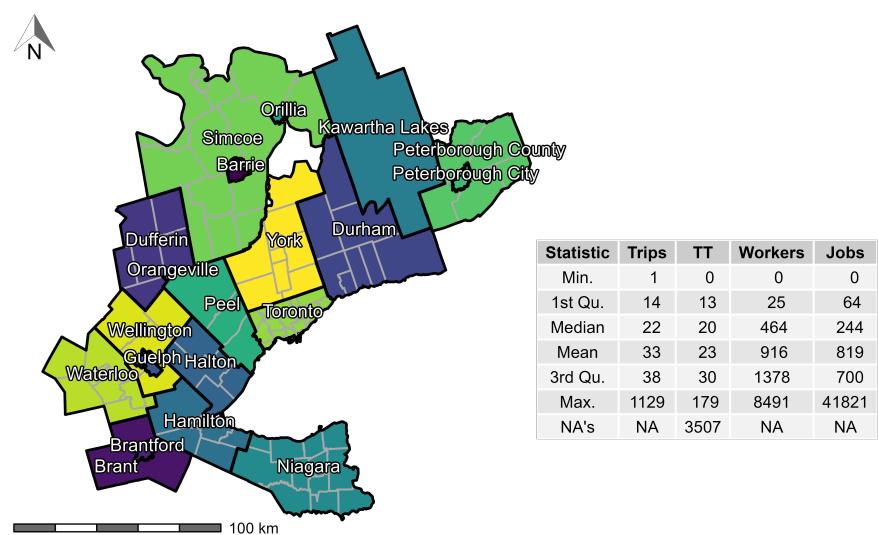


Figure 3: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

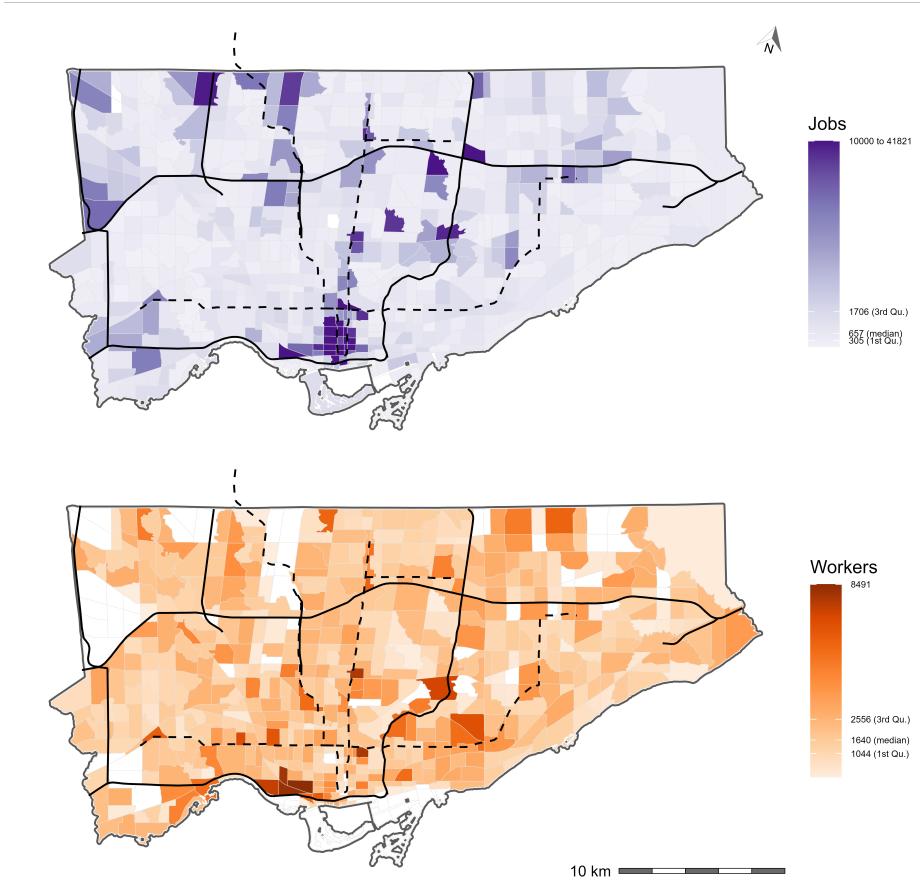


Figure 4: Spatial distribution of full-time jobs (top) and full-time working population (bottom) for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines.

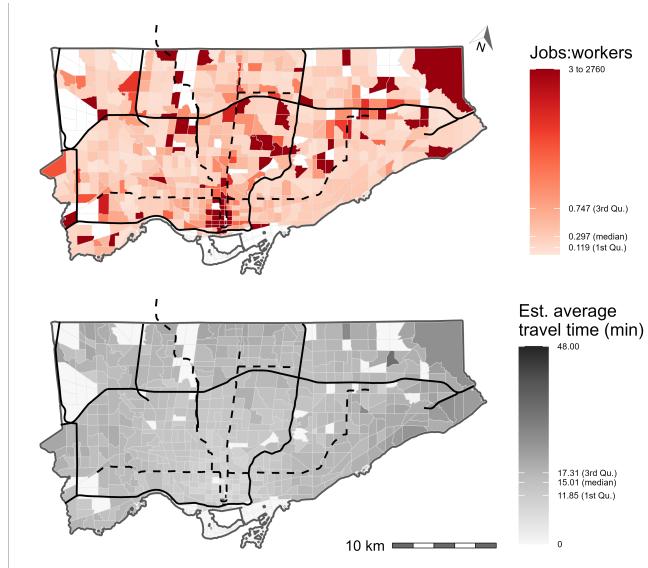


Figure 5: Spatial distribution of full-time working jobs and worker ratio (top) and car travel time to jobs estimated using R5R for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines.

487 Toronto data set, we calibrate an impedance function on the trip length distri-
 488 bution (TLD) of commute trips. Briefly, a TLD represents the proportion of
 489 trips that are taken at a specific travel cost (e.g., travel time); this distribution is
 490 commonly used to derive impedance functions in accessibility research (Batista
 491 et al., 2019; Horbachov and Svichynskyi, 2018; Lopez and Paez, 2017).

492 As mentioned, the calculations are undertaken for the city of Toronto to
 493 facilitate a comparison across measures. As such, only the work-related trips
 494 with origins and destinations within the boundaries of Toronto are selected.
 495 Specifically, edge trips are not included such as trips originating in Toronto but
 496 finishing outside of Toronto and trips originating outside of Toronto but finish-
 497 ing in Toronto. The empirical and theoretical TLD for this Toronto data set
 498 are represented in the top-left panel of Figure 6. Maximum likelihood estima-
 499 tion and the Nelder-Mead method for direct optimization available within the
 500 `{fitdistrplus}` package (Delignette-Muller and Dutang, 2015) were used. Based
 501 on goodness-of-fit criteria and diagnostics the normal distribution was selected
 502 (see Figure 6).

503 The normal distribution is defined in Equation (14), where we see that it
 504 depends on a mean parameter μ and a standard deviation parameter σ . The
 505 estimated values of these parameters are $\mu = 14.707$ and $\sigma = 7.697$.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad (14)$$

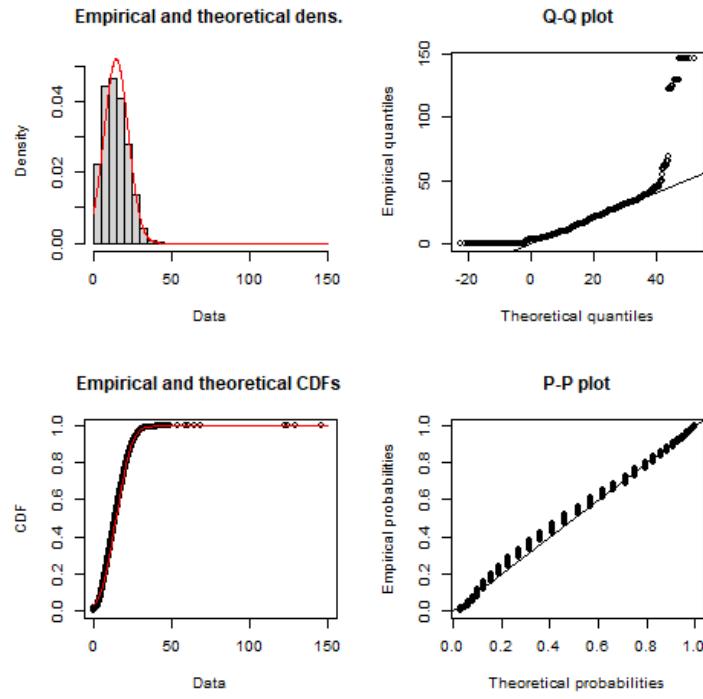


Figure 6: Car trip length distribution and calibrated normal distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

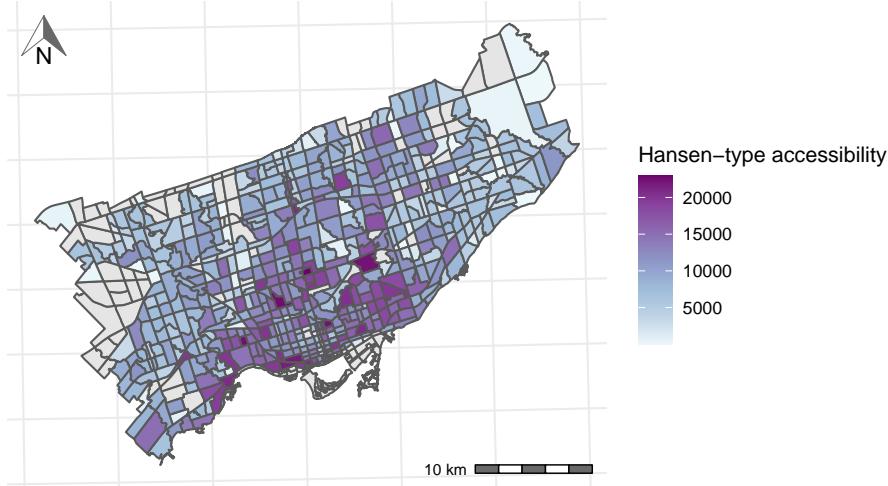


Figure 7: Estimated accessibility to employment in Toronto according to Hansen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.

$$\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

506 Figures 10, 11, and 12 are the absolute accessibility values in number of jobs
 507 accessible/available.

508 How do Shen-type internal values perform? The opportunity seeking popula-
 509 tion according to Shen-type measure greatly exceeds the population:

osp	population
1.98e+06	1.14e+06

510 The ratio of effective opportunity-seeking population to population is shown
 511 next:

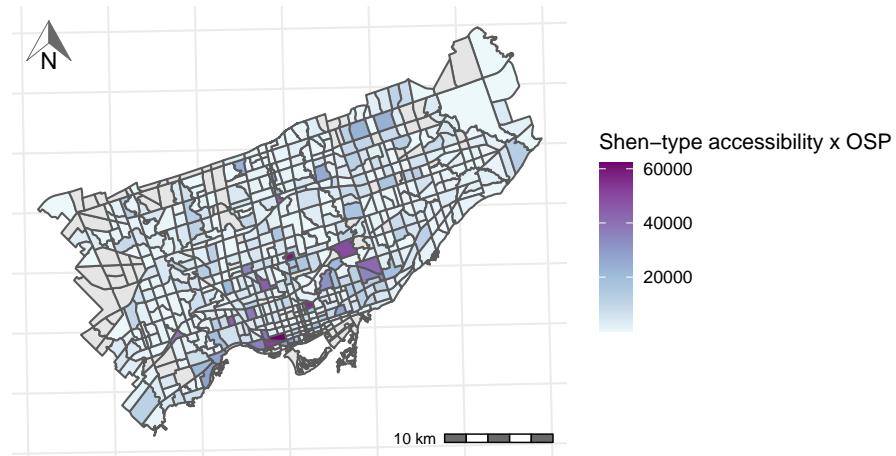


Figure 8: Estimated accessibility to employment in Toronto according to Shen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null accessibility values.

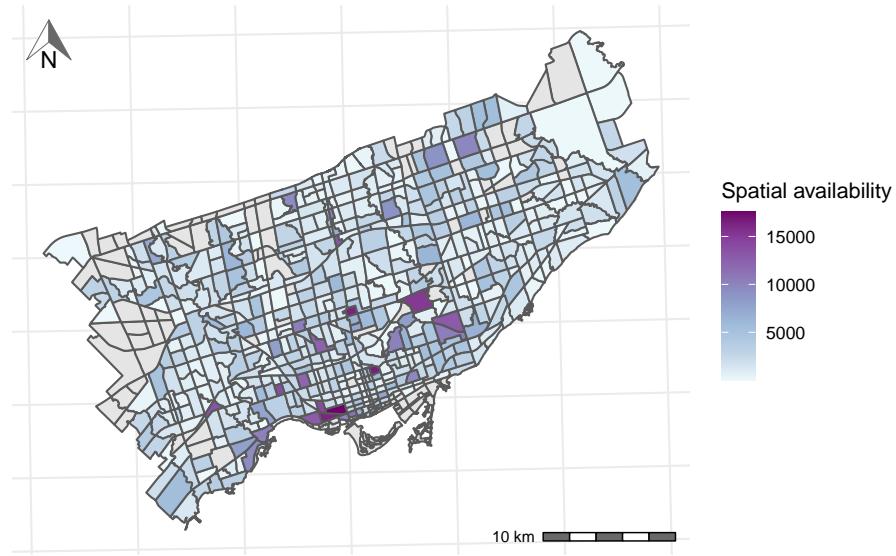


Figure 9: Estimated spatial availability of employment in Toronto. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.

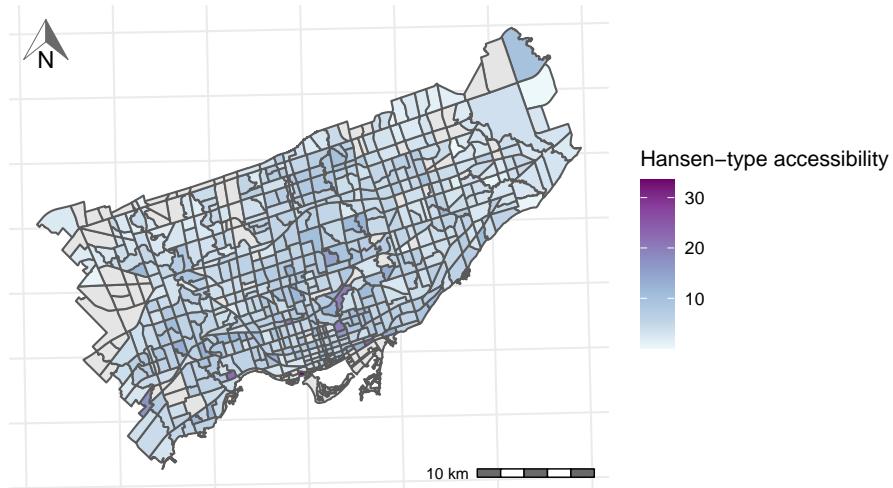
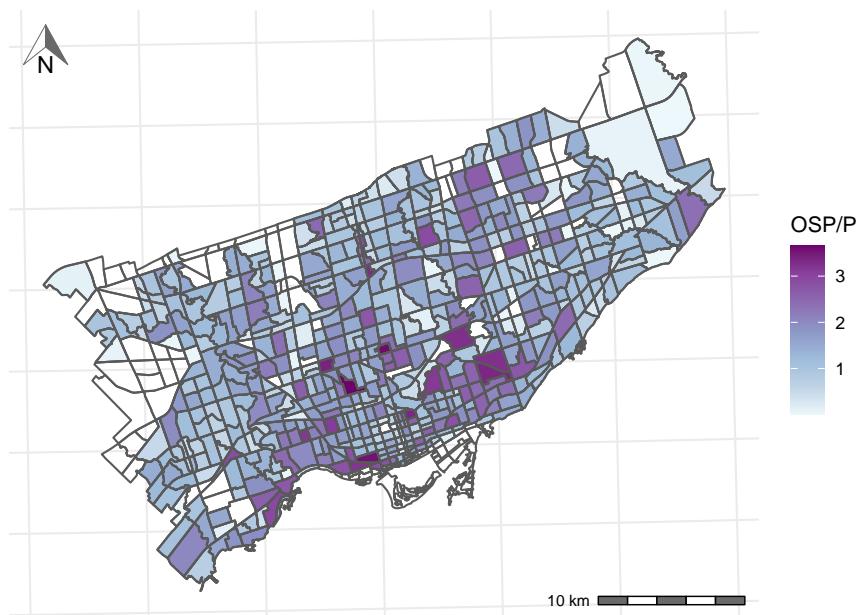


Figure 10: Estimated accessibility per capita to employment in Toronto according to Hansen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.



512

513

The effect is not constant in space.

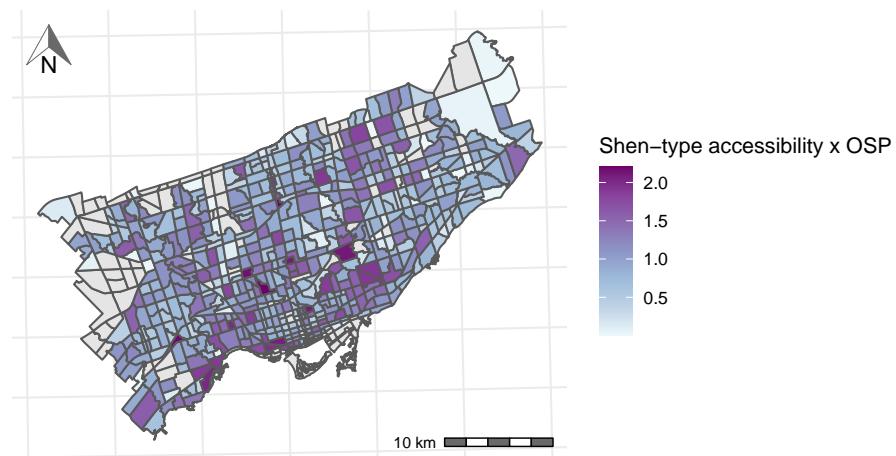


Figure 11: Estimated accessibility to employment in Toronto according to Shen-type indicator. Greyed out TAZ are zones with no residential population, i.e., with null accessibility values.

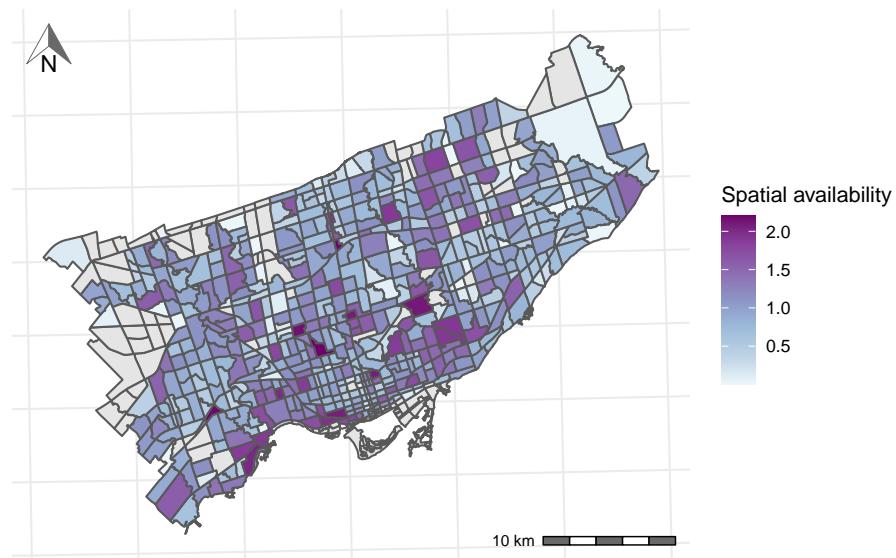


Figure 12: Estimated spatial availability of employment in Toronto. Greyed out TAZ are zones with no residential population, i.e., with null spatial availability values.

514 As a consequence of the inflated population ($osp > population$), the travel
515 time is exaggerated by Shen-type measure:

total_travel_time
668586.666173916

total_travel_time
344376.94962808

516 Why do these differences matter? Think about equity analysis! With spa-
517 tial availability, we can push accessibility analysis forward by making it more
518 interpretable

519 *4.4. Accessibility and spatial availability of jobs in Toronto*

520 Toronto is the largest city in the GGH and represents a significant subset of
521 workers and jobs in the GTHA; 33% of workers in the GTHA travel to jobs in
522 Toronto and 43% of jobs are located within Toronto.

523 **5. Discussion and Conclusions**

524 Words go here.

- 525 • Hansen-style accessibility should be used when opportunities are non-exclusive.
 526 When opportunities are perfectly exclusive (i.e., 1 spot for 1 person), spa-
 527 tial availability (i.e., accessibility with competition) should be used.
- 528 • some literature which uses Shen focuses on how travel cost discounts
 529 job supply (opportunities) and demand (opportunity-seeking population).
 530 This gets close to disentangling which population *effectively* seeking op-
 531 portunities - but comparing it to the population which doesn't seek oppor-
 532 tunities has not been made, to the authors opportunity. This is critical for
 533 equity analysis; i.e., which populations can't seek opportunities because
 534 of friction of distance? This can be plainly seen by seeing the proportion
 535 of population, in spatial availability, which is allocated to the origin

536 **6. Appendix A**

537 Equivalence of Shen-type accessibility and spatial availability

538 Population allocation factor:

$$F_{ij}^p = \frac{P_{i \in r}^\alpha}{\sum_i^K P_{i \in r}^\alpha}$$

$$F_A^p = \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha}$$

539 Cost allocation factor:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=A}^K f(c_{ij})}$$

$$F_{A1}^c = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{B1}^c = \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$F_{C1}^c = \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$$

540 Now let's put it together with P , and see how the denominators end up
 541 cancelling out:

$$v_i = \sum_j \frac{O_j}{P_{i \in r}^\alpha} \frac{\frac{P_{i \in r}^\alpha}{\sum_i^K P_{i \in r}^\alpha} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}{\frac{P_{i \in r}^\alpha}{\sum_i^K P_{i \in r}^\alpha} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}$$

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} \right)$$

$$\frac{O_2}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} \right) +$$

$$\frac{O_3}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} + \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} \right)$$

542 First, notice how the denominator on the denominator is the same across
 543 the summation? Let's simplify it:

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A1}) + f(c_{B1}) + f(c_{C1}))}} \right) + \\ \frac{O_2}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A2}) + f(c_{B2}) + f(c_{C2}))}} \right) + \\ \frac{O_3}{P_A^\alpha} \left(\frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A3}) + f(c_{B3}) + f(c_{C3}))}} \right)$$

544 See how the denominator of the denominator is the same as the denominator
 545 of the numerator's denominator for each J (J=1, J=2, and J=3)? Let's cancel
 546 those out and simplify:

$$v_A = \frac{O_1}{P_A^\alpha} \left(\frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} + \right. \\ \frac{O_2}{P_A^\alpha} \left. \frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} + \right. \\ \frac{O_3}{P_A^\alpha} \left. \frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

547 Next, see how we can cancel out the P_A^α ? Let's do that.

$$v_A = O_1 \left(\frac{f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})} + O_2 \frac{f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})} + O_3 \frac{f(c_{A3})}{P_A^\alpha \cdot f(c_{A3})} \right)$$

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