Spatial availability: a singly-constrained accessibility measure

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About the authors

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Overview

1. Hansen- and Shen- style accessibility measures
2. Shen-style accessibility's important property
3. An alternative approach: spatial availability
4. How spatial availability compares
5. An empirical example
6. Future uses

Overview of accessibility measures

Hansen-style accessibility

$$A_i = \sum_j O_j f(c_{ij})$$

- Suffers from interpretability issues *(Handy and Niemeier, 1997; Miller et al., 2018)*
- Does not include competition *(Shen, 1998; Merlin and Hu, 2018)*

Shen-style (competitive) accessibility

Shen (1998) modified accessibility popularized as the two-step floating catchment approach (2SFCA) *(Luo and Wang, 2003)*:

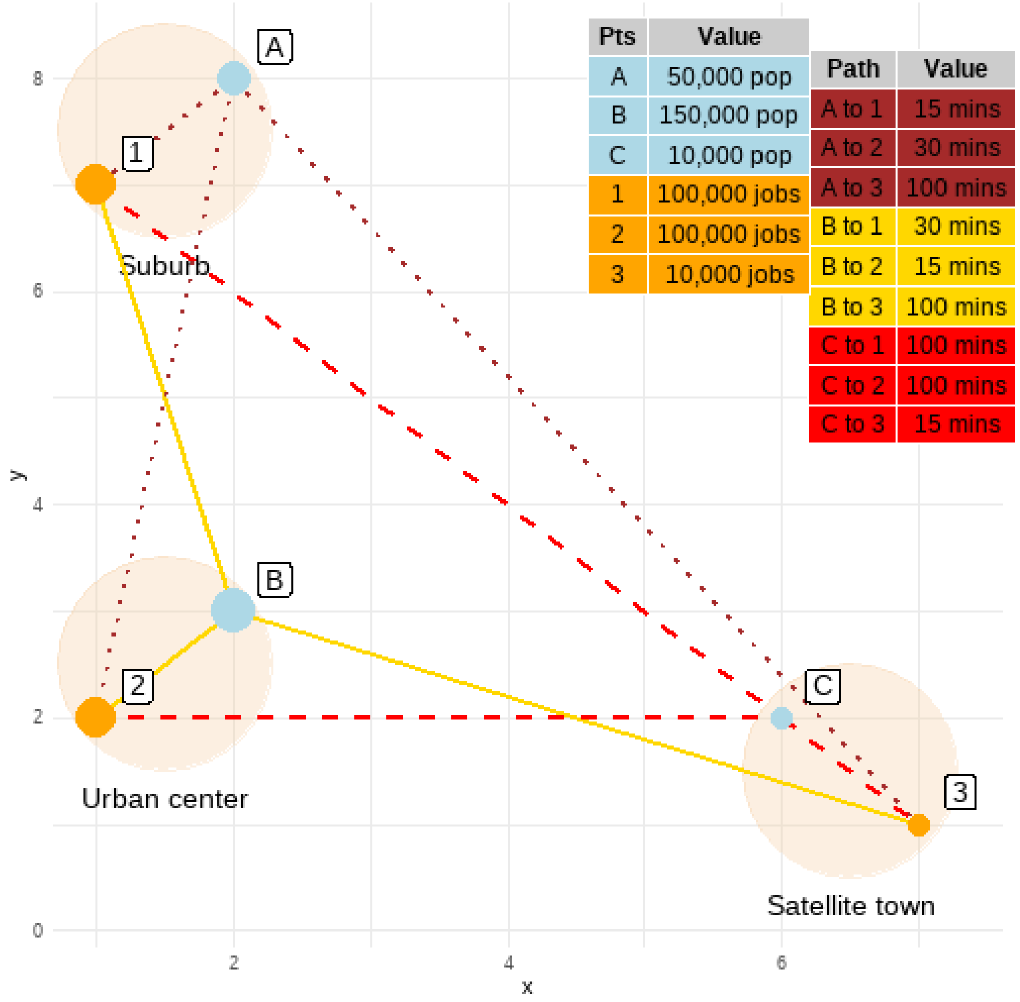
$$S_i = \sum_j \frac{O_j f(c_{ij})}{D_j}$$

with:

$$D_j = \sum_k P_k f(c_{jk})$$

- S_i is Shen-style accessibility.
- P_k is the number of people in location k seeking opportunities $k = 1, 2, \ldots, N$.

Toy example - Hansen-style accessibility



Pts	Value
A	50,000 pop
B	150,000 pop
C	10,000 pop
1	100,000 jobs
2	100,000 jobs
3	10,000 jobs

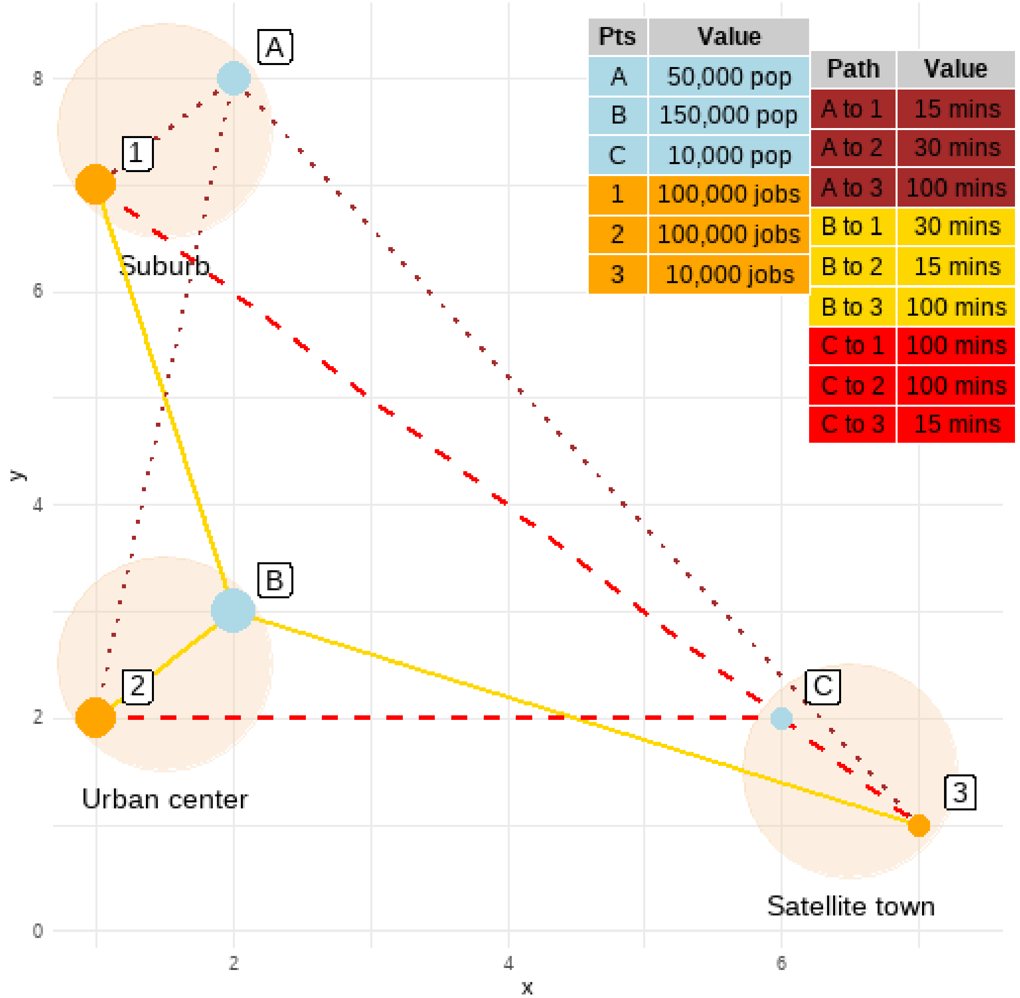
Path	Value
A to 1	15 mins
A to 2	30 mins
A to 3	100 mins
B to 1	30 mins
B to 2	15 mins
B to 3	100 mins
C to 1	100 mins
C to 2	100 mins
C to 3	15 mins

$$A_i = \sum_j O_j f(c_{ij})$$

Impedance function: $f(c_{ij}) = exp(-\beta \cdot tt)$; where $\beta = 0.1$

i	A_i
A	27292.18
B	27292.18
C	2240.38

Toy example - Shen-style accessibility



$S_i = \sum_j \frac{O_j f(c_{ij})}{D_j}$ where $D_j = \sum_k P_k f(c_{kj})$ and same $f(c_{ij})$

The intermediates $O_j f(c_{ij})$ and D_j:

i/j	1	2	3
A	22313.02	4978.71	0.45
B	4978.71	22313.02	0.45
C	4.54	4.54	2231.30

k/j	1	2	3
A|B|C	18625.02	35959.33	2240.38

Hansen- A_i and Shen- S_i style accessibility:

i	A_i	S_i
A	27292.18	1.34
B	27292.18	0.89
C	2240.38	1.00

Shen's property - with inconsistent logic

$$S_i = \sum_j \frac{O_j f(c_{ij})}{D_j}$$

- An important property: $\sum_i S_i \cdot P_i = \sum_j O_j$

i	S_i	P_i	S_i*P_i
A	1.34	50000	66833.47
B	0.89	150000	133203.36
C	1	10000	9963.17
TOTAL	3.22	210000	210000

j	O_j
1	100000
2	100000
3	10000
TOTAL	210000

Demand D_j by each origin i

i	Sum_j(D_ij)	S_i * Sum_j(D_ij)
A	13648.13	18243.04
B	40944.39	36359.54
C	2232.21	2223.99
TOTAL	56824.73	56826.57

- TOTAL D_{ij} is less than TOTAL worker population.
- TOTAL $S_i D_{ij}$ (i.e., $\frac{jobs}{worker} \cdot workers$) is less than TOTAL jobs.
- What's the value of this property?

This important property - is the basis of *spatial availability*

- Spatial availability:

$$V_i = \sum_j O_j \frac{F_i^p \cdot F_{ij}^c}{\sum_i F_i^p \cdot F_{ij}^c}$$

- Where the population allocation factor:

$$F_{ij}^p = \frac{P_i}{\sum_i P_i}$$

- And the cost allocation factor:

$$F_{ij}^c = \frac{f(c_{ij})}{\sum_i f(c_{ij})}$$

- And spatial availability per capita:

$$v_i = \sum_j \frac{O_j}{P_i} \frac{F_i^p \cdot F_{ij}^c}{\sum_i F_i^p \cdot F_{ij}^c}$$

- Normalized *spatial availability* is equivalent to Shen-style accessibility
- Both are competitive and singly-constrained accessibility measures

Spatial availability - solving for suburban center A

$$V_i = \sum_j O_j \frac{F_i^p \cdot F_{ij}^c}{\sum_i^K F_i^p \cdot F_{ij}^c}$$

Let's solve V_A,

Population allocation factor: $F_A^p = \frac{P_A}{P_A+P_B+P_C} = \frac{50000}{210000} = 0.24\ldots$

Cost allocation factor: $F_{A1}^c = \frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})} = \frac{0.22}{0.22+0.05+0.00} = 0.82\ldots$

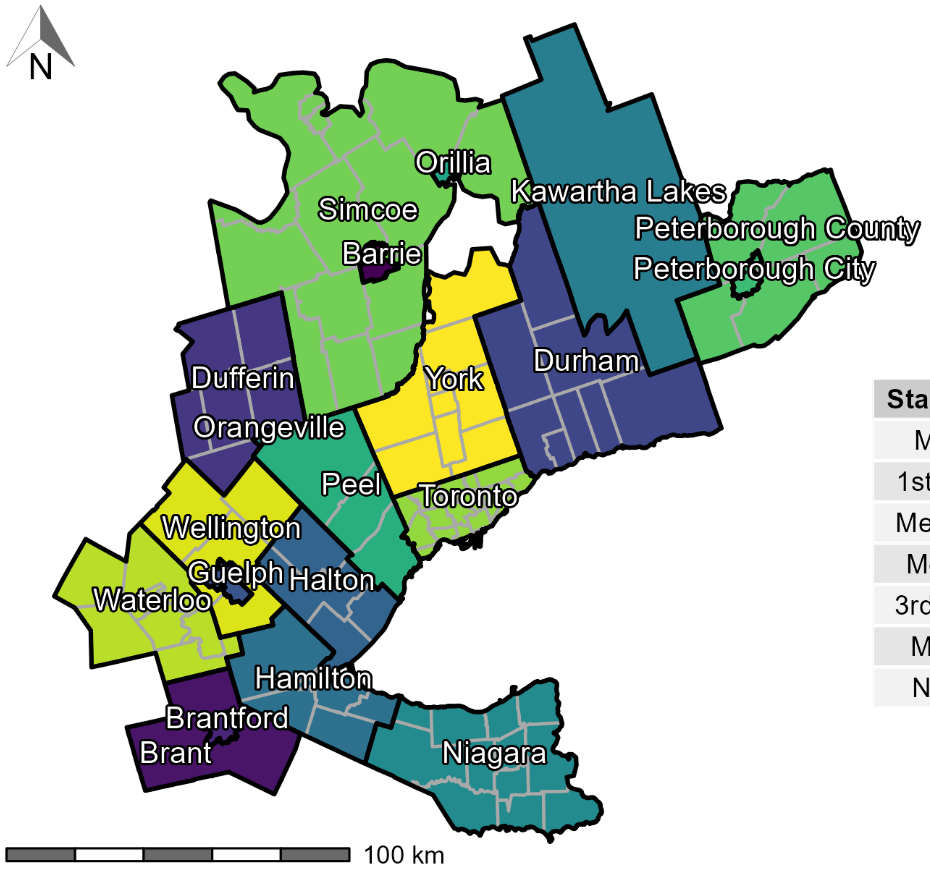
Put it together:

$$V_A = O_1 \frac{F_A^p \cdot F_{A1}^c}{F_A^p \cdot F_{A1}^c + F_B^p \cdot F_{B1}^c + F_C^p \cdot F_{C1}^c} + O_2 \frac{F_A^p \cdot F_{A2}^c}{F_A^p \cdot F_{A2}^c + F_B^p \cdot F_{B2}^c + F_C^p \cdot F_{C2}^c} + O_3 \frac{F_A^p \cdot F_{A3}^c}{F_A^p \cdot F_{A3}^c + F_B^p \cdot F_{B3}^c + F_C^p \cdot F_{C3}^c}$$

$$V_A = 100000 \frac{0.24 \cdot 0.82}{0.32} + 100000 \frac{0.24 \cdot 0.18}{0.63} + 10000 \frac{0.24 \cdot 0.00}{0.05}$$

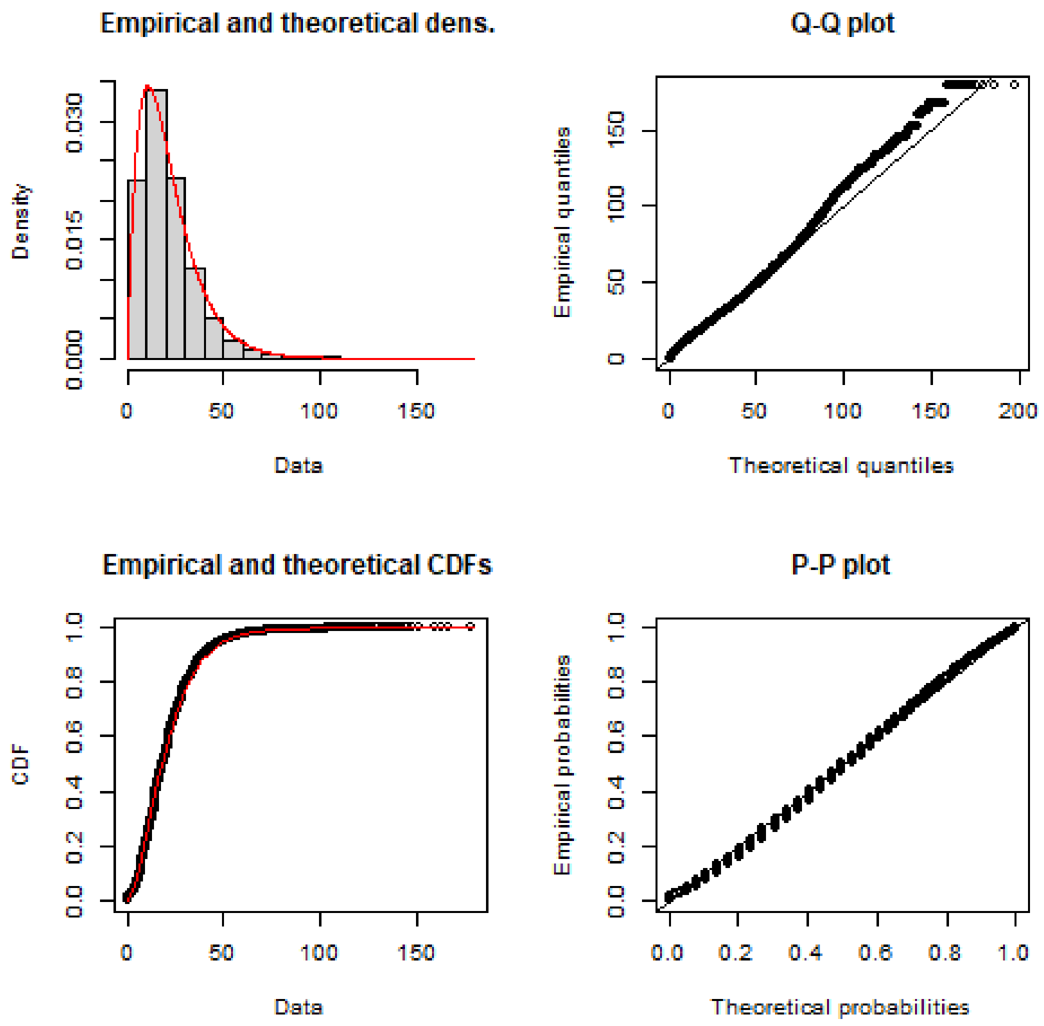
$V_A = 59900.64 + 6922.69 + 10.13 =$ 66833.46 spatially available jobs

Empirical example: 2016 Transportation Tomorrow Survey (TTS) employment data

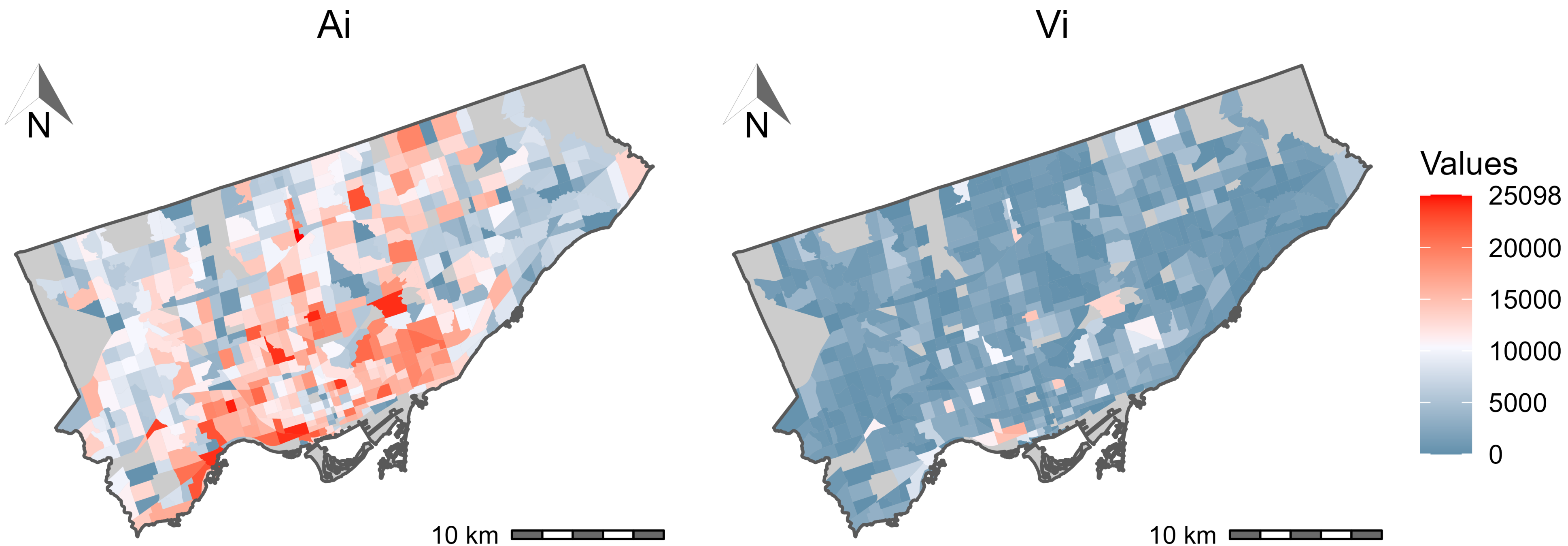


Statistic	Trips	TT	Workers	Jobs
Min.	1	0	0	0
1st Qu.	14	13	25	64
Median	22	20	464	244
Mean	33	23	916	819
3rd Qu.	38	30	1378	700
Max.	1129	179	8491	41821
NA's	NA	3507	NA	NA

Empirical example: calibrated impedance function



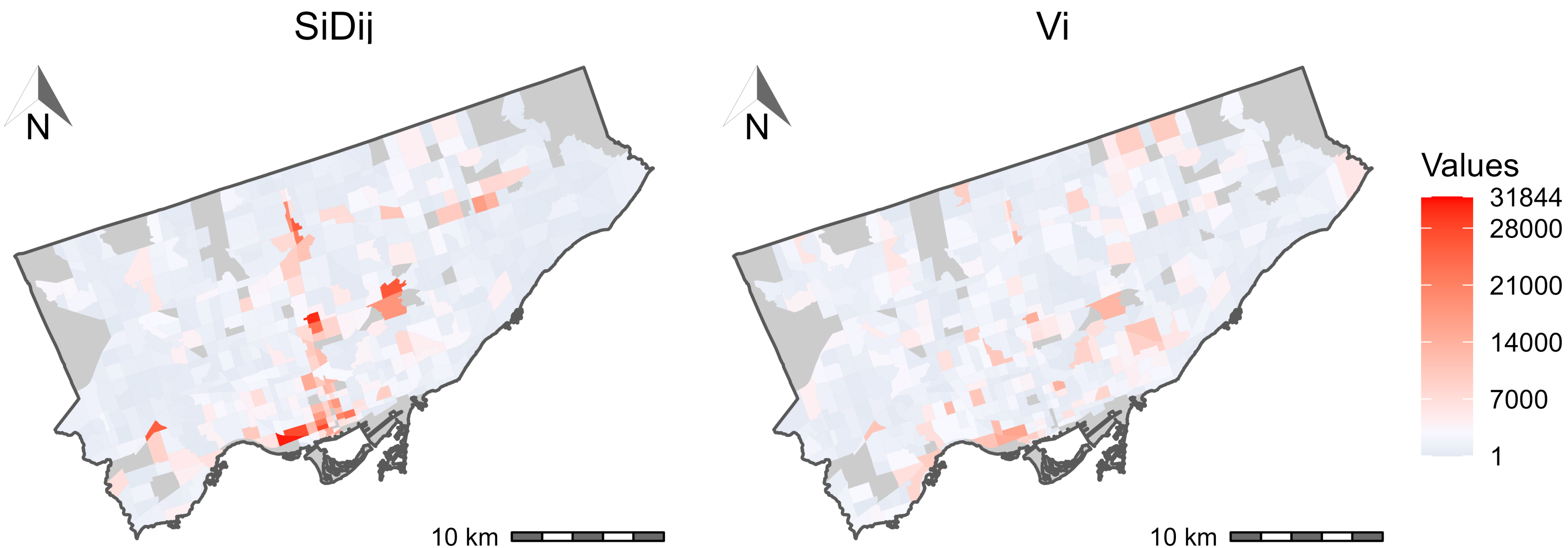
Empirical example: Hansen-style accessibility and spatial availability



$$A_i = \sum_j O_j f(c_{ij})$$

$$V_i = \sum_j O_j \frac{F_i^p \cdot F_{ij}^c}{\sum_i F_i^p \cdot F_{ij}^c}$$

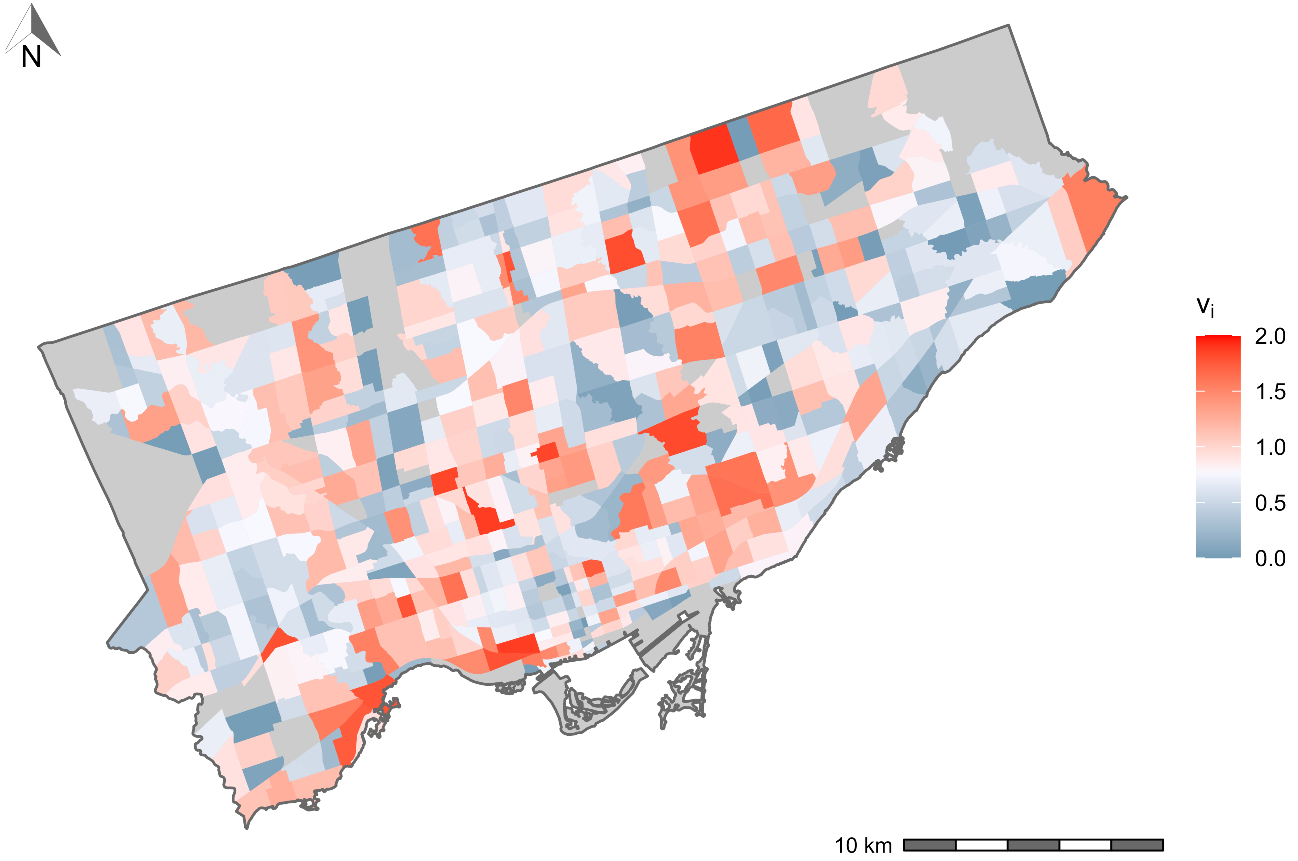
Empirical example: jobs allocated in Shen-style accessibility and spatial availability



$S_i = \sum_j \frac{O_j f(c_{ij})}{D_j}$ and $D_j = \sum_k P_k f(c_{kj})$ at each origin i

$V_i = \sum_j O_j \frac{F_i^p \cdot F_{ij}^c}{\sum_i F_i^p \cdot F_{ij}^c}$

Empirical example: spatial availability per capita



$$v_i = \frac{V_i}{P_i} = S_i = \sum_j \frac{O_j f(c_{ij})}{D_j}$$

Conclusions

Spatial availability:

- Increases interpretability of Shen-style accessibility
- Singly-constrained accessibility measures
- Can serve as a cross-regional benchmark for opportunities per capita

Acknowledgments

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Any questions? Remarks?

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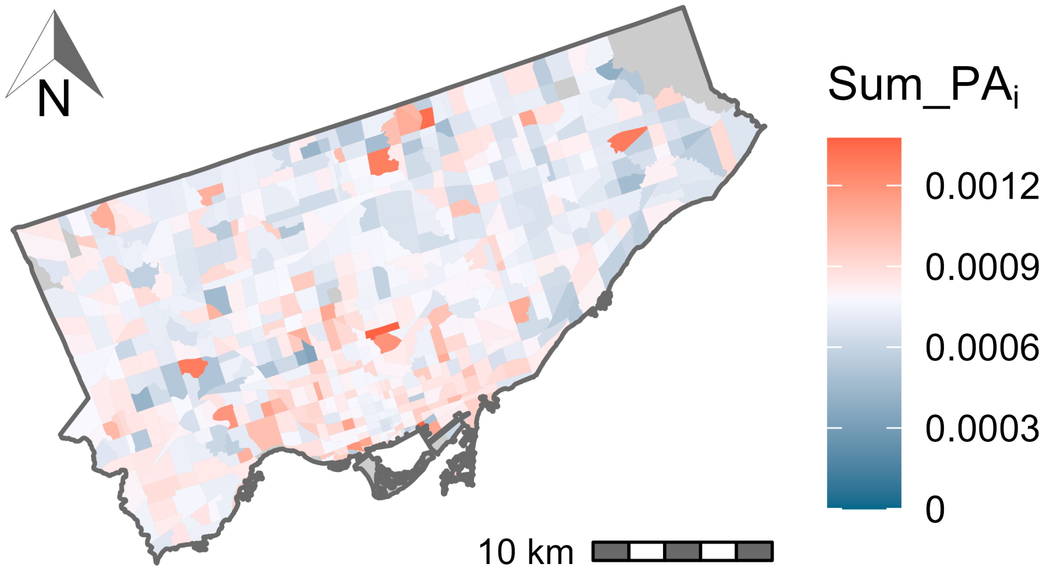
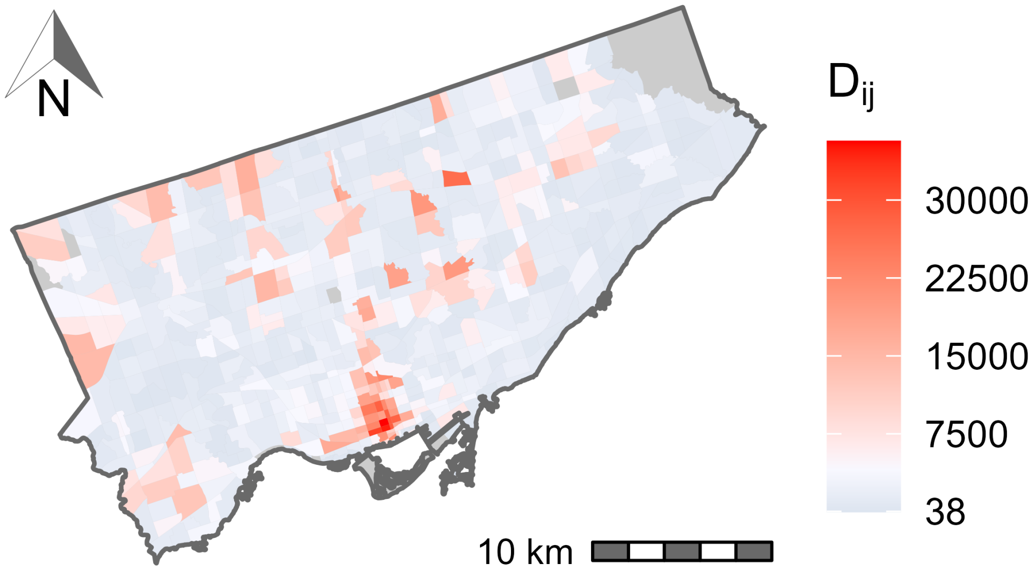
Extra slides

Empirical example: intermediates - spatial availability's proportional allocation and Shen's demand





Proof: Spatial availability cancels out into Shen-style accessibility

Population allocation factor: $F^p_{ij} = \frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}}$

$F^p_A = \frac{P^\alpha_A}{P^\alpha_A + P^\alpha_B + P^\alpha_C}$

Cost allocation factor: $F^c_{ij} = \frac{f(c_{ij})}{\sum_{i=A}^K f(c_{ij})}$

$F^c_{A1} = \frac{f(c_{A1})}{f(c_{A1}) + f(c_{B1}) + f(c_{C1})}$ $F^c_{B1} = \frac{f(c_{A2})}{f(c_{A2}) + f(c_{B2}) + f(c_{C2})}$ $F^c_{C1} = \frac{f(c_{A3})}{f(c_{A3}) + f(c_{B3}) + f(c_{C3})}$

Now let's put it together with P, and see how the denominators end up cancelling out:

$$v_i = \sum_j \frac{O_j}{P^\alpha_{i\in r}} \frac{\frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}{\sum_i^K \frac{P^\alpha_{i\in r}}{\sum_i^K P^\alpha_{i\in r}} \cdot \frac{f(c_{ij})}{\sum_i^K f(c_{ij})}}$$

$$v_A = \frac{O_1}{P_A^\alpha}\left(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}+\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{B1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}+\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{C1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}\right)+$$

$$\frac{O_2}{P_A^\alpha}\left(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}+\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{B2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}+\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{C2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}\right)+$$

$$\frac{O_3}{P_A^\alpha}\left(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}+\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{B3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}+\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{C3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}\right)$$

First, notice how the denominator on the denominator is the same across the summation? Let's simplify it:

$$v_A = \frac{O_1}{P_A^\alpha}\left(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A1})}{f(c_{A1})+f(c_{B1})+f(c_{C1})}}{\frac{P_A^\alpha\cdot f(c_{A1})+P_A^\alpha\cdot f(c_{B1})+P_A^\alpha\cdot f(c_{C1})}{(P_A^\alpha+P_B^\alpha+P_C^\alpha)\cdot(f(c_{A1})+f(c_{B1})+f(c_{C1}))}}\right)+\frac{O_2}{P_A^\alpha}\left(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A2})}{f(c_{A2})+f(c_{B2})+f(c_{C2})}}{\frac{P_A^\alpha\cdot f(c_{A2})+P_A^\alpha\cdot f(c_{B2})+P_A^\alpha\cdot f(c_{C2})}{(P_A^\alpha+P_B^\alpha+P_C^\alpha)\cdot(f(c_{A2})+f(c_{B2})+f(c_{C2}))}}\right)+\frac{O_3}{P_A^\alpha}\left(\frac{\frac{P_A^\alpha}{P_A^\alpha+P_B^\alpha+P_C^\alpha}\cdot\frac{f(c_{A3})}{f(c_{A3})+f(c_{B3})+f(c_{C3})}}{\frac{P_A^\alpha\cdot f(c_{A3})+P_A^\alpha\cdot f(c_{B3})+P_A^\alpha\cdot f(c_{C3})}{(P_A^\alpha+P_B^\alpha+P_C^\alpha)\cdot(f(c_{A3})+f(c_{B3})+f(c_{C3}))}}\right)$$

See how the denominator of the denominator is the same as the denominator of the numerator's denominator for each J (J=1, J=2, and J=3)? Let's cancel those out and simplify:

$$v_A = \frac{O_1}{P_A^\alpha}\left(\frac{P_A^\alpha\cdot f(c_{A1})}{P_A^\alpha\cdot f(c_{A1})+P_A^\alpha\cdot f(c_{B1})+P_A^\alpha\cdot f(c_{C1})}+\frac{O_2}{P_A^\alpha}\frac{P_A^\alpha\cdot f(c_{A2})}{P_A^\alpha\cdot f(c_{A2})+P_A^\alpha\cdot f(c_{B2})+P_A^\alpha\cdot f(c_{C2})}+\frac{O_3}{P_A^\alpha}\frac{P_A^\alpha\cdot f(c_{A3})}{P_A^\alpha\cdot f(c_{A3})+P_A^\alpha\cdot f(c_{B3})+P_A^\alpha\cdot f(c_{C3})}\right)$$

Next, see how we can cancel out the P_A^α? Let's do that.

$$v_A = O_1(\frac{f(c_{A1})}{P_A^\alpha \cdot f(c_{A1}) + P_B^\alpha \cdot f(c_{B1}) + P_C^\alpha \cdot f(c_{C1})} + O_2 \frac{f(c_{A2})}{P_A^\alpha \cdot f(c_{A2}) + P_B^\alpha \cdot f(c_{B2}) + P_C^\alpha \cdot f(c_{C2})} + O_3 \frac{f(c_{A3})}{P_A^\alpha \cdot f(c_{A3}) + P_B^\alpha \cdot f(c_{B3}) + P_C^\alpha \cdot f(c_{C3})})$$

Shen's accessibility:

$$S_i = \sum_j \frac{O_j f(c_{ij})}{\sum_k P_k f(c_{jk})}$$