Introducing spatial availability, a singly-constrained measure of competitive accessibility

Abstract

Accessibility measures are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of the opportunities that can be reached given the cost of movement and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve interpretability, mainly by introducing congestion and competition. In this paper we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area methods can be seen as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

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1. Introduction

The concept of accessibility in transportation studies derives its appeal from the combination of the spatial distribution of opportunities and the cost of reaching them [21, 19]. Accessibility analysis is employed in transportation, geography, public health, and many other areas, with the number of applications growing [50], especially as mobility-based planning is de-emphasized in favor of access-oriented planning [15, 20, 41, 56].

Numerous methods for calculating accessibility have been proposed in the literature [18]. Of these, gravity-type accessibility is arguably the most common; since its introduction in the literature by Hansen [21], it has been widely adopted in numerous forms [10, 38, 18, 31, 3]. Gravity-based accessibility indicators are essentially weighted sums of opportunities, with the weights given by an impedance function that depends on the cost of movement, and thus measure the *intensity of the possibility of interaction* [21]. This type of accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure [19].

Despite their utility, the interpretability of gravity-type accessibility measures can be challenging [18, 35]. Since they aggregate opportunities, the results are sensitive to the size of the region of interest (e.g., a large city has more jobs than a smaller city). As a consequence, raw outputs are not necessarily comparable across study areas [1]. This limitation becomes evident when surveying studies that implement this type of analysis. For example, Páez et al. [43] (in Montreal) and Campbell et al. [9] (in Nairobi) report accessibility as the number of health care facilities that can potentially be reached from origins. But what does it mean for a zone to have accessibility to less than 100 facilities in each of these two cities, with their different populations and number of facilities? For that matter, what does it mean for a zone to have accessibility to more than 700 facilities in Montreal, besides being "accessibility rich"? As another example, Bocarejo S. and Oviedo H. [7] (in Bogota), El-Geneidy et al. [17] (in Montreal), and Jiang and Levinson [27] (in Beijing) report accessibility as the number of jobs by origin, with accessibility values often in the hundreds of thousands, and even exceeding one million jobs for some zones in Beijing and Montreal. As indicators of urban structure, these measures are informative, but the meaning of one million accessible jobs is harder to pin down: how many jobs must any single person have access to? Clearly, the answer to this question depends on how many people demand jobs.

The interpretability of gravity-type accessibility has been discussed in numerous studies, including recently by Hu and Downs [26], Kelobonye et al. [29], and in greater depth by Merlin and Hu [34]. As hinted above, the limitations in interpretability are caused by ignoring competition - without competition, each opportunity is assumed to be equally available to every single opportunity-seeking individual that can reach it [49, 39, 29]. This assumption is appropriate when the opportunity of interest is non-exclusive, that is, if use by one unit of population does not preclude use by another. For instance, national parks with abundant space are seldom used to full capacity, so the presence of some population does not exclude use by others. When it comes to exclusive opportunities, or when operations may be affected by congestion, the solution has been to account for competition. Several efforts exist that do so. In our reckoning, the first such approach was proposed by Weibull [53], whereby the distance decay of the supply of employment and the demand for employment (by workers) were formulated under so-called axiomatic assumptions. This approach was then applied by Joseph and Bantock [28] in the context of healthcare, to quantify the availability of general practitioners in Canada. Later, Shen [49] independently re-discovered Weibull [53] formula [see footnote (7) in 49] and deconstructed it to consider accessibility for different modes. These advances were subsequently popularized as the family of two-step floating catchment area (2SFCA) methods [33] that have found widespread adoption in healthcare, education, and food systems [57, 13, 58, 12, 11].

An important development due to Shen was to show that the population-weighted sum of the accessibility measure with competition equates the number of opportunities available [footnote (7) and Appendix A in 49]. This demonstration gave the impression of a method that allocates all opportunities *exactly*. In this paper we intend to revisit accessibility with competition. We argue that Shen's proof confuses the opportunity-seeking population with total zonal population, an equivocation that results in misleading allocations of opportunities to population that are masked by the presentation of results as rates (i.e., opportunities per capita). We then propose an alternative formulation of accessibility that incorporates competition by adopting a proportional allocation mechanism. The use of balancing factors for proportional allocation is akin to imposing a single constraint on the accessibility indicator, in the spirit of Wilson's [1971] spatial interaction model.

In this way, the aim of the paper is three-fold:

- First, we aim to demonstrate that Shen-type (and thus Weibull [53] accessibility and the popular 2SFCA methods) produce misleading estimates of the opportunities allocated;

- Second, we introduce a new measure, *spatial availability*, which we submit is a more interpretable alternative to Shen-style accessibility, since opportunities in the system are preserved and proportionally allocated to each origin; and

- Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.

Discussion is supported by the use of a small synthetic example drawn from Shen [49] and empirical data drawn from the 2016 Transportation Tomorrow Survey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the spirit of openness of research in the spatial sciences [8, 42] this paper has a companion open data product [4], and all code will be available for replicability and reproducibility purposes.

2. Accesssibility measures revisited

2.1. Hansen-type accessibility

Accessibility analysis stems from the foundational works of Harris [22] and Hansen [21]. From these seminal efforts, many accessibility measures (excluding utility-based measures) have been derived, particularly after the influential work of Wilson [55] on spatial interaction. The model follows the formulation shown in Equation (1):

$$S_i = \sum_{j=1}^{J} O_j \cdot f(c_{ij}) \tag{1}$$

where:

- S is Hansen-type accessibility.
- i is a set of origin locations.
- j is a set of destination locations.
- O_j is the number of opportunities at location j; $\sum_j O_j$ is the total supply of opportunities in the study region.
- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}; it can take the form of any monotonically decreasing function chosen based on positive or normative criteria [37].

As formally defined, accessibility S_i is the sum of opportunities that can be reached from location i, weighted down by an impedance function of the cost of travel c_{ij}. Summing the opportunities in the neighborhood of i provides estimates of the number of opportunities that can *potentially* be reached from i. Several variants of this method result from using a variety of impedance functions; for example, cumulative opportunities measures are obtained when $f(\cdot)$ is a binary or indicator function [e.g., 17, 47, 18, 44]. Other measures use impedance functions modeled after any monotonically decreasing function [e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, *inter alia*, 30, 51, 46, 32]. In practice, accessibility measures with different impedance functions tend to be highly correlated [23, 48, 30].

Gravity-based accessibility has been shown to be an excellent indicator of the intersection between urban structure and transportation infrastructure [50, 46, 30]. However, beyond enabling comparisons of relative values they are not highly interpretable on their own [35]. To address the issue or interpretability, previous research has aimed to index and normalize values on a per demand-population basis [e.g., 5, 40, 52]. However, as recent research on accessibility discusses [34, 1, 39, 29], these steps do not truly adequately consider competition. In effect, when calculating S_i, every opportunity enters the weighted sum once for every origin i that can reach it. This makes interpretability opaque, and to complicate matters, can also bias the estimated landscape of opportunity.

2.2. Shen-type competitive accessibility

To account for competition, the influential works of Shen [49] and Weibull [53], as well as the widely used 2SFCA approach of Luo and Wang [33], adjust Hansen-type accessibility with the population in the region of interest. The mechanics of this approach consist of calculating, for every destination j, the population that can reach it given the impedance function $f(\cdot)$, i.e., R_j in Equation (2). Shen [49] calls this "the opportunity-seeking population". The opportunities at j are then divided by their corresponding opportunity-seeking population to obtain a measure of opportunities per capita. These values are then allocated back to the population at i, again subject to the friction represented by the impedance function. Equation (2) corresponds to the first step of this procedure, and Equation (3) to the second.

$$R_j = \frac{O_j}{\sum_i P_i \cdot f(c_{ij})} \tag{2}$$

$$A_i = \sum_j R_j \cdot f(c_{ij}) \tag{3}$$

where:

- A is Shen-type accessibility.
- i is a set of origin locations.
- j is a set of destination locations.
- O_j is the number of opportunities at location j;
- P_i is the population at location i; $\sum_j R_j$ is the total supply of opportunities in the study region.
- R_j is the provider-to-population (PPR) ratio at location j;
- c_{ij} is a measure of the cost of moving between i and j;
- $f(\cdot)$ is an impedance function of c_{ij}.

As noted above, Shen [49] furnished a demonstration that the resulting values A_i, when multiplied by the population at that origin and summed for the full study region, equates the total number of opportunities in the full study region. But is this really the case?

2.3. Example

To motivate the discussion, we begin with the example in Shen [49] (Figure 1).

Table 1 contains the information needed to calculate S_i and A_i in the example. In the table we see that population centers A and B have equal Hansen-type accessibility ($S_A = S_B =$ 27,292 jobs), despite A having a much smaller population. On the other hand, the isolated satellite town of C has low accessibility ($S_C =$ 2,240 jobs). This value is not very sensible: given its isolated position in the system and balanced jobs-population ratio, we would expect it to have better accessibility than B, with its large population and distance to

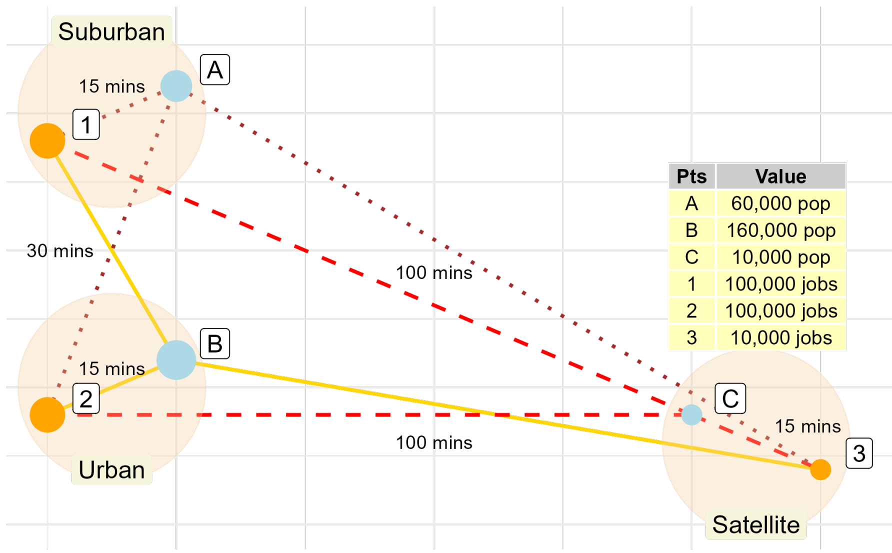


Figure 1: Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

jobs. It is difficult from these outputs to determine whether the accessibility at B is better or worse than that at A or C.

The results are much easier to interpret when we consider Shen-type accessibility. The results indicate that $A_A \approx 1.337$ jobs per capita, $A_B \approx 0.888$, and $A_C \approx 0.996$. The latter value is sensible given the job-population balance of C. Center A is relatively close to a large number of jobs (more jobs than the population of A). The opposite is true of B. According to Shen [49], the sum of the population-weighted accessibility A_i is exactly equal to the number of jobs in the region:

$$\begin{aligned}
&50,000 \times 1.3366693 \\
&+150,000 \times 0.8880224 \\
&+10,000 \times 0.9963171 = 210,000
\end{aligned}$$

As mentioned in the Introduction, this property gives the impression that jobs are allocated in their totality. However, for this property to work, the accessibility needs to be multiplied by the total population of the corresponding center. Alas, there is a logical inconsistency in this calculation, since the travel behavior, encoded in the form of the impedance function, means that the opportunity-seeking population is *not* equal to the total population. In other words, the total population is confounded with the *opportunity-seeking population*. As seen in column **Pop * f(TT)** in Table 1, the number of individuals from Center A that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2.27 respectively. Therefore, the total number of opportunity-seeking individuals is 13,647.27, which is considerably lower than the total population of A.

To ensure that the calculations are consistent with the travel behavior given

Table 1: Summary description of synthetic example: Hansen-type accessibility and Shen-type accessibility with competition

Origin	Pop.	Dest.	Jobs	TT	f(TT)	Pop * f(TT)	Jobs * f(TT)	S_i	A_i
		1	100,000	15	0.223130	11,157	22,313		
A	50,000	2	100,000	30	0.049787	2,489	4,979	27,292	1.34
		3	10,000	100	0.000045	2.27	0.454		
		1	100,000	30	0.049787	7,468	4,979		
B	150,000	2	100,000	15	0.223130	33,470	22,313	27,292	0.888
		3	10,000	100	0.000045	6.81	0.454		
		1	100,000	100	0.000045	0.454	4.54		
C	10,000	2	100,000	100	0.000045	0.454	4.54	2,240	0.996
		3	10,000	15	0.223130	2,231	2,231		

by the impedance function, the number of accessible jobs per capita should be multiplied by the population who are willing to travel to the employment center; hence, instead of the nominal number of jobs in the region, the number of jobs the method actually allocates is:

$$
\begin{aligned}
&(11,156.51 + 2,489.35 + 2.26) \times 1.3366693 \\
&+(7,468.06 + 33,469.52 + 6.81) \times 0.8880224 \\
&+(4.54 + 4.54 + 2,231.20) \times 0.9963171 \approx 56,834.59
\end{aligned}
$$

which is just a proportion of the total number of jobs in the region. Use of the total population of zones when calculating the population-weighted sum of A_i gives the impression that all jobs are allocated - however the result is inconsistent with the travel behavior embedded in the model. When the correct opportunity-seeking population is used (that is, the population weighted by the impedance), it becomes apparent that the number of jobs allocated is not equal to the total number of jobs in the region. This feature of the method is masked because the results are given in terms of opportunities per capita. In effect the method says that the jobs per capita at A are approximately 1.34 for each of 13,647.27 opportunity-seeking individuals, but zero for 36,352.73 individuals that do not reach any employment center.

These results are somewhat suspicious, because they indicate seldom-seen levels of unemployment and job vacancies, despite the seemingly-reasonable values of A_i. In the next section we propose an alternative derivation of competitive accessibility that resolves the apparent inconsistency described in this section.

3. Introducing spatial availability: a singly-constrained accessibility measure

In brief, we define the *spatial availability* at i (V_i) as the proportion of all opportunities O that are allocated to i from all destinations j. The fundamental difference with Hansen- and Shen-type accessibility is that opportunities are allocated proportionally. We begin by explaining the intuition behind the method before defining it more formally.

3.1. Proportional allocation by population

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin describing the proposed proportional allocation mechanism based on demand by population. The total population in the example is 210,000. The proportion of the population by population center is:

$$F^p_A = \frac{50{,}000}{210{,}000}$$
$$F^p_B = \frac{150{,}000}{210{,}000}$$
$$F^p_C = \frac{10{,}000}{210{,}000}$$

Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors F^p_i). In this way, employment center 1 allocates $100,000 \cdot \frac{50{,}000}{210{,}000} = 23,809.52$ jobs to A; $100,000 \cdot \frac{150{,}000}{210{,}000} = 71,428.57$ jobs to B; and $100,000 \cdot \frac{10{,}000}{210{,}000} = 7,142.857$ jobs to C. Notice how this mechanism ensures that the total number of jobs at employment center 1 is preserved at 100,000.

We can verify that the number of jobs allocated is consistent with the total number of jobs in the region:

Employment center 1:
$$100,000 \cdot \frac{50{,}000}{210{,}000} + 100,000 \cdot \frac{150{,}000}{210{,}000} + 100,000 \cdot \frac{10{,}000}{210{,}000} = 100,000$$
Employment center 2:
$$100,000 \cdot \frac{50{,}000}{210{,}000} + 100,000 \cdot \frac{150{,}000}{210{,}000} + 100,000 \cdot \frac{10{,}000}{210{,}000} = 100,000$$
Employment center 3:
$$10,000 \cdot \frac{50{,}000}{210{,}000} + 10,000 \cdot \frac{150{,}000}{210{,}000} + 10,000 \cdot \frac{10{,}000}{210{,}000} = 10,000$$

In the general case where there are N population centers and J employment centers in the region, we define the following population-based balancing factors:

$$F^p_i = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \tag{4}$$

Balancing factor F^p_i corresponds to the proportion of the population in origin i relative to the population in the region. On the right hand side of the equation, the numerator P_i^α is the population at origin i. The summation in the denominator is over $i = 1, \cdots, N$, and adds up to the total population of the region. Notice that we incorporate an empirical parameter α. The role of α is to modulate the effect of demand by population. When $\alpha < 1$, opportunities are allocated more rapidly to smaller centers relative to larger ones; in contrast, $\alpha > 1$ achieves the opposite effect.

Balancing factor F_i^p can now be used to proportionally allocate a share of available jobs at j to origin i. The number of jobs available to i from j *according to population shares* is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

The total number of jobs available from all destinations is simply the sum of V_{ij}^p over $j = 1, \cdots, J$:

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^K F_i^p}$$

Since factor F_i^p summed over $i = 1, \cdots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F_i^p = 1$), the sum of all spatially available jobs equals O, the total number of opportunities in the region:

$$\begin{aligned}
&\textstyle\sum_{i=1}^N V_i^p = \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\
&= \textstyle\sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\
&= \textstyle\sum_{j=1}^J O_j = O
\end{aligned}$$

Factors F_i^p act here as the balancing factors of the gravity model when a single constraint is imposed [i.e., to ensure the sums of columns are equal to the number of opportunities per destination, see 36, pp. 179-180 and 183-184]. As a result, the sum of spatial availability for all population centers equals the total number of opportunities.

The above concerns the mass (i.e., population size) of the gravity model. In addition, the potential for interaction is thought to decrease with increasing cost, so next we define similar balancing factors but based on the impedance.

3.2. Proportional allocation by cost

Clearly, using only balancing factors F_i^p to calculate spatial availability V_i^p does not account for the cost of reaching employment centers. Consider instead a set of balancing factors F_{ij}^c that account for the friction of distance:

$$\begin{aligned}
F_{A1}^c &= \tfrac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\
F_{B1}^c &= \tfrac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\
F_{C1}^c &= \tfrac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581
\end{aligned}$$

Balancing factors F_{ij}^c use the impedance function to proportionally allocate more jobs to closer population centers, that is, those with populations *more* *willing to reach the jobs*. Indeed, the factors F^c_{ij} can be thought of as the proportion of the population at i willing to travel to destination j conditional on the travel behavior embodied by the impedance function.

In our example, the number of jobs allocated from employment center 1 to population center A is $100,000 \times 0.8174398 = 81743.98$; to population center B is $100,000 \times 0.1823954 = 18,239.54$; and to population center C is $100,000 \times 0.0001648581 = 16.48581$. We can see once more that the total number of jobs at the employment center is preserved at 100,000. In this example, the proportional allocation mechanism assigns the largest share of jobs to population center A, which is the closest to employment center 1, and the smallest to the more distant population center C.

In the general case when there are N population centers and J employment centers in the region, we define the following cost-based balancing factors:

$$F^c_{ij} = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \tag{5}$$

The total number of jobs available to i from j according to cost is defined as follows:

$$V^c_{ij} = O_j \frac{F^c_i}{\sum_{i=1}^N F^c_i}$$

The total number of jobs available to i from all destinations is:

$$V^c_i = \sum_{j=1}^J O_j \frac{F^c_i}{\sum_{i=1}^N F^c_i}$$

Like the population-based allocation factors, F^c_i summed over $i = 1, \cdots, N$ always equals to 1 (i.e., $\sum_{i=1}^N F^c_i = 1$). As before, the sum of all spatially available jobs equals O, the total number of opportunities in the region:

$$\begin{aligned}
&\textstyle\sum_{i=1}^N V^c_i = \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F^c_i}{\sum_{i=1}^N F^c_i} \\
&\textstyle= \sum_{i=1}^N \frac{F^c_i}{\sum_{i=1}^N F^c_i} \cdot \sum_{j=1}^J O_j \\
&\textstyle= \sum_{j=1}^J O_j = O
\end{aligned}$$

We are now ready to more formally define spatial availability with due consideration to both mass and cost effects.

3.3. Putting spatial availability together

Population and the cost of travel are both part of the gravity modelling framework. Since the balancing factors defined in the preceding sections are proportions (alternatively probabilities), they can be combined multiplicatively to obtain their joint effect (alternatively, the joint probability of allocating opportunities): opportunities at a destination and demand for opportunities at the origin.

$$F_i^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \tag{6}$$

This idea is represented in Equation (7), where F_i^p is a population-based allocation factor that grants a larger share of the existing opportunities to larger centers, and F_{ij}^c is a transportation cost-based allocation factor that grants a larger share of the existing opportunities to closer centers. This is in line with the tradition of gravity modeling, and proposed framework distinguishes between opportunities at a destination and demand for opportunities at the origin.

$$V_i = \sum_{j=1}^J O_j \frac{F_i^t}{\sum_{i=1}^N F_i^t} \tag{7}$$

The terms in Equation 7 are as follows:

- V_i is the spatial availability at i.
- i is a set of origin locations in the region $i = 1, \cdots, N$.
- j is a set of destination locations in the region $j = 1, \cdots, J$.
- O_j is the number of opportunities at location j.
- F_i^t is a balancing factor as defined in Equation (6).

Notice that, unlike S_i in Equation (1), the population enters the calculation of V_i. Returning to the example in Figure 1, Table 2 contains the information needed to calculate V_i.

Table 2: Summary description of synthetic example: spatial availability

Origin	Pop.	Dest.	Jobs	TT	f(TT)	F^p	F^c	F	V__ij	V__i
		1	100,000	15	0.223130	0.238095	0.817438	0.599006	59,901	
A	50,000	2	100,000	30	0.049787	0.238095	0.182395	0.069227	6,923	66,833
		3	10,000	100	0.000045	0.238095	0.000203	0.001013	10	
		1	100,000	30	0.049787	0.714286	0.182395	0.400969	40,097	
B	150,000	2	100,000	15	0.223130	0.714286	0.817438	0.930760	93,076	133,203
		3	10,000	100	0.000045	0.714286	0.000203	0.003040	30	
		1	100,000	100	0.000045	0.047619	0.000166	0.000024	2.4	
C	10,000	2	100,000	100	0.000045	0.047619	0.000166	0.000013	1.3	9,963
		3	10,000	15	0.223130	0.047619	0.999593	0.995947	9,959	

In the table, column **V__ij** are the jobs available to each origin from each employment center. In this column V_{A1} = 59,901 is the number of jobs available at A from employment center 1. Column V_i (i.e., $\sum_{j=1}^{J} V_i j$) is the total number of jobs available to origin i. We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^{N} V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

Compare the calculated values of V_i to column **S__i** (Hansen-type accessibility) in Table 1. The spatial availability values are more intuitive. Recall that population centers A and B had equal accessibility to employment opportunities. According to V_i, population center A has greater job availability due to its close proximity to employment center 1 combined with less steep competition (i.e., a majority of the population have to travel longer distances to reach employment center 1). Job availability is lower for population center B due to much higher competition (150,000 people can reach 100,000 jobs at equal cost). And center C has almost as many jobs available as its population.

As discussed above, Hansen-type accessibility is not designed to preserve the number of jobs in the region. Shen-type accessibility is internally inconsistent: the only way it preserves the number of jobs is if the effect of the impedance function is ignored when expanding the values of jobs per capita to obtain the total number of opportunities. The proportional allocation procedure described above, in contrast, consistently returns a number of jobs available that matches the total number of jobs in the region.

Now we proceed to define a measure of spatial availability per capita:

$$v_i = \frac{V_i}{P_i} \tag{8}$$

To complete the illustrative example, the spatial availability of jobs per capita is calculated in Equation (9).

$$\begin{array}{l} v_1 = \frac{V_1}{P_1} = \frac{66,833.47}{50,000} = 1.337 \\ v_2 = \frac{V_2}{P_2} = \frac{133,203.4}{150,000} = 0.888 \\ v_3 = \frac{V_3}{P_3} = \frac{9,963.171}{10,000} = 0.996 \end{array} \tag{9}$$

Remarkably, the spatial availability per capita matches the values of A_i in Table 1. Appendix A has a proof of the mathematical equivalence between the two measures. It is interesting to note that Weibull [53], Shen [49], and this paper reach to identical expressions starting from different starting points, an effect known as *equifinality* [see 36, p. 333; 54], In effect, Shen-type accessibility and 2SFCA can be conceptualized as singly-constrained accessibility measures.

3.4. Why does proportional allocation matter?

Having shown that Shen-type and spatial availability produce equifinal results, it is reasonable to wonder whether the distinction is of any import.

Conceptually, we would argue that the internal inconsistency in the calculation of total opportunities in Shen [49] points at a more serious problem that is only evident when we consider the internal values of the method. To illustrate, Table 1 shows results of A_i that are reasonable (in fact, they match the spatial availability per capita). But when we dig deeper, these results mask potentially misleading values of jobs allocated and taken. In addition, the internal values also lead to estimates of the impact of accessibility that are deceptive. For example, the estimated system-wide cost of travel considering the jobs allocated by A_i in Table 1 is as follows:

$$\begin{aligned}
&11,157 \times 15 \text{ min} + 2,489 \times 30 \text{ min} + 2.27 \times 100 \text{ min}\\
&7,468 \times 30 \text{ min} + 33,470 \times 15 \text{ min} + 6.81 \times 100 \text{ min}\\
&0.454 \times 100 \text{ min} + 0.454 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,581 \text{ min}
\end{aligned}$$

In contrast, the estimated system-wide cost of travel according to V_i in Table 2 is as follows:

$$\begin{aligned}
&59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min}\\
&40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min}\\
&2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min}
\end{aligned}$$

Therefore, not only does the Shen-type measure effectively allocate fewer than 56,835 out of a total of 210,000 jobs in the region, it also grossly underestimates the potential cost of travel in the system by obscuring the number of jobs not allocated.

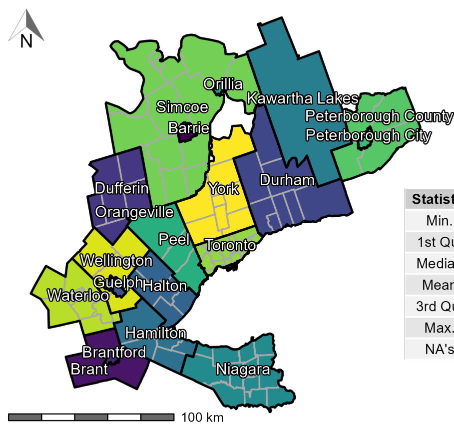
4. Empirical example of Toronto

In this section we use population and employment data from the Golden Horseshoe Area (GGH) in Ontario, Canada. This is the largest metropolitan region in Canada and includes the cities of Toronto and Hamilton. We calculate gravity accessibility, XXX, and the proposed spatial availability for Toronto after introducing the data used and calibrating an impedance function.

4.1. Data

Population and employment data are drawn from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 2) [14]. The data set includes Traffic Analysis Zones (TAZ) (n=3,764), the number of jobs (n=3,081,885) and workers (n=3,446,957) at each origin and destination. The TTS data is based on a representative sample of between 3% to 5% of households in the GGH and is weighted to reflect the population covering the study area has a whole [14].

To generate the travel cost for these trips, travel times between origins and destinations are calculated for car travel using the R package {r5r} [45] with a street network retrieved from OpenStreetMap for the GGH area. A the 3 hr travel time threshold was selected as it captures 99% of population-employment



Statistic	Trips	TT	Workers	Jobs
Min.	1	0	0	0
1st Qu.	14	13	25	64
Median	22	20	464	244
Mean	33	23	916	819
3rd Qu.	38	30	1378	700
Max.	1129	179	8491	41821
NA's	NA	3507	NA	NA

Figure 2: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

pairs (see the travel times summarized in Figure 2). This method does not account for traffic congestion or modal split, which can be estimated through other means [e.g., 2, 24]. For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions.

All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

4.2. Calibration of an impedance function

In the synthetic example introduced in a preceding section, a negative exponential function with an arbitrary parameter was used. For the empirical example, we calibrate an impedance function on the trip length distribution (TLD) of commute trips. Briefly, a TLD represents the proportion of trips that are taken at a specific travel cost (e.g., travel time); this distribution is commonly used to derive impedance functions in accessibility research [25, 6].

The empirical and theoretical TLD for this data set are represented in the top-left panel of Figure 3. Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the {fitdistrplus} package [16] were used. Based on goodness-of-fit criteria and diagnostics seen in Figure 3, the gamma distribution was selected (also see Figure **??** in Appendix XX).

```
[1] 3069541
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The gamma distribution takes the following general form where the estimated 'shape' is $\alpha = 2.019$, the estimated 'rate' is $\beta = 0.094$, and $\Gamma(\alpha)$ is defined in Equation (10).

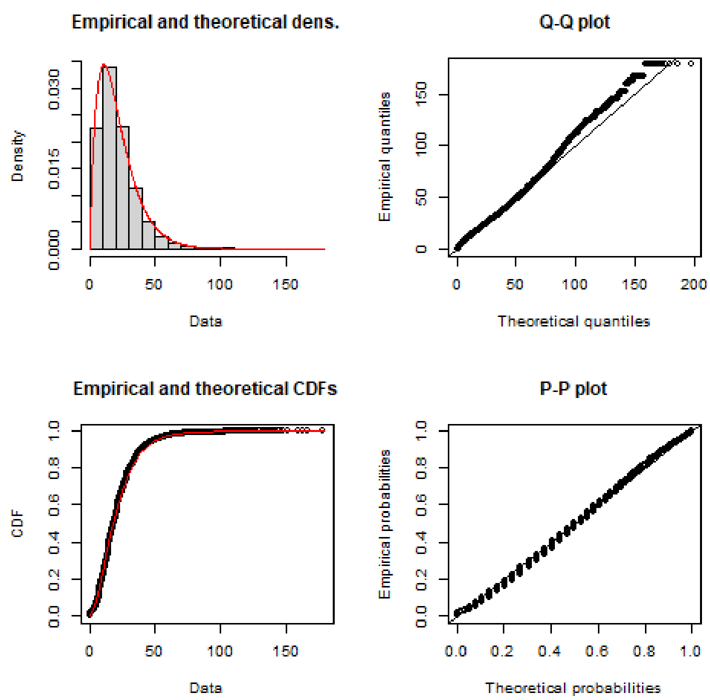

Figure 3: Car trip length distribution and calibrated gamma distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

$$
\begin{aligned}
&f(x, \alpha, \beta) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} \quad \text{for } 0 \leq x \leq \infty \\
&\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx
\end{aligned}
\tag{10}
$$

4.3. Accessibility and spatial availability of jobs in Toronto

Toronto is the largest city in the GGH and represents a significant subset of workers and jobs in the GGH; 31% of workers in the GGH travel to jobs in Toronto and 40% of jobs are located within Toronto.

5. Discussion and Conclusions

Words go here.

6. Appendix A

Equivalence of Shen-type accessibility and spatial availability

Population allocation factor:

$F^p_{ij} = \frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}}$

$F^p_A = \frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}$

Cost allocation factor:

$F^c_{ij} = \frac{f(c_{ij})}{\sum_{i=A}^K f(c_{ij})}$

$F^c_{A1} = \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}$ $F^c_{B1} = \frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}$ $F^c_{C1} = \frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}$

Now let's put it together with P, and see how the denominators end up cancelling out:

$$v_i = \sum_j \frac{O_j}{P^\alpha_{i\in r}} \frac{\frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}{\sum_i^K \frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}$$

$$v_A = \frac{O_1}{P^\alpha_A}\left(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}+\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{B1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}+\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{C1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}\right)+$$
$$\frac{O_2}{P^\alpha_A}\left(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}+\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{B2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}+\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{C2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}\right)+$$
$$\frac{O_3}{P^\alpha_A}\left(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}+\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{B3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}+\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{C3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}\right)$$

First, notice how the denominator on the denominator is the same across the summation? Let's simplify it:

$$v_A = \frac{O_1}{P^\alpha_A}\left(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P^\alpha_A\cdot f(c_{A1})+P^\alpha_A\cdot f(c_{B1})+P^\alpha_A\cdot f(c_{C1})}{(P^\alpha_A+P^\alpha_B+P^\alpha_C)\cdot(f(c_{A1})+f(c_{B1})+f(c_{C1}))}}\right)+\frac{O_2}{P^\alpha_A}\left(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P^\alpha_A\cdot f(c_{A2})+P^\alpha_A\cdot f(c_{B2})+P^\alpha_A\cdot f(c_{C2})}{(P^\alpha_A+P^\alpha_B+P^\alpha_C)\cdot(f(c_{A2})+f(c_{B2})+f(c_{C2}))}}\right)+$$
$$\frac{O_3}{P^\alpha_A}\left(\frac{\frac{P^\alpha_A}{P^\alpha_A+P^\alpha_B+P^\alpha_C}\cdot\frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P^\alpha_A\cdot f(c_{A3})+P^\alpha_A\cdot f(c_{B3})+P^\alpha_A\cdot f(c_{C3})}{(P^\alpha_A+P^\alpha_B+P^\alpha_C)\cdot(f(c_{A3})+f(c_{B3})+f(c_{C3}))}}\right)$$

See how the denominator of the denominator is the same as the denominator of the numerator's denominator for each J (J=1, J=2, and J=3)? Let's cancel those out and simplify:

$$v_A = \frac{O_1}{P^\alpha_A}\left(\frac{P^\alpha_A\cdot f(c_{A1})}{P^\alpha_A\cdot f(c_{A1})+P^\alpha_A\cdot f(c_{B1})+P^\alpha_A\cdot f(c_{C1})}+\frac{O_2}{P^\alpha_A}\frac{P^\alpha_A\cdot f(c_{A2})}{P^\alpha_A\cdot f(c_{A2})+P^\alpha_A\cdot f(c_{B2})+P^\alpha_A\cdot f(c_{C2})}+\frac{O_3}{P^\alpha_A}\frac{P^\alpha_A\cdot f(c_{A3})}{P^\alpha_A\cdot f(c_{A3})+P^\alpha_A\cdot f(c_{B3})+P^\alpha_A\cdot f(c_{C3})}\right)$$

Next, see how we can cancel out the P^α_A? Let's do that.

$$v_A = O_1\left(\frac{f(c_{A1})}{P^\alpha_A\cdot f(c_{A1})+P^\alpha_B\cdot f(c_{B1})+P^\alpha_C\cdot f(c_{C1})}+O_2\frac{f(c_{A2})}{P^\alpha_A\cdot f(c_{A2})+P^\alpha_B\cdot f(c_{B2})+P^\alpha_C\cdot f(c_{C2})}+O_3\frac{f(c_{A3})}{P^\alpha_A\cdot f(c_{A3})+P^\alpha_B\cdot f(c_{B3})+P^\alpha_C\cdot f(c_{C3})}\right)$$

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