

Probabilistic Market Budget Allocator

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Priors / Sampling Assumptions

CPM (lognormal).

$$\ln(\text{CPM}_i) \sim \mathcal{N}(\mu_i^{\text{cpm}}, (\sigma_i^{\text{cpm}})^2)$$

CTR and CVR (beta). Given mean $m \in (0, 1)$ and std s , the beta parameters are $v = s^2$, $k = \frac{m(1-m)}{v} - 1$,
 $\alpha = m k$, $\beta = (1 - m) k$,
 $X \sim \text{Beta}(\alpha, \beta)$, $X \in \{\text{CTR}_i, \text{CVR}_i\}$.

Quality multiplier (lognormal).

$$\ln(q_i) \sim \mathcal{N}(\mu_i^q, (\sigma_i^q)^2), \quad \text{with default } \mu^q = 0, \sigma^q = 0.1.$$

Capacity θ (lognormal).

$$\ln(\theta_i) \sim \mathcal{N}(\mu_i^\theta, (\sigma_i^\theta)^2).$$

AOV per sale (lognormal).

$$\ln(\text{AOV}_i^{\text{sale}}) \sim \mathcal{N}(\mu_i^{\text{aov}}, (\sigma_i^{\text{aov}})^2).$$

Distributions

CPM, AOV log normal distributions because positive and right skewed

CVR, CTR beta distribution because between 0-1

Fixed Quality multiplier from lognormal for to calculate effective values

Capacity theta as max cap from lognormal. Currently also same for all 4 channels

Marketing-to-Revenue Funnel

We model the ad spend to revenue process in four stages:

1. Spend \rightarrow Impressions

Ads are purchased at a cost-per-thousand impressions (CPM).

$$Impressions_{per\$} = \frac{1000}{CPM_{eff}}$$

where CPM_{eff} is adjusted for channel quality.

2. Impressions \rightarrow Clicks

A fraction of impressions convert to clicks at the click-through rate (CTR). The CTR is quality-adjusted and capped at 5% for realism:

$$Clicksper\$ = \frac{1000}{CPM_{\text{eff}}} \cdot CTR_{\text{eff}}$$

3. Clicks \rightarrow Leads

A fraction of clicks convert to leads at the click-to-lead conversion rate (CVR), also quality-adjusted and capped at 5%:

$$Leadsper\$(base) = \frac{1000}{CPM_{\text{eff}}} \cdot CTR_{\text{eff}} \cdot CVR_{\text{eff}}$$

This base rate defines the initial slope of the saturation curve.

4. Leads \rightarrow SQL \rightarrow Wins \rightarrow Revenue

Leads pass through two sales funnel stages:

$$p_{\ell \rightarrow s} = 0.25, \quad p_{s \rightarrow w} = 0.15$$

where $p_{\ell \rightarrow s}$ is the probability of a lead becoming a sales qualified lead (SQL), and $p_{s \rightarrow w}$ is the probability of an SQL converting to a win.

Let AOV^{sale} be the average contract value per sale. The effective average order value per lead is:

$$\text{AOV}^{\text{eff}} = \text{AOV}^{\text{sale}} \cdot p_{\ell \rightarrow s} \cdot p_{s \rightarrow w}$$

Response Curve (exponential saturation curve)

Revenue from spend S : We model leads generated from spend S using an exponential saturation curve:

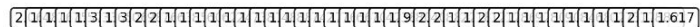
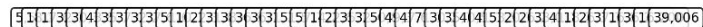
$$\text{Leads}(S) = \theta (1 - e^{-\beta S})$$

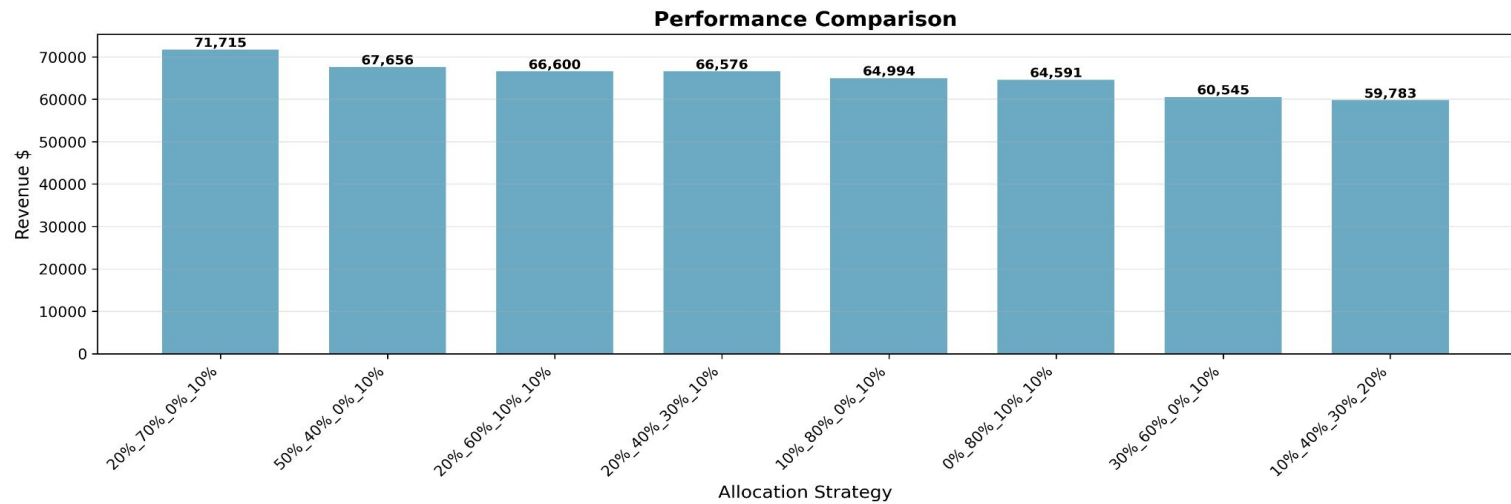
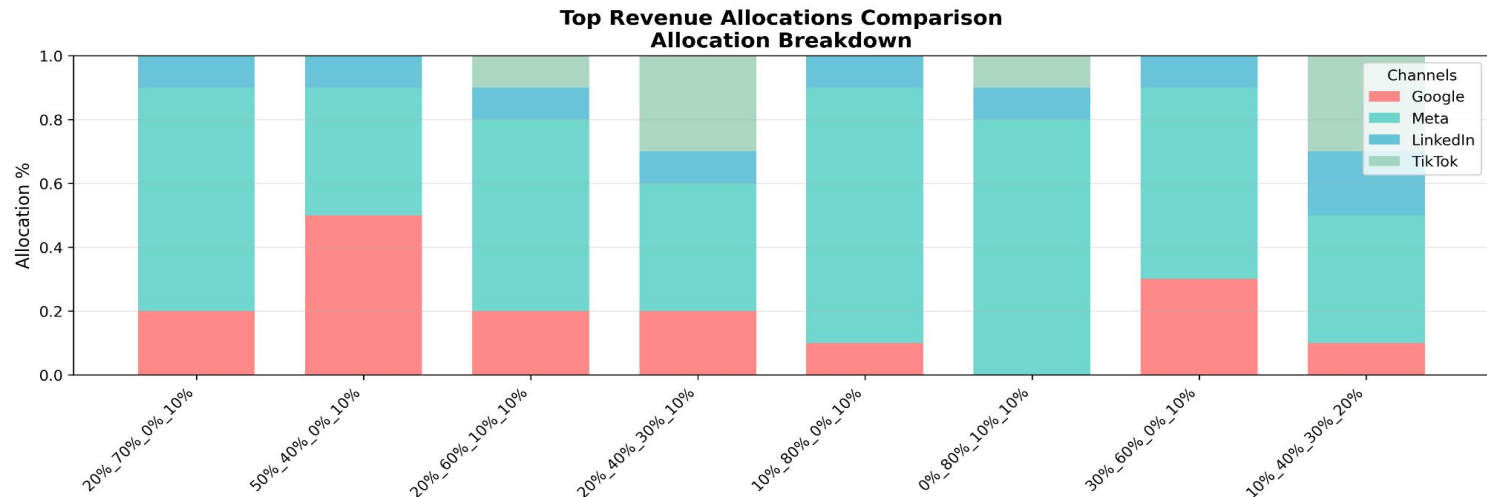
where θ is the maximum lead capacity and $\beta = \frac{\text{base}}{\theta}$ ensures the initial slope matches the base rate.

The revenue is then:

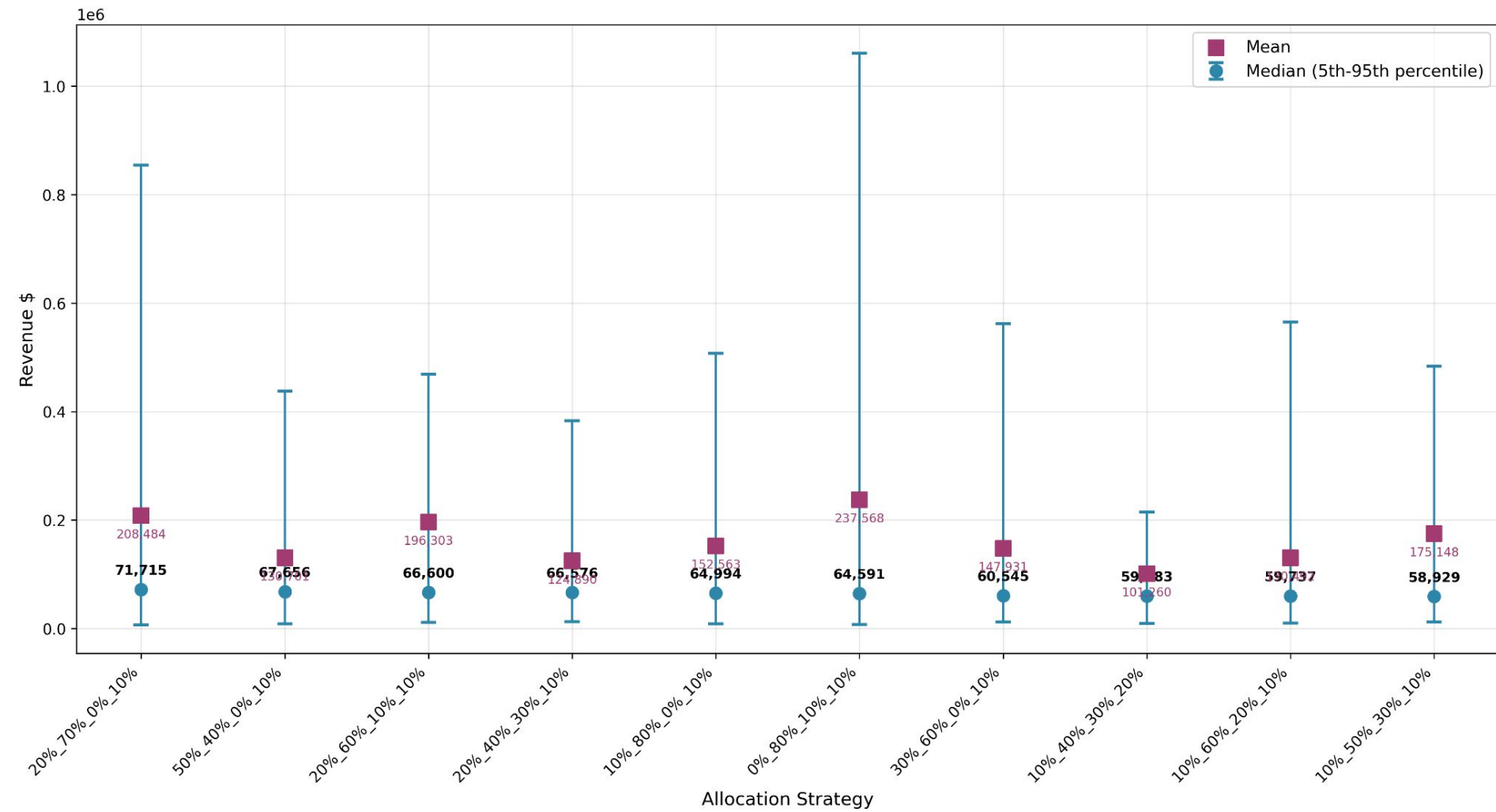
$$R(S) = \text{Leads}(S) \cdot \text{AOV}^{\text{eff}}$$

Allocation Heatmap

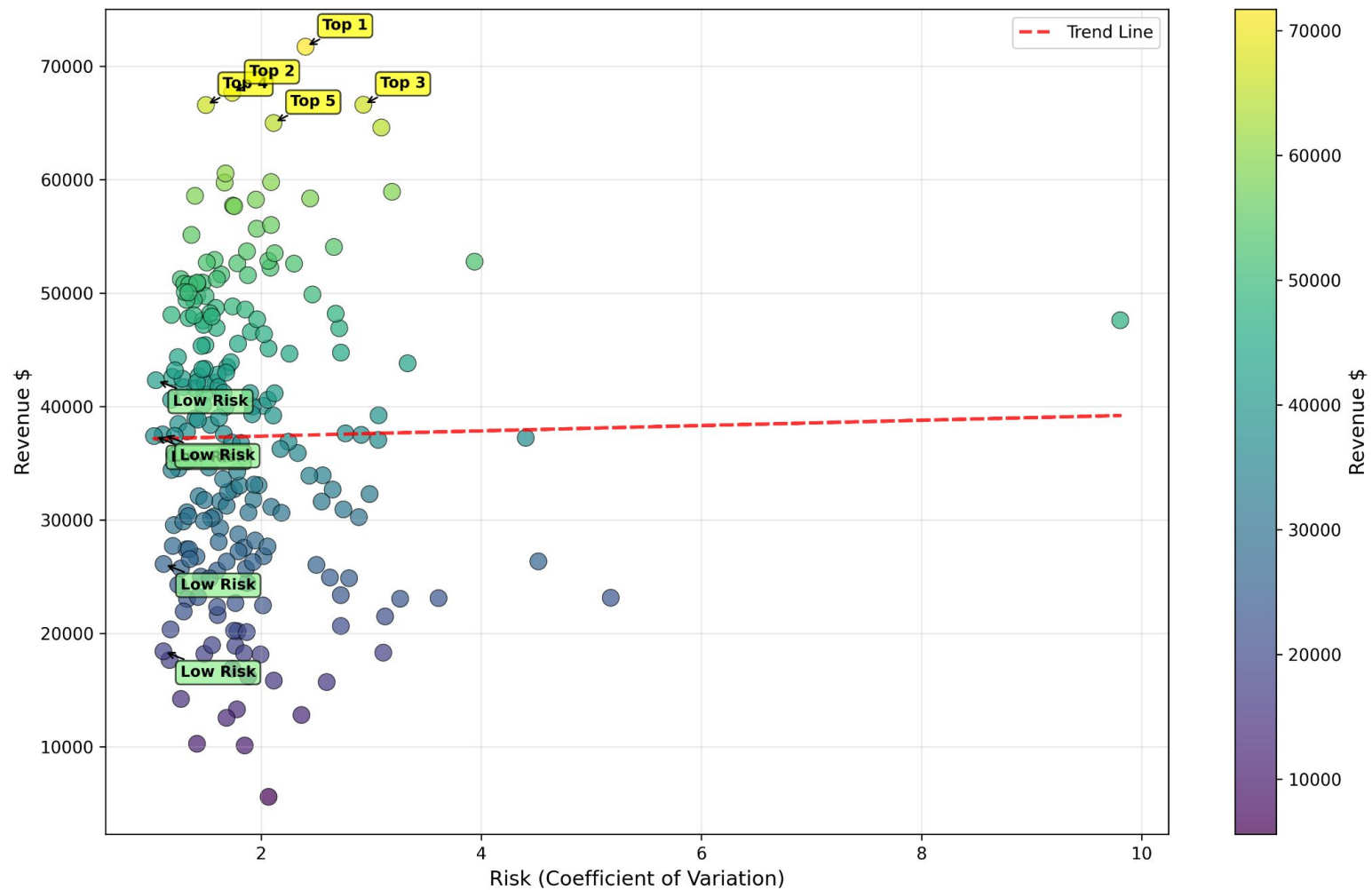




Revenue Confidence Intervals



Risk vs. Revenue Performance



Assumptions & Trade offs

Assumptions

- **Channel Independence:** Each channel's performance is modeled independently; no cross-channel cannibalization or halo effects.
- **Fixed Funnel Rates:** Lead→SQL and SQL→Win rates are constant; in reality, these may vary with spend, seasonality, or targeting.
- **Capped CTR/CVR:** Click-through and conversion rates are capped at 5% to avoid unrealistic upside scenarios, especially for B2B contexts.
- **Coarse Grid Resolution:** Budget allocations are explored in 10% increments, trading off precision for computational simplicity.

Trade-offs

- **Grid Exploration Scalability:** The combinatorial space grows quickly with finer steps, making grid search computationally expensive for higher resolution or more channels.
- **Priors Dominating:** With no observed data, priors largely determine results. This is reasonable for early modeling but limits data-specific insights.
- **High Uncertainty:** Variance in results is expected due to limited empirical constraints; uncertainty reflects lack of real-world calibration.
- **Small Simulation Size:** Current simulation count ($n=220$) is low; this limits accuracy in estimating metrics like the risk–revenue curve. Higher n ($\geq 10,000$) would yield more stable estimates.

Next Steps

1. **Increase Simulation Runs** — Move from $n=220$ to $n=10,000+$ to stabilize risk/revenue estimates and percentiles.
2. **Refine Priors with Data** — Replace generic benchmarks with channel-specific historical data to shift from prior-dominated to posterior-informed results.
3. **Experiment with Optimization Methods** — Replace coarse grid search with more efficient algorithms (e.g., genetic algorithms, Bayesian optimization, NSGA-II) for finer allocation precision without excessive computation.(if required)
4. **Incorporate Spend–Rate Interactions** — Allow CTR/CVR/funnel rates to vary with spend to capture diminishing marginal quality.
5. **Model Cross-Channel Effects** — Introduce interaction terms or constraints to capture halo/cannibalization between channels.
6. **Sensitivity Analysis** — Quantify how changes in priors or funnel assumptions impact optimal allocations to identify the most influential variables.