Introducing spatial availability, a singly-constrained competitive-access accessibility measure

Abstract

Accessibility measures are widely used in transportation, urban and healthcare planning, among other applications. These measures are weighted sums of the opportunities that can be reached given the cost of movement and are estimates of the potential for spatial interaction. These measures are useful in understanding spatial structure, but they do not properly account for competition due to multiple counting of opportunities. This leads to interpretability issues, as noted in recent research on balanced floating catchment areas (BFCA) and competitive measures of accessibility. In this paper, we respond to the limitations of accessibility analysis by proposing a new measure of *spatial availability* which is calculated by imposing a single constraint on conventional gravity-based accessibility. Similar to the gravity model from which spatial availability is derived, a single constraint ensures that the marginals at the destination are met and thus the number of opportunities are preserved. Through examples, we detail the formulation of the proposed measure. Further, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada, to contrast how the conventional accessibility measure tends to overestimate and underestimate the number of jobs *available* to workers. We conclude with some discussion of the possible uses of spatial availability and argue that, compared to conventional measures of accessibility, it can offer a more interpretable measure of opportunity access. All data and code used in this research are openly available which should facilitate testing by other reserarchers in their case studies.

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Introduction

Accessibility analysis is employed in transportation, geography, public health, and many other areas, particularly as mobility-based planning is de-emphasized in favor of access-oriented planning [13, 19, 36, 49]. The concept of accessibility derives its appeal from combining the spatial distribution of opportunities and the cost of reaching them [20].

Numerous methods for calculating accessibility have been proposed that can be broadly organized into infrastructure-, place-, person-, and utility-based measures [17]. Of these, the place-based family of measures is arguably the most common, capturing the number of opportunities reachable from an origin using the transportation network. A common type of accessibility measure is based on the gravity model of spatial interaction; since it was first developed by Hansen [20] it has been widely adopted in many forms [e.g., 8, 33, 17, 18, 28, 3]. Accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure - however, the interpretability of accessibility measures can be challenging [17, 31]. A key issue is that accessibility measures are sensitive to the number of opportunities in a region (e.g., a large city has more jobs than a smaller city), and therefore raw values cannot be easily compared across study areas [1].

Gravity-based accessibility indicators are in essence spatially smoothed estimates of the total number of opportunities in a region, but the meaning of their absolute magnitudes are unclear as they measure the *potential* for interaction (as originally defined by [1971]XXX). For instance, in the study by (XXX) high-income areas in Metropolitan Saul Paulo have high accessibility to employment (e.g., X jobs per all jobs available) and low-income areas have low accessibility to employment (e.g., Y jobs per all jobs available). Though X or Y reflect the urban structure, and correlates identify the relationship with employment, X and Y values can only be used within the context of employment accessibility in Saul Paulo. For example, in study XX which used the same employment accessibility method but for ZZ region, the X and Y accessibility values from Saul Paul cannot be immediately compared, but the X and Y low and high accessibility values in ZZ reflect the region's urban structure. That being said, decision-makers and planners find it difficult to operationalize accessibility since it reflects urban structure and is harder to interpret as an opportunity *provision* measure. For instance, if a small city creates 1000 more housing units in an employment rich neighborhood (ex., 15% of all employment opportunities are located within the neighbourhood (X)), could the city achieve a provider-provision-ratio of 1.5 jobs per person? Conventional accessibility cannot be standardized or normalized to reflect this ratio as a result of *inconsistent* opportunity-side and population-side counting.

This inconsistent counting embedded in traditional accessibility methods result in lack of clairty on how to interpret the resulting values. Returning to XX, the maximum job accessibility in Saul Paulo is XYZ, and the maximum job accessibility is XY in region ZZ. This the highest count of opportunities, weighted by associated travel costs, which a certain population can accessing the

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region. This maximum value, outside of reflecting urban structure, is difficult to interpret from a decision-maker's perspective who is deliberating planning interventions. We believe the issue in interpretation arises since opportunities are adjusted based on population-side travel and usage behaviour (i.e., travel costs, catchments, time-windows). Accessibility is not constrained to match the number of opportunities available. Put another way, traditional measures of accessibility do not capture the competition for opportunities but instead quantify access as if every person can reach every (travel and usage behaviour-adjusted) opportunity. This *inconsistency* is not necessarily problematic if a case study calls for the quantification of a accessibility assuming a "greedy" population. For instance, a greater accessibility to parks correlates with property values even though those that live in those properties don't use all the parks all the time, but this correlate suggests that the *potential* to access parks is enough. However, this *inconsistent* opportunity-adjustment can be acute when opportunities are "non-divisible" in the sense that, once taken they are no longer available to other members of the population [also discussed by 17]. Examples of these types of opportunities include jobs (e.g., when a person takes up a job, the same job cannot be taken by anyone else) and placements at schools (e.g., once a student takes a seat at a school, that opportunity is no longer available for another student). Though these non-divisible opportunities can still be modelled by conventional accessibility (e.g., Saul Paul and Z region), the measure does not reflect the *spatial availability* of the opportunity.

To re-calibrate accessibility for a more interpretable understanding of opportunity-provision, researchers have proposed several different approaches for considering competition in the conventional accessibility method. These include several approaches that first normalize the number of opportunities available at a destination by the demand from the origin zones and, second, sum the demand-corrected opportunities which can be reached from the origins [e.g. 26, 43]. These advances were popularized in the family of two-step floating catchment area (FCA) methods [30] that have found widespread adoption for calculating competitive accessibility to a variety of opportunities such as healthcare, education, and food access [50, 11, 51, 10, 9]. In principle, floating catchments purport to account for competition effects, although in practice several researchers [e.g., 14, 46] have found that they tend to over-estimate accessibility values. The underlying issue in FCAs, as demonstrated by Paez et al. [34], is the multiple counting of opportunities (ex., population 1 in a catchment accesses opportunities and population 2 in an overlapping catchment access a few opportunities which population 1 access as well as additional opportunities). The multiple counting of the *same* opportunities across populations can lead to biased estimates if not corrected [34].

Another approach which research have used to consider competition in accessibility has been the imposition of constraints on the gravity model to ensure potential interaction between zones are equal to the observed totals. Based on Wilson's [1971] entropy-derived gravity model, researchers can incorporate constraints to ensure that the modeled flows match some known quantities in the data inputs. In this way, models can be singly-constrained to match the

row- or column-marginals (i.e., the trips produced or attracted, respectively), whereas a doubly-constrained model is designed to match both marginals. Allen and Farber [2019] recently incorporated a version of the doubly-constrained gravity model within the FCA approach to calculate competitive accessibility to employment using transit across eight cities in Canada. But while such a model can account for competition, the mutual dependence of the balancing factors in a doubly-constrained model means they must be iteratively calculated which makes them more computationally-intensive. Furthermore, the double constraint means that the sum of opportunity-seekers and the sum of opportunities must match, which is not necessarily true in every potential use case (e.g., there might be more people searching for work than jobs exist in a region).

In this paper we propose an alternative approach to measuring competitive accessibility. We call it a measure of **spatial availability**, and it aims to capture the number of opportunities that are not only *accessible* but also *available* to the opportunity-seeking population, in the sense that they have not been claimed by a competing seeker of the opportunity. As we will show, spatial availability is a singly-constrained measure of accessibility. By allocating opportunities in a proportional way based on demand and distance in a single step, this method avoids the issues that result from multiple counting of opportunities in conventional accessibility analysis. This method returns a measure of the rate of available opportunities per opportunity-seeking population. Moreover, the method also returns a benchmark value for the study region against which results for individual origins can be compared both inter- and intra-regionally and used in the context of opportunity provision assessment. This novel approach comes at a time when the quantity and resolution of data is exponentially increasing and the need to operationalize accessibility methods in city-planning objectives is urgent.

It should be noted that this is a novel approach which has yet to be sufficiently refined for estimating purposes, however, there is room for an additional competitive measure as prevailing measure do *not* always reflect trends seen in empirical case studies. XXX. Similarly, conventional accessibility (i.e., potential interaction), fail to reflect empirical observations in certain contexts as well. XXX. Measures are highly depending on specific contexts and environments, thus there's conceptually room for an additional measure.

This paper is split into two main parts. The first part introduces a synthetic example that is used to calculated spatial availability alongside conventional unconstrained accessibility and popular competitive accessibility measures (XXX). The aim is to demonstrate the formulation of the proposed measure and what it represents relative to other accessibility measures. In the second part, we calculate, compare, and contrast the spatial availability, conventional accessibility, and Shen's competitive accessibility (XX) values for 2016 employment data in the city of Toronto, Canada (Transportation Tomorrow Survey (TTS)). The motivation of this part is to demonstrate how accessibility and a popular competitive accessibility measure, both suffer from a lack of interpretability. Additionally, the competitive accessibility measure (XX) and spatial availability reinforce the ways in which accessibility double-counts opportunities thus

cautioning practitioners from using it for opportunities where populations' are not greedy (i.e., 1 person can only access 1 opportunity at any given time; the presence of many opportunities doesn't matter when there are many neighbours since their utilization is normalized). Finally, we conclude by cautioning the limits of conventional accessibility measure use in its *concept* and outlining the advantages of the spatial availability measure and the breadth of potential uses from the perspective of opportunity-provision planning.

In the spirit of openness of research in the spatial sciences [7, 37] this paper has a companion open data product [4], and all code will be available for replicability and reproducibility purposes.

Part 1: Access measures on an synthetic example

We begin this part by introducing a synthetic example, calculating conventional accessibility, and demonstrating the interpretation issues associated with this widely used measure. We then present the proposed *spatial availability* measure, calculate the spatial availability values for the synthetic example and discuss how the interpretation of the resulting values from the perspective of opportunity-provision. Spatial availability values are then compared and contrasted with other competitive accessibility measures [43, 30, 25, 1] and the interpretability of spatial availability is further elaborated.

Conventional accessibility

The important works of [1954] and [1959] provide a foundation for modeling accessibility. From these seminal efforts, many accessibility measures (excluding utility-based measures) have thus been derived from [1971] influential work on the gravity model which follows the formulation shown in Equation (1).

$$A_i = \sum_{j=1}^{J} O_j \cdot f(c_{ij}) \tag{1}$$

where:

- A is accessibility.
- i is a set of origin locations.
- j is a set of destination locations.
- O_j is the number of opportunities at location j; $\sum_j O_j$ is the total supply of opportunities in the study region.
- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}; it can take the form of any monotonically decreasing function chosen based on positive or normative criteria [32].

As formally defined, accessibility A_i is the weighted sum of opportunities that can be reached from location i, given the cost of travel c_{ij}. Summing the opportunities in the neighborhood of i, as determined by the impedance function
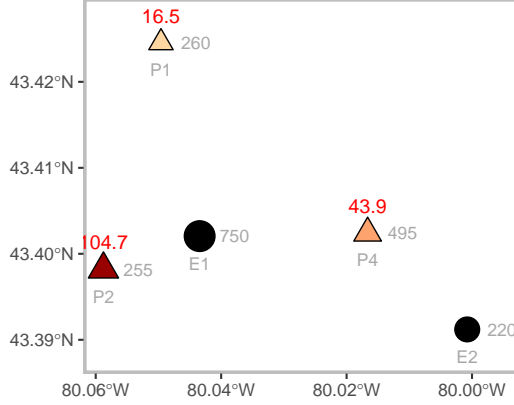
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Figure 1: Accessibility to jobs (red text) from population centers (P) to employment centers (E) for the synthetic example. Values of population and employment are shown in white text.

$f(\cdot)$, provides estimates of the number of opportunities that can be reached from i at a certain cost. The type of accessibility can be modified depending on the impedance function; for example, the measure could be cumulative opportunities [if $f(\cdot)$ is a binary or indicator function e.g., 16, 41, 17, 38] or a gravity measure using an impedance function modeled after any monotonically decreasing function [e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, *inter alia*, 27, 45, 40, 29]. In practice, the accessibility measures derived from many cumulative and gravity formulations tend to be highly correlated with one another [22, 42, 27].

The setup for our synthetic example is a system with two employment centers and three population centers. Accessibility to jobs at each population center is calculated using the accessibility measure A_i in Equation (1). In this synthetic example, we use the straight line distance between the population and jobs for c_{ij} and a negative exponential function with $\beta = 0.0015$. As noted, A_i represents the number of jobs (i.e., opportunities) that can be reached from each population center given the estimated cost as depicted in Figure 1.

The conventional gravity accessibility measure has been shown to be an excellent indicator of the intersection between urban structure and transportation infrastructure [44, 40, 27]. However, beyond enabling comparisons of relative values they are not highly interpretable on their own [31]. For instance, from Figure 1, P1 has lower accessibility than P2 but despite the accessibility value for P1 being relatively low it is still better than *zero*. On the other hand, P2 has high accessibility, but is its accessibility excellent, good, or only fair? What does it *mean* for a location to have accessibility to so many jobs?

To address this interpretability issue, previous research has aimed to index and normalize values on a per demand-population basis [e.g., 5, 35, 47]. However, as recent research on accessibility discusses [1, 34], these steps do not address the bias introduced through the uneven multiple-counting of opportunities

and/or population. The underlying issue arises as a result of the assumption that for conventional accessibility A_i, all opportunities are *available* to anyone from any origin $i = 1, \cdots, n$ who can reach them: in other words, they are assumed to be infinitely divisible and non-competitive. This results in every opportunity entering the weighted sum once for every origin i that can reach it. Put another way, if a densely populated population center pops up next to P2 this center too will have a high accessibility score since A_i does not consider competition of opportunities from neighbouring population centers. Neglecting to constrain opportunity counts (i.e., single-constraint) in addition to obscuring the interpretability of accessibility can also bias results spatially which will be discussed later on in the paper.

A review of competitive measures

Considering competition in accessibility is not new. The highly cited work of [1998] in which, in one step, he divides conventional accessibility by the travel-cost adjusted population seeking the opportunities in a given region. This work was in part popularized by the 2-step floating catchment approach (2SFCA) introduced by [2003] and used today.

A synthetic example of a three population center and two employment centre is solved using this popular competitive measure and shown in Figure XX- left. The formulation of the 2SFCA approach is shown in step 1 (Equation (2)) where the PPR R_j is calculated for each opportunity and then allocated to populations based on travel cost $f(\cdot)$ in step 2 (Equation (3)). The synthetic example is solved in detail for the 2SFCA and all other accessibility measures in the Appendix (XX).

$$R_j = \frac{O_j}{\sum_i P_i \cdot f(c_{ij})} \tag{2}$$

$$A_i = \sum_j R_j \cdot f(c_{ij}) \tag{3}$$

where:

- A is accessibility.
- i is a set of origin locations.
- j is a set of destination locations.
- O_j is the number of opportunities at location j;
- P_i is the population at location i; $\sum_j R_j$ is the total supply of opportunities in the study region.
- R_j is the provider-to-population (PPR) ratio at location j;
- c_{ij} is a measure of the cost of moving between i and j;
- $f(\cdot)$ is an impedance function of c_{ij}.

Another type of competitive measure which has been used in the literature include inverse balancing factors as used in [25] and [1]. This method involves computing the inverse balancing factor for each origin through an iterative

procedure until it converges. The iterations seek to match the travel-cost weighted opportunities to the travel-cost weighted population in the region. It requires that the total population and opportunities are equal in the region but the mean accessibilities from previous iterations can be included to standardize the disbalance. The solved synthetic example is in Figure XX - middle, and the formulation of this method is as follows in Equation (4) and (5).

$$A_i = \frac{\bar{A}^o}{\bar{A}^c} \sum_{j=1}^{J} \frac{O_j f(c_{ij})}{B_j} \tag{4}$$

$$B_i = \sum_{i=1}^{I} \frac{P_i f(c_{ij})}{A_i} \tag{5}$$

where: - B_i is the balancing factor; other variables defined in the gravity model.

Incorporating the concept of balance between the population and opportunities, a recent advancement to the 2SFCA is the balanced 2 step floating catchment approach (B2SFCA) by [34] is introduced. In both steps, the number of opportunities from the PPR to each origin, this results in a consistent number of opportunities being assigned but, ultimately, the total number of opportunities is not preserved.

In the B2SFCA, the PPR R_j for each employment center can be interpreted as the total number of jobs accessible to the total population after being *proportionally* adjusted to the travel cost. The PPR R_j is then allocated, proportionally based on travel cost, to each employment center. For this reason, the sum of all A_i adds up to the same value as the sum of all R_j. Since PPR and the subsequent A_i are proportionally allocated based travel costs, it should be noted that A_i no longer considers *potential* interaction in the how it was defined in the gravity model [20] and instead represent the allocation of PPR, based on travel time, to each population. This measure introduces some consistency in how the PPR is calculated (compared to the 2SFCA), but is still lacking interpretability in the resulting values.

The solved synthetic example is in Figure XX - right, and the formulation of this method is as follows in Equation (7) and (**??**).

$$R_j = \frac{O_j}{\sum_i P_i \frac{f(c_{ij})}{\sum_j f(c_{ij})}} \tag{6}$$

$$A_i = \sum_j R_j \frac{f(c_{ij})}{\sum_j f(c_{ij})} \tag{7}$$

Introducing spatial availability - model formulation

Here we introduce the spatial availability model formulation and demonstrate how it compares to competitive measures of accessibility.

Formally, spatial availability V_i is defined by the number of opportunities O that are proportionally allocated based on a population allocation factor F_i^p and

cost of travel allocation factor F_{ij}^c for all origins i to all destinations j as detailed in Equation (8). In line with the tradition of gravity modeling, the proposed framework distinguishes between opportunities at a destination and demand for opportunities at the origin.

$$V_i = O_j \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^{K} F_i^p \cdot F_{ij}^c} \tag{8}$$

The terms in Equation 8 are as follows:

- V_i is the spatial availability of opportunities in j to origin i.
- i is a set of origin locations in the region K.
- j is a set of destination locations in the region K.
- O_j is the number of opportunities at location j in the region K.
- F_i^p is a proportional allocation factor of the population in i.
- F_{ij}^c is a proportional allocation factor of travel cost for i; it is a product of a monotonically decreasing (i.e., impedance) function associated with the cost of travel between i and j.

Notice that, unlike A_i in Equation (1), the population in the region enters the calculation of V_i. It is important to detail the role of the two proportional allocations factors in the formulation of spatial availability. We begin by considering the population allocation factor F_i^p followed by the role of the travel cost allocation factor F_{ij}^c; then we show how both allocation factors combine in the final general form of spatial availability V_i. The calculation of spatial availability is introduced with a step-by-step example for synthetic three population centers (P_1, P_2, P_4) in the role of demand (i.e., the number of individuals in the labor market who 'demand' employment) and two employment centers (O_1, O_2) in the role of opportunities.

Population and travel cost allocation factors

We begin with allocation based on demand by population; consider an employment center j with O_j^r jobs of type r. In the general case where there are K population centers in the region, we define the following factor:

$$F_i^p = \frac{P_{i \in r}^{\alpha}}{\sum_{i=1}^{K} P_{i \in r}^{\alpha}} \tag{9}$$

The population allocation factor F_i^p corresponds to the proportion of the population in origin i relative to the population in the region. On the right hand side of the equation, the numerator $P_{i \in r}$ is the population at origin i that is eligible for and 'demands' jobs of type r (e.g., those with a certain level of training or in a designated age group). The summation in the denominator is over $i = 1, \cdots, K$, the population at origins i in the region. To modulate the effect of demand by population in this factor we include an empirical parameter α (i.e., $\alpha < 1$ places greater weight on smaller centers relative to larger ones

while $\alpha > 1$ achieves the opposite effect). This population allocation factor F_i^p can now be used to proportionally allocate a share of the jobs at j to origins.

More broadly, since the factor F_i^p is a proportion, when it is summed over $i = 1, \cdots, K$ it always equals to 1 (i.e., $\sum_i^K F_i^p = 1$). This is notable since the share of jobs at each destination j allocated to (i.e., available to) each origin, based on population, is equal to $V_i^p = O_j \cdot F_i^p$. Since the sum of F_i^p is equal to 1, it follows that $\sum_{i=1}^I V_i = O_j$. In other words, the number of jobs across the region is preserved. The result is a proportional allocation of jobs (opportunities) to origins based on the size of their populations.

For simplicity, assume that all the population in the region is eligible for these jobs, that is, that the entirety of the population is included in the set r. Also assume that $\alpha = 1$. The population allocation factors F_i^p is as follows in Equation (10).

$$
\begin{aligned}
F_1^p &= \frac{P_1^\alpha}{P_1^\alpha + P_2^\alpha + P_3^\alpha} = \frac{260}{260 + 255 + 495} = 0.257 \\
F_2^p &= \frac{P_2^\alpha}{P_1^\alpha + P_2^\alpha + P_3^\alpha} = \frac{255}{260 + 255 + 495} = 0.252 \\
F_3^p &= \frac{P_3^\alpha}{P_1^\alpha + P_2^\alpha + P_3^\alpha} = \frac{495}{260 + 255 + 495} = 0.490
\end{aligned}
\tag{10}
$$

These F_i^p values can be used to find a *partial* spatial availability in which jobs are allocated proportionally to population; this partial spatial availability V_i^p for each population center is calculated as follows in Equation (11).

$$
\begin{aligned}
V_1^p &= O_1 \cdot F_1^p + O_2 \cdot F_1^p = 750 \cdot 0.257 + 220 \cdot 0.257 = 249.29 \\
V_2^p &= O_1 \cdot F_2^p + O_2 \cdot F_2^p = 750 \cdot 0.252 + 220 \cdot 0.252 = 244.44 \\
V_3^p &= O_1 \cdot F_3^p + O_2 \cdot F_3^p = 750 \cdot 0.490 + 220 \cdot 0.490 = 475.30
\end{aligned}
\tag{11}
$$

When using only the proportional allocation factor F_i^p to calculate spatial availability (differentiated here by being defined as V_i^p instead of V_i), proportionally more jobs are allocated to the bigger population center (i.e., 2 times more jobs as it is 2 times larger in population). We can also see that the sum of spatial availability for all population centers is equal to the sum of jobs; put another way, the total opportunities are preserved.

Clearly, using only the proportional allocation factor F_i^p to calculate spatial availability does not account for how far population centers are from employment centers. It is the task of the second allocation factor F_{ij}^c to account for the friction of distance, as seen in Equation (12).

$$
F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^K f(c_{ij})}
\tag{12}
$$

Travel cost allocation factor F_{ij}^c serves to proportionally allocate more jobs to closer locations through an impedance function. c_{ij} is the cost (e.g., the distance, travel time, etc.) to reach employment center j from i and $f(\cdot)$ is an impedance function that depends on cost (c_{ij}).

To continue with the example, assume that the impedance function is a exponential function with $\beta = -0.00015$ and the distance from population

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centers to employment centers is as shown in TABLE XX. β modulates the steepness of the impedance effect and is empirically determined in the case of positive accessibility, or set by the analyst to meet a preset condition in the case of normative accessibility [32]. The proportional allocation factor F_i^p for all population centers is defined in Equation (13).

$$
\begin{aligned}
F_{1,1}^c &= \frac{\exp(\beta*2548.1)}{\exp(\beta*2548.1)+\exp(\beta*1314.1)+\exp(\beta*2170.2)} = 0.109 \\
F_{2,1}^c &= \frac{\exp(\beta*1314.1)}{\exp(\beta*2548.1)+\exp(\beta*1314.1)+\exp(\beta*2170.2)} = 0.697 \\
F_{3,1}^c &= \frac{\exp(\beta*2170.2)}{\exp(\beta*2548.1)+\exp(\beta*1314.1)+\exp(\beta*2170.2)} = 0.193 \\
F_{1,2}^c &= \frac{\exp(\beta*5419.1)}{\exp(\beta*5419.1)+\exp(\beta*2170.2)+\exp(\beta*1790.1)} = 0.004 \\
F_{2,2}^c &= \frac{\exp(\beta*4762.6)}{\exp(\beta*5419.1)+\exp(\beta*2170.2)+\exp(\beta*1790.1)} = 0.011 \\
F_{3,2}^c &= \frac{\exp(\beta*1790.1)}{\exp(\beta*5419.1)+\exp(\beta*2170.2)+\exp(\beta*1790.1)} = 0.984
\end{aligned}
\tag{13}
$$

We can see, for instance, that the proportional allocation factor for P_2 is largest for E_1 since the cost (i.e., distance) to E_1 is lowest. For E_2, P_3 has the largest proportional allocation factor similarly because it is in the closest proximity. Using the travel cost proportional allocation factors F_{ij}^c as defined in Equation (13), we can calculate the spatial availability of jobs for each population center based only on F_{ij}^c and the jobs available at each employment center, as shown in Equation (14).

$$
\begin{aligned}
V_{1,1}^c &= E_1 \cdot F_{1,1}^c = 750 \times 0.109 = 81.75 \\
V_{2,1}^c &= E_1 \cdot F_{2,1}^c = 750 \times 0.697 = 522.75 \\
V_{3,1}^c &= E_1 \cdot F_{3,1}^c = 750 \times 0.193 = 144.75 \\
V_{1,2}^c &= E_2 \cdot F_{1,2}^c = 220 \times 0.004 = 0.88 \\
V_{2,2}^c &= E_2 \cdot F_{2,2}^c = 220 \times 0.011 = 2.42 \\
V_{3,2}^c &= E_2 \cdot F_{3,2}^c = 220 \times 0.984 = 216.48
\end{aligned}
\tag{14}
$$

[1] 81.75

[1] 522.75

[1] 144.75

[1] 0.88

[1] 2.42

[1] 216.48

For instance, spatial availability defined by F_{ij}^c only (i.e., V_i^c) allocates a largest share of jobs from E_1 to P_2 since it is the closest. However, as previously discussed, P_2 has a relatively small population, so $V_{2,1}^p$ is actually the smallest value of any population center for E_1. It is necessary to combine both population and travel cost factors to better reflect demand; these two components are in line with how demand is conventionally modelled in accessibility calculations which are

re-scaled on a per demand-population basis or also consider competition [e.g., 1, 5, 50]. Fortunately, since both F_{ij}^c and F_i^p preserve the total number of opportunities as they independently sum to 1, they can be combined multiplicatively to calculate the proposed spatial availability V_i which considers demand to be based on both population and travel cost.

Putting spatial availability together

We can combine the proportional allocation factors by population F_i^p and travel cost F_{ij}^c and calculate spatial availability V_i as introduced in Equation (8) and repeated below:

$$V_i = O_j \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^K F_i^p \cdot F_{ij}^c}$$

The resulting spatial availability V_i is calculated for all population centers is calculated in Equation (15).

$$V_{1,1} = O_1 \cdot \frac{F_{1,1}^p \cdot F_{1,1}^c}{F_{1,1}^p \cdot F_{1,1}^c + F_{2,1}^p \cdot F_{2,1}^c + F_{3,1}^p \cdot F_{3,1}^c} = 750 \cdot \frac{0.26 \cdot 0.109}{0.26 \cdot 0.109 + 0.25 \cdot 0.697 + 0.49 \cdot 0.193} = 70.45$$

$$V_{2,1} = O_1 \cdot \frac{F_{2,1}^p \cdot F_{2,1}^c}{F_{1,1}^p \cdot F_{1,1}^c + F_{2,1}^p \cdot F_{2,1}^c + F_{3,1}^p \cdot F_{3,1}^c} = 750 \cdot \frac{0.25 \cdot 0.697}{0.26 \cdot 0.109 + 0.25 \cdot 0.697 + 0.49 \cdot 0.193} = 441.72$$

$$V_{3,1} = O_1 \cdot \frac{F_{3,1}^p \cdot F_{3,1}^c}{F_{1,1}^p \cdot F_{1,1}^c + F_{2,1}^p \cdot F_{2,1}^c + F_{3,1}^p \cdot F_{3,1}^c} = 750 \cdot \frac{0.49 \cdot 0.193}{0.26 \cdot 0.109 + 0.25 \cdot 0.697 + 0.49 \cdot 0.193} = 237.83$$

$$V_{1,2} = O_2 \cdot \frac{F_{1,2}^p \cdot F_{1,2}^c}{F_{1,2}^p \cdot F_{1,2}^c + F_{2,2}^p \cdot F_{2,2}^c + F_{3,2}^p \cdot F_{3,2}^c} = 220 \cdot \frac{0.26 \cdot 0.004}{0.26 \cdot 0.004 + 0.25 \cdot 0.011 + 0.49 \cdot 0.984} = 0.46$$

$$V_{2,2} = O_2 \cdot \frac{F_{2,2}^p \cdot F_{2,2}^c}{F_{1,2}^p \cdot F_{1,2}^c + F_{2,2}^p \cdot F_{2,2}^c + F_{3,2}^p \cdot F_{3,2}^c} = 220 \cdot \frac{0.25 \cdot 0.011}{0.26 \cdot 0.004 + 0.25 \cdot 0.011 + 0.49 \cdot 0.984} = 1.26$$

$$V_{3,2} = O_2 \cdot \frac{F_{1,2}^p \cdot F_{1,2}^c}{F_{1,2}^p \cdot F_{1,2}^c + F_{2,2}^p \cdot F_{2,2}^c + F_{3,2}^p \cdot F_{3,2}^c} = 220 \cdot \frac{0.49 \cdot 0.984}{0.26 \cdot 0.004 + 0.25 \cdot 0.011 + 0.49 \cdot 0.984} = 218.28$$

$$V_1 = 70.45 + 0.46 = 70.91$$
$$V_2 = 441.72 + 1.26 = 442.98$$
$$V_3 = 237.83 + 218.28 = 456.11$$

$$(15)$$

[1] 70.44885

[1] 441.7206

[1] 237.8306

[1] 0.4653881

[1] 1.254918

[1] 218.2797

[1] 70.91

[1] 442.98

```
[1] 456.11
```

Considering both population and cost allocation factors in V_i, the jobs at $E1$ that are allocated to all population centers are still preserved (i.e., $V_{1,1} + V_{2,1} + V_{3,1} = O_1$). Additionally, the sum of jobs at $E2$ are also all preserved (i.e., $V_{1,2} + V_{2,2} + V_{3,2} = O_2$). Thus the sum of V_i equals the sum of opportunities (i.e.,) Notice that V_i, allocates a number of jobs to P_1, P_2, and P_3 is between the values allocated in V_i^p and V_i^c.

When comparing V_i to the singly-constrained gravity model (see Wilson [1971]), V_i is the result of constraining A_i to match one of the marginals in the origin-destination table, the known total of opportunities. Since the sum of opportunities is preserved in the procedures above, it is possible to calculate an interpretable measure of spatial availability per capita (lower-case v_i) as shown in Equation (16).

$$v_i = \frac{V_i}{P_i} \tag{16}$$

To complete the illustrative example, the per capita spatial availability of jobs is calculated in Equation (17).

$$
\begin{aligned}
v_1 &= \frac{V_{1,1}+V_{1,2}}{P_1} = \frac{70.91}{260} = 0.272 \\
v_2 &= \frac{V_{2,1}+V_{2,2}}{P_2} = \frac{442.98}{255} = 1.737 \\
v_3 &= \frac{V_{3,1}+V_{3,2}}{P_3} = \frac{456.11}{495} = 0.921
\end{aligned}
\tag{17}
$$

```
[1] 0.2727308
```

```
[1] 1.737176
```

```
[1] 0.9214343
```

We can see that since P_2 is closest to $E1$, is similarly spaced out from $P1$ and $P2$, and is a smaller population center thus having less competition, P_2 benefits with a higher spatial availability of jobs per job-seeking population. We can also compare these values to the overall ratio of jobs-to-population in this region of two job center and three population centers is $\frac{750+220}{260+255+495} = 0.96$ jobs per person.

Part 2: Empirical example of Toronto

In this section we use population and employment data from the Golden Horseshoe Area (GGH). This is the largest metropolitan region in Canada and includes the cities of Toronto and Hamilton. We calculate gravity accessibility, XXX, and the proposed spatial availability for Toronto after introducing the data used and calibrating an impedance function.

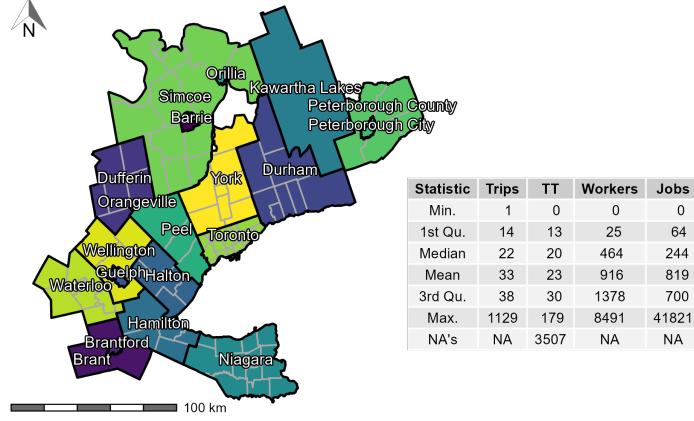| Statistic	Trips	TT	Workers	Jobs
Min.	1	0	0	0
1st Qu.	14	13	25	64
Median	22	20	464	244
Mean	33	23	916	819
3rd Qu.	38	30	1378	700
Max.	1129	179	8491	41821
NA's	NA	3507	NA	NA

Figure 2: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

Data

Population and employment data are drawn from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 2) [12]. The data set includes Traffic Analysis Zones (TAZ) (n=3,764), the number of jobs (n=3,081,885) and workers (n=3,446,957) at each origin and destination. The TTS data is based on a representative sample of between 3% to 5% of households in the GGH and is weighted to reflect the population covering the study area has a whole [12].

To generate the travel cost for these trips, travel times between origins and destinations are calculated for car travel using the R package {r5r} [39] with a street network retrieved from OpenStreetMap for the GGH area. A the 3 hr travel time threshold was selected as it captures 99% of population-employment pairs (see the travel times summarized in Figure 2). This method does not account for traffic congestion or modal split, which can be estimated through other means [e.g., 2, 23]. For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions.

All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

Calibration of an impedance function

In the synthetic example introduced in a preceding section, a negative exponential function with an arbitrary parameter was used. For the empirical example, we calibrate an impedance function on the trip length distribution
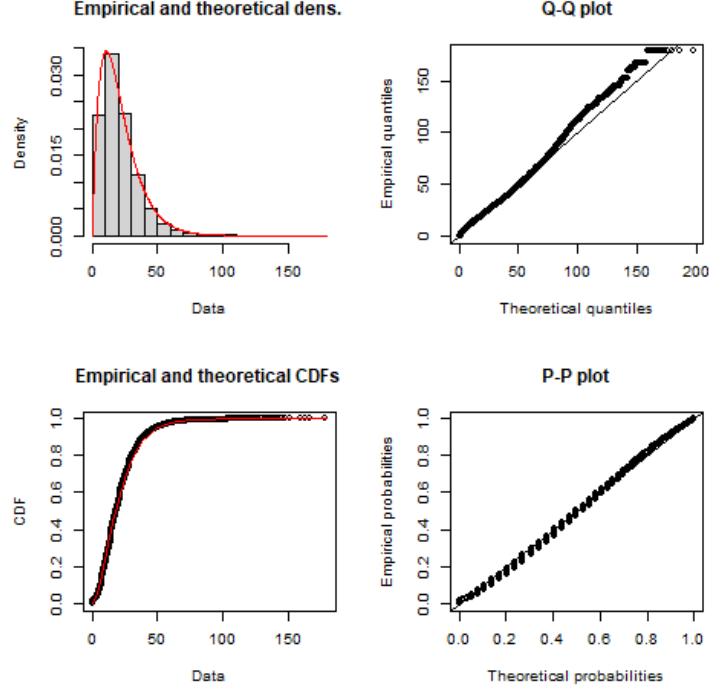
14

Figure 3: Car trip length distribution and calibrated gamma distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

(TLD) of commute trips. Briefly, a TLD represents the proportion of trips that are taken at a specific travel cost (e.g., travel time); this distribution is commonly used to derive impedance functions in accessibility research [24, 6].

The empirical and theoretical TLD for this data set are represented in the top-left panel of Figure 3. Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the {fitdistrplus} package [15] were used. Based on goodness-of-fit criteria and diagnostics seen in Figure 3, the gamma distribution was selected (also see Figure **??** in Appendix XX).

```
[1] 3069541
```

The gamma distribution takes the following general form where the estimated 'shape' is $\alpha = 2.019$, the estimated 'rate' is $\beta = 0.094$, and $\Gamma(\alpha)$ is defined in Equation (18).

$$
\begin{aligned}
f(x, \alpha, \beta) &= \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)} \quad \text{for } 0 \leq x \leq \infty \\
\Gamma(\alpha) &= \int_0^\infty x^{\alpha-1} e^{-x} \, dx
\end{aligned}
\tag{18}
$$

15

Measuring access to jobs in Toronto

Toronto is the largest city in the GGH and represents a significant subset of workers and jobs in the GGH; 31% of workers in the GGH travel to jobs in Toronto and 40% of jobs are located within Toronto.

To enhance the interpretability, spatial availability can be normalized to provide more meaningful insight into how many jobs are *available* on average for each TAZ. This normalization, shown in Figure **??**, demonstrates which TAZ have above (reds) and below (blue) the average available jobs per worker in the GGH (1.17). Similar to the spatial availability plot of the GGH jobs in Figure **??**, we can see that many average or above average jobs per worker TAZ (whites and reds) are present in southern Peel and Halton (south-west of Toronto), Waterloo and Brantford (even more south-west of Toronto), and Hamilton and Niagara (south of Toronto), however, the distribution is uneven and many TAZ within these areas do have below average values (blues).

Interestingly, when considering *competitive* job access, many areas outside of Toronto have similar jobs per worker values as TAZ in Toronto. This is contrary to the notion that since Toronto has high job access it has a significant density of employment opportunities in the GGH. Not all jobs in Toronto are *available* since Toronto has a high density of *competition* in addition to density of jobs opportunities. For instance, urban centers outside of Toronto such as those found in Brantford, Guelph, southern Peel, Halton, and Niagara have TAZ which are far above the the TTS average jobs per worker and higher than TAZ within Toronto. High job access is not seen in the accessibility plot which suggests that these less densely populated urban centers may have sufficient employment opportunities for their populations; this finding is obscured when only considering the accessibility measure for job access as will be later discussed.

It is also worth noting that there is almost two times more jobs per worker in the GGH jobs spatial availability results than the GGH Toronto spatial availability results. This suggests that all GGH people who work in the city of Toronto, on average, face more competition for jobs than all GGH people who work anywhere in the GGH .
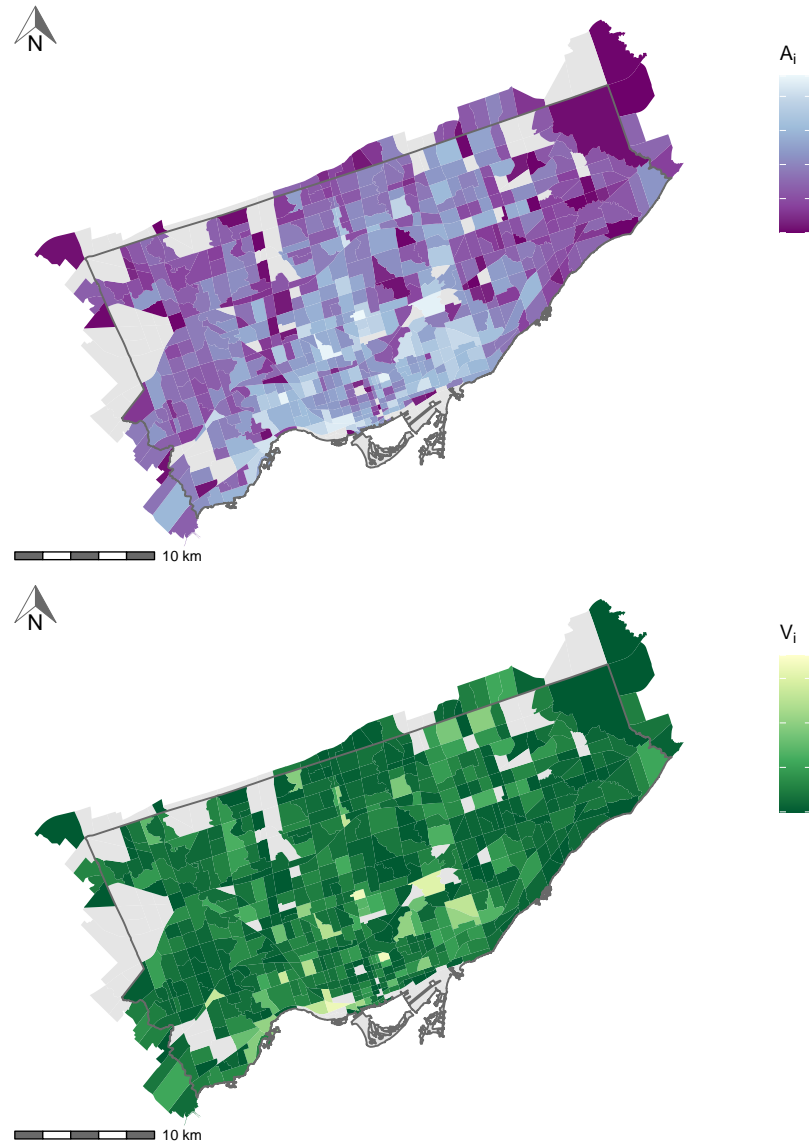
Figure 4: Calculated accessibility (top) and spatial availability (bottom) of employment from origins in destinations and origins in Toronto. Greyed out TAZ represent null accessibility and spatial availability values.
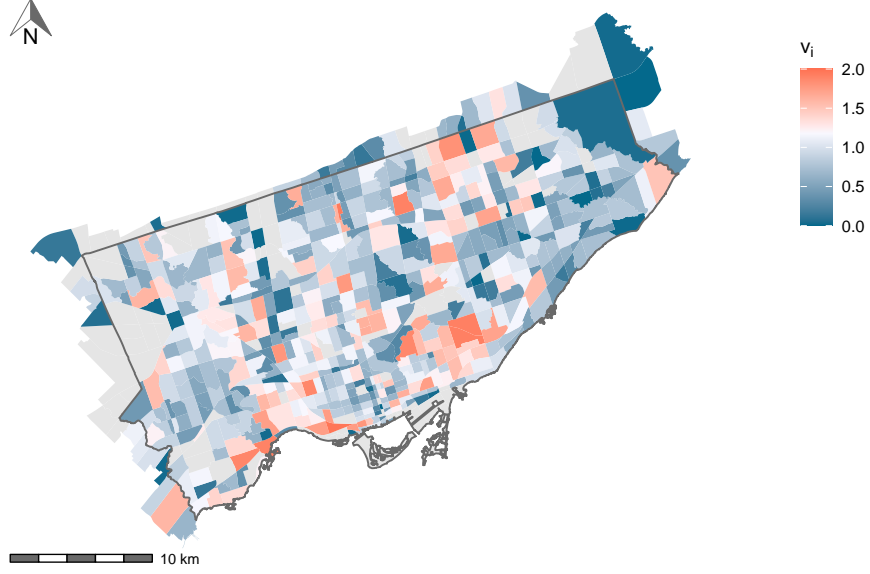
Figure 5: Spatial availability per worker, from origins to job opportunities in Toronto.

Table 1: Summary description of synthetic example

Origin	Destination	Population	Jobs	Distance
Population 1	Employment Center 1	260	750	2548.1
Population 1	Employment Center 2	260	220	5419.1
Population 2	Employment Center 1	255	750	1314.1
Population 2	Employment Center 2	255	220	4762.6
Population 4	Employment Center 1	495	750	2170.2
Population 4	Employment Center 2	495	220	1790.1
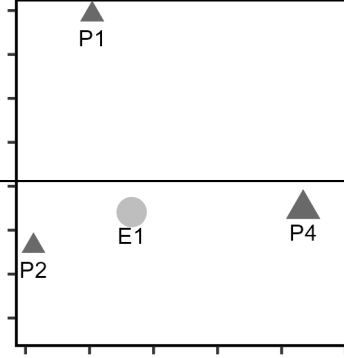
Discussion and Conclusions

Appendix A: Step-by-step accessibility calculations for synthetic example

Details for the synthetic example:

and:

$$\beta = 0.0015 \, inf(c_{ij}) = exp(\beta * distance_{ij})$$

Conventional gravity accessibiliy

$$A_i = \sum_{j=1}^{J} O_j \cdot f(c_{ij})$$

18

Solved in one step:

$$\sum_{j=1}^{J} O_j = E1 + E2 = 750 + 220 = 970 jobs$$

$A_{P1} = 750 \cdot \exp(-0.0015*2548.1) + 220 \cdot \exp(-0.0015*5419.1) = 16.5 A_{P2} = 750 \cdot \exp(-0.0015*1314.1) + 220 \cdot \exp(-$

A_{P1}, A_{P2}, and A_{P3} values represent the number of travel-cost adjusted opportunities accessible to each population. Specifically, only a proportion of opportunities are allocated to population centers based on their travel cost value (higher the travel cost lower the number of opportunities). The population is not considered in this measure and the allocation of opportunities is not constrained, it is only adjusted based on the weight of the travel cost. With our negative exponential distance decay, accessibility can be as high as 970 (the total number of opportunities in the region) and as low as essentially 0.

However, in many instances being close to opportunities doesn't necessarily mean much practically to an individual nor can this scale of 0 to the maximum number of total opportunities in the region be operationalized by decision-makers. However, correlates have been found (XX) so it is a strong indicator of urban structure, but practically what does it mean for an individual to live in a population center of $A_{P4} = 43.9$ jobs? On a scale of 0 to 970 ($f(c_{ij}) = 0$ to $f(c_{ij}) = 1$), this value is low but of the three population centers it is around average. However, A_{P4} also has the largest population of all population centers. It has a population that is less than two times the population center of A_{P1} but an accessibility value that is greater than two times A_{P1}'s accessibility value. Does this mean that accessibility, after adjusting for population, is greater than in $P4$? It is hard to say since the populations have different travel costs to the opportunities. From this perspective, competitive measures such as the FCA were introduced with the most recently popularized 2SFCA (XX) discussed as follows.

2 step floating catchment approach (2SFCA)

Step one:

$$\begin{aligned}
R_j &= \frac{O_j}{\sum_i P_i \cdot f(c_{ij})} \\
R_{E1} &= \frac{750}{260 \cdot \exp(-0.0015*2548.1) + 255 \cdot \exp(-0.0015*1314.1) + 495 \cdot \exp(-0.0015*2170.2)} \\
R_{E1} &= 12.4 jobs per travel cost adjust. pop \\
R_{E2} &= \frac{220}{260 \cdot \exp(-0.0015*5419.1) + 255 \cdot \exp(-0.0015*4762.6) + 495 \cdot \exp(-0.0015*1790.1)} \\
R_{E2} &= 6.5 jobs per travel cost adjust. pop
\end{aligned} \qquad (19)$$

Step two:

$A_i = \sum_j R_j \cdot f(c_{ij})$
$A_{P1} = 12.4 \cdot \exp(-0.0015 * 2548.1) + 6.46 \cdot \exp(-0.0015 * 5419.1) = 0.27$
$A_{P2} = 12.4 \cdot \exp(-0.0015 * 1314.1) + 6.46 \cdot \exp(-0.0015 * 4762.6) = 1.73$
$A_{P4} = 12.4 \cdot \exp(-0.0015 * 2170.2) + 6.46 \cdot \exp(-0.0015 * 1790.1) = 0.92$

$$(20)$$

We see that the PPR R_j for each employment center can be interpreted as the total number of jobs accessible to the total travel-cost adjusted population. This step recognizes that not all opportunities can be distributed to the entire population evenly since not *all* opportunities can be reached by *all* population centers. It is assumed that all population and employment centers are in the same catchment. In step two, A_i values represent the travel-cost adjusted PPR for each population center. Put another way, here A_{P1}, A_{P2}, and A_{P3} values represent the number of jobs accessible to each population center after being travel-cost adjusted from both the opportunities-perspective and population-perspective. The value could theoretically be on a scale of 0 to the maximum total number of PPR in the catchment (i.e., $f(c_{ij}) = 0$ to $f(c_{ij}) = 1$); in this case that value is 18.9.

The method assumes unconstrained allocation based on travel cost thus results can be interpreted in a similar way as the conventional gravity based accessibility but from the perspective of PPR instead of opportunities. This means that PPR is allocated based on the *potential* for interaction.

Looking at $P4$, the largest and most central (to both employment centers) population center, it has a 3.4 times greater 2SFCA value compared to $P1$ (conventional accessibility is only 2.6 times greater). $P4$ also has an 2SFCA value which is closer in the value to $P2$, it is only 0.53 times smaller than $P2$ (conventional accessibility is 0.42 times smaller). Since the 2SFCA adjusts accessibility from both population-side and opportunity-side travel-costs, small populations which are close to big employment centers ($P2$ close to $E1$) but also are in competition with relatively close and large population centers ($P4$) have less relative accessibility ($P2$ vs. $P4$) than conventional accessibility. However, as discussed by Paez et al. [34], the PPR calculation in the first step and allocation of PPR to origins in the second step is not *proportional* to the total population seeking opportunities. Though the 'potential' for interaction is being consistently allocated in these two steps, when looking to interpret the measure from the perspective of allocation, the resulting values are difficult to interpret. This issue of interpretability has been attempted to be remedied by adjusting the population and opportunities in both steps by a *proportional* travel cost in the B2SFCA as follows.

balanced 2 step floating catchment approach (B2SFCA)

Step one:

$$R_j = \frac{O_j}{\sum_i P_i \frac{f(c_{ij})}{\sum_j f(c_{ij})}}$$

$$R_{E1} = \frac{750}{260\frac{\exp(\beta*2548.1)}{\exp(\beta*2548.1)+\exp(\beta*5419.1)}+255\frac{\exp(\beta*1314.1)}{\exp(\beta*1314.1)+\exp(\beta*4762.6)}+495\frac{\exp(\beta*2170.2)}{\exp(\beta*2170.2)+\exp(\beta*1790.1)}}$$

$$R_{E2} = \frac{220}{260\frac{\exp(\beta*5419.1)}{\exp(\beta*2548.1)+\exp(\beta*5419.1)}+255\frac{\exp(\beta*4762.6)}{\exp(\beta*1314.1)+\exp(\beta*4762.6)}+495\frac{\exp(\beta*1790.1)}{\exp(\beta*2170.2)+\exp(\beta*1790.1)}}$$

$$R_{E1} = 1.09 jobsperproportionaltravelcostadjust.pop$$

$$R_{E2} = 0.68 jobsperproportionaltravelcostadjust.pop$$

(21)

Step two:

$$A_i = \sum_j R_j \frac{f(c_{ij})}{\sum_j f(c_{ij})}$$

$$A_{P1} = 1.09\frac{\exp(\beta*2548.1)}{\exp(\beta*2548.1)+\exp(\beta*1314.1)+\exp(\beta*2170.2)} + 0.68\frac{\exp(\beta*5419.1)}{\exp(\beta*5419.1)+\exp(\beta*2170.2)+\exp(\beta*1790.1)}$$

$$A_{P2} = 1.09\frac{\exp(\beta*1314.1)}{\exp(\beta*2548.1)+\exp(\beta*1314.1)+\exp(\beta*2170.2)} + 0.68\frac{\exp(\beta*4762.6)}{\exp(\beta*5419.1)+\exp(\beta*2170.2)+\exp(\beta*1790.1)}$$

$$A_{P4} = 1.09\frac{\exp(\beta*2170.2)}{\exp(\beta*2548.1)+\exp(\beta*1314.1)+\exp(\beta*2170.2)} + 0.68\frac{\exp(\beta*1790.1)}{\exp(\beta*5419.1)+\exp(\beta*2170.2)+\exp(\beta*1790.1)}$$

$$A_{P1} = 0.12$$

$$A_{P2} = 0.77$$

$$A_{P4} = 0.88$$

(22)

In the B2SFCA, the PPR R_j for each employment center can be interpreted as the total number of jobs accessible to the total population after being *proportionally* adjusted to the travel cost. The PPR R_j is then allocated, proportionally based on travel cost, to each employment center. For this reason, the sum of all A_i adds up to 1.77, the same value as the sum of all R_j.

It should be also noted that when using the *proportional* travel costs to adjust opportunities and population, instead of using the raw travel costs as in the 2SFCA, the resulting A_i values are different. For instance, A_{P2} is the highest value (1.88 times her than P4) in the 2SFCA but in the B2SFCA A_{P4} has the highest value. In both 2SFCA and conventional accessibility, A_{P2} is the highest value since P2 is the most populous population center and has the closest proximity (low travel cost) to the most populous E1. Though population size is considered in the first step of 2SFCA, the low population size of P2 is not significant enough to result in a large enough decrease in A_{P2} relative to A_{P4} (compared to conventional accessibility in which the acccesibility values between P2 and P4 are relatively greater). However, in B2SFCA, the smaller population size of P2 has a dramatic impact on the A_{P2} value. This is because the travel-cost adjustment is proportional so the proportionally large population size of P4 results in a smaller PPR for both employment centers and thus a smaller amount of PPR from E1 is allocated to P2 in step 2. Since PPR and the subsequent A_i are proportionally allocated based travel costs, values no longer consider *potential* interaction and instead represent the allocation of employment center PPR, based on travel time, to each population center.

This measure introduces some consistency in how the PPR is calculated and

is allocated to P1, P2, and P4, but is still lacking interpretability in the resulting values.

Inverse Balancing accessibility, doubly-constrained

I don't get how to calculate this!!!

This measure results in a opportunities per person metric, however, it constraints opportunities and population from both sides, estimating accessibility iteratively. The formulation requires the number of opportunities equals the population so they propose the iterative estimates are standardized such that opportunities equals population and the disbalanced is caried through as a factor. The formulation for the synthetic example would take the following form:

$$
\begin{aligned}
A_i &= \frac{\bar{A}^o}{\bar{A}^c} \sum_{j=1}^{J} \frac{O_j f(c_{ij})}{L_j} \\
L_i &= \sum_{i=1}^{I} \frac{P_i f(c_{ij})}{A_i}
\end{aligned}
\tag{23}
$$

Iteration 1:

$$
\begin{aligned}
A_{P1} &= (1) * \frac{750 \exp(\beta * 2548.1) + 220 * \exp(\beta * 5419.1)}{1} = 16.47 \\
A_{P2} &= (1) * \frac{750 \exp(\beta * 1314.1) + 220 * \exp(\beta * 4762.6)}{1} = 104.65 \\
A_{P4} &= (1) * \frac{750 \exp(\beta * 2170.2) + 220 * \exp(\beta * 1790.1)}{1} = 43.93 \\
L_{E1} &= \frac{260 \exp(\beta * 2548.1)}{16.47} + \frac{255 * \exp(\beta * 1314.1)}{104.65} + \frac{495 * \exp(\beta * 2170.2)}{43.93} = 1.12 \\
L_{E2} &= \frac{260 \exp(\beta * 5419.1)}{16.47} + \frac{255 * \exp(\beta * 4762.6)}{104.65} + \frac{495 * \exp(\beta * 1790.1)}{43.93} = 0.78
\end{aligned}
\tag{24}
$$

We can complete ~6 more iterations until we reach close to convergence. I skip writing them out but the final A_i values appear like:

$$
\begin{aligned}
A_{P1} &= 14.6 \\
A_{P2} &= 92.4 \\
A_{P4} &= 47.0
\end{aligned}
\tag{25}
$$

Here we see that A_{P1}, A_{P2}, and A_{P3} values represent the number of opportunities, after being adjusted based on opportunity-side and population-side travel costs, which are accessible to each population. Compared to conventional accessibility, the population is enche population is not considered in this measure and the allocation of opportunities is not constrained, it is only adjusted based on the weight of the travel cost. With our negative exponential distance decay, accessibility can be as high as 970 (the total number of opportunities in the region) and as low as essentially 0.

However, in many instances being close to opportunities doesn't necessarily mean much practically to an individual nor can this scale of 0 to the maximum number of total opportunities in the region be operationalized by decision-makers. However, correlates have been found (XX) so it is a strong indicator of urban structure, but practically what does it mean for an individual to live in a population center of $A_{P4} = 43.9$ jobs? On a scale of 0 to 970 ($f(c_{ij}) = 0$ to $f(c_{ij}) = 1$), this value is low but of the three population centers it is around average. However, A_{P4} also has the largest population of all population centers.

It has a population that is less than two times the population center of A_{P1} but an accessibility value that is greater than two times A_{P1}'s accessibility value. Does this mean that accessibility, after adjusting for population, is greater than in $P4$? It is hard to say since the populations have different travel costs to the opportunities. From this perspective, the FCA were introduced with the most recently popularized 2SFCA (XX) discussed as follows.

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