Estimating spatial availability/mismatch using singly constrained accessibility measures

Author One[a], Author Two[,a]

[a] *Address*

Abstract

This is the abstract.

It consists of two paragraphs.

*Corresponding Author

Email addresses: `author.1@example.com` (Author One), `author.2@example.com` (Author Two)

Introduction

The concept of accessibility is a relatively simple one whose appeal derives from combining the spatial distribution of opportunities and the cost of reaching them (Hansen, 1959). Numerous methods for calculating accessibility have been proposed that can be broadly organized into infrastructure-, place-, person-, and utility-based measures (Geurs and van Wee, 2004). Of these, the place-based family of measures is arguably the most common, capturing the number of opportunities reachable from an origin using the transportation network. This type of measure is also referred to as a gravity-based measure of accessibility that captures the potential for interaction.

Accessibility analysis is widely employed in transportation, geography, public health, and many other areas, and there is increasing emphasis on a shift from mobility-oriented to accessibility-oriented planning (Deboosere et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021). However, while these types of accessibility measures are excellent indicators of the intersection between urban structure and transportation infrastructure, they have been criticized in the past for not being highly interpretable. Previous research has highlighted how the weighting of opportunities using an impedance function can make gravity measures more difficult for planners and policymakers to interpret compared to simpler cumulative opportunity measures (Geurs and van Wee, 2004; Miller, 2018). Moreover, because place-based accessibility measures sensitive to the number of opportunities and the characteristics of the transportation network, raw accessibility values cannot be easily compared across study areas (Allen and Farber, 2019).

Intra- and inter-regional comparisons are challenging because gravity-based accessibility indicators are spatially smoothed estimates of the total number of opportunities, however, the meaning of their magnitudes is unclear. This is evident when we consider the "total accessibility" in the region, a quantity that is not particularly meaningful since it is not constrained to resemble, let alone match the number of opportunities available. Furthermore, while accessibility depends on the supply of destination opportunities weighted by the travel costs associated with reaching them, the calculated accessibilities are not sensitive to the demand for those opportunities at the origins. Put another way, traditional measures of place-based accessibility do not capture the competition for opportunities. This theoretical shortcoming (Geurs and van Wee, 2004) is particularly problematic when those opportunities are "non-divisible" in the sense that, once they have been taken by someone, are no longer available to other members of the population. Examples of indivisible opportunities include jobs (when a person takes up a job, the same job cannot be taken by someone else) and placements at schools (once a student takes a seat at a school, that particular opportunity is no longer there for another student). From a different perspective, employers may see workers as opportunities, so when a worker takes a job, this particular individual is no longer in the available pool of candidates for hiring.

To remedy these issues, researchers have proposed several different approaches for calculating competitive accessibility measures. On the one hand, this includes several approaches that first normalize the number of opportunities available at a destination by the demand for them from the origin zones and, second, sum the demand-corrected opportunities reachable from the origins (e.g. Joseph and Bantock, 1984; Shen, 1998). These advances were popularized in the family of two-step floating catchment area methods (Luo and Wang, 2003) that have found widespread adoption for calculating competitive accessibility to healthcare and other uses. In principle floating catchment areas purport to account for competition/congestion effects, although in practice several researchers (e.g., Delamater, 2013; Wan et al., 2012) have found that they tend to over-estimate the level of demand and/or service. The underlying issue, as demonstrated by Paez et al. (2019), is the multiple counting of both population and level of service, which can lead to biased estimates if not corrected.

A second approach is to impose constraints on the gravity model to ensure flows between zones are equal to the observed totals. Based on Wilson's (1971) entropy-derived gravity model, researchers can incorporate constraints to ensure that the modelled flows match some known quantities in the data inputs. In this way, models can be singly-constrained to match the row- or column-marginals (the trips produced or attracted, respectively), whereas a doubly-constrained model is designed to match both marginals. Allen and Farber (2019) recently incorporated a version of the doubly-constrained gravity model within the floating catchment area approach to calculate competitive accessibility to employment using transit across eight cities in Canada. But while such a model can account for competition, the mutual dependence of the balancing factors in a doubly-constrained model means they must be iteratively calculated which makes them more computationally-intensive. Furthermore, the double constraint means that the sum of opportunity-seekers and the sum of opportunities must match, which is not necessarily true in every case (e.g., there might be more people searching for work than jobs exist in a region).

In this paper we propose an alternative approach to measuring competitive accessibility. We call it a measure of *spatial availability* (SA), and it aims to capture the number of indivisible opportunities that are not only *accessible* but also *available* to the opportunity-seeking population, in the sense that they have not been claimed by a competing seeker of the opportunity. As we will show, spatial availability is a singly-constrained measure of accessibility. By allocating opportunities in a proportional way based on demand and distance, this method avoids the issues of inflation that result from multiple counting of opportunities in traditional accessibility measures. The method returns meaningful accessibilities that correspond to the rate of available opportunities per person. Moreover, the method also returns a benchmark value for the region under study against which results for individual origins can be compared.

In the following sections we will describe and illustrate this new measure using simple numerical examples. First, we will describe the measure. Second, we will calculate the SA using a simple hypothetical population and employment centers data set for three use-cases: one of jobs from the perspective of the population, another of workers from the perspective of employers, and another considering catchment restrictions. Thirdly, we calculate the SA using real world data for the Transportation Tomorrow Survey (TTS) home-to-work commute in 2016 for the Greater Golden Horseshoe (GGH) area in Ontario, Canada. Finally, we discuss the differences between accessibility estimates to the proposed measure of SA and the potential range of uses of the SA measure.

Background

Most accessibility measures (excluding utility-based measures) are derived from the gravity model, and are known as *gravity-based* accessibility. Briefly, consider the following accessibility measure A_i :

$$A_i = \sum_{j=1}^{J} O_j f(c_{ij})$$

where:

- i is a set of origin locations.
- j is a set of destination locations.
- O_j is the number of opportunities at location j. These are opportunities for activity and add some sort of *supply* to the area;
- c_{ij} is a measure of the cost of moving between i and j
- $f(\cdot)$ is an impedance (or distance-decay) function (a monotonically non-increasing or decreasing function of c_{ij}).

The accessibility value A_i, it can be seen, is the weighted sum of opportunities that can be reached from location i, given the cost of travel c_{ij} and an impedance function. Summing the opportunities in the neighborhood of i (the neighborhood is defined by the impedance function) estimates of the total number of opportunities that can be reached from i at a certain cost. Depending on the impedance function, the measure could be cumulative opportunities (if $f(\cdot)$ is a binary or indicator function) or a more traditional gravity measure, for instance with a Gaussian impedance function or an inverse cost impedance function .

We use a simple numerical example to introduce the key concepts, and we will use the usual accessibility measure for comparison. In this way, we aim to show the differences between accessibility and spatial availability, which helps to explain how spatial availability can improve interpretability in the analysis of spatially dispersed opportunities.

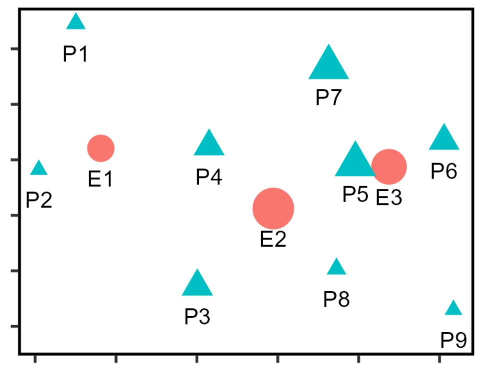
Numerical Example

In this section we present a simple numerical example. The setup for the example is a system with three employment centers and nine population centers, as seen in Table 1.

The accessibility to employment of each of the population centers can be calculated using the expression above for A_i. As noted, this yields the number

Table 1: Numerical example

id	number	type
E1	750	jobs
E2	2250	jobs
E3	1500	jobs
P1	260	population
P2	255	population
P3	510	population
P4	495	population
P5	1020	population
P6	490	population
P7	980	population
P8	260	population
P9	255	population



of jobs (opportunities) that are accessible (i.e., can be reached) from each population center, given the cost. In this example we use the straight line distance between the population and jobs for c_{ij}, and a negative exponential function with $\beta = 0.0015$.

Figure 1 shows the three employment centers locations (black circles), where the size of the symbol is in proportion to the number of jobs at each location. We also see nine population centers (triangles), where the size of the symbol is proportional to the accessibility (

$$A_i$$

) to jobs. At a glance:

- Population centers (triangles) in the middle of the map are relatively close to all three employment centers and thus have the highest levels of job accessibility. Population center P5 is relatively central and close to all employment centers, and it is the closest population to the second largest employment center in the region. Unsurprisingly, this population center has the highest accessibility 680.64);

- Population centers (triangles) near the left edge of the map (only in proximity to the small employment center) have the lowest levels of job accessibility. Population center P1 is quite peripheral and the closest employment center is also the smallest one. Consequently, it has the lowest accessibility with $A_i = 17.12$);

What are the Issues?

Accessibility measures are excellent indicators of the intersection between urban structure and transportation infrastructure. However, beyond enabling comparisons of relative values they are not highly interpretable on their own. For

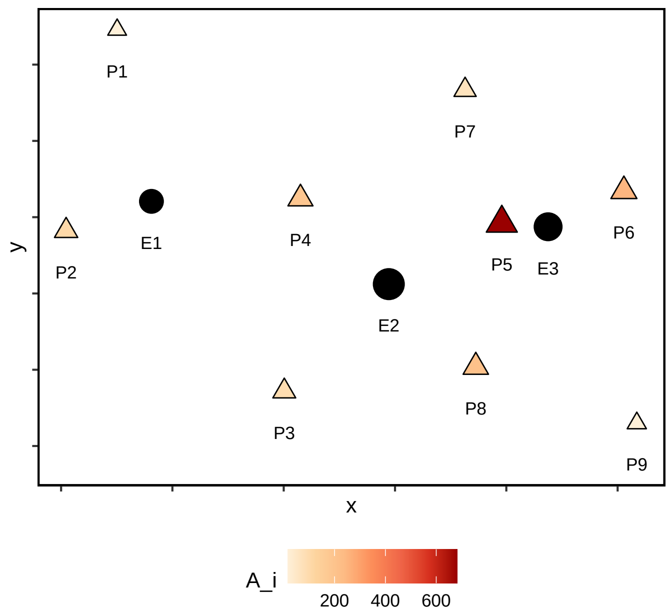


Figure 1: Accessibility results using simple numerical example

instance, from Figure 1, P1 has lower accessibility than P5, however, despite the accessibility value for P1 being low it is still better than *zero*. This evaluation leaves decision makers unclear on how to interpret accessibility values, particularly extreme values and particularly when comparing across scenarios or regions . The obscurity in interpretation of accessibility values arises from the following two ways.

First, origins with accessibility to a high density of opportunities are susceptible to having disproportionately higher accessibility values than origins with a lower density of opportunities. This occurs since total accessibility ($\sum_{i=1}^{I} A_i$) depends on the number of origins; every additional origin in the analysis causes at least one and possibly more opportunities at destinations to be multiple-counted. As opportunities are multiple-counted, origins which are in proximity to high density of opportunities have accessibility values which are relatively conglomerated compared to origins in proximity to a lower density of opportunities. This issue means total accessibility is vulnerable to the modifiable areal unit problem (MAUP) as opportunities and origins which are spatially aggregated are multiple-counted inconsistently.

Second, the calculated accessibilities do not take competition into account. For example, an individual at P5 has accessibility to 680.6373657 jobs. But since this is also a large population center, there is potentially large competition for those accessible jobs. In other words, the value of A_i is not sensitive to the size of population at the origin seeking the opportunity (in this case jobs), let alone the population at other locations. This unfortunately limits the interpretability of the measure. Floating catchment areas purport to account for competition/congestion effects, but as discussed by Paez, Higgins, and Vivona (2019), they are vulnerable to conglomeration, which makes them prone to bias unless corrected.

To address these two shortcomings of the accessibility measure, we propose a singly-constrained gravity measure that corresponds to the concept of *spatial availability*.

Spatial Availability: Proportional Allocation of Opportunities Based on Demand

Analytical Framework

As recent research on accessibility shows (see for instance Paez, Higgins, Vivona (2019) and Allen and Farber (2019)), accounting for competitive access in a meaningful way requires the proportional allocations of quantities in the accessibility calculations, or their normalization. At issue is the fact that multiple-counting is commonplace when calculating conventional accessibility A_i for $i = 1, \cdots, n$ as every opportunity enters the weighted sum once for every origin i that can reach it. This has the unfortunate effect of obscuring the interpretability of A_i and fails to answer for a individual at a specific population center the question: *"many jobs are accessible, but the same jobs are also accessible to my (possibly) numerous neighbors...what does high accessibility actually mean to me?"*

In the spatial availability framework proposed, and in line with the gravity tradition, we distinguish between opportunities at a destination and demand for opportunities at the origin. To explain the analytical framework, the example of access to employment is illustrative, with "population" in the role of demand (i.e., the number of individuals in the labour market who "demand" employment) and "jobs" in the role of opportunities.

As an overview, spatial availability (V_{ij}) is the proportional allocation of opportunities (O) and allocation factor for population (f^p) and cost of travel (f^c). Since spatial availability (V_{ij}) consists of these two allocation factors, this example first details how the population allocation factor f^p_{ij} produces V^p_{ij}, next details the role of travel cost allocation factor f^c_{ij} in producing V^c_{ij}, and finally combines both allocation factors in the final general form of spatial availability V_{ij} as follows:

$$V_{ij} = O_j \frac{f^p_{ij} \cdot f^c_{ij}}{\sum_{k=1}^{K} f^p_{kj} \cdot f^c_{kj}}$$

Population Allocation Factor

We begin with allocation based on demand; consider an employment center j with O^r_j jobs of type r. In the general case where there are K population centers in the region, the following factor can be defined:

$$f^p_{ij} = \frac{P^\alpha_{i\in r}}{\sum_{k=1}^K P^\alpha_{k\in r}}$$

On the right hand side of the equation, $P_{i\in r}$ is the population at location i that is eligible for jobs of type r (maybe those with a certain level of training, or in a designated age group). The summation in the bottom is over $k = 1, \cdots, K$, the number of population centers in the region. The resulting factor f^p_{ij} corresponds to the proportion of the population in zone i relative to the rest of the region's population centres K. The factors f^p_{ij} satisfy the property that $\sum_i^I f^p_{ij} = 1$. We can also add an empirical parameter α that can be used to modulate the effect of size in the calculations (i.e., $\alpha < 1$ places greater weight on smaller centres relative to larger ones while $\alpha > 1$ achieves the opposite effect).

This factor (f^p_{ij}) can now be used to proportionally allocate a share of the jobs at the employment centre j to population center i and population center k. The share of jobs at j allocated to (i.e., available to) each population center is:

$$V^p_{ij} = O_j f^p_{ij}$$

and since $\sum_i^I f^p_{ij} = 1$ it follows that:

$$\sum_{i=1}^I V_{ij} = O_j$$

In other words, the number of jobs is preserved. The result is a proportional allocation of available jobs to population centres based on demand. As an example, consider an employment center j in a region with two population centers (say i and k). For simplicity, assume that the all the population in the region is eligible for these jobs, that is, that the entirety of the population is included in the set r. The allocation factors for the jobs at j would be:

$$\begin{aligned} f^p_{ij} &= \tfrac{P^\alpha_i}{P^\alpha_i + P^\alpha_k} \\ f^p_{kj} &= \tfrac{P^\alpha_k}{P^\alpha_i + P^\alpha_k} \end{aligned}$$

Suppose that there are three hundred jobs in the employment center ($W_j = 300$), and that the populations are $P_i = 240$ and $P_k = 120$. The jobs are allocated as follows (assuming that $\alpha = 1$):

$$\begin{aligned} V^p_{ij} &= O_j f^p_{ij} = O_j \tfrac{P^\alpha_i}{P^\alpha_i + P^\alpha_k} = 300 \tfrac{240}{240+120} = 300 \tfrac{240}{360} = 200 \\ V^p_{kj} &= O_j f^p_{kj} = O_j \tfrac{P^\alpha_k}{P^\alpha_i + P^\alpha_k} = 300 \tfrac{120}{240+120} = 300 \tfrac{120}{360} = 100 \end{aligned}$$

It can be seen that proportionally more jobs are allocated to the bigger center and also that the total number of jobs is preserved. However, the factors above account for the total number of opportunities at the destination (i.e., the number of jobs at the employment center), but they do not account for their location relative to the population centers. The proportional allocation procedure above is insensitive to how far population centers i or k are from employment center j.

To account for this effect we define a second set of allocation factors based on distance to the employment centers.

Travel Cost Allocation Factor

These are defined as:

$$f^c_{ij} = \frac{f(c_{ij})}{\sum_{k=1}^{K} f(c_{ij})}$$

where c_{ij} is the cost (e.g., the distance, travel time, etc.) from population center i to employment center j, and $f(\cdot)$ is an impedance function that is a monotonically decreasing function of cost (c_{ij}); in other words, this allocation factor (f^c_{ij}) serves to proportionally allocates more jobs to closer locations through an impedance function. To illustrate, assume that the impedance function is a negative exponential function as follows, and assume that β (which modulates the steepness of the impedance effect and is an empirical parameter) is the value of 1:

$$f(c_{ij}) = \exp(-\beta c_{ij})$$

Continuing the example, suppose that the distance from population center i to employment center j is 0.6 km, and the distance from population center k to employment center j is 0.3 km. Being closer, we would expect more jobs to be allocated to the population of k. The jobs would be sorted as follows:

$$\begin{aligned} f^c_{ij} &= \tfrac{\exp(-\beta D_{ij})}{\exp(-\beta D_{ij})+\exp(-\beta D_{kj})} \\ f^c_{kj} &= \tfrac{\exp(-\beta D_{kj})}{\exp(-\beta D_{ij})+\exp(-\beta D_{kj})} \end{aligned}$$

This step normalizes the impedance between i and j and k and j by the total impedance in the study area. Numerically, the jobs allocation is:

$$\begin{aligned} V^c_{ij} = O_j f^c_{ij} &= O_j \tfrac{\exp(-D_{ij})}{\exp(-D_{ij})+\exp(-D_{kj})} = 300 \tfrac{\exp(-0.6)}{\exp(-0.6)+\exp(-0.3)} = 3 \times 0.426 = 127.8 \\ V^c_{kj} = O_j f^c_{kj} &= O_j \tfrac{\exp(-D_{kj})}{\exp(-D_{ij})+\exp(-D_{kj})} = 300 \tfrac{\exp(-0.3)}{\exp(-0.6)+\exp(-0.3)} = 3 \times 0.574 = 172.2 \end{aligned}$$

A larger share of jobs (172.2 jobs) is allocated to the population center (k) that is closest as assumed. As before, the sum of jobs allocated to the population centers matches the total number of jobs available. Nevertheless, in isolation, this step does not account for the allocation of jobs based on demand.

Putting Spatial Availability Together

We can combine the proportional allocation factors by population and distance and calculated spatial availability (V_{ij}) as follows:

$$V_{ij} = O_j \frac{f^p_{ij} \cdot f^c_{ij}}{\sum_{k=1}^{K} f^p_{kj} \cdot f^c_{kj}}$$

When applied to the example of population center i and k (i.e., the demand) traveling to employment center j (i.e., the opportunity O):

$$V_{ij} = O_j \cdot \frac{f^p_{ij} \cdot f^c_{ij}}{f^p_{ij} \cdot f^c_{ij} + f^p_{kj} \cdot f^c_{kj}} = 300 \frac{\left(\frac{2}{3}\right)\left(0.426\right)}{\left(\frac{2}{3}\right)\left(0.426\right) + \left(\frac{1}{3}\right)\left(0.574\right)} = \left(300\right)\left(\frac{0.284}{0.475}\right) = 179.4$$
$$V_{kj} = O_j \cdot \frac{f^p_{kj} \cdot f^c_{kj}}{f^p_{ij} \cdot f^c_{ij} + f^p_{ik} \cdot f^c_{ik}} = 300 \frac{\left(\frac{1}{3}\right)\left(0.574\right)}{\left(\frac{2}{3}\right)\left(0.426\right) + \left(\frac{1}{3}\right)\left(0.574\right)} = \left(300\right)\left(\frac{0.191}{0.475}\right) = 120.6$$

Notice how fewer jobs are allocated to population center i compared to the allocation by population only, to account for the higher cost of reaching the employment center. On the other hand, distance alone allocated more jobs to the closest population center (i.e., k), but since it is smaller, it also gets a smaller share of the jobs overall. Again, the sum of jobs at employment center j that are allocated to population centers i and k simultaneously based on *population*- and *distance*- based allocation is preserved (i.e., $W_{ij} + W_{kj} = W_j$).

Availability is simply the sum of the above by origin:

$$V_i = \sum_{j=1}^{J} V_{ij}$$

This quantity represents opportunities (e.g., jobs) that can be reached from i (i.e., they are accessible), and that are *not* allocated to a competitor: therefore the weighted sum of available opportunities. Compare V_i to the singly-constrained gravity model (see Wilson (1971)). In essence, V_i is the result of constraining A_i to match one of the marginals in the origin-destination table, the known total of opportunities.

Since the sum of opportunities is preserved in the procedures above, it is possible to calculate a highly interpretable measure of spatial availability per capita (call it lower-case v_i) as follows:

$$v_i = \frac{V_i}{P_i}$$

In the example above:

$$v_{ij} = \frac{V_{ij}}{P_i} = \frac{179.4}{240}$$
$$v_{kj} = \frac{V_{ik}}{P_k} = \frac{120.6}{120}$$

Less competition (P_k is the smallest population center in the region) and being closer to the jobs clearly works in favor of individuals at k. Where the overall ratio of jobs to population in the region is $300/(240 + 120) = 0.83$, the spatially available jobs per capita at k is closer to unity.

In the following sections we use the same numerical example presented above to illustrate how availability V_i is calculated. We conclude by contrasting the two measures in the final section.

1st Use Case: Available Jobs for The Working Population

In the following examples we use the same impedance function that was used to illustrate the accessibility calculations in the numerical example. The spatial availability calculations are implemented in the function `sp_avail`. The inputs are an Origin-Destination table with labels for the origins (`o_id`), labels for the destinations (`d_id`), the population (`pop`) and number of opportunities (`opp`), an indicator for catchments or other eligibility constraints (`r`), and a pre-calculated impedance function (`f`). For this example, we assume that there are no catchment restrictions by setting `r` to 1.

The value of the function (its output) is a vector with V_ij given the inputs, that is, the opportunities available to i from j:

```
# A tibble: 9 x 2
  Origin          V_i
  <fct>         <dbl>
1 Population 1   67.0
2 Population 2  414.
3 Population 3  336.
4 Population 4  745.
5 Population 5 2081.
6 Population 6  270.
7 Population 7  256.
8 Population 8  316.
9 Population 9   14.9
```

The following plot shows the estimates of spatial availability:

We see that population center 5 has the highest level of spatial availability, due to being a large population center that is moreover relatively close to jobs. To improve the interpretability of this measure, we first note that the regional measure of jobs per capita is 0.994. We then calculate the spatially available jobs per person at each population center:

Plot the spatial availability per person:

Some population centers have almost two jobs available per person (compared to the overall regional value of approximately one job per capita), while others have less than one job available per person. This does not mean that people are not taking some of the jobs. It means that controlling for the cost of reaching jobs, they are worse off than those with more jobs spatially available.

2nd Use Case: Available Workers for Employment Centers

We can also examine the pool of workers available to each employment center by considering the workers as the opportunities and the jobs as the population.

Calculate the spatial availability by proportionally allocating jobs to workers (we refer to spatial availability in this case as W_ji) as follows:

Plot the availability estimates:

Plot the spatial availability of workers per job:

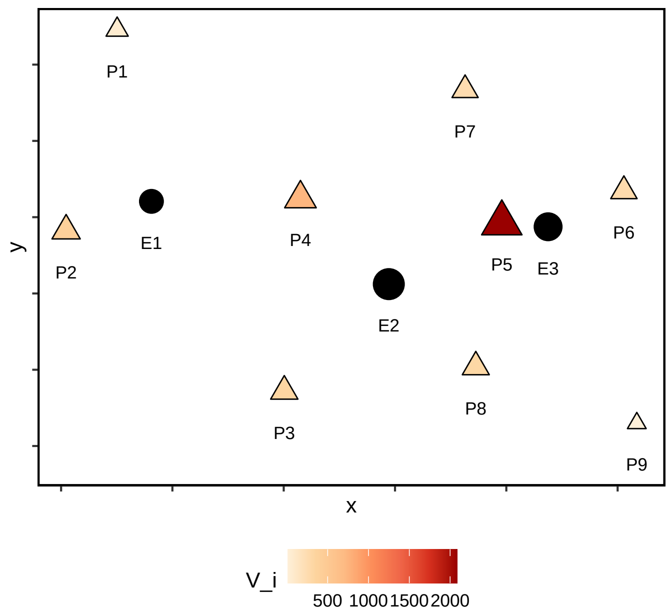


Figure 2: Spatial availability of jobs

3rd Use Case: Available Jobs for Specialized Working Populations

In this section we introduce catchment/eligibility constraints. Due to differences in educational achievement among the population, the jobs in Employment Center 1 can only be taken by individuals in population centers 1 and 2. Jobs in Employment Center 2 can be taken by individuals in population centers 3, 4, 5, 7, and 8. Lastly, jobs in Employment Center 3 require qualifications available only among individuals in population centers 5, 6, 8, and 9.

Calculate the spatial availability by proportionally allocating *specialized* workers to jobs (we refer to spatial availability in this case as V_ij_r):

Plot the availability estimates:

Available jobs per person with catchment/eligibility conditions:

The plot in Fig. 8 shows the availability per person without and with catchment restrictions.

We can see that when there are catchment restrictions population center 2, despite being relatively peripheral, has higher spatial availability due to spatial specialization. With catchments, the spatial availability of jobs declines from the perspective of population center 4: the population here has skills required for jobs at a small employment center, where they face substantial competition from other population centers.

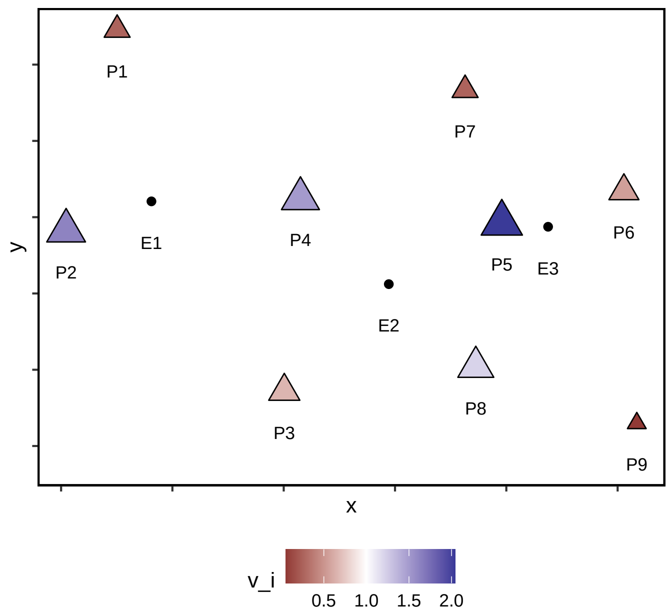


Figure 3: Spatial availability of jobs per capita

Empirical Example: Spatial Availability and Accessibility of Jobs in Toronto

For the reasons demonstrated in the hypothetical toy example; the spatial availability measure produces different and more interpretable results than accessibility. In this section we will demonstrate both measures and a comparision using empirical data for home-based work trips to places of employment in Toronto, Ontario.

Data

The 2016 Transportation Tomorrow Survey (TTS) data for 20 municipalities contained within the the Greater Golden Horseshoe (GGH) area in the province of Ontario, Canada (43.6°N 79.73°W) is analysed (Figure 9). This data set includes home origins and work destinations defined by centroids of Traffic Analysis Zones (TAZ) of varying areas (n=3764), the number of jobs (n=3081900) and workers (3446957) at each origin and destination, and the trips from origin to destination for the morning home-to-work commute (n=3069541). Also included are calculated travel times by car (calculated via `r5r`) and a derived impedance function values corresponding to the cost of travel based on the trip length distribution (TLD). The descriptive statistics are presented in Table 2 and a data-package is available to explore the data in greater detail.

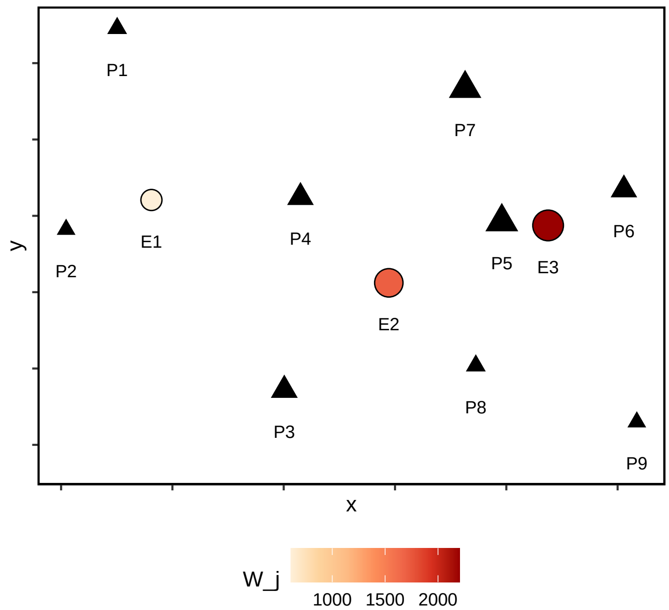


Figure 4: Spatial availability of workers

Table 2: Descriptive statistics of the TTS 2016 dataset for the Greater Golden Horshoe Area

	Trips	Travel_Time
	Min. : 1.00	Min. : 0.10
	1st Qu.: 14.00	1st Qu.: 13.00
	Median : 22.00	Median : 20.00
	Mean : 30.83	Mean : 23.39
	3rd Qu.: 37.00	3rd Qu.: 30.00
	Max. :1129.00	Max. :179.00

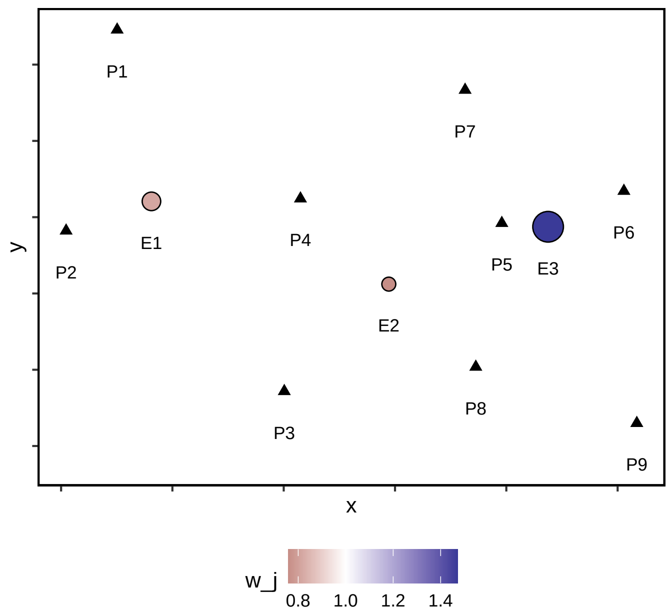


Figure 5: Spatial availability of workers per job

Calibrating an Impedance Function

In the hypothetical toy data set, an arbitrary negative exponential function describing the increase in travel cost as distance increases was used as the impedance function to derive both accessibility and availability. In this data set, an impedance function can be derived from the empirical trip length distribution (TLD) as the number of trips and their travel cost (in the case, travel time in minutes) are known (see black points in Figure **??**).

The TLD density plot appears to follow a gamma distribution, as such, this theoretical distribution along with other common TLD distributions such as log-normal and exponential distributions were fitted to the empirical TLD using the maximum likelihood estimation method and Nelder-Mead method for direct optimization available within the [‘fitdistrplus’ package] (https://cran.r-project.org/web/packages/fitdistrplus/fitdistrplus.pdf) in R . Based on goodness-of-fit criteria, the gamma distribution was selected (see red line in Figure **??**) with the following shape and rate parameters (Table **??**) in the general form of:

$$f(x,\alpha,\beta) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} \quad \text{for } 0 \leq x \leq \infty$$

where the estimated ‘shape’ is α,the estimated ‘rate’ is β, and the $\Gamma(\alpha)$ is defined as:

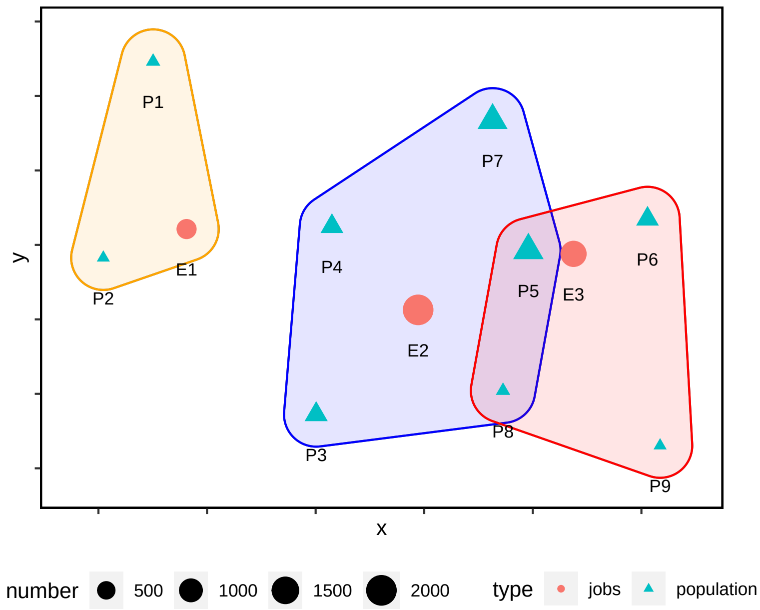


Figure 6: Catchments areas in numeric example

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} \, dx$$

The calibrated impedance function has a shape parameter of 2.019 and a rate parameter of 0.094. The function and diagnostics are plotted in Fig. 10.

Discussion

Comparing Accessibility and Spatial Availability

Side by side plots of accessibility and SA which are indexed by 0-100. Use text below for inspiration.

Firstly, Accessibility - not interpetable, (XX accessible jobs per person.. so what?) vs. Availability (XX available jobs per person). We can use the boundary benchmark to determine if an allocation is unjust if it is below XX we couldn't do it with Accessibility because we can compare by capita.

Secondly, scales back competition.

Discussion

Comparing Accessibility and Spatial Availability

Below we compare accessibility and the first case of spatial availability illustrated above. To facilitate the comparison, we index the measures so that

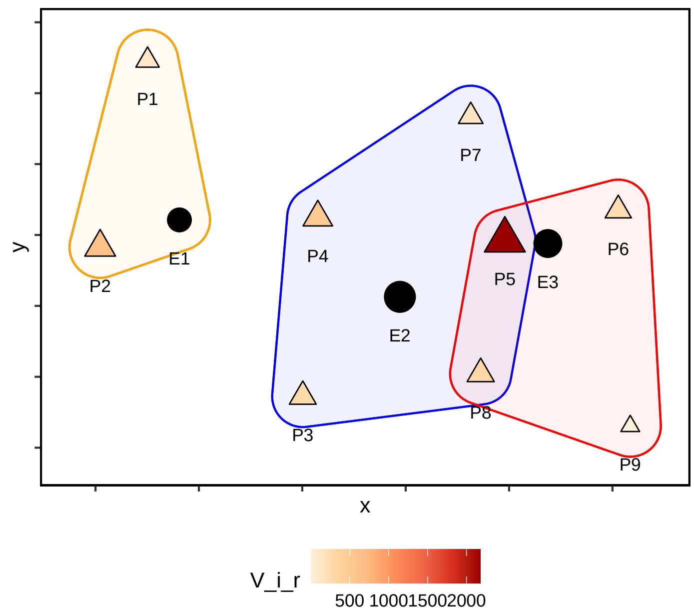


Figure 7: Spatial availability of jobs with catchment restrictions

the lowest value corresponds to an index of 100:

The figure suggests that high accessibility does not necessarily mean high availability.

How are the these measures different?

The sum of all spatial availability values is equal to the total number of opportunities within the region. As such, the value for each population center reflects how many opportunities are *available* accounting for the indivisibility of the opportunities. This provides greater interpretability of the results. Comparatively, accessibility values associated with each population centers reflects how many jobs can *potentially* be reached by each population center; these values are not adjusted proportionally to the number of population (i.e. workers).

Secondly, spatial availability is not robust to the modifiable unit problem, since the estimates account for the population at the centers. If more population centers are added, the availability adjusts accordingly by allocating opportunities proportionally.

Further, the measure of spatial availability can be a useful way to distinguish between high accessibility/high population centers (which potentially can result in lower availability due to competition), and contrariwise, low accessibility/low population centers, which may enjoy higher availability than the accessibility calculations may suggest. Remote, smaller population centers can be sufficiently

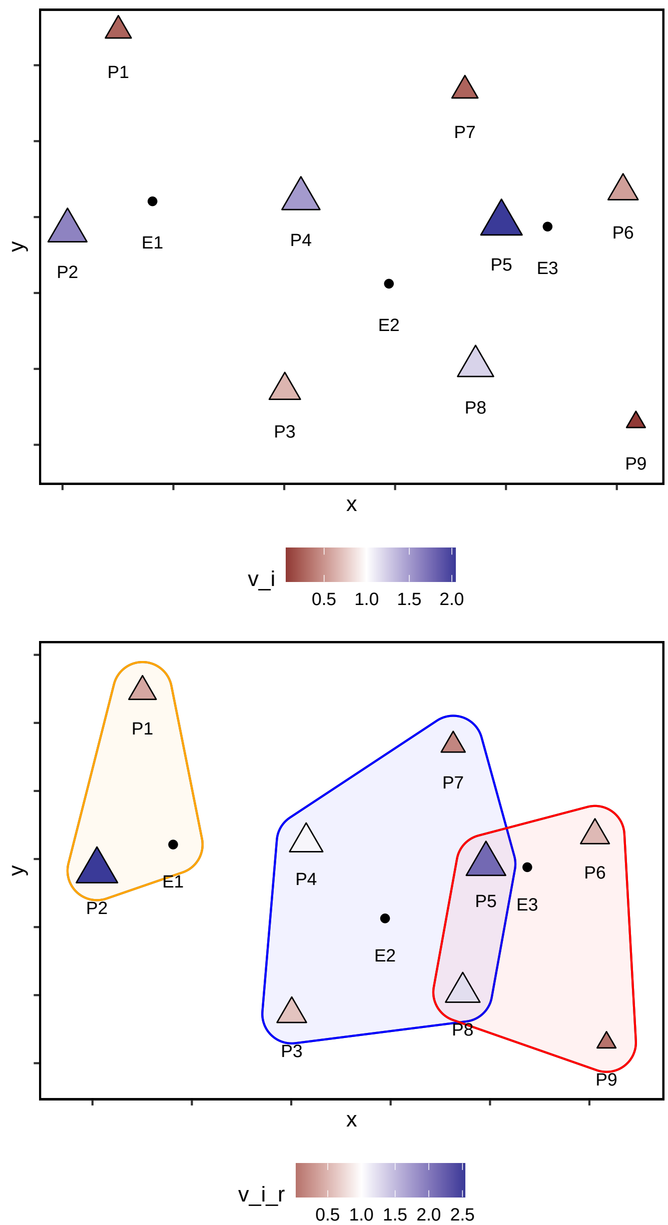


Figure 8: Spatial availability of jobs per capita with and without catchment restrictions

Figure 9: The TTS 2016 study area within the Greater Golden Horseshoe in Ontario, Canada.

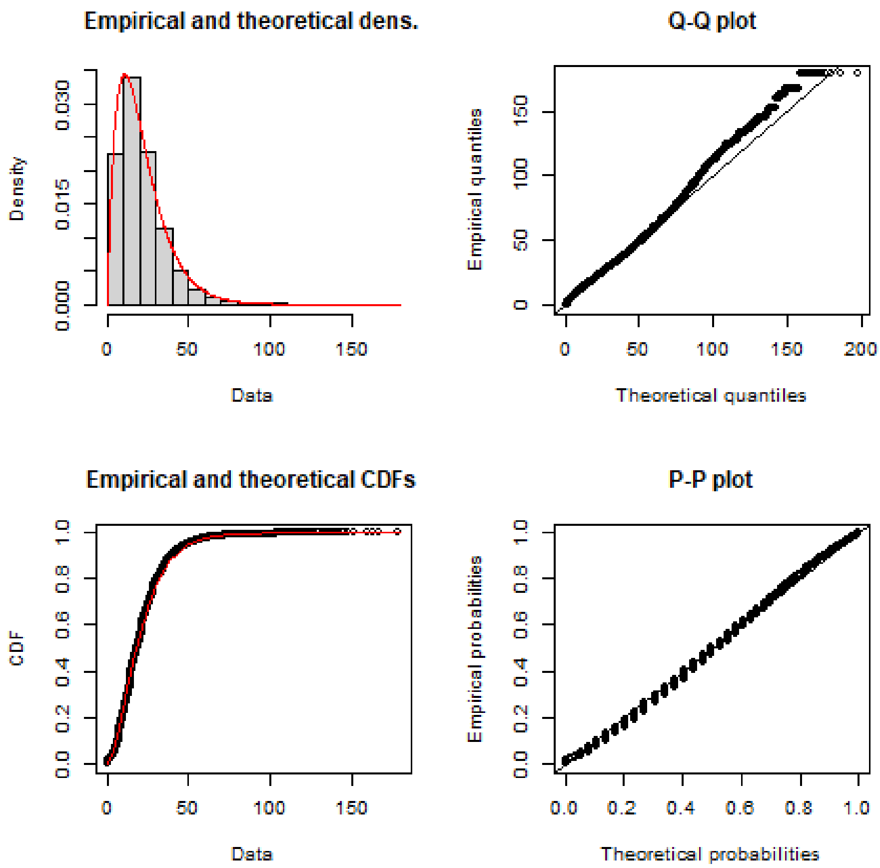


Figure 10: Impedance function and diagnostics.

accessible and in close proximity to the smaller employment centers; however this "sufficiency" is obscured by accessibility measure by over-inflating the accessibly of population centers which are more central to more (and larger) employment centers. Conventional accessible does not shed light on how sufficiently accessible opportunities are available to the population.

As a final point, we note that the measure proposed, by producing a concrete and interpretable number of opportunities available, can be meaningfully compared to the total number of opportunities in the region. Likewise for opportunities available per capita. This is an important topic in equity analysis. For instance, considering the two remote small population centers in the top left corner it is evident that for individuals in population center 2, despite facing low nominal accessibility, have a relatively high number of available jobs per capita.

It is a form of *spatial mismatch* and related to *balanced floating catchment?* approach.

Use Cases and Limitations for Spatial Availability

Conclusion

Words go here.

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