

# PLOS ONE

## Introducing spatial availability, a singly-constrained measure of competitive accessibility --Manuscript Draft--

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<b>Full Title:</b>	Introducing spatial availability, a singly-constrained measure of competitive accessibility
<b>Short Title:</b>	Introducing spatial availability, a singly-constrained measure of competitive accessibility
<b>Corresponding Author:</b>	Anastasia Soukhov McMaster University Toronto, ON CANADA
<b>Keywords:</b>	competitive accessibility; spatial availability; Spatial Analysis; urban studies
<b>Abstract:</b>	Accessibility indicators are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability: one of those methodological additions have been the introduction of competition. In this paper, we focus on competition, but first demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call _spatial availability_. This measure relies on proportional allocation balancing factors (friction of distance and population competition) that are equivalent to imposing a single constraint on conventional gravity-based accessibility. In other words, the proportional allocation of opportunities results in a _spatially available opportunities_ value which is assigned to each origin that, when all origin values are summed, equals the total number of opportunities in the region. We also demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods are equivalent to spatial availability and can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada. Spatial availability is an important contribution since it clarifies the interpretation of accessibility with competition and paves the way for future applications in equity analysis.
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•Thank you for this comment. The data set is available from <https://soukhova.github.io/TTS2016R/> The full manuscript and the code are included in this repository: <https://github.com/soukhova/Spatial-Availability-Measure>. Both links are provided as footnotes in the Introduction section.

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•Thank you for this comment. All images are original and were generated by us using the data sources stated in the manuscript using code found in the .Rmd file in the repository (<https://github.com/soukhova/Spatial-Availability-Measure/blob/main/Spatial-Availability-Paper/Spatial-Availability-Paper.Rmd>). The packages used to produce the figures are cited.

#### Reviewer #1

The authors demonstrated how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population.

1.Please highlight the key points in the abstract.

•Thanks for this comment. We modified the abstract to highlight the key points of this paper. The abstract now reads as follows:

Accessibility indicators are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability: one of those methodological additions have been the introduction of competition. In this paper, we focus on competition, but first demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative

formulation of accessibility with competition, a measure we call *\_spatial availability\_*. This measure relies on proportional allocation balancing factors (friction of distance and population competition) that are equivalent to imposing a single constraint on conventional gravity-based accessibility. In other words, the proportional allocation of opportunities results in a *\_spatially available opportunities\_* value which is assigned to each origin that, when all origin values are summed, equals the total number of opportunities in the region. We also demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods are equivalent to spatial availability and can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada. Spatial availability is an important contribution since it clarifies the interpretation of accessibility with competition and paves the way for future applications in equity analysis.

2.The introduction should further highlight the scientific problems, motivations, and possible innovations of the paper.

•Thank you for these points. We modified the text as follows to include the highlighted motivations (as a result of the scientific problems) at the end of the introduction.

The key motivations of this paper are as follows:

- To address and improve on the interpretability of Hansen-type accessibility measure; and

- To consider competition from the perspective of the population for opportunities within an accessibility measure.

•We'd like to draw your attention to the summary of the objectives of the paper in the revised version of the manuscript and included as follows. We believe these correctly highlight the possible innovations of this paper.

In this way, the aim of the paper is three-fold:

- First, we aim to demonstrate that Shen-type (and thus @weibull\_axiomatic\_1976 accessibility and the popular 2SFCA methods) produce equivocal estimates of opportunities allocated as the result is presented as a rate (i.e., opportunities per capita);

- Second, we introduce a new measure, *\_spatial availability\_*, which we submit is a more interpretable alternative to Shen-type accessibility, since opportunities in the system are preserved and proportionally allocated to the population; and

- Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.

3.What is the contribution and why the contribution is important?

•An important question to ask indeed! We believe we highlight the contribution and its importance throughout the paper, particularly in subsection "Why does proportional allocation matter?" as well as in the conclusion. The contribution is in the conclusion is summarized as follows:

In this paper we show how a widely used measure of accessibility with competition (Shen-type accessibility) obscures some important internal values of opportunities taken. This is caused by confounding the population of zones with the *\_effective opportunity-seeking population\_*. We then propose an alternative derivation of accessibility with competition that we call spatial availability. This measure ensures that opportunities are allocated in a proportional way and preserved in the regional total. We also show that spatial availability and Shen-type accessibility are equifinal: formally the equations are the same (along with 2SFCA) and can be consider as singly-constrained measures.

•In addition, following your suggestion, we edited the first few sentences of the conclusion's second paragraph to make the importance of the contribution more clear. It now reads:

Spatial availability matters because competition is an important consideration for certain opportunity types and conventional Hansen-type accessibility does not capture it [@merlin2017competition]. Spatial availability also brings forward a different interpretation to competition than the Shen-type measure through its output value, the

proportional allocation factors, and the output values per capita. In equity analysis and policy planning, an analyst might be interested in the internal values of their accessibility analysis, for example travel times, and who pays how much for accessibility. The increased interpretability and internal consistency of spatial availability can help to push accessibility analysis forward. Hansen-type measure tend to result in values which are very extreme as a result of multiple-counting opportunities as shown in empirical example. Multiple-counting may not be an issue if the opportunity-type is non-exclusive, but with the case of employment where one worker can only take one job, the resulting values are difficult to interpret (though it can be interpreted relatively to speak about urban form). In this paper, we also demonstrated how attempting to disentangle the absolute values of opportunities from the Shen-type measure is difficult as a result of Shen's definition which confounds the population with the effective-opportunity seeking population.

•We also added a sentence to the abstract which states the contribution of the paper more clearly, as inspired by this comment. The most current version of the abstract is included in reply to your comment 1.

4.The results should be better described, discussed and justified. I suggest the results should be better discussed and justified, such as whether they are consistent with previous studies or analyzing the reasons for the empirical results. please read:

- The influence of high-speed rail on ice–snow tourism in northeastern China. *Tourism Management* ( 2020 ) , doi:10.1016/j.tourman.2019.104070
- Study on the Impact of High-speed Railway Opening on China's Accessibility Pattern and Spatial Equality[J].*Sustainability* 2018,10,2943. doi:10.3390/su10082943
- The Impact of Accessibility on House Prices Based on a Geographically Weighted Regression Model[J]. *Chinese Geographical Science* , 2018 , 28 ( 3 ) : 505-515. doi: 10.1007/s11769-018-0954-6

•Thank you for these references. They have been included in the revised version of the paper as examples of the important role that accessibility analysis plays in different disciplines.

#### Reviewer #2

The authors used the data from Transportation Tomorrow Survey to explore the application of spatial availability and compare it to other relevant measures. The manuscript is valid and innovative. However, there also have some problems need to be solved.

1.In introduction, the authors only inserted the serial number of the references. The presentation is omitted. It is not standard enough.

•Thank you for this comment. We are advised to use the in-text referencing style of PLoS ONE, which uses serial number of the reference.

2.There are many methods to measure the accessibility. What's the advantage and disadvantage of the improved methods compared with other methods, such as space syntax and potential model.

•Thank you for these points, discussing the advantages and disadvantages of similar methods is important to contextualize the improved method. In this paper, we do discuss other accessibility measures at length in the 'Accessibility measures revisited' section. We outline Hansen-type accessibility and then Shen-type accessibility, both of which are forms of potential accessibility. We introduce spatial availability in the following section and then discuss its advantages in the subsection 'Why does proportional allocation matter?'.

•Our aim is to present a new accessibility measure, spatial availability, and we discuss its properties in reference to two comparable measures, Hansen-type accessibility, and Shen-type accessibility (2SFCA). These two are forms of potential accessibility (see for an example Zhang et al., 2021). We reviewed the space syntax model. As noted by Zhang et al. (2022, p. 9434). "[a] higher level of integration implies spatial clustering and better topological accessibility". Some space syntax measures correlate well with accessibility, however accessibility is sufficiently distinct that it is not an appropriate comparator.

3.Some relevant references should be cited as follow:

•The influence of the spatial pattern of urban road networks on the quality of business

	<p>environments: the case of Dalian City. Environment, Development and Sustainability (2022) 24:9429–9446. <a href="https://doi.org/10.1007/s10668-021-01832-z">https://doi.org/10.1007/s10668-021-01832-z</a></p> <ul style="list-style-type: none"> <li>• Differences in Accessibility of Public Health Facilities in Hierarchical Municipalities and the Spatial Pattern Characteristics of Their Services in Doumen District, China. Land 2021, 10, 1249. <a href="https://doi.org/10.3390/land10111249">https://doi.org/10.3390/land10111249</a>.</li> <li>• Transportation Accessibility Evaluation of Educational Institutions Conducting Field Environmental Education Activities in Ecological Protection Areas: A Case Study of Zhuhai City. Sustainability 2021, 13, 9392. <a href="https://doi.org/10.3390/su13169392">https://doi.org/10.3390/su13169392</a></li> </ul> <p>Thank you for these references, which we have added to the bibliography in the paper.</p> <p>We trust that you will find that these revisions satisfy all the comments communicated. We are grateful for your insightful and constructive feedback, and appreciate that it helped to improve the paper.</p>
<b>Additional Information:</b>	
<b>Question</b>	<b>Response</b>
<b>Financial Disclosure</b>	The Social Sciences and Humanities Research Council's partnership grant: Mobilizing justice: towards evidence-based transportation equity policy.
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- Provide the name of the Institutional Animal Care and Use Committee (IACUC) or other relevant ethics board that reviewed the study protocol, and indicate whether they approved this research or granted a formal waiver of ethical approval
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<p><b>Describe where the data may be found in full sentences. If you are copying our sample text, replace any instances of XXX with the appropriate details.</b></p> <ul style="list-style-type: none"> <li>• If the data are <b>held or will be held in a public repository</b>, include URLs, accession numbers or DOIs. If this information will only be available after acceptance, indicate this by ticking the box below. For example: <i>All XXX files are available from the XXX database (accession number(s) XXX, XXX.).</i></li> <li>• If the data are all contained <b>within the manuscript and/or Supporting Information files</b>, enter the following: <i>All relevant data are within the manuscript and its Supporting Information files.</i></li> <li>• If neither of these applies but you are able to provide <b>details of access elsewhere</b>, with or without limitations, please do so. For example:</li> </ul> <p><i>Data cannot be shared publicly because of [XXX]. Data are available from the XXX Institutional Data Access / Ethics Committee (contact via XXX) for researchers who meet the criteria for access to confidential data.</i></p> <p><i>The data underlying the results presented in the study are available from (include the name of the third party</i></p>	<p>All relevant data are within the manuscript and its Supporting Information files.</p>

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Additional data availability information:



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McMaster University, 1280 Main Street West  
Hamilton, Ontario, Canada, L8S 4L7

September 22nd, 2022

**Dear Dr. Emily Chenette,**  
*Editor-in-Chief, PLOS ONE*

This letter is submitted in support of the research paper ***Introducing spatial availability, a singly-constrained measure of competitive accessibility***. I am the lead and corresponding author, and a doctoral graduate student in the School of Earth, Environment and Society at McMaster University. The co-authors are Prof. Antonio Paez, from the School of Earth, Environment and Society at McMaster University, Prof. Christopher D. Higgins, from the Department of Geography & Planning at University of Toronto Scarborough, and Prof. Moataz Mohamed, from the Department of Civil Engineering at McMaster University.

Thank you and the reviewers for all the valuable feedback. The following points have been changed:

- Modified the abstract to highlight the paper's important contributions;
- Clarified the article's contribution within the Introduction and Conclusion sections; and
- Added additional relevant literature for the reader's reference.

As mentioned in the journal requirements comments, please update the Role of Funder statement as follows:

*This research has received funding from the Social Sciences and Humanities Research Council of Canada's (SSHRC) partnership grant Mobilizing Justice: towards evidence-based transportation equity policy as well as the Canada Graduate Scholarship – Doctoral (CGS-D) awarded to Anastasia Soukhov. The funder was not involved in the decision to submit the manuscript for publication, nor will they be involved in any aspect of the design or conduct of the manuscript.*

We look forward to hearing again from you and your expert reviewers. On behalf of my co-authors, I would like to thank you in advance for your attention. This article is currently available as a preprint on the OSF server here: <https://osf.io/sjd5n/>

**Sincerely,**

**Anastasia Soukhov**, MASC, B.Eng.

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# Introducing spatial availability, a singly-constrained measure of competitive accessibility

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## Abstract

Accessibility indicators are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability: one of those methodological additions have been the introduction of competition. In this paper, we focus on competition, but first demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure relies on proportional allocation balancing factors (friction of distance and population competition) that are equivalent to imposing a single constraint on conventional gravity-based accessibility. In other words, the proportional allocation of opportunities results in a *spatially available opportunities* value which is assigned to each origin that, when all origin values are summed, equals the total number of opportunities in the region. We also demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods are equivalent to spatial availability and can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada. Spatial availability is an important contribution since it clarifies the interpretation of accessibility with competition and paves the way for future applications in equity analysis.

# Introduction

The concept of accessibility in transportation studies derives its appeal from the combination of the spatial distribution of opportunities and the cost of reaching them [1,2]. Accessibility analysis is employed in geography [3,4], public health [5–9], real estate valuation [10], tourism [11], and transportation [12,13] among other areas, with the number of applications growing [14], especially as mobility-based planning is de-emphasized in favor of access-oriented planning [15–18].

Accessibility analysis stems from the foundational works of [19] and [1]. From these seminal efforts, many accessibility measures have been derived, particularly after the influential work of [20] on spatial interaction<sup>1</sup>. Of these, gravity-type accessibility is arguably the most common; since its introduction in the literature, it has been widely adopted in numerous forms [13,21–25]. Hansen-type accessibility indicators are essentially weighted sums of opportunities, with the weights given by an impedance function that depends on the cost of movement, and thus measure the *intensity of the possibility of interaction* [1]. This type of accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure [2].

Despite their usefulness, the interpretability of Hansen-type accessibility measures can be challenging [13,26]. Since they aggregate opportunities, the results are sensitive to the size of the region of interest (e.g., a large city has more jobs than a smaller city). As a consequence, raw outputs are not necessarily comparable across study areas [27]. This limitation becomes evident when surveying studies that implement this type of analysis. For example, [5] (in Montreal) and [28] (in Nairobi) report accessibility as the number of health care facilities that can potentially be reached from origins. But what does it mean for a zone to have accessibility to less than 100 facilities in each of these two cities, with their different populations and number of facilities? For that matter, what does it mean for a zone to have accessibility to more than 700 facilities in Montreal, besides being “accessibility rich”? As another example, [29] (in Bogota), [30] (in Montreal), and [31] (in Beijing) report accessibility as numbers of jobs, with accessibility values often in the hundreds of thousands, and even exceeding one million jobs for some zones in Beijing and Montreal. As indicators of urban structure, these measures are informative, but the meaning of one million accessible jobs is harder to pin down: how many jobs must any single person have access to? Clearly, the answer to this question depends on how many people demand jobs.

The interpretability of Hansen-type accessibility has been discussed in numerous studies, including recently by [32], [33], and in greater depth by [34]. As hinted above, the limitations in interpretability are frequently caused by ignoring competition; without competition, each opportunity is assumed to be equally available to every single opportunity-seeking individual that can reach it [33,35,36]. This assumption is appropriate when the opportunity of interest is non-exclusive, that is, if use by one unit of population does not preclude use by another. For instance, national parks with abundant space are seldom used to full capacity, so the presence of some population does not exclude use by others. When it comes to exclusive opportunities, or when operations may be affected by congestion, the solution has been to account for competition. Several efforts exist that do so. In our reckoning, the first such approach was proposed by [37], whereby the distance decay of the supply of employment and the demand for employment (by workers) were formulated under so-called axiomatic assumptions. This approach was then applied by [38] in the context of healthcare, to quantify the availability of general practitioners in Canada. About two decades later, [35] independently re-discovered Weibull’s [37] formula [see footnote (7) in 35] and

<sup>1</sup>Utility-based measures derive from a very different theoretical framework, random utility maximization

deconstructed it to consider accessibility for different modes. These advances were subsequently popularized as the family of Two-Stage Floating Catchment area (2SFCA) methods [39] that have found widespread adoption in healthcare, education, and food systems [40–44].

An important development contained in Shen’s work is a proof that the population-weighted sum of the accessibility measure with competition equates to the number of opportunities available [footnote (7) and Appendix A in 35]. This demonstration gives the impression that Shen-type accessibility allocates *all* opportunities to the origins, however to the authors’ knowledge, it has not been interpreted by literature in this way. For instance, [45], [34], and [46] all use Shen-type accessibility to calculate job access but report values as ‘competitive accessibility scores’ or simply ‘job accessibility’. These works do not explicitly recognize that jobs that are assigned to each origin are in fact a proportion of *all* the opportunities in the system. This recognition, we argue, is critical to interpreting the meaning of the final result. Thus, in this paper we intend to revisit accessibility with competition within the context of disentangling how opportunities are allocated. We first argue that Shen’s competitive accessibility misleadingly refers to the total zonal population to equal the travel-cost discounted opportunity-seeking population. This equivocation, we believe, results in an ambiguous interpretation of what Shen-type accessibility represents as the allocation of opportunities to population is masked by the results presenting as rates (i.e., opportunities per capita). We then propose an alternative formulation of accessibility that incorporates competition by adopting a proportional allocation mechanism; we name this measure *spatial availability*. The use of balancing factors for proportional allocation is akin to imposing a single constraint on the accessibility indicator, in the spirit of Wilson’s [20] spatial interaction model.

The key motivations of this paper are as follows:

- To address and improve on the interpretability of Hansen-type accessibility measure; and
- To consider competition from the perspective of the population for opportunities within an accessibility measure.

In this way, the paper’s aim is three-fold:

- First, we aim to demonstrate that Shen-type (and thus [37] accessibility and the popular 2SFCA methods) produce equivocal estimates of opportunities allocated as the result is presented as a rate (i.e., opportunities per capita);
- Second, we introduce a new measure, *spatial availability*, which we submit is a more interpretable alternative to Shen-type accessibility, since opportunities in the system are preserved and proportionally allocated to the population; and
- Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.

Discussion is supported by the use of the small synthetic example of [35] and empirical data drawn from the 2016 Transportation Tomorrow Survey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the spirit of openness of research in the spatial sciences [47,48] this paper has a companion open data product<sup>2</sup> [49], and all code is available for replicability and reproducibility purposes<sup>3</sup>.

<sup>2</sup><https://soukhova.github.io/TTS2016R/>

<sup>3</sup><https://github.com/soukhova/Spatial-Availability-Measure>

## Accessibility measures revisited

In this section we revisit Hansen-type and Shen-type accessibility indicators. We adopt the convention of using a capital letter for absolute values (number of opportunities) and lower case for rates (opportunities per capita).

### Hansen-type accessibility

Hansen-type accessibility measures follow the general formulation shown in Equation (1):

$$S_i = \sum_{j=1}^J O_j \cdot f(c_{ij}) \quad (1)$$

where:

- $c_{ij}$  is a measure of the cost of moving between  $i$  and  $j$ .
- $f(\cdot)$  is an impedance function of  $c_{ij}$ ; it can take the form of any monotonically decreasing function chosen based on positive or normative criteria [50].
- $i$  is a set of origin locations ( $i = 1, \dots, N$ ).
- $j$  is a set of destination locations ( $j = 1, \dots, J$ ).
- $O_j$  is the number of opportunities at location  $j$ ;  $O = \sum_{j=1}^J O_j$  is the total supply of opportunities in the study region.
- $S$  is Hansen-type accessibility as weighted sum of opportunities.

As formally defined, accessibility  $S_i$  is the sum of opportunities that can be reached from location  $i$ , weighted down by an impedance function of the cost of travel  $c_{ij}$ .

Summing the opportunities in the neighborhood of  $i$  provides estimates of the number of opportunities that can *potentially* be reached from  $i$ . Several measures result from using a variety of impedance functions; for example, cumulative opportunities measures are obtained when  $f(\cdot)$  is a binary or indicator function [13,e.g., 30,51,52]. Other measures use impedance functions modeled after any monotonically decreasing function [e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, *inter alia*, 53,54–56]. In practice, accessibility measures with different impedance functions tend to be highly correlated [53,57,58].

Gravity-based accessibility has been shown to be an excellent indicator of the intersection between spatially distributed opportunities and transportation infrastructure [14,53,55]. However, beyond enabling comparisons of relative values, they are not highly interpretable on their own [26]. To address the issue of interpretability, previous research has aimed to index and normalize values on a per demand-population basis [e.g., 59,60,61]. However, as recent research on accessibility discusses [27,33,e.g., 34,36], these steps do not adequately consider competition. In effect, when calculating  $S_i$ , every opportunity enters the weighted sum once for every origin  $i$  that can reach it. This makes interpretability opaque, and to complicate matters, can also bias the estimated landscape of opportunity.

### Shen-type competitive accessibility

To account for competition, the influential works of [35] and [37], as well as the widely used 2SFCA approach of [39], adjust Hansen-type accessibility to account for the population's demand for opportunities in the region of interest. The mechanics of this approach consist of calculating, for every destination  $j$ , the population that can reach the opportunity(ies) given the impedance function  $f(\cdot)$ ; let us call this the *effective*

*opportunity-seeking population* (Equation (2)). This value can be seen as the Hansen-type *market area* (accessibility to population) of  $j$ . The opportunities at  $j$  are divided by the sum of the effective opportunity-seeking population to obtain a measure of opportunities per capita, i.e.,  $R_j$  in Equation (3). This can be thought of as the *level of service* at  $j$ . Per capita values are then allocated back to the population at  $i$ , again subject to the impedance function as seen in Equation (4); this is accessibility with competition.

$$P_{ij}^* = P_i \cdot f(c_{ij}) \quad (2)$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^*} \quad (3)$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \quad (4)$$

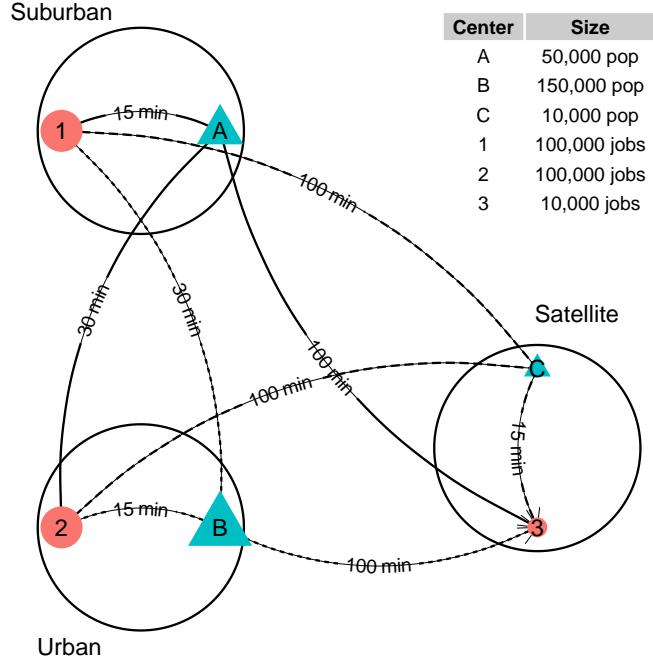
where:

- $a$  is Shen-type accessibility as weighted sum of opportunities per capita (or weighted level of service).
- $c_{ij}$  is a measure of the cost of moving between  $i$  and  $j$ .
- $f(\cdot)$  is an impedance function of  $c_{ij}$ .
- $i$  is a set of origin locations ( $i = 1, \dots, N$ ).
- $j$  is a set of destination locations ( $j = 1, \dots, J$ ).
- $O_j$  is the number of opportunities at location  $j$ ;  $O = \sum_{j=1}^J O_j$  is the total supply of opportunities in the study region.
- $P_i$  is the population at location  $i$ .
- $P_{ij}^*$  is the population at location  $i$  that can reach destination  $j$  according to the impedance function; we call this the *effective opportunity-seeking population*.
- $R_j$  is the ratio of opportunities at  $j$  to the sum over all origins of the *effective opportunity-seeking population* that can reach  $j$ ; in other words, this is the total number of opportunities per capita found at  $j$ .

Shen describes  $P_i$  as the “*the number of people in location  $i$  seeking opportunities*” [35]. In our view, this is somewhat equivocal and where misinterpretation of the final results may arise. Consider a population center where the population is only willing to take an opportunity if the trip required is less than or equal to 60 minutes. This travel behaviour is captured by the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 & \text{if } c_{ij} \leq 60 \text{ min} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

If an employment center is less than 60 minutes away, the population can seek opportunities there (i.e.,  $f(c_{ij}) = 1$ ). But are these people still part of the opportunity-seeking population for jobs located two hours away? How about four hours away? We assume that they are not part of the opportunity-seeking population because their travel behaviour, as represented by the impedance function, would yield  $f(c_{ij}) = 0$ , eliminating them from the effective opportunity-seeking population  $P_{ij}^*$ . We see Shen’s definition as ambiguous because, for the purpose of calculating accessibility, the impedance function defines what constitutes the population that effectively can seek opportunities at remote locations. Thus,  $P_i$  should be plainly understood as the population at location  $i$  (as defined above) and not the “*the number of people in location  $i$  seeking opportunities*”. In other words,  $P_i$  and  $P_{ij}^*$  are confounded.



**Fig 1.** Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

Furthermore, an identical misunderstanding can be described for  $O_j$  which is defined as “*the number of relevant opportunities in location  $j$* ” in [35] (our emphasis).  $O_j$  is adjusted by the same  $f(c_{ij})$  in Equation (4), so the *relevancy* is determined by the travel behaviour associated with the impedance function and not only by  $O_j$ . For this reason,  $O_j$  should be understood plainly as the opportunities at location  $j$  (as defined above).

Misunderstanding  $P_i$  and  $O_j$  may lead to a misleading interpretation of the final result  $a_i$ , especially as expressed in Shen’s proof (see Equation (6)).

$$\sum_{i=1}^N a_i P_i = \sum_{j=1}^J O_j \quad (6)$$

Confounding  $P_i$  with the effective opportunity-seeking population and confounding  $O_j$  with the jobs taken may cause us to misunderstand  $a_i$  as “*relevant opportunities*” per “*people in location  $i$  seeking opportunities*”. Instead, as mathematically expressed in Shen’s proof,  $a_i$  is a proportion of the opportunities available to the population, since multiplying  $a_i$  by the population at  $i$  and summing for all origins in the system equals to the total number of opportunities in the system. Embedded in  $a_i$  is already the travel behaviour, so  $P_i$  and  $O_j$  must be plainly understood as the population at  $i$  and opportunities at  $j$  for Equation (6) to hold true.

### Shen’s synthetic example

In this section we use the synthetic example in [35] to highlight the importance of understanding  $P_i$  and  $O_j$  as simply the population at the origin  $i$  and the opportunities at destination  $j$ , respectively. This is critical to understanding how opportunities are allocated to the population based on the impedance function.

Table 1 contains the information needed to calculate  $S_i$  and  $a_i$  for this example. We

**Table 1.** Summary description of the synthetic example: Hansen-type accessibility  $S_i$ , Shen-type accessibility  $a_i$ , and spatial availability  $V_i$  with beta = 0.1 (light yellow) and beta = 0.6 (light grey).

Origin	A			B			C		
Dest.	1	2	3	1	2	3	1	2	3
Pop.	50000	50000	50000	150000	150000	150000	10000	10000	10000
Jobs	100000	100000	10000	100000	100000	10000	100000	100000	10000
TT	15	30	100	30	15	100	100	100	15
f(TT)	0.223	0.050	< 0.001	0.050	0.223	< 0.001	< 0.001	< 0.001	0.223
Pop * f(TT)	11156.5	2489.4	2.3	7468.1	33469.5	6.8	0.5	0.5	2231.3
Jobs * f(TT)	22313.0	4978.7	0.5	4978.7	22313.0	0.5	4.5	4.5	2231.3
S_i	27292.2	27292.2	27292.2	27292.2	27292.2	27292.2	2240.4	2240.4	2240.4
a_i	1.337	1.337	1.337	0.888	0.888	0.888	0.996	0.996	0.996
f(TT)	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
Pop * f(TT)	6.170	< 0.001	< 0.001	0.002	18.511	< 0.001	< 0.001	< 0.001	1.234
Jobs * f(TT)	12.341	0.002	< 0.001	0.002	12.341	< 0.001	< 0.001	< 0.001	1.234
S_i	12.343	12.343	12.343	12.343	12.343	12.343	1.234	1.234	1.234
a_i	1.999	1.999	1.999	0.667	0.667	0.667	1.000	1.000	1.000
F^c	0.238	0.238	0.238	0.714	0.714	0.714	0.048	0.048	0.048
F^p	0.817	0.182	< 0.001	0.182	0.817	< 0.001	< 0.001	< 0.001	1.000
F	0.599	0.069	0.001	0.401	0.931	0.003	< 0.001	< 0.001	0.996
V_ij	59900.6	6922.7	10.1	40096.9	93076.0	30.4	2.4	1.3	9959.5
V_i	66833.5	66833.5	66833.5	133203.4	133203.4	133203.4	9963.2	9963.2	9963.2

use a negative exponential impedance function with  $\beta = 0.1$  as also used in [35, see footnote (5)]:

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

In Table 1, we see that population centers  $A$  and  $B$  have equal Hansen-type accessibility ( $S_A = S_B = 27,292$  jobs). On the other hand, the isolated satellite town of  $C$  has low accessibility ( $S_C = 2,240$  jobs). But center  $B$ , despite its high accessibility, is a large population center.  $C$ , in contrast, is smaller but also relatively isolated and has a balanced ratio of jobs (10,000 jobs) to population (10,000 people). It is difficult from these outputs to determine whether accessibility at  $C$  is better or worse than that at  $A$

or  $B$ .

The results are easier to interpret when we consider Shen-type accessibility. The results indicate that  $a_A \approx 1.337$  jobs per capita,  $a_B \approx 0.888$ , and  $a_C \approx 0.996$ . The latter value is sensible given the jobs-population balance of  $C$ . Center  $A$  is relatively close to a large number of jobs (more jobs than the population of  $A$ ). The opposite is true of  $B$ . According to [35], the sum of the population-weighted accessibility  $a_i$  is exactly equal to the number of jobs in the region following Shen's proof:

$$\begin{aligned}\sum_{i=1}^N a_i P_i &= \sum_{j=1}^J O_j \\ 50,000 \times 1.3366693 &\\ +150,000 \times 0.8880224 &\\ +10,000 \times 0.9963171 &= 210,000\end{aligned}$$

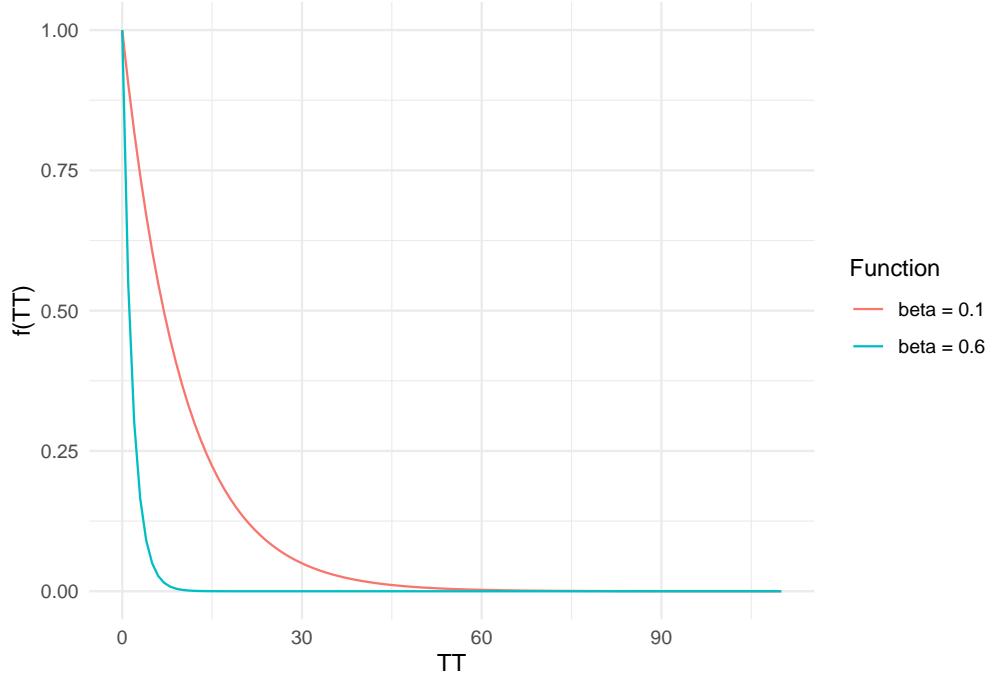
As mentioned earlier, this property under Shen's definition of  $P_i$  "people in location  $i$  seeking opportunities", gives the impression that all jobs sought are allocated to the people located at each origin  $i$ . In other words, Shen defines  $P_i$  to mean  $P_{ij}^*$  (i.e., the effective opportunity-seeking population which is already adjusted by travel behaviour) instead of defining it to simply be the full population at  $i$  (i.e.,  $P_i$ ). As seen in column **Pop \* f(TT)** in Table 1 (i.e.,  $P_{ij}^* = P_i \cdot f(c_{ij})$ ), the number of individuals from population center  $A$  that are willing to reach employment centers 1, 2, and 3 are 11,156, 2,489, and 2.27 respectively. Therefore, the total effective opportunity-seeking population at  $A$  is  $P_A^* = \sum_j P_{Aj}^*$ , that is, 13,647.27 people, which is considerably lower than the total population of  $A$  (i.e.,  $P_A = 50,000$  people). Demonstrated as follows, using  $P_{ij}^*$  in the calculations associated with this proof results in only 56,834.59 jobs being allocated to the population, instead of the nominal number of jobs in the region that is over three times this number (i.e., 210,000 jobs).

$$\begin{aligned}\sum_{i=1}^N a_i P_{ij}^* &= \\ (11,156.51 + 2,489.35 + 2.26) \times 1.3366693 &\\ +(7,468.06 + 33,469.52 + 6.81) \times 0.8880224 &\\ +(4.54 + 4.54 + 2,231.20) \times 0.9963171 &\approx 56,834.59\end{aligned}$$

Furthermore, even when Shen's  $P_i$  is understood plainly as the total population at  $i$ , the meaning of the proof may still be ambiguous. The proof can still give the impression that all jobs are allocated to the total population, since total population ( $\sum_{i=1}^N P_i$ ) goes into the equation and total jobs ( $\sum_{j=1}^J O_j$ ) in the region is the result. However, this impression is incomplete as it does not reflect the amount of population which takes jobs and the number of people being considered for jobs: these magnitudes are a product of being weighted down by the impedance function. The magnitudes are not obvious because the result,  $a_i$ , is a rate (i.e., opportunities per capita).

Let us consider a modification to the travel behaviour of the example discussed to illustrate how the presentation of  $a_i$  as a rate obscures the magnitude of the effective opportunity-seeking population. We modify the example by increasing the  $\beta$  to 0.6 (compared to the previous value of 0.1; see Figure 2). This modification increases the cost of travel and thus the impedance function, which is an expression of the population's relative willingness to travel to opportunities, reflects a population which is relatively less willing to travel to opportunities further away compared to the previous  $\beta$  value. The Hansen-type and Shen-type values are presented in the yellow rows of Table 1.

As expected, Hansen-type accessibility drops quite dramatically after this  $\beta$  modification: the friction of distance is so high that few opportunities are within reach. In contrast, Shen-type accessibility converges to the jobs:population ratio (i.e., origin  $A$  is  $\frac{100,000}{50,000} = 2$ ). This is explained by the way the impedance function excludes the



**Fig 2.** Comparison of two negative exponential impedance functions used in the synthetic example. The x-axis represents the travel time (in mins) and the y-axis represents the impedance function at each travel time.

population in droves, thus reducing the competition for jobs: as seen in Table 1, the effective opportunity-seeking population from  $A$  is only about equal to 6.17; likewise, the number of jobs at 1 weighted by the impedance is only 12.341. In other words, competition is low because jobs are expensive to reach, but those willing to reach jobs enjoy relatively high accessibility (in the limit, the jobs:population ratio). On the other hand, the accessibility is effectively zero for those in the population prevented by the impedance from reaching any jobs.

In what follows, we propose an alternative derivation of [35] accessibility with competition that explicitly clarifies the opportunities allocated to the *effective opportunity-seeking population* within its formulation. Hence, the results are not only more interpretable, but also extend the potential of accessibility analysis.

## Introducing spatial availability: a singly-constrained measure of accessibility

In brief, we define the *spatial availability* at  $i$  ( $V_i$ ) as the proportion of all opportunities  $O$  that are allocated to  $i$  from all destinations  $j$ :

$$V_i = \sum_{j=1}^N O_j F_{ij}^t$$

where:

- $F_{ij}^t$  is a balancing factor that depends on the population and cost of movement in the system.

- $O_j$  is the number of opportunities at  $j$ . 262
- $V_i$  is the number of spatially available opportunities from the perspective of  $i$ . 263

The general form of spatial availability is also as a sum, and the fundamental difference with Hansen- and Shen-type accessibility is that opportunities are allocated proportionally. Balancing factor  $F_{ij}^t$  consists of two components: a population-based balancing factor  $F_i^p$  and an impedance-based balancing factor  $F_{ij}^c$  which, respectively, allocate opportunities to  $i$  in proportion to the size of the population of the different competing centers (the mass effect of the gravity model) and the cost of reaching opportunities (the impedance effect). In the next two subsections, we explain the intuition behind the method before defining it in full. 264  
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## Proportional allocation by population 272

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by the population. Recall, the total population in the example is 210,000. The proportion of the population by population center is as follows: 273  
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$$F_A^p = \frac{50,000}{210,000}$$

$$F_B^p = \frac{150,000}{210,000}$$

$$F_C^p = \frac{10,000}{210,000}$$

Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors  $F_i^p$ . In this way, employment center 1 allocates  $100,000 \cdot \frac{50,000}{210,000} = 23,809.52$  jobs to  $A$ ;  $100,000 \cdot \frac{150,000}{210,000} = 71,428.57$  jobs to  $B$ ; and  $100,000 \cdot \frac{10,000}{210,000} = 7,142.857$  jobs to  $C$ . Notice how this mechanism ensures that the total number of jobs at employment center 1 is preserved at 100,000. 278  
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We can verify that the number of jobs allocated is consistent with the total number of jobs in the system: 286  
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Employment center 1 to population centers A, B, and C:  
 $100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$

Employment center 2 to population centers A, B, and C:  
 $100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$

Employment center 3 to population centers A, B, and C:  
 $10,000 \cdot \frac{50,000}{210,000} + 10,000 \cdot \frac{150,000}{210,000} + 10,000 \cdot \frac{10,000}{210,000} = 10,000$

In the general case where there are  $N$  population centers in the region, we define the following population-based balancing factors in Equation (7): 288  
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$$F_i^p = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \quad (7)$$

Balancing factor  $F_i^p$  corresponds to the proportion of the population in origin  $i$  relative to the population in the region. On the right hand side of the equation, the numerator  $P_i^\alpha$  is the population at origin  $i$ . The summation in the denominator is over  $i = 1, \dots, N$ , and adds up to the total population of the region. Notice that we 288  
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incorporate an empirical parameter  $\alpha$ . The role of  $\alpha$  is to modulate the effect of demand by population. When  $\alpha < 1$ , opportunities are allocated more rapidly to smaller centers relative to larger ones;  $\alpha > 1$  achieves the opposite effect.

Balancing factor  $F_i^p$  can now be used to proportionally allocate a share of available jobs at  $j$  to origin  $i$ . The number of jobs available to  $i$  from  $j$  balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

In the general case where there are  $J$  employment centers, the total number of jobs available from all destinations to  $i$  is simply the sum of  $V_{ij}^p$  over  $j = 1, \dots, J$ :

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

Since the factor  $F_i^p$ , when summed over  $i = 1, \dots, N$  always equals to 1 (i.e.,  $\sum_{i=1}^N F_i^p = 1$ ), the sum of all spatially available jobs equals  $O$ , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^p &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\ &= \sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

The terms  $F_i^p$  act here as the balancing factors of the gravity model when a single constraint is imposed [i.e., to ensure that the sums of columns are equal to the number of opportunities per destination, see 62 and 183-184]. As a result, the sum of spatial availability for all population centers equals the total number of opportunities.

The discussion so far concerns only the mass effect (i.e., population size) of the gravity model. In addition, the potential for interaction is thought to decrease with increasing cost, so next we define similar balancing factors but based on the impedance.

## Proportional allocation by cost

Clearly, using only balancing factor  $F_i^p$  to calculate spatial availability  $V_i^p$  does not account for the cost of reaching employment centers. Consider instead a set of balancing factors  $F_{ij}^c$  that account for the friction of distance for our example. Recall that the impedance function  $f(c_{ij})$  equals  $\exp(-\beta \cdot c_{ij})$  where  $\beta = 0.1$  and travel time  $c_{ij}$  is either 15, 30 or 60 minutes. For instance, the impedance-based balancing factors  $F_{ij}^c$  would be the following for employment center 1 (employment center 2 and 3 have their own balancing factor values for each origin  $i$  as will be discussed later):

$$\begin{aligned} F_{A1}^c &= \frac{0.223130}{0.223130 + 0.049787 + 0.000045} = 0.8174398 \\ F_{B1}^c &= \frac{0.049787}{0.223130 + 0.049787 + 0.000045} = 0.1823954 \\ F_{C1}^c &= \frac{0.000045}{0.223130 + 0.049787 + 0.000045} = 0.0001648581 \end{aligned}$$

Balancing factors  $F_{ij}^c$  use the impedance function to proportionally allocate more jobs to closer population centers, that is, to those with populations *more willing to reach the jobs*. Indeed, the factors  $F_{ij}^c$  can be thought of as the proportion of the population at  $i$  willing to travel to destination  $j$ , conditional on the travel behavior as given by the impedance function. For instance, 81.74398% of jobs from employment center 1 are allocated to population center  $A$  based on impedance.

So as follows from our example, of the 100,000 jobs at employment center 1 the number of jobs allocated to population center  $A$  is  $100,000 \times 0.8174398 = 81,743.98$

jobs; the number allocated to population center  $B$  is  $100,000 \times 0.1823954 = 18,239.54$   
 jobs; and the number allocated to population center  $C$  is  
 $100,000 \times 0.0001648581 = 16.48581$  jobs. We see once more that the total number of  
 jobs at the employment center is preserved at 100,000. In this example, the  
 proportional allocation mechanism assigns the largest share of jobs to population center  
 $A$ , which is the closest to employment center 1, and the smallest to the more distant  
 population center  $C$ . 326  
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In the general case where there are  $N$  population centers and  $J$  employment centers  
 in the region, we define the following impedance-based balancing factors:  
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$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \quad (8)$$

The total number of jobs available to  $i$  from  $j$  according to impedance is defined as  
 follows:  
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$$V_{ij}^c = O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

The total number of jobs available to  $i$  from all destinations is:  
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$$V_i^c = \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

Like the population-based allocation factors,  $F_i^c$  summed over  $i = 1, \dots, N$  always  
 equals to 1 (i.e.,  $\sum_{i=1}^N F_{ij}^c = 1$ ). As before, the sum of all spatially available jobs equals  
 $O$ , the total number of opportunities in the region:  
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$$\begin{aligned} \sum_{i=1}^N V_i^c &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c} \\ &= \sum_{i=1}^N \frac{F_i^c}{\sum_{i=1}^N F_i^c} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

We are now ready to more formally define spatial availability with due consideration  
 to both population and travel cost effects.  
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## Assembling mass and impedance effects

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Population and the cost of travel are both part of the gravity modelling framework.  
 Since the balancing factors defined in the preceding sections are proportions  
 (alternatively, can be understood as probabilities), they can be combined  
 multiplicatively to obtain their joint effect. This multiplicative relationship can  
 alternatively be understood as the joint probability of allocating opportunities and is  
 captured by Equation (9), where  $F_i^p$  is the population-based balancing factor that  
 grants a larger share of the existing opportunities to larger centers and  $F_{ij}^c$  is the  
 impedance-based balancing factor that grants a larger share of the existing  
 opportunities to closer centers. This is in line with the tradition of gravity modeling.  
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$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \quad (9)$$

with  $F_i^p$  and  $F_{ij}^c$  as defined in Equations (7) and (8) respectively. The combined  
 balancing factor  $F_{ij}^t$  is used to proportionally allocate jobs from  $j$  to  $i$ . Hence, spatial  
 availability is given by Equation (10).  
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$$V_i = \sum_{j=1}^J O_j F_{ij}^t \quad (10)$$

The terms in Equation 10:

- $F_{ij}^t$  is a balancing factor as defined in Equation (9). 357
- $i$  is a set of origin locations in the region  $i = 1, \dots, N$ . 358
- $j$  is a set of destination locations in the region  $j = 1, \dots, J$ . 359
- $O_j$  is the number of opportunities at location  $j$ . 360
- $V_i$  is the spatial availability at  $i$ . 361

Notice that, unlike  $S_i$  in Hansen-type accessibility (Equation (1)), the population enters the calculation of  $V_i$  through  $F_i^p$ . Returning to the example in Figure 1, Table 1 also contains the information needed to calculate  $V_i$ , with  $\beta$  set again to 0.1. Column **V\_ij** are the jobs available to each origin from each employment center. In this column  $V_{A1} = 59,901$  is the number of jobs available at  $A$  from employment center 1. Column **V\_i** (i.e.,  $\sum_{j=1}^J V_{ij}$ ) gives the total number of jobs available to origin  $i$ . We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

Compared the calculated values of  $V_i$  to column **S.i** (Hansen-type accessibility) in Table 1. The spatial availability values are more intuitive. Recall that population centers  $A$  and  $B$  had identical Hansen-type accessibility to employment opportunities. According to  $V_i$ , population center  $A$  has greater job availability due to: 1) its close proximity to employment center 1; combined with 2) less competition (i.e., a majority of the population have to travel longer distances to reach employment center 1). Job availability is lower for population center  $B$  due to much higher competition (150,000 people can reach 100,000 jobs at equal cost). And center  $C$  has almost as many jobs available as it has population.

As discussed above, Hansen-type accessibility is not designed to preserve the number of jobs in the region. Shen-type accessibility ends up preserving the number of jobs in the region but the definitions of variables are internally obscured; the only way it preserves the number of jobs is if the effect of the impedance function is ignored when expanding the values of jobs per capita to obtain the total number of opportunities. The proportional allocation procedure described above, in contrast, consistently returns a number of jobs available that matches the total number of jobs in the region.

Since the jobs spatially available are consistent with the jobs in the region, it is possible to define a measure of spatial availability per capita as presented in Equation (11):

$$v_i = \frac{V_i}{P_i} \quad (11)$$

And, since the jobs are preserved, it is possible to use the regional jobs per capita ( $\frac{\sum_{j=1}^J O_j}{\sum_{i=1}^N P_i}$ ) as a benchmark to compare the spatial availability of jobs per capita at each origin.

In the example, since the population is equal to the number of jobs, the regional value of jobs per capita is 1.0. To complete the illustrative example, the spatial availability of jobs per capita by origin is:

$$\begin{aligned}
v_1 &= \frac{V_1}{P_1} = \frac{66,833.47}{50,000} = 1.337 \\
v_2 &= \frac{V_2}{P_2} = \frac{133,203.4}{150,000} = 0.888 \\
v_3 &= \frac{V_3}{P_3} = \frac{9,963.171}{10,000} = 0.996
\end{aligned} \tag{12}$$

We can see that population center  $A$  has fewer jobs per capita than the regional benchmark, center  $B$  has more, and center  $C$  is at parity. Remarkably, the spatial availability per capita matches the values of  $a_i$  in Table 1. Appendix A has a proof of the mathematical equivalence between the two measures. It is interesting to notice how [37], [35], as well as this paper, all reach identical expressions starting from different assumptions; this effect is known as *equifinality* [see 62, and 63]. This result means that Shen-type accessibility and 2SFCA can be re-conceptualized as singly-constrained accessibility measures along with the proposed spatial availability measure.

## Why does proportional allocation matter?

We have shown that Shen-type accessibility and spatial availability produce equifinal results when accessibility per-capita is computed. At this point it is reasonable to ask whether the distinction between these two measures is of any importance.

Conceptually, we would argue that the confounded populations in Shen-type accessibility leads to internal inconsistency in the calculation of total opportunities in [35]: this points to a deeper issue that is only evident when we consider the intermediate values of the method. To illustrate, Table 1 shows results of  $a_i$  that are reasonable (and they match exactly the spatial availability per capita). But when we dig deeper, these results mask potentially misleading values for the jobs allocated and the number of jobs taken. For instance, a region with a high jobs:population ratio but a prohibitive transportation network that results in a high cost of travel may yield a high  $a_i$  value. This value, however, can conceal a low *effective opportunity-seeking population* and a proportionally low number of allocated jobs, while also obscuring the magnitude of the population that does and does *not* take jobs.

In addition, the intermediate accessibility values of  $a_i$  (Shen-type measure) may also lead to impact estimates that are deceptive [see 64]. For example, the estimated region-wide cost of travel considering the jobs allocated by  $a_i$  in Table 1 (i.e.,  $Jobs * f(TT)$ ) is as follows:

$$\begin{aligned}
&22,313 \times 15 \text{ min} + 4,979 \times 30 \text{ min} + 0.454 \times 100 \text{ min} \\
&4,979 \times 30 \text{ min} + 22,313 \times 15 \text{ min} + 0.454 \times 100 \text{ min} \\
&4.54 \times 100 \text{ min} + 4.54 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,594 \text{ min}
\end{aligned}$$

In contrast, the estimated region-wide cost of travel according to  $V_i$  in Table 1 is as follows:

$$\begin{aligned}
&59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min} \\
&40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min} \\
&2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min}
\end{aligned}$$

Often referred to as ‘the supply of jobs’ (or simply Hansen-type accessibility) in the Shen-type measure:  $Jobs * f(TT)$  cannot be used to understand the region-wide cost of travel. Recall how we define  $Pop * f(TT)$  as the *effective opportunity-seeking population* ( $P_{ij}^*$ ),  $Jobs * f(TT)$  similarly represents the *effective opportunities allocated* and sums to approximately 56,824 out of a total of 210,000 jobs. Like  $Pop * f(TT)$ , the *effective opportunities allocated* to each origin is only a reflection of the impedance function and not the *actual* number of opportunities allocated to each origin. Therefore, the resulting 1,002,594 min is not a meaningful measure of the cost of travel in the system.

However, since spatial availability allocates the *actual* number of opportunities to each origin; the 3,859,054 min can be used to quantify the system-wide impacts of competitive accessibility in this region. We know spatial availability's output is the number of opportunities at each  $i$  since the combined balancing factors allocate a proportional amount of the total opportunities to each  $i$  such that the number of opportunities allocated to each  $i$  sum to equal the total opportunities in the region.

## Empirical example of Toronto

In this section we illustrate the application of spatial availability through an empirical example. For this, we use full-time employment flows from the Greater Golden Horseshoe (GGH) area in Ontario, Canada. Contained with the GGH is the Greater Toronto and Hamilton (GTHA) which forms the most populous metropolitan regions in Canada and the core urban agglomeration in the GGH.

The GTHA contains the city of Toronto, the most populous city in Canada. The city of Toronto is the focus of this empirical example; it will be used to demonstrate the application of the proposed spatial availability measure along with how it compares to Hansen- and Shen-type measures. We begin this section by explaining the data and then detailing the calculated comparisons.

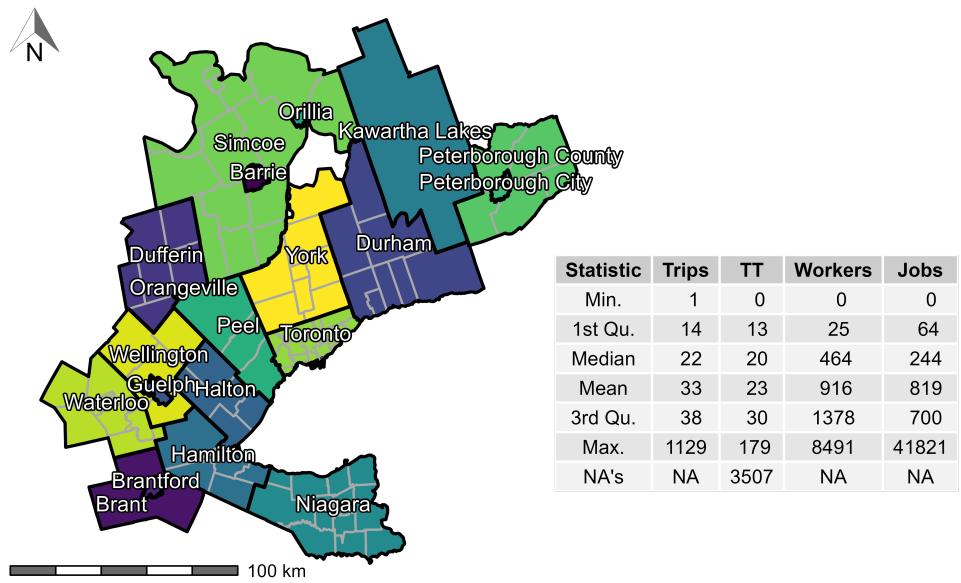
### GGH Data

We obtained full-time employment flows from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 3) [65] every five years. The data set includes origin to destination flows associated with full-time employment trips; the number of jobs ( $n=3,081,885$ ) and workers ( $n=3,446,957$ ) (i.e., the number of originating trips and destination trips) at each origin and destination are represented at the level of Traffic Analysis Zones (TAZ) ( $n=3,764$ ). TAZ are a unit of spatial analysis which are defined as part of the TTS, however, TAZ are commonly used to ascribe production and attraction of trips in the context of transportation planning modelling. In the GGH data set, the TAZ contain on average 916 workers and jobs 819 with more detailed descriptive statistics discussed later. The TTS data is based on a representative sample of between 3% to 5% of households in the GGH and is weighted to reflect the population covering the study area as a whole [65].

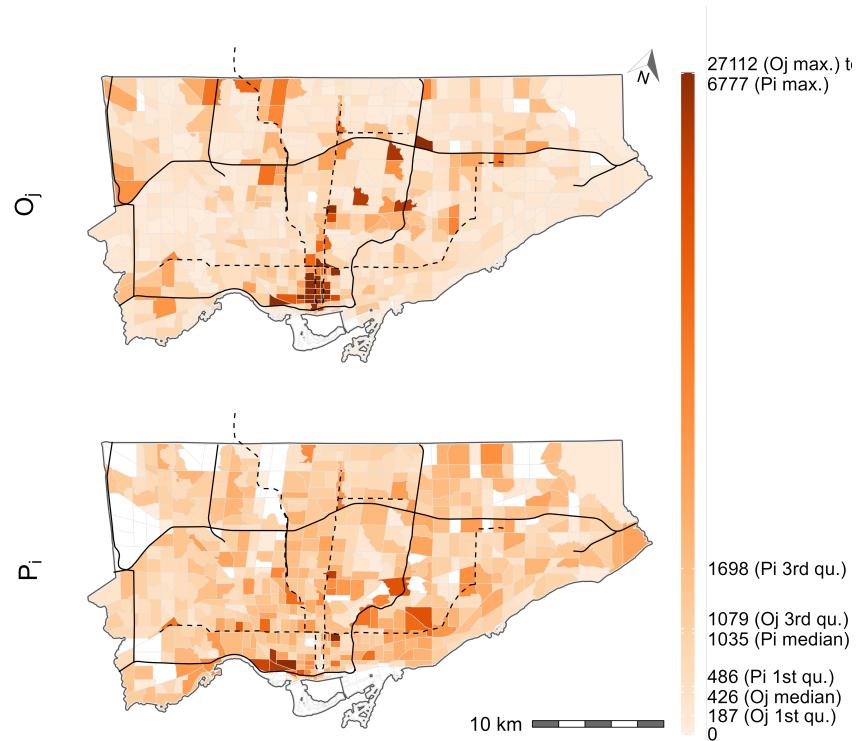
To generate the travel cost for the full-time employment trips, travel times between origins and destinations (i.e., centroids of the TAZ) are calculated for car travel using the R package {r5r} [66] with a street network retrieved from OpenStreetMap. It is also assumed that intra-TAZ trips are equal to 0.1 minutes. For inter-TAZ trips, a 3 hr travel time threshold was selected as it captures 99% of population-employment pairs (see the travel times summarized in Figure 3). This method does not account for traffic congestion or modal split, which can be estimated through other means [e.g., 67,68]. For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions. All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

### Spatial employment characteristics in Toronto

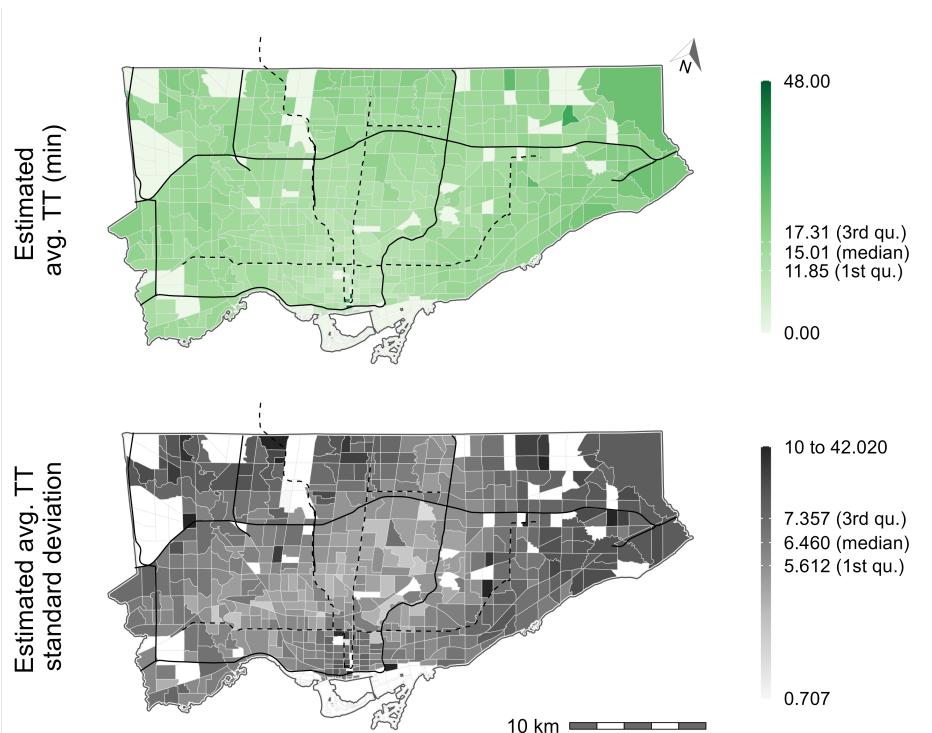
As mentioned, the focus of this empirical example is on the city of Toronto. It is the largest city in the GGH and represents a significant subset of workers and jobs in the GGH; 22% of workers in the GGH live in Toronto and 25% of jobs that these workers



**Fig 3.** TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).



**Fig 4.** Spatial distribution of full-time jobs (top) and full-time working population (bottom) at each TAZ for Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs.



**Fig 5.** Spatial distribution of full-time working population to jobs ratio (top) and car travel time to jobs estimated using R5R (bottom) for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. White TAZ represent a TAZ with no workers thus no travel time for the top plot, for the bottom plot they represent no travel time TAZ and TAZ with only 1 travel time.

take are located within Toronto. The spatial distribution of jobs and workers is shown  
 in Figure 4. It can be seen that a large cluster of jobs can be found in the central  
 southern part of Toronto (the downtown core). Spatial trends in the distribution of  
 workers is more even relative to the distribution of jobs.

Next, the spatial distribution of the estimated car travel time (green) and the  
 associated standard deviation (grey) is visualized in Figure 5. It can be seen that the  
 car travel time is lower within the downtown core and, unexpectedly higher as the TAZ  
 is further from the downtown core. These travel time estimations are to be expected, as  
 these car travel time are calculated using an uncongested OpenStreetMaps road network  
 from the centroid of origin TAZ to destination TAZ. Since within Toronto trips are only  
 considered, trips which originate from the center of Toronto, an area with high job  
 density, relatively closer proximity to all other Toronto TAZ, and high road connectivity,  
 travel times are lower than outside in areas further from the downtown core. In terms of  
 the variability of the travel times, the center TAZ of Toronto have lower variability than  
 TAZ closer to the borders of Toronto. Trends from both plots indicate that trips  
 originating from within the center of Toronto are shorter and more similar in length  
 than trips originating from closer to the border of Toronto.

Nonetheless, the point of these visualizations is to demonstrate the spatial  
 distribution of worker and job data in the city of Toronto to contextualize spatial  
 availability and Shen- and Hansen- type measures.

## Calibration of an impedance function for Toronto

In the synthetic example introduced before, we used a negative exponential function  
 with the parameter reported by [35]. For the empirical Toronto data set, we calibrate an  
 impedance function on the trip length distribution (TLD) of commute trips. Briefly, a  
 TLD represents the proportion of trips that are taken at a specific travel cost (e.g.,  
 travel time); this distribution is commonly used to derive impedance functions in  
 accessibility research [69–71].

As mentioned, the calculations are undertaken for the city of Toronto using only the  
 employed population in the city and jobs taken by residents of Toronto. Specifically,  
 edge trips are not included, such as trips originating in Toronto but finishing outside of  
 Toronto and trips originating outside of Toronto but finishing in Toronto. The empirical  
 and theoretical TLD for this Toronto data set are represented in the top-left panel of  
 Figure 6. Maximum likelihood estimation and the Nelder-Mead method for direct  
 optimization available within the `{fitdistrplus}` package [72] were used. Based on  
 goodness-of-fit criteria and diagnostics, the normal distribution was selected (see Figure  
 6).

For reference, the normal distribution is defined in Equation (13), where we see that  
 it depends on a mean parameter  $\mu$  and a standard deviation parameter  $\sigma$ . The  
 estimated values of these parameters are  $\mu = 14.169$  and  $\sigma = 7.369$ .

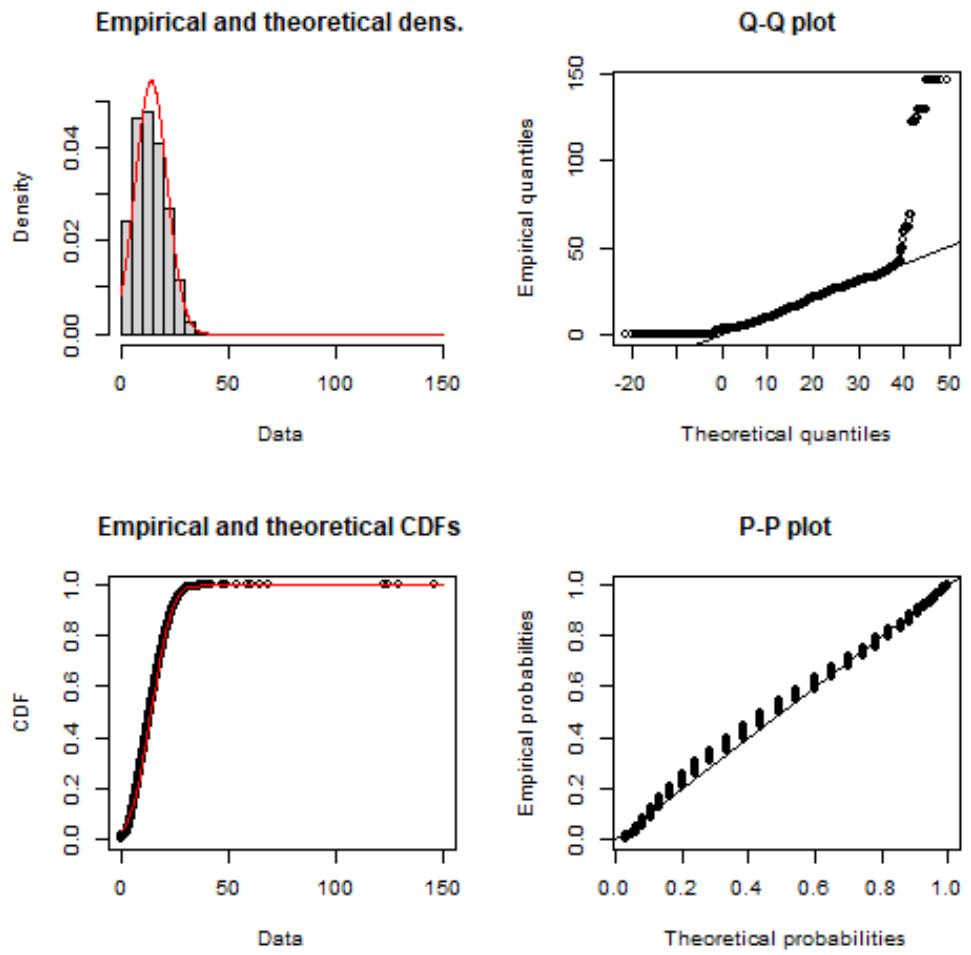
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad (13)$$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

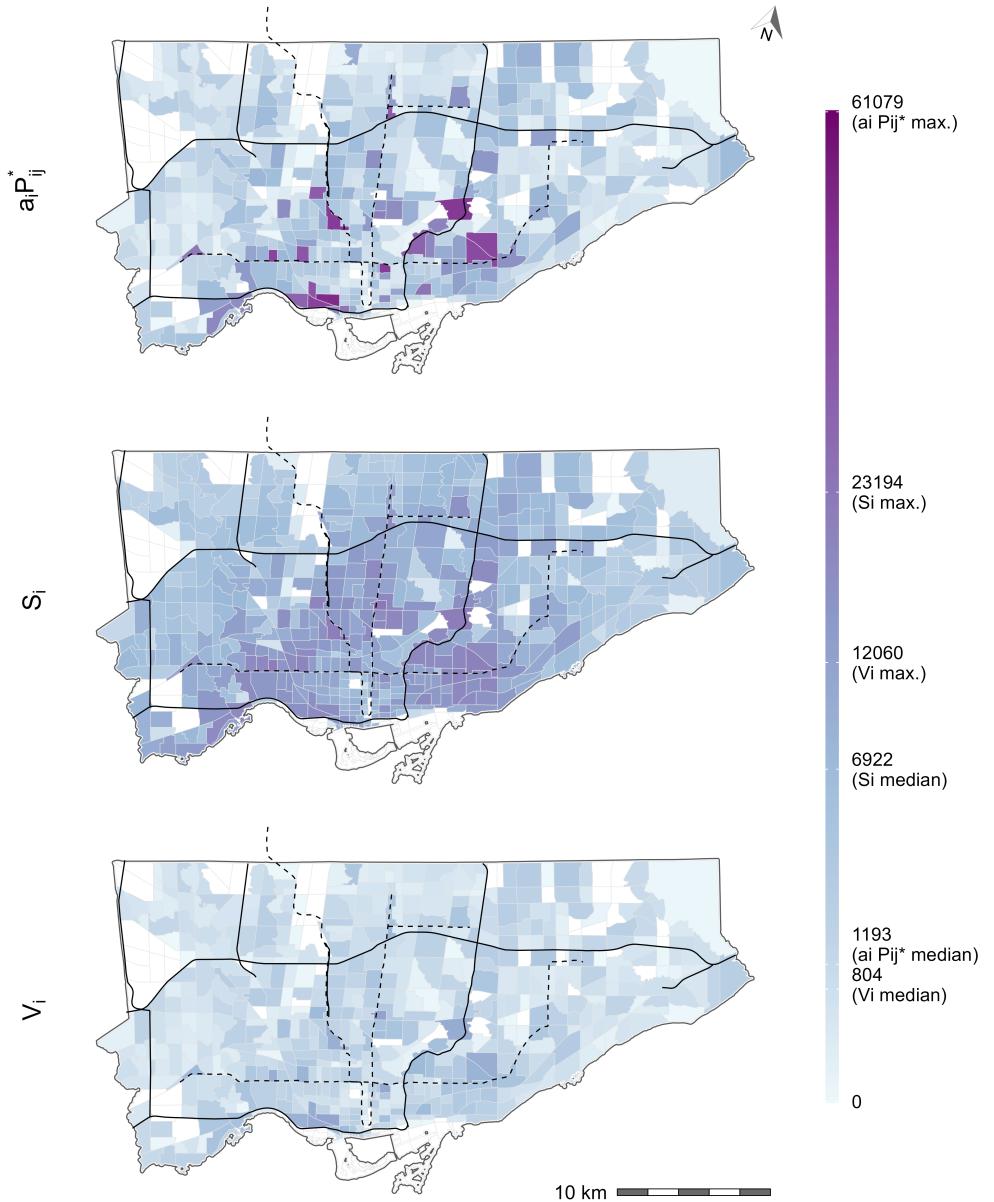
## Accessibility and spatial availability of jobs in Toronto

### Absolute opportunity values

Figure 7 contains the number of jobs accessible using Shen-type accessibility,  
 Hansen-type accessibility, and the number of jobs *available* using the spatial availability



**Fig 6.** Car trip length distribution and calibrated normal distribution impedance function (red line) with associated Q-Q and P-P plots. Based on the estimated car travel times for full-time employment and workers in Toronto from the TTS 2016.

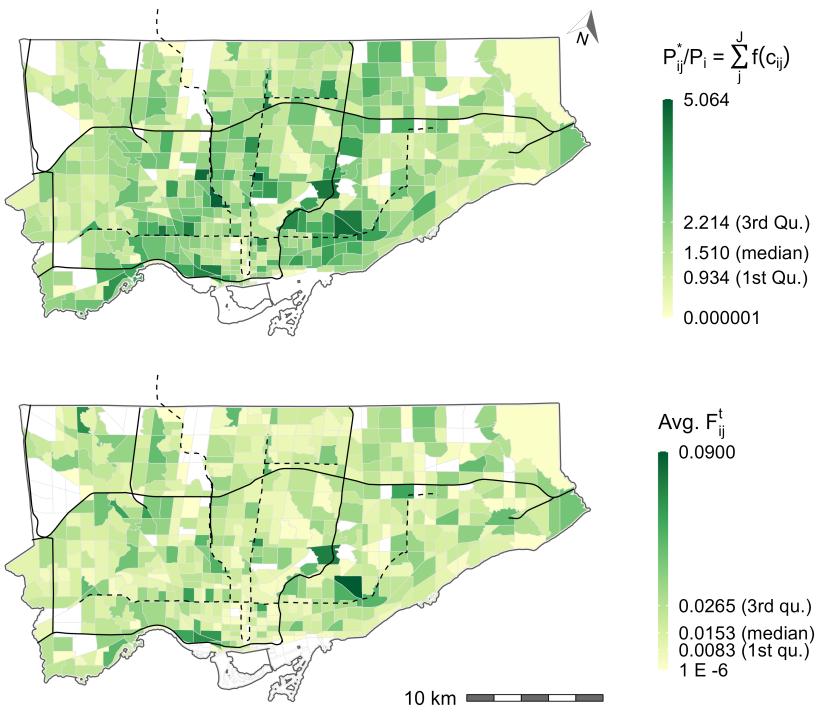


**Fig 7.** Estimated accessibility to jobs (# of jobs) in Toronto according to Shen-type measure multiplied by the effective opportunity-seeking population (top), Hansen-type measure (middle), and spatial availability (bottom). Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values. Legend scale is square root transformed to effectively visualize the spread range.

measure. The values from all these measures are represented on the same axis as they measure the absolute value of *jobs* accessible to the workers in the origin. In the top plot, the Shen-type accessibility is multiplied by the *effective opportunity-seeking population* to yield a value that corresponds to absolute number of accessible jobs (considering competition) according to Shen's definition. In the middle plot, the Hansen-type accessibility is an unconstrained case of accessibility in which all jobs which are in-reach of each origin (according to the impedance function); each value corresponds to the number of jobs which can be reach at each origin assuming no competition. Lastly, in the bottom plot, the proposed spatial availability measure is a constrained case of accessibility which yields the number of jobs, at each origin, considering competition from the population in nearby origin and the relative travel cost (according to the impedance function).

What is notable about the bottom plot is that the proportional allocation mechanism of spatial availability ensures that the job availability value for each origin all sums to the city-wide total of 769,231 jobs (i.e., the number of destination flows from Toronto origins to Toronto destinations). The number of accessible jobs at each origin can therefore be interpreted as the number of *available* jobs to each origin based on the relative travel behaviour and density of competition for jobs (i.e., worker population). A proportion of each of the 769,231 jobs in Toronto are only allocated once to each origin. In terms fo the middle plot, the city-wide total for Hansen-type accessibility is 4,366,743 jobs, which as a value is meaningless since the measure is unconstrained; it represents the sum of opportunities that have been counted anywhere from 1 to many times depending on the impedance function. As previously discussed, unconstrained counting of the same opportunity by all origins is not an issue if the opportunity itself is non-exclusive, but since one job can only be given to one worker (especially since the worker and job data is derived from origin-destination flows), it is inappropriate to use unconstrained measures to capture employment characteristics. Comparing the middle and bottom plots, it is evident that the unconstrained counting of opportunities (Hansen-type) results in absolute values that are higher throughout the city, particularly in TAZ that are in proximity to high job density (recall Figure 4). These same trends are not present in the spatial availability bottom plot, as the absolute value is lower than Hansen-type accessibility as the proximity to high job density and competition from worker density is proportionally metered; the resulting values are thus lower than the middle plot and reflect the spatial distribution trends of both the workers and job density (recall Figure 4).

Lastly, the top plot that visualizes the *absolute* Shen-type measure (as understood by Shen's definition of  $P_i$  being equal to  $P_{ij}^*$  sums to the city-wide value of 2,117,774 by multiplying  $a_i$  by the *effective opportunity-seeking population*  $P_{ij}^*$  (i.e., the denominator of the rate ). This plot thus demonstrates how confounding  $P_i$  with  $P_{ij}^*$  yields an *incorrect* number of competitively accessible jobs: it is evidently incorrect because the sum of  $a_i P_{ij}^*$  greatly exceeds the city-wide total of workers (i.e.,  $2,117,774 > 769,231$ ). To the authors' knowledge, literature has not attempted to convert Shen-type accessibility to the absolute value of accessible jobs in the way demonstrated in the top plot: we suspect this is the case because of the ambiguous definition that conflates  $P_{ij}^*$  with  $P_i$ . If  $a_i$  is multiplied by  $P_i$ , it yields the same value as  $V_i$ , but since the definition of Shen-type measure is equivocal doing so is not clear since the denominator of  $a_i$  (which is a rate) is *not*  $P_i$ . The resulting plot, spatially, is similar to spatial availability (bottom plot) but certain TAZ have exceptionally high values in an inconsistent way. This is because  $a_i$  uses the impedance function values for both access to jobs (numerator) and the competition from neighboring workers (denominator  $P_{ij}^*$ ) to adjust their impact: using  $P_{ij}^*$  does not *consistently* isolate the absolute value of accessible jobs. However, if  $a_i$  is multiplied by  $P_i$  it yields the same values at  $V_i$  (bottom plot) (the



**Fig 8.** The ratio of the effective opportunity seeking population to the population (top) and the average spatial availability's balancing factor (Equation (9)) (bottom) for Toronto TAZ. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

proof for mathematically equivalency is in Appendix A). As also mentioned earlier, the formulation of the denominator and numerator of  $a_i$  is ambiguous so to presume that multiplying it by  $P_i$  would disintangle the rate and yield the absolute value of accessible and available (i.e., considering competition) jobs is unclear.

### Internal values

Carrying on the discussion on how to retrieve the absolute value of *available* jobs using the Shen-type measure ( $a_i$ ), Figure 8 highlights how the differences between  $P_i$  and  $P_{ij}^*$  are not uniform across space; the values at each origin are equivalent to  $\sum_j f(c_{ij})$ . Recall,  $P_i$  is the number of workers at each TAZ (city-wide sum of 769,231) while  $P_{ij}^*$  is the number of workers who seek jobs (city-wide sum of 1,770,609) in that TAZ based on their travel behaviour.  $P_{ij}^*$  is an internal value of  $a_i$  and the top plot presents the ratio of  $P_{ij}^*$  to  $P_i$  which reflects how the effective opportunity-seeking population is sometimes inflated (i.e., impedance values is greater than 1) and others deflated (i.e., impedance value is less than 1) by the Shen-type measure ( $a_i$ ). As such, using  $P_{ij}^*$  to untangle the absolute job availability from  $a_i$  instead of  $P_{ij}^*$  can lead to exaggerating the total travel time in the city since it does not represent the *actual* number of workers but the *effective* number of workers. For instance, when trying to calculate the city-wide travel time using  $a_i P_{ij}^*$ , Shen-type accessibility yields 499,740.1 [h] instead of the city-wide travel time of 183,736.8 [h] that corresponds to the *absolute* (i.e., the total number of jobs in the city is preserved) number of available jobs from  $V_i$ . The absolute number of opportunities cannot be easily disentangled from  $a_i$ .

By contrast, not only are the absolute values a direct result of  $V_i$ , the internal

combined balancing factor  $F_{ij}^t$  (Equation (9)) can be used for analysis. The bottom plot shows the average  $F_{ij}^t$  for each TAZ which is the proportional allocation mechanism of opportunities to origins in the  $V_i$  calculation. Practically, the visualized values corresponds to the average *proportion* of opportunities available that are claimed by the zone based on travel behaviour and population competition for opportunities. These values can allow the analyst to understand the magnitude of the *proportion of opportunities* that the origin TAZ is assigned based on the opportunities located at reachable destination TAZ. For instance, the TAZ with the maximum value of 0.090 has many origin to destination trips (112 trips, upper 3rd quantile), many workers (5,538 workers, upper 3rd quantile), and is located centrally within Toronto. Averaging  $F_{ij}^t$  demonstrates that this TAZ claims on average a high proportion of jobs from reachable TAZs. This does not necessarily mean TAZ with a high  $V_i$  have an exceptionally high average  $F_{ij}^t$ ; for instance, many TAZ around the downtown core have high  $V_i$  values but do not have exceptionally high average  $F_{ij}^t$ . The average  $F_{ij}^t$  can thus be used to identify relatively “greedy” areas that could possibly withstand reductions in availability, if that meant increasing spatial availability in areas with a deficit of jobs available. The balancing factor is an interesting feature of spatial availability which opens up avenues for future analysis; alas, there does not seem to be an equivalent for the Shen-type measure.

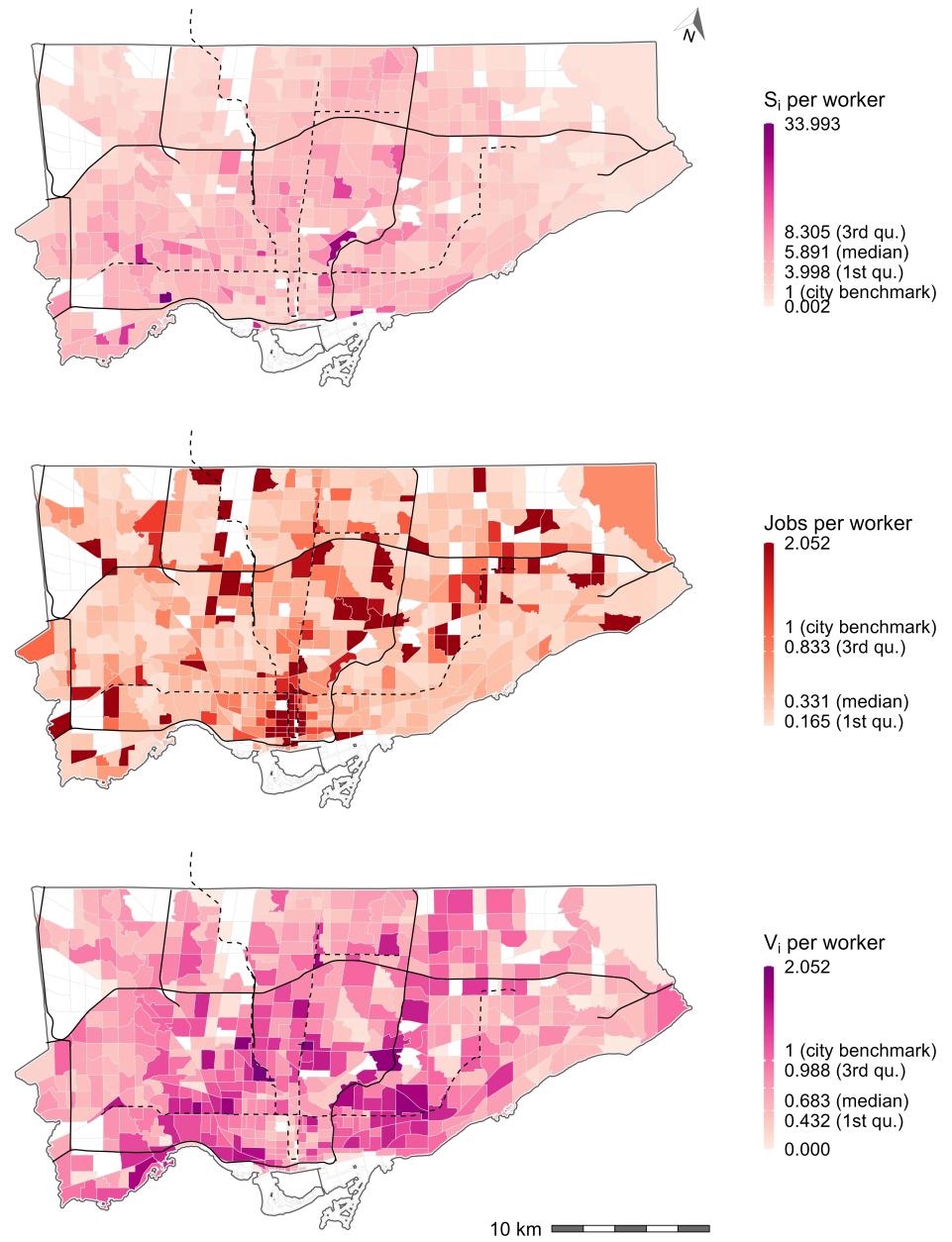
### Benchmarking opportunity availability

Figure 9 presents the number of jobs per capita for Hansen-type accessibility (top plot), the raw number of jobs per capita (middle plot), and the spatially available jobs per capita (bottom plot). In addition to clarifying the meaning of internal values, spatial availability can also be divided by population at each origin and expressed as a rate: this rate can be used as a benchmark for equity analysis and compared directly to the raw number of jobs per capita.

The bottom plot features a value which is mathematically equivalent to Shen-type measure, but with stronger interpretability thanks to the proportional allocation mechanism. This mechanism makes clear that all the opportunities are allocated proportionally to origins, which improves interpretability since the  $V_i$  values are the absolute value of *opportunity availability*. The value can thus be directly divided by the population at the origin and expressed as opportunities per capita. When spatial availability is compared to Hansen-type measure (top plot), dividing the output by population directly yields a more difficult to interpret number of *unconstrained* accessible jobs per capita. For instance, the median light-pink shaded TAZ corresponds to approximately 5.89 unconstrained accessible jobs per capita; this value is difficult to intercept, because as discussed in the introduction, jobs are *exclusive* opportunity types so their accessibility value should take into consideration competition.

The bottom plot also displays the spatially available jobs per capita. It can be interpreted as a benchmark its values can be compared directly to the raw number of jobs per capita (middle plot) since the total number of opportunities are preserved (and the population, in this case, is equivalent to the number of opportunities). For instance, a TAZ with a  $v_i > 1$  have more *available jobs* (based on travel behaviour and competition) than their working population. This TA has sufficient employment opportunities (under the assumptions of the input data), while TAZ with a  $v_i < 1$  do not have sufficient employment opportunities. From an equity perspective,  $v_i$  can be used to target where residential housing, job opportunities, and/or transportation system improvements should be created.

For TAZ with  $v_i$  values significantly greater than 1 (dark pinks), constructing more residential housing for the type of workers who occupy the *available jobs* in the proximate TAZ should be considered. Assuming the input data is correct, increasing the



**Fig 9.** Hansen-type accessible jobs per capita (top), number of jobs to population ratio (middle), and spatially available jobs per capita (bottom) for Toronto. The city benchmark corresponds to total number of jobs in Toronto divided by the number of workers in Toronto, since this is equal the value is 1. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

competition in the area will decrease the  $v_i$  score, but it can be decreased up to threshold of  $v_i = 1$ . For TAZ with  $v_i$  values significantly less than 1 (light pinks), constructing more employment opportunities for the type of workers who live in proximate TAZ and/or prioritizing transportation network improvements to create more favourable travel time conditions.

Depending on the raw jobs per worker ratio, different approaches are appropriate. For instance, adding more residential locations near the downtown core (bottom center on the bottom plot) could be a good approach to increasing  $v_i$  as there is already a high jobs per worker ratio (middle plot). However, doing so will decrease the  $v_i$  availability in areas near the border of the city, so in addition to doing so, adding more employment opportunities to areas with low raw jobs per worker ratio and low  $v_i$  is needed. In addition to these changes, the travel time landscape would also influence the resulting  $v_i$  score, so transportation network improves to areas with low  $v_i$  could also be considered. This is to say,  $v_i$  is dependent on the magnitude and spatial distribution of residential housing, job opportunities, and transportation system so the region could be optimized to achieve thresholds of specific  $v_i$  values and thus the difference in residential housing, job opportunities, and transportation system can become policy targets. It should also be kept in mind, that though  $v_i = 1$  and the comparison to the raw jobs per worker values can be used for policy planning,  $v_i$  can easily be transformed back to  $V_i$  to understand the magnitude of the job availability within that origin.

## Conclusion

In this paper we show how a widely used measure of accessibility with competition (Shen-type accessibility) obscures some important internal values of opportunities taken. This is caused by confounding the population of zones with the *effective opportunity-seeking population*. We then propose an alternative derivation of accessibility with competition that we call spatial availability. This measure ensures that opportunities are allocated in a proportional way and are preserved in the regional total. We also show that spatial availability and Shen-type accessibility are equifinal: formally the equations are the same (along with 2SFCA) and can be considered as singly-constrained measures.

Spatial availability matters because competition is an important consideration for certain opportunity types and conventional Hansen-type accessibility does not capture it [34]. Through its intermediate values, Spatial availability also brings forward a different interpretation to competition than the Shen-type measure. In equity analysis and policy planning, an analyst might be interested in the internal values of their accessibility analysis, for example travel times, and who pays how much for accessibility. The increased interpretability and internal consistency of spatial availability can help to push accessibility analysis forward. Hansen-type measure tends to result in values which are very extreme as a result of multiple-counting opportunities as shown in the empirical example. Multiple-counting may not be an issue if the opportunity-type is non-exclusive: but in the case of employment, where one worker can only take one job, the resulting values are difficult to interpret (though it can be interpreted relatively to comment on urban form). In this paper, we also demonstrated how attempting to disentangle the absolute values of opportunities from the Shen-type measure is difficult as a result of Shen's definition, which confounds the population with the effective-opportunity seeking population.

As demonstrated in this paper, spatial availability increases interpretability by first presenting the absolute value of *available* jobs and then by dividing the available jobs value by the number of working population. This rate is equivalent to Shen-type measure but contains internal values, such as the proportional allocation mechanism, that yield more realistic estimates of opportunities taken. Spatial availability's formulation also includes a set of balancing factors that can be used to better understand the absolute and rate values obtained.

Based on the research presented, we suggest the following guidelines for the application of spatial availability and the topic of future work:

- 1) The Hansen-type accessibility should be used when opportunities are non-exclusive. When opportunities are perfectly exclusive (i.e., 1 spot for 1 person), spatial availability (i.e., accessibility with competition) should be used.
- 2) Shen-type accessibility can be used to compute the availability of jobs (the rate and the absolute values if the original definition is corrected), however, if the analyst is interested in internal values and secondary analysis of the results, spatial availability should be considered.
- 3) With the renewed interpretability of what the absolute *opportunity availability* is at each origin, the spatial availability per capita  $v_i$  value of 1 can be used as a policy goal. For areas with a value below 1, targeted increases to the quantity of opportunities, residential housing, and transportation system improvements can be considered such that the number of *available jobs* per capita in the zone is at least equal to 1. Since spatial availability per capita implicitly preserves the number of opportunities in the region, it can be directly compared to the region's raw jobs to population ratio to inform policy. Additionally, the absolute values of

spatial availability can be used to understand the magnitude of the opportunity  
availability deficit (or surplus).  
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- 4) Spatial availability per capita can also be compared directly to other regions as  
done by literature using Shen-type measure/2SFCA [9,e.g., 73,74]. However, as a  
result of the renewed interpretation, the magnitude of *spatially available*  
opportunities can be quantified.  
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- 5) Lastly, since opportunities are preserved, many new avenues of analysis can be  
pursued. This is especially important in light of emerging concerns with equity.  
For instance, the population and opportunities can be segmented (i.e., transit  
users, active transportation users, low income, low education, new comers,  
children) and their spatial availability to opportunities can be assessed,  
benchmarked, and corresponding policy to target inequities can be theorized. As  
another example, the combined balancing factor can be analysed to identify which  
populations currently do not seek opportunities because of friction of distance.  
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## Appendix A

In this appendix, we solve spatial per population ( $v_i$ ) for population center  $A$  (Shen's synthetic example as discussed in Section 2.3) to demonstrate the mathematical equivalence of Shen-type accessibility measure ( $a_i$ ) and spatial availability per person ( $v_i$ ). The demonstration is shown in the following four steps.

**First step:** the population-based balancing factor  $F_i^p$  used in  $V_i$  is defined as:

$$F_i^p = \frac{P_i^\alpha}{\sum_i^N P_i^\alpha}$$

For population center  $A$ ,  $F_A^p$  is equal to:

$$F_A^p = \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha}$$

**Second step:** the impedance-based balancing factor  $F_{ij}^c$  in  $V_i$  is defined as:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=A}^N f(c_{ij})}$$

In this synthetic example, combinations of workers from population center  $A$  are permitted to go to all employment centers (1, 2, 3), so their relative impedance value is experienced in all of the nine origin-destination trip combinations. Therefore, all nine  $F_{ij}^c$  are computed as follows, since they all consider the impact of population center  $A$  trip combinations (i.e., either  $A1$ ,  $A2$ ,  $A3$ ).

$$F_{A1}^c = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{B1}^c = \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{C1}^c = \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{A2}^c = \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$F_{B2}^c = \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$F_{C2}^c = \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$F_{A3}^c = \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$$

$$F_{B3}^c = \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$$

$$F_{C3}^c = \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$$

**Third step:** when the balancing factors ( $F_i^p$  and  $F_{ij}^c$ ) concerning population center  $A$  are assembled and divided by  $P_i$ , the denominators of the denominators cancel out. The following equation is the assigned general form, with the strike-through indicating which values cancel out:

$$v_i = \sum_j \frac{O_j}{P_i^\alpha} \frac{\frac{P_i^\alpha}{\sum_i^N P_i^\alpha} \cdot \frac{f(c_{ij})}{\sum_t^N f(c_{ij})}}{\sum_i^N \frac{P_i^\alpha}{\sum_i^N P_i^\alpha} \cdot \frac{f(c_{ij})}{\sum_t^N f(c_{ij})}}$$

To demonstrate that the strike-through terms cancel out, the following following terms for  $v_A$  are subbed into the general form:

$$\begin{aligned} v_A &= \frac{O_1}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} \right) + \\ &\quad \frac{O_2}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} \right) + \\ &\quad \frac{O_3}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} \right) \end{aligned}$$

$v_A$  simplifies to the following:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{\cancel{P_A^\alpha} \cdot f(c_{A1})}{\cancel{P_A^\alpha} \cdot f(c_{A1}) + \cancel{P_A^\alpha} \cdot f(c_{B1}) + \cancel{P_A^\alpha} \cdot f(c_{C1})} \right) + \frac{O_2}{P_A^\alpha} \left( \frac{\cancel{P_A^\alpha} \cdot f(c_{A2})}{\cancel{P_A^\alpha} \cdot f(c_{A2}) + \cancel{P_A^\alpha} \cdot f(c_{B2}) + \cancel{P_A^\alpha} \cdot f(c_{C2})} \right) + \frac{O_3}{P_A^\alpha} \left( \frac{\cancel{P_A^\alpha} \cdot f(c_{A3})}{\cancel{P_A^\alpha} \cdot f(c_{A3}) + \cancel{P_A^\alpha} \cdot f(c_{B3}) + \cancel{P_A^\alpha} \cdot f(c_{C3})} \right)$$

Notice, the denominator of the denominator is the same as the denominator of the numerator for each  $j$  ( $j=1$ ,  $j=2$ , and  $j=3$ ). Now, we remove those strike-through terms (as indicated at the beginning of this step) and re-write  $v_a$  as follows:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} \right) + \frac{O_2}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} \right) + \frac{O_3}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

**Fourth step:** We can now cancel out one more term,  $P_A^\alpha$  as follows:

$$v_A = \frac{O_1}{\cancel{P_A^\alpha}} \left( \frac{\cancel{P_A^\alpha} \cdot f(c_{A1})}{\cancel{P_A^\alpha} \cdot f(c_{A1}) + \cancel{P_A^\alpha} \cdot f(c_{B1}) + \cancel{P_A^\alpha} \cdot f(c_{C1})} \right) + \frac{O_2}{\cancel{P_A^\alpha}} \left( \frac{\cancel{P_A^\alpha} \cdot f(c_{A2})}{\cancel{P_A^\alpha} \cdot f(c_{A2}) + \cancel{P_A^\alpha} \cdot f(c_{B2}) + \cancel{P_A^\alpha} \cdot f(c_{C2})} \right) + \frac{O_3}{\cancel{P_A^\alpha}} \left( \frac{\cancel{P_A^\alpha} \cdot f(c_{A3})}{\cancel{P_A^\alpha} \cdot f(c_{A3}) + \cancel{P_A^\alpha} \cdot f(c_{B3}) + \cancel{P_A^\alpha} \cdot f(c_{C3})} \right)$$

Which can be expressed as:

$$v_A = \left( \frac{O_1 \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})} + \frac{O_2 \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})} + \frac{O_3 \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_B^\alpha \cdot f(c_{B3}) + P_C^\alpha \cdot f(c_{C3})} \right)$$

And generalized to be formally identical to the Shen-type accessibility measure with competition as follows:

$$v_i = a_i = \sum_j \frac{O_j \cdot f(c_{ij})}{\sum_i P_i \cdot f(c_{ij})}$$

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# Introducing spatial availability, a singly-constrained measure of competitive accessibility

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## Abstract

Accessibility indicators are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability, mainly by introducing competition. In this paper, we demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call *spatial availability*. This measure results from using balancing factors that are equivalent to imposing a single constraint on conventional gravity-based accessibility. Further, we demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada.

# Introduction

The concept of accessibility in transportation studies derives its appeal from the combination of the spatial distribution of opportunities and the cost of reaching them [1,2]. Accessibility analysis is employed in transportation, geography, public health, and many other areas, with the number of applications growing [3], especially as mobility-based planning is de-emphasized in favor of access-oriented planning [4–7].

Accessibility analysis stems from the foundational works of [8] and [1]. From these seminal efforts, many accessibility measures have been derived, particularly after the influential work of [9] on spatial interaction<sup>1</sup>. Of these, gravity-type accessibility is arguably the most common; since its introduction in the literature, it has been widely adopted in numerous forms [10–14]. Hansen-type accessibility indicators are essentially weighted sums of opportunities, with the weights given by an impedance function that depends on the cost of movement, and thus measure the *intensity of the possibility of interaction* [1]. This type of accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure [2].

Despite their usefulness, the interpretability of Hansen-type accessibility measures can be challenging [12,15]. Since they aggregate opportunities, the results are sensitive to the size of the region of interest (e.g., a large city has more jobs than a smaller city). As a consequence, raw outputs are not necessarily comparable across study areas [16]. This limitation becomes evident when surveying studies that implement this type of analysis. For example, [17] (in Montreal) and [18] (in Nairobi) report accessibility as the number of health care facilities that can potentially be reached from origins. But what does it mean for a zone to have accessibility to less than 100 facilities in each of these two cities, with their different populations and number of facilities? For that matter, what does it mean for a zone to have accessibility to more than 700 facilities in Montreal, besides being “accessibility rich”? As another example, [19] (in Bogota), [20] (in Montreal), and [21] (in Beijing) report accessibility as numbers of jobs, with accessibility values often in the hundreds of thousands, and even exceeding one million jobs for some zones in Beijing and Montreal. As indicators of urban structure, these measures are informative, but the meaning of one million accessible jobs is harder to pin down: how many jobs must any single person have access to? Clearly, the answer to this question depends on how many people demand jobs.

The interpretability of Hansen-type accessibility has been discussed in numerous studies, including recently by [22], [23], and in greater depth by [24]. As hinted above, the limitations in interpretability are frequently caused by ignoring competition – without competition, each opportunity is assumed to be equally available to every single opportunity-seeking individual that can reach it [23,25,26]. This assumption is appropriate when the opportunity of interest is non-exclusive, that is, if use by one unit of population does not preclude use by another. For instance, national parks with abundant space are seldom used to full capacity, so the presence of some population does not exclude use by others. When it comes to exclusive opportunities, or when operations may be affected by congestion, the solution has been to account for competition. Several efforts exist that do so. In our reckoning, the first such approach was proposed by [27], whereby the distance decay of the supply of employment and the demand for employment (by workers) were formulated under so-called axiomatic assumptions. This approach was then applied by [28] in the context of healthcare, to quantify the availability of general practitioners in Canada. About two decades later, [25] independently re-discovered Weibull’s [27] formula [see footnote (7) in 25] and deconstructed it to consider accessibility for different modes. These advances were

<sup>1</sup>Utility-based measures derive from a very different theoretical framework, random utility maximization

subsequently popularized as the family of Two-Stage Floating Catchment area (2SFCA) methods [29] that have found widespread adoption in healthcare, education, and food systems [30–34].

An important development contained in Shen’s work is a proof that the population-weighted sum of the accessibility measure with competition equates to the number of opportunities available [footnote (7) and Appendix A in 25]. This demonstration gives the impression that Shen-type accessibility allocates *all* opportunities to the origins, however to the authors’ knowledge, it has not interpreted by literature in this way. For instance, [35], [24], and [36] all use Shen-type accessibility to calculate job access but report values as ‘competitive accessibility scores’ or simply ‘job accessibility’. These works do not explicitly recognize that jobs that are assigned to each origin are in fact a proportion of *all* the opportunities in the system. This recognition, we argue, is critical to interpreting the meaning of the final result. Thus, in this paper we intend to revisit accessibility with competition within the context of disentangling how opportunities are allocated. We first argue that Shen’s competitive accessibility misleadingly refers to the total zonal population to equal the travel-cost discounted opportunity-seeking population. This equivocation, we believe, results in a ambiguous interpretation of what Shen-type accessibility represents as the allocation of opportunities to population is masked by the results presenting as rates (i.e., opportunities per capita). We then propose an alternative formulation of accessibility that incorporates competition by adopting a proportional allocation mechanism; we name this measure *spatial availability*. The use of balancing factors for proportional allocation is akin to imposing a single constraint on the accessibility indicator, in the spirit of Wilson’s [9] spatial interaction model.

In this way, the aim of the paper is three-fold:

- First, we aim to demonstrate that Shen-type (and thus [27] accessibility and the popular 2SFCA methods) produce equivocal estimates of opportunities allocated as the result is presented as a rate (i.e., opportunities per capita);
- Second, we introduce a new measure, *spatial availability*, which we submit is a more interpretable alternative to Shen-type accessibility, since opportunities in the system are preserved and proportionally allocated to the population; and
- Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.

Discussion is supported by the use of the small synthetic example of [25] and empirical data drawn from the 2016 Transportation Tomorrow Survey of the Greater Toronto and Hamilton Area in Ontario, Canada. In the spirit of openness of research in the spatial sciences [37,38] this paper has a companion open data product [39], and all code is available for replicability and reproducibility purposes at <https://github.com/soukhova/Spatial-Availability-Measure>.

## Accessibility measures revisited

In this section we revisit Hansen-type and Shen-type accessibility indicators. We adopt the convention of using a capital letter for absolute values (number of opportunities) and lower case for rates (opportunities per capita).

### Hansen-type accessibility

Hansen-type accessibility measures follow the general formulation shown in Equation (1):

$$S_i = \sum_{j=1}^J O_j \cdot f(c_{ij}) \quad (1)$$

where:

- $c_{ij}$  is a measure of the cost of moving between  $i$  and  $j$ . 96
- $f(\cdot)$  is an impedance function of  $c_{ij}$ ; it can take the form of any monotonically 97  
decreasing function chosen based on positive or normative criteria [40]. 98
- $i$  is a set of origin locations ( $i = 1, \dots, N$ ). 100
- $j$  is a set of destination locations ( $j = 1, \dots, J$ ). 101
- $O_j$  is the number of opportunities at location  $j$ ;  $O = \sum_{j=1}^J O_j$  is the total supply 102  
of opportunities in the study region. 103
- $S$  is Hansen-type accessibility as weighted sum of opportunities. 104

As formally defined, accessibility  $S_i$  is the sum of opportunities that can be reached from location  $i$ , weighted down by an impedance function of the cost of travel  $c_{ij}$ . 105  
Summing the opportunities in the neighborhood of  $i$  provides estimates of the number 106  
of opportunities that can *potentially* be reached from  $i$ . Several measures result from 107  
using a variety of impedance functions; for example, cumulative opportunities measures 108  
are obtained when  $f(\cdot)$  is a binary or indicator function [12,e.g., 20,41,42]. Other 109  
measures use impedance functions modeled after any monotonically decreasing function 110  
[e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, 111  
*inter alia*, 43,44–46]. In practice, accessibility measures with different impedance 112  
functions tend to be highly correlated [43,47,48]. 113

Gravity-based accessibility has been shown to be an excellent indicator of the 115  
intersection between spatially distributed opportunities and transportation 116  
infrastructure [3,43,45]. However, beyond enabling comparisons of relative values they 117  
are not highly interpretable on their own [15]. To address the issue of interpretability, 118  
previous research has aimed to index and normalize values on a per demand-population 119  
basis [e.g., 49,50,51]. However, as recent research on accessibility discusses [16,23,e.g., 120  
24,26], these steps do not adequately consider competition. In effect, when calculating 121  
 $S_i$ , every opportunity enters the weighted sum once for every origin  $i$  that can reach it. 122  
This makes interpretability opaque, and to complicate matters, can also bias the 123  
estimated landscape of opportunity. 124

### Shen-type competitive accessibility

To account for competition, the influential works of [25] and [27], as well as the widely 125  
used 2SFCA approach of [29], adjust Hansen-type accessibility with the population in 126  
the region of interest. The mechanics of this approach consist of calculating, for every 127  
destination  $j$ , the population that can reach it given the impedance function  $f(\cdot)$ ; let us 128  
call this the *effective opportunity-seeking population* (Equation (2)). This value can be 129  
seen as the Hansen-type *market area* (accessibility to population) of  $j$ . The 130  
opportunities at  $j$  are then divided by the sum of the effective opportunity-seeking 131  
population to obtain a measure of opportunities per capita, i.e.,  $R_j$  in Equation (3). 132  
This can be thought of as the *level of service* at  $j$ . Per capita values are then allocated 133  
back to the population at  $i$ , again subject to the impedance function as seen in 134  
Equation (4); this is accessibility with competition. 135

$$P_{ij}^* = P_i \cdot f(c_{ij}) \quad (2)$$

$$R_j = \frac{O_j}{\sum_i P_{ij}^*} \quad (3)$$

$$a_i = \sum_j R_j \cdot f(c_{ij}) \quad (4)$$

where:

- $a$  is Shen-type accessibility as weighted sum of opportunities per capita (or weighted level of service). 138  
139
- $c_{ij}$  is a measure of the cost of moving between  $i$  and  $j$ . 140
- $f(\cdot)$  is an impedance function of  $c_{ij}$ . 141
- $i$  is a set of origin locations ( $i = 1, \dots, N$ ). 142
- $j$  is a set of destination locations ( $j = 1, \dots, J$ ). 143
- $O_j$  is the number of opportunities at location  $j$ ;  $O = \sum_{j=1}^J O_j$  is the total supply 144  
of opportunities in the study region. 145
- $P_i$  is the population at location  $i$ . 146
- $P_{ij}^*$  is the population at location  $i$  that can reach destination  $j$  according to the 147  
impedance function; we call this the *effective opportunity-seeking population*. 148
- $R_j$  is the ratio of opportunities at  $j$  to the sum over all origins of the *effective* 149  
*opportunity-seeking population* that can reach  $j$ ; in other words, this is the total 150  
number of opportunities per capita found at  $j$ . 151

[25] describes  $P_i$  as the “*the number of people in location  $i$  seeking opportunities*”. In our view, this is somewhat equivocal and where misinterpretation of the final results may arise. Consider a population center where the population is only willing to take an opportunity if the trip required is less than or equal to 60 minutes. This is identical to the following impedance function:

$$f(c_{ij}) = \begin{cases} 1 & \text{if } c_{ij} \leq 60 \text{ min} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

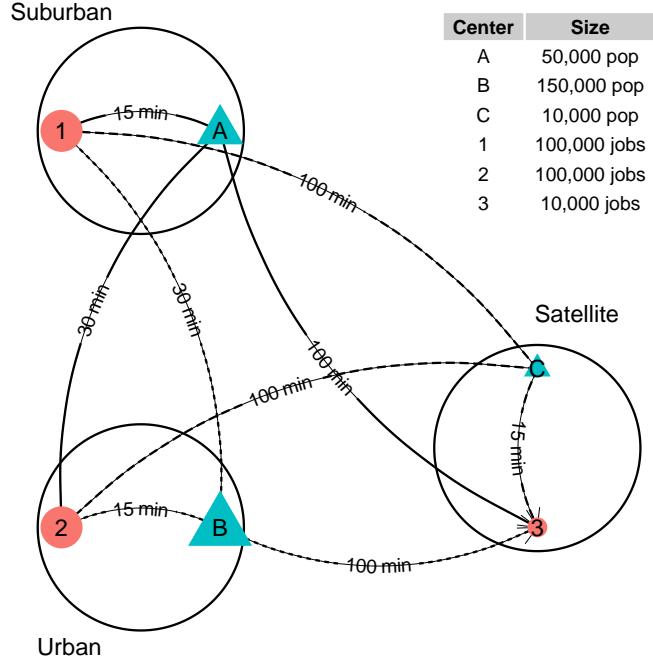
If an employment center is less than 60 minutes away, the population can seek opportunities there (i.e.,  $f(c_{ij}) = 1$ ). But are these people still part of the opportunity-seeking population for jobs located two hours away? Four hours? Ten hours? We assume that they are not because their travel behaviour, as represented by the impedance function would yield  $f(c_{ij}) = 0$ , eliminating them from the effective opportunity-seeking population  $P_{ij}^*$ . We see Shen’s definition as ambiguous because, for the purpose of calculating accessibility, the impedance function defines what constitutes the population that effectively can seek opportunities at remote locations. Thus  $P_i$  should be plainly understood as the population at location  $i$  (as defined above) and not the “*the number of people in location  $i$  seeking opportunities*”. In other words,  $P_i$  and  $P_{ij}^*$  are confounded.

Furthermore, an identical misunderstanding can be described for  $O_j$  which is defined as “*the number of relevant opportunities in location  $j$* ” in [25] (our emphasis).  $O_j$  is adjusted by the same  $f(c_{ij})$  in Equation (4), so the *relevancy* is determined by the travel behaviour associated with the impedance function not purely by  $O_j$  itself. For this reason,  $O_j$  should be understand plainly as the opportunities at location  $j$  (as we also defined them above).

Misunderstanding  $P_i$  and  $O_j$  may lead to a misleading interpretation of the final result  $a_i$ , especially as expressed in Shen’s proof (see Equation (6)).

$$\sum_{i=1}^N a_i P_i = \sum_{j=1}^J O_j \quad (6)$$

Notice, confounding  $P_i$  with the effective opportunity-seeking population and  $O_j$  with the jobs taken may cause us to misunderstand  $a_i$  as “*relevant opportunities*” per



**Fig 1.** Shen (1998) synthetic example with locations of employment centers (in orange), population centers (in blue), number of jobs and population, and travel times.

“people in location  $i$  seeking opportunities”. Instead, as mathematically expressed in the proof,  $a_i$  is a proportion of the opportunities available to the population, since multiplying  $a_i$  by the population at  $i$  and summing for all origins in the system equals to the total number of opportunities in the system. Embedded in  $a_i$  is already the travel behaviour so  $P_i$  and  $O_j$  must be plainly understood as population at  $i$  and opportunities at  $j$  to have Equation (6) hold true.

### Shen’s synthetic example

In this section we use the example in [25] to detail the importance of understanding  $P_i$  and  $O_j$  as simply the population at the origin  $i$  and the opportunities at destination  $j$  respectively. This is critical to understanding how the opportunities are allocated to the population based on the impedance function.

Table 1 contains the information needed to calculate  $S_i$  and  $a_i$  for this example. We use a negative exponential impedance function with  $\beta = 0.1$  as done in [25, see footnote (5)]:

$$f(c_{ij}) = \exp(-\beta \cdot c_{ij})$$

In Table 1, we see that population centers  $A$  and  $B$  have equal Hansen-type accessibility ( $S_A = S_B = 27,292$  jobs). On the other hand, the isolated satellite town of  $C$  has low accessibility ( $S_C = 2,240$  jobs). But center  $B$ , despite its high accessibility, is a large population center.  $C$ , in contrast, is smaller but also relatively isolated and has a balanced ratio of jobs (10,000 jobs) to population (10,000 people). It is difficult from these outputs to determine whether the accessibility at  $C$  is better or worse than that at  $A$  or  $B$ .

The results are easier to interpret when we consider Shen-type accessibility. The results indicate that  $a_A \approx 1.337$  jobs per capita,  $a_B \approx 0.888$ , and  $a_C \approx 0.996$ . The latter value is sensible given the jobs-population balance of  $C$ . Center  $A$  is relatively

**Table 1.** Summary description of synthetic example: Hansen-type accessibility  $S_i$ , Shen-type accessibility  $a_i$ , and spatial availability  $V_i$  with beta = 0.1 (light yellow) and beta = 0.6 (light grey).

Origin	A			B			C		
Dest.	1	2	3	1	2	3	1	2	3
Pop.	50000	50000	50000	150000	150000	150000	10000	10000	10000
Jobs	100000	100000	10000	100000	100000	10000	100000	100000	10000
TT	15	30	100	30	15	100	100	100	15
f(TT)	0.223	0.050	< 0.001	0.050	0.223	< 0.001	< 0.001	< 0.001	0.223
Pop * f(TT)	11156.5	2489.4	2.3	7468.1	33469.5	6.8	0.5	0.5	2231.3
Jobs * f(TT)	22313.0	4978.7	0.5	4978.7	22313.0	0.5	4.5	4.5	2231.3
S_i	27292.2	27292.2	27292.2	27292.2	27292.2	27292.2	2240.4	2240.4	2240.4
a_i	1.337	1.337	1.337	0.888	0.888	0.888	0.996	0.996	0.996
f(TT)	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
Pop * f(TT)	6.170	< 0.001	< 0.001	0.002	18.511	< 0.001	< 0.001	< 0.001	1.234
Jobs * f(TT)	12.341	0.002	< 0.001	0.002	12.341	< 0.001	< 0.001	< 0.001	1.234
S_i	12.343	12.343	12.343	12.343	12.343	12.343	1.234	1.234	1.234
a_i	1.999	1.999	1.999	0.667	0.667	0.667	1.000	1.000	1.000
F^c	0.238	0.238	0.238	0.714	0.714	0.714	0.048	0.048	0.048
F^p	0.817	0.182	< 0.001	0.182	0.817	< 0.001	< 0.001	< 0.001	1.000
F	0.599	0.069	0.001	0.401	0.931	0.003	< 0.001	< 0.001	0.996
V_ij	59900.6	6922.7	10.1	40096.9	93076.0	30.4	2.4	1.3	9959.5
V_i	66833.5	66833.5	66833.5	133203.4	133203.4	133203.4	9963.2	9963.2	9963.2

close to a large number of jobs (more jobs than the population of A). The opposite is true of B. According to [25], the sum of the population-weighted accessibility  $a_i$  is exactly equal to the number of jobs in the region following Shen's proof:

$$\begin{aligned} \sum_{i=1}^N a_i P_i &= \sum_{j=1}^J O_j \\ 50,000 \times 1.3366693 &+ 150,000 \times 0.8880224 \\ + 10,000 \times 0.9963171 &= 210,000 \end{aligned}$$

As mentioned earlier, this property under Shen's definition of  $P_i$  "people in location i" is

*seeking opportunities*" , gives the impression that all jobs sought are allocated to the people located at each origin  $i$ . In other words, Shen defines  $P_i$  to mean  $P_{ij}^*$  (i.e., the effective opportunity-seeking population which is already adjusted by travel behaviour) instead of defining it to simply be the full population at  $i$  (i.e.,  $P_i$ ). As seen in column **Pop \* f(TT)** in Table 1 (i.e.,  $P_{ij}^* = P_i \cdot f(c_{ij})$ ), the number of individuals from population center  $A$  that are *willing to reach* employment centers 1, 2, and 3 are 11,156, 2,489, and 2.27 respectively. Therefore, the total effective opportunity-seeking population at  $A$  is  $P_A^* = \sum_j P_{Aj}^*$ , that is, 13,647.27 people, which is considerably lower than the total population of  $A$  (i.e.,  $P_A = 50,000$  people). Demonstrated as follows, using  $P_{ij}^*$  in the calculations associated with this proof results in only 56,834.59 jobs being allocated to the population, instead of the nominal number of jobs in the region that is over three times this number (i.e., 210,000 jobs).

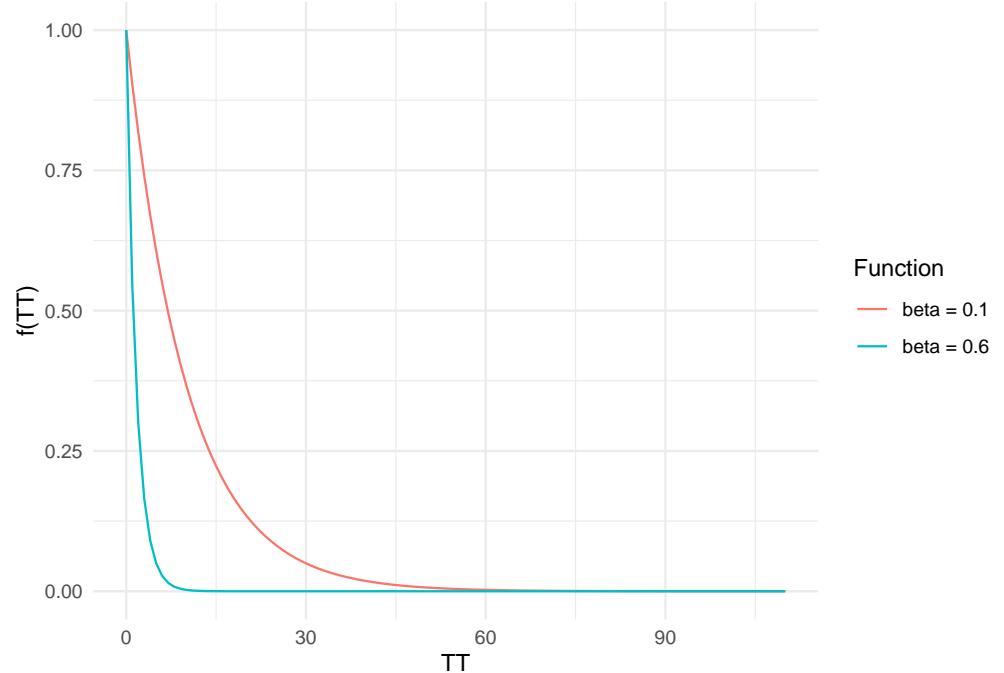
$$\begin{aligned}\sum_{i=1}^N a_i P_{ij}^* &= \\ (11,156.51 + 2,489.35 + 2.26) \times 1.3366693 &\\ +(7,468.06 + 33,469.52 + 6.81) \times 0.8880224 &\\ +(4.54 + 4.54 + 2,231.20) \times 0.9963171 &\approx 56,834.59\end{aligned}$$

Furthermore, even when Shen's  $P_i$  is understood plainly as the total population at  $i$ , the meaning of the proof may still be ambiguous. The proof can still give the impression that all jobs are allocated to the total population since total population ( $\sum_{i=1}^N P_i$ ) goes into the equation and total jobs ( $\sum_{j=1}^J O_j$ ) in the region is the result. However, this impression is incomplete since it does not reflect the amount of population which takes jobs and the number of people being considered for jobs; these magnitudes are a product of being weighted down by the impedance function. These magnitudes are not obvious from  $a_i$  is because the result is presented as a rate (i.e., opportunities per capita).

Let us consider a modification to the travel behaviour of the example discussed to illustrate how the presentation of  $a_i$  as a rate obscures the magnitude of the effective opportunity-seeking population. We modify the example by increasing the  $\beta$  to 0.6 (compared to the previous value of 0.1; see Figure 2). This modification increases the cost of travel and thus the impedance function, which is an expression of the population's relative willingness to travel to opportunities, reflects a population which is relatively less willing to travel to opportunities further away compared to the previous  $\beta$  value. The Hansen-type and Shen-type values are presented in the yellow rows of Table 1.

As expected, Hansen-type accessibility drops quite dramatically after this  $\beta$  modification: the friction of distance is so high that few opportunities are within reach. In contrast, Shen-type accessibility converges to the jobs:population ratio (i.e., origin  $A$  is  $\frac{100,000}{50,000} = 2$ ). This is explained by the way the impedance function excludes the population in droves, thus reducing the competition for jobs: as seen in Table 1, the effective opportunity-seeking population from  $A$  is only about equal to 6.17; likewise, the number of jobs at 1 weighted by the impedance is only 12.341. In other words, competition is low because jobs are expensive to reach, but those willing to reach jobs enjoy relatively high accessibility (in the limit, the jobs/population ratio). On the other hand, the accessibility is effectively zero for those in the population prevented by the impedance from reaching any jobs.

In what follows, we propose an alternative derivation of [25] accessibility with competition that explicitly clarifies the opportunities allocated to the *effective opportunity-seeking population* within its formulation. Hence, the results are not only more interpretable, but also extend the potential of accessibility analysis.



**Fig 2.** Comparison of two negative exponential impedance functions used in the synthetic example. The x-axis represents the travel time (in mins) and the y-axis represents the impedance function at each travel time.

## Introducing spatial availability: a singly-constrained measure of accessibility

In brief, we define the *spatial availability* at  $i$  ( $V_i$ ) as the proportion of all opportunities  $O$  that are allocated to  $i$  from all destinations  $j$ :

$$V_i = \sum_{j=1}^N O_j F_{ij}^t$$

where:

- $F_{ij}^t$  is a balancing factor that depends on the population and cost of movement in the system.
- $O_j$  is the number of opportunities at  $j$ .
- $V_i$  is the number of spatially available opportunities from the perspective of  $i$ .

The general form of spatial availability is also as a sum, and the fundamental difference with Hansen- and Shen-type accessibility is that opportunities are allocated proportionally. Balancing factor  $F_{ij}^t$  consists of two components: a population-based balancing factor  $F_i^p$  and an impedance-based balancing factor  $F_{ij}^c$  which, respectively, allocate opportunities to  $i$  in proportion to the size of the population of the different competing centers (the mass effect of the gravity model) and the cost of reaching opportunities (the impedance effect). In the next two subsections, we explain the intuition behind the method before defining it in full.

## Proportional allocation by population

According to the gravity modelling framework, the potential for interaction depends on the mass (i.e., the population) and the friction of distance (i.e., the impedance function). We begin by describing the proposed proportional allocation mechanism based on demand by population. Recall, the total population in the example is 210,000. The proportion of the population by population center is as follows:

$$F_A^p = \frac{50,000}{210,000}$$

$$F_B^p = \frac{150,000}{210,000}$$

$$F_C^p = \frac{10,000}{210,000}$$

Jobs are allocated proportionally from each employment center to each population center depending on their population sizes as per the balancing factors  $F_i^p$ . In this way, employment center 1 allocates  $100,000 \cdot \frac{50,000}{210,000} = 23,809.52$  jobs to  $A$ ;  $100,000 \cdot \frac{150,000}{210,000} = 71,428.57$  jobs to  $B$ ; and  $100,000 \cdot \frac{10,000}{210,000} = 7,142.857$  jobs to  $C$ . Notice how this mechanism ensures that the total number of jobs at employment center 1 is preserved at 100,000.

We can verify that the number of jobs allocated is consistent with the total number of jobs in the system:

Employment center 1 to population centers A, B, and C:  
 $100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$

Employment center 2 to population centers A, B, and C:  
 $100,000 \cdot \frac{50,000}{210,000} + 100,000 \cdot \frac{150,000}{210,000} + 100,000 \cdot \frac{10,000}{210,000} = 100,000$

Employment center 3 to population centers A, B, and C:  
 $10,000 \cdot \frac{50,000}{210,000} + 10,000 \cdot \frac{150,000}{210,000} + 10,000 \cdot \frac{10,000}{210,000} = 10,000$

In the general case where there are  $N$  population centers in the region, we define the following population-based balancing factors in Equation (7):

$$F_i^p = \frac{P_i^\alpha}{\sum_{i=1}^N P_i^\alpha} \quad (7)$$

Balancing factor  $F_i^p$  corresponds to the proportion of the population in origin  $i$  relative to the population in the region. On the right hand side of the equation, the numerator  $P_i^\alpha$  is the population at origin  $i$ . The summation in the denominator is over  $i = 1, \dots, N$ , and adds up to the total population of the region. Notice that we incorporate an empirical parameter  $\alpha$ . The role of  $\alpha$  is to modulate the effect of demand by population. When  $\alpha < 1$ , opportunities are allocated more rapidly to smaller centers relative to larger ones;  $\alpha > 1$  achieves the opposite effect.

Balancing factor  $F_i^p$  can now be used to proportionally allocate a share of available jobs at  $j$  to origin  $i$ . The number of jobs available to  $i$  from  $j$  balanced by population shares is defined as follows:

$$V_{ij}^p = O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

In the general case where there are  $J$  employment centers, the total number of jobs available from all destinations to  $i$  is simply the sum of  $V_{ij}^p$  over  $j = 1, \dots, J$ :

$$V_i^p = \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p}$$

Since the factor  $F_i^p$ , when summed over  $i = 1, \dots, N$  always equals to 1 (i.e.,  $\sum_{i=1}^N F_i^p = 1$ ), the sum of all spatially available jobs equals  $O$ , the total number of opportunities in the region:

$$\begin{aligned}\sum_{i=1}^N V_i^p &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_i^p}{\sum_{i=1}^N F_i^p} \\ &= \sum_{i=1}^N \frac{F_i^p}{\sum_{i=1}^N F_i^p} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O\end{aligned}$$

The terms  $F_i^p$  act here as the balancing factors of the gravity model when a single constraint is imposed [i.e., to ensure that the sums of columns are equal to the number of opportunities per destination, see 52 and 183-184]. As a result, the sum of spatial availability for all population centers equals the total number of opportunities.

The discussion so far concerns only the mass effect (i.e., population size) of the gravity model. In addition, the potential for interaction is thought to decrease with increasing cost, so next we define similar balancing factors but based on the impedance.

## Proportional allocation by cost

Clearly, using only balancing factors  $F_i^p$  to calculate spatial availability  $V_i^p$  does not account for the cost of reaching employment centers. Consider instead a set of balancing factors  $F_{ij}^c$  that account for the friction of distance for our example. Recall that the impedance function  $f(c_{ij})$  equals  $\exp(-\beta \cdot c_{ij})$  where  $\beta = 0.1$  and travel time  $c_{ij}$  is either 15, 30 or 60 minutes. For instance, the impedance-based balancing factors  $F_{ij}^c$  would be the following for employment center 1 (employment center 2 and 3 have their own balancing factor values for each origin  $i$  as will be discussed later):

$$\begin{aligned}F_{A1}^c &= \frac{0.223130}{0.223130+0.049787+0.000045} = 0.8174398 \\ F_{B1}^c &= \frac{0.049787}{0.223130+0.049787+0.000045} = 0.1823954 \\ F_{C1}^c &= \frac{0.000045}{0.223130+0.049787+0.000045} = 0.0001648581\end{aligned}$$

Balancing factors  $F_{ij}^c$  use the impedance function to proportionally allocate more jobs to closer population centers, that is, to those with populations *more willing to reach the jobs*. Indeed, the factors  $F_{ij}^c$  can be thought of as the proportion of the population at  $i$  willing to travel to destination  $j$ , conditional on the travel behavior as given by the impedance function. For instance, 81.74398% of jobs from employment center 1 are allocated to population center  $A$  based on impedance.

So as follows from our example, of the 100,000 jobs at employment center 1 the number of jobs allocated to population center  $A$  is  $100,000 \times 0.8174398 = 81,743.98$  jobs; the number allocated to population center  $B$  is  $100,000 \times 0.1823954 = 18,239.54$  jobs; and the number allocated to population center  $C$  is  $100,000 \times 0.0001648581 = 16.48581$  jobs. We see once more that the total number of jobs at the employment center is preserved at 100,000. In this example, the proportional allocation mechanism assigns the largest share of jobs to population center  $A$ , which is the closest to employment center 1, and the smallest to the more distant population center  $C$ .

In the general case where there are  $N$  population centers and  $J$  employment centers in the region, we define the following impedance-based balancing factors:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=1}^N f(c_{ij})} \quad (8)$$

The total number of jobs available to  $i$  from  $j$  according to impedance is defined as

follows:

$$V_{ij}^c = O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

The total number of jobs available to  $i$  from all destinations is:

$$V_i^c = \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c}$$

Like the population-based allocation factors,  $F_i^c$  summed over  $i = 1, \dots, N$  always equals to 1 (i.e.,  $\sum_{i=1}^N F_{ij}^c = 1$ ). As before, the sum of all spatially available jobs equals  $O$ , the total number of opportunities in the region:

$$\begin{aligned} \sum_{i=1}^N V_i^c &= \sum_{i=1}^N \sum_{j=1}^J O_j \frac{F_{ij}^c}{\sum_{i=1}^N F_{ij}^c} \\ &= \sum_{i=1}^N \frac{F_i^c}{\sum_{i=1}^N F_i^c} \cdot \sum_{j=1}^J O_j \\ &= \sum_{j=1}^J O_j = O \end{aligned}$$

We are now ready to more formally define spatial availability with due consideration to both population and travel cost effects.

## Assembling mass and impedance effects

Population and the cost of travel are both part of the gravity modelling framework. Since the balancing factors defined in the preceding sections are proportions (alternatively, can be understood as probabilities), they can be combined multiplicatively to obtain their joint effect. This multiplicative relationship can alternatively be understood as the joint probability of allocating opportunities and is captured by Equation (9), where  $F_i^p$  is the population-based balancing factor that grants a larger share of the existing opportunities to larger centers and  $F_{ij}^c$  is the impedance-based balancing factor that grants a larger share of the existing opportunities to closer centers. This is in line with the tradition of gravity modeling.

$$F_{ij}^t = \frac{F_i^p \cdot F_{ij}^c}{\sum_{i=1}^N F_i^p \cdot F_{ij}^c} \quad (9)$$

with  $F_i^p$  and  $F_{ij}^c$  as defined in Equations (7) and (8) respectively. The combined balancing factor  $F_{ij}^t$  is used to proportionally allocate jobs from  $j$  to  $i$ . Hence, spatial availability is given by Equation (10).

$$V_i = \sum_{j=1}^J O_j F_{ij}^t \quad (10)$$

The terms in Equation 10 are as follows:

- $F_{ij}^t$  is a balancing factor as defined in Equation (9).
- $i$  is a set of origin locations in the region  $i = 1, \dots, N$ .
- $j$  is a set of destination locations in the region  $j = 1, \dots, J$ .
- $O_j$  is the number of opportunities at location  $j$ .
- $V_i$  is the spatial availability at  $i$ .

Notice that, unlike  $S_i$  in Hansen-type accessibility (Equation (1)), the population enters the calculation of  $V_i$  through  $F_i^p$ . Returning to the example in Figure 1, Table 1 also contains the information needed to calculate  $V_i$ , with  $\beta$  set again to 0.1. Column

**V\_ij** are the jobs available to each origin from each employment center. In this column  $V_{A1} = 59,901$  is the number of jobs available at  $A$  from employment center 1. Column **V\_i** (i.e.,  $\sum_{j=1}^J V_{ij}$ ) gives the total number of jobs available to origin  $i$ . We can verify that the total number of jobs available is consistent with the total number of jobs in the region (with some small rounding error):

$$\sum_{i=1}^N V_i = 66,833 + 133,203 + 9,963 \approx 210,000$$

Compare the calculated values of  $V_i$  to column **S.i** (Hansen-type accessibility) in Table 1. The spatial availability values are more intuitive. Recall that population centers  $A$  and  $B$  had identical Hansen-type accessibility to employment opportunities. According to  $V_i$ , population center  $A$  has greater job availability due to: 1) its close proximity to employment center 1; combined with 2) less competition (i.e., a majority of the population have to travel longer distances to reach employment center 1). Job availability is lower for population center  $B$  due to much higher competition (150,000 people can reach 100,000 jobs at equal cost). And center  $C$  has almost as many jobs available as it has population.

As discussed above, Hansen-type accessibility is not designed to preserve the number of jobs in the region. Shen-type accessibility ends up preserving the number of jobs in the region but the definitions of variables are internally obscured; the only way it preserves the number of jobs is if the effect of the impedance function is ignored when expanding the values of jobs per capita to obtain the total number of opportunities. The proportional allocation procedure described above, in contrast, consistently returns a number of jobs available that matches the total number of jobs in the region.

Since the jobs spatially available are consistent with the jobs in the region, it is possible to define a measure of spatial availability per capita as presented in Equation (11):

$$v_i = \frac{V_i}{P_i} \quad (11)$$

And, since the jobs are preserved, it is possible to use the regional jobs per capita ( $\frac{\sum_{j=1}^J O_j}{\sum_{i=1}^N P_i}$ ) as a benchmark to compare the spatial availability of jobs per capita at each origin.

In the example, since the population is equal to the number of jobs, the regional value of jobs per capita is 1.0. To complete the illustrative example, the spatial availability of jobs per capita by origin is:

$$\begin{aligned} v_1 &= \frac{V_1}{P_1} = \frac{66,833.47}{50,000} = 1.337 \\ v_2 &= \frac{V_2}{P_2} = \frac{133,203.4}{150,000} = 0.888 \\ v_3 &= \frac{V_3}{P_3} = \frac{9,963.171}{10,000} = 0.996 \end{aligned} \quad (12)$$

We can see that population center  $A$  has fewer jobs per capita than the regional benchmark, center  $B$  has more, and center  $C$  is at parity. Remarkably, the spatial availability per capita matches the values of  $a_i$  in Table 1. Appendix A has a proof of the mathematical equivalence between the two measures. It is interesting to notice how [27], [25], as well as this paper, all reach identical expressions starting from different assumptions; this effect is known as *equifinality* [see 52, and 53]. This result means that Shen-type accessibility and 2SFCA can be re-conceptualized as singly-constrained accessibility measures.

## Why does proportional allocation matter?

We have shown that Shen-type accessibility and spatial availability produce equifinal results when accessibility per-capita is computed. At this point it is reasonable to ask whether the distinction between these two measures is of any importance.

Conceptually, we would argue that the confounded populations in Shen-type accessibility leads to internal inconsistency in the calculation of total opportunities in [25]: this points to a deeper issue that is only evident when we consider the intermediate values of the method. To illustrate, Table 1 shows results of  $a_i$  that are reasonable (and they match exactly the spatial availability per capita). But when we dig deeper, these results mask potentially misleading values for the jobs allocated and the number of jobs taken. For instance, a region with a high jobs:population ratio but a prohibitive transportation network that results in a high cost of travel may yield a high  $a_i$  value. This value, however, can conceal a low *effective opportunity-seeking population* and proportionally low number of allocated jobs while additionally obscuring the number of population which does *not* take jobs and the jobs *not* taken.

In addition, the intermediate accessibility values of  $a_i$  (Shen-type measure) may also lead to impact estimates that are deceptive [see 54]. For example, the estimated region-wide cost of travel considering the jobs allocated by  $a_i$  in Table 1 (i.e.,  $Jobs * f(TT)$ ) is as follows:

$$\begin{aligned} & 22,313 \times 15 \text{ min} + 4,979 \times 30 \text{ min} + 0.454 \times 100 \text{ min} \\ & 4,979 \times 30 \text{ min} + 22,313 \times 15 \text{ min} + 0.454 \times 100 \text{ min} \\ & 4.54 \times 100 \text{ min} + 4.54 \times 100 \text{ min} + 2,231 \times 15 \text{ min} = 1,002,594 \text{ min} \end{aligned}$$

In contrast, the estimated region-wide cost of travel according to  $V_i$  in Table 1 is as follows:

$$\begin{aligned} & 59,901 \times 15 \text{ min} + 6,923 \times 30 \text{ min} + 10 \times 100 \text{ min} \\ & 40,097 \times 30 \text{ min} + 93,076 \times 15 \text{ min} + 30 \times 100 \text{ min} \\ & 2.4 \times 100 \text{ min} + 1.3 \times 100 \text{ min} + 9,959 \times 15 \text{ min} = 3,859,054 \text{ min} \end{aligned}$$

Often referred to as ‘the supply of jobs’ (or simply Hansen-style accessibility) in the Shen-type measure:  $Jobs * f(TT)$  cannot be used to understand the region-wide cost of travel. Recall how we define  $Pop * f(TT)$  as the *effective opportunity-seeking population* ( $P_{ij}^*$ ),  $Jobs * f(TT)$  similarly represents the *effective opportunities allocated* and sums to approximately 56,824 out of a total of 210,000 jobs. Like  $Pop * f(TT)$ , the *effective opportunities allocated* to each origin is only a reflection of the impedance function and not the *actual* number of opportunities allocated to each origin. Therefore, the resulting 1,002,594 min is not a meaningful measure of the cost of travel in the system.

However, since spatial availability allocates the *actual* number of opportunities to each origin; the 3,859,054 min can be used to quantify the system-wide impacts of competitive accessibility in this region. We know spatial availability’s output is the number of opportunities at each  $i$  since the combined balancing factors allocate a proportional amount of the total opportunities to each  $i$  such that the number of opportunities allocated to each  $i$  sum to equal the total opportunities in the region.

## Empirical example of Toronto

In this section we illustrate the application of spatial availability through an empirical example. For this, we use full-time employment flows from the Greater Golden Horseshoe (GGH) area in Ontario, Canada. Contained with the GGH is the Greater Toronto and Hamilton (GTHA) which forms the most populous metropolitan regions in Canada and the core urban agglomeration in the GGH.

The GTHA contains the city of Toronto, the most populous city in Canada. The city of Toronto is the focus of this empirical example, it will be used to demonstrate the application of the proposed spatial availability measure along with how it compares to Hansen- and Shen-type measures. We begin this section by explaining the data and then detailing the calculated comparisons.

## GGH Data

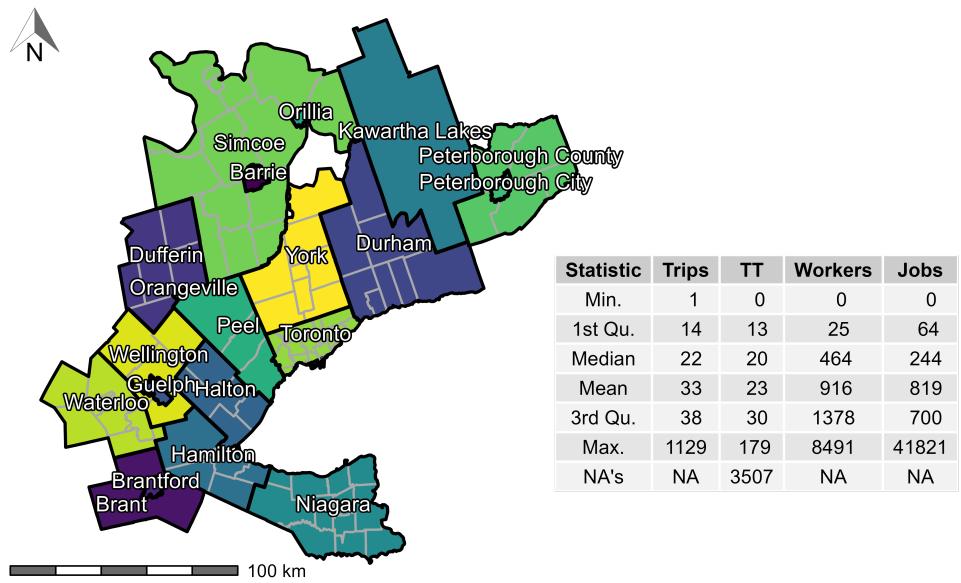
We obtained full-time employment flows from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 3) [55] every five years. The data set includes origin to destination flows associated with full-time employment trips; the number of jobs ( $n=3,081,885$ ) and workers ( $n=3,446,957$ ) (i.e., the number of originating trips and destination trips) at each origin and destination are represented at the level of Traffic Analysis Zones (TAZ) ( $n=3,764$ ). TAZ are a unit of spatial analysis which are defined as part of the TTS, however, TAZ are commonly used to ascribe production and attraction of trips in the context of transportation planning modelling. In the GGH data set, the TAZ contain on average 916 workers and jobs 819 with more detailed descriptive statistics discussed later. The TTS data is based on a representative sample of between 3% to 5% of households in the GGH and is weighted to reflect the population covering the study area as a whole [55].

To generate the travel cost for the full-time employment trips, travel times between origins and destinations (i.e., centroids of the TAZ) are calculated for car travel using the R package {r5r} [56] with a street network retrieved from OpenStreetMap. It is also assumed that intra-TAZ trips are equal to 0.1 minutes. For inter-TAZ trips, a 3 hr travel time threshold was selected as it captures 99% of population-employment pairs (see the travel times summarized in Figure 3). This method does not account for traffic congestion or modal split, which can be estimated through other means [e.g., 57,58]. For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions. All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

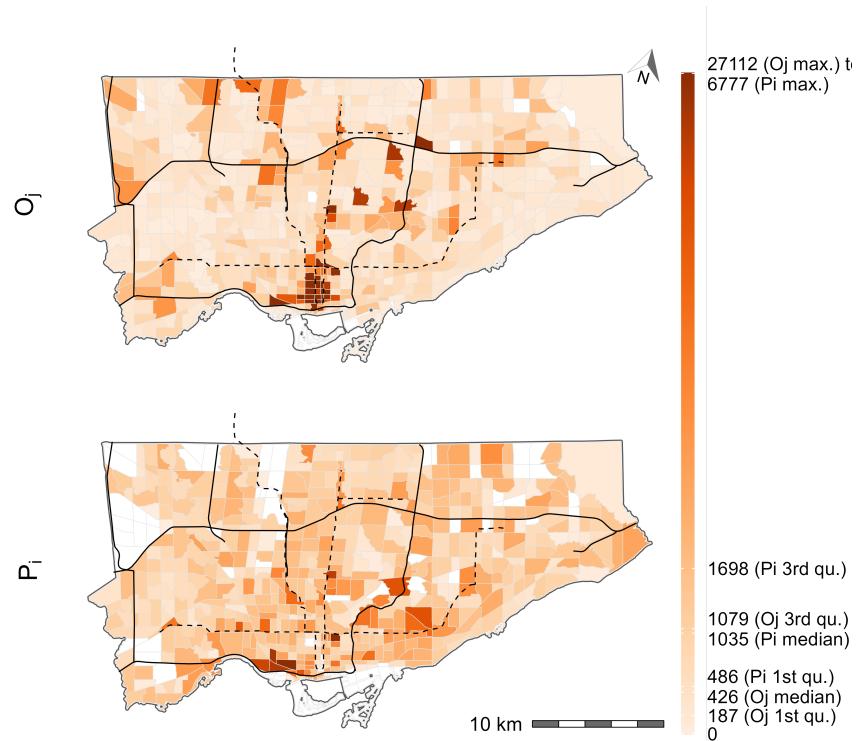
## Spatial employment characteristics in Toronto

As mentioned, the focus of this empirical example is on the city of Toronto. It is the largest city in the GGH and represents a significant subset of workers and jobs in the GGH; 22% of workers in the GGH live in Toronto and 25% of jobs that these workers take are located within Toronto. The spatial distribution of jobs and workers is shown in Figure 4. It can be seen that a large cluster of jobs can be found in the central southern part of Toronto (the downtown core). Spatial trends in the distribution of workers is more even relative to the distribution of jobs.

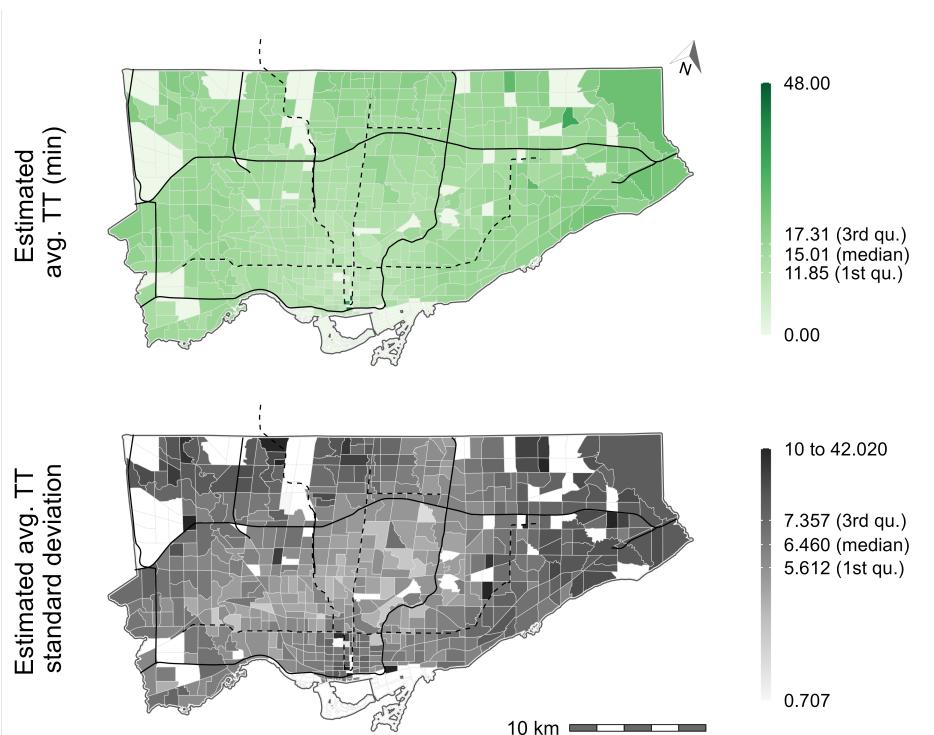
Next, the spatial distribution of the estimated car travel time (green) and the associated standard deviation (grey) is visualized in Figure 5. It can be seen that the car travel time is lower within the downtown core and, unexpectedly higher as the TAZ is further from the downtown core. These travel time estimations are to be expected, as these car travel time are calculated using an uncongested OpenStreetMaps road network from the centroid of origin TAZ to destination TAZ. Since within Toronto trips are only considered, trips which originate from the center of Toronto, an area with high job density, relatively closer proximity to all other Toronto TAZ, and high road connectivity, travel times are lower than outside in areas further from the downtown core. In terms of the variability of the travel times, the center TAZ of Toronto have lower variability than TAZ closer to the borders of Toronto. Trends from both plots indicate



**Fig 3.** TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).



**Fig 4.** Spatial distribution of full-time jobs (top) and full-time working population (bottom) at each TAZ for Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs.



**Fig 5.** Spatial distribution of full-time working population to jobs ratio (top) and car travel time to jobs estimated using R5R (bottom) for the city of Toronto as provided by the 2016 TTS. Black lines represent expressways and black dashed lines represent subway lines. White TAZ represent a TAZ with no workers thus no travel time for the top plot, for the bottom plot they represent no travel time TAZ and TAZ with only 1 travel time.

that trips originating from within the center of Toronto are shorter and more similar in length than trips originating from closer to the border of Toronto.

Nonetheless, the point of these visualizations is to demonstrate the spatial distribution of worker and job data in the city of Toronto to contextualize spatial availability and Shen- and Hansen- type measures.

## Calibration of an impedance function for Toronto

In the synthetic example introduced before, we used a negative exponential function with the parameter reported by [25]. For the empirical Toronto data set, we calibrate an impedance function on the trip length distribution (TLD) of commute trips. Briefly, a TLD represents the proportion of trips that are taken at a specific travel cost (e.g., travel time); this distribution is commonly used to derive impedance functions in accessibility research [59–61].

As mentioned, the calculations are undertaken for the city of Toronto using only the employed population in the city and jobs taken by residents of Toronto. Specifically, edge trips are not included such as trips originating in Toronto but finishing outside of Toronto and trips originating outside of Toronto but finishing in Toronto. The empirical and theoretical TLD for this Toronto data set are represented in the top-left panel of Figure 6. Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the `{fitdistrplus}` package [62] were used. Based on goodness-of-fit criteria and diagnostics the normal distribution was selected (see Figure 6).

The normal distribution is defined in Equation (13), where we see that it depends on a mean parameter  $\mu$  and a standard deviation parameter  $\sigma$ . The estimated values of these parameters are  $\mu = 14.169$  and  $\sigma = 7.369$ .

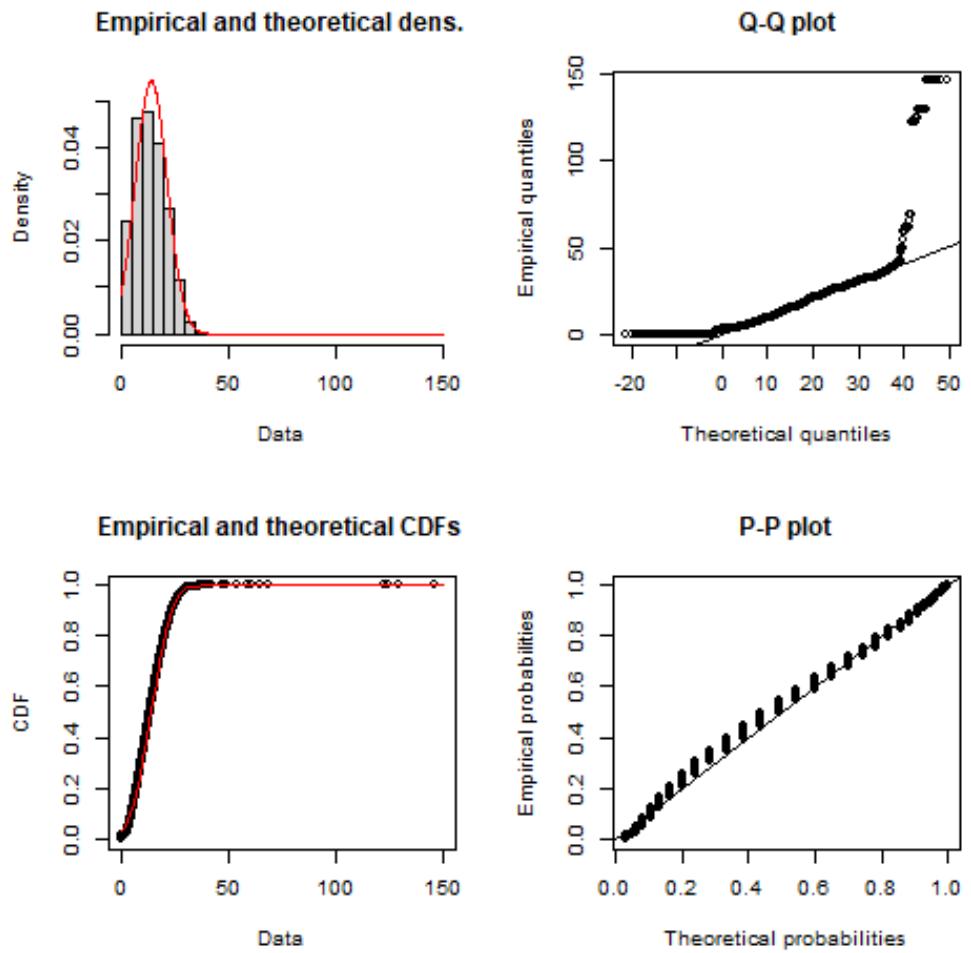
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad (13)$$
$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

## Accessibility and spatial availability of jobs in Toronto

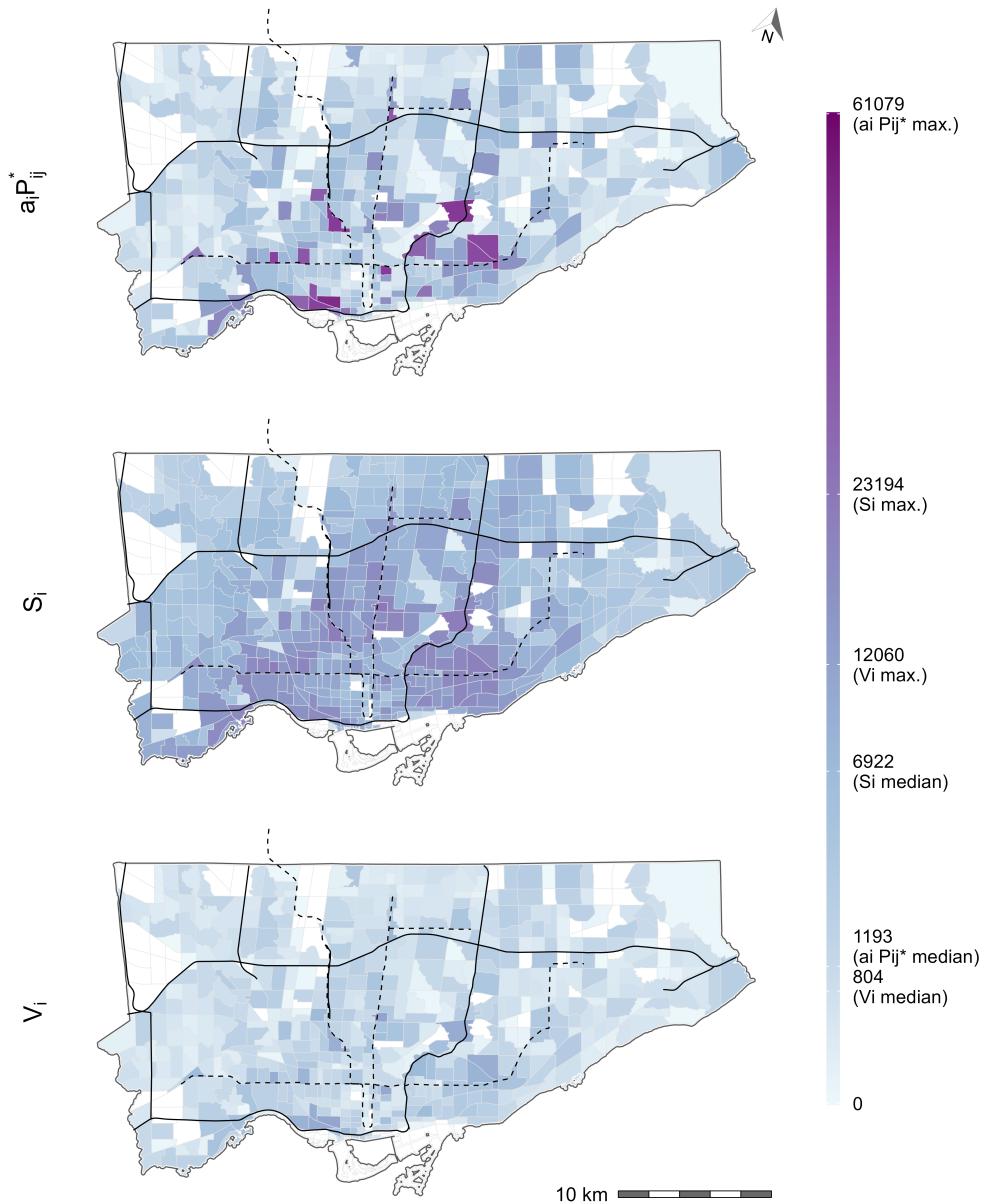
### Absolute opportunity values

Figure 7 contains the number of jobs accessible using Shen-type accessibility, Hansen-type accessibility, and the number of jobs *available* using the spatial availability measure. The values from all these measures are represented on the same axis as they are comparable as they measure the absolute value of *jobs* accessible to the workers in the origin. In the top plot, the Shen-type accessibility is multiplied by the *effective opportunity-seeking population* to yield a value that corresponds to absolute number of accessible jobs (considering competition) according to Shen's definition. In the middle plot, the Hansen-type accessibility is an unconstrained case of accessibility in which all jobs which are in-reach of each origin (according to the impedance function); each value corresponds to the number of jobs which can be reach at each origin assuming no competition. Lastly, in the bottom plot, the spatial availability measure is a constrained case of accessibility which yields the number of jobs, at each origin, considering competition from the population in nearby origin and the relative travel cost (according to the impedance function).

What is notable about the bottom plot is that the proportional allocation mechanism of spatial availability ensures that the job availability value for each origin all sums to the city-wide total of 769,231 jobs (i.e., the number of destination flows from



**Fig 6.** Car trip length distribution and calibrated normal distribution impedance function (red line) with associated Q-Q and P-P plots. Based on the estimated car travel times for full-time employment and workers in Toronto from the TTS 2016.



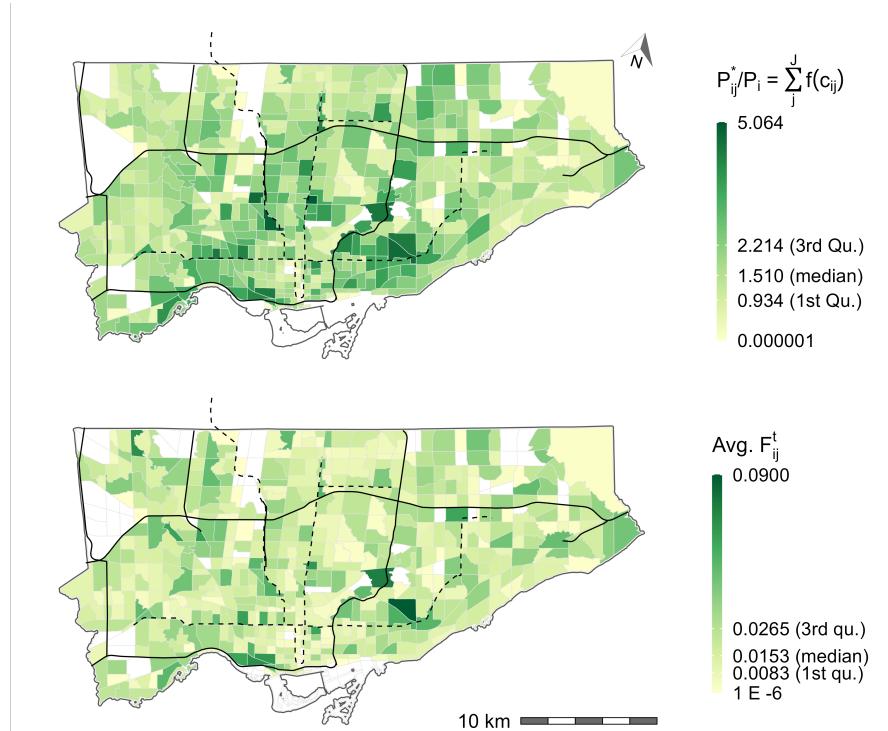
**Fig 7.** Estimated accessibility to jobs (# of jobs) in Toronto according to Shen-type measure times effective opportunity-seeking population (top), Hansen-type measure (middle), and spatial availability (bottom). Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values. Legend scale is square root transformed to effectively visualize the spread range.

Toronto origins to Toronto destinations). The number of accessible jobs at each origin can therefore be interpreted as the number of *available* jobs to each origin based on the relative travel behaviour and density of competition for jobs (i.e., worker population). A proportion of each of the 769,231 jobs in Toronto are only allocated once to each origin. In terms of the middle plot, the city-wide total for Hansen-type accessibility is 4,366,743 jobs, which as a value is meaningless since the measure is unconstrained; it represents the sum of opportunities that have been counted anywhere from 1 to many times depending on the impedance function. As previously discussed, unconstrained counting of the same opportunity by all origins is not an issue if the opportunity itself is non-exclusive, but since one job can only be given to one worker (especially since the worker and job data is derived from origin-destination flows), it is inappropriate to use unconstrained measures to capture employment characteristics. Comparing the middle and bottom plots, it is evident that the unconstrained counting of opportunities (Hansen-style) results in absolute values that are higher throughout the city, particularly in TAZ that are in proximity to high job density (recall Figure 4). These same trends are not present in the spatial availability bottom plot, as the absolute value is lower than Hansen-style accessibility as the proximity to high job density and competition from worker density is proportionally metered; the resulting values are thus lower than the middle plot and reflect the spatial distribution trends of both the workers and job density (recall Figure 4).

Lastly, the top plot that visualizes the *absolute* Shen-type measure (as understood by Shen's definition of  $P_i$  being equal to  $P_{ij}^*$  sums to the city-wide value of 2,117,774 by multiplying  $a_i$  by the *effective opportunity-seeking population*  $P_{ij}^*$  (i.e., the denominator of the rate)). This plot thus demonstrates how confounding  $P_i$  with  $P_{ij}^*$  yields an *incorrect* number of competitively accessible jobs: it is evidently incorrect because the sum of  $a_i P_{ij}^*$  greatly exceeds the city-wide total of workers (i.e.,  $2,117,774 > 769,231$ ). To the authors' knowledge, literature has not attempted to convert Shen-type accessibility to the absolute value of accessible jobs in the way demonstrated in the top plot: we suspect this is the case because of the ambiguous definition that conflates  $P_{ij}^*$  with  $P_i$ . If  $a_i$  is multiplied by  $P_i$ , it yields the same value as  $V_i$ , but since the definition of Shen-type measure is equivocal doing so is not clear since the denominator of  $a_i$  (which is a rate) is *not*  $P_i$ . The resulting plot, spatially, is similar to spatial availability (bottom plot) but certain TAZ have exceptionally high values in an inconsistent way. This is because  $a_i$  uses the impedance function values for both access to jobs (numerator) and the competition from neighboring workers (denominator  $P_{ij}^*$ ) to adjust their impact: using  $P_{ij}^*$  does not *consistently* isolate the absolute value of accessible jobs. However, if  $a_i$  is multiplied by  $P_i$  it yields the same values at  $V_i$  (bottom plot) (the proof for mathematically equivalency is in Appendix A). As also mentioned earlier, the formulation of the denominator and numerator of  $a_i$  is ambiguous so to presume that multiplying it by  $P_i$  would disintangle the rate and yield the absolute value of accessible *and available* (i.e., considering competition) jobs is unclear.

### Internal values

Carrying on the discussion on how to retrieve the absolute value of *available* jobs using the Shen-type measure ( $a_i$ ), Figure 8 highlights how the differences between  $P_i$  and  $P_{ij}^*$  are not uniform across space; the values at each origin are equivalent to  $\sum_j f(c_{ij})$ . Recall,  $P_i$  is the number of workers at each TAZ (city-wide sum of 769,231) while  $P_{ij}^*$  is the number of workers who *seek* jobs (city-wide sum of 1,770,609) in that TAZ based on their travel behaviour.  $P_{ij}^*$  is an internal value of  $a_i$  and the top plot presents the ratio of  $P_{ij}^*$  to  $P_i$  which reflects how the effective opportunity-seeking population is sometimes inflated (i.e., impedance values is greater than 1) and others deflated (i.e., impedance value is less than 1) by the Shen-type measure ( $a_i$ ). As such, using  $P_{ij}^*$  to



**Fig 8.** The ratio of the effective opportunity seeking population to the population (top) and the average spatial availability's balancing factor (Equation (9)) (bottom) for Toronto TAZ. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

untangle the absolute job availability from  $a_i$  instead of  $P_{ij}^*$  can lead to exaggerating the total travel time in the city since it does not represent the *actual* number of workers but the *effective* number of workers. For instance, when trying to calculate the city-wide travel time using  $a_i P_{ij}^*$ , Shen-type accessibility yields 499,740.1 [h] instead of the city-wide travel time of 183,736.8 [h] that corresponds to the *absolute* (i.e., the total number of jobs in the city is preserved) number of available jobs from  $V_i$ . The absolute number of opportunities cannot be easily disentangled from  $a_i$ .

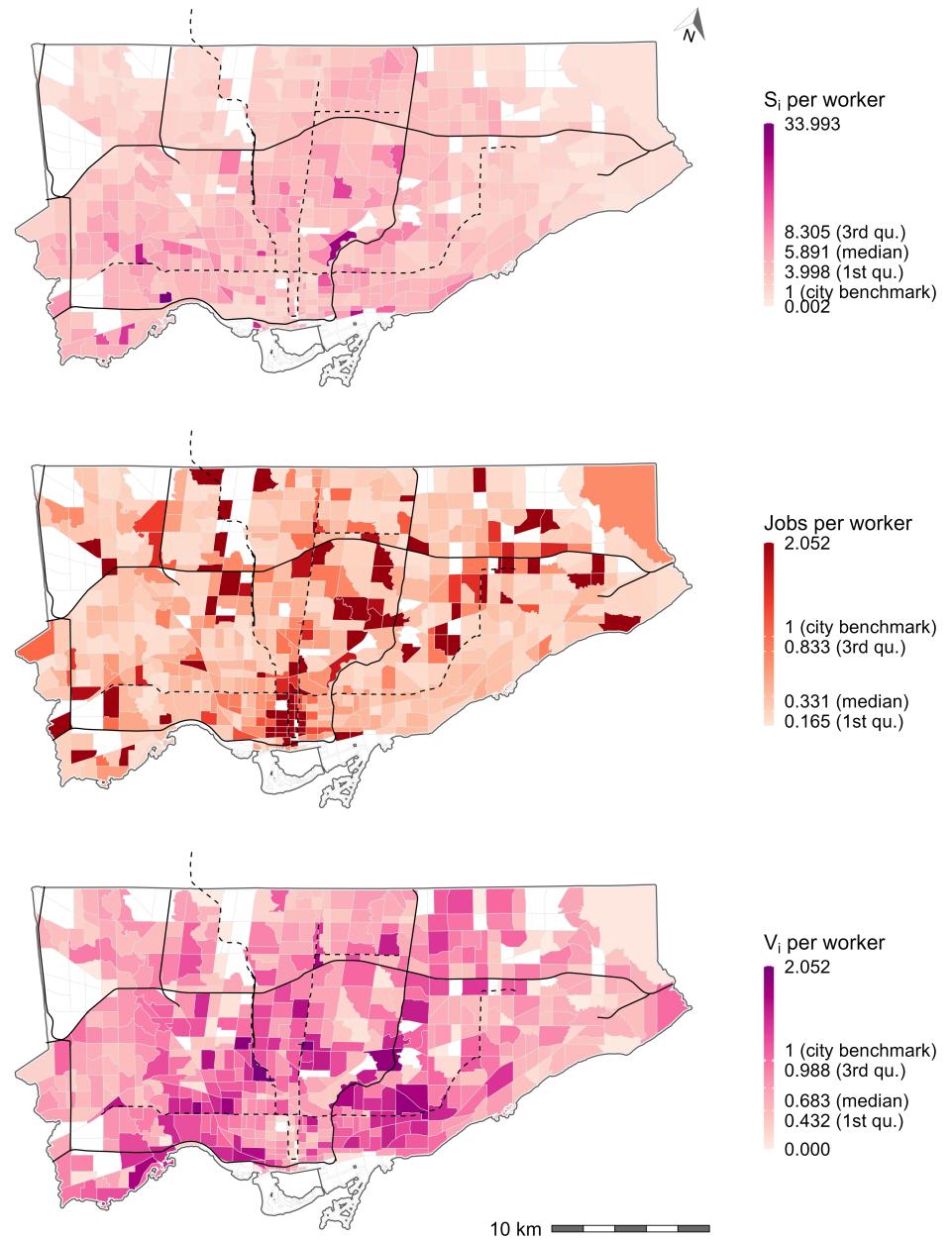
By contrast, not only are the absolute values a direct result of  $V_i$ , the internal combined balancing factor  $F_{ij}^t$  (Equation (9)) can be used for analysis. The bottom plot shows the average  $F_{ij}^t$  for each TAZ which is the proportional allocation mechanism of opportunities to origins in the  $V_i$  calculation. Practically, the visualized values corresponds to the average of *proportion* of opportunities available that are claimed by the zone based on travel behaviour and population competition for opportunities. These values can allow the analyst to understand the magnitude of the *proportion of opportunities* that the origin TAZ is assigned based on the opportunities located at reachable destination TAZ. For instance, the TAZ with the maximum value of 0.090 has many origin to destination trips (112 trips, upper 3rd quantile), many workers (5538 workers, upper 3rd quantile), and located centrally within Toronto. Averaging  $F_{ij}^t$  demonstrates that this TAZ claims on average a high proportion of jobs from reachable TAZs. This does not necessarily mean TAZ with a high  $V_i$  have an exceptionally high average  $F_{ij}^t$ ; for instance, many TAZ around the downtown core have high  $V_i$  values but do not have exceptionally high average  $F_{ij}^t$ . The average  $F_{ij}^t$  can thus be used to identify relatively “greedy” areas that could possibly withstand reductions in availability, if that meant increasing spatial availability in areas with a deficit of jobs available. The balancing factor is an interesting feature of spatial availability which opens up avenues for future analysis; alas, there does not seem to be an equivalent for the Shen-type measure.

### Benchmarking opportunity availability

Figure 9 presents the number of jobs per capita for Hansen-type accessibility (top plot), the raw number of jobs per capita (middle plot), and the spatially available jobs per capita (bottom plot). In addition to clarifying the meaning of internal values, spatial availability can also be divided by population at each origin and expressed as a rate: this rate can be used as a benchmark for equity analysis and compared directly to the raw number of jobs per capita.

The bottom plot features a value which is mathematically equivalent to Shen-type measure, but with stronger interpretability thanks to the proportional allocation mechanism. This mechanism makes clear that all the opportunities are allocated proportionally to origins, which improves interpretability since the  $V_i$  values are the absolute value of *opportunity availability*. The value can thus be directly divided by the population at the origin and expressed as opportunities per capita. When spatial availability is compared to Hansen-type measure (top plot), dividing the output by population directly yield's a more difficult to interpret number of *unconstrained* accessible jobs per capita. For instance, the median light-pink shaded TAZ corresponds to approximately 5.89 unconstrained accessible jobs per capita; this value is difficult to intercept, because as discussed in the introduction, jobs are *exclusive* opportunity types so their accessibility value should take into consideration competition.

The bottom plot displays the spatially available jobs per capita. It can be interpreted as a benchmark its values can be compared directly to the raw number of jobs per capita (middle plot) since the total number of opportunities are preserved (and the population, in this case, is equivalent to the number of opportunities). For instance, a TAZ with a  $v_i > 1$  have more *available jobs* (based on travel behaviour and



**Fig 9.** Hansen-type accessible jobs per capita (top), number of jobs to population ratio (middle), and spatially available jobs per capita (bottom) for Toronto. The city benchmark corresponds to total number of jobs in Toronto divided by the number of workers in Toronto, since this is equal the value is 1. Black lines represent expressways and black dashed lines represent subway lines. All white TAZ have no worker population or jobs, i.e., with null accessibility values.

competition) than their working population. This TA has sufficient employment opportunities (under the assumptions of the input data), while TAZ with a  $v_i < 1$  do not have sufficient employment opportunities. From an equity perspective,  $v_i$  can be used to target where residential housing, job opportunities, and/or transportation system improvements should be created. For TAZ with  $v_i$  values significantly greater than 1 (dark pinks), constructing more residential housing for the type of workers who occupy the *available jobs* in the proximate TAZ should be considered. Assuming the input data is correct, increasing the competition in the area will decrease the  $v_i$  score but if it can be decreased up to threshold of  $v_i = 1$ . For TAZ with  $v_i$  values significantly less than 1 (light pinks), constructing more employment opportunities for the type of workers who live in proximate TAZ and/or prioritizing transportation network improvements to create more favourable travel time conditions. Depending on the raw jobs per worker ratio, different approaches are appropriate. For instance, adding more residential locations near the downtown core (bottom center on the bottom plot) could be a good approach to increasing  $v_i$  as there is already a high jobs per worker ratio (middle plot). However, doing so will decrease the  $v_i$  availability in areas near the border of the city, so in addition to doing so, adding more employment opportunities to areas with low raw jobs per worker ratio and low  $v_i$  is needed. In addition to these changes, the travel time landscape would also influence the resulting  $v_i$  score, so transportation network improves to areas with low  $v_i$  could also be considered. This is to say,  $v_i$  is dependent on the magnitude and spatial distribution of residential housing, job opportunities, and transportation system so the region could be optimized to achieve thresholds of specific  $v_i$  values and thus the difference in residential housing, job opportunities, and transportation system can become policy targets. It should also be kept in mind, that though  $v_i = 1$  and the comparison to the raw jobs per worker values can be used for policy planning,  $v_i$  can easily be transformed back to  $V_i$  to understand the magnitude of the job availability within that origin.

## Conclusion

In this paper we show how a widely used measure of accessibility with competition obscures some important internal values of opportunities taken. This is caused by confounding the population of zones with the *effective opportunity-seeking population*. We then propose an alternative derivation of accessibility with competition that we call spatial availability. This measure ensures that opportunities are allocated in a proportional way and preserved in the regional total. We also show that spatial availability and Shen-type accessibility are equifinal: formally the equations are the same (along with 2SFCA) and can be consider as singly-constrained measures.

Why do differences between Hansen-style measure and the interpretation of Shen-type measure matter? In equity analysis and policy planning, an analyst might be interested in the internal values of their accessibility analysis, for example travel times, and who pays how much for accessibility. The increased interpretability and internal consistency of spatial availability can help to push accessibility analysis forward. Hansen-type measure tend to result in values which are very extreme as a result of multiple-counting opportunities as shown in empirical example. Multiple-counting may not be an issue if the opportunity-type is non-exclusive, but with the case of employment where one worker can only take one job, the resulting values are difficult to interpret (though it can be interpreted relatively to speak about urban form). In this paper, we also demonstrated how attempting to disentangle the absolute values of opportunities from the Shen-type measure is difficult as a result of Shen's definition which confounds the population with the effective-opportunity seeking population. As demonstrated in this paper, spatial availability increases interpretability by presenting first, the absolute value of *available* jobs and then by dividing the available jobs value by the number of working population. This rate is equivalent to Shen-type measure but contains internal values, such as the proportional allocation mechanism, that yield more realistic estimates of opportunities taken, as well as a set of balancing factors that can be used to better understand the absolute and rate values obtained.

Based on this research we suggest the following guidelines for the application of spatial availability and the topic of future work:

- 1) The Hansen-style accessibility should be used when opportunities are non-exclusive. When opportunities are perfectly exclusive (i.e., 1 spot for 1 person), spatial availability (i.e., accessibility with competition) should be used.
- 2) Shen-type accessibility can be used to compute the availability of jobs (the rate and the absolute values if the original definition is corrected), however, if the analyst is interested in internal values and secondary analysis of the results, spatial availability should be considered.
- 3) With the renewed interpretability of what the absolute *opportunity availability* is at each origin, the spatial availability per capita  $v_i$  value of 1 can be used as a policy goal. For areas with a value below 1, targeted increases to the quantity of opportunities, residential housing, and transportation system improvements can be considered such that the number of *available jobs* per capita in the zone is at least equal to 1. Since spatial availability per capita implicitly preserves the number of opportunities in the region, it can be directly compared to the the region's raw jobs to population ratio to inform policy. Additionally, the absolute values of spatial availability can be used to understand the magnitude of the opportunity availability deficit (or surplus).
- 4) Spatial availability per capita can also be compared directly to other regions as done by literature using Shen-type measure/2SFCA (e.g., [63]). However, as a

result of the renewed interpretation, the magnitude of *spatially available*  
opportunities can be quantified.

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- 5) Lastly, since opportunities are preserved, many new avenues of analysis can be pursued. This is especially important in light of emerging concerns with equity. For instance, the population and opportunities can be segmented (i.e., transit users, active transportation users, low income, low education, new comers, children) and their spatial availability to opportunities can be assessed, benchmarked, and corresponding policy to target inequities can be theorized. As another example, the combined balancing factor can be analysed to identify which populations currently do not seek opportunities because of friction of distance. This is a topic for future research.

## Appendix A

The mathematical equivalence of Shen-type accessibility measure and spatial availability is provided in 4 steps in this appendix. Spatial availability per population  $v_i$  is solved for population center  $A$  (Shen's synthetic example as discussed in Section 2.3): this value is represented by  $v_A$  and is equivalent to  $a_A$  as follows.

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**First step:** the population-based balancing factor  $F_i^p$  used in  $V_i$  is defined as:

$$F_i^p = \frac{P_i^\alpha}{\sum_i^N P_i^\alpha}$$

For population center  $A$ ,  $F_i^p$  is equal to:

$$F_A^p = \frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha}$$

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**Second step:** the impedance-based balancing factor  $F_{ij}^c$  in  $V_i$  is defined as:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_{i=A}^N f(c_{ij})}$$

In this synthetic example, combinations of workers from population center  $A$  are permitted to go to all employment centers (1, 2, 3), so their relative impedance value is experienced in all of the nine origin-destination trip combinations. Therefore, all nine  $F_{ij}^c$  are computed as follows, since they all consider the impact of population center  $A$  trip combinations (i.e., either  $A1$ ,  $A2$ ,  $A3$ ).

$$F_{A1}^c = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{B1}^c = \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{C1}^c = \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$$

$$F_{A2}^c = \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$$

$$\begin{aligned}
F_{B2}^c &= \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} \\
F_{C2}^c &= \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})} \\
F_{A3}^c &= \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} \\
F_{B3}^c &= \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})} \\
F_{C3}^c &= \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}
\end{aligned}$$

**Third step:** when these balancing factors ( $F_i^p$  and  $F_{ij}^c$ ) concerning population center  $A$  are assembled and divided by  $P_i$  allow the denominators of the denominators to cancel out. The following equation is the assigned general form, with the strike-through indicating which values cancel out:

$$v_i = \sum_j \frac{O_j}{P_i^\alpha} \frac{\frac{P_i^\alpha}{\sum_i^N P_i^\alpha} \cdot \frac{f(c_{ij})}{\sum_i^N f(c_{ij})}}{\sum_i^N \frac{P_i^\alpha}{\sum_i^N P_i^\alpha} \cdot \frac{f(c_{ij})}{\sum_i^N f(c_{ij})}}$$

To demonstrate that those denominator terms cancel out, the following following terms for  $v_A$  are subbed into the general form as follows:

$$\begin{aligned}
v_A &= \frac{O_1}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}} \right) + \\
&\quad \frac{O_2}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}} \right) + \\
&\quad \frac{O_3}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{B3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} + \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha + P_B^\alpha + P_C^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{C3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}} \right)
\end{aligned}$$

$v_A$  simplifies to the following:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}}{\frac{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A1}) + f(c_{B1}) + f(c_{C1}))}} \right) + \frac{O_2}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}}{\frac{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A2}) + f(c_{B2}) + f(c_{C2}))}} \right) + \frac{O_3}{P_A^\alpha} \left( \frac{\frac{P_A^\alpha}{P_A^\alpha + P_B^\alpha + P_C^\alpha} \cdot \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}}{\frac{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})}{(P_A^\alpha + P_B^\alpha + P_C^\alpha) \cdot (f(c_{A3}) + f(c_{B3}) + f(c_{C3}))}} \right)$$

Now, notice how the denominator of the denominator is the same as the denominator of the numerator for each  $j$  ( $j=1$ ,  $j=2$ , and  $j=3$ )? We remove those cancelled out terms (as indicated at the beginning of this step) and re-write  $v_a$  as follows:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} \right) + \frac{O_2}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} \right) + \frac{O_3}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

**Fourth step:** We can now cancel out one more term,  $P_A^\alpha$ , from the denominator and numerator as follows:

$$v_A = \frac{O_1}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_A^\alpha \cdot f(c_{B1}) + P_A^\alpha \cdot f(c_{C1})} \right) + \frac{O_2}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_A^\alpha \cdot f(c_{B2}) + P_A^\alpha \cdot f(c_{C2})} \right) + \frac{O_3}{P_A^\alpha} \left( \frac{P_A^\alpha \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_A^\alpha \cdot f(c_{B3}) + P_A^\alpha \cdot f(c_{C3})} \right)$$

Which can be expressed as:

$$v_A = \left( \frac{O_1 \cdot f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})} + \frac{O_2 \cdot f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})} + \frac{O_3 \cdot f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_B^\alpha \cdot f(c_{B3}) + P_C^\alpha \cdot f(c_{C3})} \right)$$

And generalized to be formally identical to the Shen-type accessibility measure with competition as follows:

$$v_i = a_i = \sum_j \frac{O_j \cdot f(c_{ij})}{\sum_i P_i \cdot f(c_{ij})}$$

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## Journal Requirements and Reviewers Response Letter

The authors thank the reviewer for their time and thoughtful commentary in response to our paper. We have extensively revised the paper to clarify all the remaining minor comments in the paper. In this respect, we present the comments in **blue text**, our responses and comments in black text, and the relevant revised text is presented in **red**.

### Journal Requirements

1. Please ensure that your manuscript meets PLOS ONE's style requirements, including those for file naming. The PLOS ONE style templates can be found at <https://journals.plos.org/plosone/s/file?id=wjVg/PLOSOne%20formatting%20sample%20main%20body.pdf> and <https://journals.plos.org/plosone/s/file?id=ba62/PLOSOne%20formatting%20sample%20title%20authors%20affiliations.pdf>
- Thank you for this comment. We produced the manuscript using the .tex style template for PLOS ONE.
2. Thank you for stating the following financial disclosure: "The Social Sciences and Humanities Research Council's partnership grant: Mobilizing justice: towards evidence-based transportation equity policy."

Please state what role the funders took in the study. If the funders had no role, please state: "The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript." If this statement is not correct you must amend it as needed. Please include this amended Role of Funder statement in your cover letter; we will change the online submission form on your behalf.

- Thank you for this comment. We've updated the statement accordingly and included it in the revised cover letter.
- 3. In your Data Availability statement, you have not specified where the minimal data set underlying the results described in your manuscript can be found. PLOS defines a study's minimal data set as the underlying data used to reach the conclusions drawn in the manuscript and any additional data required to replicate the reported study findings in their entirety. All PLOS journals require that the minimal data set be made fully available. For more information about our data policy, please see <http://journals.plos.org/plosone/s/data-availability>.

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<http://journals.plos.org/plosone/s/data-availability#loc-recommended-repositories>. Any potentially identifying patient information must be fully anonymized.

Important: If there are ethical or legal restrictions to sharing your data publicly, please explain these restrictions in detail. Please see our guidelines for more information on what we consider unacceptable restrictions to publicly sharing data: <http://journals.plos.org/plosone/s/data-availability#loc-unacceptable-data-access-restrictions>. Note that it is not acceptable for the authors to be the sole named individuals responsible for ensuring data access.

We will update your Data Availability statement to reflect the information you provide in your cover letter.

- Thank you for this comment. The data set is available from <https://soukhova.github.io/TTS2016R/>. The full manuscript and the code are included in this repository: <https://github.com/soukhova/Spatial-Availability-Measure>. Both links are provided as footnotes in the Introduction section.
4. We note that Figures 3,4,5,7,8 and 9 in your submission contain copyrighted images. All PLOS content is published under the Creative Commons Attribution License (CC BY 4.0), which means that the manuscript, images, and Supporting Information files will be freely available online, and any third party is permitted to access, download, copy, distribute, and use these materials in any way, even commercially, with proper attribution. For more information, see our copyright guidelines: <http://journals.plos.org/plosone/s/licenses-and-copyright>.

We require you to either (1) present written permission from the copyright holder to publish these figures specifically under the CC BY 4.0 license, or (2) remove the figures from your submission.

- Thank you for this comment. All images are original and were generated by us using the data sources stated in the manuscript using code found in the .Rmd file in the repository (<https://github.com/soukhova/Spatial-Availability-Measure/blob/main/Spatial-Availability-Paper/Spatial-Availability-Paper.Rmd>). The packages used to produce the figures are cited.

## Reviewer #1

The authors demonstrated how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population.

1. Please highlight the key points in the abstract.

- Thanks for this comment. We modified the abstract to highlight the key points of this paper. The abstract now reads as follows:

*Accessibility indicators are widely used in transportation, urban, and healthcare planning, among many other applications. These measures are weighted sums of reachable opportunities from a given origin conditional on the cost of movement, and are estimates of the potential for spatial interaction. Over time, various proposals have been forwarded to improve their interpretability: one of those methodological additions have been the introduction of competition. In this paper, we focus on competition, but first demonstrate how a widely used measure of accessibility with congestion fails to properly match the opportunity-seeking population. We then propose an alternative formulation of accessibility with competition, a measure we call \_spatial availability\_. This measure relies on proportional allocation balancing factors (friction of distance and population competition) that are equivalent to imposing a single constraint on conventional gravity-based accessibility. In other words, the proportional allocation of opportunities results in a \_spatially available opportunities\_ value which is assigned to each origin that, when all origin values are summed, equals the total number of opportunities in the region. We also demonstrate how Two-Stage Floating Catchment Area (2SFCA) methods are equivalent to spatial availabilty and can be reconceptualized as singly-constrained accessibility. To illustrate the application of spatial availability and compare it to other relevant measures, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada. Spatial availability is an important contribution since it clarifies the interpretation of accessibility with competition and paves the way for future applications in equity analysis.*

2. The introduction should further highlight the scientific problems, motivations, and possible innovations of the paper.

- Thank you for these points. We modified the text as follows to include the highlighted motivations (as a result of the scientific problems) at the end of the introduction.

*The key motivations of this paper are as follows:*

- *To address and improve on the interpretability of Hansen-type accessibility measure; and*
- *To consider competition from the perspective of the population for opportunities within an accessibility measure.*

- We'd like to draw your attention to the summary of the objectives of the paper in the revised version of the manuscript and included as follows. We believe these correctly highlight the possible innovations of this paper.

*In this way, the aim of the paper is three-fold:*

- *First, we aim to demonstrate that Shen-type (and thus @weibull\_axiomatic\_1976 accessibility and the popular 2SFCA methods) produce equivocal estimates of opportunities allocated as the result is presented as a rate (i.e., opportunities per capita);*
- *Second, we introduce a new measure, \_spatial availability\_ which we submit is a more interpretable alternative to Shen-type accessibility, since opportunities in the system are preserved and proportionally allocated to the population; and*
- *Third, we show how Shen-type accessibility (and 2SFCA methods) can be seen as measures of singly-constrained accessibility.*

### 3. What is the contribution and why the contribution is important?

- An important question to ask indeed! We believe we highlight the contribution and its importance throughout the paper, particularly in subsection "Why does proportional allocation matter?" as well as in the conclusion. The contribution is in the conclusion is summarized as follows:

*In this paper we show how a widely used measure of accessibility with competition (Shen-type accessibility) obscures some important internal values of opportunities taken. This is caused by confounding the population of zones with the \_effective opportunity-seeking population\_. We then propose an alternative derivation of accessibility with competition that we call spatial availability. This measure ensures that opportunities are allocated in a proportional way and preserved in the regional total. We also show that spatial availability and Shen-type accessibility are equifinal: formally the equations are the same (along with 2SFCA) and can be consider as singly-constrained measures.*

- In addition, following your suggestion, we edited the first few sentences of the conclusion's second paragraph to make the importance of the contribution more clear. It now reads:

*Spatial availability matters because competition is an important consideration for certain opportunity types and conventional Hansen-type accessibility does not capture it [@merlin2017competition]. Spatial availability also brings forward a different interpretation to competition than the Shen-type measure through its output value, the proportional allocation factors, and the output values per capita. In equity analysis and policy planning, an analyst might be interested in the internal values of their accessibility analysis, for example travel times, and who pays how much for accessibility. The increased interpretability and internal consistency of spatial availability can help to push accessibility analysis forward.*

*Hansen-type measure tend to result in values which are very extreme as a result of multiple-counting opportunities as shown in empirical example. Multiple-counting may not be an issue if the opportunity-type is non-exclusive, but with the case of employment where one worker can only take one job, the resulting values are difficult to interpret (though it can be interpreted relatively to speak about urban form). In this paper, we also demonstrated how attempting to disentangle the absolute values of opportunities from the Shen-type measure is difficult as a result of Shen's definition which confounds the population with the effective-opportunity seeking population.*

- We also added a sentence to the abstract which states the contribution of the paper more clearly, as inspired by this comment. The most current version of the abstract is included in reply to your comment 1.
  
- 4. The results should be better described, discussed and justified. I suggest the results should be better discussed and justified, such as whether they are consistent with previous studies or analyzing the reasons for the empirical results. please read:
  - The influence of high-speed rail on ice–snow tourism in northeastern China. *Tourism Management* (2020) , doi:10.1016/j.tourman.2019.104070
  - Study on the Impact of High-speed Railway Opening on China's Accessibility Pattern and Spatial Equality[J].*Sustainability* 2018,10,2943. doi:10.3390/su10082943
  - The Impact of Accessibility on House Prices Based on a Geographically Weighted Regression Model[J]. *Chinese Geographical Science*, 2018, 28 (3) : 505-515. doi: 10.1007/s11769-018-0954-6
  
  - Thank you for these references. They have been included in the revised version of the paper as examples of the important role that accessibility analysis plays in different disciplines.

## Reviewer #2

The authors used the data from Transportation Tomorrow Survey to explore the application of spatial availability and compare it to other relevant measures. The manuscript is valid and innovative. However, there also have some problems need to be solved.

1. In introduction, the authors only inserted the serial number of the references. The presentation is omitted. It is not standard enough.
  - Thank you for this comment. We are advised to use the in-text referencing style of PLoS ONE, which uses serial number of the reference.
  
2. There are many methods to measure the accessibility. What's the advantage and disadvantage of the improved methods compared with other methods, such as space syntax and potential model.
  - Thank you for these points, discussing the advantages and disadvantages of similar methods is important to contextualize the improved method. In this paper, we do discuss other accessibility measures at length in the 'Accessibility measures revisited' section. We outline Hansen-type accessibility and then Shen-type accessibility, both of which are forms of potential accessibility. We introduce spatial availability in the following section and then discuss its advantages in the subsection 'Why does proportional allocation matter?'.
  - Our aim is to present a new accessibility measure, spatial availability, and we discuss its properties in reference to two comparable measures, Hansen-type accessibility, and Shen-type accessibility (2SFCA). These two are forms of potential accessibility (see for an example Zhang et al., 2021). We reviewed the space syntax model. As noted by Zhang et al. (2022, p. 9434). "[a] higher level of integration implies spatial clustering and better topological accessibility". Some space syntax measures correlate well with accessibility, however accessibility is sufficiently distinct that it is not an appropriate comparator.
  
3. Some relevant references should be cited as follow:
  - The influence of the spatial pattern of urban road networks on the quality of business environments: the case of Dalian City. Environment, Development and Sustainability (2022) 24:9429–9446. <https://doi.org/10.1007/s10668-021-01832-z>
  - Differences in Accessibility of Public Health Facilities in Hierarchical Municipalities and the Spatial Pattern Characteristics of Their Services in Doumen District, China. Land 2021, 10, 1249. <https://doi.org/10.3390/land10111249>.
  - Transportation Accessibility Evaluation of Educational Institutions Conducting Field Environmental Education Activities in Ecological Protection Areas: A Case Study of Zhuhai City. Sustainability 2021, 13, 9392. <https://doi.org/10.3390/su13169392>

Thank you for these references, which we have added to the bibliography in the paper.

We trust that you will find that these revisions satisfy all the comments communicated. We are grateful for your insightful and constructive feedback, and appreciate that it helped to improve the paper.

Data Review URL



Click here to download Data Review URL  
<http://github.com/soukhova/Spatial-Availability-Measure>



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<http://soukhova.github.io/TTS2016R/>