Introduction to computational linguistics

Exercise session 1: "Algorithms for matching"

Thursday May 7 2009

- a. Show how to compute Z_i stepwise for i > 1 (using the notion of Z-boxes) for the following strings:
 - i. AABCAABXAAZ
 - ii. ABCDXABCYABDXY
- Apply the Knuth-Morris-Pratt algorithm to find occurrences of ABXYABXZ in XABXYABXYABXZABXZABXYABXZA

S = AABCAABXAAZ

Step 0)

Compute $Z_2(S)$ by comparing left-to-right S[2..ISI] and S[1..ISI] until a mismatch is found; $Z_2(S)$ is the length of that string. If $Z_2(S) > 0$ then $r=r2=Z_2(S)+1$ and l=2, else l=r=0

S	Α	Α	В	С	Α	Α	В	Х	Α	Α	Z	
	Ι	2	3	4	5	6	7	8	9	10	П	
$Z_{i}(S)$		I										

$$Z_2(S)=1: \{ A A B ... \} \text{ so } I=2, r=Z_2(S)+1=1+1=2$$

k > r: 3 > (r=2) so find $Z_3(S)$ by comparing S[3...ISI] to S[1..ISI] until a mismatch is found; if $Z_3(S) > 0$ then I=3, r=3+ $Z_3(S)$ -1

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 $S(3)='B' \neq S(1)='A'$, hence $Z_3(S)=0$, I and r remain as they are: I=r=2

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S	Α	Α	В	С	Α	Α	В	Х	Α	Α	Z	
	I	2	3	4	5	6	7	8	9	10	П	
$Z_{i}(S)$		I	0									

$$Z_3(S)=0$$
 so $l=2$, $r=2$

k > r: 4 > (r=2) so find $Z_4(S)$ by comparing S[4...ISI] to S[1..ISI] until a mismatch is found; if $Z_4(S) > 0$ then I=4, $r=4+Z_4(S)-1$

k > r: 4 > (r=2) so find $Z_4(S)$ by comparing S[4...ISI] to S[1..ISI] until a mismatch is found; if $Z_4(S) > 0$ then I=4, $r=4+Z_4(S)-1$

 $S(4)='C' \neq S(1)='A'$, hence $Z_4(S)=0$, I and r remain as they are: I=r=2

k > r: 4 > (r=2) so find $Z_4(S)$ by comparing S[4...ISI] to S[1..ISI] until a mismatch is found; if $Z_4(S) > 0$ then I=4, $r=4+Z_4(S)-1$

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S	Α	Α	В	С	Α	Α	В	X	Α	Α	Z	
	I	2	3	4	5	6	7	8	9	10	Π	
$Z_{i}(S)$		I	0	0								

$$Z_4(S)=0$$
 so $I=2$, $r=2$

k > r: 5 > (r=2) so find $Z_5(S)$ by comparing S[5...ISI] to S[1..ISI] until a mismatch is found; if $Z_5(S) > 0$ then I=5, r=5+ $Z_5(S)$ -1

k > r: 5 > (r=2) so find $Z_5(S)$ by comparing S[5...ISI] to S[1..ISI] until a mismatch is found; if $Z_5(S) > 0$ then I=5, r=5+ $Z_5(S)$ -1

S[5..7]="A A B" matches S[1..3]="A A B", hence $Z_5(S)$ =3, and I and r are set as follows: I=5, r=5+ $Z_5(S)$ -1=5+3-1=7

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	—	2	3	4	5	6	7	8	9	10	П	
$Z_{i}(S)$		Ι	0	0	3							

$$Z_5(S)=3$$
 so $l=5$, $r=7$

 $6 \le (r=7)$: position k=6 is contained in a Z-box (namely, "AAB"=S[5..7], with S(6)='A').

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Furthermore, there must be a match to a prefix of S of length minimum $[Z_2(S), IS[2..3]I]$, i.e. minimum [1,r-k+1=2] = 2

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Step 2a)

 $6 \le (r=7)$: position k=6 is contained in a Z-box (namely, "AAB"=S[5..7], with S(6)='A').

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 $Z_6(S)=Z_2(S)=1$ which is smaller than the length of S[2..3], hence I and r stay the same

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	I	2	3	4	5	6	7	8	9	10	П	
$Z_{i}(S)$		I	0	0	3	I						

 $Z_6(S)=Z_2(S)=1$ so I and r remain the same: I=5, r=7

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Furthermore, there must be a match to a prefix of S of length minimum $[Z_3(S), IS[3..3]]$, i.e. minimum [0,r-k+1=1] = 1

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 $Z_7(S)=Z_3(S)=0$ which is smaller than the length of S[3..3], hence I and r stay the same

S	Α	Α	В	С	Α	Α	В	Х	Α	Α	Z	
	I	2	3	4	5	6	7	8	9	10	П	
$Z_{i}(S)$		I	0	0	3	ı	0					

 $Z_7(S)=Z_3(S)=0$ so I and r remain the same: I=5, r=7

$$k=8 > (r=7)$$
 so step 1:

match S[8..ISI] to S[1..ISI]: mismatch, so $Z_8(S)=0$, I and r remain the same

S	Α	Α	В	С	Α	Α	В	X	Α	Α	Z	7 ₀ (9	S)=0 so l=5, r=7
	I	2	3	4	5	6	7	8	9	10	П	-8/	
$Z_{i}(S)$		I	0	0	3	I	0	0					

$$k=9 > (r=7)$$
 so step 1:

match S[9..ISI] to S[1..ISI]: match S[9..10]=S[1..2], so $Z_9(S)=2$, I=9 and r=10

 $k=10 \le (r=10)$ so step 2:

S(10) contained in S[9..10]; S(10) matches S(10-9+1)=S(2)='A'; $Z_2(S)=1 \ge I$ S[10..10]I=10-10+1=1, hence **Step 2b)** but mismatch

S	Α	Α	В	С	Α	Α	В	Х	Α	Α	Z	$Z_{10}(S)=1$
	ı	2	3	4	5	6	7	8	9	10	П	-10 ⁽³⁾
$Z_{i}(S)$		I	0	0	3	I	0	0	2	ı		

k=11 > (r=10) so step 1:

match S[11..ISI] to S[1..ISI]: mismatch so $Z_{11}(S)=0$





_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	$Z_{2}(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z	_i (S)		0													

$$Z_2(S)$$
: S(2) \neq S(1) so $Z_2(S)$ =0, r=l=0

_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	Х	Υ	$Z_2(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	2/3/3
Z	(S)		0													

i=3..5:
$$Z_i(S)$$
: $S(i) \neq S(1)$ so $Z_i(S)$ =0, r=l=0



	S	Α	В	С	D	X	Α	В	C	Υ	Α	В	D	X	Y	$Z_2(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z	i(S)		0													

i=3..5: $Z_i(S)$: $S(i) \neq S(1)$ so $Z_i(S)$ =0, r=l=0

	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	Χ	Y	$Z_{i=35}(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	==35 ⁽³⁾
Z _i (S)	I	0	0	0	0										



_	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	X	Υ	$Z_{2}(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z	i(S)	1	0													

i=3..5: $Z_i(S)$: $S(i) \neq S(1)$ so $Z_i(S)$ =0, r=l=0

	S	Α	В	U	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	$Z_{i=35}(S)=0$
		Ι	2	3	4	5	6	7	8	9	10	П	12	13	14	Z ₁₌₃₅ (3)
Z_{i}	(S)		0	0	0	0										

 $Z_6(S)$: S(6) = S(1): S[6..8] matches S[1..3], so $Z_6(S)$ =3, l=6 and r=8

a.ii) Z_i for ABCDXABCYABDXY

 $Z_{2}(S)$: S(2) \neq S(1) so $Z_{2}(S)$ =0, r=l=0

_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	Х	Υ	$Z_2(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z	i(S)		0													

i=3..5: $Z_i(S)$: $S(i) \neq S(1)$ so $Z_i(S)$ =0, r=l=0

	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	X	Y	$Z_{i=35}(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	Z ₁₌₃₅ (3)
Z _i (S)	I	0	0	0	0										

 $Z_6(S)$: S(6) = S(1): S[6..8] matches S[1..3], so $Z_6(S)$ =3, l=6 and r=8

S	Α	В	U	Δ	X	Α	В	С	Υ	Α	В	D	X	Y	$Z_{6}(S)=3$
	I	2	3	4	5	6	7	8	9	10	П	12	13	14	6(3)
$Z_{i}(S)$		0	0	0	0	3									









_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	Z ₇ (S)=0
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z	_i (S)	-	0	0	0	0	3	0								



_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	Z ₇ (S)=0
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	2/(3)
Z	_i (S)		0	0	0	0	3	0								

 $Z_8(S)$: 8 \leq (r=8) hence S(8)=S(8-6+1)=S(3)='C', $Z_3(S)$ =0 whereas IS[8..8]I=1, hence $Z_8(S)$ = $Z_3(S)$ =0 and I and remain as they are: I=6 and r=8



S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	X	Υ	Z ₇ (S)=0
	I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z _i (S)		0	0	0	0	3	0								

 $Z_8(S)$: 8 \leq (r=8) hence S(8)=S(8-6+1)=S(3)='C', $Z_3(S)$ =0 whereas IS[8..8]I=1, hence $Z_8(S)$ = $Z_3(S)$ =0 and I and remain as they are: I=6 and r=8

_	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	X	Υ	$Z_{8}(S)=0$
		—	2	3	4	5	6	7	8	9	10	П	12	13	14	_8(0)
Z	i(S)	I	0	0	0	0	3	0	0							



_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	Z ₇ (S)=0
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z	_i (S)	-	0	0	0	0	3	0								

 $Z_8(S)$: 8 \leq (r=8) hence S(8)=S(8-6+1)=S(3)='C', $Z_3(S)$ =0 whereas IS[8..8]I=1, hence $Z_8(S)$ = $Z_3(S)$ =0 and I and remain as they are: I=6 and r=8

	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	X	Υ	Z ₈ (S)=0
		I	2	3	4	5	6	7	8	9	10		12	13	14	_8(3)
Z_{i}	(S)		0	0	0	0	3	0	0							

 $Z_9(S)$: 9 > (r=8) but S(9) \neq S(1) hence $Z_9(S)$ =0 and I and remain as they are: I=6 and r=8

a.ii) Z_i for ABCDXABCYABDXY

 $Z_7(S)$: 7 \leq (r=8) hence S(7)=S(7-6+1)=S(2)='B', $Z_2(S)$ =0 whereas IS[7..8]I=2, hence $Z_7(S)$ = $Z_2(S)$ =0 and I and remain as they are: I=6 and r=8

5	3	Α	В	U	D	X	Α	В	С	Υ	Α	В	D	X	Υ	$Z_7(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	
Z _i (S	3)		0	0	0	0	3	0								

 $Z_8(S)$: 8 \leq (r=8) hence S(8)=S(8-6+1)=S(3)='C', $Z_3(S)$ =0 whereas IS[8..8]I=1, hence $Z_8(S)$ = $Z_3(S)$ =0 and I and remain as they are: I=6 and r=8

_	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	Χ	Υ	Z ₈ (S)=0
		-	2	3	4	5	6	7	8	9	10	П	12	13	14	_8(3)
Z	(S)	I	0	0	0	0	3	0	0							

 $Z_{9}(S)$: 9 > (r=8) but S(9) \neq S(1) hence $Z_{9}(S)$ =0 and I and remain as they are: I=6 and r=8

_	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	Χ	Υ	$Z_{9}(S)=0$
		Ι	2	3	4	5	6	7	8	9	10	П	12	13	14	<u> </u>
Zi	(S)		0	0	0	0	3	0	0	0						



 $Z_{10}(S)$: 10 > (r=8), S(10)=S(1), match S[10..1] with S[1..2], hence $Z_{10}(S)$ =2 and I=10 and r=11

 $Z_{10}(S)$: 10 > (r=8), S(10)=S(1), match S[10..1] with S[1..2], hence $Z_{10}(S)$ =2 and l=10 and r=11

_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	$Z_{10}(S)=0$
		_	2	3	4	5	6	7	8	9	10	П	12	13	14	-10 ⁽³⁾
Z	_i (S)		0	0	0	0	3	0	0	0	2					

 $Z_{10}(S)$: 10 > (r=8), S(10)=S(1), match S[10..1] with S[1..2], hence $Z_{10}(S)$ =2 and l=10 and r=11

_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	X	Υ	$Z_{10}(S)=0$
		_	2	3	4	5	6	7	8	9	10	П	12	13	14	-10 ⁽³⁾
Z	_i (S)		0	0	0	0	3	0	0	0	2					

 $Z_{11}(S)$: 11 \leq (r=11) hence S(11)=S(11-10+1)=S(2)='B', $Z_2(S)$ =0 whereas IS[11..11]I=1, hence $Z_{11}(S)$ = $Z_2(S)$ =0 and I and remain as they are: I=10 and r=11

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_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	Х	Υ	$Z_{10}(S)=0$
		_	2	3	4	5	6	7	8	9	10	П	12	13	14	-10 ⁽³⁾
Z	i(S)		0	0	0	0	3	0	0	0	2					

 $Z_{11}(S)$: 11 \leq (r=11) hence S(11)=S(11-10+1)=S(2)='B', $Z_2(S)$ =0 whereas IS[11..11]I=1, hence $Z_{11}(S)$ = $Z_2(S)$ =0 and I and remain as they are: I=10 and r=11

_	S	Α	В	U	D	X	Α	В	U	Υ	Α	В	D	X	Y	Z ₁₁ (S)=0
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	-11 ⁽³⁾
Z_{i}	(S)		0	0	0	0	3	0	0	0	2	0				

a.ii) Z_i for ABCDXABCYABDXY

 $Z_{10}(S)$: 10 > (r=8), S(10)=S(1), match S[10..1] with S[1..2], hence $Z_{10}(S)$ =2 and l=10 and r=11

	S	Α	В	С	D	Х	Α	В	C	Υ	Α	В	D	X	Υ	$Z_{10}(S)=0$
		_	2	3	4	5	6	7	8	9	10	П	12	13	14	-10 ⁽³⁾
Z	_i (S)	I	0	0	0	0	3	0	0	0	2					

 $Z_{11}(S)$: 11 \leq (r=11) hence S(11)=S(11-10+1)=S(2)='B', $Z_2(S)$ =0 whereas IS[11..11]I=1, hence $Z_{11}(S)$ = $Z_2(S)$ =0 and I and remain as they are: I=10 and r=11

	s	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	Х	Υ	Z ₁₁ (S)=0
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	711(9)
Z _i (S)		0	0	0	0	3	0	0	0	2	0				

 $i=12..14: Z_i(S)=0$

a.ii) Z_i for ABCDXABCYABDXY

 $Z_{10}(S)$: 10 > (r=8), S(10)=S(1), match S[10..1] with S[1..2], hence $Z_{10}(S)$ =2 and I=10 and r=11

_	S	Α	В	С	D	Х	Α	В	С	Υ	Α	В	D	Х	Υ	$Z_{10}(S)=0$
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	710(0)
Z_{i}	(S)		0	0	0	0	3	0	0	0	2					

 $Z_{11}(S)$: 11 \leq (r=11) hence S(11)=S(11-10+1)=S(2)='B', $Z_2(S)$ =0 whereas IS[11..11]I=1, hence $Z_{11}(S)$ = $Z_2(S)$ =0 and I and remain as they are: I=10 and r=11

_	S	Α	В	U	D	X	Α	В	С	Υ	Α	В	D	X	Υ	Z ₁₁ (S)=	:0
		I	2	3	4	5	6	7	8	9	10	П	12	13	14	-11(9)	
Z_{i}	(S)		0	0	0	0	3	0	0	0	2	0					

 $i=12..14: Z_i(S)=0$

	S	Α	В	С	D	X	Α	В	С	Υ	Α	В	D	Х	Υ	i=1214: Z _i (S)=0
		Ι	2	3	4	5	6	7	8	9	10	11	12	13	14	
Z _i ((S)		0	0	0	0	3	0	0	0	2	0	0	0	0	

b) Knuth-Morris-Pratt



- "Apply the Knuth-Morris-Pratt algorithm to find occurrences of ABXYABXZ in XABXYABXYABXZABXZABXYABXZA"
- Pre-processing
 - For each position i in the pattern we need to define sp_i(P) to be the length of the longest proper suffix of P[1..i] that matches a prefix of P.
 - Optimization: let sp'_i(P) be sp_i(P) with the added condition that characters P(i+1) and P(sp'_i+1) are unequal
 - Compute sp'_i(P) on the basis of the Z-values for the pattern;
 compute the failure function on the basis of the sp'_i(P) values

- Basic idea
 - Shift smarter than the naive method does

- A mismatch with P(8) means we can shift 4 places
- Deduction on P alone: no need to know T, or how P and T are aligned
- Complexity of the algorithm
 - The algorithm is linear, not -possibly- sublinear like Boyer-Moore
 - Extension: the Aho-Corasick algorithm for matching sets of patterns

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- Deduction on P alone: no need to know T, or how P and T are aligned
- Complexity of the algorithm
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- Basic idea
 - Shift smarter than the naive method does

- A mismatch with P(8) means we can shift 4 places (like good suffix rule!)
- Deduction on P alone: no need to know T, or how P and T are aligned
- Complexity of the algorithm
 - The algorithm is linear, not -possibly- sublinear like Boyer-Moore
 - Extension: the Aho-Corasick algorithm for matching sets of patterns



Definition

For each position i in P, define $sp_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P.

Optimization

For each position i in P, define $sp'_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P, with the added condition that characters P(i+1) and P(sp'_i+1) are unequal.



Definition

For each position i in P, define $sp_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P.

Optimization

For each position i in P, define $sp'_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P, with the added condition that characters P(i+1) and P(sp'_i+1) are unequal.

- Alignment of P and T, left-to-right matching
- The shift rule:

For any alignment of P and T, if the first mismatch (comparing from left to right) occurs in position i+1 of P and position k of T, then shift P to the right (relative to T) so that P[1..sp_i'] aligns with T[k-sp_i'..k-1]. In other words, shift P exactly i+1-(sp_i'+1)=i-sp_i' places to the right, so that character sp_i'+1 of P will align with character k in T. In the case that an occurrence of P has been found (no mismatch), shift P by n-sp_i' places.

Preprocessing using the Z values

Position j > 1 maps to i if $i=j+Z_j(P)-1$. That is, j maps to i if i is the right end of a Z-box starting at j.

Z-based Knuth-Morris-Pratt

```
for i := 1 to n do

sp_i' := 0;

for j := n downto 2 do

i := j + Z_j(P) - 1;

sp_i' := Z_j
```

Preprocessing using the Z values

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$$S = \begin{bmatrix} \alpha & & & & \\ & \alpha & & \\ & & Z_{l_k} & & \\ & & &$$

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) - 1;$
 $sp_i' := Z_j$

Position j > 1 maps to i if $i=j+Z_j(P)-1$. That is, j maps to i if i is the right end of a Z-box starting at j.

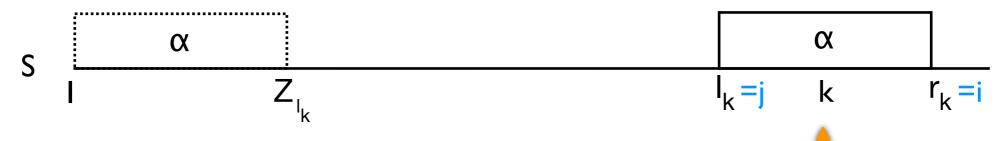
$$S = \begin{bmatrix} \alpha & & & & \\ & \alpha & & \\ & & Z_{l_k} & & \\ & & &$$

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) - 1;$
 $sp_i' := Z_j$

Position j > 1 maps to i if $i=j+Z_j(P)-1$. That is, j maps to i if i is the right end of a Z-box starting at j.



Z-based Knuth-Morris-Pratt

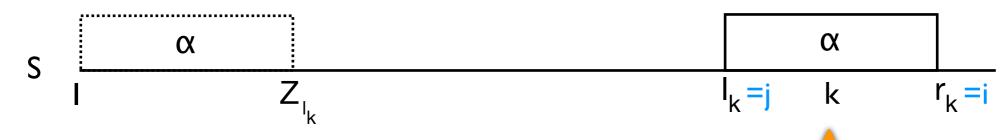
 $sp_i' := 0;$

$$i := j + Z_{j}(P) -1;$$

$$sp_i' := Z_j$$

sp'_i(P) is the length of the longest proper suffix of P[1...i] i.e. the length of the Z-box that starts at j (the suffix)

Position j > 1 maps to i if $i=j+Z_j(P)-1$. That is, j maps to i if i is the right end of a Z-box starting at j.



Z-based Knuth-Morris-Pratt

for
$$i := 1$$
 to n do

$$sp_i' := 0;$$

for j := n downto 2 do

$$i := j + Z_j(P) -1;$$

$$sp_i' := Z_j$$

sp'_i(P) is the length of the longest proper suffix of P[1...i] i.e. the length of the Z-box that starts at j (the suffix)

Preliminaries

- Shifts through pointers: p points into P, c points into T
- For each position i from 1 to n+1, define the failure function F'(i) to be sp'_{i-1} + 1 (and define F(i)=sp_{i-1} +1); let sp₀' and sp₀ be 0.

The algorithm

```
preprocess P to find F'(k)=sp'_{k-1}+1 for k from 1 to n+1

c:=1;

p:=1;

while c+(n-p) \le m do

while P(p)=T(c) and p \le n

p:=p+1;

c:=c+1;

if p=n+1 then

report an occurrence of P starting at position c-n of T if p=1 then c:=c+1

p:=F'(p)
```

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_j$

S	Α	В	Х	Υ	Α	В	Х	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

i				
J				
7 (0)				
Z _j (S)				
,				
i				
•				
sn'				
sp _i '				

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

j	8				
Z _j (S)	0				
i	8+0- I				
sp _i '	0				

The Z-values are as follows, we only have a Z-box starting at l=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

j	8	7			
Z _j (S)	0	0			
i	8+0- I	7+0- I			
sp _i '	0	3			

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	Ι	2	3	4	5	6	7	8	
$Z_{i}(S)$	1	0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

i	8	7	6			
Z _j (S)	0	0	0			
i	8+0- I	7+0- I	6+0- I			
sp _i '	0	3	0			

The Z-values are as follows, we only have a Z-box starting at l=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i$$
' := 0;
for j := n downto 2 do
 $i := j + Z_j(P) -1$;
 sp_i ' := Z_i

j	8	7	6	5		
Z _j (S)	0	0	0	3		
i	8+0- I	7+0- I	6+0- I	5+3- I		
sp _i '	0	3	0	0		

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	Ι	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

j	8	7	6	5	4		
Z _j (S)	0	0	0	3	0		
i	8+0- I	7+0- I	6+0- I	5+3- I	4+0- I		
sp _i '	0	3	0	0	0		

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

j	8	7	6	5	4	3	
Z _j (S)	0	0	0	3	0	0	
i	8+0- I	7+0- I	6+0- I	5+3- I	4+0- I	3+0- I	
sp _i '	0	3	0	0	0	0	

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$	-	0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

j	8	7	6	5	4	3	2	
Z _j (S)	0	0	0	3	0	0	0	
i	8+0- I	7+0- I	6+0- I	5+3- I	4+0- I	3+0- I	2+0- I	
sp _i '	0	3	0	0	0	0	0	

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i$$
' := 0;
for j := n downto 2 do
 $i := j + Z_j(P) -1$;
 sp_i ' := Z_i

j	8	7	6	5	4	3	2	-
Z _j (S)	0	0	0	3	0	0	0	0
i	8+0- I	7+0- I	6+0- I	5+3- I	4+0- I	3+0- I	2+0- I	
sp _i '	0	3	0	0	0	0	0	0

The Z-values are as follows, we only have a Z-box starting at I=5: $Z_5(S)=3$

S	Α	В	Х	Υ	Α	В	X	Z	
	I	2	3	4	5	6	7	8	
$Z_{i}(S)$		0	0	0	3	0	0	0	

for i := 1 to n do

$$sp_i' := 0;$$

for j := n downto 2 do
 $i := j + Z_j(P) -1;$
 $sp_i' := Z_i$

$$sp_7'=Z_8(P)=0; sp_6'=Z_7(P)=0;$$

 $sp_5'=Z_6(P)=0; sp_7'=Z_5(P)=3;$
 $sp_3'=Z_4(P)=0;...; sp_1'=Z_2(P)=0$

Failure function F'(k)=sp'_{k-1}+1 for k from 1 to n+1

k		I	2	3	4	5	6	7	8	9
sp _i '	0	0	0	0	0	2	2	0	0	
F'(k)		I	I	I	I	I	I	I	4	I

```
preprocess P to find F'(k)=sp'_{k-1}+1 for k from 1 to n+1
   c := 1;
   p := 1;
   while c + (n-p) \le m do
       while P(p) = T(c) and p \le n
          p := p+1;
          c := c+1;
       if p = n+1 then
          report an occurrence of P starting at position c-n of T
       if p = 1 then c := c+1
       p := F'(p)
```

```
preprocess P to find F'(k)=sp'_{k-1}+1 for k from 1 to n+1
   c := 1;
   p := 1;
   while c + (n-p) \le m do
                                                      MatchChar (MC)
       while P(p) = T(c) and p \le n
          p := p+1;
          c := c+1;
       if p = n+1 then
          report an occurrence of P starting at position c-n of T
       if p = 1 then c := c+1
       p := F'(p)
```

```
preprocess P to find F'(k)=sp'_{k-1}+1 for k from 1 to n+1
   c := 1;
   p := 1;
   while c + (n-p) \le m do
                                                      MatchChar (MC)
       while P(p) = T(c) and p \le n
          p := p+1;
          c := c+1;
                                                    MatchPattern (MP)
       if p = n+1 then
          report an occurrence of P starting at position c-n of T
       if p = 1 then c := c+1
       p := F'(p)
```

```
preprocess P to find F'(k)=sp'_{k-1}+1 for k from 1 to n+1
   c := 1;
   p := 1;
   while c + (n-p) \le m do
                                                       MatchChar (MC)
       while P(p) = T(c) and p \le n
          p := p+1;
          c := c+1;
                                                    MatchPattern (MP)
       if p = n+1 then
          report an occurrence of P starting at position c-n of T
       if p = 1 then c := c+1
                                                    MismatchStart (F0)
       p := F'(p)
```

```
preprocess P to find F'(k)=sp'<sub>k-1</sub> + 1 for k from 1 to n+1 c := 1; p := 1; while c + (n-p) \le m do
```

```
while P(p) = T(c) and p \le n
p := p+1;
c := c+1;
if p = n+1 then
report \text{ an occurrence of P starting at position c-n of T}
if p = 1 then c := c+1
p := F'(p)
MismatchInternal (Fi)
```

$$c=1, p=1:F0$$

c=9, p=8: Fi: F'(8)=4
$$\Rightarrow$$
 p=4

$$c=10, p=5: MC$$

$$c=12, p=7: MC$$

$$c=13, p=8: MC$$

$$c=14, p=9: MP$$



X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z

c=1, p=1: F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	Х	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25



c=1, p=1: F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	Х	Z
Ι	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	

c=14, p=9: MP



$$c=1, p=1:F0$$

c=9, p=8: Fi: F'(8)=4
$$\Rightarrow$$
 p=4

$$c=10, p=5: MC$$

$$c=12, p=7: MC$$

$$c=13, p=8: MC$$

$$c=14, p=9: MP$$

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
ı	 								<u> </u>			13		15		17					22			
A	В	Х	Υ	Α	В	Х	Z																	
	A	В	X	Υ	Α	В	X	Z																



c=1, p=1:F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC

																							-	
X	Α	В	Х	Υ	Α	В	Х	Υ	Α	В	Х	Z	Α	В	Х	Z	Α	В	X	Υ	Α	В	Х	Z
I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	
	A	В	Х	Υ	Α	В	X	Z																
	Α	В	Х	Υ	Α	В	Х	Z																
							L	<u> </u>	L				L			L				L				

c=14, p=9: MP



X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	
	A	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	X	Z																
	Α	В	X	Υ	Α	В	X	Z																



c=1, p=1: F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	Х	Υ	Α	В	Х	Z																	
	A	В	Х	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	X	Z																
	Α	В	X	Υ	Α	В	X	Z																
	Α	В	X	Y	Α	В	X	Z																



X	Α	В	Х	Υ	Α	В	Х	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	
	A	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	X	Z																
	Α	В	X	Y	Α	В	Х	Z																
	Α	В	Х	Υ	Α	В	Х	Z																

c=13, p=8: MC

c=14, p=9: MP



X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	
	A	В	Х	Υ	Α	В	Х	Z																
	Α	В	Х	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Y	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																



c=1, p=1: F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	
	A	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
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	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	X	Z																



X	Α	В	X	Υ	Α	В	Х	Υ	Α	В	X	Z	Α	В	Х	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	Х	Z																	
	A	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Y	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	Х	Z																
	Α	В	X	Υ	Α	В	X	Z																
	Α	В	X	Υ	Α	В	Х	Z																



c=1, p=1: F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4
$$\Rightarrow$$
 p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	Х	Υ	Α	В	X	Z																	
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					Α	В	Х	Y	Α	В	Х	Z												
			-				-																	$\overline{}$

c=12, p=7: MC

c=13, p=8: MC

c=14, p=9: MP



c=1, p=1:F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
ı	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
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X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
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X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
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X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
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					Α	В	X	Υ	Α	В	X	Z												

c=13, p=8: MC

c=14, p=9: MP



c=1, p=1:F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	X	Z																	
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					Α	В	X	Υ	Α	В	X	Z												
					Α	В	X	Υ	Α	В	Х	Z												
					A	В	X	Y	A	В	X	Z												



c=1, p=1:F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	X	Z																	
	A	В	X	Y	Α	В	X	Z																
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	Α	В	X	Υ	Α	В	X	Z																
	Α	В	X	Y	Α	В	X	Z																
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	Α	В	X	Υ	Α	В	X	Z																
					Α	В	X	Y	Α	В	X	Z												
					Α	В	X	Υ	A	В	Х	Z												
					Α	В	X	Υ	Α	В	X	Z												
					Α	В	X	Υ	Α	В	X	Z												
					Α	В	X	Υ	Α	В	Х	Z												
					A	В	X	Y	A	В	X	Z												



c=1, p=1:F0
c=2, p=1: MC
c=3, p=2: MC
c=4, p=3: MC
c=5, p=4: MC
c=6, p=5: MC
c=7, p=6: MC
c=8, p=7: MC
c=9, p=8: Fi: F'(8)=4 \Rightarrow p=4
c=9, p=4: MC
c=10, p=5: MC
c=11, p=6: MC
c=12, p=7: MC
c=13, p=8: MC
c=14, p=9: MP

X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z
I	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	В	X	Υ	Α	В	X	Z																	
	A	В	X	Y	Α	В	X	Z																
	Α	В	X	Υ	Α	В	X	Z																
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					A	В	X	Y	A	В	X	Z												
					Α	В	Х	Υ	Α	В	Х	Z												
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I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	16	17	18	19	20	21	22	23	24	25
X	Α	В	X	Υ	Α	В	X	Υ	Α	В	X	Z	Α	В	X	Z	Α	В	X	Υ	Α	В	X	Z

```
public int match (String pattern, String text) {
  int p = 0;
  int s = 0;
  int t = 0;
  int matches = 0; // number of matches, return value
  while (t < text.length()) {
     if (p < pattern.length()) System.out.println("pattern("+p+") \t"+pattern.charAt(p));
     System.out.println("text("+t+") \t"+text.charAt(t));
     if (pattern.charAt(p) == text.charAt(t)) {
       // make sure to check against length-1 else OutOfRange exception!
       if (p < pattern.length()-1) {
          p = p+1;
          t = t+1;
       } else {
          System.out.println("Match found at position "+(s+1));
          p = 0;
          s = s+1;
          t = s;
          matches = matches+1;
       } // end if..else check for full occurrence of P
    } else {
       p = 1;
       s = s+1;
       t = s;
    } // end if..else check for character match
  } // end while over the text
  return matches;
} // end match
```

```
public int match (String pattern, String text) {
  // -----
  // Initialization
  int p = 0;
  int s = 0;
  int t = 0;
  int matches = 0; // number of matches, return value
  Vector patternVec = new Vector();
  Vector textVec = new Vector ();
  // Represent the pattern as a sequence of words
  StringTokenizer pst = new StringTokenizer(pattern);
  while (pst.hasMoreTokens()) {
    String word = (String)pst.nextToken();
    patternVec.addElement(word);
  } // end while
  // Represent the text as a sequence of words
  StringTokenizer tst = new StringTokenizer(text);
  while (tst.hasMoreTokens()) {
    String word = (String)tst.nextToken();
    textVec.addElement(word);
  } // end
```

```
// Loop
  // Note that the conditions now refer to the vectors, not to the original strings.
  while (t < textVec.size()) {
     if (p < patternVec.size()) System.out.println("pattern("+p+") \t<"+(String)patternVec.elementAt(p)+">");
     System.out.println("text("+t+") \t<"+(String)textVec.elementAt(t)+">");
     if (((String)patternVec.elementAt(p)).equals((String)textVec.elementAt(t))) {
       // make sure to check against length-1 else OutOfRange exception!
       if (p < patternVec.size()-1) {
          p = p+1;
          t = t+1;
       } else {
          System.out.println("Match found at position "+(s+1));
          p = 0;
          s = s+1;
          t = s:
          matches = matches+1;
       } // end if..else check for full occurrence of P
     } else {
       p = 0;
       s = s+1;
       t = s:
     } // end if..else check for character match
  } // end while over the text
  return matches:
} // end match
```

KeywordTreeNode.java

- records:
 - the label on the edge to the vertex *v*
 - the path, i.e. the concatenation of the words on the path to v: L(v)
 - the parent of the vertex
 - the children of a vertex i
- basic accessor methods for adding, getting and setting

```
public KeywordTreeNode buildBranch (Vector pvec) {
  KeywordTreeNode broot = new KeywordTreeNode ();
  KeywordTreeNode parent = broot;
  boolean rootSet = false;
  String path = "";
  Iterator pvlter = pvec.iterator();
  while (pvlter.hasNext()) {
     String word = (String)pvlter.next();
     // check whether the root has been set; if not, initialize
     // the root, otherwise create a new node, and add it to
     // the current parent.
     if (!rootSet) {
       broot.setEdge(word);
       rootSet = true;
       broot.setPath(word);
       path = word;
     } else {
       KeywordTreeNode node = new KeywordTreeNode(word);
       path = path+" "+word;
       node.setPath(path);
       node.setParent(parent);
       parent.addChild(node);
       // Set the parent to be the current node
       parent = node;
     } // end if..else check for root or child
  } // end while
  return broot;
} // end
```

```
KeywordTreeNode branch = this.buildBranch(patternVec);
  // Next, go down the tree as far as possible to find the lowest attachment point for this branch. From the root
  // of the branch we go down tree, until we get to a point where none of the children on the branch would be
  // matched; that is where we insert the (remainder of the) branch.
  KeywordTreeNode branchnode = branch; // the current branch node
  KeywordTreeNode attachment = treeroot; // the node where to attach
  boolean golower = (treeroot.isLeaf())?false:true;
  while (golower) {
    // cycle over the children of the current attachment node
    boolean matchfound = false:
     Iterator chlter = attachment.getChildren();
    while (chlter.hasNext() && !matchfound) {
       KeywordTreeNode child = (KeywordTreeNode) chlter.next();
       // if this child has the same edge, and the same
       // path, as the current node in the branch, then
       // decend one node down the branch and set the
       // current node as the attachment point for the
       // remainder of the branch.
       if (child.getEdge().equals(branchnode.getEdge()) && child.getPath().equals(branchnode.getPath())) {
          matchfound = true;
          attachment = child:
          branchnode = branchnode.getFirstChild();
       } // end if.. check whether match found
    } // end while over children
    // continue if we found a matching child, and descended accordingly
    golower = matchfound;
  } // end while
  attachment.addChild(branchnode);
} // end for
```