Macroeconometrics - UK variables report

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1 Preliminaries

In this project, we aim to analyze the development of the Gross Domestic Product (GDP), exchange rates, and trade balance of the United Kingdom (UK). We collect our data from the OECD open data portal. We use data on a quarterly basis, starting latest in 1997. This period covers important financial events such as the Global Financial Crisis (GFC) in 2007, the Brexit referendum in 2016 and UK's final EU leave in 2020 as well as the COVID-19 pandemic from 2020-2022.

2 Univariate Analysis

2.1 Macro trends

Figure 1 illustrates the evolution of the UK's macroeconomic indicators—GDP, Trade Balance, and Exchange Rate—in both levels and first differences.

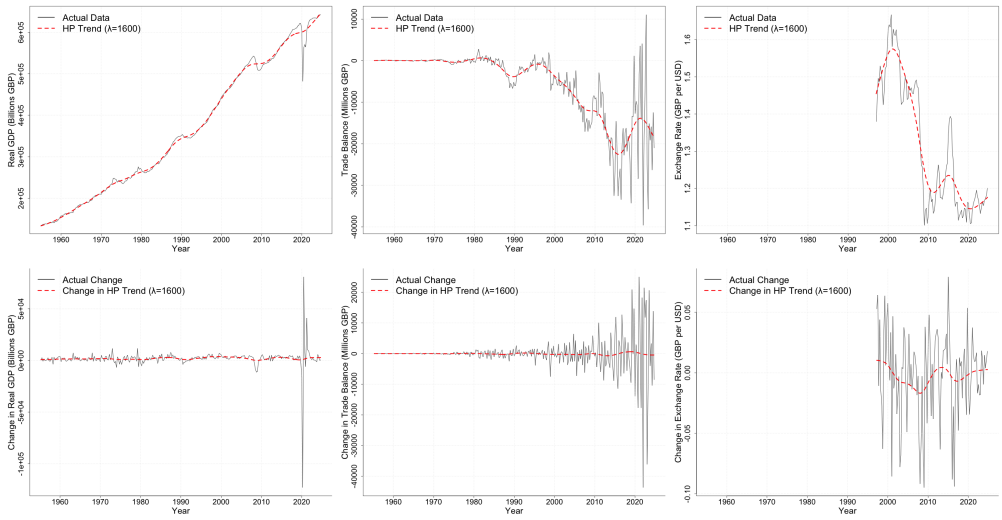
Trend Dynamics and Structural Breaks in Level Series. Data on UK's GDP, available on a quarterly basis since 1955, exhibits a clear, upwards trajectory with only a few shocks such as the 2007 financial crisis or the 2020 COVID-19 pandemic interrupting the general trend. Hence, the GDP of the UK is clearly not stationary. However, the trend seems to be linear, suggesting that the first-differences time series of the GDP might be stationary. Similarly, data on the trade balance is available on a quarterly basis starting in 1955.We see that the trade balance fluctuates around 0 until the 1990s where a negative trend seems to set in continueing until the 2010s, when trade balance starts fluctuating around a low, negative value. However, since the observed spikes are getting much bigger over time, we also see an increase in fluctuation around the respective stationary mean.

Hence, the data seems to be stationary in the beginning and in the end with a negative trend being observed between the 1990s and the 2010s. Data on the exchange rate is available since 1997 on a quarterly basis. It exhibits stationarity between 1997 and 2007, as well as from 2007 onwards. In 2007, a shock seems to have shifted the mean of the stationary process downwards. The bottom row highlights first-differenced series, which strip away trends to reveal stationary fluctuations and cyclical patterns, supporting the idea that differencing mitigates non-stationarity. Across all panels, Hodrick-Prescott filtered trends ($\lambda = 1600$, standard for quarterly data) visually disentangle long-term trajectories from cyclical noise.

2.2 Unit roots and stationarity tests

In this section, we aim to formally conduct stationarity tests for the series. As explained in the last section, we have reason to doubt that our series are entirely stationary. However, for our analysis, we are relying

Figure 1: Trends in GDP, trade balance, and exchange rate



Notes: This figure presents levels (top row) and first differences (bottom row) of key UK macroeconomic indicators: real GDP (left), trade balance (center), and exchange rate against the US dollar (right), from 1955 to 2023. All series are shown alongside Hodrick-Prescott (HP) trends with a smoothing parameter of $\lambda = 1600$.
Source: Authors' computation from the UK Statistical Office.

on stationarity properties of the series. Hence, after identifying the non-stationary series formally, we will conduct the first-difference transformation to obtain stationary series for our analysis.

Table 1 presents the results of unit root and stationarity tests for three key UK macroeconomic variables—Gross Domestic Product (GDP), Trade Balance, and xchange Rate—analyzed in both levels and first differences. The Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and ERS DF-GLS tests evaluate the null hypothesis of a unit root (non-stationarity), while the KPSS test assesses the null of stationarity. For GDP, the level series exhibits non-stationarity across all tests (e.g., ADF p-value = 0.353, KPSS statistic = 4.732*), but its first-differenced series shows strong evidence of stationarity (ADF statistic = -8.020*, KPSS = 0.086). Similarly, the Trade Balance in levels displays persistent non-stationarity (KPSS = 3.395*), with stationarity achieved after first differencing (ADF = -8.171*). The Exchange Rate shows mixed results in levels (e.g., ADF p-value = 0.418), but clear stationarity in differences (PP = -82.885***)

	Test Statistics			
Series/Test	**ADF**	**PP**	**ERS DF-GLS**	**KPSS**
Gross Domestic Product				
Levels	−2.531 (0.353)	−21.533*** (0.048)	3.074	4.732***
Differences	−8.020*** (0.010)	−321.613*** (0.010)	−7.941***	0.086
Trade Balance				
Levels	−2.225 (0.481)	−157.415*** (0.010)	−0.672	3.395***
Differences	−8.171*** (0.010)	−300.088*** (0.010)	−12.912***	0.035
Exchange Rate				
Levels	−2.382 (0.418)	−13.158 (0.354)	−1.415	1.754***
Differences	−5.041*** (0.010)	−82.885*** (0.010)	−2.188	0.084

Notes: Null hypotheses—ADF/PP/ERS: series has a unit root (non-stationary); KPSS: series is stationary. To establish stationarity: reject ADF/PP/ERS null (significant ***/**) *and* fail to reject KPSS null (statistic < critical value). P-values in parentheses. Critical values (1% level): ADF/PP = −3.43, ERS DF-GLS = −2.57, KPSS = 0.739. $^{***}p < 0.01$, $^{**}p < 0.05$. First differences calculated as $\Delta y_t = y_t - y_{t-1}$.
Source: Author's calculations using data from the UK Statistical Office.

Table 1: Unit Root and Stationnary Test Results

We highlight that first-differencing effectively mitigates non-stationarity, as evidenced by statistically significant rejections of unit root hypotheses (***p<0.01) and failure to reject KPSS stationarity for differenced series.These results justify the use of differenced series for subsequent analysis, ensuring compliance with the stationarity assumptions underlying any further econometric treatment. Critical values and p-values are reported to validate the robustness of conclusions.

2.3 Model Estimation

GDP: For the GDP, we use the stationary first-differenced data. Our tests suggest an ARIMA(1,0,4) process:

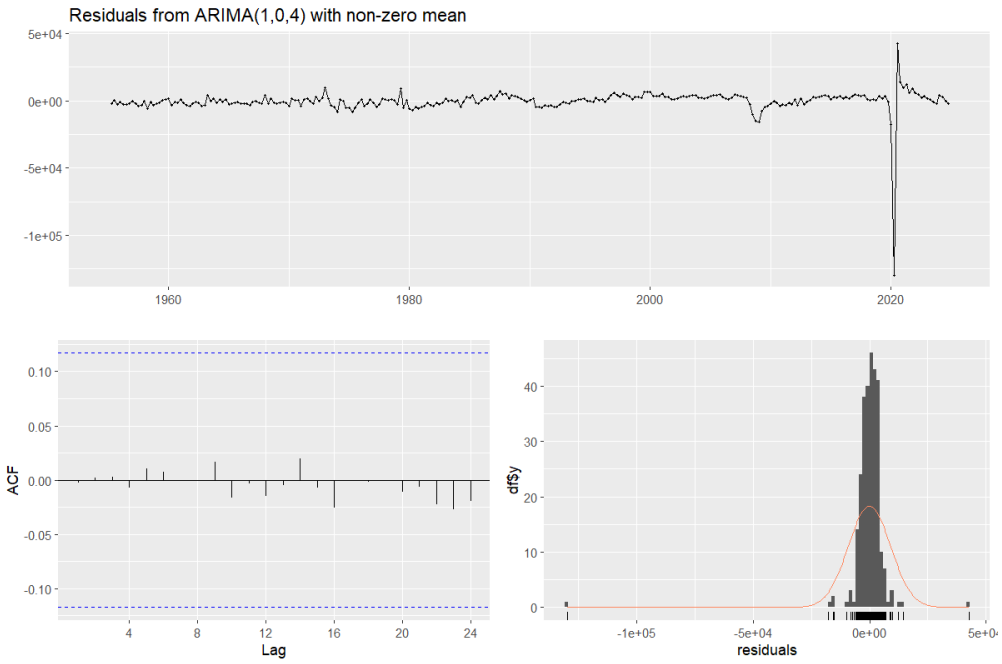


Figure 2: Model estimation - GDP

The coefficients of the model are as follows:

term	estimate	std.error
ar1	0.4886750	0.2015697
ma1	-0.7703784	0.1988893
ma2	0.0925464	0.0930497
ma3	0.1352912	0.0747171
ma4	-0.2405810	0.0610664
intercept	1832.9872955	232.7539989

Table 2: Model coefficients GDP

Trade Balance: For the Trade Balance, we use the stationary level data. Our tests suggests an ARIMA(0,0,4) process:

The coefficients of the model are as follows:

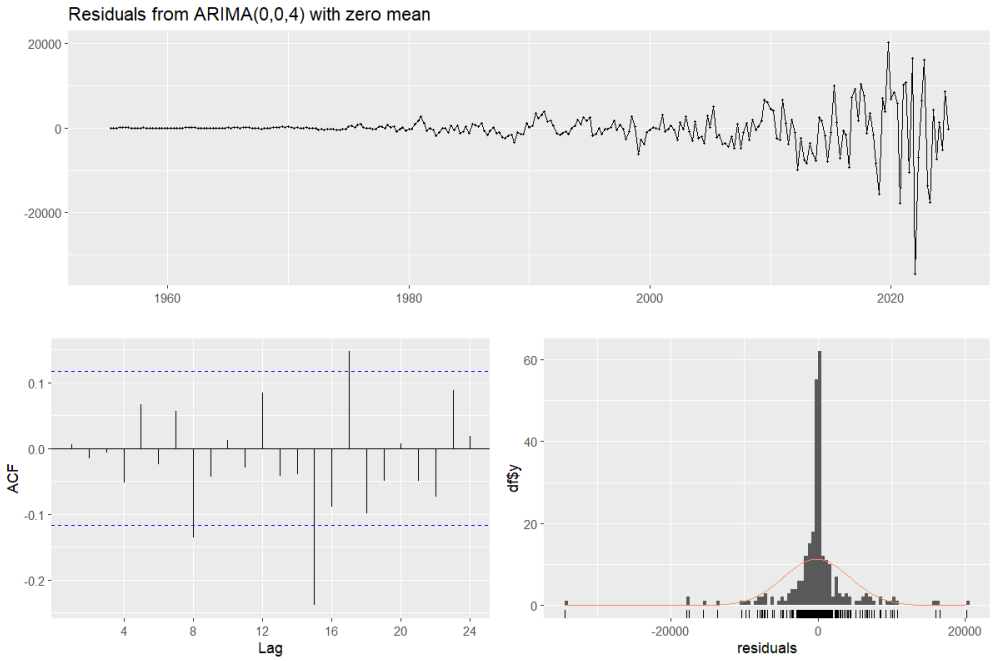


Figure 3: Model estimation - Trade Balance

term	estimate	std.error
ma1	-0.7749122	0.0571450
ma2	-0.1000937	0.0735727
ma3	-0.0300321	0.0804010
ma4	0.2784463	0.0640672

Table 3: Model coefficients Trade Balance

Exchange rate: For the Exchange Rate, we use te stationary level data. Our tests suggests an ARIMA(1,0,0) process:

The coefficients of the model are as follows:

term	estimate	std.error
ar1	0.2592112	0.0921888

Table 4: Model coefficients Exchange Rate

2.4 Forecasting

We should probably focus on one series in the final report, I'll just include all so you can decide

GDP

The fit of the in-sample GDP prediction is shown in the following figure:

The formal measures of the fit are estimated as:

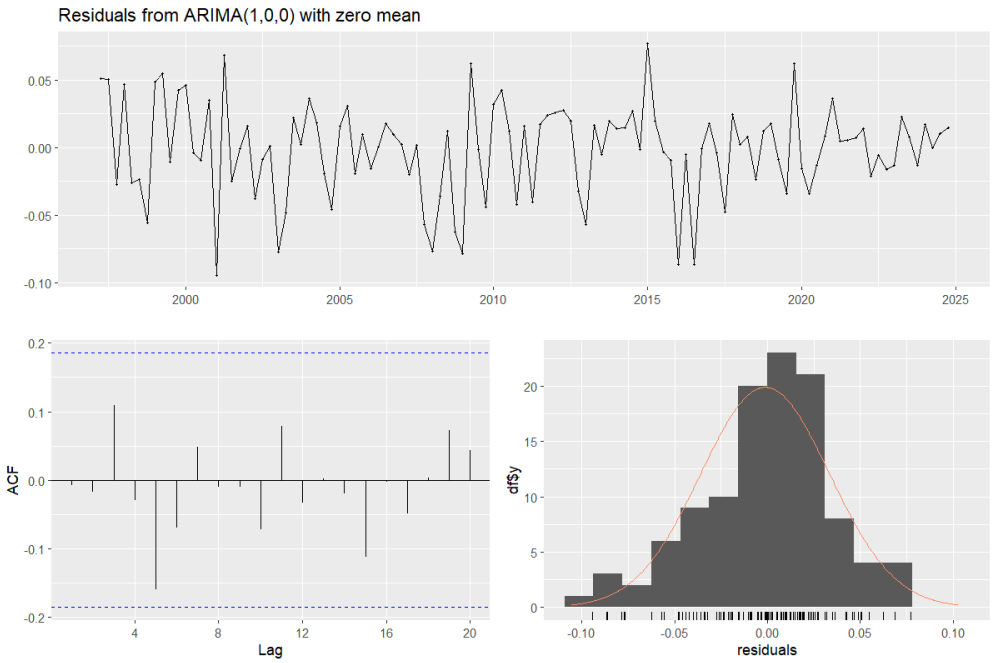


Figure 4: Model estimation - Exchange Rate

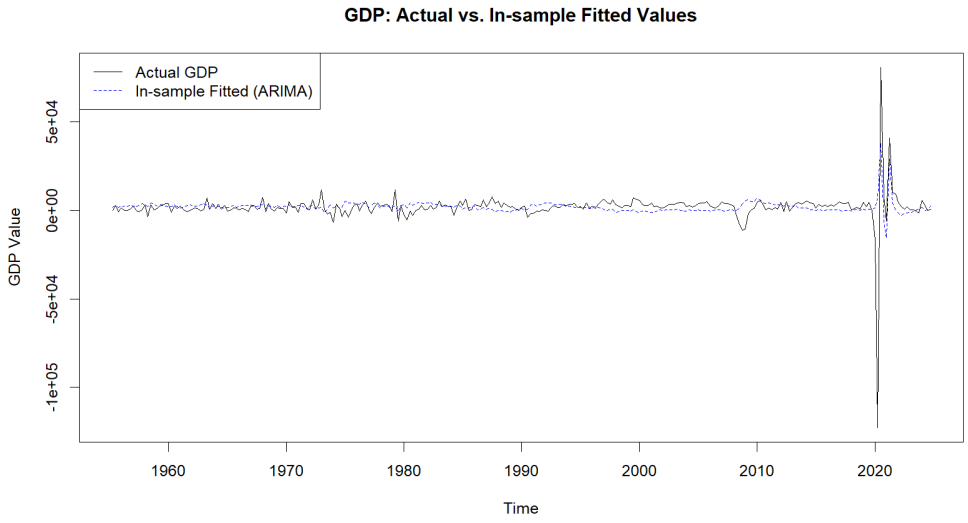


Figure 5: GDP - in-sample prediction

set	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set	-30.63153	9021.217	3505.39	-106.5096	452.5585	0.7699877	-0.0027098

Table 5: GDP - accuracy metrics

The forecast of the GDP series is:

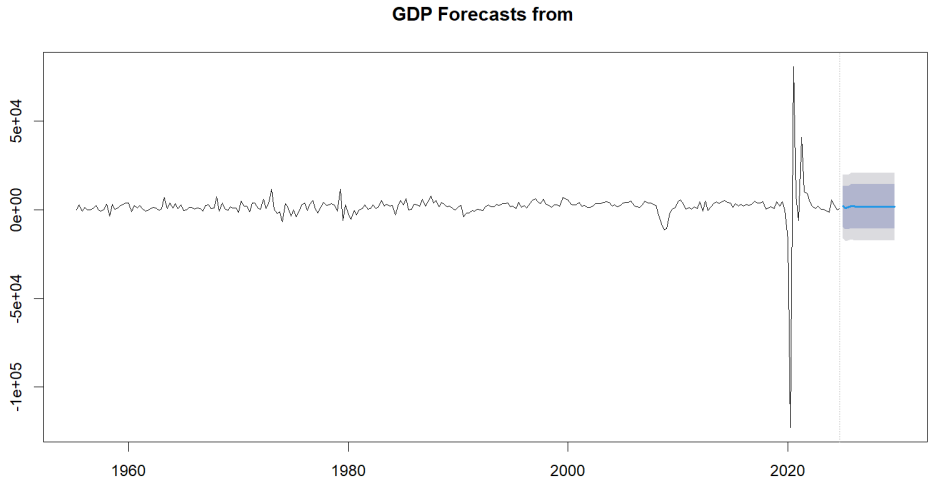


Figure 6: GDP - Forecast

Trade Balance:

The in-sample fit of the Trade Balance is described by the following metrics:

set	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set	-196.0811	4513.033	2255.195	Inf	Inf	0.6855749	0.0055908

Table 6: Trade Balance - accuracy metrics

The forecast of the GDP series is:

Exchange Rate:

The in-sample fit of the Trade Balance is described by the following metrics:

set	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set	-0.0011617	0.034616	0.0265389	137.7543	152.9467	0.6818584	-0.0075285

Table 7: Exchange Rate - accuracy metrics

The forecast of the GDP series is:

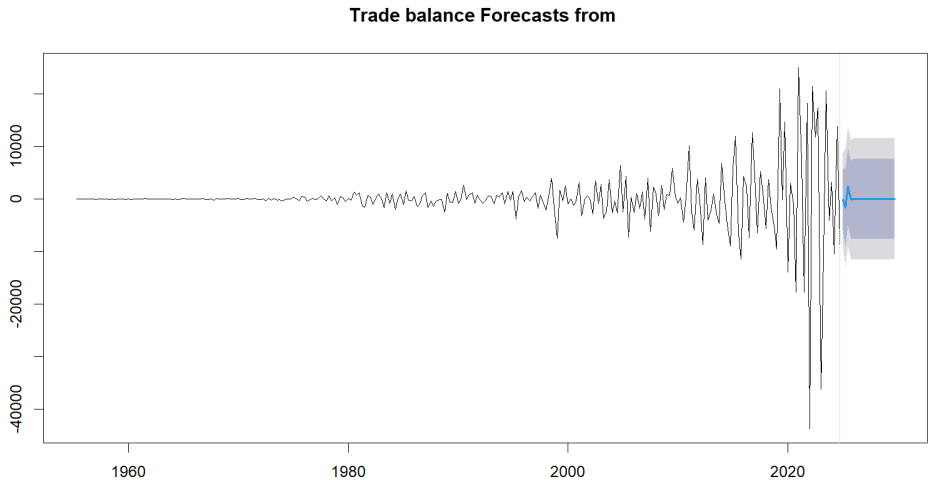


Figure 7: GDP - Forecast

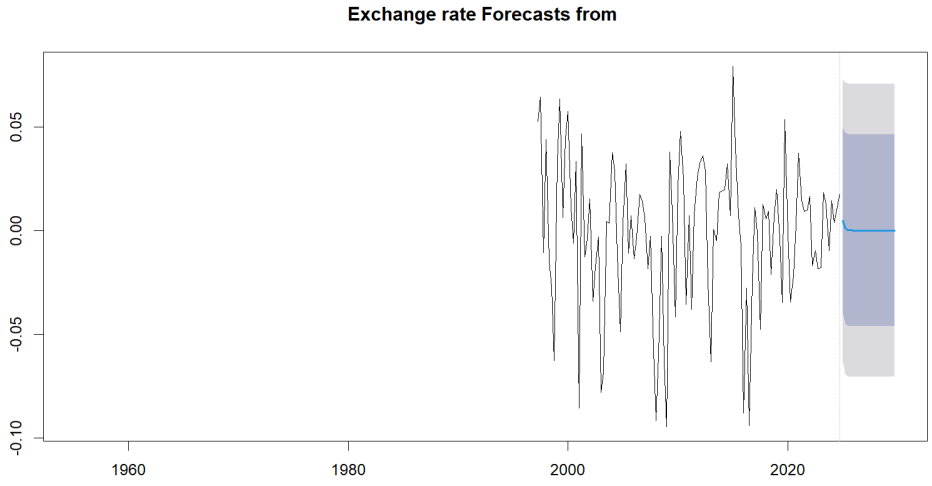


Figure 8: Exchange Rate - Forecast

3 Multivariate Analysis

3.1 Optimal lag selection

c.f. R script. Different tests suggest different lags as can be seen in the following table:

AIC(n)	HQ(n)	SC(n)	FPE(n)
7	1	1	7

Table 8: Lag selection

We decide to use a lag of 7, ensuring better forecasting fits at the expense of a loss of power.

3.2 Residual testing

The test statistics for the behavior of our residuals are summarized in the following table:

Test	ChiSq	df	p.value
Serial Correlation	70.6377	45	0.0086591
ARCH	290.9468	288	0.4403334
Normality	7261.2574	6	0.0000000

Table 9: Residuals tests

3.3 VAR forecasts

We should probably also add an in-sample forecast as Idann and Kenan did?

The VAR forecast of the first-differenced GDP is visualized by:

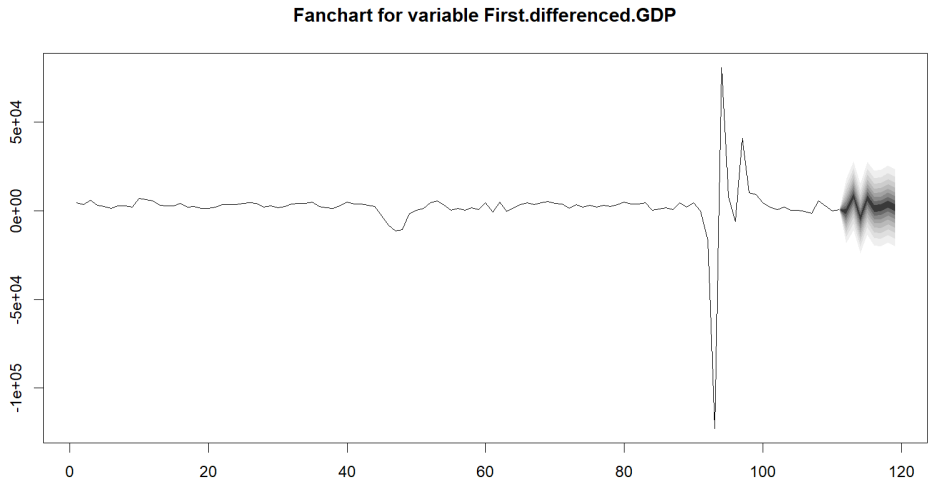


Figure 9: GDP - VAR Forecast

3.4 Cholesky decomposition

3.4.0.1 Economic reasoning.

3.5 Impulse response functions

The impulse response functions for GDP are depicted as:

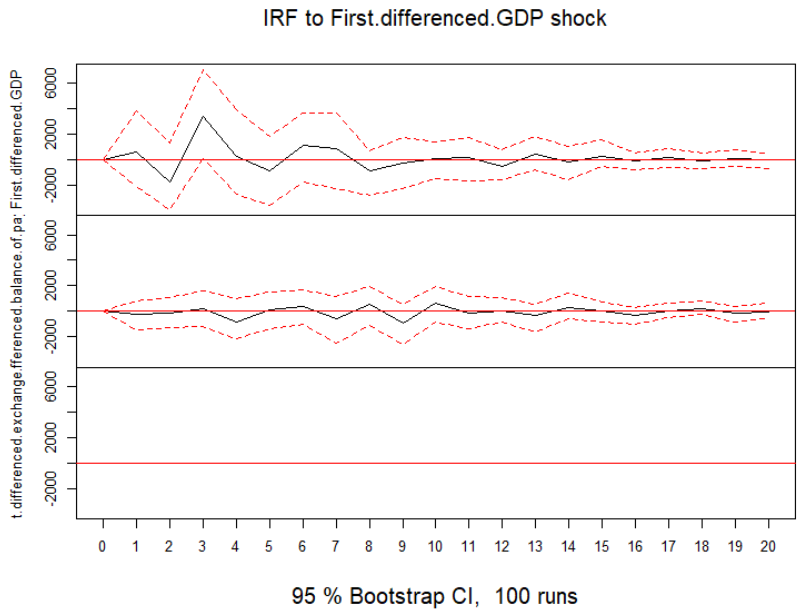


Figure 10: GDP - Shock reactions

The impulse response functions for Trade Balance are depicted as:

The impulse response functions for Exchange Rate are depicted as:

3.6 Modify the ordering of the variables

After modifying the ordering, the IRFs look as follows:

The impulse response functions for GDP are depicted as:

The impulse response functions for Trade Balance are depicted as:

The impulse response functions for Exchange Rate are depicted as:

4 Concluding Remarks

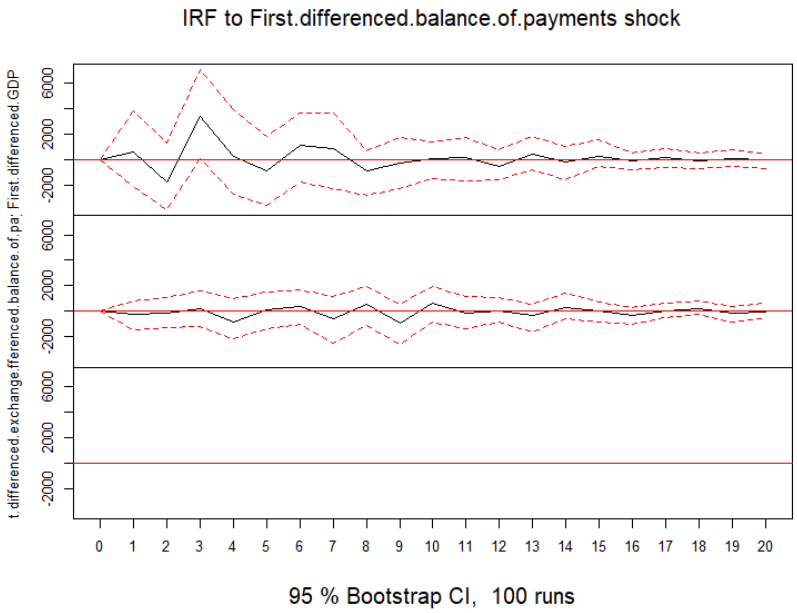


Figure 11: Trade Balance - Shock reactions

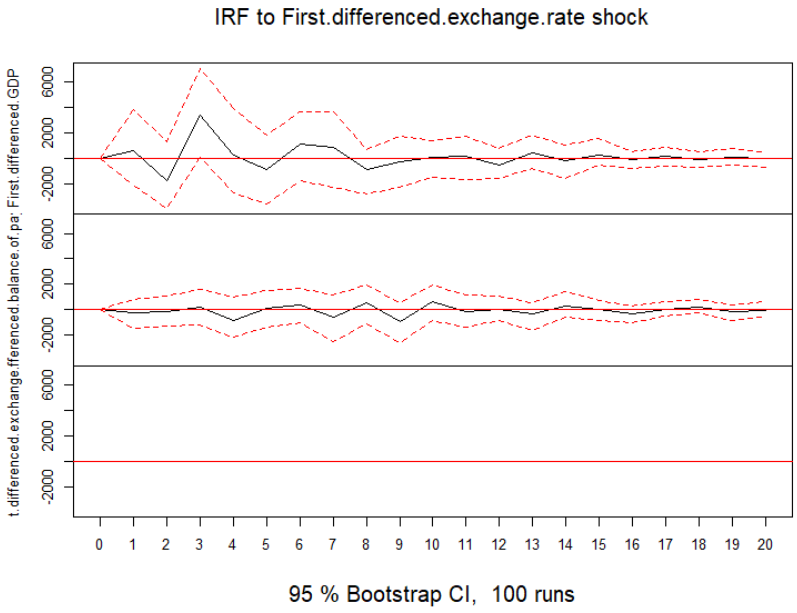


Figure 12: Exchange Rate - Shock reactions

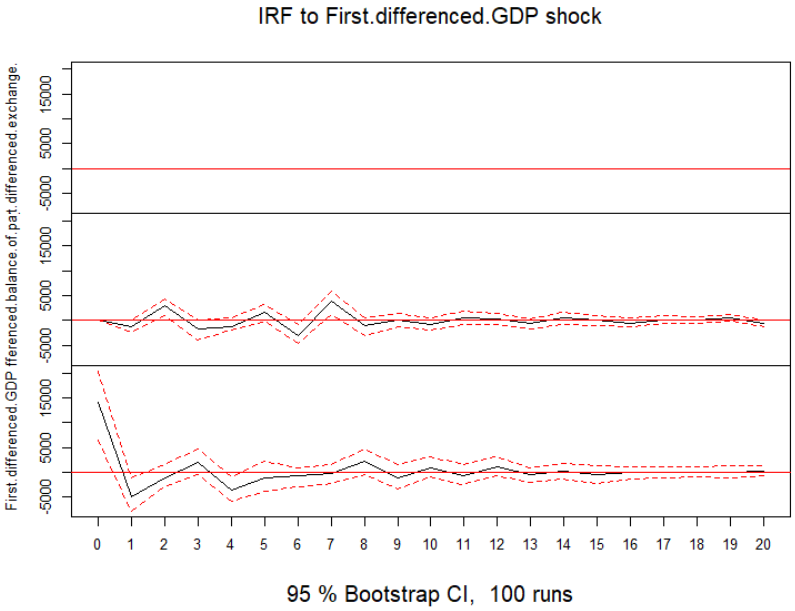


Figure 13: GDP - Shock reactions

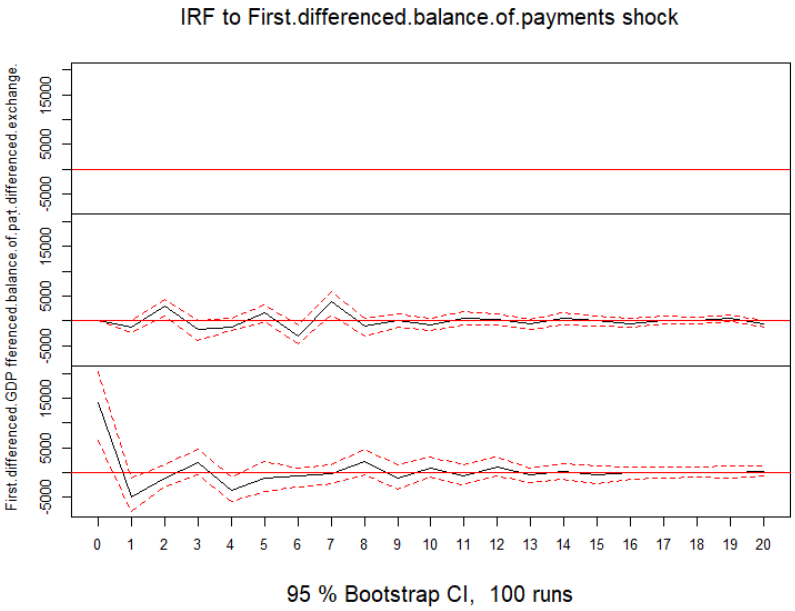


Figure 14: Trade Balance - Shock reactions

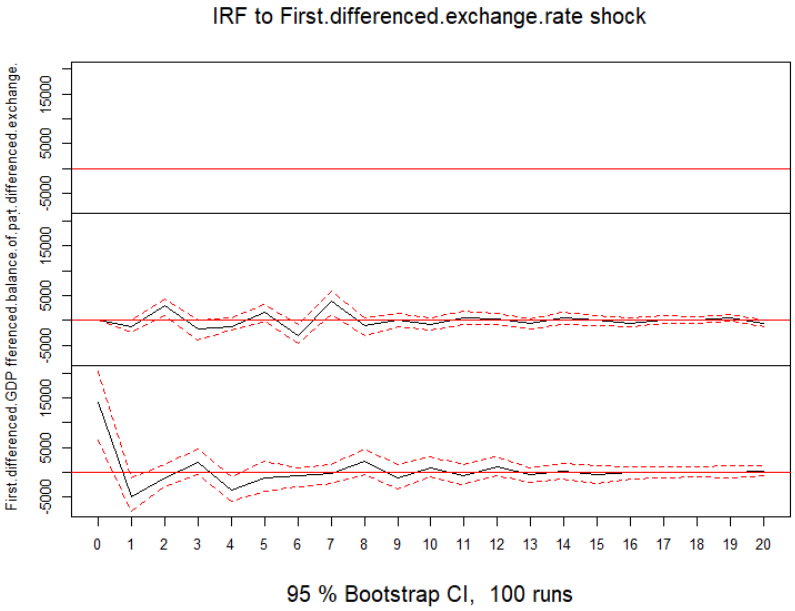


Figure 15: Exchange Rate - Shock reactions