Introducing spatial availability, a singly-constrained competitive-access accessibility measure

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Abstract

Accessibility measures are widely used in transportation, urban, and healthcare planning, among other applications. These measures are weighted sums of the opportunities that can be reached given the cost of movement and are interpreted to represent the potential for spatial interaction. Though useful to understanding spatial structure, recent research on balanced floating catchment areas (BFCA) and competitive measures of accessibility raises questions about their interpretability. In this paper, we propose a new measure of *spatial availability* which is calculated by imposing a single constraint on gravity-based accessibility. Similar to the gravity model from which spatial availability is derived, a single constraint ensures that a single set of marginals are met and thus the number of opportunities are preserved. Through examples, we detail the formulation of the proposed measure. Further, we use data from the 2016 Transportation Tomorrow Survey of the Greater Golden Horseshoe area in southern Ontario, Canada, to contrast how the conventional accessibility measure tends to overestimate and underestimate the number of jobs *available* to workers. We conclude with some discussion on the possible uses of spatial availability and argue that, compared to conventional measures of accessibility, it can offer a more meaningful and interpretable measure of opportunity access. All data and code used in this research are openly available.

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Introduction

Accessibility analysis is employed in transportation, geography, public health, and many other areas, particularly as mobility-based planning is de-emphasized in favor of access-oriented planning (Deboosere et al., 2018; Handy, 2020; Proffitt et al., 2017; Yan, 2021). The concept of accessibility derives its appeal from combining the spatial distribution of opportunities and the cost of reaching them (Hansen, 1959).

Numerous methods for calculating accessibility have been proposed that can be broadly organized into infrastructure-, place-, person-, and utility-based measures (Geurs and van Wee, 2004). Of these, the place-based family of measures is arguably the most common, capturing the number of opportunities reachable from an origin using the transportation network. A common type of placed-based measure is the gravity measure of accessibility that captures the potential for interaction; since it was first developed by Hansen (1959) it has been widely adopted in many forms (Arranz-López et al., 2019; e.g., Cervero et al., 2002; Geurs and van Wee, 2004; Handy and Niemeier, 1997; Levinson, 1998; Paez, 2004). Accessibility analysis offers a powerful tool to study the intersection between urban structure and transportation infrastructure - however, the interpretability of accessibility measures can be challenging (Geurs and van Wee, 2004; Miller, 2018). A key issue is that accessibility measures are sensitive to the number of opportunities in a region (e.g., a large city has more jobs than a smaller city), and therefore raw values cannot be easily compared across study areas (Allen and Farber, 2019).

Gravity-based accessibility indicators are in essence spatially smoothed estimates of the total number of opportunities in a region, but the meaning of their magnitudes is unclear. This is evident when we consider the "total accessibility" in the region, a quantity that is not particularly meaningful since it is not constrained to resemble, let alone match, the number of opportunities available. Furthermore, while accessibility depends on the number of opportunities weighted by the travel costs associated with reaching them, the calculated accessibilities are not sensitive to the demand for those opportunities at the origins. Put another way, traditional measures of accessibility do not capture the competition for opportunities. This shortcoming (see Geurs and van Wee, 2004) is particularly acute when opportunities are "non-divisible" in the sense that, once taken they are no longer available to other members of the population. Examples of these types of opportunities include jobs (e.g., when a person takes up a job, the same job cannot be taken by anyone else) and placements at schools (e.g., once a student takes a seat at a school, that opportunity is no longer available for another student). From a different perspective, employers may see workers as opportunities, so when a worker takes a job, this particular individual is no longer in the available pool of candidates for hiring.

To remedy these issues, researchers have proposed several different approaches for calculating competitive accessibility values. On the one hand, this includes several approaches that first normalize the number of opportunities available at a destination by the demand for them from the origin zones and, second, sum the demand-corrected opportunities which can be reached from the origins (e.g. Joseph and Bantock, 1984; Shen, 1998). These advances were popularized in the family of two-step floating catchment area (FCA) methods (Luo and Wang, 2003) that have found widespread adoption for calculating competitive accessibility to a variety of opportunities such as healthcare, education, and food access (B. Y. Chen et al., 2020; Chen, 2019; Z. Chen et al., 2020; Yang et al., 2006; Ye et al., 2018). In principle, floating catchments purport to account for competition/congestion effects, although in practice several researchers (e.g., Delamater, 2013; Wan et al., 2012) have found that they tend to over-estimate the level of demand and/or service. The underlying issue, as demonstrated by Paez et al. (2019), is the multiple counting of both population and level of service, which can lead to biased estimates if not corrected.

A second approach is to impose constraints on the gravity model to ensure potential interaction between zones are equal to the observed totals. Based on Wilson's (1971) entropy-derived gravity model, researchers can incorporate constraints to ensure that the modeled flows match some known quantities in the data inputs. In this way, models can be singly-constrained to match the row- or column-marginals (i.e., the trips produced or attracted, respectively), whereas a doubly-constrained model is designed to match both marginals. Allen and Farber (2019) recently incorporated a version of the doubly-constrained gravity model within the FCA approach to calculate competitive accessibility to employment using transit across eight cities in Canada. But while such a model can account for competition, the mutual dependence of the balancing factors in a doubly-constrained model means they must be iteratively calculated which makes them more computationally-intensive. Furthermore, the double constraint means that the sum of opportunity-seekers and the sum of opportunities must match, which is not necessarily true in every potential use case (e.g., there might be more people searching for work than jobs exist in a region).

In this paper we propose an alternative approach to measuring competitive accessibility. We call it a measure of **spatial availability**, and it aims to capture the number of opportunities that are not only *accessible* but also *available* to the opportunity-seeking population, in the sense that they have not been claimed by a competing seeker of the opportunity. As we will show, spatial availability is a singly-constrained measure of accessibility. By allocating opportunities in a proportional way based on demand and distance, this method avoids the issues that result from multiple counting of opportunities in conventional accessibility analysis. The method returns a measure of the rate of available opportunities per opportunity-seeking population. Moreover, the method also returns a benchmark value for the study region against which results for individual origins can be compared both inter- and intra-regionally.

In the following sections we introduce and illustrate the proposed measure using a synthetic example and an empirical example. First, we describe the analytical framework of the measure. Second, we calculate the spatial availability using data from the Transportation Tomorrow Survey (TTS) home-to-work commute in 2016 for the Greater Golden Horse (GGH) area in Ontario, Canada, and discuss differences with conventional unconstrained accessibility analysis.

Third, we return to the synthetic example and calculate spatial availability for two additional use-cases: one use case for jobs from the perspective of the population considering catchment restrictions and another use case for workers from the perspective of employers. Finally, we conclude by discussing the advantages of the spatial availability measure and the breadth of potential uses.

In the spirit of openness of research in the spatial sciences (Brunsdon and Comber, 2021; Páez, 2021) this paper has a companion open data product (Arribas-Bel et al., 2021), and all code will be available for replicability and reproducibility purposes.

Background

Most accessibility measures (excluding utility-based measures) are derived from the gravity model and follow the formulation shown in Equation (1). The limitations associated with this common and widely used measure, namely issues in interpretation and spatial bias, are the motivation for the proposed *spatial availability* measure. We begin this section by conceptually demonstrating the issues associated with conventional accessibility analysis using a simple synthetic example.

$$A_i = \sum_{j=1}^{J} O_j f(c_{ij}) \tag{1}$$

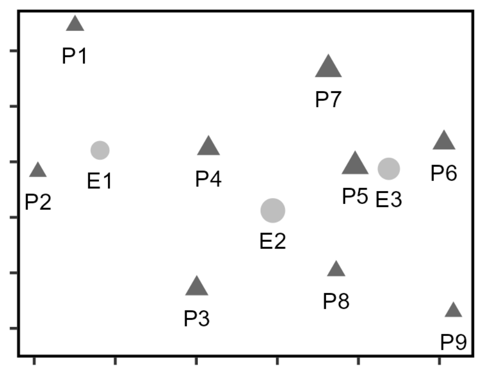
where:

- A is accessibility.
- i is a set of origin locations.
- j is a set of destination locations.
- O_j is the number of opportunities at location j; $\sum_j O_j$ is the total supply of opportunities in the study region.
- c_{ij} is a measure of the cost of moving between i and j.
- $f(\cdot)$ is an impedance function of c_{ij}; it can take the form of any monotonically decreasing function chosen based on positive or normative criteria (Paez et al., 2012).

As formally defined, accessibility A_i is the weighted sum of opportunities that can be reached from location i, given the cost of travel c_{ij}. Summing the opportunities in the neighborhood of i, as determined by the impedance function $f(\cdot)$, provides estimates of the number of opportunities that can be reached from i at a certain cost. The type of accessibility can be modified depending on the impedance function; for example, the measure could be cumulative opportunities (if $f(\cdot)$ is a binary or indicator function e.g., El-Geneidy et al., 2016; Geurs and van Wee, 2004; Qi et al., 2018; Rosik et al., 2021) or a gravity measure using an impedance function modeled after any monotonically decreasing function (e.g., Gaussian, inverse power, negative exponential, or log-normal, among others, see, *inter alia*, Kwan, 1998; Li et al., 2020; Reggiani et al., 2011; Vale and Pereira,

Table 1: Summary description of synthetic example

ID	Number	Location Type
E1	750	jobs
E2	2250	jobs
E3	1500	jobs
P1	260	population
P2	255	population
P3	510	population
P4	495	population
P5	1020	population
P6	490	population
P7	980	population
P8	260	population
P9	255	population



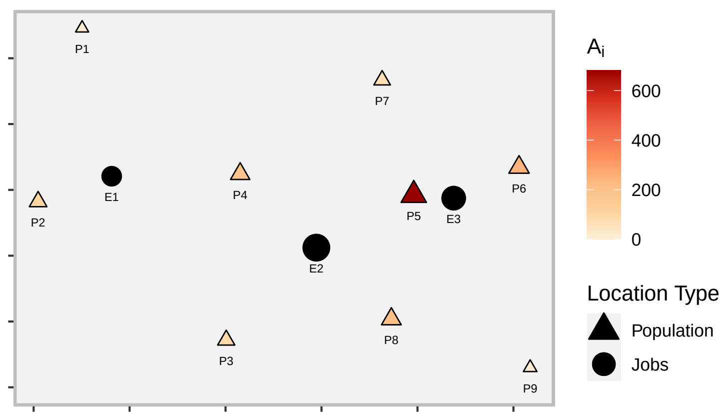


Figure 1: Accessibility to jobs from population centers in the synthetic example

2017). In practice, the accessibility measures derived from many cumulative and gravity formulations tend to be highly correlated with one another (Higgins, 2019; Kwan, 1998; Santana Palacios and El-geneidy, 2022).

Synthetic example

The setup for our synthetic example is a system with three employment centers and nine population centers, as summarized in Table 1. Accessibility to jobs at each population center is calculated using the accessibility measure A_i in Equation (1). In this simple example, we use the straight line distance between the population and jobs for c_{ij} and a negative exponential function with $\beta = 0.0015$. As noted, A_i represents the number of jobs (i.e., opportunities) that can be reached from each population center given the estimated cost as depicted in Figure 1.

Figure 1 shows the locations of the three employment centers (black circles), where the size of the symbol is in proportion to the number of jobs at each location. We also see nine population centers (triangles), where the size of the symbol is proportional to the accessibility (A_i) to jobs. The accessibility values illustrate the following:

- Population centers (triangles) in the middle of the plot are relatively close to all three employment centers (circles) and thus have the highest levels of job accessibility. Population center P5 is relatively central and close to all employment centers, and it is the closest population to the second largest employment center in the region. Unsurprisingly, this population center has the highest accessibility ($A_i = 680.64$).

- Population centers near the left edge of the map (only in proximity to the small employment center) have the lowest levels of job accessibility. Population center P1 is quite peripheral and the closest employment center is also the smallest one. Consequently, it has the lowest accessibility with $A_i = 17.12$.

The effect of competition for opportunities

Accessibility measures are excellent indicators of the intersection between urban structure and transportation infrastructure (Kwan, 1998; Reggiani et al., 2011; Shi et al., 2020). However, beyond enabling comparisons of relative values they are not highly interpretable on their own (Miller, 2018). For instance, from Figure 1, P1 has lower accessibility than P5 but despite the accessibility value for P1 being relatively low it is still better than *zero*. On the other hand, P5 has high accessibility, but is its accessibility excellent, good, or only fair? What does it *mean* for a location to have accessibility to so many jobs?

To address this interpretability issue, previous research has aimed to index and normalize values on a per demand-population basis (e.g., Barboza et al., 2021; Pereira et al., 2019; Wang et al., 2021). However, as recent research on accessibility discusses (Allen and Farber, 2019; Paez et al., 2019), these steps do not address the bias introduced through the uneven multiple-counting of opportunities and/or population. This is similar to the congestion effect that FCA methods aim to address, although these methods do not necessarily solve the issue completely (see Paez et al., 2019). The underlying issue arises as a result of the assumption that for conventional accessibility A_i, all opportunities are *available* to anyone from any origin $i = 1, \cdots, n$ who can reach them: in other words, they are assumed to be infinitely divisible and non-competitive. This results in every opportunity entering the weighted sum once for every origin i that can reach it. Put another way, if a densely populated population center pops up next to P5 this center too will have a high accessibility score since A_i does not consider competition of opportunities from neighbouring population centers. Neglecting to constrain opportunity counts (i.e., single-constraint) in addition to obscuring the interpretability of accessibility can also bias results in two ways:

1) Demand centers in less dense outer limits of cores may be assigned disproportionately *high* accessibility values. These periphery areas are traditionally located in proximity to more dense urban demand centers and large urban opportunity centers and thus may have low travel cost to these large opportunity centers. Accessibility A_i does not consider opportunity-constraints and, as such, these periphery demand centers benefit from the high accessibility to opportunities without competition considerations from their more dense and more centrally located neighbours.
2) Remote areas which are still within the region of the analysis and are near relative smaller opportunity centers may be assigned disproportionately *low* accessibility values, despite facing low competition for opportunities. These more remote areas may be sufficiently supplied with opportunities proportionate to their demand but this relationship is obscured by the artificially high accessibility awarded to demand- and opportunity-rich areas in which competition disproportionately occurs.

The spatially uneven multiple-counting of opportunities (i.e., the competition effect) makes accessibility estimates difficult. Recent accessibility measures which seek to improve interpretability are either vulnerable to this impact or require heroic assumptions. As previously mentioned, the FCA method increases interpretability by purporting to account for competition, however, as discussed by Paez et al. (2019), FCA methods are vulnerable to a similar multiple-counting effect. On this same note, the doubly-constrained gravity model proposed by Allen and Farber (2019) which is based on the FCA method, accounts for competition, but requires that the magnitude of demand matches the opportunities or be re-scaled to match. As noted, this assumption is not always realistic for many opportunity types such as in the case of job seekers and jobs.

To address this competition effect and to more accurately account for non-divisible opportunities, we propose a singly-constrained gravity measure called **spatial availability**. This measure seeks to address the following questions from the perspective of an individual at a specific demand center: *"many opportunities are accessible, but the same opportunities are also accessible to my (possibly) numerous neighbors... what does high accessibility actually mean to me?"* and *"few opportunities are accessible to me but I am located in a remote area with proportionally few neighbors... what does low accessibility mean to me?"*. Beyond the individual, spatial availability, can also be used as-is to evaluate the *spatial mismatch* of accessible opportunities and demand-seeking population within regions and between regions.

Analytical framework

Next, we introduce the analytical framework of spatial availability and highlight the differences between the measures to demonstrate how the proposed method improves the interpretability of opportunity access.

Formally, spatial availability V_{ij} is defined by the number of opportunities O that are proportionally allocated based on a population allocation factor F^p_{ij} and cost of travel allocation factor F^c_{ij} for all origins i to all destinations j as detailed in Equation (2). In line with the tradition of gravity modeling, the proposed framework distinguishes between opportunities at a destination and demand for opportunities at the origin.

$$V_{ij} = O_j \frac{F^p_{ij} \cdot F^c_{ij}}{\sum_{i=1}^{K} F^p_{ij} \cdot F^c_{ij}} \tag{2}$$

The terms in Equation 2 are as follows:

- V_{ij} is the spatial availability of opportunities in j to origin i.
- i is a set of origin locations in the region K.
- j is a set of destination locations in the region K.
- O_j is the number of opportunities at location j in the region K.
- F^p_{ij} is a proportional allocation factor of the population in i.
- F^c_{ij} is a proportional allocation factor of travel cost for i; it is a product of a monotonically decreasing (i.e., impedance) function associated with the cost of travel between i and j.

Notice that, unlike A_i in Equation (1), the population in the region enters the calculation of V_{ij}. It is important to detail the role of the two proportional allocations factors in the formulation of spatial availability. We begin by considering the population allocation factor F^p_{ij} followed by the role of the travel cost allocation factor F^c_{ij}; then we show how both allocation factors combine in the final general form of spatial availability V_{ij}. The calculation of spatial availability is introduced with a step-by-step example for two population centers (P_1 and P_2) in the role of demand (i.e., the number of individuals in the labor market who 'demand' employment) and one employment center (O_1) in the role of opportunities.

Population and travel cost allocation factors

We begin with allocation based on demand by population; consider an employment center j with O^r_j jobs of type r. In the general case where there are K population centers in the region, we define the following factor:

$$F^p_{ij} = \frac{P^\alpha_{i\in r}}{\sum_{i=1}^{K} P^\alpha_{i\in r}} \tag{3}$$

The population allocation factor F^p_{ij} corresponds to the proportion of the population in origin i relative to the population in the region. On the right hand side of the equation, the numerator $P_{i\in r}$ is the population at origin i that is eligible for and 'demands' jobs of type r (e.g., those with a certain level of training or in a designated age group). The summation in the denominator is over $i = 1, \cdots, K$, the population at origins i in the region. To modulate the effect of demand by population in this factor we include an empirical parameter

α (i.e., $\alpha < 1$ places greater weight on smaller centers relative to larger ones while $\alpha > 1$ achieves the opposite effect). This population allocation factor F^p_{ij} can now be used to proportionally allocate a share of the jobs at j to origins.

More broadly, since the factor F^p_{ij} is a proportion, when it is summed over $i = 1, \cdots, K$ it always equals to 1 (i.e., $\sum_i^K F^p_{ij} = 1$). This is notable since the share of jobs at each destination j allocated to (i.e., available to) each origin, based on population, is equal to $V^p_{ij} = O_j \cdot F^p_{ij}$. Since the sum of F^p_{ij} is equal to 1, it follows that $\sum_{i=1}^I V_{ij} = O_j$. In other words, the number of jobs across the region is preserved. The result is a proportional allocation of jobs (opportunities) to origins based on the size of their populations.

To illustrate the population allocation factor, suppose that the employment center in the example has 300 jobs ($O_1 = 300$), and that the two population centers have 240 and 120 people, respectively, ($P_1 = 240$ and $P_2 = 120$). For simplicity, assume that all the population in the region is eligible for these jobs, that is, that the entirety of the population is included in the set r. Also assume that $\alpha = 1$. The population allocation factors F^p_{ij} for the jobs at O_1 for each population center P_1 and P_2 are as follows in Equation (4).

$$\begin{aligned} F^p_{1,1} &= \tfrac{P_1^\alpha}{P_1^\alpha + P_2^\alpha} = \tfrac{240}{240+120} = \tfrac{240}{360} \\ F^p_{2,1} &= \tfrac{P_2^\alpha}{P_1^\alpha + P_2^\alpha} = \tfrac{120}{240+120} = \tfrac{120}{360} \end{aligned} \tag{4}$$

These F^p_{ij} values can be used to find a *partial* spatial availability in which jobs are allocated proportionally to population; this partial spatial availability V^p_{ij} for each population center is calculated as follows in Equation (5).

$$\begin{aligned} V^p_{1,1} &= O_1 \cdot F^p_{1,1} = 300 \cdot \tfrac{240}{360} = 200 \\ V^p_{2,1} &= O_1 \cdot F^p_{2,1} = 300 \cdot \tfrac{120}{360} = 100 \end{aligned} \tag{5}$$

When using only the proportional allocation factor F^p_{ij} to calculate spatial availability (differentiated here by being defined as V^p_{ij} instead of V_{ij}), proportionally more jobs are allocated to the bigger population center (i.e., 2 times more jobs as it is 2 times larger in population). We can also see that the sum of spatial availability for all population centers is equal to the sum of jobs; put another way, the total opportunities are preserved.

Clearly, using only the proportional allocation factor F^p_{ij} to calculate spatial availability does not account for how far population centers P_1 or P_2 are from employment center O_1. It is the task of the second allocation factor F^c_{ij} to account for the friction of distance, as seen in Equation (6).

$$F^c_{ij} = \frac{f(c_{ij})}{\sum_{i=1}^K f(c_{ij})} \tag{6}$$

Travel cost allocation factor F^c_{ij} serves to proportionally allocate more jobs to closer locations through an impedance function. c_{ij} is the cost (e.g., the distance, travel time, etc.) to reach employment center j from i and $f(\cdot)$ is an impedance function that depends on cost (c_{ij}).

To continue with the example, assume that the impedance function is a negative exponential function with $\beta = 1$. β modulates the steepness of the impedance effect and is empirically determined in the case of positive accessibility, or set by the analyst to meet a preset condition in the case of normative accessibility (Paez et al., 2012). Also suppose that the distance from population center P_1 to employment center O_1 is 0.6 km, and the distance from population center P_2 to employment center O_1 is 0.3 km. The proportional allocation factor F_{ij}^p for the jobs at O_1 for both population centers P_1 and P_2 is defined as follows in Equation (7).

$$
\begin{aligned}
F_{1,1}^c &= \frac{\exp(-\beta \cdot D_{1,1})}{\exp(-\beta \cdot D_{1,1}) + \exp(-\beta \cdot D_{2,1})} = \frac{\exp(-0.6)}{\exp(-0.6)+\exp(-0.3)} = 0.426 \\
F_{2,1}^c &= \frac{\exp(-\beta \cdot D_{2,1})}{\exp(-\beta \cdot D_{1,1}) + \exp(-\beta \cdot D_{2,1})} = \frac{\exp(-0.3)}{\exp(-0.6)+\exp(-0.3)} = 0.574
\end{aligned}
\tag{7}
$$

We can see that the proportional allocation factor for P_2 is larger than P_1 since the cost (i.e., distance) to O_1 is lower. Using the travel cost proportional allocation factors F_{ij}^c as defined in Equation (7), we can calculate the spatial availability of jobs for each population center based only on F_{ij}^c and the jobs available at O_1, as shown in Equation (8).

$$
\begin{aligned}
V_{1,1}^c = O_1 \cdot F_{1,1}^c = 300 \times 0.426 = 127.8 \\
V_{2,1}^c = O_1 \cdot F_{2,1}^c = 300 \times 0.574 = 172.2
\end{aligned}
\tag{8}
$$

As seen above, spatial availability defined by F_{ij}^c only (i.e., V_{ij}^c) allocates a larger share of jobs to P_2 since the population center is closer to O_1. However, as previously discussed, P_2 has a smaller population than P_1, so P_1 receives a larger share of jobs when spatial availability is defined by F_{ij}^p (i.e., V_{ij}^p). It is necessary to combine both population and travel cost factors to better reflect demand; these two components are in line with how demand is conventionally modelled in accessibility calculations which are re-scaled on a per demand-population basis or also consider competition (e.g., Allen and Farber, 2019; Barboza et al., 2021; Yang et al., 2006). Fortunately, since both F_{ij}^c and F_{ij}^p preserve the total number of opportunities as they independently sum to 1, they can be combined multiplicatively to calculate the proposed spatial availability V_{ij} which considers demand to be based on both population and travel cost.

Putting spatial availability together

We can combine the proportional allocation factors by population F_{ij}^p and travel cost F_{ij}^c and calculate spatial availability V_{ij} as introduced in Equation (2) and repeated below:

$$
V_{ij} = O_j \frac{F_{ij}^p \cdot F_{ij}^c}{\sum_{i=1}^K F_{ij}^p \cdot F_{ij}^c}
$$

To complete the illustrative example of employment center O_1 and population centers P_1 and P_2, the resulting spatial availability V_{ij} is calculated for both population centers is calculated in Equation (9).

$$
\begin{aligned}
V_{1,1} = O_1 \cdot \frac{F^p_{1,1} \cdot F^c_{1,1}}{F^p_{1,1} \cdot F^c_{1,1} + F^p_{2,1} \cdot F^c_{2,1}} = 300 \frac{\left(\frac{2}{3}\right)\left(0.426\right)}{\left(\frac{2}{3}\right)\left(0.426\right) + \left(\frac{1}{3}\right)\left(0.574\right)} = 179.4 \\
V_{2,1} = O_1 \cdot \frac{F^p_{2,1} \cdot F^c_{2,1}}{F^p_{1,1} \cdot F^c_{1,1} + F^p_{ik} \cdot F^c_{ik}} = 300 \frac{\left(\frac{1}{3}\right)\left(0.574\right)}{\left(\frac{2}{3}\right)\left(0.426\right) + \left(\frac{1}{3}\right)\left(0.574\right)} = 120.6
\end{aligned} \tag{9}
$$

In this example, fewer jobs are allocated to population center P_1 compared to the allocation by population only, to account for the higher cost of reaching the employment center. On the other hand, distance alone allocated more jobs to the closest population center (i.e., P_2), but since it is smaller, it also gets a smaller share of the jobs overall. To reiterate, the sum of jobs at employment center O_1 that are allocated to population centers P_1 and P_2 simultaneously based on *population-* and *travel cost* allocation factors are preserved (i.e., $V_{1,1} + V_{2,1} = O_1$).

In the common case that population centers have multiple destination opportunities j, spatial availability is simply the sum of Equation (2) for all opportunities J (i.e., $V_i = \sum_{j=1}^{J} V_{ij}$). The resulting value of V_i represents opportunities that can be accessed from origin i and that are *not* allocated to any other competing origin: V_i is thus the weighted sum of available opportunities. When comparing V_i to the singly-constrained gravity model (see Wilson (1971)), V_i is the result of constraining A_i to match one of the marginals in the origin-destination table, the known total of opportunities.

Since the sum of opportunities is preserved in the procedures above, it is possible to calculate an interpretable measure of spatial availability per capita (lower-case v_i) as shown in Equation (10).

$$
v_i = \frac{V_i}{P_i} \tag{10}
$$

To complete the illustrative example, the per capita spatial availability of jobs is calculated in Equation (11).

$$
\begin{aligned}
v_{1,1} = \frac{V_{1,1}}{P_2} = \frac{179.4}{240} = 0.8 \\
v_{2,1} = \frac{V_{2,1}}{P_2} = \frac{120.6}{120} = 1.0
\end{aligned} \tag{11}
$$

We can see that since P_2 is closer to O_1 and has less competition (as it has a smaller population than P_1), P_2 benefits with a higher spatial availability of jobs per job-seeking population. We can also compare these values to the overall ratio of jobs-to-population in this region of one job center and two population centers is $300/(240 + 120) = 0.83$.

Empirical example: spatial accessibility and availability of jobs in the GGH

In this section we use population and employment data from the Golden Horseshoe Area (GGH). This is the largest metropolitan region in Canada and includes the cities of Toronto and Hamilton. We present two scales of analysis. The first example demonstrates how accessibility broadly overestimates *job access* (relative to spatial availability) and particularly overestimates job access for areas in the less dense outer limits of the urban core. The second example demonstrates how accessibility underestimates job access for areas in the urban periphery. We first introduce the data used, then calibrate an impedance function, and finally discuss the results.

Data

Population and employment data are drawn from the 2016 Transportation Tomorrow Survey (TTS). This survey collects representative urban travel information from 20 municipalities contained within the GGH area in the southern part of Ontario, Canada (see Figure 2) (Data Management Group, 2018). The data set includes Traffic Analysis Zones (TAZ) (n=3,764), the number of jobs (n=3,081,885) and workers (n=3,446,957) at each origin and destination. The TTS data is based on a representative sample of between 3% to 5% of households in the GGH and is weighted to reflect the population covering the study area has a whole (Data Management Group, 2018).

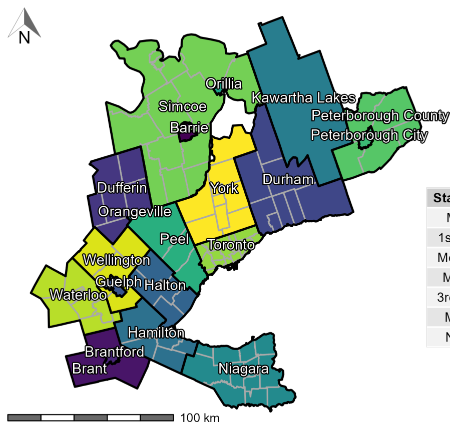
To generate the travel cost for these trips, travel times between origins and destinations are calculated for car travel using the R package {r5r} (Rafael H. M. Pereira et al., 2021) with a street network retrieved from OpenStreetMap for the GGH area. A the 3 hr travel time threshold was selected as it captures 99% of population-employment pairs (see the travel times summarized in Figure 2). This method does not account for traffic congestion or modal split, which can be estimated through other means (e.g., Allen and Farber, 2021; Higgins et al., 2021). For simplicity, we carry on with the assumption that all trips are taken by car in uncongested travel conditions.

All data and data preparation steps are documented and can be freely explored in the companion open data product {TTS2016R}.

Calibration of an impedance function

In the synthetic example introduced in a preceding section, a negative exponential function with an arbitrary parameter was used. For the empirical example, we calibrate an impedance function on the trip length distribution (TLD) of commute trips. Briefly, a TLD represents the proportion of trips that are taken at a specific travel cost (e.g., travel time); this distribution is commonly used to derive impedance functions in accessibility research (Batista et al., 2019; Horbachov and Svichynskyi, 2018).

The empirical and theoretical TLD for this data set are represented in the top-left panel of Figure 3. Maximum likelihood estimation and the Nelder-Mead method for direct optimization available within the {fitdistrplus} package (Delignette-Muller and Dutang, 2015) were used. Based on goodness-of-fit criteria and diagnostics seen in Figure 3, the gamma distribution was selected (also see Figure 11 in Appendix A).



Statistic	Trips	TT	Workers	Jobs
Min.	1	0	0	0
1st Qu.	14	13	25	64
Median	22	20	464	244
Mean	33	23	916	819
3rd Qu.	38	30	1378	700
Max.	1129	179	8491	41821
NA's	NA	3507	NA	NA

Figure 2: TTS 2016 study area (GGH, Ontario, Canada) along with the descriptive statistics of the trips, calculated origin-destination car travel time (TT), workers per TAZ, and jobs per TAZ. Contains 20 regions (black boundaries) and sub-regions (dark gray boundaries).

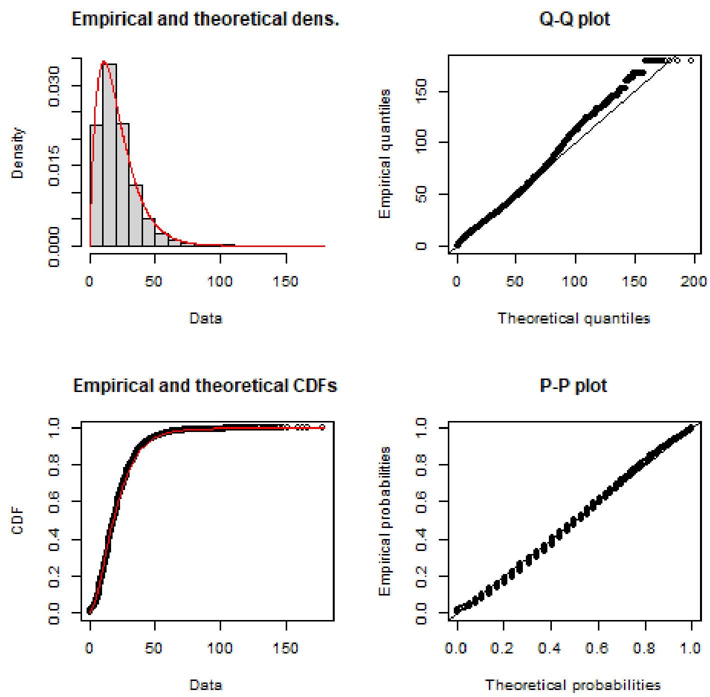


Figure 3: Car trip length distribution and calibrated gamma distribution impedance function (red line) with associated Q-Q and P-P plots. Based on TTS 2016.

The gamma distribution takes the following general form where the estimated 'shape' is $\alpha = 2.019$, the estimated 'rate' is $\beta = 0.094$, and $\Gamma(\alpha)$ is defined in Equation (12).

$$\begin{array}{l} f(x, \alpha, \beta) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} \quad \text{for } 0 \leq x \leq \infty \\ \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx \end{array} \tag{12}$$

Measuring access to jobs in the GGH

Toronto is the largest city in the GGH and represents a significant subset of workers and jobs in the GGH; 51% of workers in the GGH travel to jobs in Toronto and 73% of jobs are located within Toronto. As will be discussed, when accessibility and spatial availability values are compared, this significant subset of jobs in Toronto illustrates both issues associated with the competition effect. Specifically, since accessibility does not include the single-opportunity constraint like spatial availability does, it *overestimates* the jobs available for most TAZ in proximity to Toronto (i.e., GTA) and *underestimates* the jobs available for TAZ in the periphery of the GGH.

Figure 4 presents the accessibility and spatial availability for the full TTS data set. Conventionally, higher accessibility is interpreted as the ability to reach more opportunities. Within the accessibility plot, job access values follow a radial trend where a few TAZ with a high values are strictly located within the boundaries of Toronto and values radially decrease further from the boundaries of Toronto. This general trend is echoed in qualitative studies which find the further from Toronto the longer the employment commute (Axisa et al., 2012) and the closer to core of Toronto the more opportunities are accessible (for some to certain types of jobs, see Páez et al., 2013).

Next, the spatial availability measure is presented alongside the accessibility plot in Figure 4. Similar to the accessibility plot, the higher the value, the more access that TAZ has to jobs in the GGH. Since spatial availability constrains its total to match the total number of opportunities, high values of spatial availability can be seen as higher access to *available* jobs (i.e., competitive job access) and we can observe which TAZ have spatial availability values that are above or below the regional average of 642. It is worth noting that the spatial availability and accessibility plots do not follow the same spatial distribution. Within the spatial availability plot, job access appears more evenly assigned throughout the GGH. Particularly, job access values, as measured by spatial availability, are higher around the north east and south west periphery TAZ and more moderate in and around Toronto than compared to accessibility.

Note that in Figure 4 it can be observed that a few TAZ are greyed out; this corresponds to a null accessibility and spatial availability. Overall, 22% of TAZ contain zero home-to-work GGH trips and as such are allocated a null accessibility and spatial availability. The majority of these TAZ contain no worker population, specifically, 95% have zero workers while only 18% of these TAZ have zero jobs (17% have both zero workers and jobs).

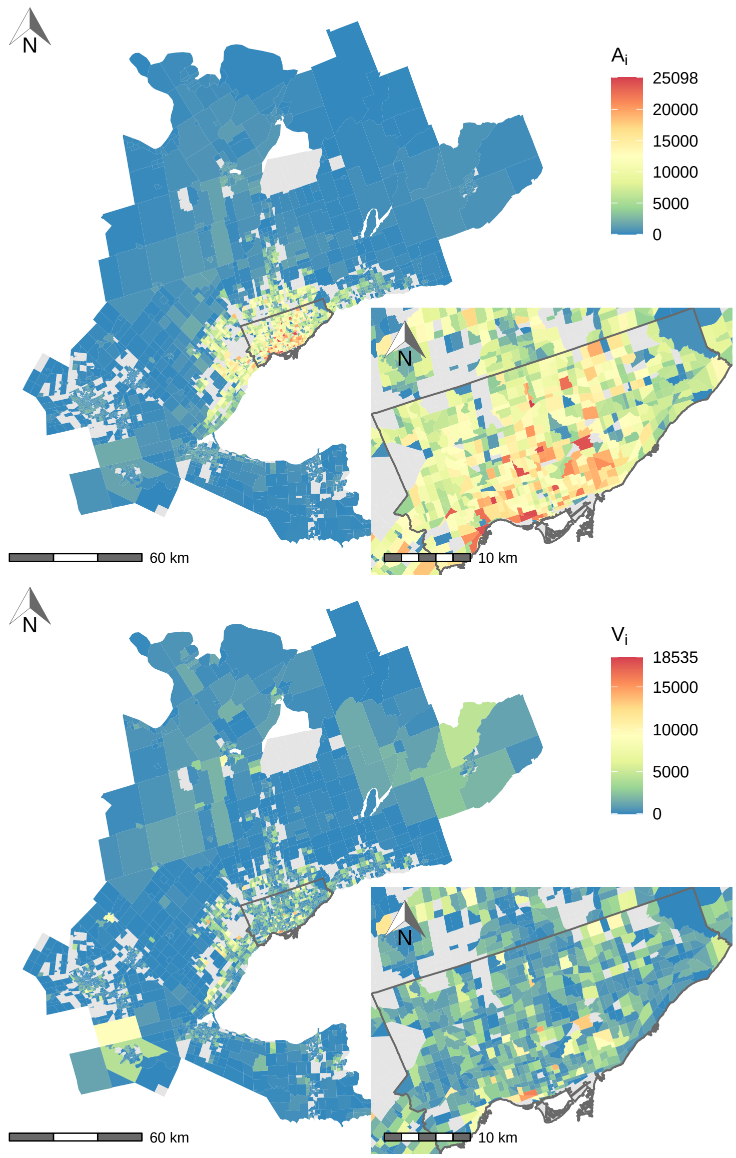


Figure 4: Calculated accessibility (top) and spatial availability (bottom) of employment from origins in the GGH to destinations in the GGH. Greyed out TAZ represent null accessibility and spatial availability values.

To enhance the interpretability, spatial availability can be normalized to provide more meaningful insight into how many jobs are *available* on average for each TAZ. This normalization, shown in Figure 5, demonstrates which TAZ have above (reds) and below (blue) the average available jobs per worker in the GGH (0.89). Similar to the spatial availability plot of the GGH jobs in Figure 4, we can see that many average or above average jobs per worker TAZ (whites and reds) are present in southern Peel and Halton (south-west of Toronto), Waterloo and Brantford (even more south-west of Toronto), and Hamilton and Niagara (south of Toronto), however, the distribution is uneven and many TAZ within these areas do have below average values (blues).

Interestingly, when considering *competitive* job access, many areas outside of Toronto have similar jobs per worker values as TAZ in Toronto. This is contrary to the notion that since Toronto has high job access it has a significant density of employment opportunities in the GGH. Not all jobs in Toronto are *available* since Toronto has a high density of *competition* in addition to density of jobs opportunities. For instance, urban centers outside of Toronto such as those found in Brantford, Guelph, southern Peel, Halton, and Niagara have TAZ which are far above the the TTS average jobs per worker and higher than TAZ within Toronto. High job access is not seen in the accessibility plot which suggests that these less densely populated urban centers may have sufficient employment opportunities for their populations; this finding is obscured when only considering the accessibility measure for job access as will be later discussed.

It is also worth noting that there is almost two times more jobs per worker in the GGH jobs spatial availability results than the GGH Toronto spatial availability results. This suggests that all GGH people who work in the city of Toronto, on average, face more competition for jobs than all GGH people who work anywhere in the GGH .

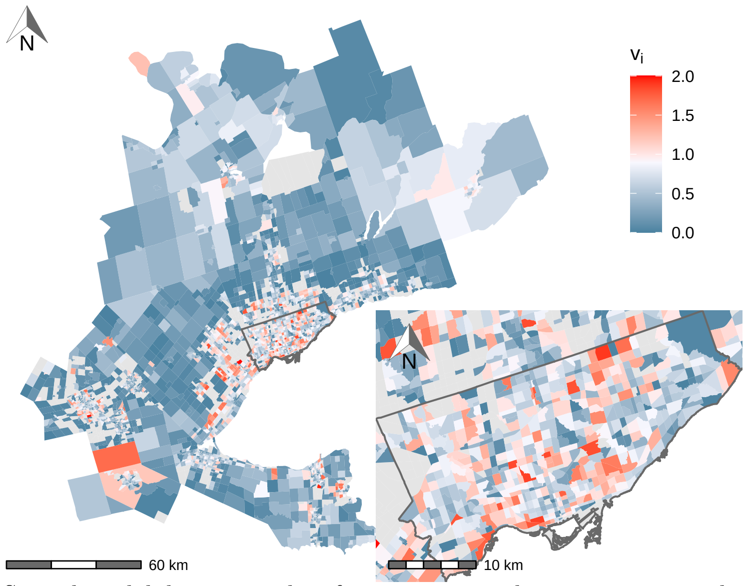


Figure 5: Spatial availability per worker, from origins to job opportunities in the GGH.

Discussion

In the preceding section we used both spatial accessibility and availability measures to explore the landscape of employment opportunities in the GGH. To build on these findings, we return to the two issues associated with accessibility. As noted earlier, accessibility has the tendency to overestimate job access for areas which may experience high competition and underestimate job access for areas with low competition.

To compare both measures, we calculate their relative magnitudes by re-scaling values from 0 to 100 as described by A_{ij}^I and V_{ij}^I in Equation 13. The ratio between A_{ij}^I and V_{ij}^I is then calculated as shown in Equation 14. This ratio represents how many times over- or under- estimated job access is for a TAZ when measured by the re-scaled accessibility relative to the re-scaled spatial availability (e.g., accessibility is X times *larger* than spatial availability thus job access is *overestimated*).

$$
\begin{aligned}
A_{ij}^I &= \tfrac{A_{ij}}{\max(A_{ij})} \cdot 100 \\
V_{ij}^I &= \tfrac{V_{ij}}{\max(V_{ij})} \cdot 100
\end{aligned}
\tag{13}
$$

$$
Ratio_{ij} = \frac{A_{ij}^I}{V_{ij}^I} \tag{14}
$$

Accessibility's role in overestimating and underestimating access

The majority of TAZ (83%) have a ratio above 1 and are thus overestimated. Overestimated TAZ have a median ratio of 4.43 and range between 1 to 1889 with an upper quartile value of 10.54. Overestimated ratio values have a large spread, so values in the 95th percentile (i.e., 69 to 1889) may occur in part as a result of high competition and multiple-counting of opportunities. In other words, these TAZ contain exceptionally low spatial availability values and relatively high accessibility values.

The TAZ that are underestimated are also plotted in Figure 6 for comparison. Of the underestimated TAZ, the median ratio is 0.605 and ranges between 0.006 to 0.995. Similarly, ratio values have a large spread at underestimated extremes; TAZ with values in the 5th percentile (i.e., 0.198 and lower) may partially occur as a result of high competition and multiple-counting of opportunities in *other* TAZ in the region (i.e., 5th percentile underestimated TAZ are assigned exceptionally low relative accessibility values but moderate spatial availability values).

As observed from the top plot in Figure 6, the TAZ with overestimated job access are coloured in varying shades of red. TAZ with the highest degree of overestimation are concentrated within Toronto and the periphery of the GTA. Differences between the number of workers and travel time can only partially explain the overestimation, with TAZ that have a lower number of workers and higher travel time having a larger overestimation. Specifically, the correlations between workers and overestimation, and travel time and overestimation is -0.17 and 0.13, respectively.

In addition to TAZ-specific worker and travel time values, the number of jobs and the value of all three of these variables in neighbouring TAZ also factor into the calculation for both measures. From the perspective of accessibility, the overestimated TAZ are awarded higher values since they are more centrally located near opportunities (i.e., low travel costs) and are not discounted from being in proximity to highly competitive TAZ (i.e., high density of workers). On the other hand, spatial availability considers this competition by proportionally allocating jobs to TAZ relative to their travel cost and worker population. In other words, TAZ appears *overestimated* when the impact of travel cost alone outpaces the impact of both travel cost and workers thus the proportional allocation of opportunity results in a significantly smaller job access than the resulting value for accessibility. This may explain why TAZ which are exceptionally overestimated (dark reds) are on the periphery of more densely populated urban cities. These TAZ are located far enough away from workers in densely populated TAZ but still relatively close enough to jobs in Toronto. As such, their high accessibility value is a result of counting many more opportunities than the single-constrained spatial availability proportionally allocates.

Turning to the bottom plot in Figure 6, TAZ with underestimated job access are located on the periphery of the GGH area and in proximity to other, smaller, employment areas within smaller municipalities outside the GTA. It can be understood that the discussed impact of accessibility's multiple-counting of opportunities effectively deflates the job access in these TAZ particularly because their accessibility values are lower compared to their multiple-counted GTA TAZ neighbours. This multiple-counting impact is starkly apparent when we revisit Figure 5 which demonstrates that the spatial availability of jobs per

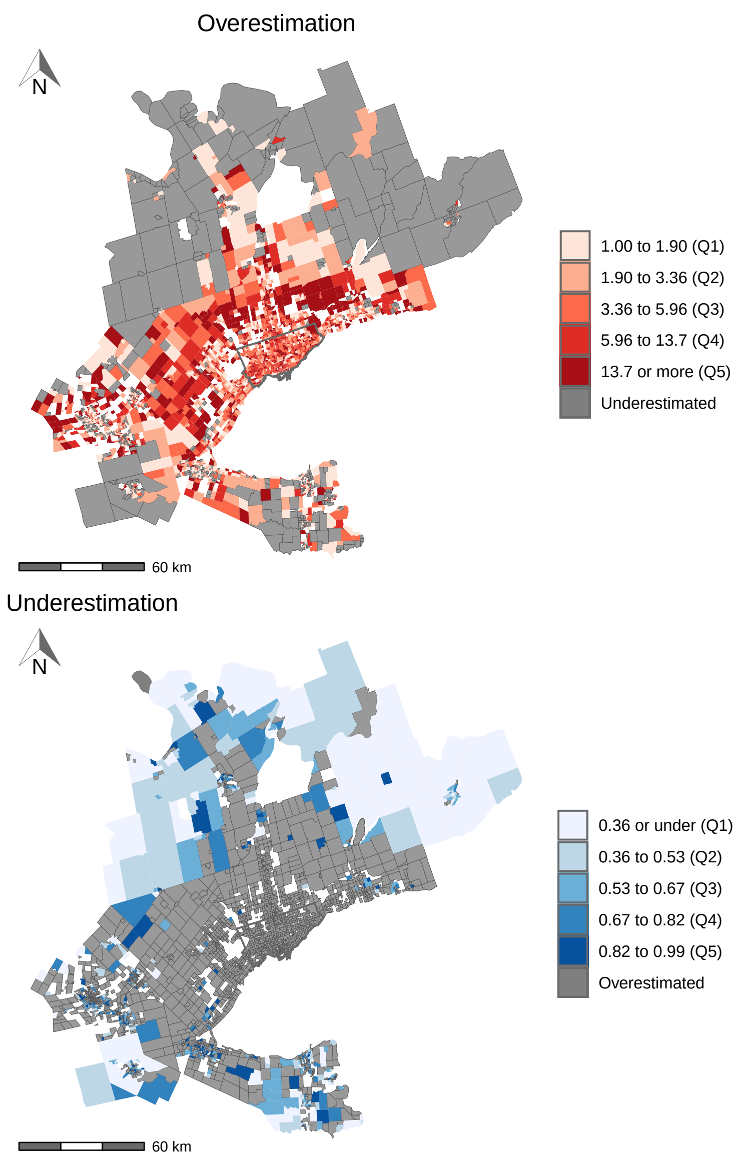


Figure 6: Overestimated (top) and underestimated (bottom) ratios of re-scaled accessibility and spatial availability measures for all origins to job opportunites in the GGH. Values are expressed in five quantiles. Greyed TAZ have values which are underestimated in the top plot and are overestimated in the bottom plot. TAZ with no trips and thus no spatial availability or accessibility values are not drawn.

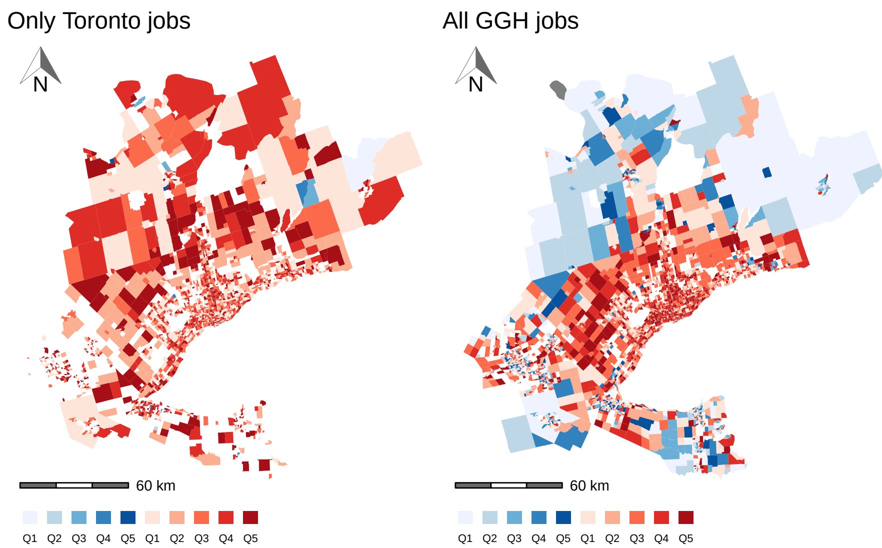


Figure 7: Ratios of the re-scaled accessibility to re-scaled spatial availability for Toronto only jobs (left) and all GGH jobs (right) represented in quantiles. Red represent overestimated differences and blue represents underestimated diffeences. TAZ with no trips and thus no spatial availability or accessibility values are not drawn.

worker for many municipalities have similar job access values as within areas in the GTA; this is not the case for the accessibility measure when contrasted with discussed trends in Figure 4.

Comparing differences in job access when only considering Toronto jobs in the GGH

To reinforce the impact that competition has on the spatial availability measure, Figure 6 presents a consolidated plot of those included in Figure 6 and another plot calculated only using the subset of jobs in Toronto. This subset of jobs is significant as 51% of workers in the GGH travel to jobs in Toronto and 73% of jobs are located within Toronto. It can be observed, that when subseting the analysis, almost all TAZ are overestimated (i.e., accessibility is larger than spatial availability). Notably, we can see virtually all underestimation disappears (blues) and the TAZ in the GHG periphery are now all overestimated (reds).

Focusing on the GGH peripheral TAZ in the left plot (Figure 6), these TAZ have relatively low spatial availability since they have relatively high travel times to jobs in Toronto and have a moderately low worker population. Similarly, these peripheral TAZ also have relatively low accessibility (compared to the region) as a result of the multiple-counting which occurs within and around the high density of jobs in Toronto. However, when spatial availability and accessibility are compared, accessibility is still *higher* than spatial availability thus the difference appears *overestimated* (red).

Interestingly, the difference in the two measures flips when the workers who work in areas outside of Toronto are re-introduced to these peripheral TAZ as represented in right plot (Figure 6). Since more workers (more competition) and more jobs with lower travel costs (less competition) are introduced, it can be inferred that these peripheral TAZ experience moderately higher job access as measured with spatial availability. Conversely, for the accessibility measure, the introduction of additional workers does not meter the accessibility gained by the introduction of additional jobs and as such, the re-scaled accessibility values are *higher* than the spatial availability resulting in underestimated TAZ.

As such, we see that when including all the jobs in the region back into the analysis (right plot in Figure 6) - these periphery TAZ are much less *job deprived* when measured using spatial availability than when measured using accessibility. Overall, within the full set of jobs in the GGH, the first and second issue associated with accessibility's competition effect are observed.

Other use cases of spatial availability

In addition to measuring access to jobs for workers, spatial availability can be used to measure many other opportunity types and scenarios. In this section we briefly discuss two ways in which spatial availability can be applied in practice. Since spatial availability singly-constraints opportunities which are allocated to the demand seeking population, opportunities or demand seeking populations can be subset while the resulting access measure does not lose any interpretability. As such, we firstly present an example with specialized population subsets and specialized employment centers. We then present an example where the focus is changed to employers seeking worker.

To illustrate these two additional examples, we return to the synthetic example initially introduced in the Background section.

Specialized employment opportunities

Suppose that population centers P1 through P8 are not all eligible for employment at the three employment centers E1, E2, and E3. This can be due to education attainment or more simply a geographic barrier (e.g., river without a bridge) making it impossible for certain populations to be employed at certain employment centers. In this example, we consider that only population center P1 and P2 are eligible for employment at employment center E1. Next assume that jobs in employment center E2 can be taken by individuals in population centers P3, P4, P5, P7, and P8. Lastly, jobs in employment center E3 require qualifications available only among individuals in population centers P5, P6, P8, and P9. In essence, eligibility criteria creates catchments as seen in Figure 8.

For higher interpretability, we can inspect Figure 9 which presents plots of spatial availability per capita with and without catchments.

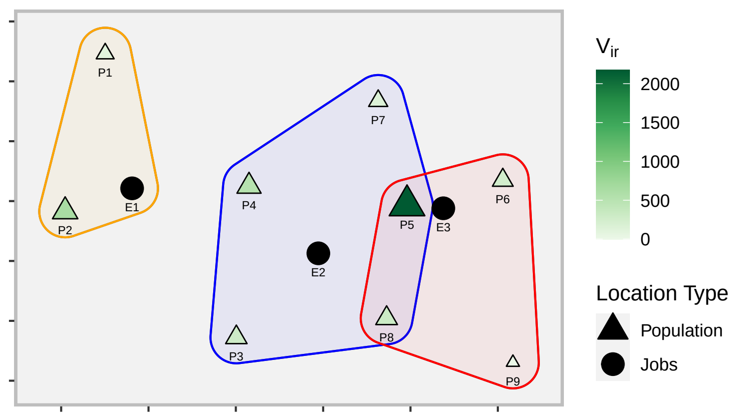


Figure 8: Spatial availability of jobs from population centers assuming catchment restrictions (r) for the simple synthetic example

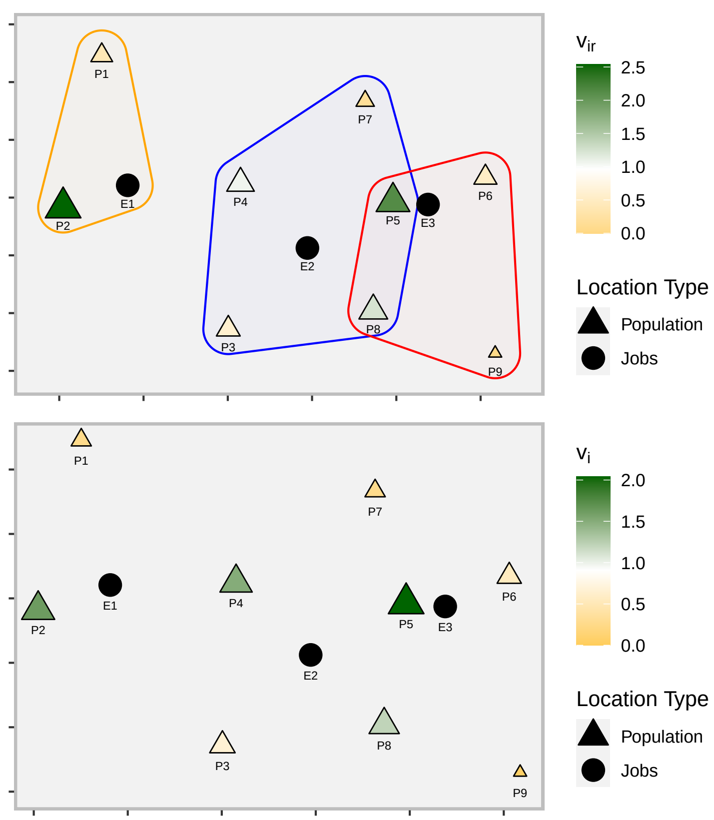


Figure 9: Spatial availability of jobs per capita for each population centers with catchment restrictions (r) (top) and without catchment restrictions (bottom) for the synthetic example

In the bottom plot of Figure 9, we see that population center P5 has the highest level of spatial availability, due to being a large population center that is relatively close to jobs. We also see population centers which are further from employment centers have low spatial availability (P1, P3, P6, P7, P9) and population centers which are more central have, evidently, higher job access. We can also discern that the regional spatial availability per capita is 0.908. In contrast, when catchments are introduced, as shown in the top plot of Figure 9, we see that the number of opportunities available for population centers in the blue catchment decreases as job competition increases. However, we also observe that access marginally increases for the population centers in the yellow catchment and the regional spatial availability per capita increases slightly to 0.964.

Spatial availability of the labour pool

In this use case, we remove the catchments and switch the demand and opportunity roles of the employment centers and population centers. In this way, the spatial availability measure reflects the opportunities to find workers from the perspective of employers. In this example, workers are allocated proportionally to employment centers based on travel cost (i.e., distance) and demand population (i.e., jobs at each employment center). Figure 10 presents spatial availability generally and per capita for this use case.

As shown in Figure 10, the top plot demonstrates the spatial availability which measures accessibility to labour for the three employment centers. As with all spatial availability calculations, the number of all opportunities are proportionally allocated to each demand center so employment center E3, E2 and E1 have access to 2205, 1710, and 610 workers respectively. Again, this totals to 4525 workers in the area. In the bottom plot, these spatial availability values are normalized per demand and we can see that employment center E3 has an above average worker access of 1.47 workers per job (i.e., 2205 workers to 1500 jobs) while E2 and E1 have 0.76 and 0.81 workers per job respectively.

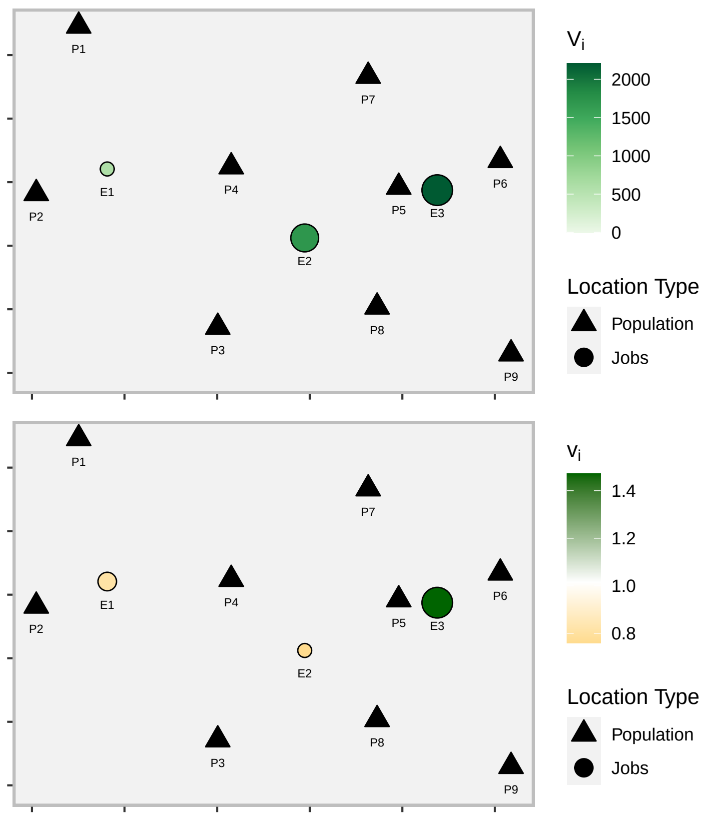


Figure 10: Spatial availability of workers for each employment center (top) and the spatial availability of workers per job for each employment center (bottom) for the simple synthetic example

Concluding remarks

In this paper, we introduce a singly-constrained accessibility measure that we term *spatial availability*. This measure is an alternative to conventional accessibility calculations that provides more meaningful estimates than accessibility by ensuring that the estimates match the observed number of opportunities. This enables the calculation of per capita measures and measures relative to regional averages. This development makes it easier to compare opportunity landscapes across regions and/or time.

We demonstrated the use of spatial availability and compared it to spatial accessibility in an empirical example of workers and jobs in the GGH. We also discussed two additional use cases to emphasize the versatility of our new measure. In all examples, we highlight how intuitive the per demand population access values are and how the measure can be interpreted.

Fundamentally, accessibility measure's methodology results in the overestimation or underestimation of opportunity access as a result of the competition effect. Accessibility does not include factors to bound the summation so origins, hypothetically, can have infinite opportunity access. By including an intuitive single-sided constraint in the new spatial availability measure, it can be used to calculate access to non-divisible opportunities. In this paper, we focused on employment, but any competitive opportunities such as schools, hospitals, and other essential services with finite capacity at a given time can be studied. Additionally, opportunities that are non-competitive in the sense that they have always larger capacities than population, can still benefit from this measure as access will not be over- or under-estimated as a result of the competition effect seen in accessibility; non-competitive opportunities can include large natural parks and beaches. As such, values measured for opportunities of any type can benefit from an availability perspective through the reduction in competition effect (from accessibility) and a meaningful benchmark of demand per population unit.

Appendix A

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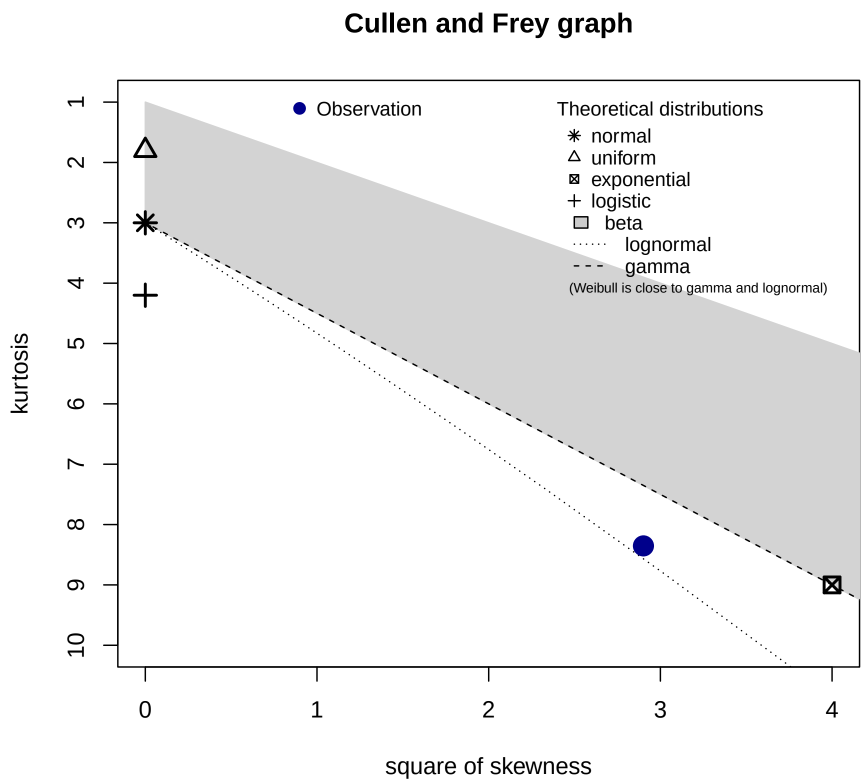

Figure 11: Cullen and frey graphy for the 2016 TTS calculated travel times.

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