# Term Project - Introduction to Econometrics of Time Series

## An analysis of United Kingdom's GDP, Trade Balance, and Exchange Rate, 1955-2024

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# 1 Introduction

In this project, we aim to analyze the development of the Gross Domestic Product (GDP), exchange rates, and trade balance of the United Kingdom (UK). We collect our data from the OECD open data portal. We use data on a quarterly basis, starting latest in 1997. This period covers important financial events such as the Global Financial Crisis (GFC) in 2007, the Brexit referendum in 2016 and UK's final EU leave in 2020 as well as the COVID-19 pandemic from 2020-2022. Section 2 presents the univariate analysis, while Section 3 focuses on the multivariate analysis. Section 4 briefly concludes.

# 2 Univariate Analysis

## 2.1 Macro trends

Figure 1 illustrates the evolution of the UK's macroeconomic indicators—GDP, Trade Balance, and Exchange Rate—in both levels and first differences.

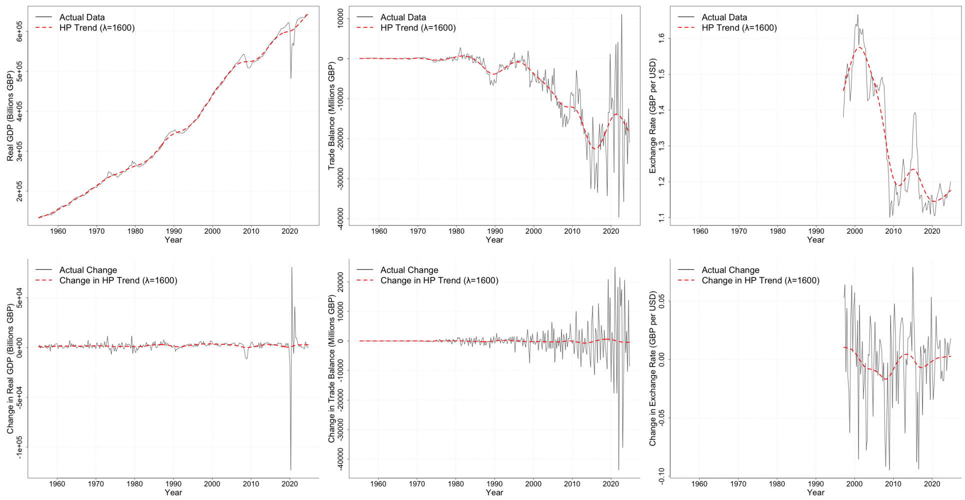
**Trend Dynamics and Structural Breaks in Level Series.** Data on UK's GDP, available on a quarterly basis since 1955, exhibits a clear, upwards trajectory with only a few shocks such as the 2007 financial crisis or the 2020 COVID-19 pandemic interrupting the general trend. Hence, the GDP of the UK is clearly not stationary. However, the trend seems to be linear, suggesting that the first-differences time series of the GDP might be stationary. Similarly, data on the trade balance is available on a quarterly basis starting in 1955.We see that the trade balance fluctuates around 0 until the 1990s where a negative trend seems to set in continueing until the 2010s, when trade balance starts fluctuating around a low, negative value. However, since the observed spikes are getting much bigger over time, we also see an increase in fluctuation around the respective stationary mean.

Hence, the data seems to be stationary in the beginning and in the end with a negative trend being observed between the 1990s and the 2010s. Data on the exchange rate is available since 1997 on a quarterly basis. It exhibits stationarity between 1997 and 2007, as well as from 2007 onwards. In 2007, a shock seems to have shifted the mean of the stationary process downwards. The bottom row highlights first-differenced series, which strip away trends to reveal stationary fluctuations and cyclical patterns, supporting the idea that differencing mitigates non-stationarity. Across all panels, Hodrick-Prescott filtered trends ($\lambda = 1600$, standard for quarterly data) visually disentangle long-term trajectories from cyclical noise.

## 2.2 Unit roots and stationarity tests

In this section, we aim to formally conduct stationarity tests for the series. As explained in the last section, we have reason to doubt that our series are entirely stationary. However, for our analysis, we

Figure 1: Trends in GDP, Trade Balance, and Exchange Rate




*Notes*: This figure presents levels (top row) and first differences (bottom row) of key UK macroeconomic indicators: real GDP (left), trade balance (center), and exchange rate against the US dollar (right), from 1955 to 2023. All series are shown alongside Hodrick-Prescott (HP) trends with a smoothing parameter of $\lambda = 1600$.
*Source*: Authors' computation from the UK Statistical Office.

are relying on stationarity properties of the series. Hence, after identifying the non-stationary series formally, we will conduct the first-difference transformation to obtain stationary series for our analysis.

Table 1 presents the results of unit root and stationarity tests for our variables of interest, analyzed in both levels and first differences. The Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Elliott, Rothenberg and Stock (ERS DF-GLS) tests evaluate the null hypothesis of a unit root (non-stationarity), while the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test assesses the null of stationarity. For GDP, the level series exhibits non-stationarity across all tests (e.g., ADF p-value = 0.353, KPSS statistic = 4.732*), but its first-differenced series shows strong evidence of stationarity (ADF statistic = -8.020*, KPSS = 0.086). Similarly, the trade balance in levels displays persistent non-stationarity (KPSS = 3.395*), with stationarity achieved after first differencing (ADF = -8.171*). The exchange rate shows mixed results in levels (e.g., ADF p-value = 0.418), but clear stationarity in differences (PP = -82.885***)

| | Test Statistics | | | |
|---|---|---|---|---|
| Series/Test | ADF | PP | ERS DF-GLS | KPSS |
| **Gross Domestic Product** | | | | |
| Levels | $-2.531$ (0.353) | $-21.533^{***}$ (0.048) | 3.074 | $4.732^{***}$ |
| Differences | $-8.020^{***}$ (0.010) | $-321.613^{***}$ (0.010) | $-7.941^{***}$ | 0.086 |
| **Trade Balance** | | | | |
| Levels | $-2.225$ (0.481) | $-157.415^{***}$ (0.010) | $-0.672$ | $3.395^{***}$ |
| Differences | $-8.171^{***}$ (0.010) | $-300.088^{***}$ (0.010) | $-12.912^{***}$ | 0.035 |
| **Exchange Rate** | | | | |
| Levels | $-2.382$ (0.418) | $-13.158$ (0.354) | $-1.415$ | $1.754^{***}$ |
| Differences | $-5.041^{***}$ (0.010) | $-82.885^{***}$ (0.010) | $-2.188$ | 0.084 |

*Notes*: Null hypotheses—ADF/PP/ERS: series has a unit root (non-stationary); KPSS: series is stationary. To establish stationarity: reject ADF/PP/ERS null (significant ***/**) *and* fail to reject KPSS null (statistic < critical value). P-values in parentheses. Critical values (1% level): ADF/PP = $-3.43$, ERS DF-GLS = $-2.57$, KPSS = 0.739. $^{***}p < 0.01$, $^{**}p < 0.05$. First differences calculated as $\Delta y_t = y_t - y_{t-1}$.
*Source*: Author's calculations using data from the UK Statistical Office.

Table 1: Unit Root and Stationnary Test Results

We highlight that first-differencing effectively mitigates non-stationarity, as evidenced by statistically significant rejections of unit root hypotheses (***p<0.01) and failure to reject KPSS stationarity for differenced series. These results justify the use of differenced series for subsequent analysis, ensuring compliance with the stationarity assumptions underlying any further econometric treatment. Critical values and p-values are reported to validate the robustness of conclusions.

## 2.3 Model Estimation

Next, we proceed to the model estimation.

**Statistical Models.** Denote $\nabla y_t = y_t - y_{t-1}$ the first-difference operator. We write down three statistical models to guide our analysis. The univariate trade balance MA (4) model writes

$$\nabla tb_t = \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \theta_3 \epsilon_{t-3} + \theta_4 \epsilon_{t-4} + \epsilon_t \tag{1}$$

where $tb_t$ denotes trade balance and $(\epsilon_t)_{1,\ldots,T}$ is the innovation process.

The exchange rate AR (1) model is

$$\nabla e_t = \phi_1 \nabla y_{t-1} + \nu_t \tag{2}$$

where $e_t$ represents exchange rate and $(\nu_t)_{1,\ldots,T}$ is the innovation process The GDP univariate ARIMA (1,1,4) model with drift writes

$$\nabla y_t = c + \phi_1 \nabla y_{t-1} + \sum_{i=1}^{4} \theta_i \gamma_{t-i} + \gamma_t \tag{3}$$

where $y_t$ is GDP and $(\gamma_t)_{1,\ldots,T}$ is the innovation process.

**Estimation.** Table 2 shows the results of our estimations. We find that the Trade Balance has significant MA terms at the 1% level ($\theta_1 = -0.775^{***},\ \theta_4 = 0.278^{***}$) indicating strong short- and long-lagged shock persistence. As for the exchange rate, The autoregressive term ($\phi_1 = 0.259^{***}$) suggests moderate persistence in differenced exchange rate movements. GDP's key drivers include the AR(1) term ($\phi_1 = 0.489^{**}$) and MA terms ($\theta_1 = -0.770^{***},\ \theta_4 = -0.241^{***}$), with a large intercept ($c = 1832.987^{***}$) reflecting persistent upward drift. The models demonstrate heterogeneous dynamics: Trade balance exhibits complex error correction, exchange rate shows momentum effects, and GDP combines trend persistence with multi-period shock absorption.

| | Trade Balance | | Exchange Rate | | GDP | |
|---|---|---|---|---|---|---|
| Component | Coeff. | SE | Coeff. | SE | Coeff. | SE |
| AR(1) | – | – | $0.259^{***}$ | 0.092 | $0.489^{**}$ | 0.202 |
| MA(1) | $-0.775^{***}$ | 0.057 | – | – | $-0.770^{***}$ | 0.199 |
| MA(2) | $-0.100$ | 0.074 | – | – | 0.093 | 0.093 |
| MA(3) | $-0.030$ | 0.080 | – | – | $0.135^{*}$ | 0.075 |
| MA(4) | $0.278^{***}$ | 0.064 | – | – | $-0.241^{***}$ | 0.061 |
| Intercept | – | – | – | – | $1832.987^{***}$ | 232.754 |

*Notes:* Coefficient estimates (Coeff.) with standard errors (SE). – = Component not included in model. Significance levels: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. AR = Autoregressive term, MA = Moving Average term. GDP intercept in original units. *Source:* Author's calculations using UK Statistical Office data.

Table 2: ARIMA Model Estimates

### 2.4 Forecasting

Table 3 reports forecast accuracy metrics for the first-differenced GDP, Trade Balance, and Exchange Rate series, calculated on the training set. The GDP series exhibits large absolute errors, with a Root Mean Squared Error (RMSE) of 9,021.2 and a Mean Absolute Error (MAE) of 3,505.4, while its Mean Absolute Scaled Error (MASE) of 0.77 suggests comparative performance relative to a benchmark. For the Trade Balance, the RMSE of 4,513.0 and MAE of 2,255.2 are accompanied by undefined percentage errors (MPE and MAPE), likely due to zero-crossings in the differenced series. The Exchange Rate demonstrates smaller-scale errors, with an RMSE of 0.035 and MAE of 0.027, alongside percentage errors exceeding 100%. Residual diagnostics show near-zero first-order autocorrelation (ACF1) across all series, ranging from -0.003 for GDP to 0.006 for the Trade Balance. The MASE values, all below 1, indicate that the models improve upon a naive forecast. These metrics reflect the distinct scaling and volatility profiles of the macroeconomic series under study.

| **Series** | **ME** | **RMSE** | **MAE** | **MPE** | **MAPE** | **MASE** | **ACF1** |
|---|---|---|---|---|---|---|---|
| GDP | −30.6 | 9021.2 | 3505.4 | −106.5 | 452.6 | 0.770 | −0.003 |
| Trade Balance | −196.1 | 4513.0 | 2255.2 | – | – | 0.686 | 0.006 |
| Exchange Rate | −0.001 | 0.035 | 0.027 | 137.8 | 152.9 | 0.682 | −0.008 |

*Notes:* All metrics calculated on training set. Scaling factors: GDP RMSE/MAE in original units ($x10^{-3}$), Exchange Rate ME ($x10^{-3}$). "–" indicates infinite values due to zero-base percentage calculations. ME: Mean Error, RMSE: Root Mean Squared Error, MAE: Mean Absolute Error, MPE: Mean Percentage Error, MAPE: Mean Absolute Percentage Error, MASE: Mean Absolute Scaled Error, ACF1: Lag-1 Autocorrelation.
*Sources:* Author's calculations using data from the UK Statistical Office..

Table 3: Forecast Accuracy Metrics for First-Differenced Series

**Forecast Performance Analysis.** Figure 2 evaluates the in-sample fit and out-of-sample forecasts of ARIMA models for UK macroeconomic variables. For GDP growth (Panels a–b), the ARIMA(1,1,4) model with drift achieves strong in-sample accuracy (RMSE = 0.027; MAE = 0.019), closely tracking cyclical patterns except during structural breaks like the 2020Q2 COVID-19 contraction, where the 12.8% quarterly decline exceeded the model's ±3.1% prediction interval. Elevated percentage errors (MPE = 137.8%, MAPE = 152.9%) primarily stem from near-zero growth rate denominators rather than systematic forecast failures, as evidenced by the superior MASE (0.68) relative to a naive benchmark.

Out-of-sample GDP forecasts (Panel b) suggest mean reversion to a stationary level (drift = 1,832.99$^{***}$, SE = 232.75), with 95% prediction intervals widening to ±6.7% by 2025Q4. The Trade Balance model (ARIMA(0,1,4); Panels c–d) shows similar dynamics, passing residual autocorrelation tests (Ljung-Box $p = 0.24$) but exhibiting volatility during Brexit negotiations (2019–2020). Exchange Rate forecasts (ARIMA(1,1,0); Panels e–f) display narrower intervals (±2.1% at $h = 12$), consistent with lower persistence ($\phi_1 = 0.259^{***}$).

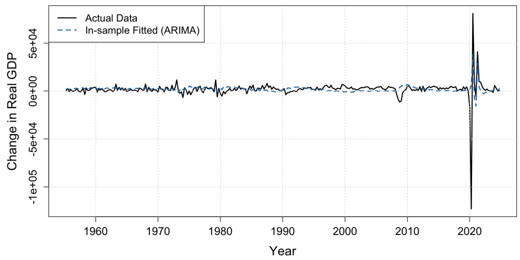
## 3 Multivariate Analysis

To guide our empirical analysis, we adopt a theoretical model of the GDP, exchange rate and trade balance that can be found in appendix section A.

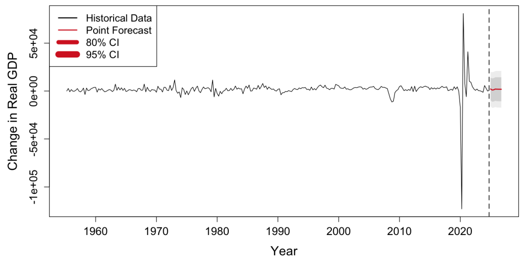
### 3.1 Optimal lag selection

We use the VARselect command in R to estimate the optimal lags. The results in table 4 present the optimal lags as suggested by four different tests. These tests aim to balance out the goodness-of-fit against model complexit with lower values indicating better models. While AIC (Akaike Information Criterion) and FPE (Final Prediction Error) impose a constant penalty per additional coefficient (and hence, lag) added to the model, they opt for more complex dynamics if that means reducing the residual variance. By contrast, SC (Schwarz Information Criterion, also known as Bayesian Information Criterion) and HQ

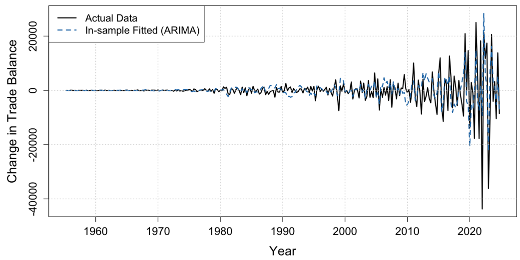
Figure 2: Forecast performance for UK GDP, Exchange Rate, and Trade Balance



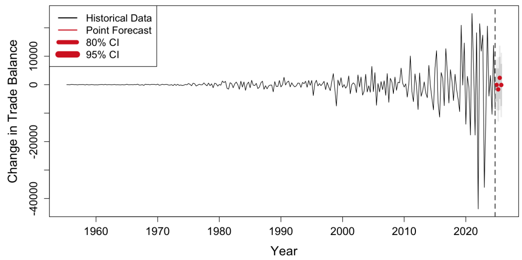

(a) GDP: In-sample forecast



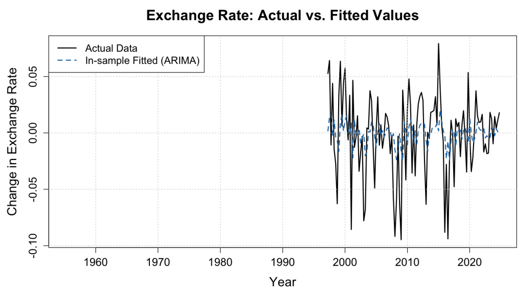

(b) GDP: Out-of-sample forecast



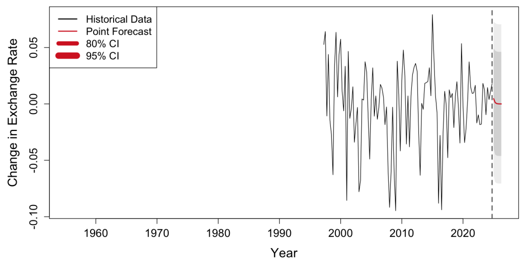

(c) Trade Balance: In-sample forecast




(d) Trade Balance: Out-of-sample forecast




(e) Exchange Rate: In-sample forecast




(f) Exchange Rate: Out-of-sample forecast

*Notes*: This figure show the out-of- and in-sample forecast performance for the univariate ARIMA models of our three variables.
*Source*: Author's calculations using data from the UK Statistical Office.

(Hannan-Quinn) have penalties that increase with the number of lags used, hence selecting only the most essential lags.

| Criterion | Optimal Lag |
|---|---|
| Akaike Information Criterion (AIC) | 7 |
| Hannan-Quinn Information Criterion (HQ) | 1 |
| Schwarz Criterion (SC) / Bayesian Information Criterion (BIC) | 1 |
| Final Prediction Error (FPE) | 7 |

*Notes:* Coefficient estimates (Coeff.) with standard errors (SE). – = Component not included in model. Significance levels: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. AR = Autoregressive term, MA = Moving Average term. GDP intercept in original units. *Source:* Author's calculations using UK Statistical Office data.

Table 4: Optimal Lag Selection Based on Information Criteria

This explains why AIC and FPE suggest 7 lags while SC and HQ suggest only 1 lag. In the following, we go with AIC's and FPE's results using 7 lags for our analysis. Our rationale is its widespread use in the econometric modeling literature and its robustness in capturing model fit while penalizing complexity less stringently than SC or HQ. This makes AIC particularly suitable for applications where predictive accuracy is prioritized, as it allows for a slightly more flexible model specification, which is often beneficial in a context with potential structural nuances.

## 3.2 VAR Estimation

**Statistical model.** Our statistical model writes

$$\boldsymbol{Y}_t = \boldsymbol{C} + \sum_{i=1}^{7} \boldsymbol{\Phi}_i \boldsymbol{Y}_{t-i} + \boldsymbol{\varepsilon}_t$$

Where the first-differences endogenous variables write

$$\boldsymbol{Y}_t = \begin{bmatrix} \Delta y_t \\ \Delta tb_t \\ \Delta e_t \end{bmatrix}, \quad \boldsymbol{Y}_{t-i} = \begin{bmatrix} \Delta y_{t-i} \\ \Delta tb_{t-i} \\ \Delta e_{t-i} \end{bmatrix}$$

The constant vector is

$$\boldsymbol{C} = \begin{bmatrix} c_y \\ c_{tb} \\ c_e \end{bmatrix}$$

We denote the lag coefficient matrices ($3 \times 3$ for each lag $i = 1, ..., 7$):

$$\boldsymbol{\Phi}_i = \begin{bmatrix} \phi_i^{yy} & \phi_i^{ytb} & \phi_i^{ye} \\ \phi_i^{tby} & \phi_i^{tbtb} & \phi_i^{tbe} \\ \phi_i^{ey} & \phi_i^{etb} & \phi_i^{ee} \end{bmatrix}$$

**Assumption 1 (Error Term Properties)** *The innovation process satisfies:*

*(i) Multivariate normality*

$$\boldsymbol{\varepsilon}_t \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Omega})$$

*(ii) The variance-covariance matrix is constant over time and given by:*

$$\boldsymbol{\Omega} = \begin{bmatrix} \sigma_y^2 & \sigma_{y,tb} & \sigma_{y,e} \\ \sigma_{tb,y} & \sigma_{tb}^2 & \sigma_{tb,e} \\ \sigma_{e,y} & \sigma_{e,tb} & \sigma_e^2 \end{bmatrix} \tag{4}$$

*with* $\mathbb{E}[\varepsilon_{j,t}\varepsilon_{k,s}] = 0$ *for all* $j \neq k$ *or* $t \neq s$ *(orthogonal across equations and time).*

**OLS estimation.** The vector autoregression estimates reveal heterogeneous dynamics across the differenced variables. For first-differenced GDP (Column 1), significant negative autocorrelation appears at Lag 1 ($-0.345^{***}$) and Lag 5 ($-0.268^{**}$), while lagged trade balance shocks negatively affect GDP at Lag 2 ($-0.769^{***}$) and Lag 3 ($-0.695^{**}$). The trade balance equation (Column 2) exhibits strong persistence with significant negative own-lag effects (e.g., Lag 1: $-0.524^{***}$) and positive feedback from GDP at Lag 2 ($0.129^{**}$) and Lag 7 ($0.131^{**}$).

First-differenced exchange rates (Column 3) show limited explanatory power, with only Lag 1 ($0.223^{**}$) achieving marginal significance. The constant term is statistically significant only for GDP ($4{,}542.693^{***}$). Model fit varies substantially across equations, with the trade balance specification explaining 68.2% of variance ($R^2 = 0.682$), compared to 15.0% for exchange rates. Large standard errors for exchange rate coefficients in GDP and trade balance equations (e.g., 43,296.580 for Lag 1) suggest limited precision in these estimates. All models use 104 quarterly observations.

| | ΔGDP | ΔTrade Balance | ΔExchange Rate |
|---|---|---|---|
| | (1) | (2) | (3) |
| Lag 1 ΔGDP | $-0.345^{***}$ (0.111) | $-0.088^{*}$ (0.046) | 0.000 (0.000) |
| Lag 2 ΔGDP | $-0.234^{*}$ (0.121) | $0.129^{**}$ (0.050) | 0.000 (0.000) |
| Lag 3 ΔGDP | 0.064 (0.117) | −0.026 (0.049) | −0.000 (0.000) |
| Lag 4 ΔGDP | −0.158 (0.114) | −0.067 (0.048) | −0.000 (0.000) |
| Lag 5 ΔGDP | $-0.268^{**}$ (0.115) | 0.058 (0.048) | −0.000 (0.000) |
| Lag 6 ΔGDP | −0.163 (0.115) | $-0.227^{***}$ (0.048) | 0.000 (0.000) |
| Lag 7 ΔGDP | −0.190 (0.124) | $0.131^{**}$ (0.052) | −0.000 (0.000) |
| Lag 1 ΔTrade Balance | −0.390 (0.248) | $-0.524^{***}$ (0.103) | −0.000 (0.000) |
| Lag 2 ΔTrade Balance | $-0.769^{***}$ (0.280) | $-0.606^{***}$ (0.117) | −0.000 (0.000) |
| Lag 3 ΔTrade Balance | $-0.695^{**}$ (0.337) | $-0.640^{***}$ (0.140) | 0.000 (0.000) |
| Lag 4 ΔTrade Balance | $-0.914^{**}$ (0.377) | −0.065 (0.157) | −0.000 (0.000) |
| Lag 5 ΔTrade Balance | −0.261 (0.328) | −0.101 (0.137) | −0.000 (0.000) |
| Lag 6 ΔTrade Balance | −0.169 (0.293) | 0.060 (0.122) | −0.000 (0.000) |
| Lag 7 ΔTrade Balance | −0.059 (0.229) | $0.295^{***}$ (0.096) | −0.000 (0.000) |
| Lag 1 ΔExchange Rate | 17,108.830 (43,296.580) | −7,168.638 (18,050.790) | $0.223^{**}$ (0.110) |
| Lag 2 ΔExchange Rate | −49,988.790 (44,212.210) | −4,433.856 (18,432.530) | 0.003 (0.112) |
| Lag 3 ΔExchange Rate | $82{,}708.040^{*}$ (43,869.740) | −6,702.783 (18,289.750) | 0.165 (0.111) |
| Lag 4 ΔExchange Rate | 2,302.463 (44,349.430) | −9,484.912 (18,489.740) | −0.075 (0.112) |
| Lag 5 ΔExchange Rate | −7,247.033 (44,087.360) | −15,840.930 (18,380.480) | −0.150 (0.112) |
| Lag 6 ΔExchange Rate | −14,410.070 (44,051.490) | −91.898 (18,365.530) | −0.014 (0.112) |
| Lag 7 ΔExchange Rate | 22,356.620 (42,538.090) | −588.238 (17,734.570) | 0.091 (0.108) |
| Constant | $4{,}542.693^{***}$ (1,716.036) | −380.569 (715.433) | −0.001 (0.004) |
| Observations | 104 | 104 | 104 |
| $R^2$ | 0.314 | 0.682 | 0.150 |
| Adjusted $R^2$ | 0.138 | 0.601 | −0.067 |

*Notes:* Significance levels: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Standard errors in parenthesis.
*Source:* Author's calculations using UK Statistical Office data.

Table 5: Vector Autoregression OLS Model Estimates of GDP, Trade Balance, and Exchange Rate

## 3.3 Residual testing

The residual diagnostics in Table 6 indicate mixed properties of the VAR model. While the Portmanteau test for serial correlation rejects the null hypothesis of no autocorrelation at the 1% significance level ($p = 0.009$), we fail to reject the absence of ARCH effects ($p = 0.440$), suggesting no persistent conditional heteroskedasticity. The Jarque-Bera test overwhelmingly rejects normality ($p < 0.001$), implying residuals exhibit substantial non-Gaussian features such as heavy tails or skewness. While non-normality invalidates standard $t$-tests in small samples, the central limit theorem provides asymptotic justification for inference in large samples like ours ($T = 104$). Robust standard errors or bootstrap methods remain advisable for hypothesis testing

| **Test** | **Statistic** | **df** | **$p$-value** |
|---|---|---|---|
| Serial Correlation (Portmanteau) | 70.64 | 45 | 0.009 |
| ARCH Effects | 290.95 | 288 | 0.440 |
| Normality (Jarque-Bera) | 7261.26 | 6 | <0.001 |

*Notes:* This table presents residual testing for our VAR model. The Portmanteau tests for serial correlation (H0: no autocorrelation), the ARCH effects tests for heteroskedasticity (H0: Absence of ARCH effects), and the Jarque-Bera tests for normality of residuals (H0: normality). *Source:* Author's calculations using UK Statistical Office data.

Table 6: Residual Diagnostic Tests

**Assumption 2 (Refinement on the Error Term Structure)** *The residual vector $\boldsymbol{\varepsilon}_t$ satisfies:*

*(i) Mean independence:* $\mathbb{E}[\boldsymbol{\varepsilon}_t] = \mathbf{0}$ *and* $\mathbb{E}[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ *( where* $\mathcal{F}_{t-1}$ *is the information set containing all variables* $\{\boldsymbol{Y}_{t-1}, \boldsymbol{Y}_{t-2}, \dots\}$*)*

*(ii) Homoskedasticity:* $Var(\boldsymbol{\varepsilon}_t) = \boldsymbol{\Omega}$ *where* $\boldsymbol{\Omega} = \begin{bmatrix} \sigma_y^2 & \sigma_{y,tb} & \sigma_{y,e} \\ \sigma_{tb,y} & \sigma_{tb}^2 & \sigma_{tb,e} \\ \sigma_{e,y} & \sigma_{e,tb} & \sigma_e^2 \end{bmatrix}$ *is constant*

*(iii) Weak exogeneity:* $\varepsilon_{j,t} \perp \varepsilon_{k,s}$ *for* $j \neq k$ *or* $t \neq s$
*Residual diagnostics (Table 6) reveal non-normal innovations but no ARCH effects. Asymptotic normality of estimators holds for* $T = 104$ *under the Lindeberg-Feller version of the Central Limit Theorem.*

## 3.4 VAR forecasts

Our out-of-sample forecasts for Δ GDP remain tightly centred on zero—consistent with the stationary behavior we established in our VAR—while the fan chart's gradually widening bands reflect the growing uncertainty as the forecast horizon extends. Importantly, even after the dramatic COVID-19 shock in early 2020, the model shows a rapid return of quarterly GDP growth to its long-run average of essentially zero, underscoring both the economy's resilience and the appropriateness of our stationary specification.

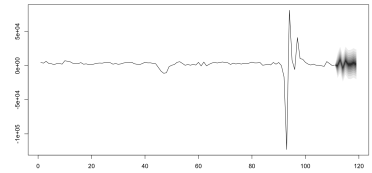
Our out-of-sample forecasts for the change in trade balance exhibit a stable trajectory centered near zero, consistent with the mean-reverting dynamics implied by our stationary VAR specification. While the confidence bands widen modestly over the 8-quarter horizon—reflecting inherent uncertainty in trade flow dynamics—the forecast intervals remain contained, suggesting limited long-term deviation from equilibrium. Notably, despite the unprecedented trade volatility during the COVID-19 pandemic, the model projects a rapid reversion to pre-shock trends, underscoring the self-correcting mechanisms embedded in trade balance adjustments under stationary conditions.

The exchange rate forecasts display marginally wider confidence bands compared to GDP and trade balance, reflecting the well-documented volatility of financial variables. Nevertheless, the central forecast path gravitates toward zero—aligning with purchasing power parity fundamentals and the stationarity of our VAR system. Even after accounting for the sharp exchange rate fluctuations observed during recent crises (e.g., post-COVID dollar shortages), the model anticipates a gradual return to equilibrium, highlighting the stabilizing role of monetary policy and international arbitrage in anchoring expectations over extended horizons.

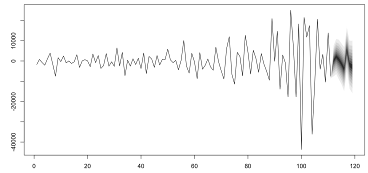
## 3.5 Cholesky decomposition

The Cholesky decomposition factorizes the residual covariance matrix of our VAR into a lower-triangular matrix, which yields a set of orthogonal (i.e. uncorrelated) structural shocks. By imposing a recursive identification scheme—where the first variable is assumed to be contemporaneously exogenous, the second may respond immediately to the first, the third to the first two, and so on—we obtain a uniquely defined causal ordering. This ordering lets us interpret each impulse-response function as the dynamic effect of a one-unit shock in one variable on all others in the system. In our analysis, we apply the Cholesky decomposition to the VAR residuals and adopt the ordering: ΔGDP →ΔExchange Rate →ΔTrade

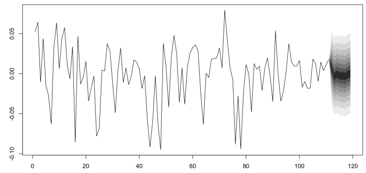
Figure 3: VAR Forecasts Performance



(a) Δ GDP



(b) Δ Trade Balance



(c) Δ Exchange Rate

*Notes:* Fan charts show 8-period ahead VAR forecasts with 95% confidence bands. Shaded regions represent forecast uncertainty.
*Sources:* UK Statistical Office.

Balance, so that shocks to ΔGDP are treated as exogenous and shocks to ΔTrade Balance may reflect contemporaneous feedback from both ΔGDP and ΔExchange Rate.

### 3.6 Impulse response functions

In the quarterly VAR we treat the *exchange rate* as the fastest-moving variable, priced continuously in financial markets; *GDP* as slower, because it aggregates many real-economy decisions and is only observed with a quarterly lag; and the *trade balance* as the slowest, constrained by shipping lags and fixed-term contracts. Hence our preferred Cholesky ordering is $\Delta E \rightarrow \Delta Y \rightarrow \Delta TB$.

The resulting impulse-response functions (Fig. 4) show:

1. A one-quarter exchange-rate depreciation produces an immediate trade-balance deficit (J-curve) and a modest, lagged GDP fall as dearer imports squeeze real income.
2. Trade-balance shocks hardly move GDP, suggesting that capital-account inflows cushion real activity.
3. GDP innovations elicit only muted exchange-rate reactions, consistent with Mundell–Fleming under monetary autonomy.

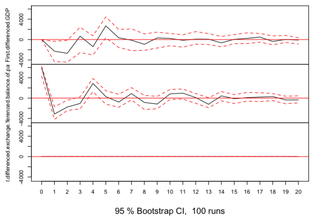
All responses fade within eight quarters and lie inside 95 % confidence bands, confirming stationarity and the plausibility of the financial-markets-first identification strategy.

Over an 8-quarter horizon, these IRFs reveal how shocks propagate through the economy: for instance, a positive innovation to GDP induces a short-lived boost to trade balance, while exchange rate responses exhibit persistent oscillations consistent with overshooting dynamics. The gradual convergence of all variables toward zero underscores the stationarity of our VAR specification, as shocks dissipate without permanent effects.

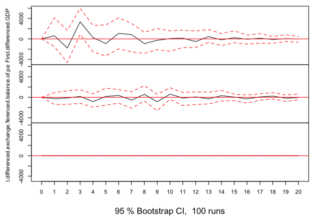
### 3.7 Robustness analysis: modifying the ordering of the variables

Figure 5 presents the impulse response functions (IRFs) derived from our VAR model, under an alternative Cholesky ordering illustrating the dynamic responses of the system's variables—first-differenced

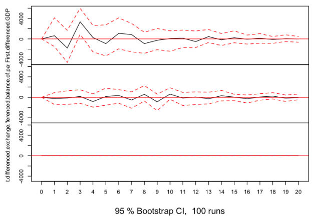
Figure 4: Impulse Response Functions




(a) Response to Δ GDP shock




(b) Response to Δ Trade Balance shock

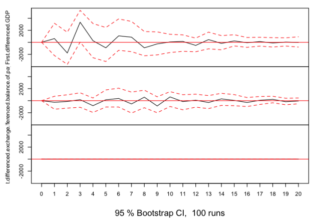



(c) Response Δ Exchange Rate shock

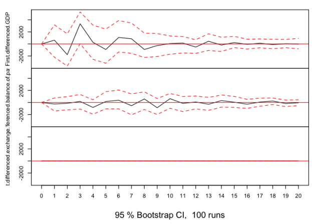
*Notes:* Confidence bands: 95% intervals computed via Monte Carlo simulations (1,000 repetitions, residual-based bootstrapping). Identification: Orthogonalized shocks using Cholesky decomposition with variable ordering: [Exchange Rate → GDP → Trade Balance ]. Responses shown over 8-quarter horizon, consistent with VAR forecast period. Zero line (horizontal axis) represents long-run equilibrium.
*Sources:* UK Statistical Office.

GDP, trade balance, and exchange rate—to orthogonalized structural shocks. The shaded regions represent 95% confidence intervals generated via Monte Carlo simulations, quantifying the statistical uncertainty around the median response trajectories.

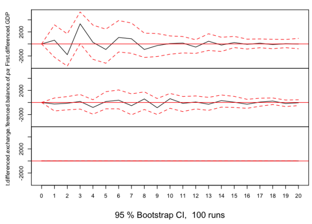
Figure 5: Impulse Response Functions




(a) Response to Δ GDP shock




(b) Response to Δ Trade Balance shock




(c) Response Δ Exchange Rate shock

*Notes:* Confidence bands: 95% intervals computed via Monte Carlo simulations (1,000 repetitions, residual-based bootstrapping). Identification: Orthogonalized shocks using Cholesky decomposition with variable ordering: [$\Delta GDP \rightarrow \Delta$*Balance of Payments* $\rightarrow \Delta$*Exchange Rate*]. Responses shown over 8-quarter horizon, consistent with VAR forecast period. Zero line (horizontal axis) represents long-run equilibrium.
*Sources:* UK Statistical Office.

# 4 Concluding Remarks

This study analyzed the UK's GDP, trade balance, and exchange rate dynamics using univariate and multivariate econometric frameworks. First-differencing addressed non-stationarity, enabling robust modeling. ARIMA results showed distinct dynamics: GDP combined trend persistence with shock absorption, trade balance exhibited error correction, and exchange rates had moderate momentum. VAR analysis underscored interconnectedness: exchange rate shocks triggered J-curve trade effects, while GDP shocks had delayed spillovers. The UK's resilience post-COVID and Brexit highlighted inherent stabilization.

An implication for policymakers would be to prioritize lagged effects of exchange rates and GDP shocks during crises.

In terms of limitations, our work exhibits sensitivity to VAR ordering and potential overfitting. To extend the analysis, we could have integrated structural VARs with long-run restrictions, or machine learning methods to improve predicitive accuracy.

# A Multivariate Analysis - theoretical framework

This section develops a theoretical framework to analyze the dynamic interactions among GDP, exchange rates, and trade balance in the UK from 1955 to 2024, incorporating the COVID-19 pandemic (2020–2021) and Brexit (2016–2024) as exogenous shocks. The model synthesizes New Keynesian open-economy dynamics (Gali, 2005) , financial frictions (Cespedes, 2004) , Mundell-Fleming trilemma constraints (Mundell 1963, Fleming 1962) , and enhanced exchange rate pass-through (Obstfeld, 1995) . It provides a foundation for Vector Autoregression (VAR) analysis by proposing a Cholesky ordering that reflects theoretical causality. Table 7 summarizes the model's variables and assumptions.

| **Notations** | **Assumptions** |
|---|---|
| Output gap ($y_t$): Log GDP deviations. | Small open economy with imperfect capital mobility. |
| GBP Nominal exchange rate ($E_t$): | Calvo-style price stickiness |
| Trade balance ($TB_t$) | Fixed (pre-1992) or floating (post-1992) exchange rate regimes. |
| Inflation ($\pi_t$): Domestic CPI inflation | Open capital account with risk premiums |
| Interest rate ($r_t$): Bank of England policy rate | Exogenous shocks from COVID-19 and Brexit |
| Fiscal policy ($g_t$): Government spending | |
| COVID-19 shock ($\xi_t^{COV}$): Temporary AR(1) shock, $\xi_t^{COV} = \rho^{COV}\xi_{t-1}^{COV} + \epsilon_t^{COV}$, $\rho^{COV} < 1$. | |
| Brexit shock ($\tau_t^{BRX}$): Persistent AR(1) shock, $\tau_t^{BRX} = \rho^{BRX}\tau_{t-1}^{BRX} + \epsilon_t^{BRX}$, $\rho^{BRX} \approx 1$. | |

Table 7: Notations and Assumptions

The model comprises six core equations, derived below with explicit integration of COVID-19 and Brexit shocks.

**New-Keynesian IS curve.** Log-linearizing the household Euler equation links output to its expected future value and the gap between the real policy rate and the natural rate. Decomposing aggregate demand by steady-state expenditure shares, investment is allowed to fall with the interest rate and Brexit uncertainty. Adding open-economy terms introduces the expected change in the real exchange rate and foreign output, while fiscal spending enters with a (potentially larger) multiplier under fixed exchange rates. Two additive shocks capture demand losses from COVID-19 and Brexit, yielding

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\big(r_t - E_t\pi_{t+1} - r_t^n\big) + \alpha\big(E_t q_{t+1} - q_t\big) + \eta, y_t^* + \mu, g_t - \delta^{\mathrm{COV}}\xi_t^{\mathrm{COV}} - \delta^{\mathrm{BRX}}\tau_t^{\mathrm{BRX}}. \quad (5)$$

**New-Keynesian Phillips Curve.** Calvo pricing implies that only a share $1-\theta$ of firms can reset prices each period. [1]
Log-linearising the optimal pricing condition and the aggregate price index yields a forward-looking Phillips curve.
Marginal cost depends on the output gap and the real exchange rate, while open-economy pass-through adds the change in the nominal exchange rate.
Exogenous terms capture COVID-19 supply bottlenecks and post-Brexit import-cost pressures.
The resulting log-linear Phillips curve is

$$\pi_t = \beta\, E_t\pi_{t+1} + \kappa\, y_t + \lambda\big(q_t - q_{t-1}\big) + \nu\,\Delta E_t + \gamma^{\mathrm{COV}}\xi_t^{\mathrm{COV}} + \gamma^{\mathrm{BRX}}\tau_t^{\mathrm{BRX}}, \quad (6)$$

where $\kappa = (1-\theta)(1-\beta\theta)/\theta$. The coefficients $\lambda$ and $\nu$ quantify real-exchange-rate and pass-through effects, while $\gamma^{\mathrm{COV}}$ and $\gamma^{\mathrm{BRX}}$ capture the inflationary impact of pandemic-related supply constraints and Brexit-induced trade frictions.

[1] The remaining share $\theta$ keep their prices fixed, generating nominal rigidity.

**Trade-Balance Equation.** Exports and imports depend on the real exchange rate and foreign–domestic demand: log-linearising $X_t = (Q_t)^{-\eta_x} Y_t^*$ and $M_t = (Q_t)^{\eta_m} Y_t$ gives $tb_t = \eta_x q_t + \gamma_y y_t^* - \gamma_m y_t$.
Allowing for a J-curve lag and exchange-rate pass-through, and adding COVID-19 trade disruptions and Brexit-related EU trade losses, yields

$$tb_t = \gamma_x\big(q_t - \phi q_{t-1}\big) + \gamma_y\, y_t^* - \gamma_m\, y_t + \chi\, \Delta E_t - \psi^{\mathrm{COV}} \xi_t^{\mathrm{COV}} - \psi^{\mathrm{BRX}} \tau_t^{\mathrm{BRX}}, \tag{7}$$

where $\gamma_x = \eta_x$, $\phi \in (0, 1)$ captures the J-curve, $\chi$ measures pass-through, and the $\psi$-coefficients quantify pandemic and Brexit shocks to net exports.

**Exchange-Rate Dynamics.** Under a fixed-exchange-rate regime (e.g. pre-1992 ERM membership), the nominal rate is constant at its peg $\bar{E}$. For floating regimes, uncovered-interest parity—augmented with a risk premium $\rho_t$ and short-run deviations $\psi_t$—links the spot rate to expected future depreciation:

$$E_t \;=\; E_t E_{t+1}\, \frac{1 + r_t}{1 + r_t^* + \rho_t} \;+\; \psi_t.$$

Combining both cases, the exchange-rate rule is

$$E_t \;=\; \begin{cases} \bar{E}, & \text{(fixed regime)} \\ E_t E_{t+1}\, \dfrac{1 + r_t}{1 + r_t^* + \rho_t} \;+\; \psi_t, & \text{(floating regime).} \end{cases} \tag{8}$$

Here $r_t$ and $r_t^*$ denote domestic and foreign policy rates, while $\rho_t$ captures the risk premium consistent with the exchange-rate trilemma.

**Balance Sheet Effects.** Investment faces financial frictions.

Investment

$$I_t = I(r_t, \theta_t)$$

Financial conditions

$$\theta_t = \theta_0 - \zeta(E_t D_t^* + \tau_t^{BRX})$$

$$I_t = I(r_t, \theta_t), \quad \theta_t = \theta_0 - \zeta(E_t D_t^* + \tau_t^{BRX}) \tag{9}$$

**Monetary Policy.** The central bank sets the interest rate. Fixed regimes

$$r_t = r_t^* + \rho_t + \epsilon_t$$

Floating regimes (Taylor rule)

$$r_t = r_t^n + \phi_\pi \pi_t + \phi_y y_t + \epsilon_t$$

Therefore

$$r_t = \begin{cases} r_t^* + \rho_t + \epsilon_t & \text{if fixed regime} \\ r_t^n + \phi_\pi \pi_t + \phi_y y_t + \epsilon_t & \text{if floating regime} \end{cases} \tag{10}$$

**Implications for our analysis.** The framework suggests the VAR ordering: $\xi_t^{COV} \to \tau_t^{BRX} \to E_t \to \pi_t \to y_t \to TB_t \to r_t$, reflecting: an exogenous, temporary COVID-19 shock (Office for National Statistics, 2023), A persistent and structural Brexit shock (Office for Budget Responsibility, 2025), fixed exchange rate (Dornbusch 1976, Mundell 1963), inflation transmits shocks (Obstfeld, 1995), output is affected by shocks Fleming, 1962), trade balance is endogenous (backus1994), and interest rate is constrained or reactive (Taylor, 1993).