Algorithms for matching

Introduction to Computational Linguistics

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Today’s lecture

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- Objective:
  - Efficient algorithms for finding matches of *patterns* (strings) in *texts*.
  - The focus is on *exact* matching
  - We deal with chars/Strings, but this generalizes to words/Strings

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- Objective:
  - Efficient algorithms for finding matches of *patterns* (strings) in *texts*.
  - The focus is on *exact* matching
  - We deal with chars/Strings, but this generalizes to words/Strings
- Why efficient methods for pattern matching?
  - Applications of pattern matching in *search* (web search for IR, IE, Q/A), *tagging* (named entity recognition), *shallow processing* (parsing)
  - Efficiency pays off when dealing with large amounts of data!
  - Furthermore: preliminaries for finite-state automata, dynamic programming/memoization techniques in parsing

What will you learn?

Dan Gusfield. *Algorithms on Strings, Trees and Sequences*. CUP, 1997:
Chapters 1, 2 ,3[pp52-65]; Chapter 11

What will you learn?

1. The naive method for exact string matching
    - Method for finding matches of a pattern P in a text T using $O(|P| \cdot |T|)$ comparisons

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   - Smart shifts in sublinear $O(|P|+|T|)$ time (B-M) or linear time (K-M-P)

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   - *Keyword trees* to represent sets of patterns; their use in a generalization of Knuth-Morris-Pratt

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- This week points 1-3 up to Booyer-Moore, next week KMP and trees.

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Preliminaries

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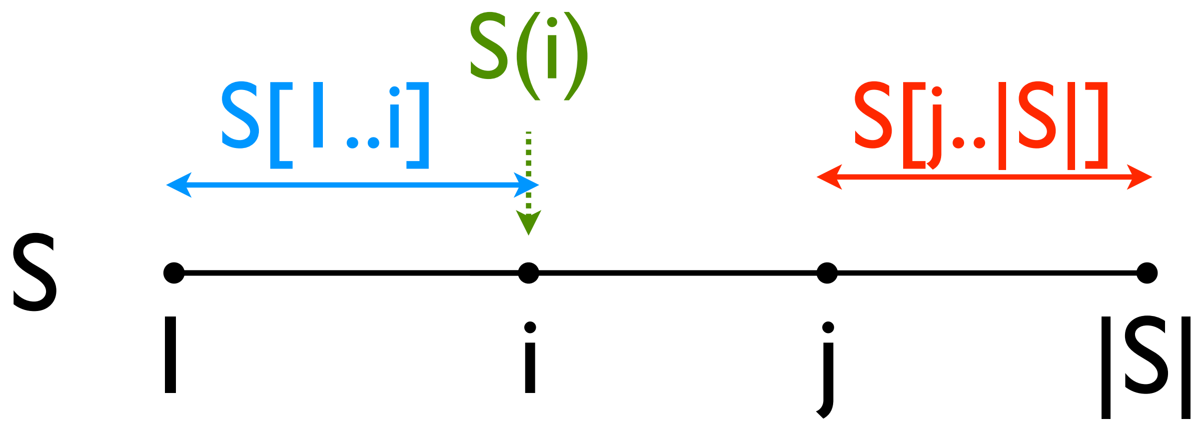

The naive method for matching

- Given
  - a **pattern** P, and a text **T** in which we are looking for matches of **P**
  - **Pointers:** *p* to position in P; *t* to position in T; *s* to start of matching P in T
- Algorithm

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T: set position in P, p=1; set position in T, t=1
    2. Set the current left-alignment position in T to s=1
[Loop]
    2. Compare the character at P(p) with the character at T(t)
    3. If P(p) == T(t),
        4. If p < |P| then set p=p+1 and set t=t+1; else report match, and set p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
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The naive method for matching

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The naive method for matching

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

```
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    1. Align the left of P with the left of T:
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The naive method for matching

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

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[Start: p=1, t=1,s=1]
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	✗												

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	✗												
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The naive method for matching

⇩ (at position 2)

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✘												
		A	B	X	Y	A	B	X	Z				

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
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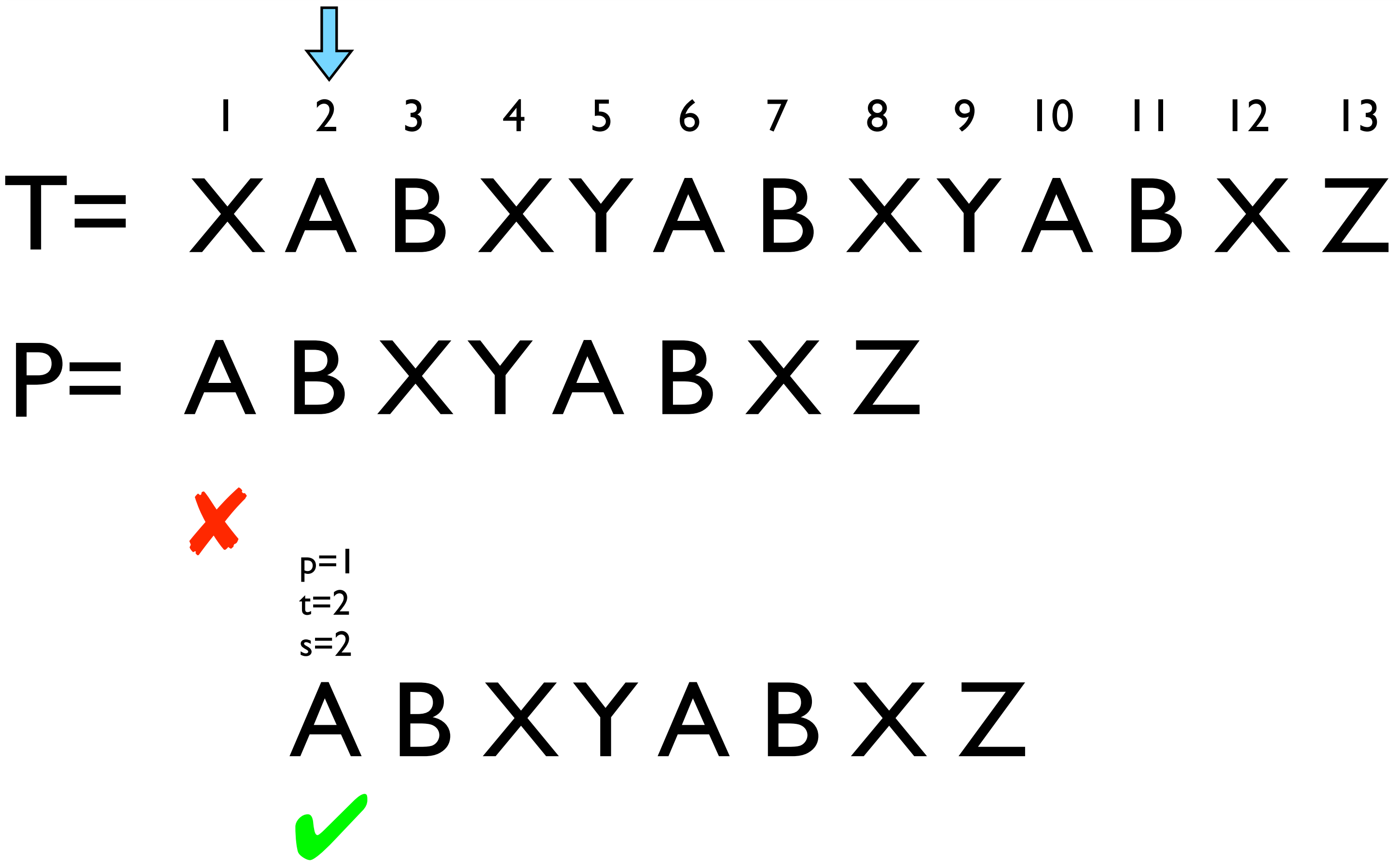

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P=	A	B	X	Y	A	B	X	Z					
	✗	p=1 t=2 s=2											
		A	B	X	Y	A	B	X	Z				
		✓											

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	✗												
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		✓											

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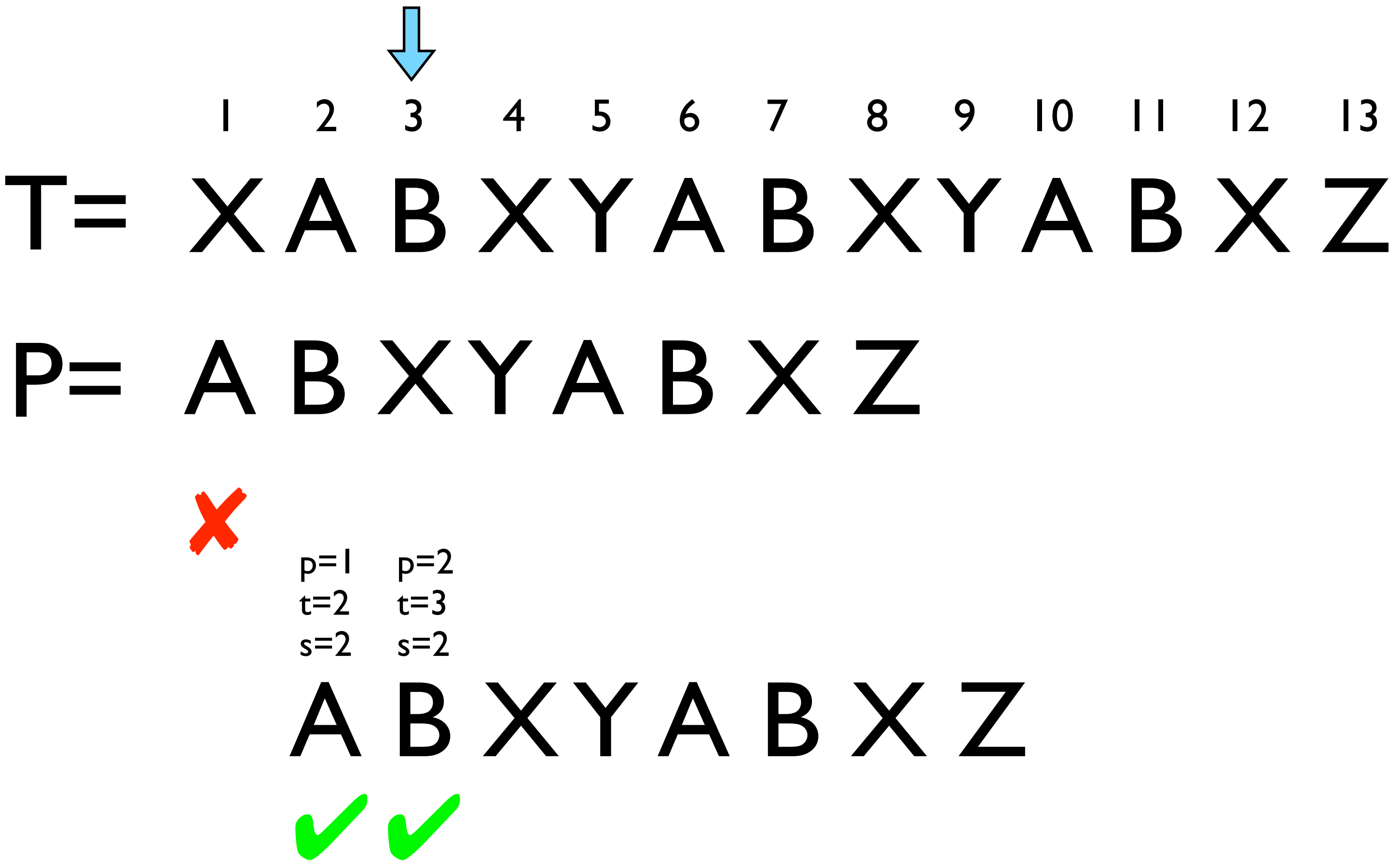

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	✗												
		p=1 t=2 s=2	p=2 t=3 s=2										
		A	B	X	Y	A	B	X	Z				
		✓	✓										

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The naive method for matching

↓ (at 4)

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2 | p=2 t=3 s=2

A B X Y A B X Z

✓ ✓

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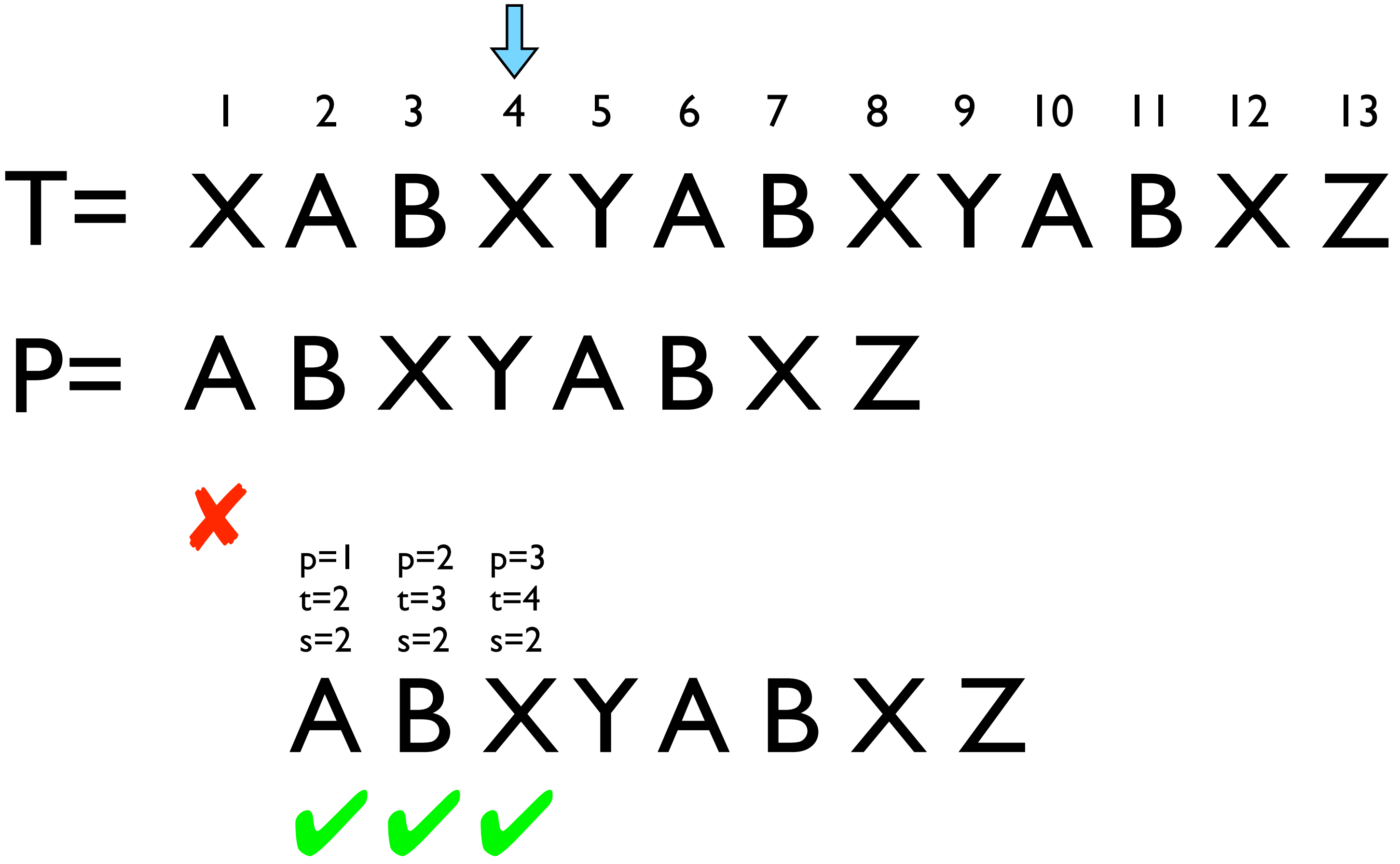

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2   p=2 t=3 s=2   p=3 t=4 s=2

A B X Y A B X Z

✓✓✓

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[Start: p=1, t=1,s=1]
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P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2									
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓									

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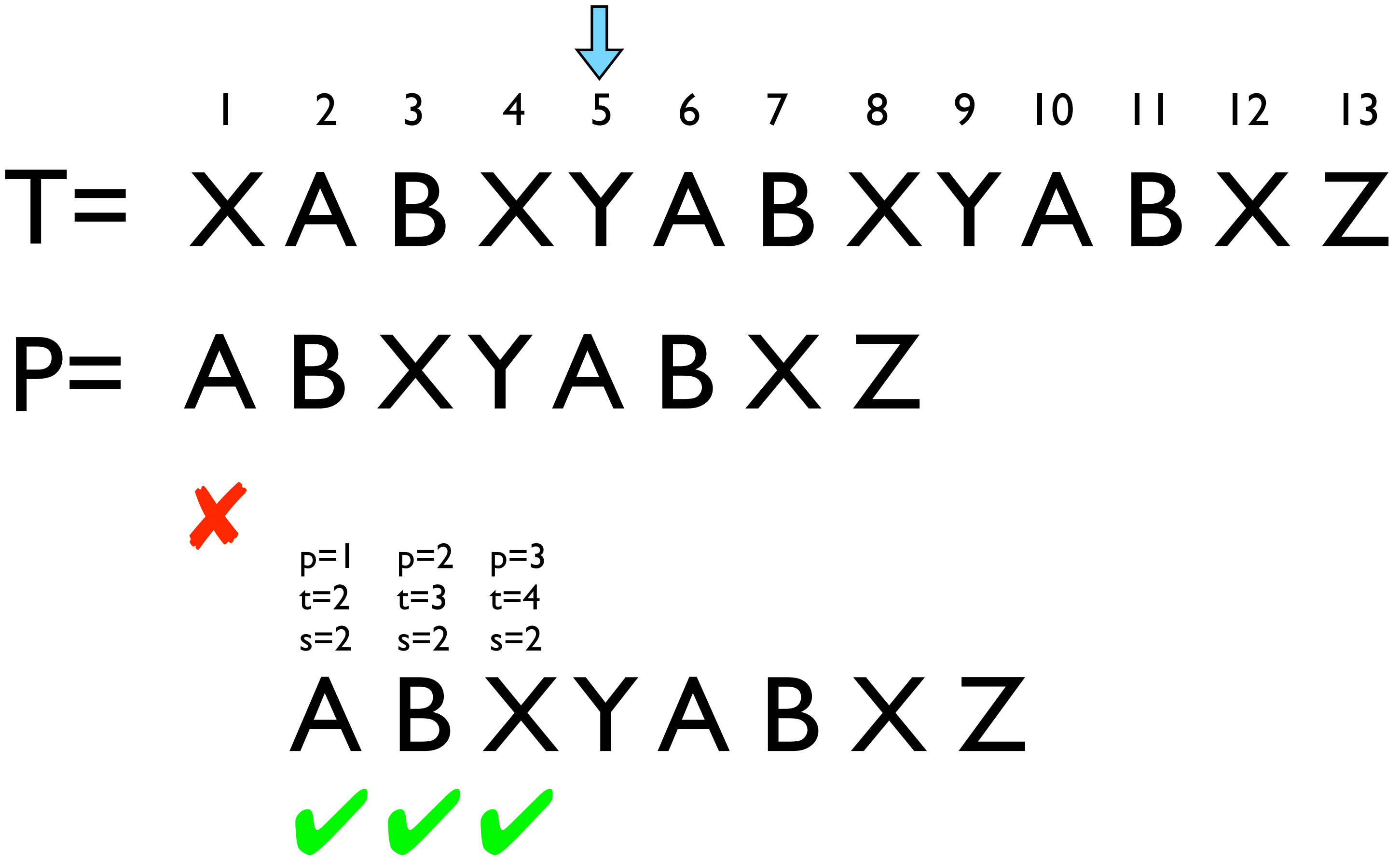

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2 | p=2 t=3 s=2 | p=3 t=4 s=2

A B X Y A B X Z

✓ ✓ ✓

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[Start: p=1, t=1,s=1]
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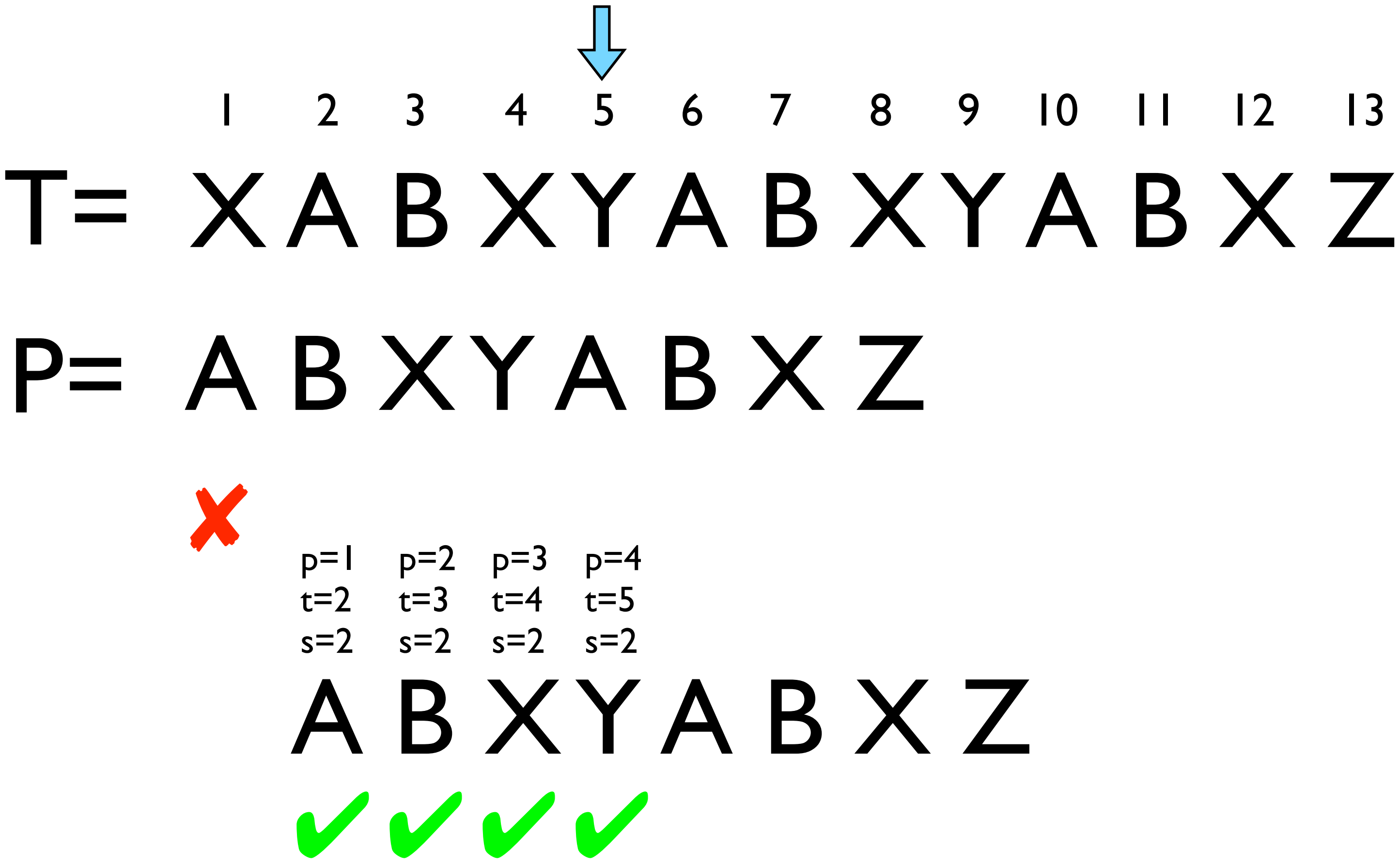

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P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2								
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓								

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P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2								
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓								

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[Start: p=1, t=1,s=1]
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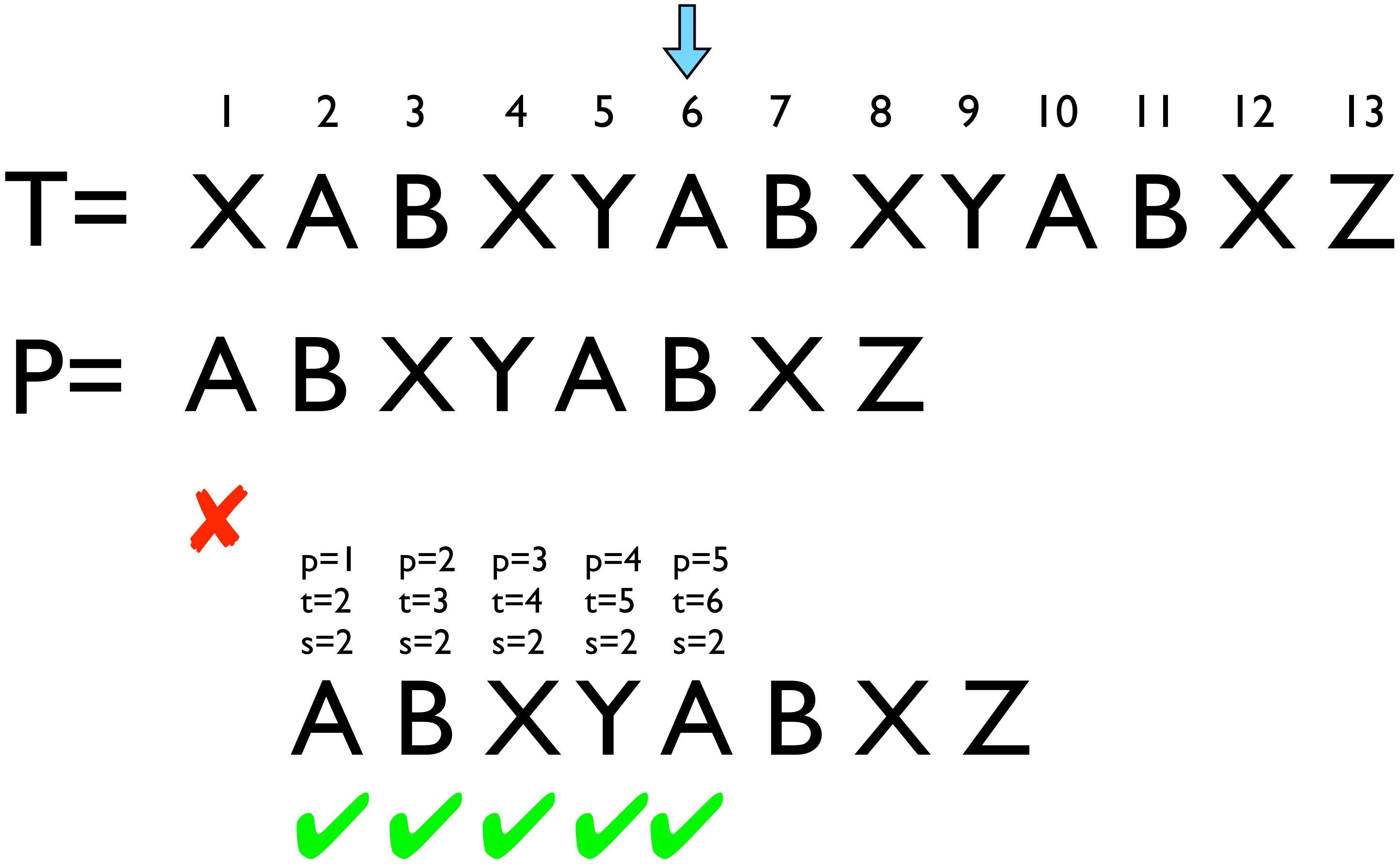
The naive method for matching



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T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2			
A	B	X	Y	A	B	X	Z
✓	✓	✓	✓	✓			

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        2. Compare P(p) with T(t)
        3. If P(p) == T(t),
                4. If p < |P|
                        then p=p+1 and t=t+1;
                        else report match, and p=1, s=s+1, t=s;
                5. Else p=1 and s=s+1, t=s
```
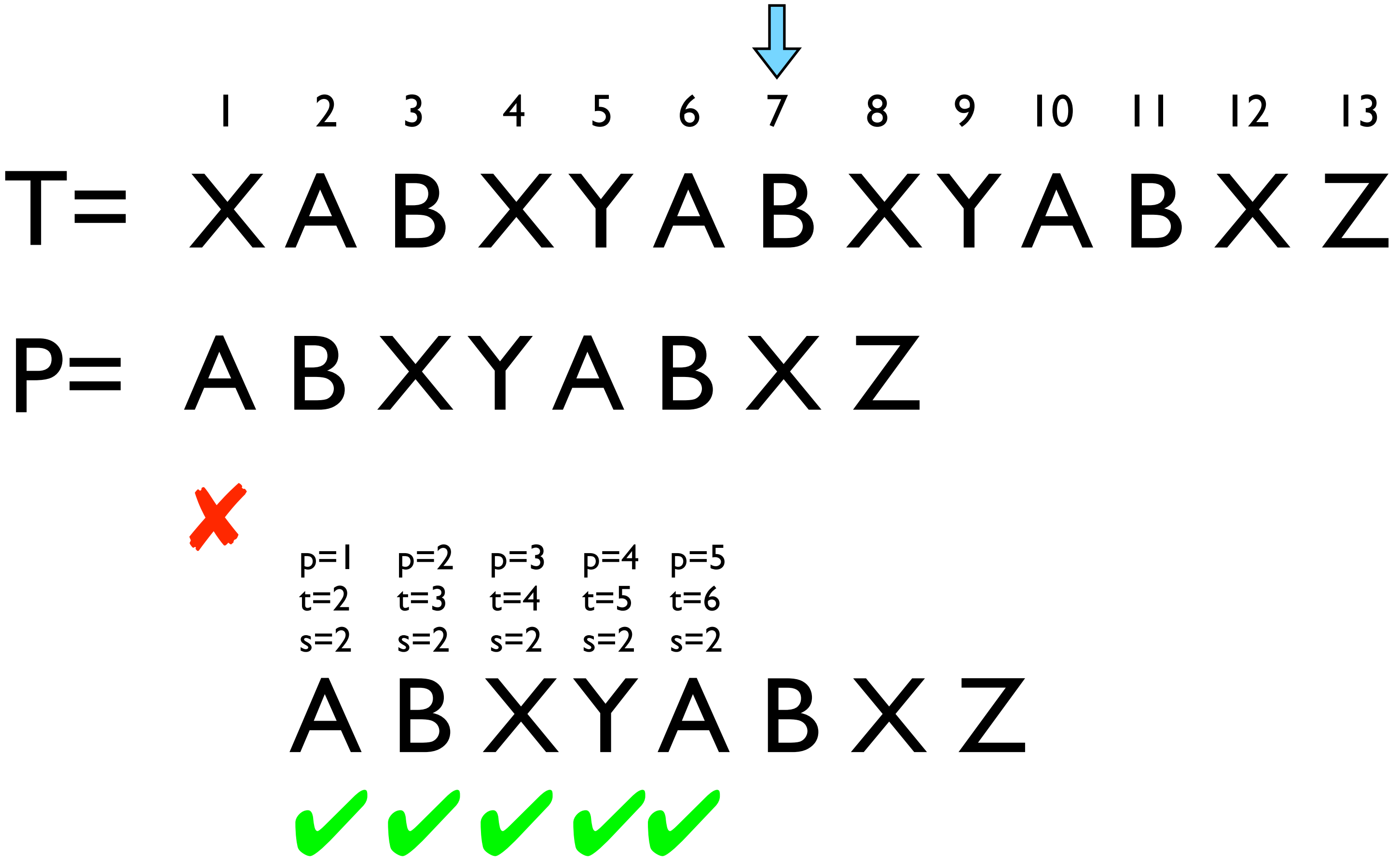
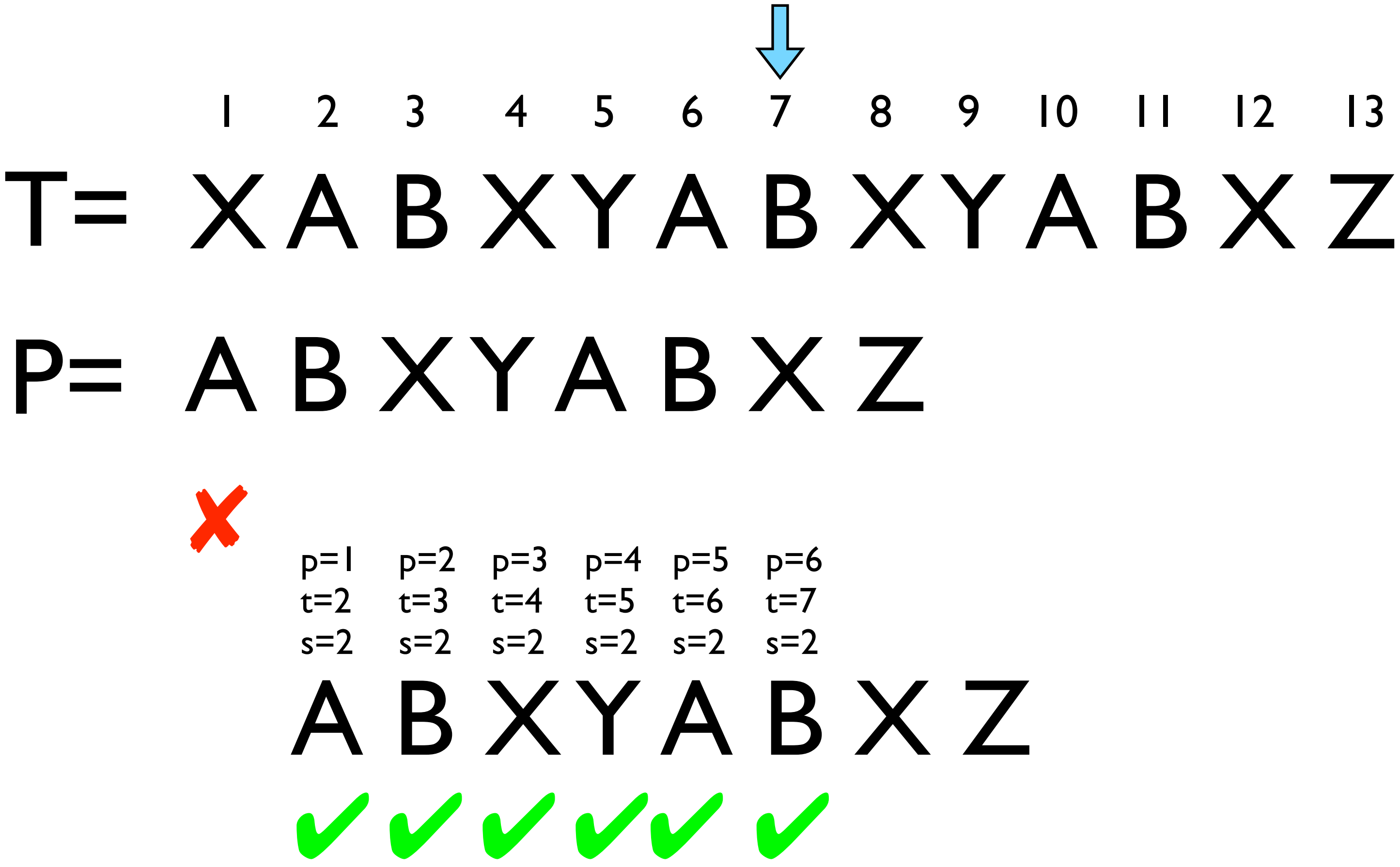

The naive method for matching



```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6
t=2	t=3	t=4	t=5	t=6	t=7
s=2	s=2	s=2	s=2	s=2	s=2
A	B	X	Y	A	B
✓	✓	✓	✓	✓	✓

A B X Y A B X Z

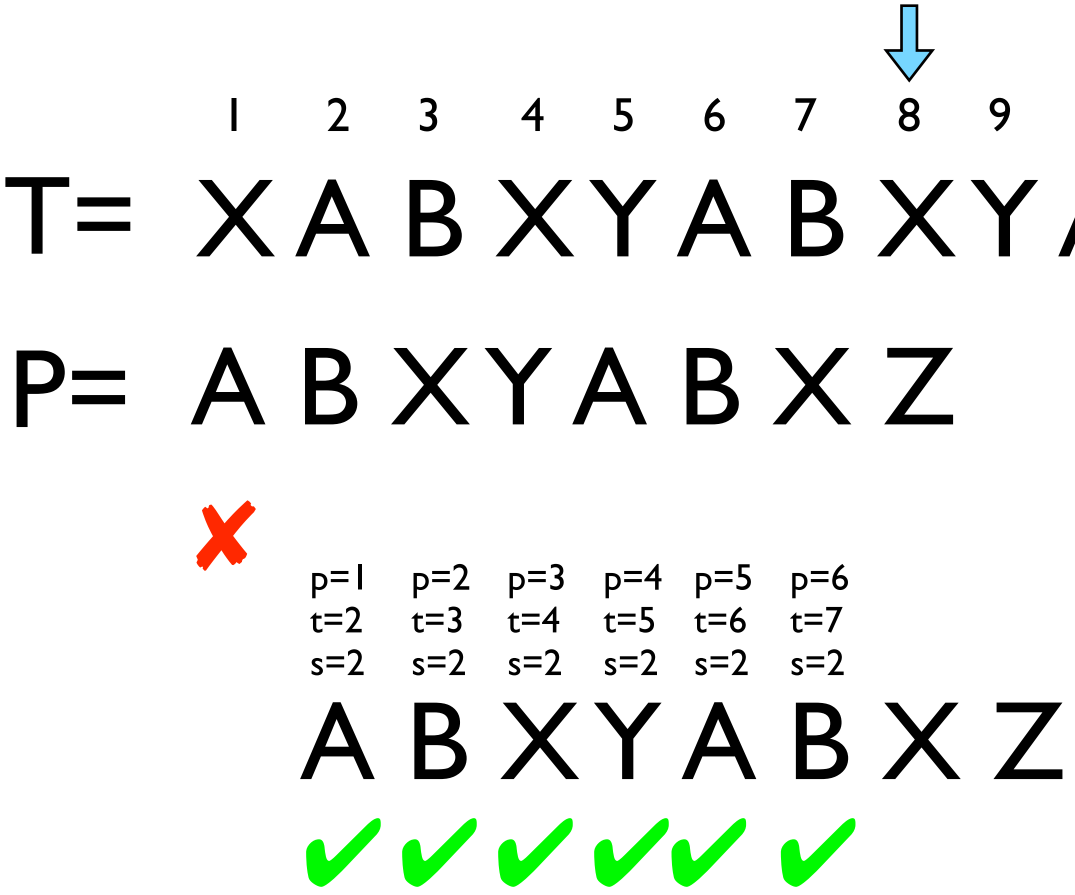
```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2						
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓						

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

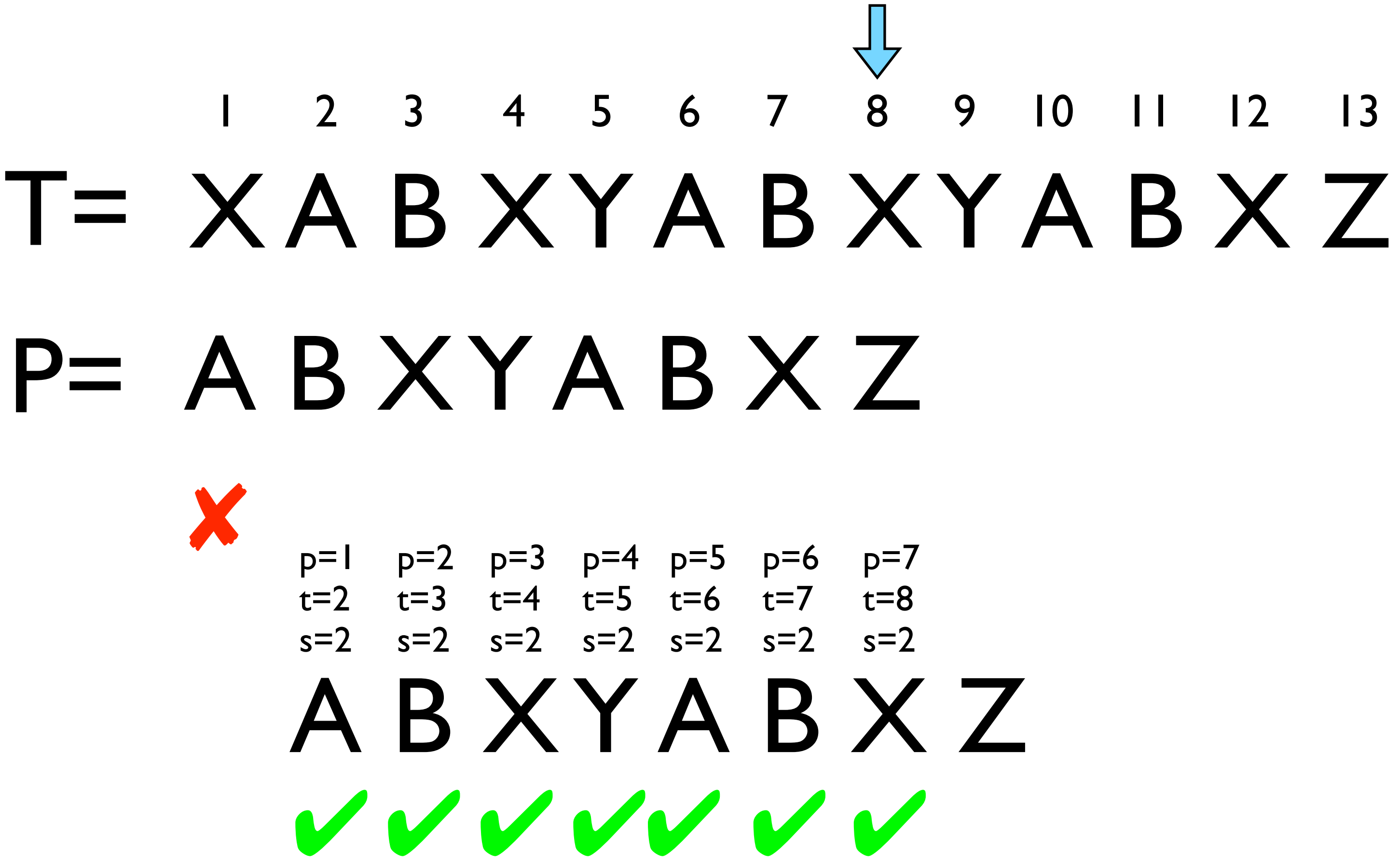
The naive method for matching



p=1	p=2	p=3	p=4	p=5	p=6
t=2	t=3	t=4	t=5	t=6	t=7
s=2	s=2	s=2	s=2	s=2	s=2

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



								⇩					
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2					
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓					

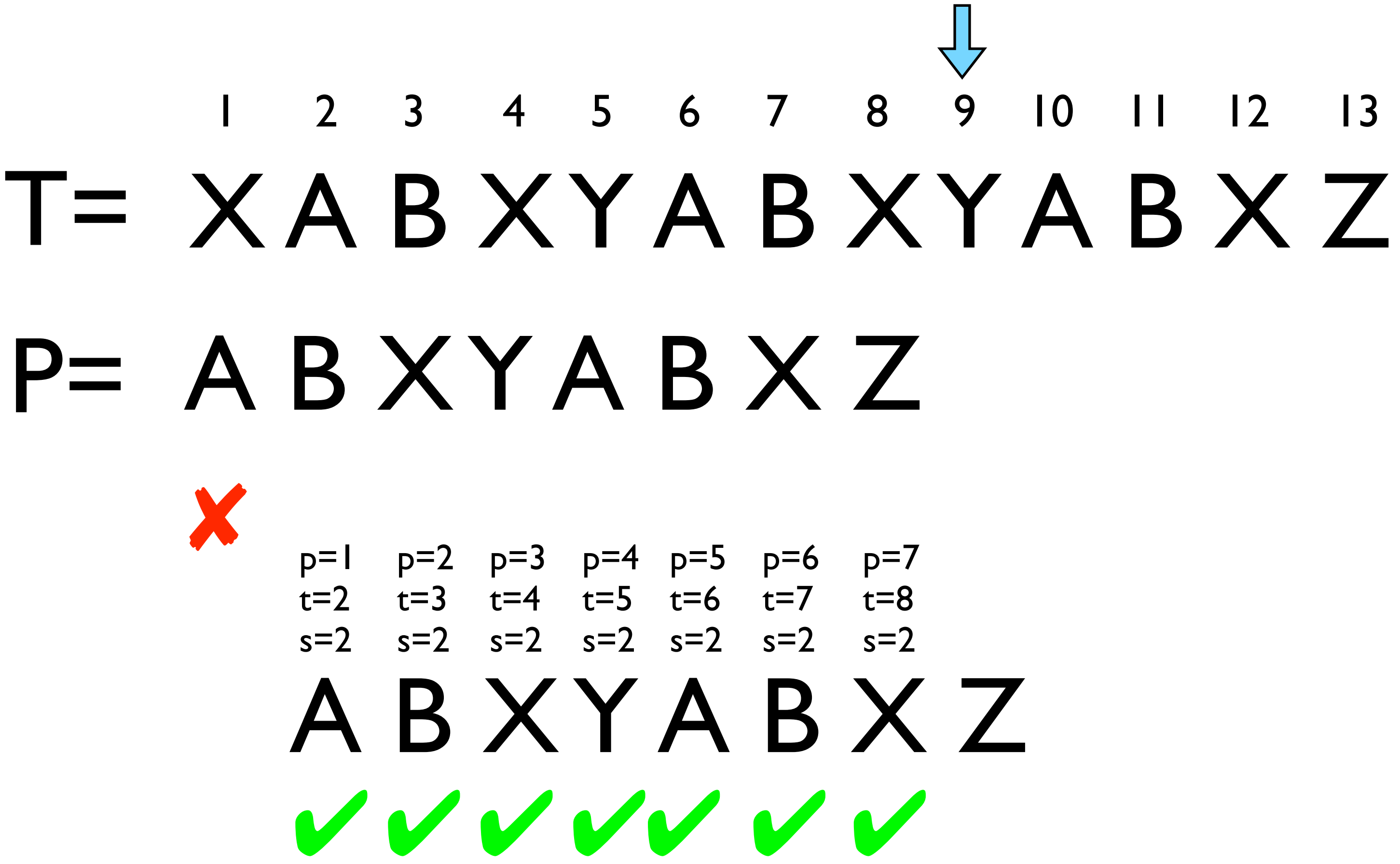
```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2					
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓					

```
[Start: p=1, t=1,s=1]
        1. Align the left of P with the left of T:
        2. p=1; t=1; s=1
[Loop]
        2. Compare P(p) with T(t)
        3. If P(p) == T(t),
                4. If p < |P|
                        then p=p+1 and t=t+1;
                        else report match, and p=1, s=s+1, t=s;
                5. Else p=1 and s=s+1, t=s
```

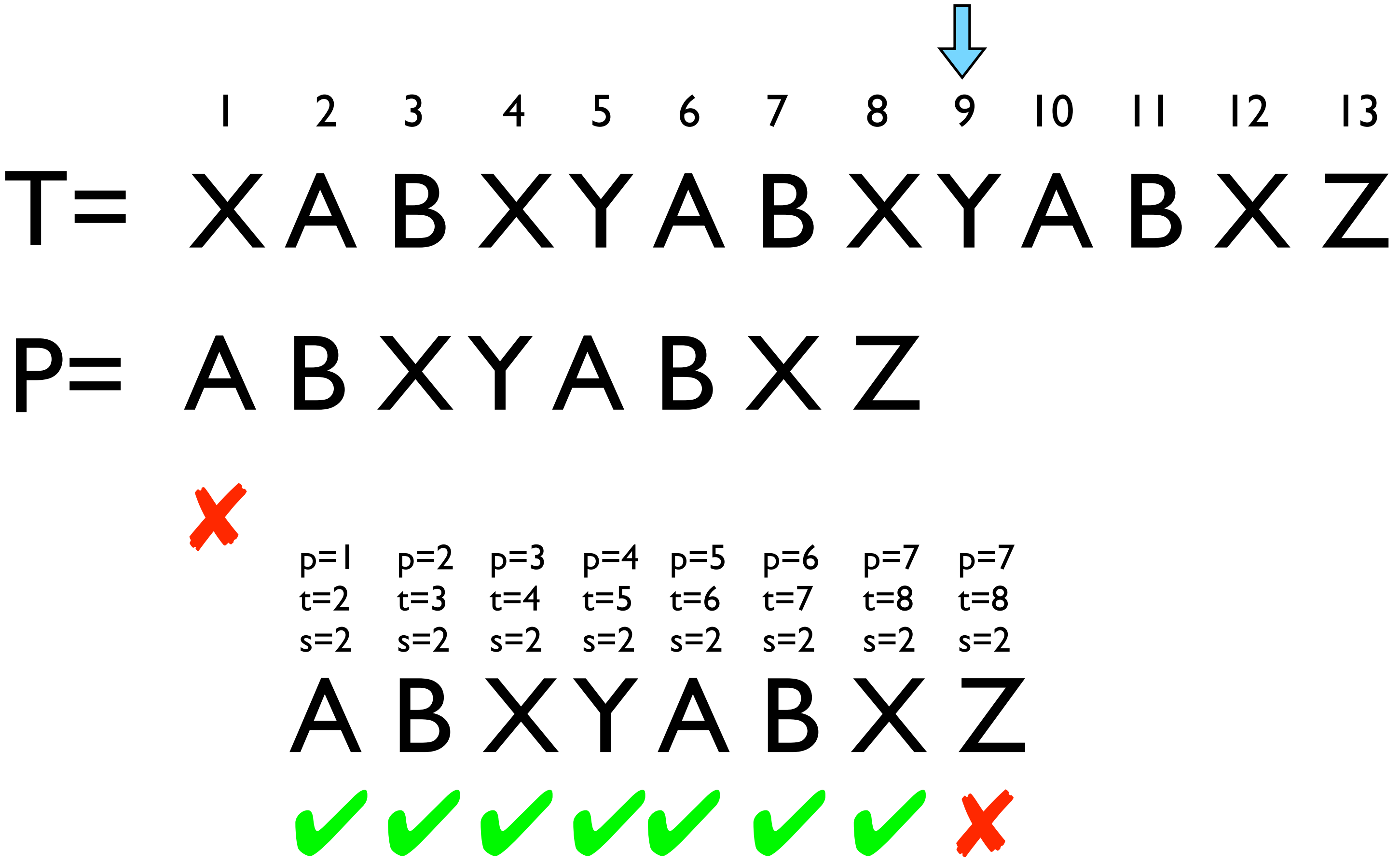
The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2					
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓					

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2
A	B	X	Y	A	B	X	Z
✓	✓	✓	✓	✓	✓	✓	✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching

									⇩				
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

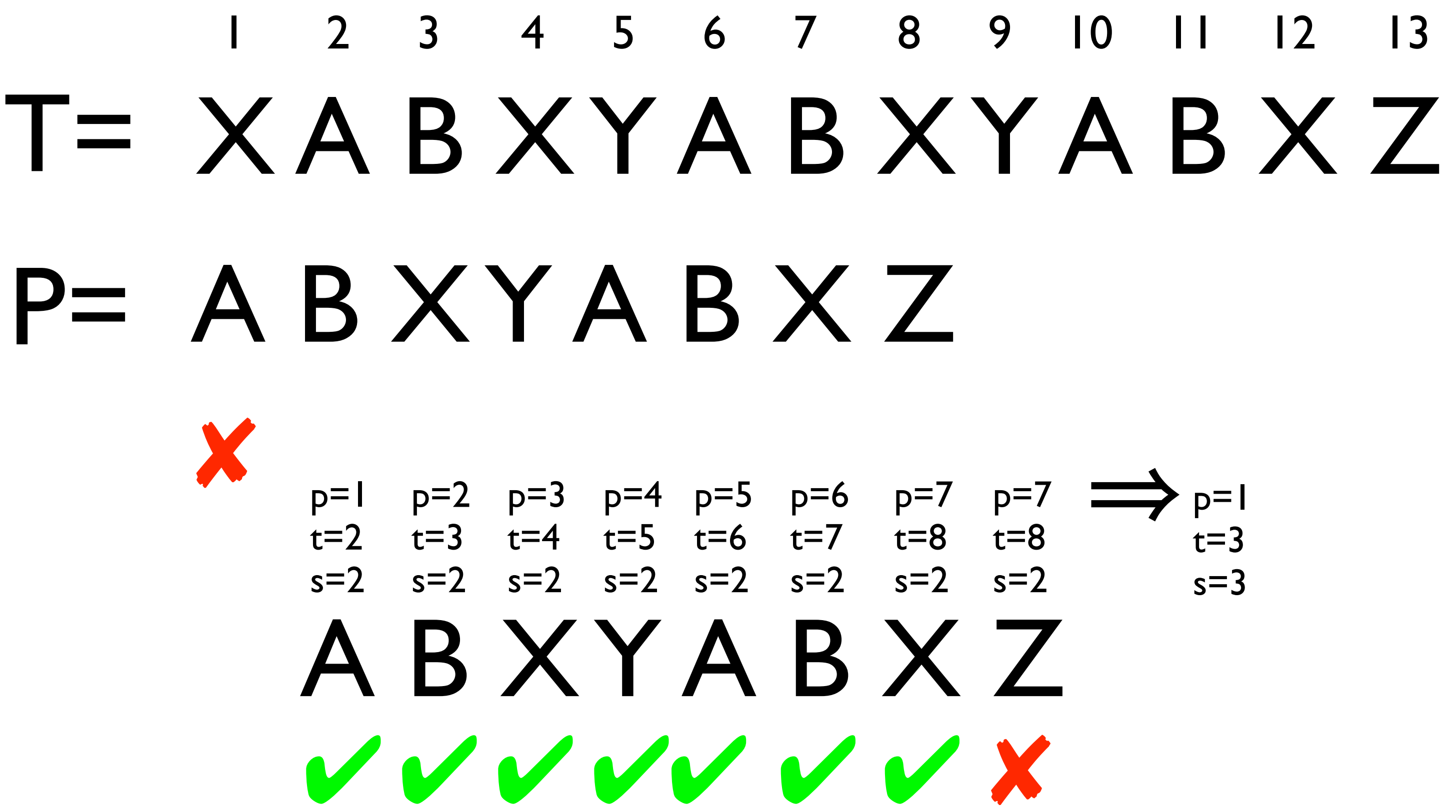
P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3
A	B	X	Y	A	B	X	Z		
✓	✓	✓	✓	✓	✓	✓	✗		

```
[Start: p=1, t=1,s=1]
        1. Align the left of P with the left of T:
        2. p=1; t=1; s=1
[Loop]
        2. Compare P(p) with T(t)
        3. If P(p) == T(t),
                4. If p < |P|
                        then p=p+1 and t=t+1;
                        else report match, and p=1, s=s+1, t=s;
                5. Else p=1 and s=s+1, t=s
```

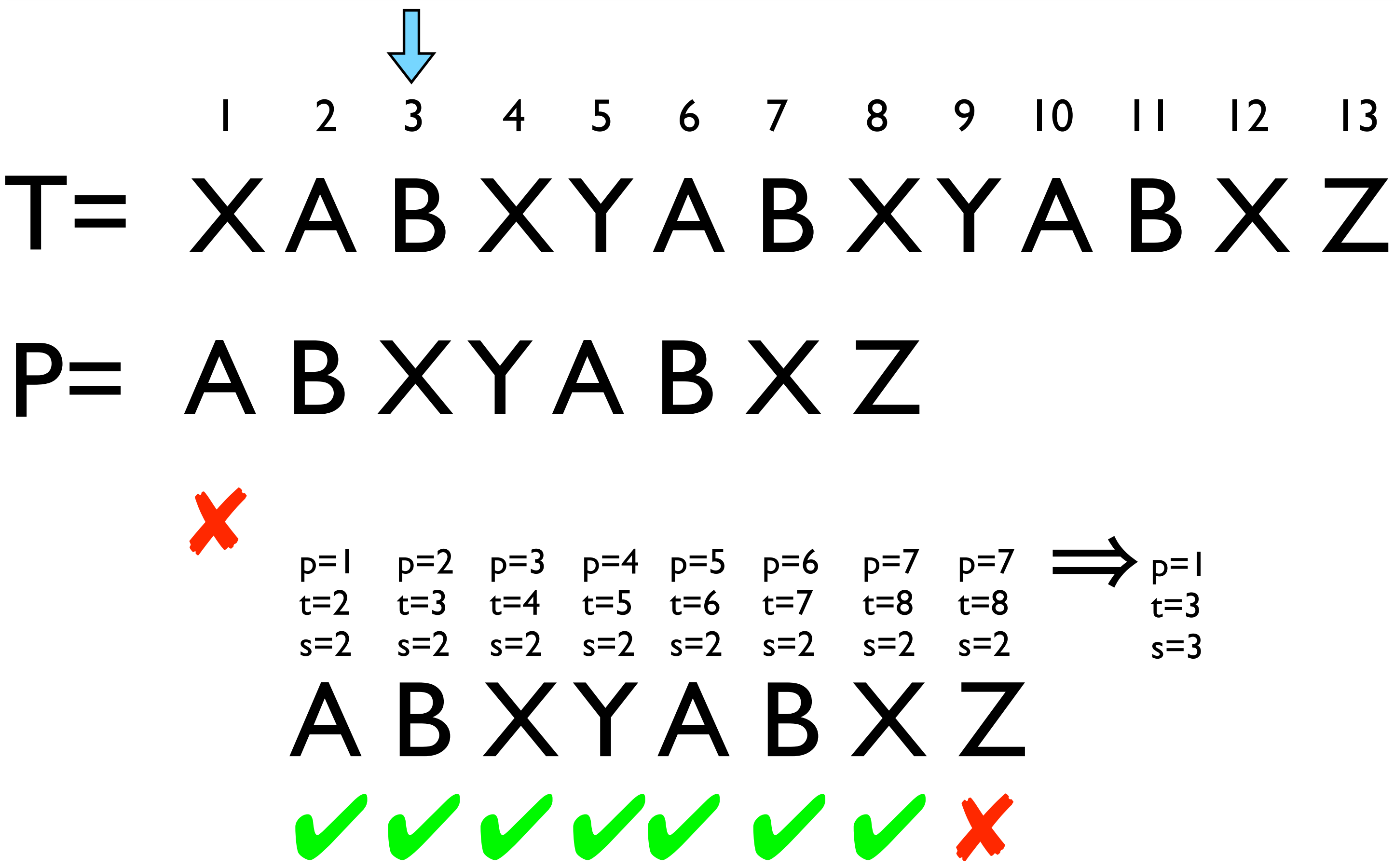
The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13	
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z	
P=	A	B	X	Y	A	B	X	Z						
	✗													
		p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2					⇒ p=1 t=3 s=3
		A	B	X	Y	A	B	X	Z					
		✓	✓	✓	✓	✓	✓	✓	✗					

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

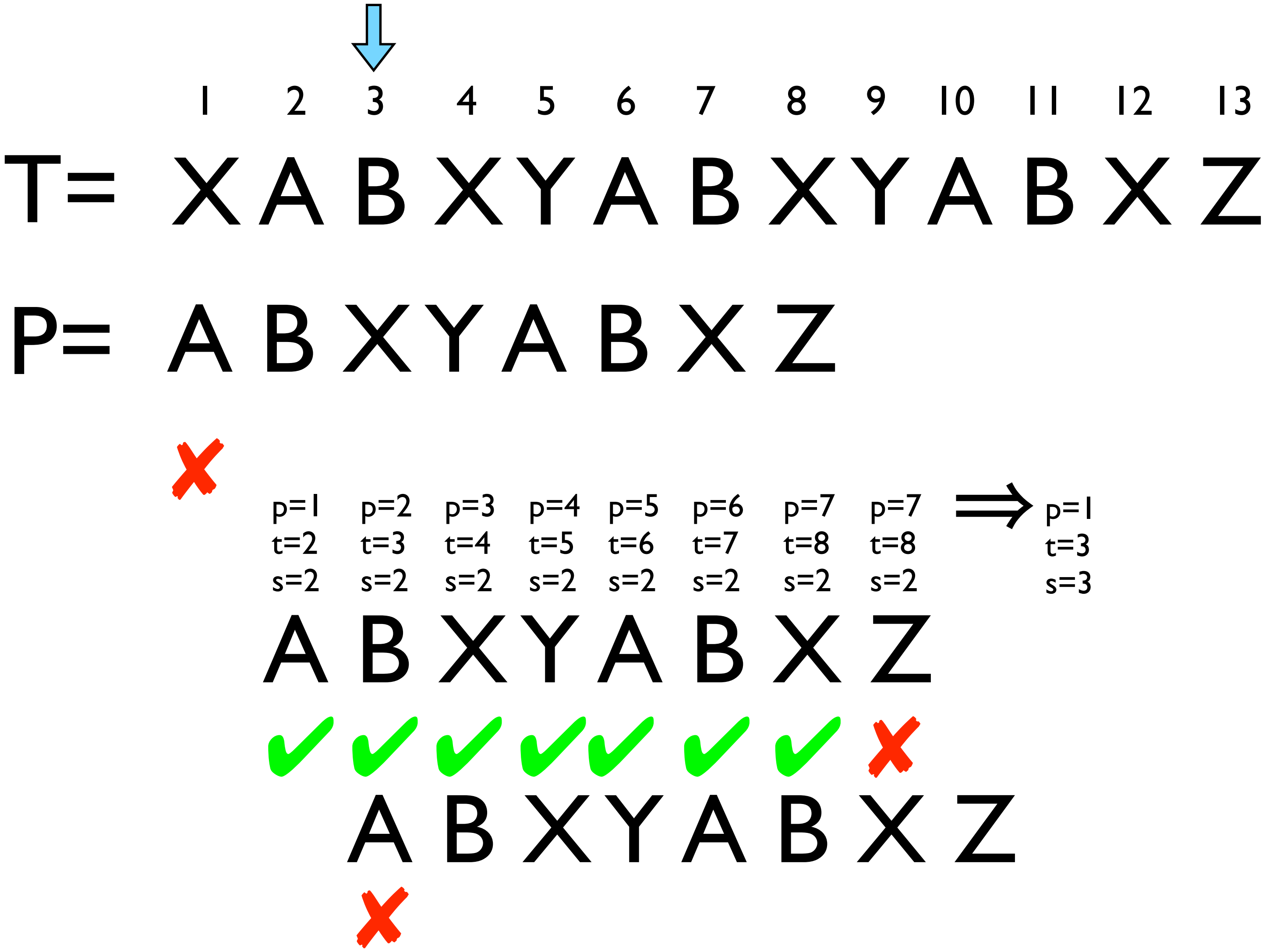
P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3
A	B	X	Y	A	B	X	Z		
✓	✓	✓	✓	✓	✓	✓	✗		

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

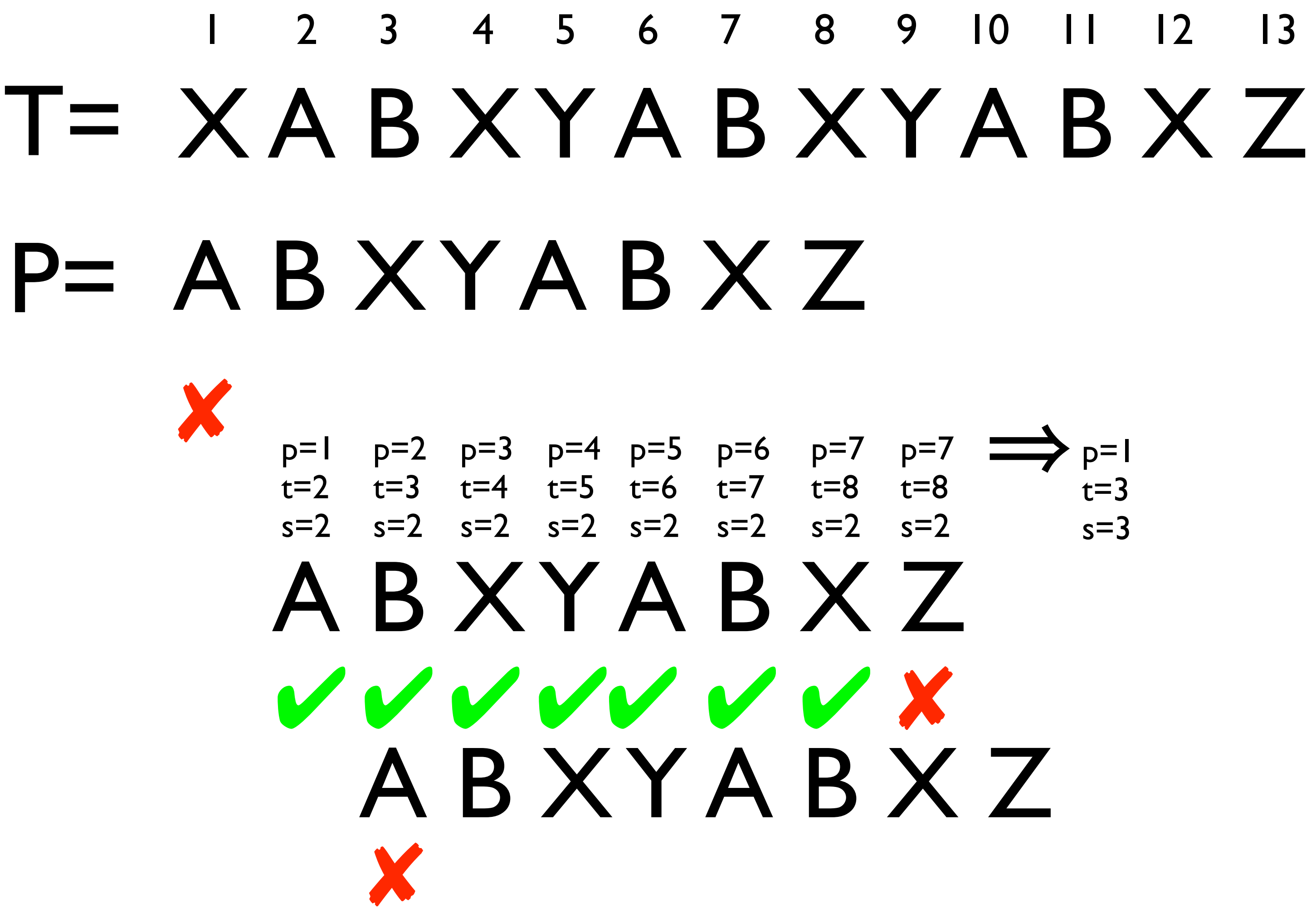
✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3

A B X Y A B X Z

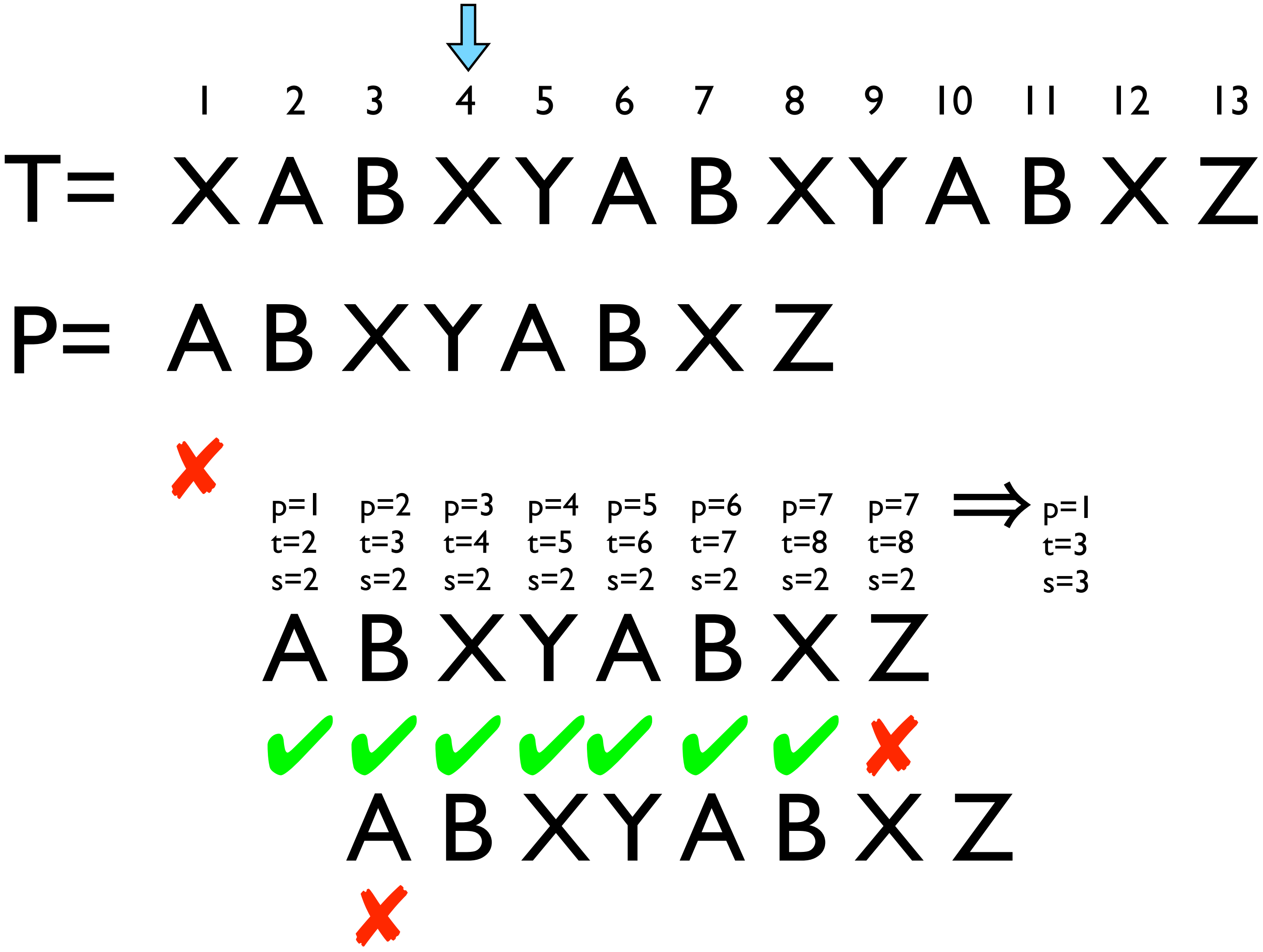
✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
        1. Align the left of P with the left of T:
        2. p=1; t=1; s=1
[Loop]
        2. Compare P(p) with T(t)
        3. If P(p) == T(t),
                4. If p < |P|
                        then p=p+1 and t=t+1;
                        else report match, and p=1, s=s+1, t=s;
                5. Else p=1 and s=s+1, t=s
```

The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

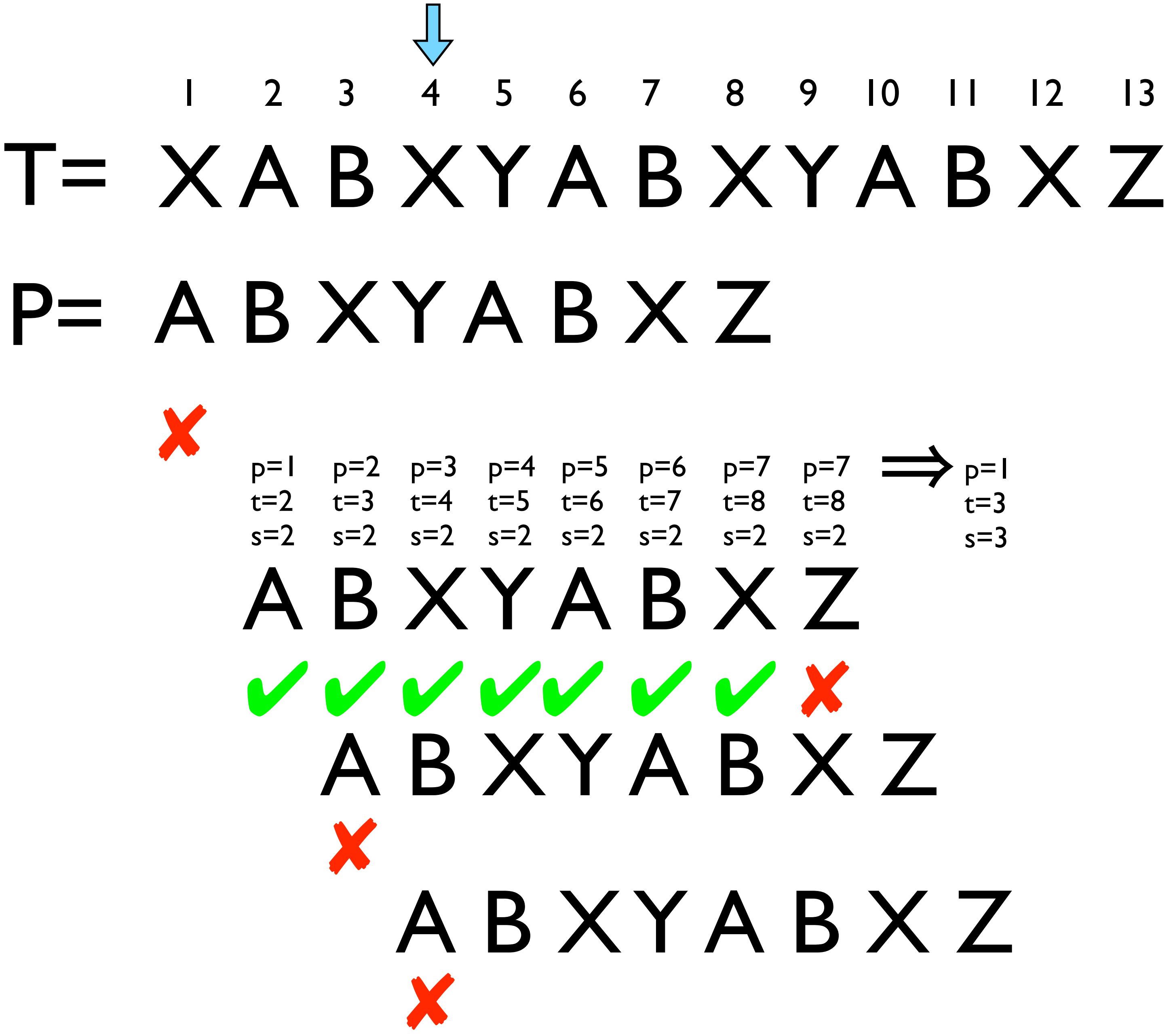
p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3
A	B	X	Y	A	B	X	Z		
✓	✓	✓	✓	✓	✓	✓	✗		

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

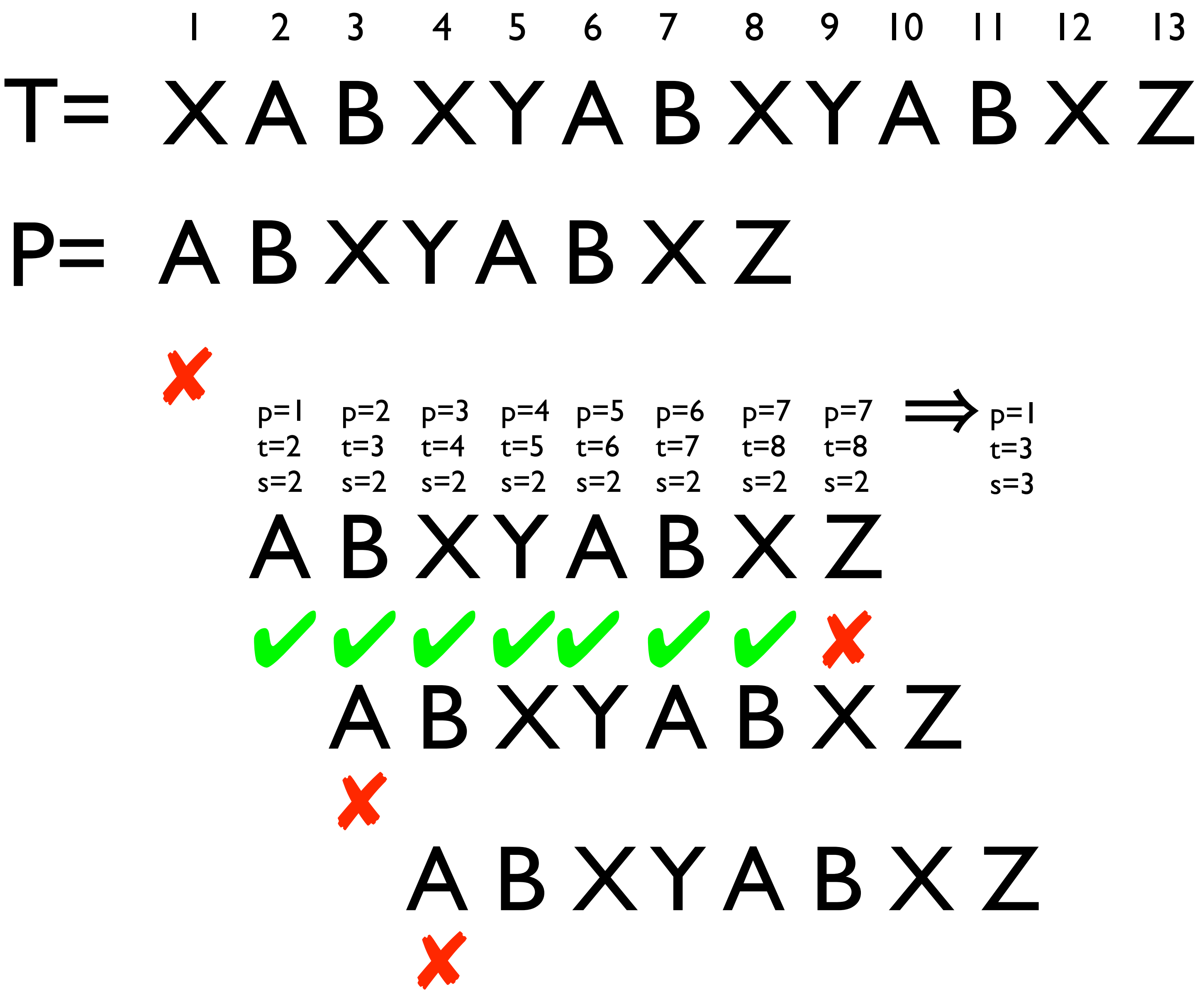
✗

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3
A	B	X	Y	A	B	X	Z		
✓	✓	✓	✓	✓	✓	✓	✗		

A B X Y A B X Z

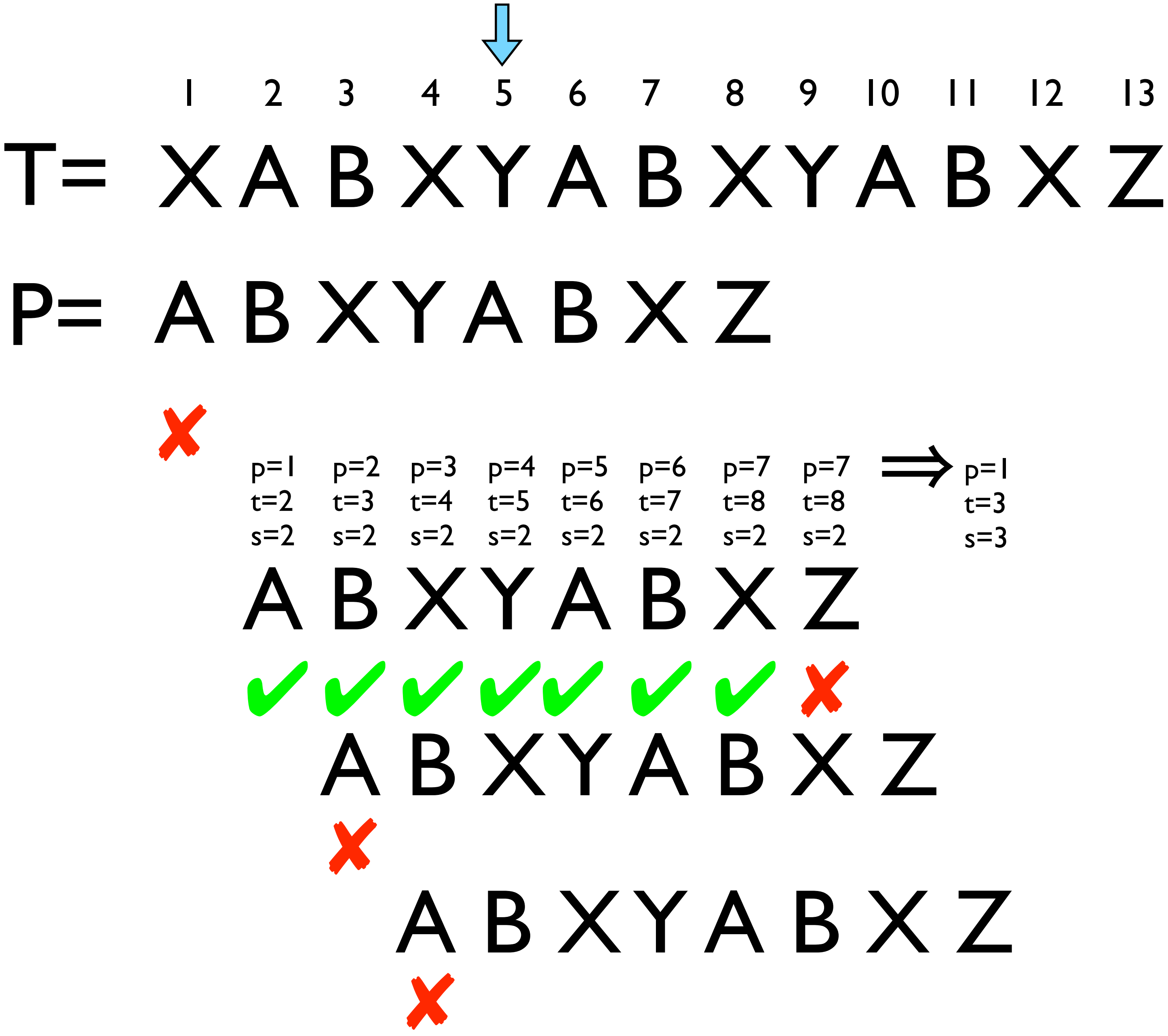
✗

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

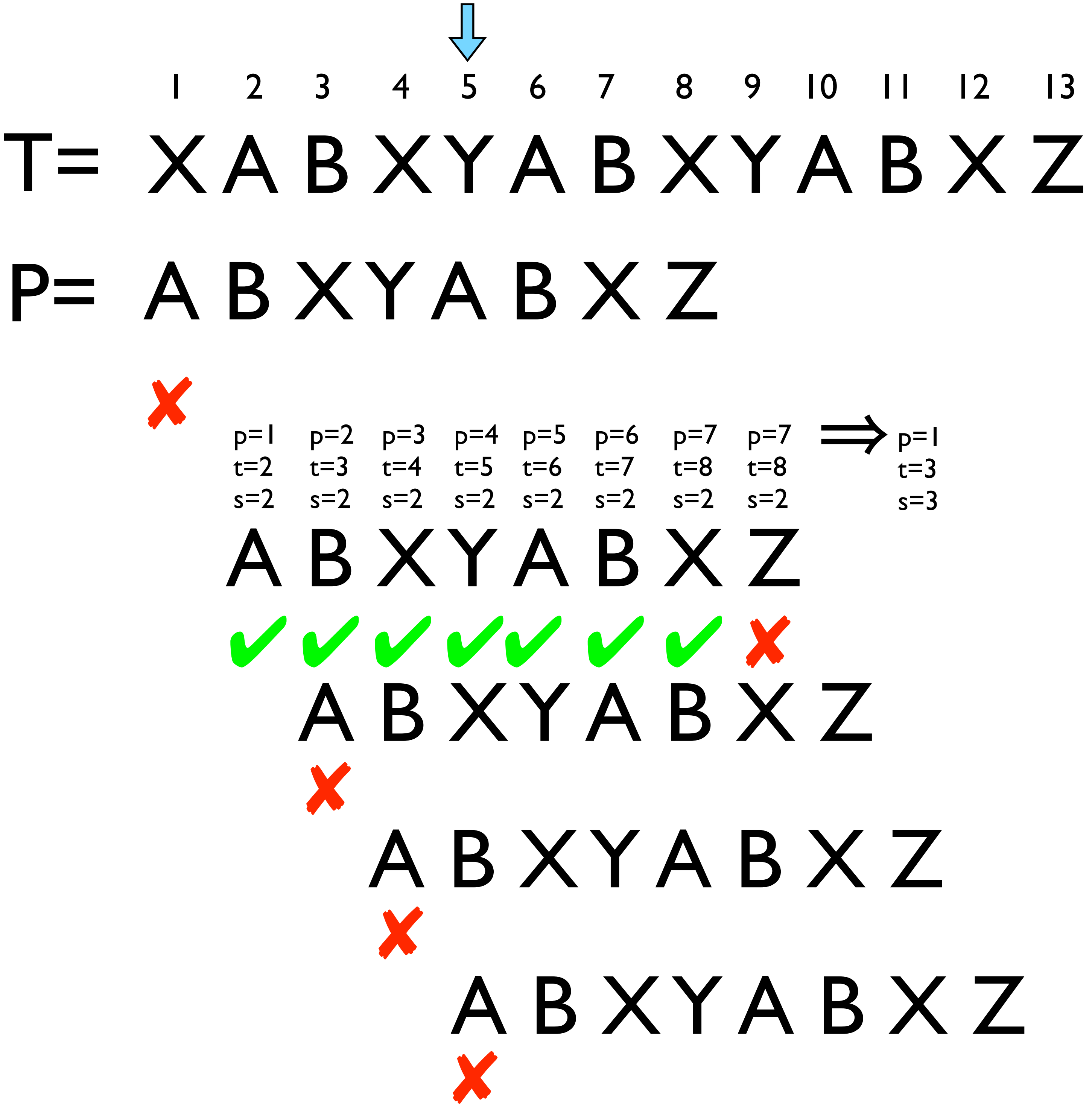
✗

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

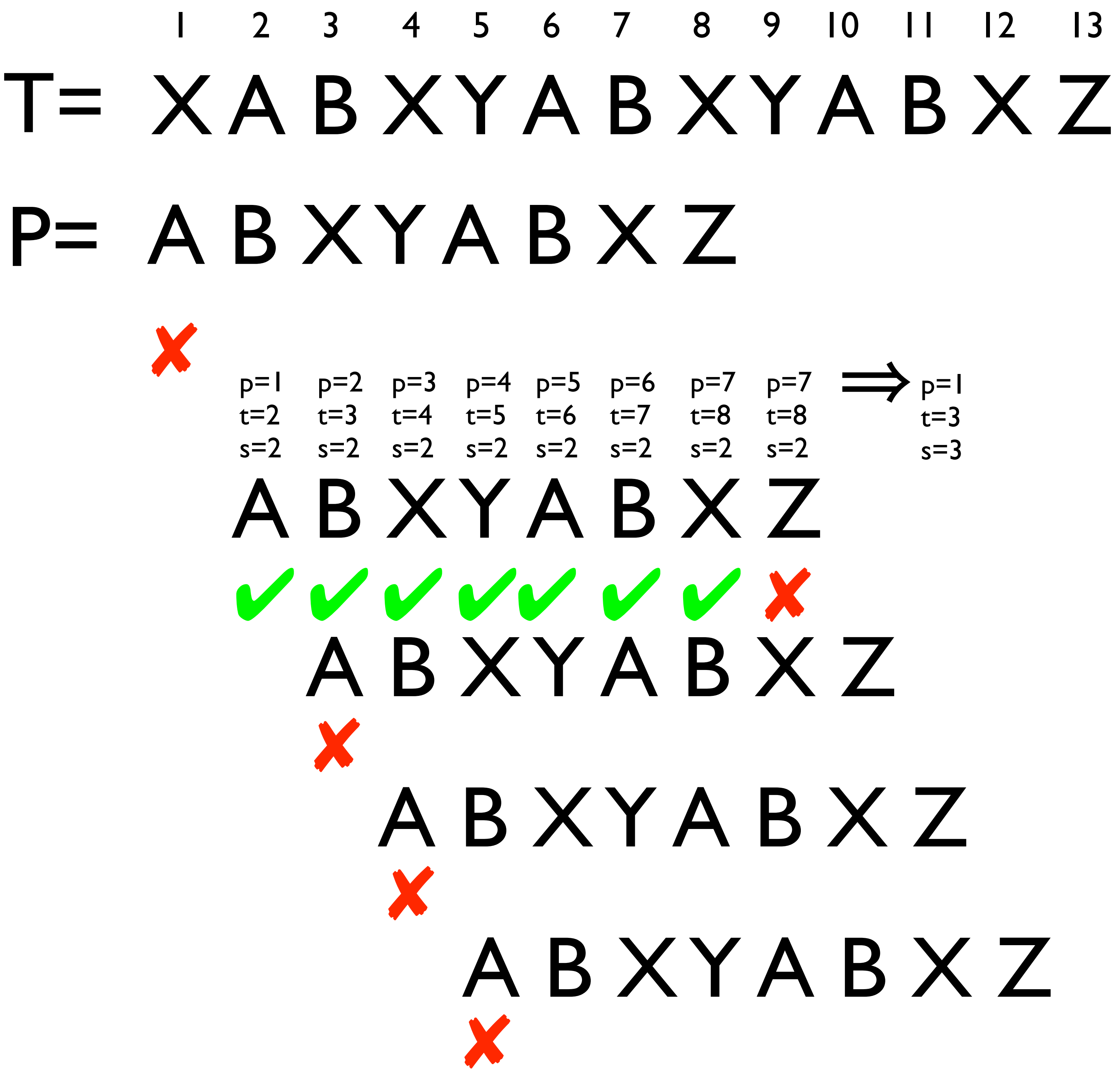
✗

A B X Y A B X Z

✗

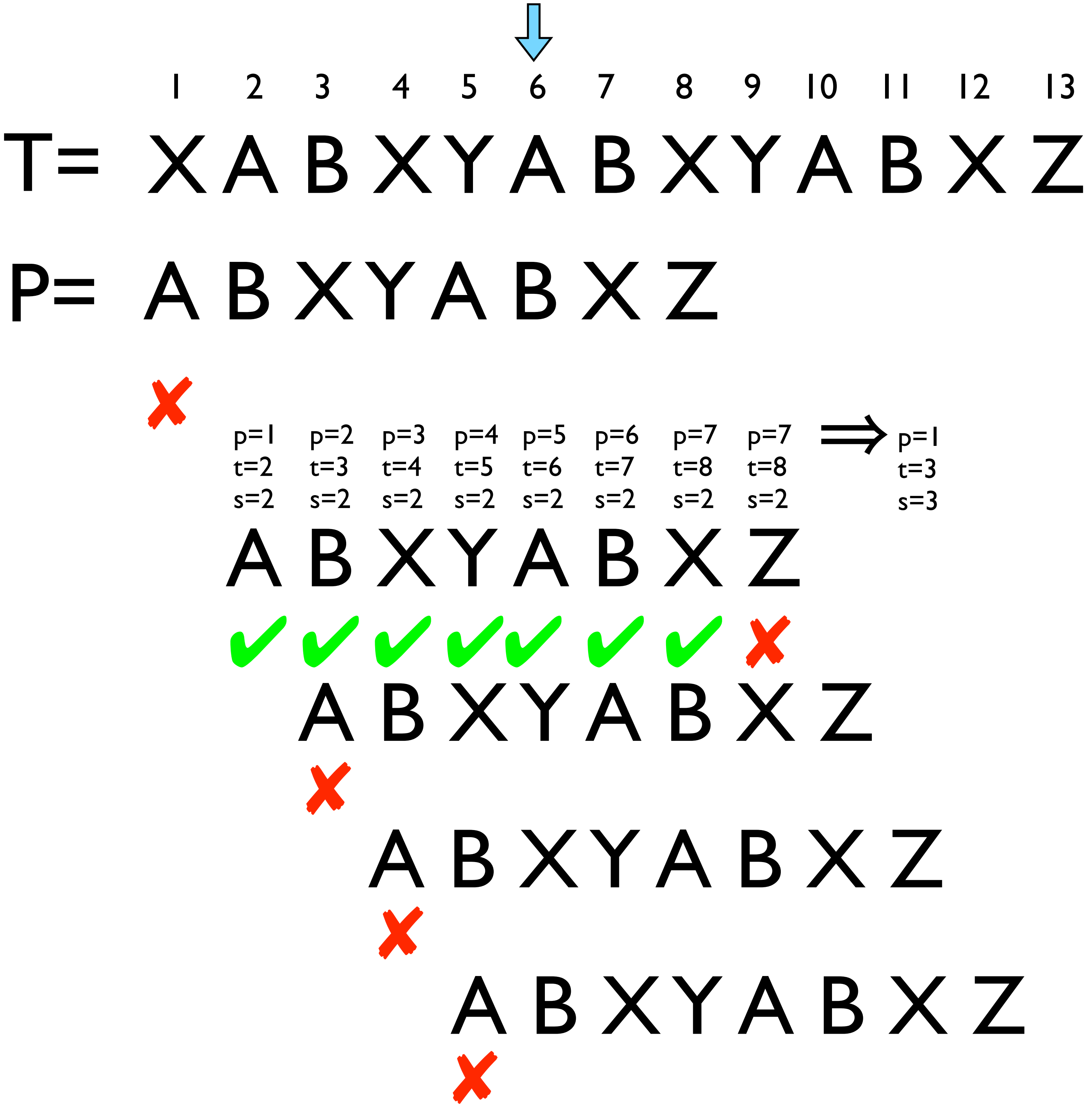
```
[Start: p=1, t=1,s=1]
        1. Align the left of P with the left of T:
        2. p=1; t=1; s=1
[Loop]
        2. Compare P(p) with T(t)
        3. If P(p) == T(t),
                4. If p < |P|
                        then p=p+1 and t=t+1;
                        else report match, and p=1, s=s+1, t=s;
                5. Else p=1 and s=s+1, t=s
```

The naive method for matching



```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



⇩

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

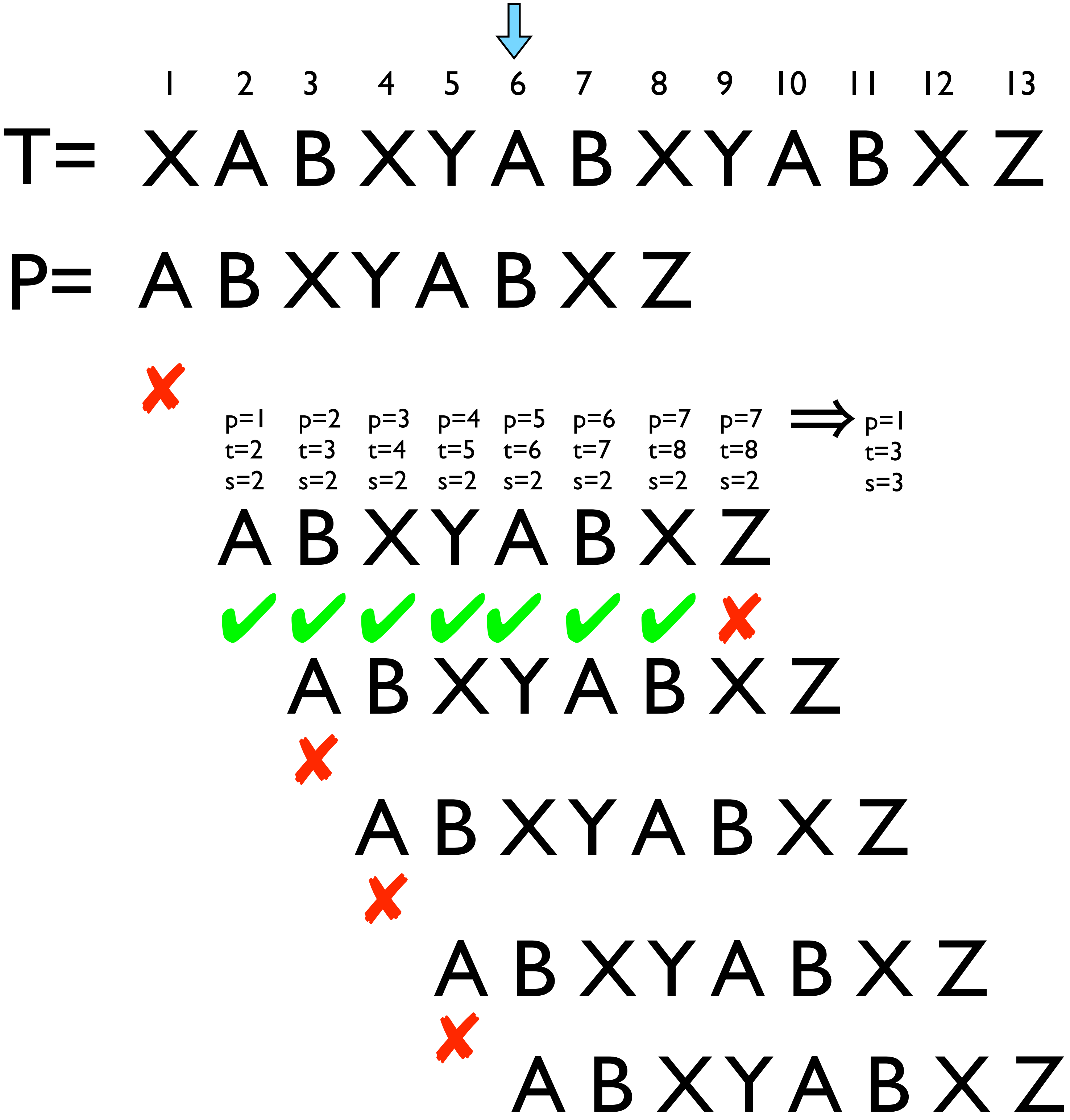
✗

A B X Y A B X Z

✗

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
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        5. Else p=1 and s=s+1, t=s
```
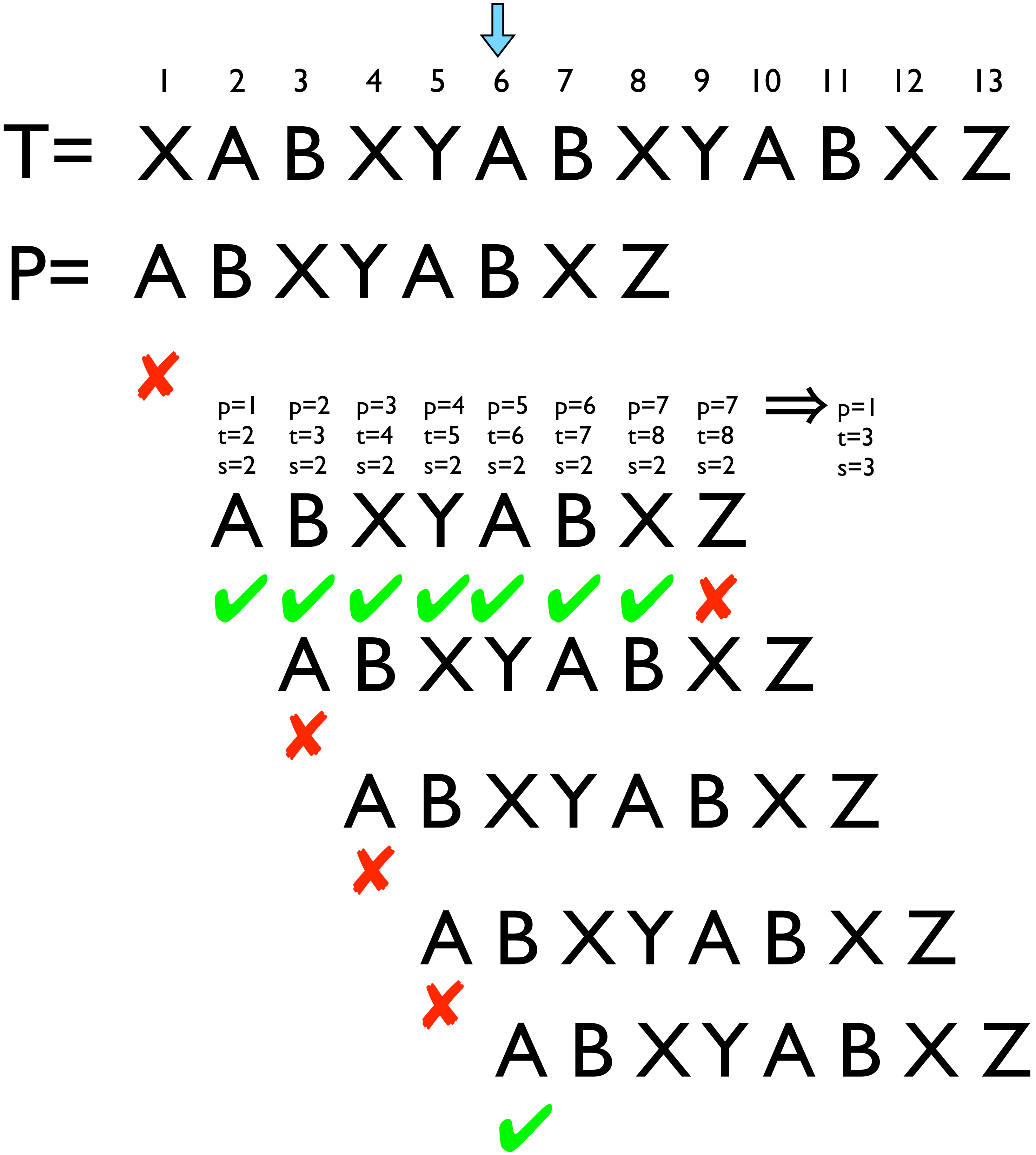

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

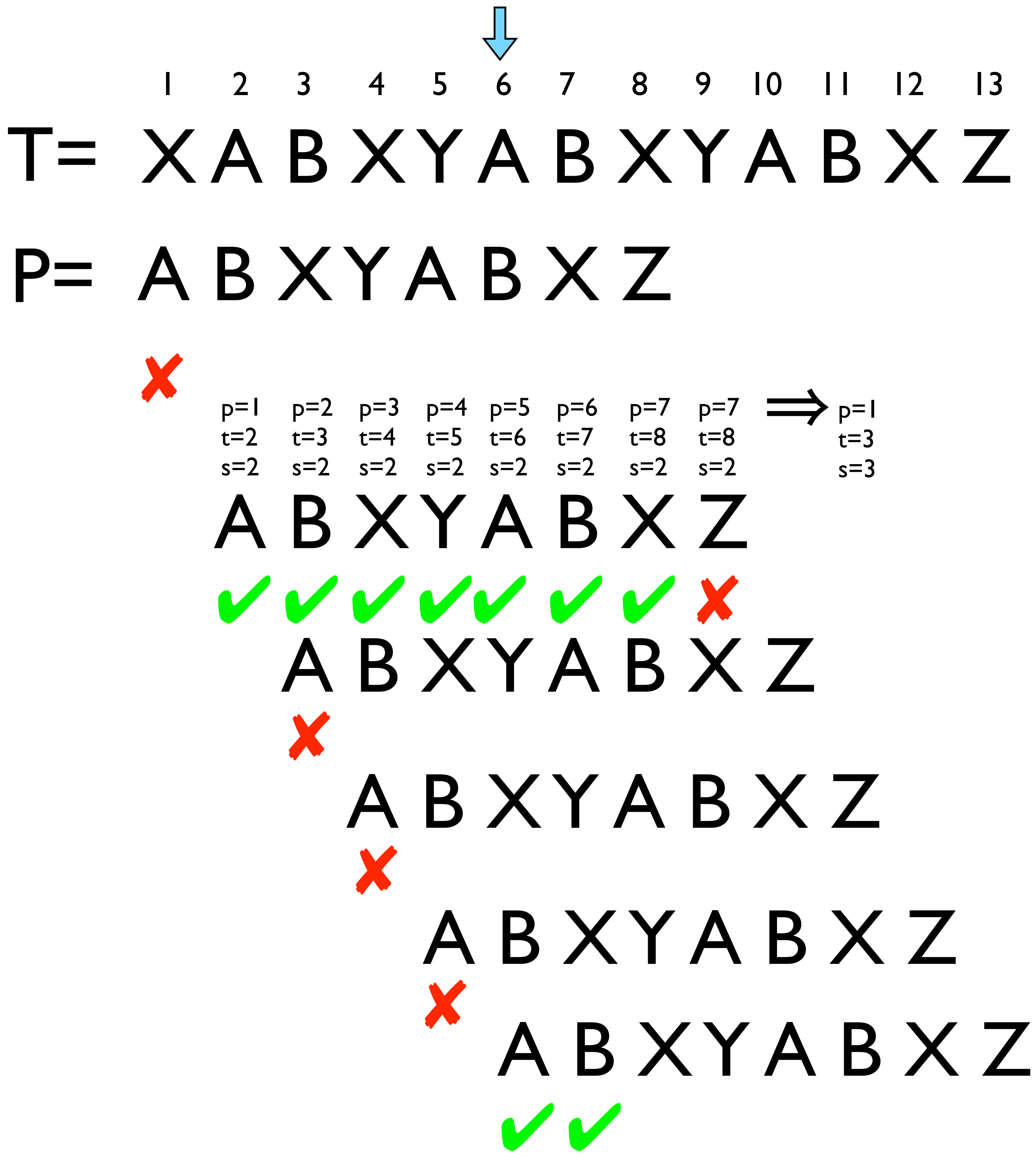
✗

A B X Y A B X Z

✓

```
[Start: p=1, t=1,s=1]
        1. Align the left of P with the left of T:
        2. p=1; t=1; s=1
[Loop]
        2. Compare P(p) with T(t)
        3. If P(p) == T(t),
                4. If p < |P|
                        then p=p+1 and t=t+1;
                        else report match, and p=1, s=s+1, t=s;
                5. Else p=1 and s=s+1, t=s
```

The naive method for matching



```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
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[Loop]
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```
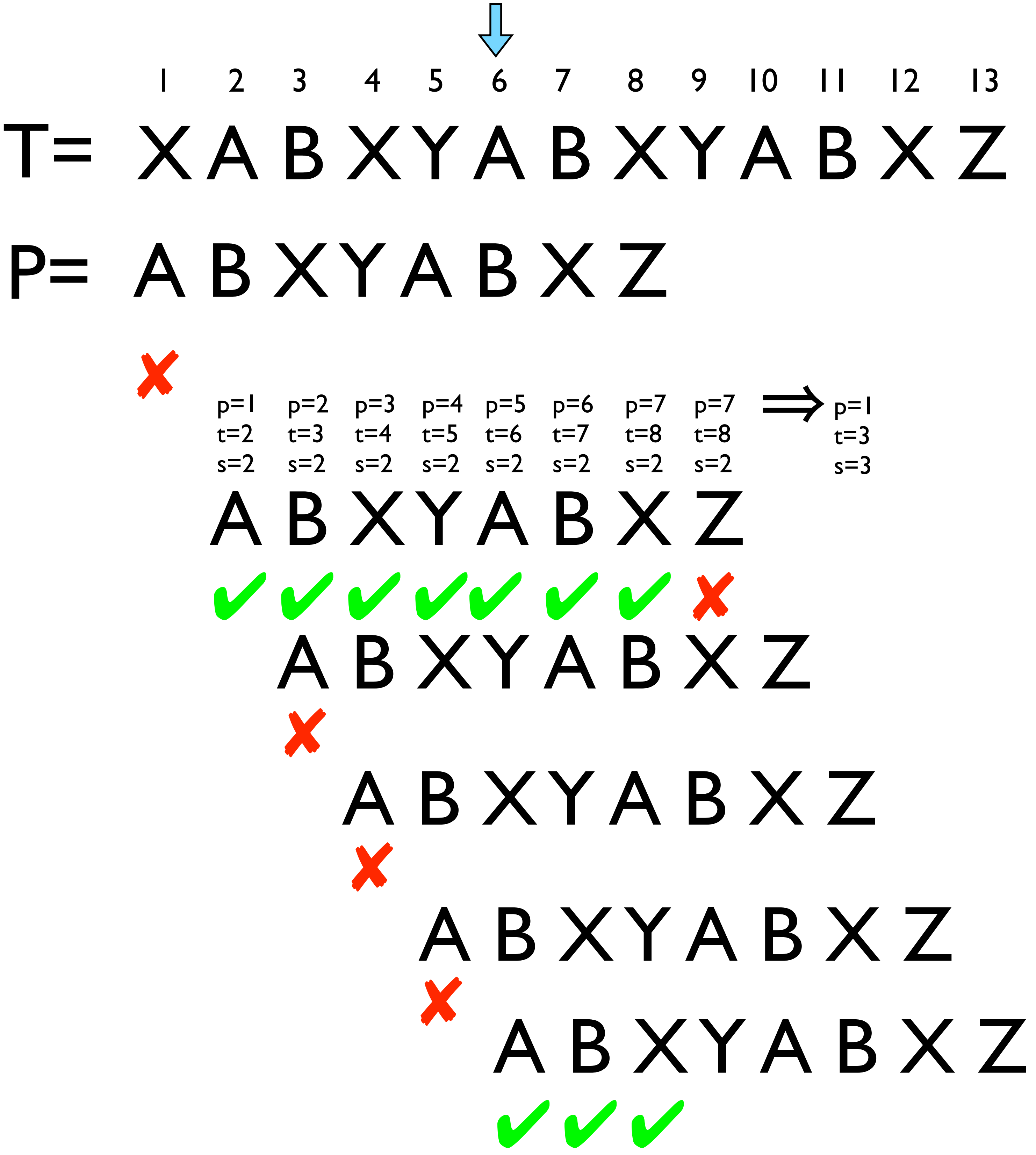

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
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[Loop]
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```
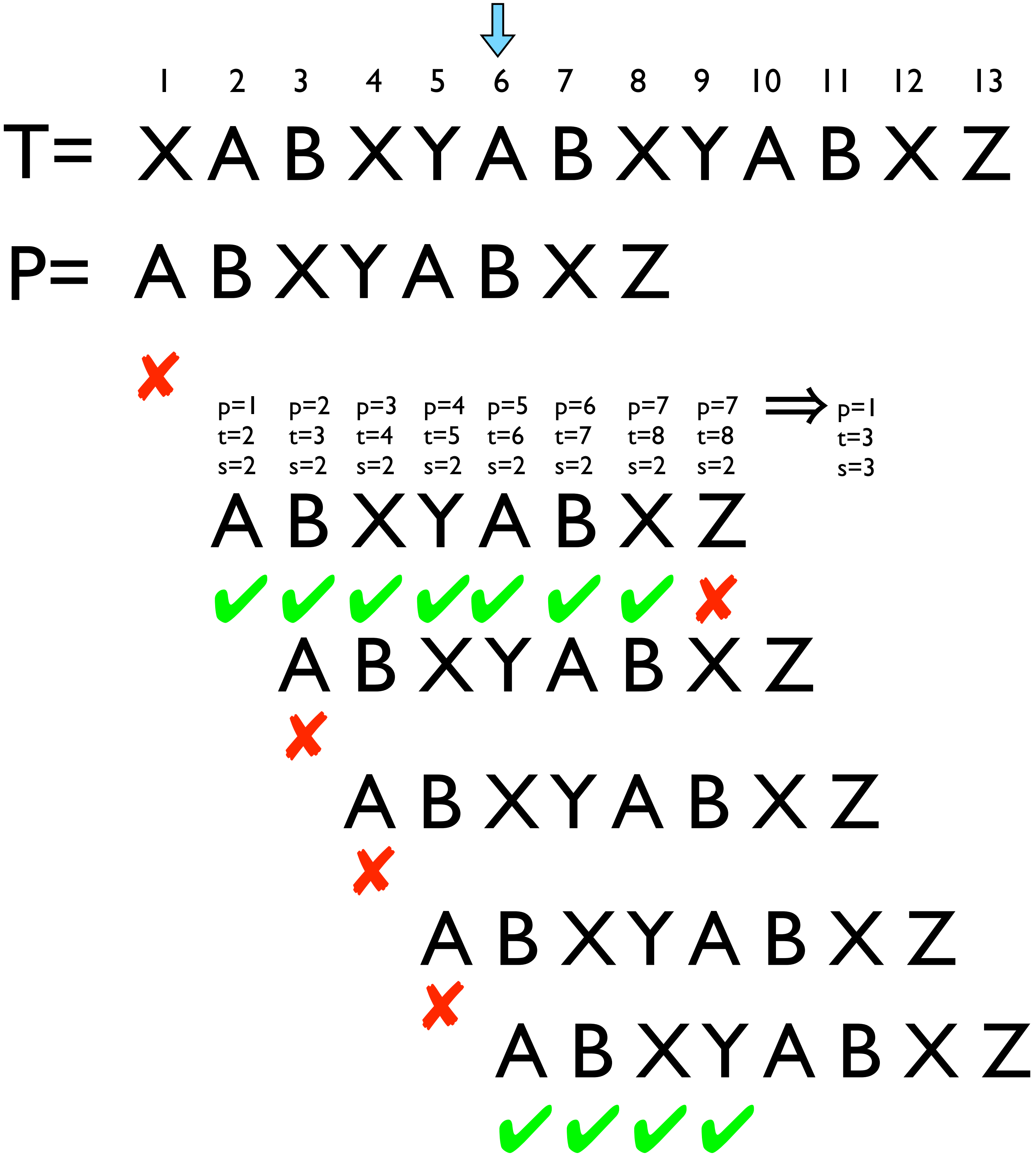

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✘

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✘

A B X Y A B X Z

✘

A B X Y A B X Z

✘

A B X Y A B X Z

✘

A B X Y A B X Z

✓ ✓ ✓ ✓

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
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```
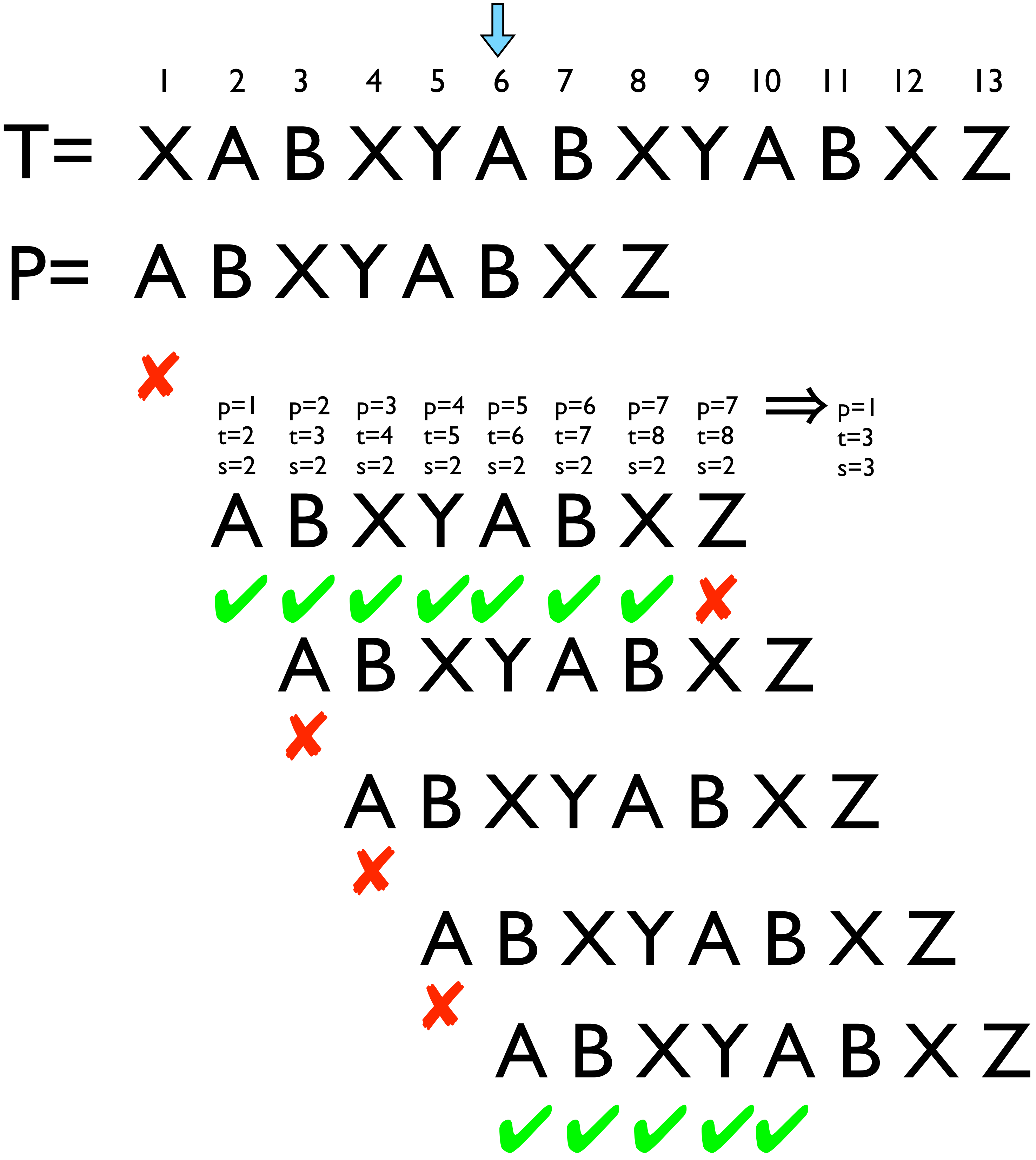

The naive method for matching



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓

```
[Start: p=1, t=1,s=1]
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```
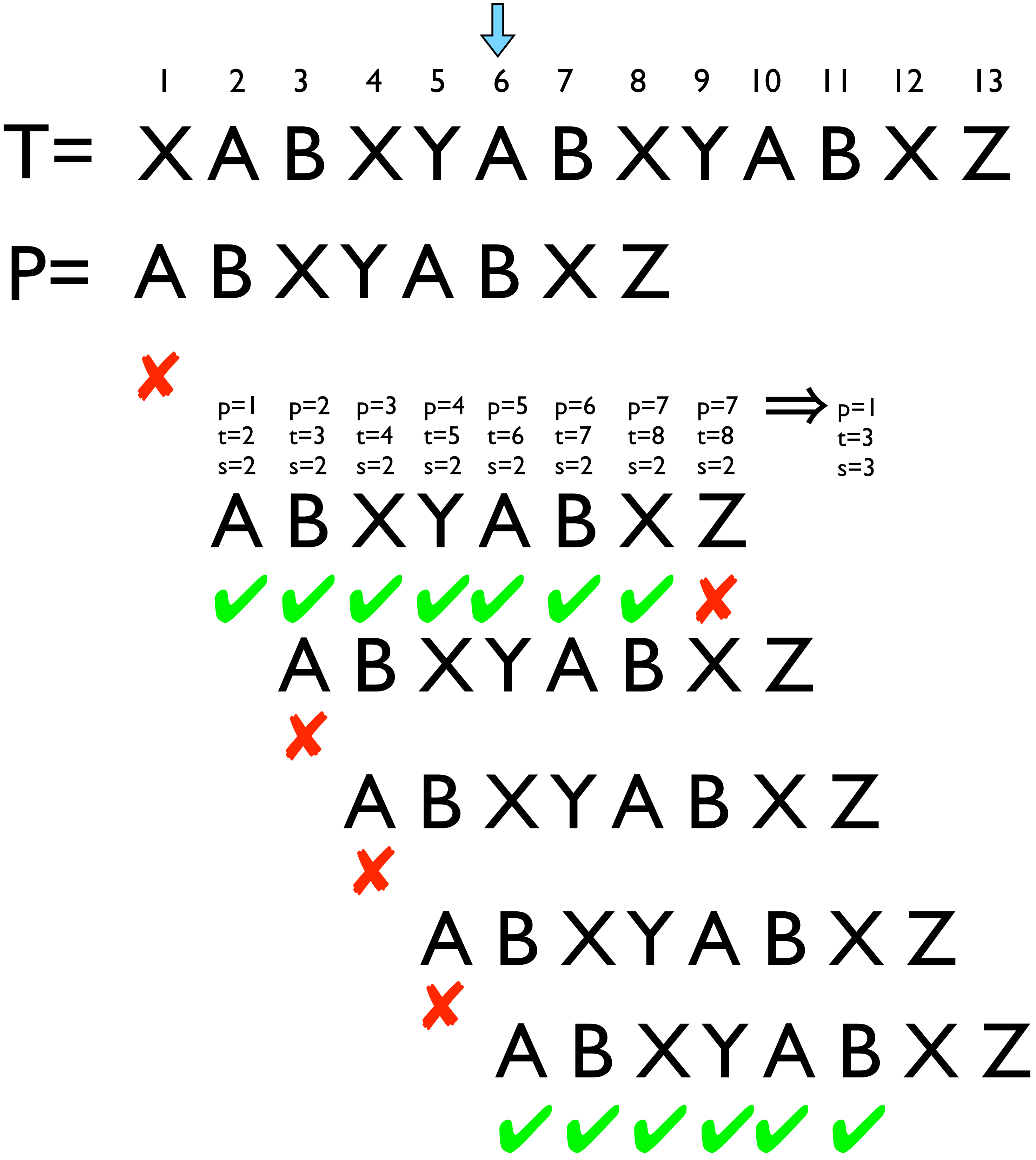

The naive method for matching



```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
    2. p=1; t=1; s=1
[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
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            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```
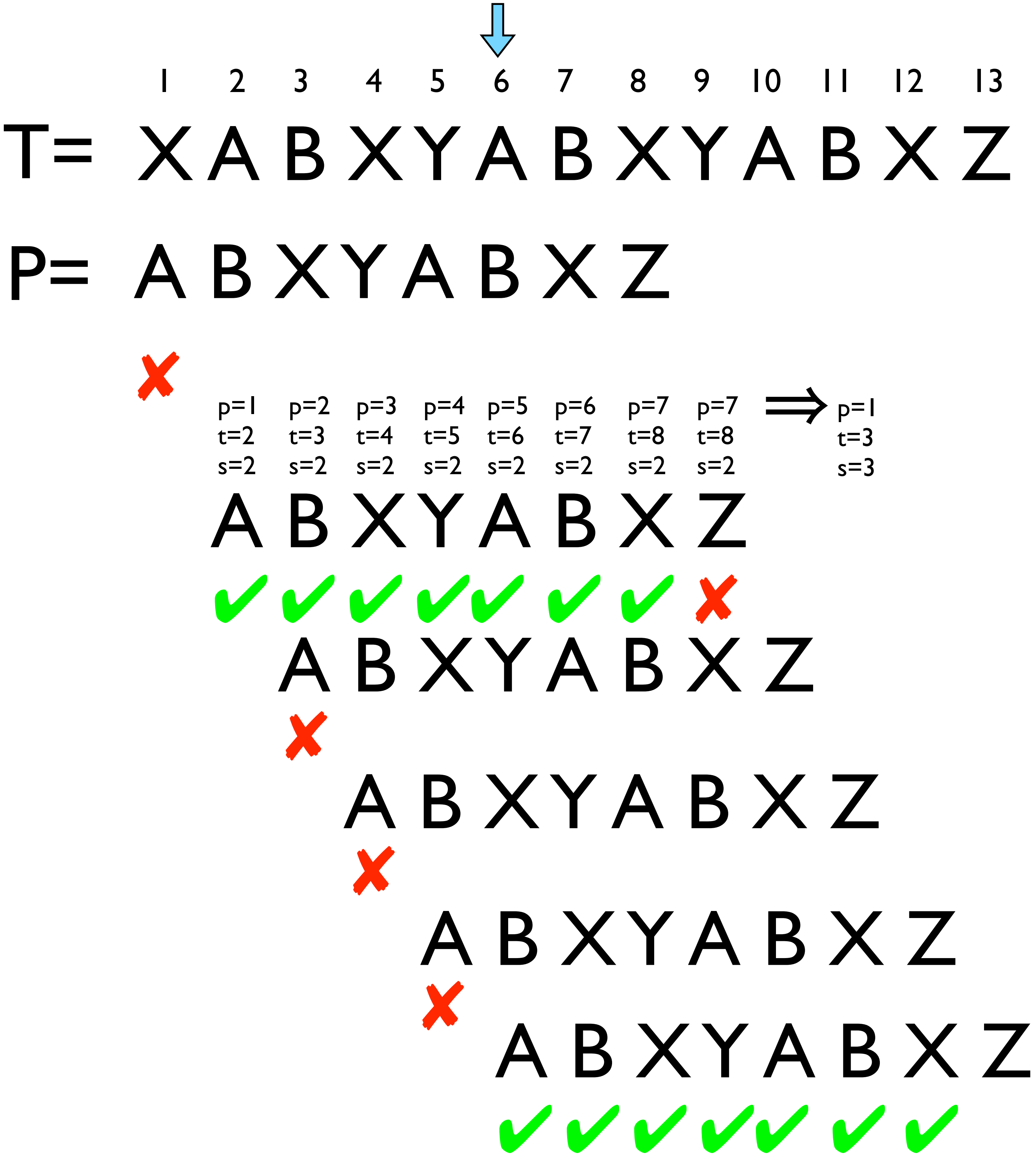

The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=7	⇒	p=1
t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=8		t=3
s=2	s=2	s=2	s=2	s=2	s=2	s=2	s=2		s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓

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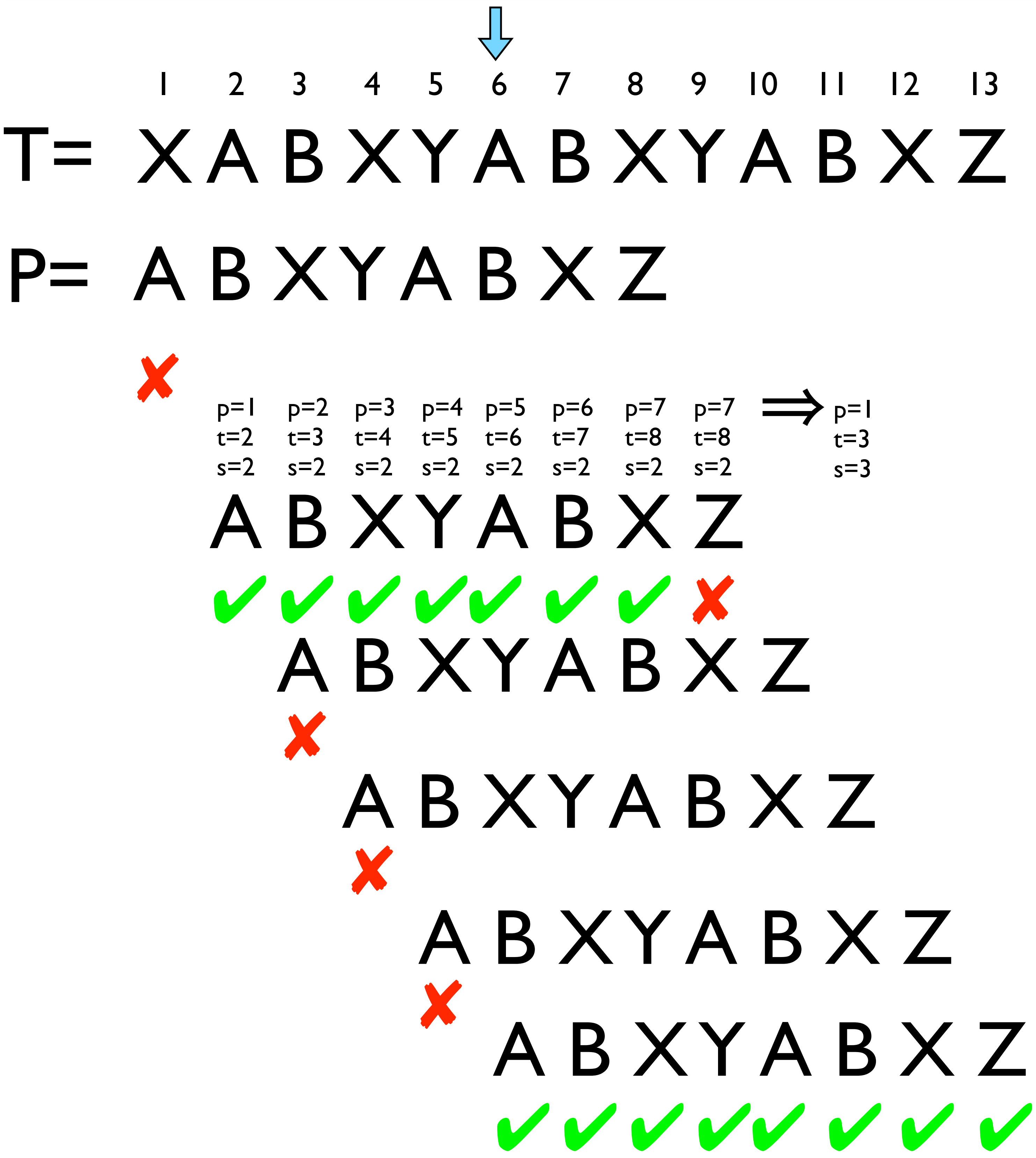
The naive method for matching



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

p=1 t=2 s=2	p=2 t=3 s=2	p=3 t=4 s=2	p=4 t=5 s=2	p=5 t=6 s=2	p=6 t=7 s=2	p=7 t=8 s=2	p=7 t=8 s=2	⇒	p=1 t=3 s=3

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
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        5. Else p=1 and s=s+1, t=s
```
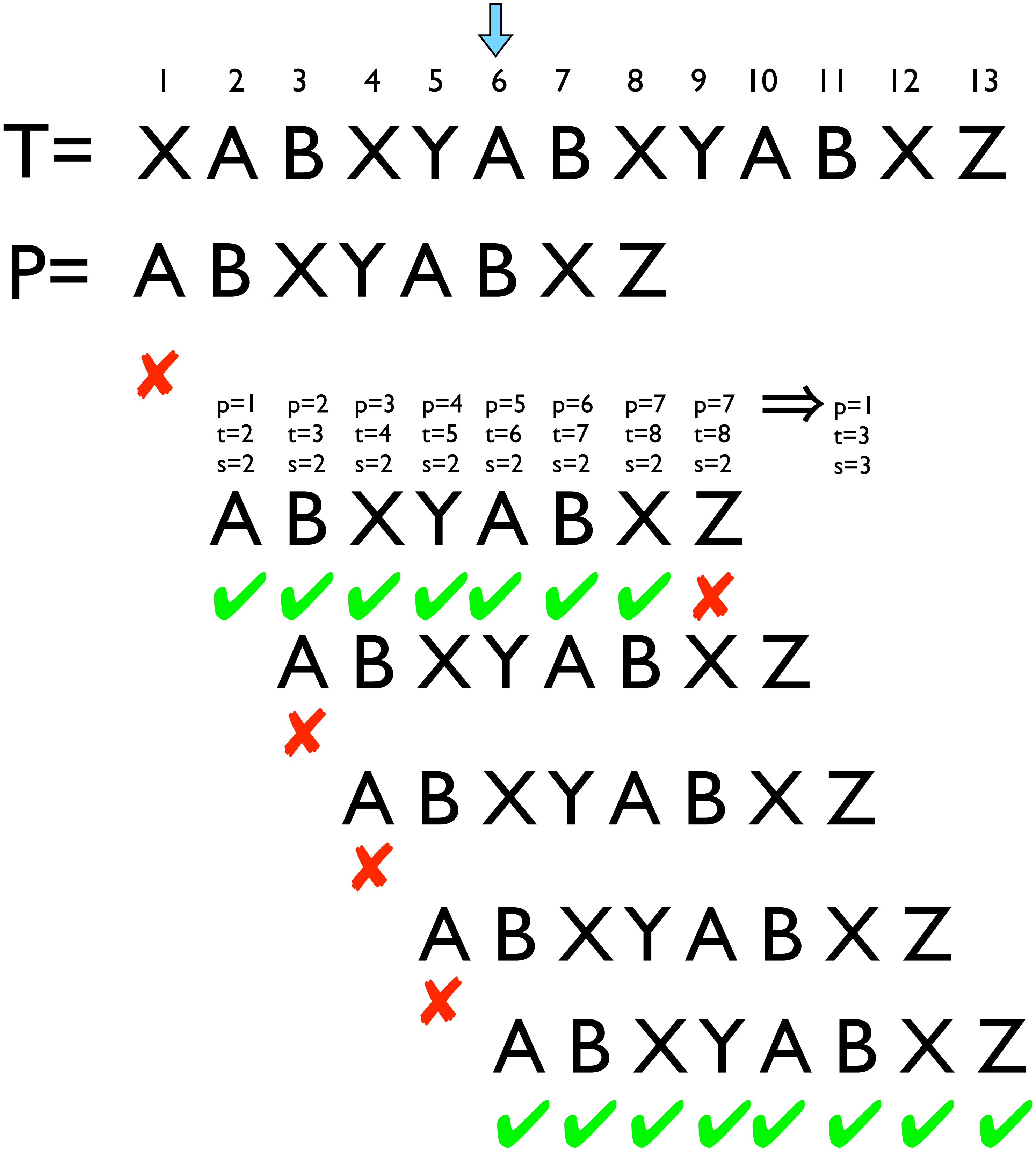

The naive method for matching



```
[Start: p=1, t=1,s=1]
    1. Align the left of P with the left of T:
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[Loop]
    2. Compare P(p) with T(t)
    3. If P(p) == T(t),
        4. If p < |P|
            then p=p+1 and t=t+1;
            else report match, and p=1, s=s+1, t=s;
        5. Else p=1 and s=s+1, t=s
```

In total,
we had to make
20
comparisons

The naive method for matching

The naive method for matching

- Observations
  - The worst-case number of comparisons is $O(|P| \cdot |T|)$
  - This is not so useful in real-life applications!
  - E.g. |P|=30 and |T|=200K: 6M comparisons; with 1ms per comparison this would mean 6000s, or 100 minutes, i.e. 1:40h. If we manage to get linear complexity $O(|P|+|T|)$ we are down to 3.33min!

The naive method for matching

- Observations
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- Ideas for speeding up the naive method
  - Try to shift further when a mismatch occurs, but never so far as to miss an occurrence of P in T

Speeding up thru smarter shifting

Speeding up thru smarter shifting

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z

Speeding up thru smarter shifting

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

Speeding up thru smarter shifting

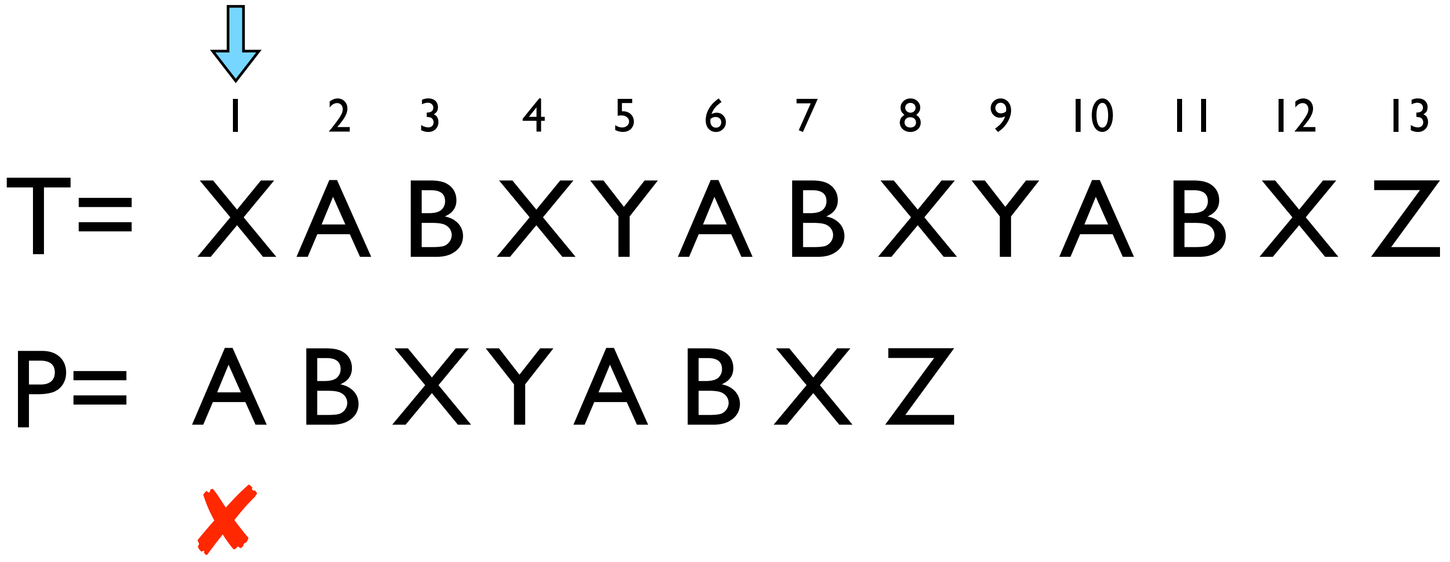
⇩

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

Speeding up thru smarter shifting



	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												

Speeding up thru smarter shifting

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				
		✓											

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✘												
		A	B	X	Y	A	B	X	Z				
		✔	✔										

Speeding up thru smarter shifting

⇩ (above 2)

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

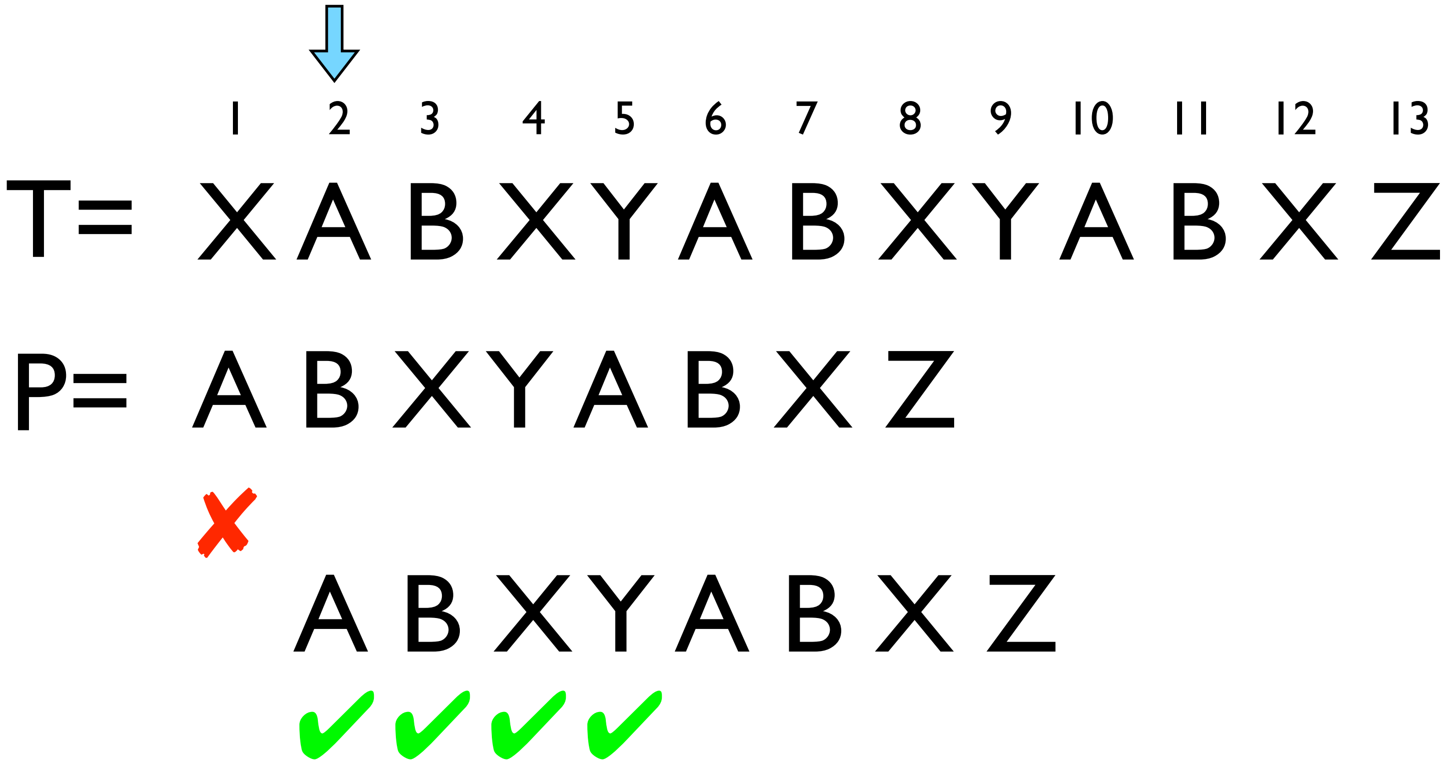
P= A B X Y A B X Z

✗

A B X Y A B X Z

✓✓✓

Speeding up thru smarter shifting



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

A B X Y A B X Z

✓✓✓✓

Speeding up thru smarter shifting

⇩ (2)

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

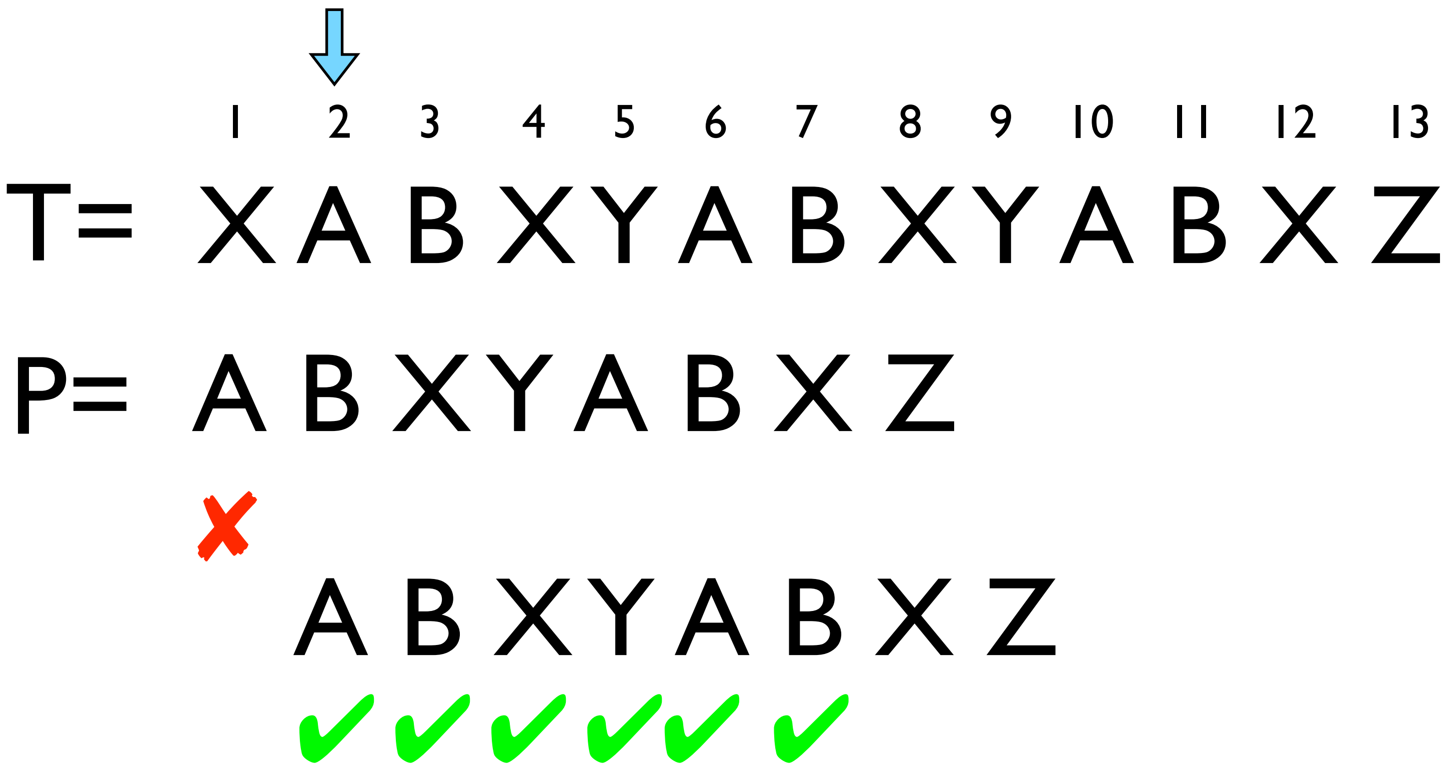
P= A B X Y A B X Z

✗

A B X Y A B X Z

✓✓✓✓✓

Speeding up thru smarter shifting



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

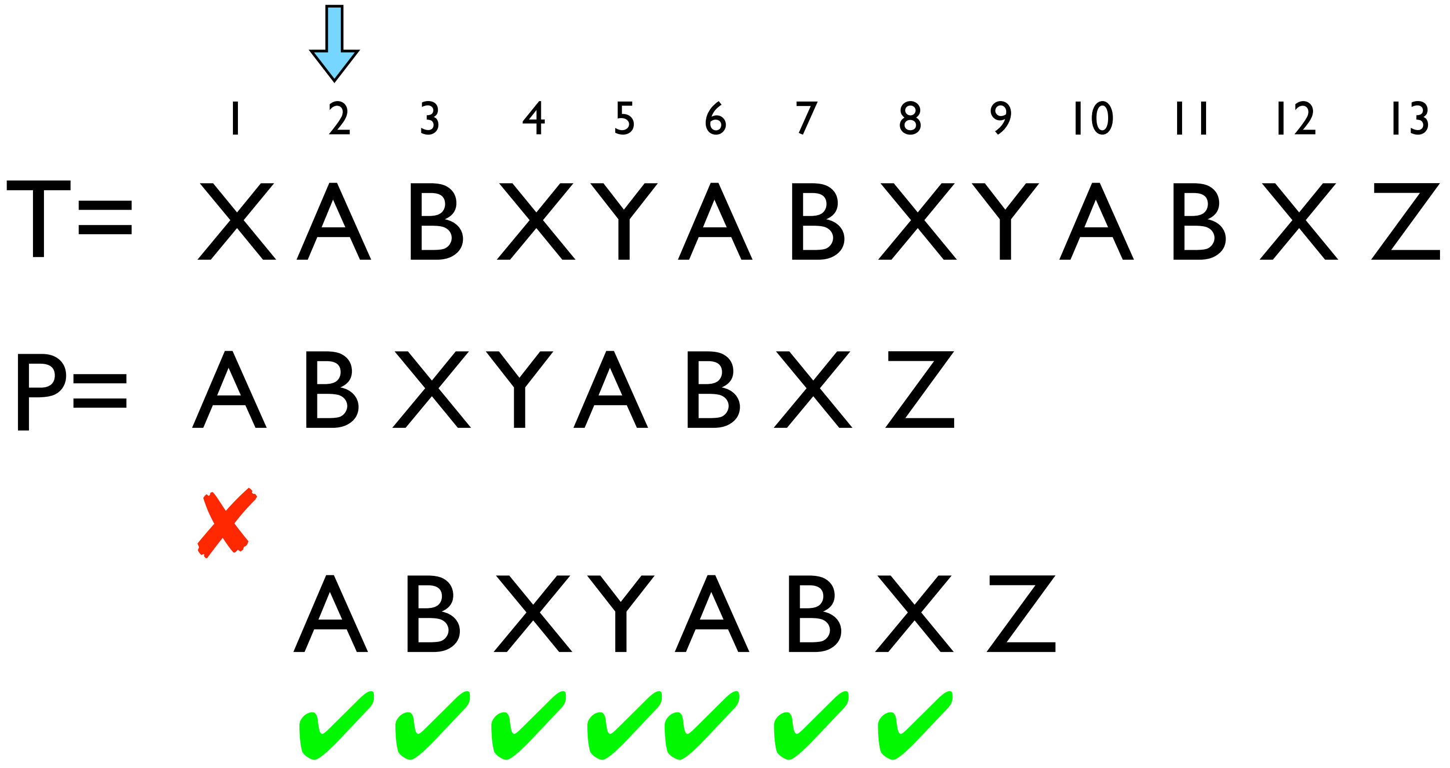
P= A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓

Speeding up thru smarter shifting



1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

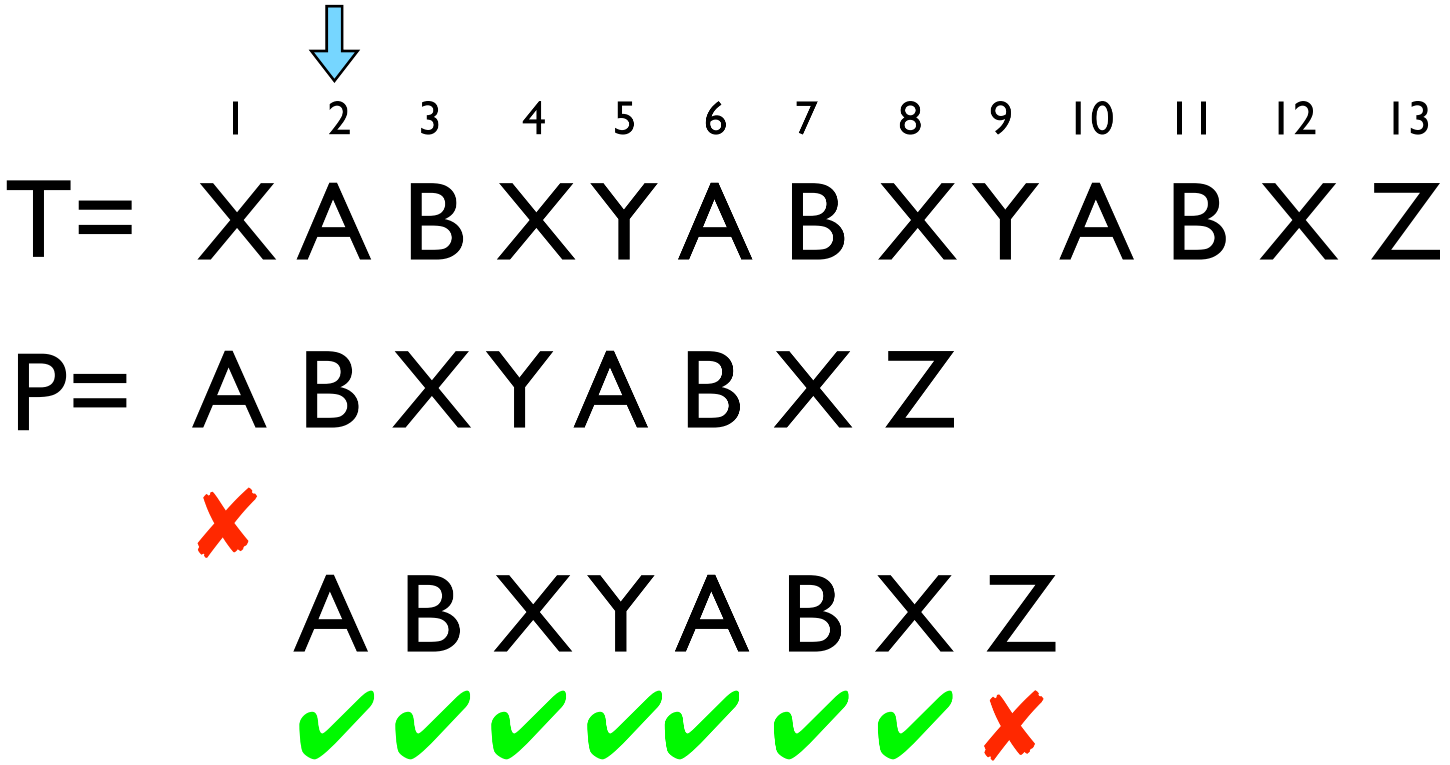
P= A B X Y A B X Z

✘

A B X Y A B X Z

✔ ✔ ✔ ✔ ✔ ✔ ✔

Speeding up thru smarter shifting



		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓	✗				

Speeding up thru smarter shifting

⇩ (at 2)

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

The next occurrence of P(1)=”A” in T is not before position 5 in T, so shift to position 6!

Speeding up thru smarter shifting

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓	✗				

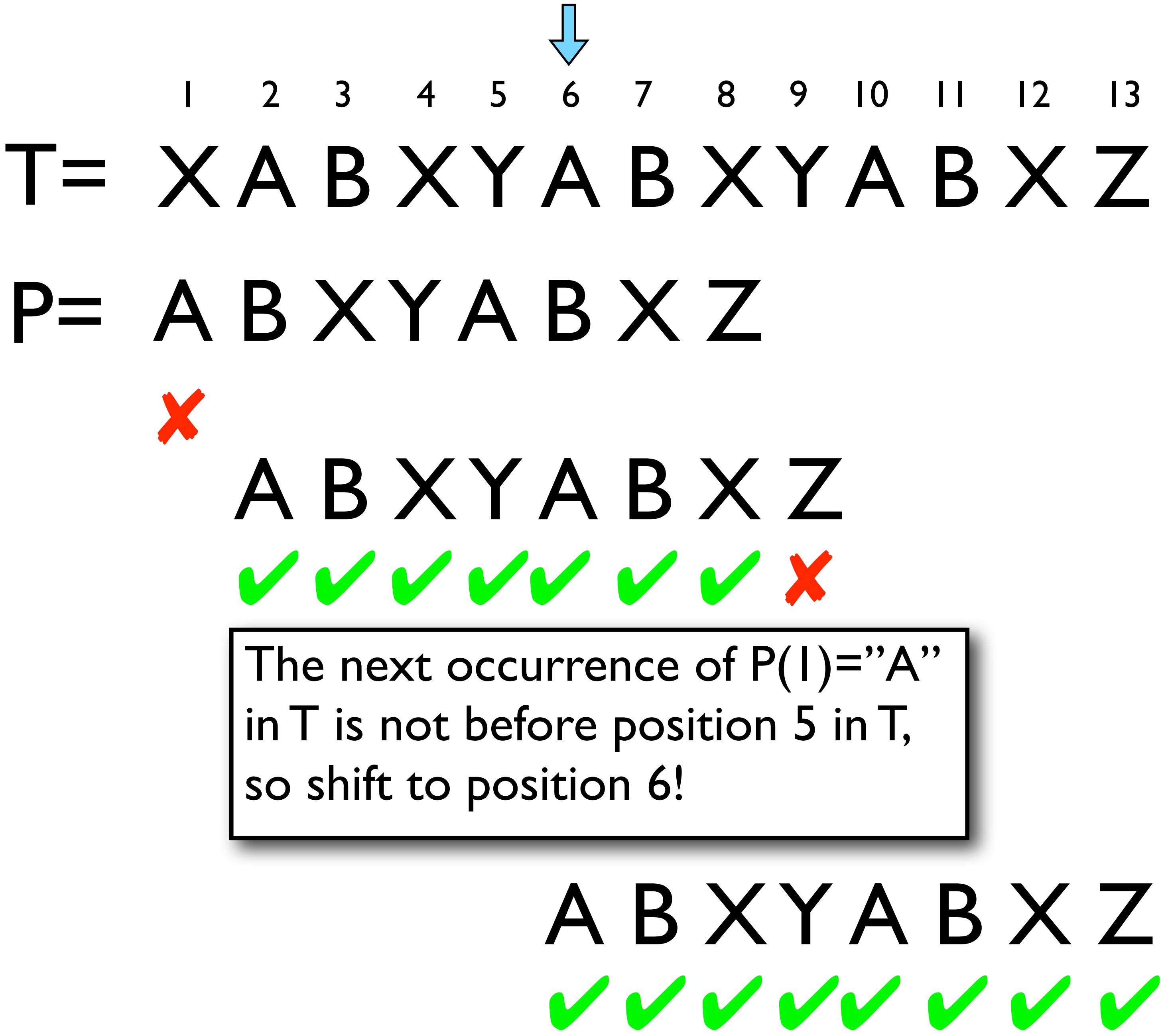
The next occurrence of P(1)=”A” in T is not before position 5 in T, so shift to position 6!

Speeding up thru smarter shifting

						⇩							
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✘												
		A	B	X	Y	A	B	X	Z				
		✔	✔	✔	✔	✔	✔	✔	✘				

The next occurrence of P(1)=”A” in T is not before position 5 in T, so shift to position 6!

Speeding up thru smarter shifting



↓

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

The next occurrence of P(1)=”A” in T is not before position 5 in T, so shift to position 6!

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

Speeding up thru smarter shifting

						⇩							
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓	✗				
						A	B	X	Y	A	B	X	Z
						✓	✓	✓	✓	✓	✓	✓	✓

The next occurrence of P(1)=”A” in T is not before position 5 in T, so shift to position 6!

In total,
we had to make
17
comparisons

Speeding up thru smarter shifting

1 2 3 4 5 6 7 8 9 10 11 12 13

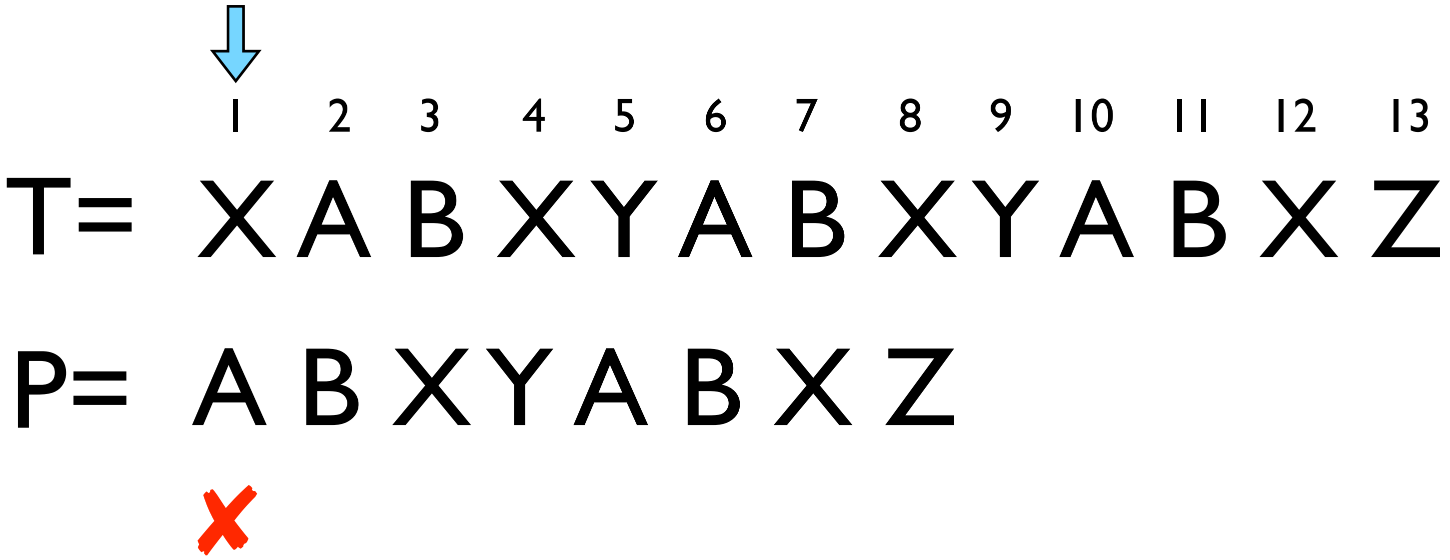
T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

Speeding up thru smarter shifting

	⇩												
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					

Speeding up thru smarter shifting



⇩

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✘

Speeding up thru smarter shifting

	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✘												

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓	✓	✓	✓	✗				

Speeding up thru smarter shifting

		⇩											
	1	2	3	4	5	6	7	8	9	10	11	12	13
T=	X	A	B	X	Y	A	B	X	Y	A	B	X	Z
P=	A	B	X	Y	A	B	X	Z					
	✗												
		A	B	X	Y	A	B	X	Z				
		✓	✓	✓	✓✓		✓	✓	✗				

Assume: we know prefix P[1..3]=”A B X” starts at T(6). P[1..3]=T[6..8]; align at T(6) but start matching P(4) against T(9)

Speeding up thru smarter shifting

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

✘

A B X Y A B X Z

✔ ✔ ✔ ✔✔ ✔ ✔ ✘

Assume: we know prefix P[1..3]=”A B X” starts at T(6). P[1..3]=T[6..8]; align at T(6) but start matching P(4) against T(9)

Speeding up thru smarter shifting

⇩

1 2 3 4 5 6 7 8 9 10 11 12 13

T= X A B X Y A B X Y A B X Z

P= A B X Y A B X Z

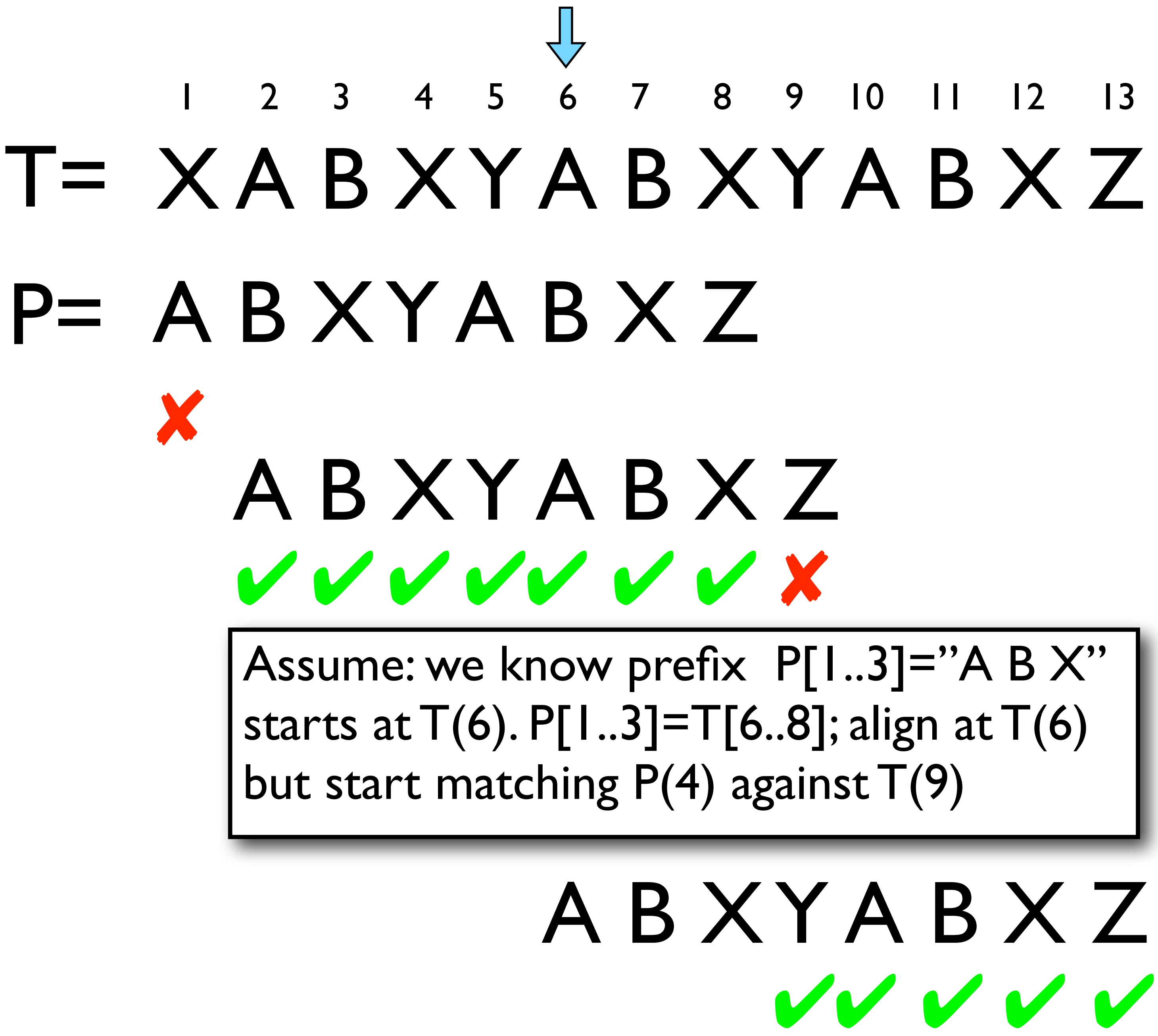
✗

A B X Y A B X Z

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✗

Assume: we know prefix P[1..3]="A B X" starts at T(6). P[1..3]=T[6..8]; align at T(6) but start matching P(4) against T(9)

Speeding up thru smarter shifting



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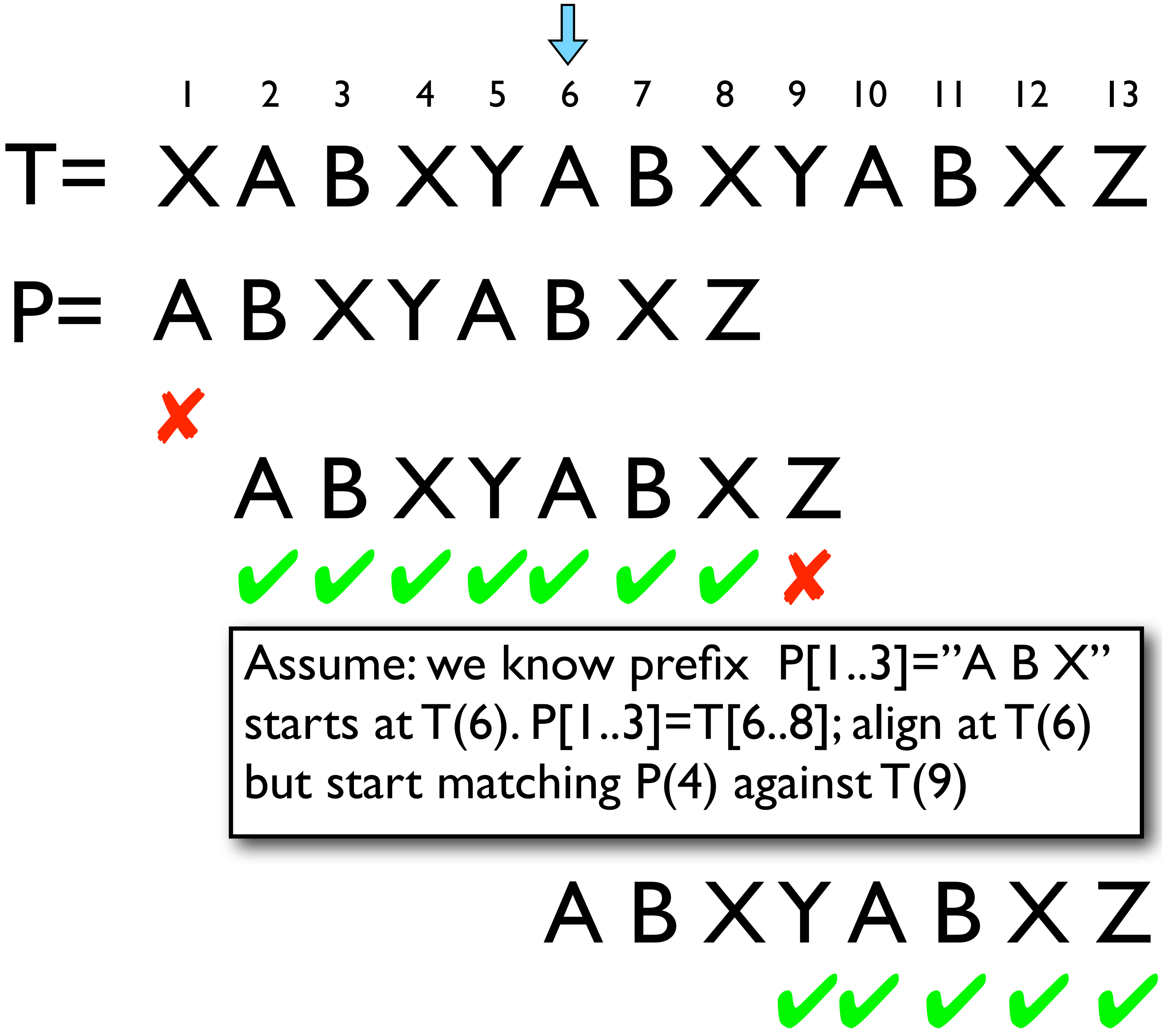

Assume: we know prefix P[1..3]="A B X" starts at T(6). P[1..3]=T[6..8]; align at T(6) but start matching P(4) against T(9)

In total, we had to make **14** comparisons

Smarter shifting thru preprocessing

- Before searching, preprocess P (or T, or P+T)
- Fundamental preprocessing of a string S
  - At S(i), i > 1 compute length of longest prefix of S[i..|S|] that is a prefix of S
  - Let $Z_i(S)$ be that length at i

Smarter shifting thru preprocessing

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	1	2	3	4	5	6	7	8	9	10	11
S =	A	A	B	C	A	A	B	X	A	A	Z

Smarter shifting thru preprocessing

- Before searching, preprocess P (or T, or P+T)
- Fundamental preprocessing of a string S
    - At S(i), i > 1 compute length of longest prefix of S[i..|S|] that is a prefix of S
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1 2 3 4 5 6 7 8 9 10 11

S = A A B C A A B X A A Z

$Z_5(S)=3$: (A A B C...A A B X)

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S =	A	A	B	C	A	A	B	X	A	A	Z

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$Z_6(S)=1$: (A A ...A B)

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S =	A	A	B	C	A	A	B	X	A	A	Z

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$Z_7(S)=Z_8(S)=0$

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$$\begin{array}{rccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ S = & A & A & B & C & A & A & B & X & A & A & Z \end{array}$$

$Z_5(S)=3$: (A A B C...A A B X)

$Z_6(S)=1$: (A A ...A B)

$Z_7(S)=Z_8(S)=0$

$Z_9(S)=2$: (A A B ...A A Z)

Smarter shifting thru preprocessing

- Given a string S=P\$T
  - The dollar sign \$ is not in the languages for P or T
  - $|P|=n$, $|T|=m$, $n \leq m$, so $S=n+m+1$
- Compute $Z_i(S)$ for $2 < i < n+m+1$
  - Because "\$" is not in the language for P, $Z_i(S) \leq n$ for every $i > 1$
  - $Z_i(S)=n$ for $i > n+1$ identifies an occurrence of P starting at $i-(n+1)$ in T
  - Also: If P occurs in T starting at position j, then it must be that $Z_{(n+1)+j}(S)=n$
- If $Z_i(S)$ is computable in linear time, then we have linear time matching
  - Matching = search ⇒ matching = preprocessing + search

Computing $Z_i(S)$ in linear time

- The task: Compute $Z_i(S)$ in linear time, i.e. *O(|S|)*
- The notion of a **Z-box**
  - For every $i > 1$ with $Z_i(S) > 0$, define a Z-box to be the substring from i until $i+Z_i(S)-1$, i.e. $S[i...i+Z_i(S)-1]$
  - For every $i > 1$, r_i is the right-most endpoint of the Z-boxes that begin at or before i;
  - i.e, r_i is the largest value of $j+Z_j(S)-1$ for all $1 < j \leq i$ such that $Z_i(S) > 0$

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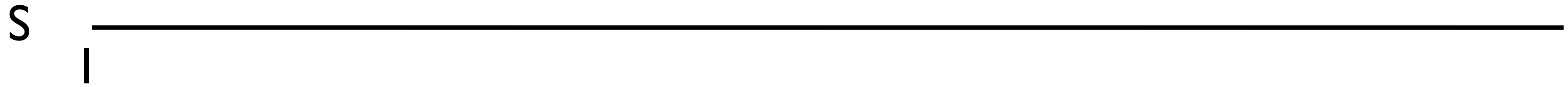

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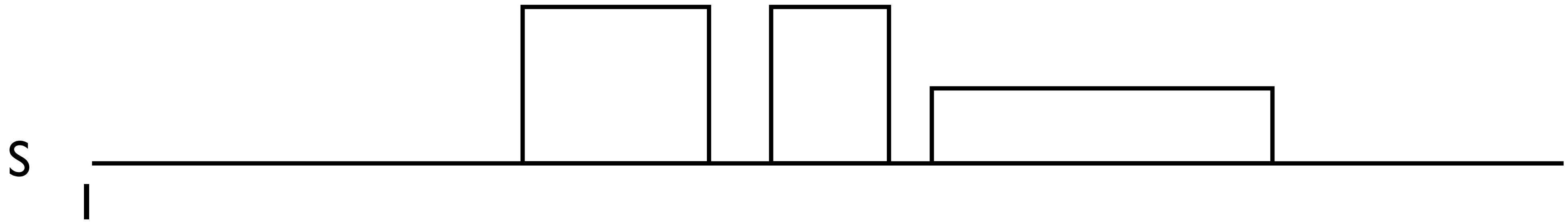

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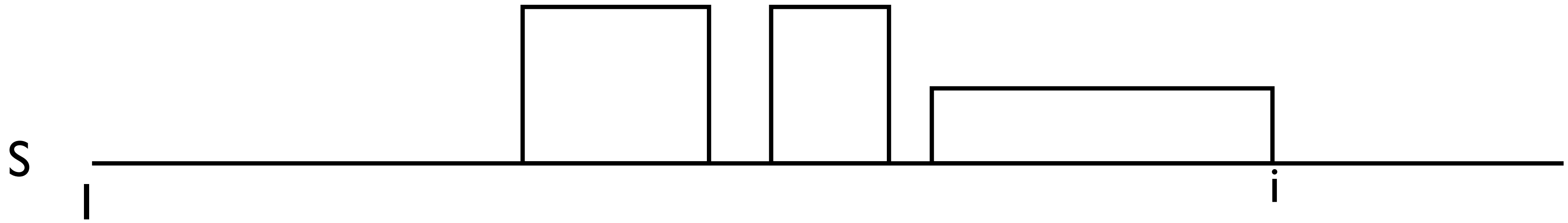

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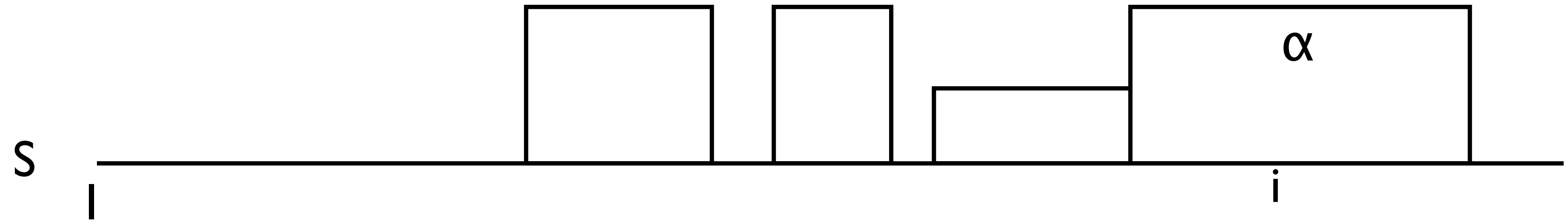

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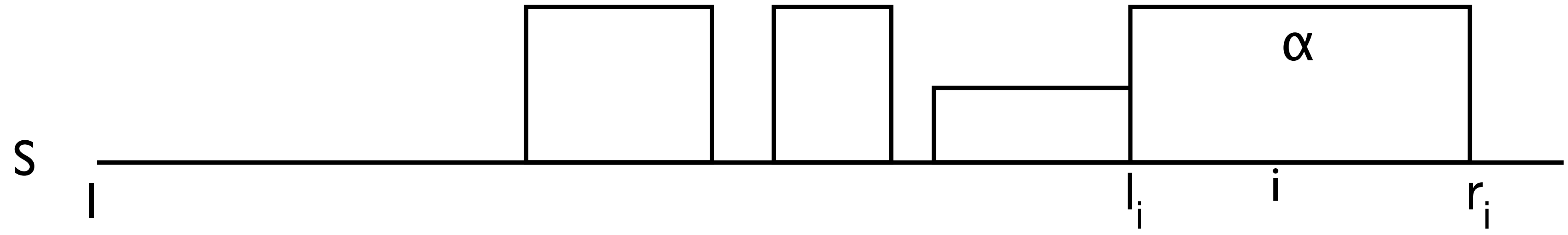

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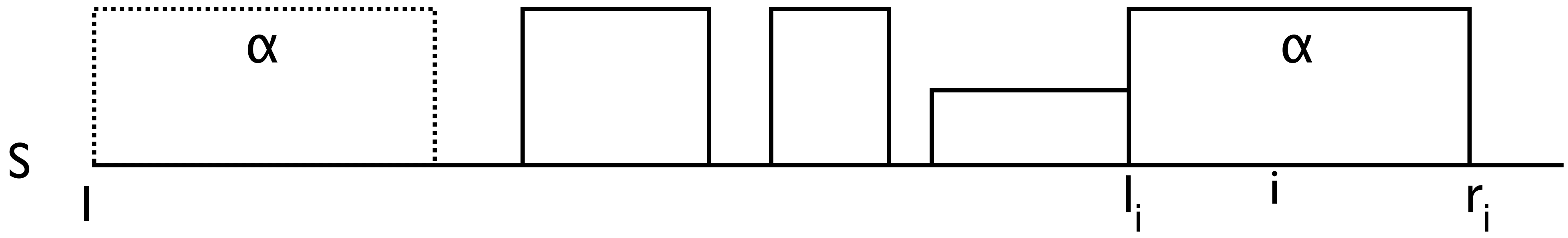

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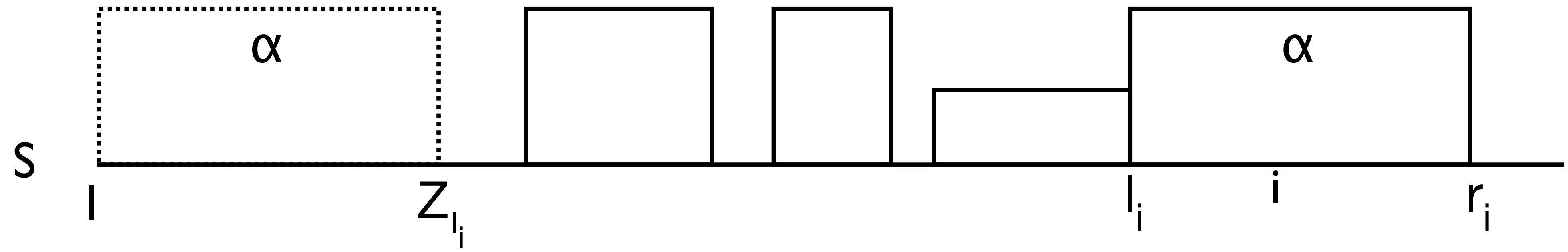


Sketch of the algorithm

- We need to compute $Z_i(S)$, r_i and l_i for every $i > 2$
- In any iteration i, we only need r_j and l_j for j=i-1; i.e just r, l
- If we discover a new Z-box at i, set r to the end of that Z-box, which is the right-most position of any Z-box discovered so far
- Step 0

  Find $Z_2(S)$ by comparing left to right S[2..|S|] and S[1..|S|] until a mismatch is found; $Z_2(S)$ is the length of that string. If $Z_2(S) > 0$ then set $r=r_2$ to $Z_2(S)+1$ and $l=l_2$, else r=l=0
- Induction hypothesis: we have correct $Z_i(S)$ for i up to k-1>1, r, l
  - Next, compute $Z_i(S)$ from the already computed Z values

Compute $Z_i(S)$ from $Z_i(S)$, $2 < j < i$

- Simplest case: inclusion
- E.g. for $k=121$, we have $Z_2(S) \ldots Z_{120}(S)$, and $r_{120}=130$, $l_{120}=100$
  - Thus: a substring of length 31 starting at position 100, matching S[1..31]
  - And: the substring of length 10 starting at 121 must match S[22..31], so Z_{22} could help!
  - For example, if Z_{22} is 3, then Z_{121} must also be 3

Compute $Z_i(S)$ from $Z_j(S)$, $2 < j < i$

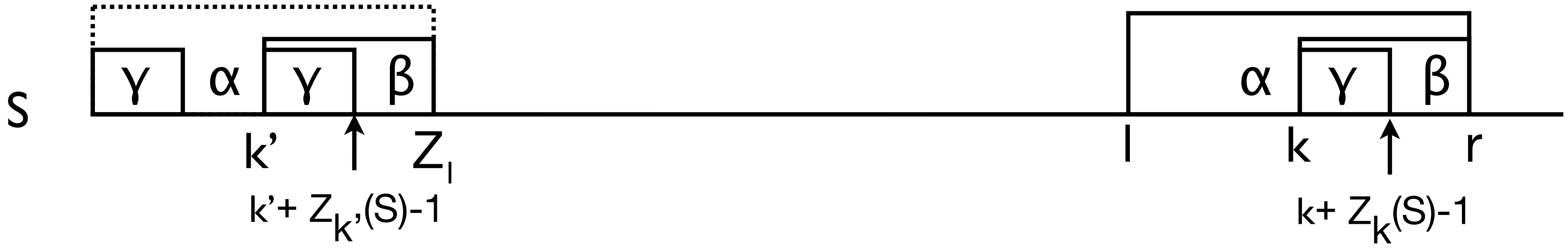
- Given $Z_i(S)$ for all $1 < i \leq k-1$, and the current values of $Z_k(S)$, r, and l; compute the updated r and l
- Step 1:
  - if $k > r$, then find $Z_k(S)$ by comparing the characters starting at k to the characters starting at position 1 in S, until a mismatch is found. The length of the match is $Z_k(S)$. If $Z_k(S) > 0$, set $r=k+Z_k(S)-1$, and $l=k$.

Compute $Z_i(S)$ from $Z_i(S)$, $2 < j < i$

- Step 2
  - If $k \leq r$, then position k is contained in a Z-box, and hence S(k) is contained in a substring S[l..r] (call it α) such that $l > 1$ and α matches a prefix of S.
  - Therefore, character S(k) also appears in position k'=k-l+1 of S.
  - By the same reasoning, the substring S[k..r] (call it β) must match substring S[k'..Z_l(S)]. (*Remember the example with Z_{22}(S), r=121!*)
  - Hence, the substring at position k must match a prefix of S of length at least the *minimum* of $Z_{k'}$(S) and |β| (which is r-k+1).

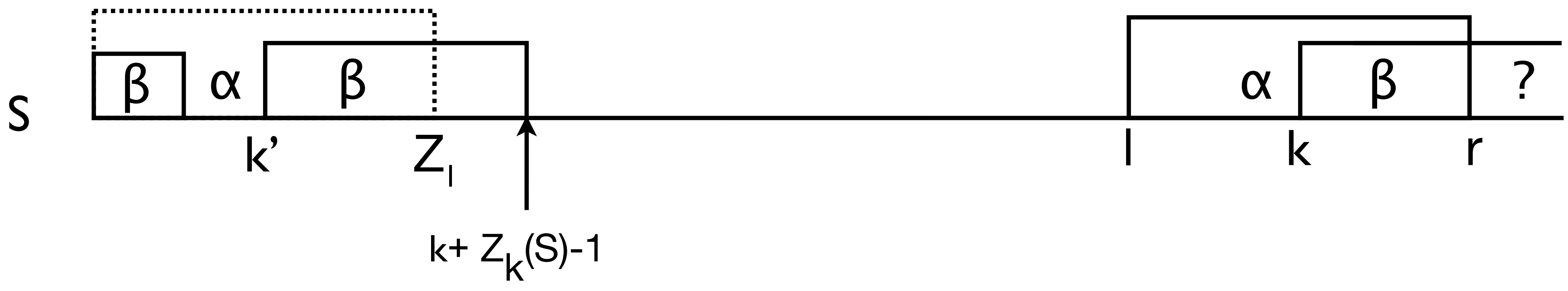
Two cases given the minimum

- Case 1: If $Z_{k'}(S) < |\beta|$
  - then position k is a Z-box (call it γ) contained within a larger Z-box
  - set $Z_k(S)=Z_{k'}(S)$ and leave r and l as they are



Two cases given the minimum

- Case 2: If $Z_{k'}(S) \geq |\beta|$
  - then the entire substring S[k..r] must be a prefix of S and $Z_k(S) \geq |\beta| = r-k+1$
  - However, $Z_k(S)$ may be strictly larger, so compare characters starting at r +1 of S to the characters starting at $|\beta|+1$ of S until a mismatch occurs (*Remember the second smart improvement over the naive method!*)
  - Say the mismatch is at $q \geq r+1$. Then $Z_k(S)=q-k$, $r=q-1$, and $l=k$



Linear time computation

- “The algorithm computes all the $Z_i(S)$ values in O(|S|) time”

  The time is proportional to the number of iterations, |S|, plus the number of character comparisons. Each comparison is either a match or a mismatch. Each iteration that performs any character comparisons at all ends the first time it finds a mismatch; hence there are at most |S| mismatches during the entire algorithm. To bound the number of mismatches, note first that $r_k \geq r_{k-1}$ for every iteration k. Now, let k be an iteration where $q > 0$ matches occur. Then r_k is set to r_k+q at least. Finally, $r_k \leq |S|$ so the total number of matches that can occur during any execution of the algorithm is at most |S|.

- “Computing $Z_i(S)$ on S=P$T finds matches of P in T in O(|T|)”

The Boyer-Moore algorithm

- Like the naive method
  - Align P with T, check whether characters in P and T match
  - After the check is complete, P is shifted rightwards relative to T
- Smarter shifting
  - For an alignment, check whether P occurs in T scanning right-to-left in P
  - The **bad character** shift: shift right beyond the bad character
  - The **good suffix** shift: shift right using the match of the good suffix of P

Boyer-Moore: right-to-left scan

- For any alignment of P against T, check P right-to-left

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								

- For example,
  - P(7)=T(9) ... but P(3) ≠ T(5)
  - Upon a mismatch, shift P right relative to T
- The linear nature of the algorithm is in the shifts
  - Scanning right-to-left still yields an algorithm running in O(nm) time

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
									✔								

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T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
								✓	✓								

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T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
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							✓	✓	✓								

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
						✔	✔	✔	✔								

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Boyer-Moore: right-to-left scan

- For any alignment of P against T, check P right-to-left

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
					✘	✓	✓	✓	✓								

- For example,
  - P(7)=T(9) ... but P(3) ≠ T(5)
  - Upon a mismatch, shift P right relative to T
- The linear nature of the algorithm is in the shifts
  - Scanning right-to-left still yields an algorithm running in O(nm) time

B-M: The bad character shift

- The basic idea
  - Suppose the rightmost character in P is y, aligned to x in T with x≠y
  - If x is in P, then we can shift P so that the rightmost x is below x in T
  - If x is not in P, then we can shift P completely beyond the x in T
- Possibly sublinear matching: not all characters in T may need to be compared

B-M: The bad character shift

- Store the right-most position of each character

  For each character x in the alphabet, let R(x) be the rightmost position of x in P. R(x) is defined to be 0 if x is not in P.

- The **bad character shift rule** makes use of R

  Suppose for an alignment of P against T, the rightmost n-i characters of P match against T, but the character at P(i) is a mismatch with the character T(k). Now, we can shift P right by *max*[1,i-R(T(k))] places; i.e. if the right-most occurrence in P of the character T(k) is in position j < i (possibly with j=0), then shift P so that the character j of P is below character k of T. Else, shift P by 1.

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								

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T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
					✘	✔	✔	✔	✔								

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T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
					✘	✔	✔	✔	✔								

R(“T”)=1

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T =	X	P	B	C	T	B	X	A	B	P	Q	X	C	T	B	P	Q
P =			T	P	A	B	X	A	B								
			1	2	3	4	5	6	7								
					✘	✓	✓	✓	✓								
					T	P	A	B	X	A	B						

R("T")=1

B-M: the good suffix rule

- The basic idea:
  - Given the character T(k) against which P mismatches,
  - Take the good suffix t of P, i.e. the part that matched against T
  - Look in P for the left-most copy t’ of t, such that the character k’ to the immediate left of t’ differs from T(k); *else the shift would yield the same mismatch!*
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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

T = P R S T A B S T U B A B V Q X R S T

P = Q C A B D A B D A B

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T =	P	R	S	T	A	B	S	T	U	B	A	B	V	Q	X	R	S	T
P =			Q	C	A	B	D	A	B	D	A	B						
			1	2	3	4	5	6	7	8	9	10						
										✗	✓	✓						

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T =	P	R	S	T	A	B	S	T	U	B	A	B	V	Q	X	R	S	T
P =			Q	C	A	B	D	A	B	D	A	B						
			1	2	3	4	5	6	7	8	9	10						
										✗	✓	✓						

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T =	P	R	S	T	A	B	S	T	U	B	A	B	V	Q	X	R	S	T
P =			Q	C	A	B	D	A	B	D	A	B						
			1	2	3	4	5	6	7	8	9	10						
										✗	✓	✓						

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T =	P	R	S	T	A	B	S	T	U	B	A	B	V	Q	X	R	S	T
P =		Q	C	A	B	D	A	B	D	A	B							
		1	2	3	4	5	6	7	8	9	10							
									✗	✓	✓							

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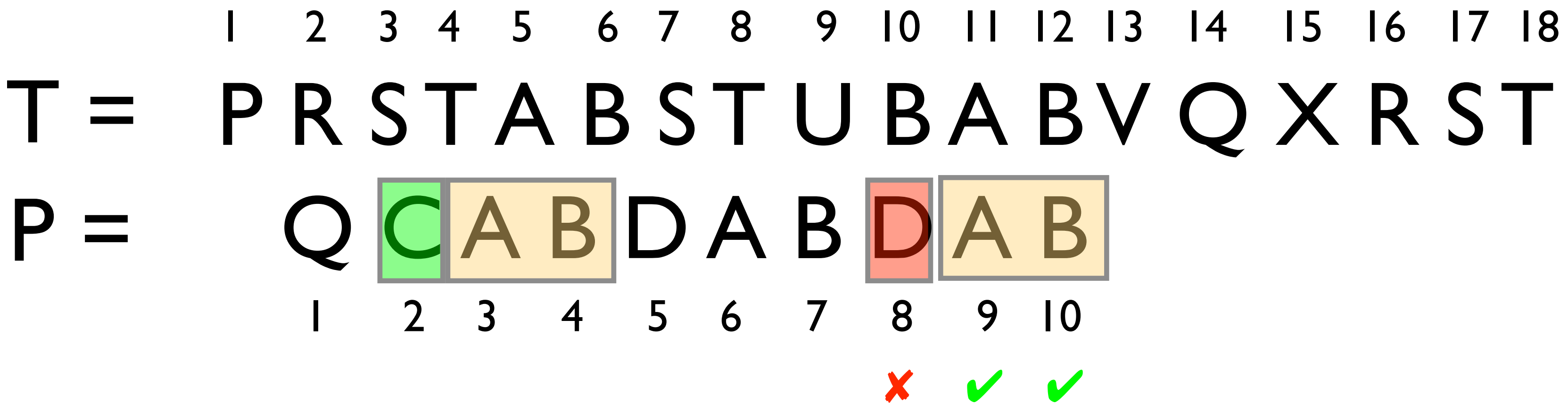

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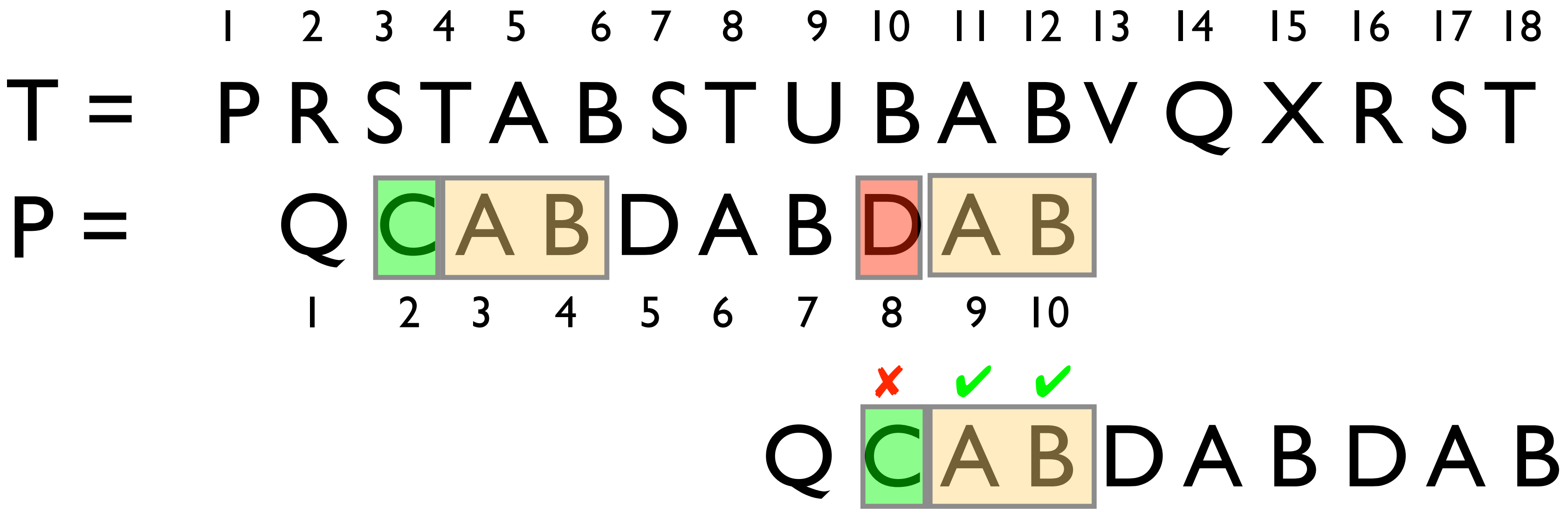

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T = P R S T A B S T U B A B V Q X R S T

P = Q C A B D A B D A B

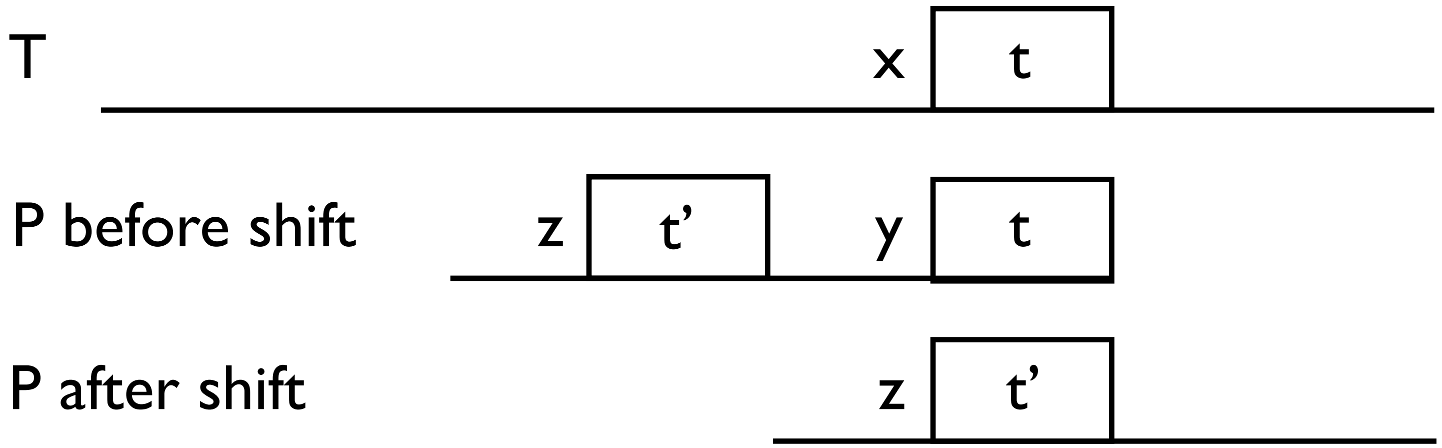
1 2 3 4 5 6 7 8 9 10

✗ ✓ ✓

Q C A B D A B D A B

B-M: the good suffix rule

Suppose for a given alignment of P and T, a substring t of T matches a suffix of P, but a mismatch occurs at the next comparison to the left. Then find, if it exists, the right-most copy t' of t in P such that t' is not a suffix of P and the character to the left of t' in P differs from the character to the left of t in P. Shift P to the right so that the substring t' in P is below substring t in T. If t' does not exist, then shift the left end of P past the left end of t in T by the least amount so that a prefix of the shifted pattern matches a suffix of t in T. If no such shift is possible, then shift P by n places to the right. If an occurrence of P is found, then shift P by the least amount so that a proper prefix of the shifted P matches a suffix of the occurrence of P in T. If no such shift is possible, then shift P by n places, past t in T.



Preprocessing

- We need some preprocessing for the good suffix rule
  - We need to compute the positions of copies of suffixes of P
  - whereby a copy differs from the suffix in its immediate left character
- Definition

For each i, L(i) is the largest position less than n such that string P[i..n] matches a suffix of P[1..L(i)]. Let L(i) be zero if there is no position satisfying the conditions. For each i, L'(i) is the largest position less than n such that string P[i..n] matches a suffix of P[1..L'(i)] and such that the character preceding the suffix is not equal to P(i-1). Let L'(i) be 0 if there is no position satisfying the conditions.

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P =	C	A	B	D	A	B	D	A	B
	1	2	3	4	5	6	7	8	9

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P =	C	A	B	D	A	B	D	A	B	L(8)=6
	1	2	3	4	5	6	7	8	9	

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  - whereby a copy differs from the suffix in its immediate left character
- Definition

For each i, L(i) is the largest position less than n such that string P[i..n] matches a suffix of P[1..L(i)]. Let L(i) be zero if there is no position satisfying the conditions. For each i, L’(i) is the largest position less than n such that string P[i..n] matches a suffix of P[1..L’(i)] and such that the character preceding the suffix is not equal to P(i-1). Let L’(i) be 0 if there is no position satisfying the conditions.

P =	C	A	B	D	A	B	D	A	B	L(8)=6	L’(8)=3
	1	2	3	4	5	6	7	8	9		

Preprocessing

- Computing L'(i)
  - For string P, $N_j(P)$ is the length of the longest suffix of the substring P[1...j] that is also a suffix of the full string P.

P =	C	A	B	D	A	B	D	A	B
	1	2	3	4	5	6	7	8	9

- We can compute $N_i(S)$ from $Z_i(S)$
  - Recall that $Z_i(S)$ is the length of the longest substring of S that starts at i and is a *prefix* of S
  - $N_i(S)$ is the reverse of Z: if P^r is the reverse of P, then $N_j(P)=Z_{n-j+1}(P^r)$
  - Hence we can obtain the values for N using the linear algorithm for Z

Preprocessing

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  - For string P, $N_j(P)$ is the length of the longest suffix of the substring P[1...j] that is also a suffix of the full string P.

P =	C	A	B	D	A	B	D	A	B
	1	2	3	4	5	6	7	8	9

$N_3(P)=2$

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  - For string P, $N_j(P)$ is the length of the longest suffix of the substring P[1...j] that is also a suffix of the full string P.

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1 2 3 4 5 6 7 8 9

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P =	C	A	B	D	A	B	D	A	B	$N_3(P)=2$
	1	2	3	4	5	6	7	8	9	

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  - For string P, $N_j(P)$ is the length of the longest suffix of the substring P[1...j] that is also a suffix of the full string P.

P =	C	A	B	D	A	B	D	A	B
	1	2	3	4	5	6	7	8	9

$N_3(P)=2$ $N_6(P)=5$

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  - For string P, $N_j(P)$ is the length of the longest suffix of the substring P[1...j] that is also a suffix of the full string P.

P =	C	A	B	D	A	B	D	A	B
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  - Hence we can obtain the values for N using the linear algorithm for Z

Preprocessing: from N to L’

- Z-based Boyer-Moore for obtaining L’(i) from $N_i(P)$

```
for i := 1 to n do L’(i) := 0
for j := 1 to n-1 do
    begin
    i := n - $N_j(P)$ + 1
    L’(i) := j
    end
```

- Intuition
  - We have computed the lengths of the longest suffixes as $N_j(P)$
  - Cycle over P right-to-left, looking at where the longest suffixes start
  - Assign to L’(i) the largest index j such that $N_j(P) = |P[i..n]| = (n-i+1)$
  - Those L’(i) for which there is no such index have been initialized to 0.

The final preprocessing detail ...

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- Let l’(i) denote the longest suffix of P[i..n] that is also a prefix of P, if one exists. If none exists, let l’(i) be zero.

The final preprocessing detail ...

- Let l’(i) denote the longest suffix of P[i..n] that is also a prefix of P, if one exists. If none exists, let l’(i) be zero.
- Once more, all the preprocessing and rules:
  - Bad character rule: given a mismatch on x in T, shift P right to align with an x in P (if any)
    - Compute R(x), the right-most occurrence of x in P
  - Good suffix rule: shift P right to a copy of the matching suffix but with a different character to its immediate left
    - Use $Z_j(P)$ to compute $N_j(P)$, the length of the longest suffix of P[1..j] that is a suffix of P
    - Use $N_j(P)$ to compute L’(i), the largest position less than n s.t. P[i..n] matches a suffix of P[1..L’(i)]
    - Compute l’(i), to deal with the case when we have L’(i) = 0 or when an occurrence of P is found

The Boyer-Moore algorithm

[Preprocessing stage]

Given the pattern P

Compute L’(i) and l’(i) for each position i of P

and compute R(x) for each character x ∈ Σ

[Search stage]

```
k := n
while k ≤ m do
    i := n
    h := k
    while i > 0 and P(i) = T(h) do
        i := i-1
        h := h-1
    if i = 0 then
        report an occurrence of P in T ending at position k
        k := k+n - l’(2)
    else
        shift P (increase k) by the maximum amount determined by the bad character rule
        and the good suffix rule
```

Next week

Knuth-Morris-Pratt,
extensions to tree-based search
and inexact matching

More matching

Knuth-Morris-Pratt,
extensions to tree-based search
and inexact matching

What we saw last week

- Improve search by making smart “jumps” upon mismatches
- Jumps based on computation of the Z-function
  - At each position *i* in a String, the score $Z_i(S)$ indicates the size of the Z-box at that position,
  - i.e. the length of the string starting at i which is a prefix of S (of that size)
- Booyer-Moore algorithm
  - Bad character, good suffix rules

What we will look at today

- Knuth-Morris-Pratt algorithm
  - Left-to-right matching of P against T
  - Smart jumps: look at suffix of P[1..i] which is also a prefix, jump beyond the prefix (like the good suffix role in B-M)
- Aho-Crosick algorithm: search using sets of patterns
  - Definition of keyword trees over sets of patterns
  - Extension of KMP, using failure links in keyword trees
- Inexact matching
  - A little bit of dynamic programming to find edit distances

The Knuth-Morris-Pratt algorithm

- Basic idea
  - Shift smarter than the naive method does

	1	2	3	4	5	6	7	8	9
P=	A	B	C	X	A	B	C	D	E

  - A mismatch with P(8) means we can shift 4 places
  - Deduction on P alone: no need to know T, or how P and T are aligned
- Complexity of the algorithm
  - The algorithm is linear, not -possibly- sublinear like Boyer-Moore
  - Extension: the Aho-Corasick algorithm for matching sets of patterns

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  - Deduction on P alone: no need to know T, or how P and T are aligned
- Complexity of the algorithm
  - The algorithm is linear, not -possibly- sublinear like Boyer-Moore
  - Extension: the Aho-Corasick algorithm for matching sets of patterns

The Knuth-Morris-Pratt algorithm

- Basic idea
  - Shift smarter than the naive method does

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				⇧					

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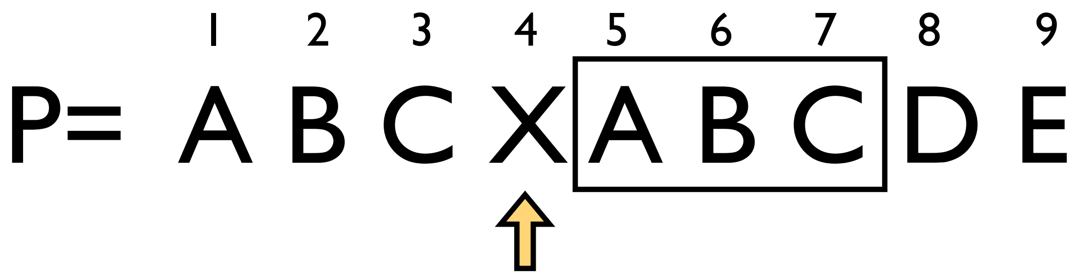

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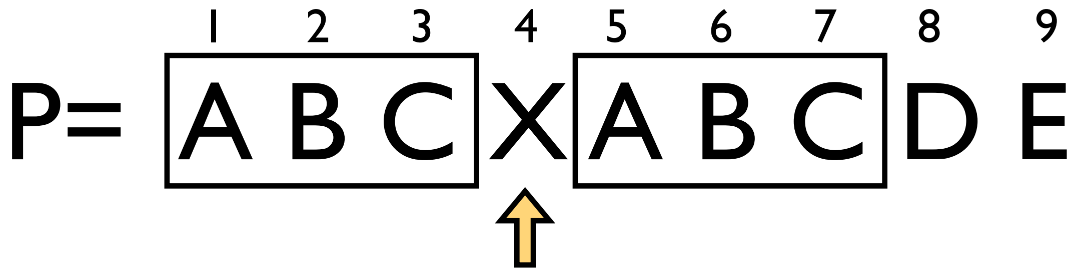

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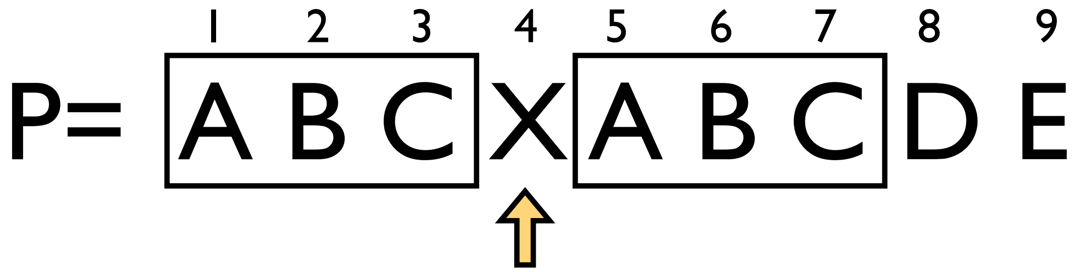

  - A mismatch with P(8) means we can shift 4 places (*like good suffix rule!*)
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  - The algorithm is linear, not -possibly- sublinear like Boyer-Moore
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Formalization

- Definition

For each position i in P, define $sp_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P.

	1	2	3	4	5	6	7	8	9	10	11
P=	A	B	C	A	E	A	B	C	A	B	D

- Optimization

For each position i in P, define $sp'_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P, with the added condition that characters P(i+1) and P(sp'_i+1) are unequal.

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sp	0	0	0	1	0	1	2	3	4	2	0

- Optimization

For each position i in P, define $sp'_i(P)$ to be the length of the longest proper suffix of P[1...i] that matches a prefix of P, with the added condition that characters P(i+1) and $P(sp'_i+1)$ are unequal.

The Knuth-Morris-Pratt shift rule

- Alignment of P and T, left-to-right matching
- The shift rule:

For any alignment of P and T, if the first mismatch (comparing from left to right) occurs in position i+1 of P and position k of T, then shift P to the right (relative to T) so that P[1..sp_i'] aligns with T[k-sp_i'..k-1]. In other words, shift P exactly i+1-(sp_i'+1)=i-sp_i' places to the right, so that character sp_i'+1 of P will align with character k in T. In the case that an occurrence of P has been found (no mismatch), shift P by n-sp_i' places.

Computing the sp_i' values

- Preprocessing using the Z values

  Position j > 1 maps to i if $i=j+Z_j(P)-1$. That is, j maps to i if i is the right end of a Z-box starting at j.

- Z-based Knuth-Morris-Pratt

```
for i := 1 to n do
    $sp_i$' := 0;
for j := n downto 2 do
    i := j + $Z_j$(P) -1;
    $sp_i$' := $Z_j$
```

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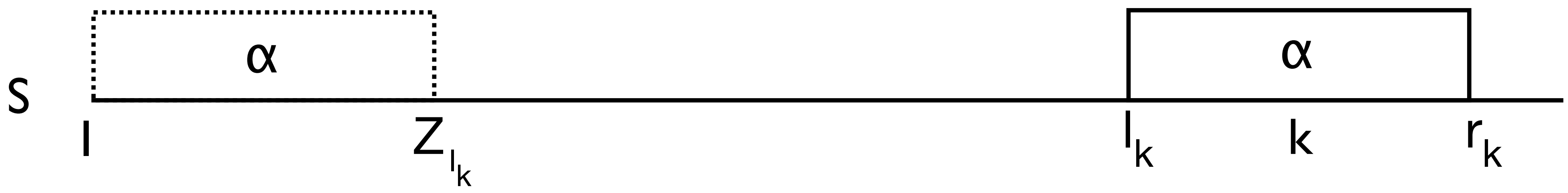

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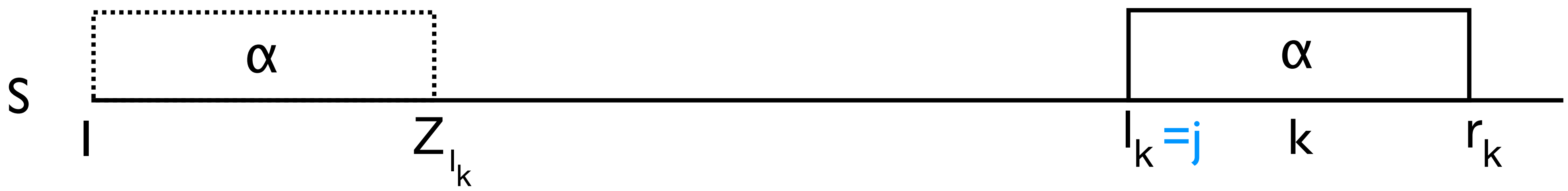

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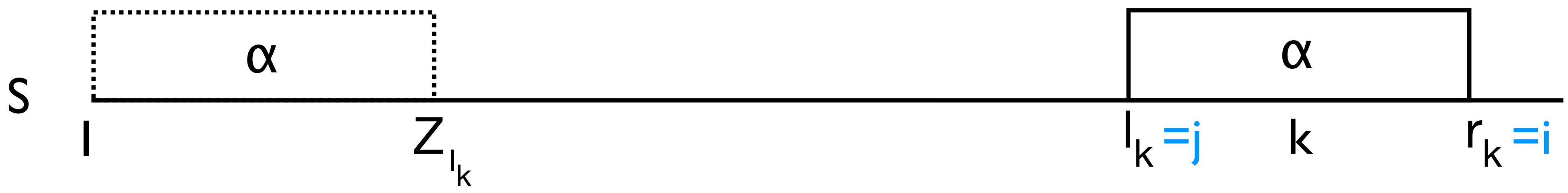

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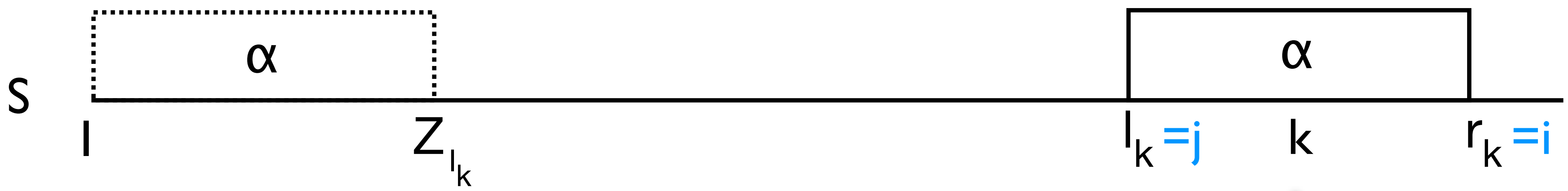

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for i := 1 to n do
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$sp'_i(P)$ is the length of the longest proper suffix of P[1...i] i.e. the length of the Z-box that starts at j (the suffix)

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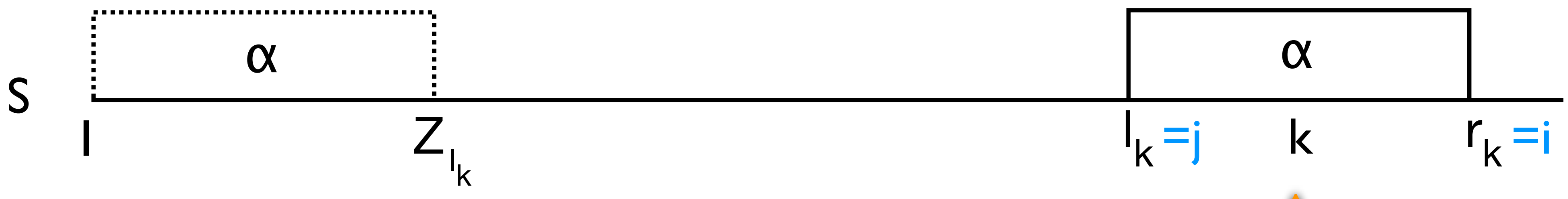

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The algorithm

- Preliminaries
  - Shifts through pointers: p points into P, c points into T
  - For each position i from 1 to n+1, define the failure function F'(i) to be $sp'_{i-1} + 1$ (and define $F(i)=sp_{i-1} +1$); let sp_0' and sp_0 be 0.
- The algorithm

```
preprocess P to find $F'(k)=sp'_{k-1} + 1$ for k from 1 to n+1
          c := 1;
          p := 1;
          while c + (n-p) ≤ m do
                    while P(p) = T(c) and p ≤ n
                              p := p+1;
                              c := c+1;
                    if p = n+1 then
                              report an occurrence of P starting at position c-n of T
                    if p = 1 then c:=c+1
                    p := F'(p)
```

Matching sets of patterns

- Generalizing the exact matching problem
  - Match against a set of patterns *P* = {P1,...,Pz}
  - If n is the total length of the patterns, and m the length of T, then we can solve this problem in *O(n+zm)* time
- Representing a set of patterns as a keyword tree

The keyword tree for a set of patterns *P* is a rooted directed tree satisfying the following conditions: (1) each edge is labeled with exactly one character; (2) any two edges out of the same node have distinct labels; (3) every pattern Pi in *P* maps to some node v in the tree such that the characters on the path from the root of the tree to v exactly spell out Pi, and every leaf of the tree is mapped to by some pattern in *P*.

Keyword tree

P = {robot, roller, science, school}

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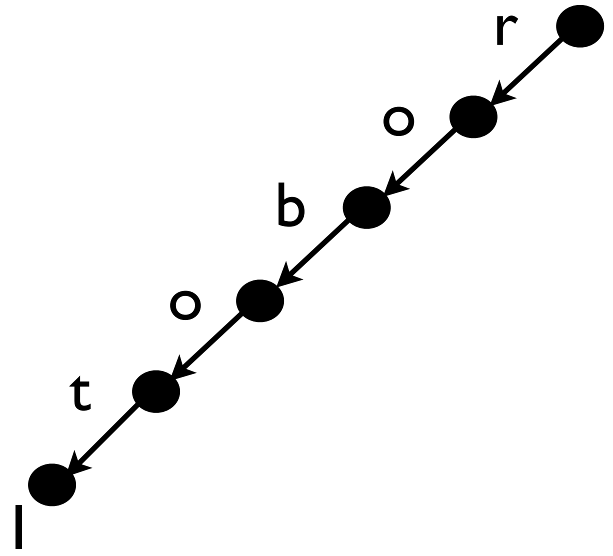

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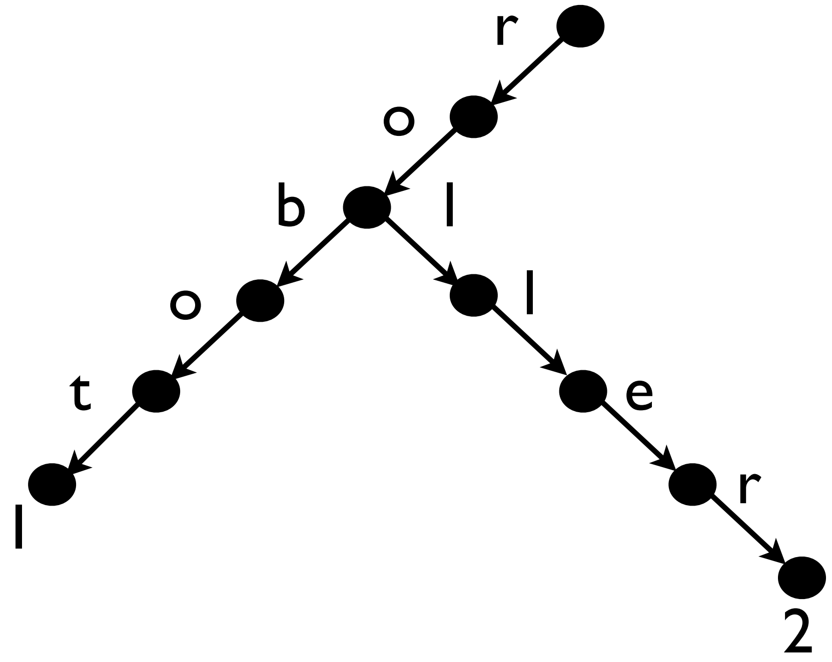

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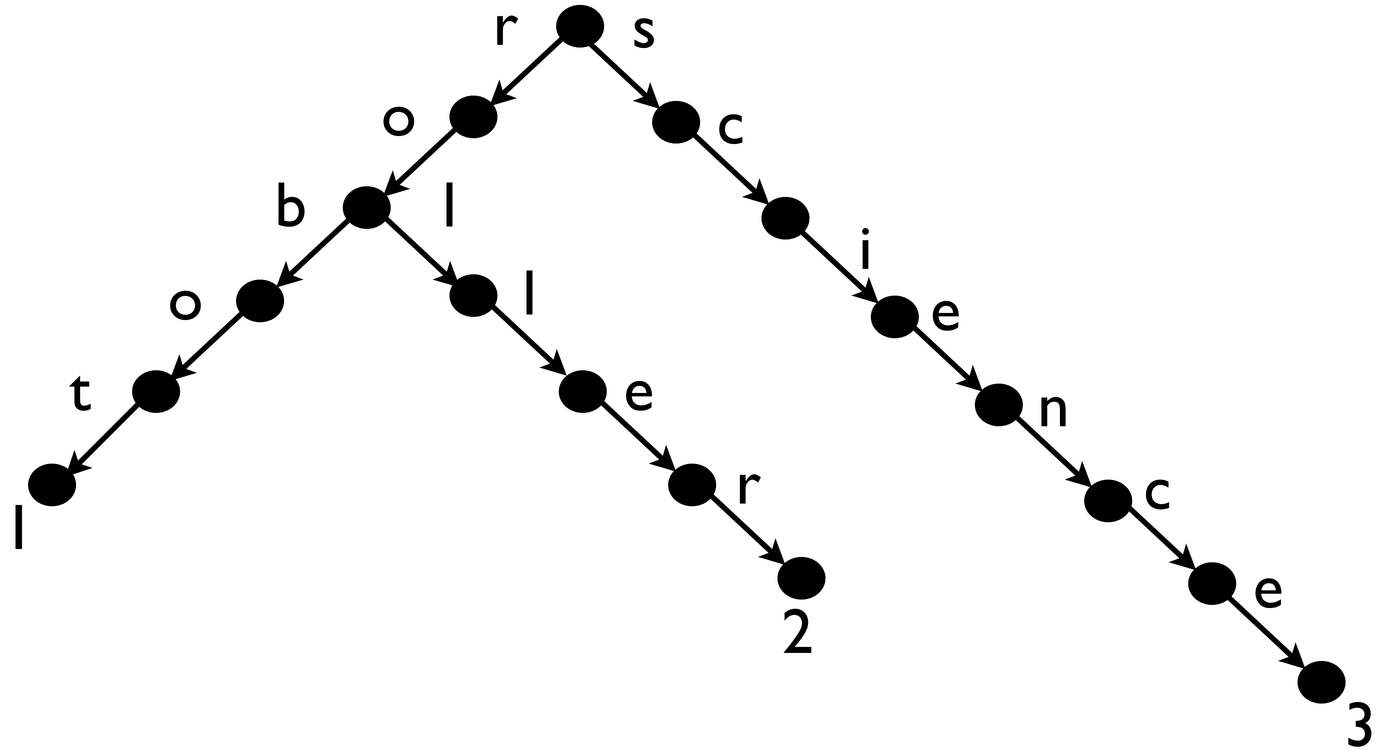

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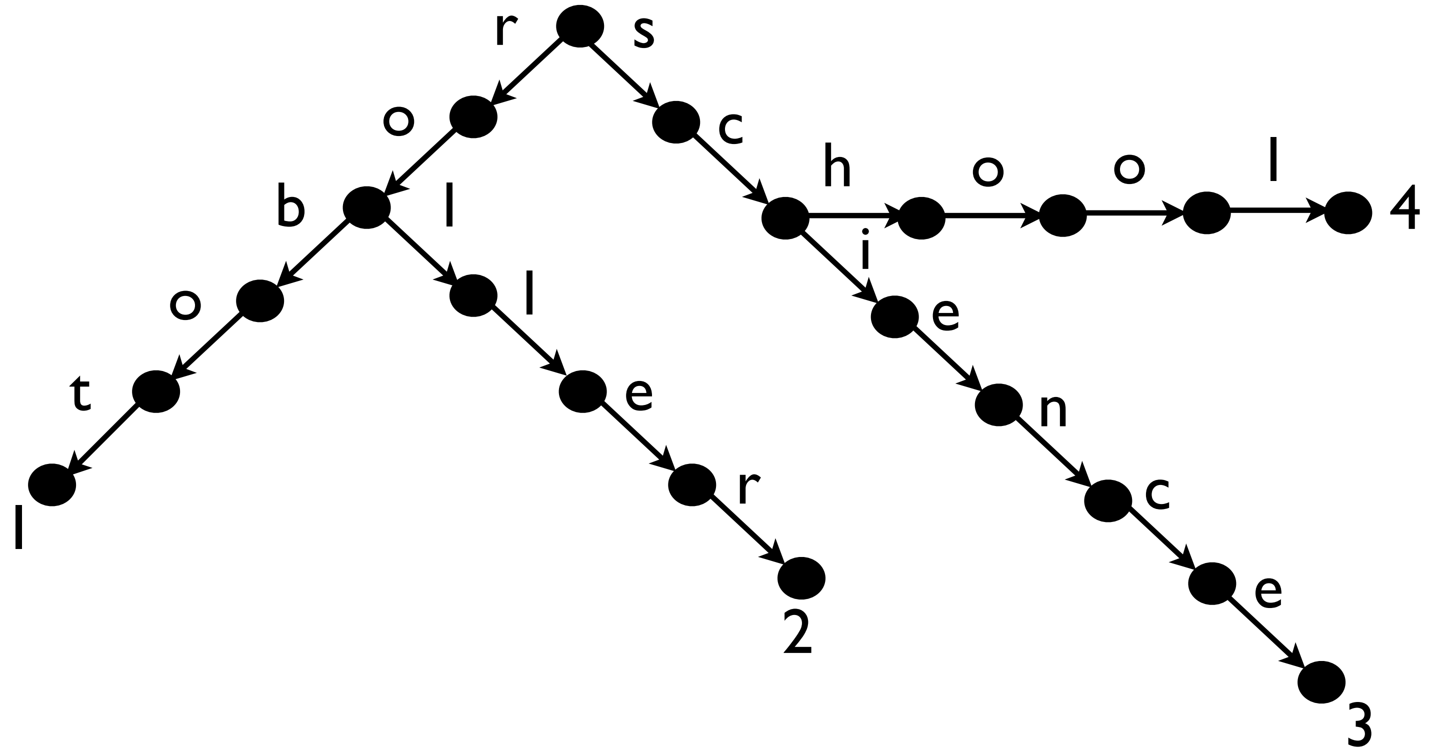


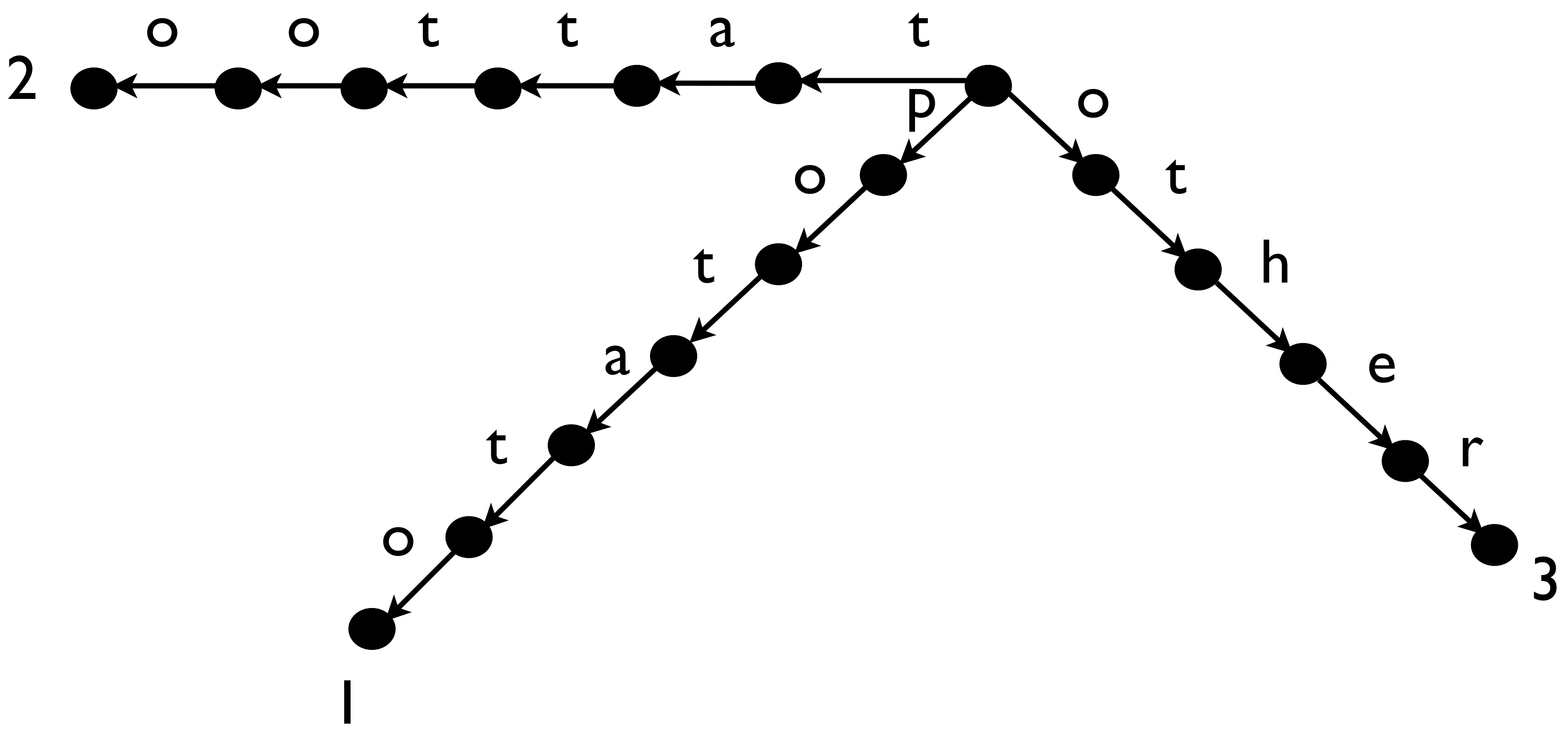
Naive use of keyword trees

- E.g: search for occurrences of patterns beginning at T(1)
  - Follow the unique path in the tree matching a prefix of T as far as
  - If we get to a node number i, pattern Pi has been found starting at T(1)
  - (More patterns are found if some patterns are prefixes of other patterns)
- General approach
  - To find patterns starting at position l in T, following unique paths
  - Numbered nodes indicate patterns found at position l in T
  - Given l, traversing the tree takes time proportional to m and n; incrementing l from 1 to m and traversing the tree thus takes *O(mn)* time
- Specialization: the dictionary problem
  - Given a known dictionary and a text, find a string from T in the dictionary

Speeding up set matching

- Speed up through a generalization of Knuth-Morris-Pratt
    - Knuth-Morris-Pratt improved on the naive method by longer shifts
    - Aho-Corasick shifts the position in l skipping over parts of paths
- Basic idea behind the Aho-Corasick algorithm
    - Generalization of Knuth-Morris-Pratt, particularly: generalize *sp*
    - Label each node v with the concatenation of the chars on its path, L(v)
    - For each node v, lp(v) is the length of the longest proper suffix of L(v) that is a prefix of some pattern in *P*.
    - Given α the lp(v)-length suffix of L(v), there is a unique node in the tree labeled with α. Let this node be n_v; (v,n_v) is called the **failure link.**

Failure links



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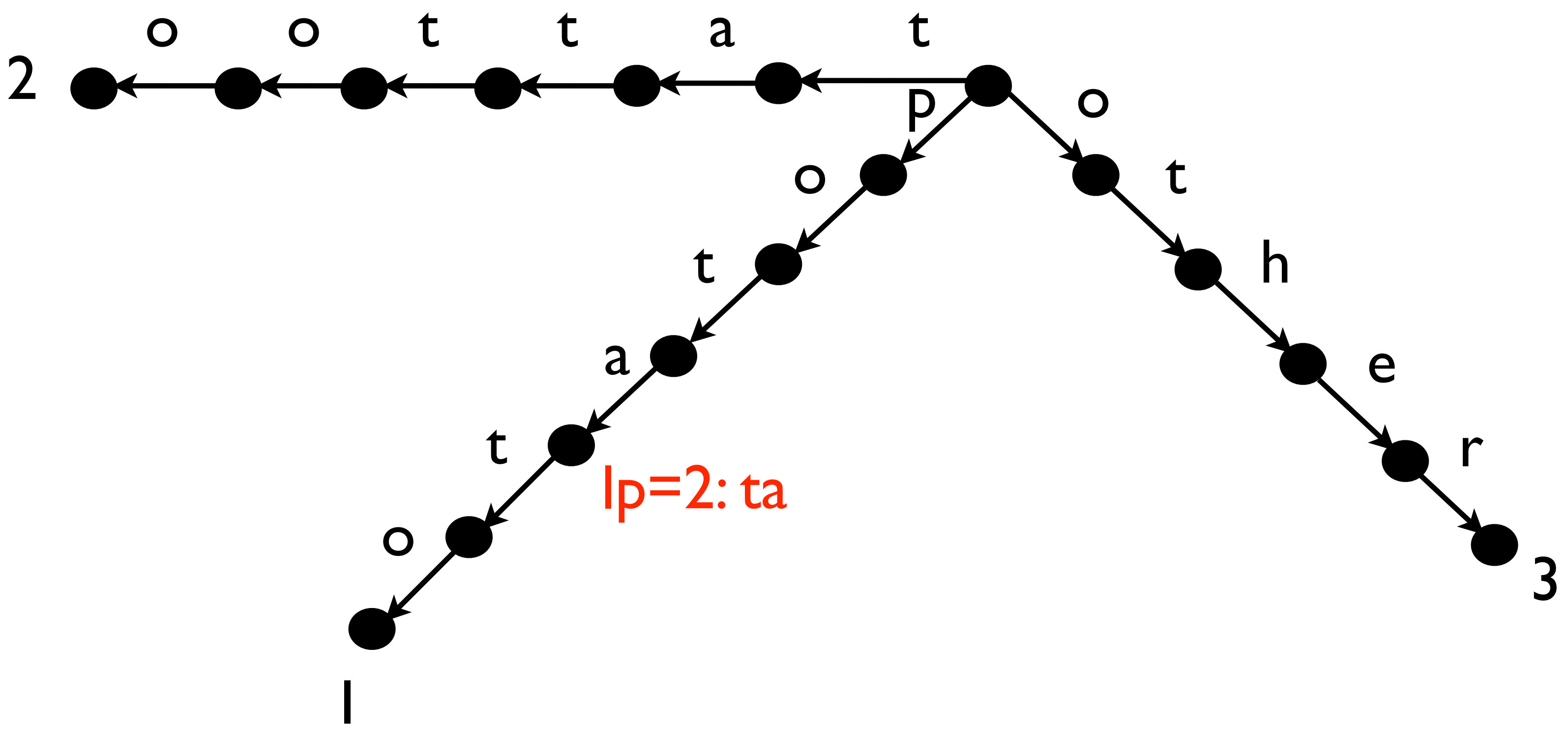

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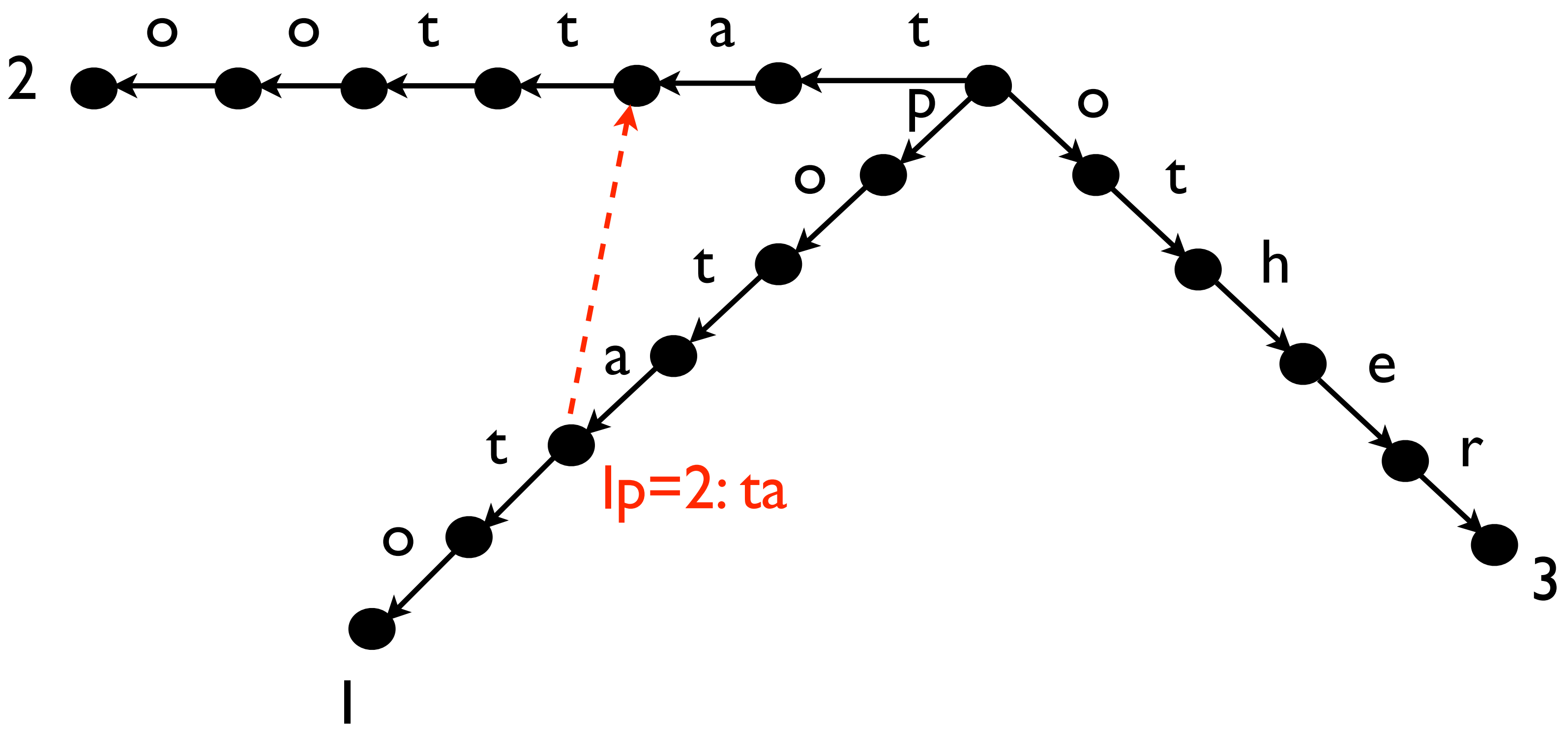

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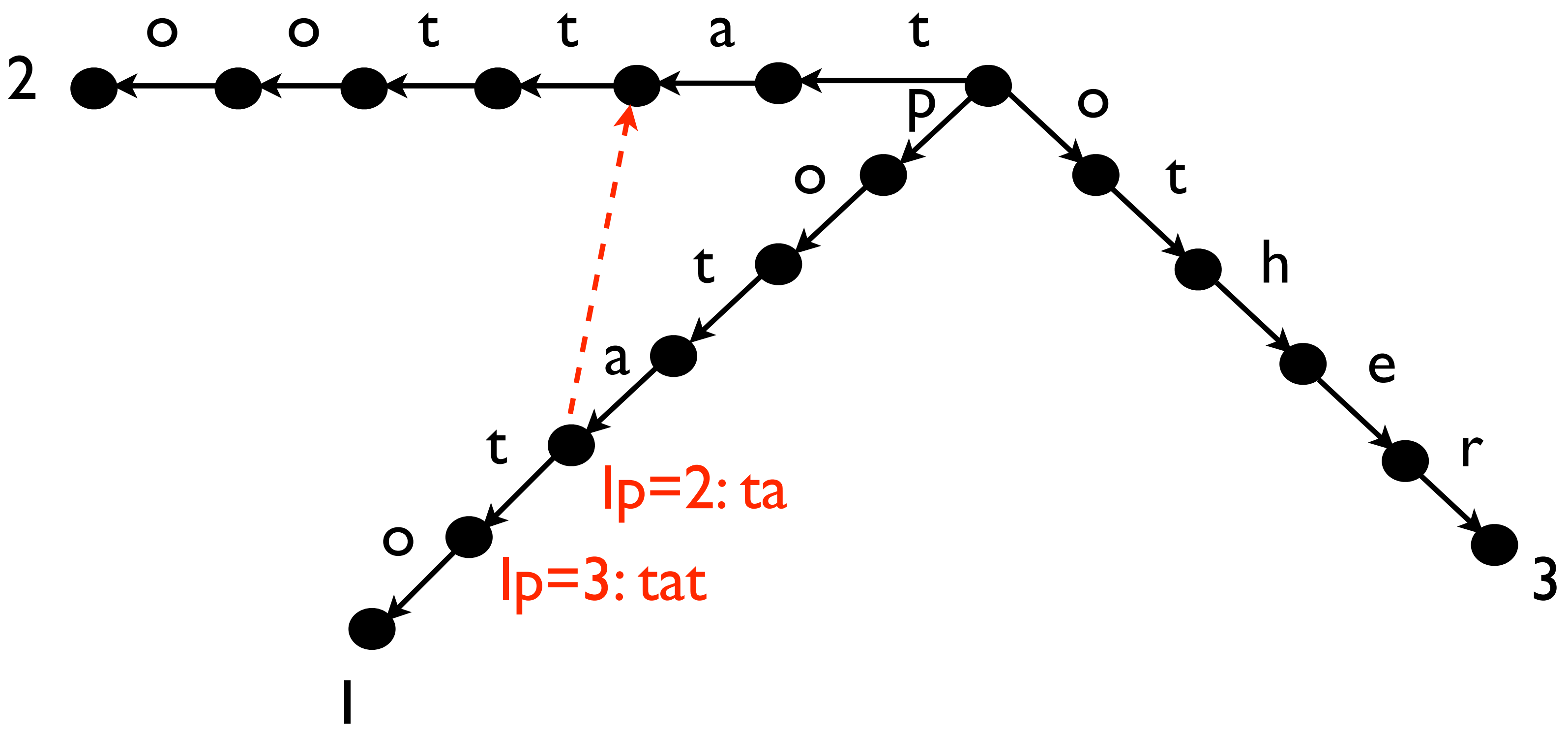

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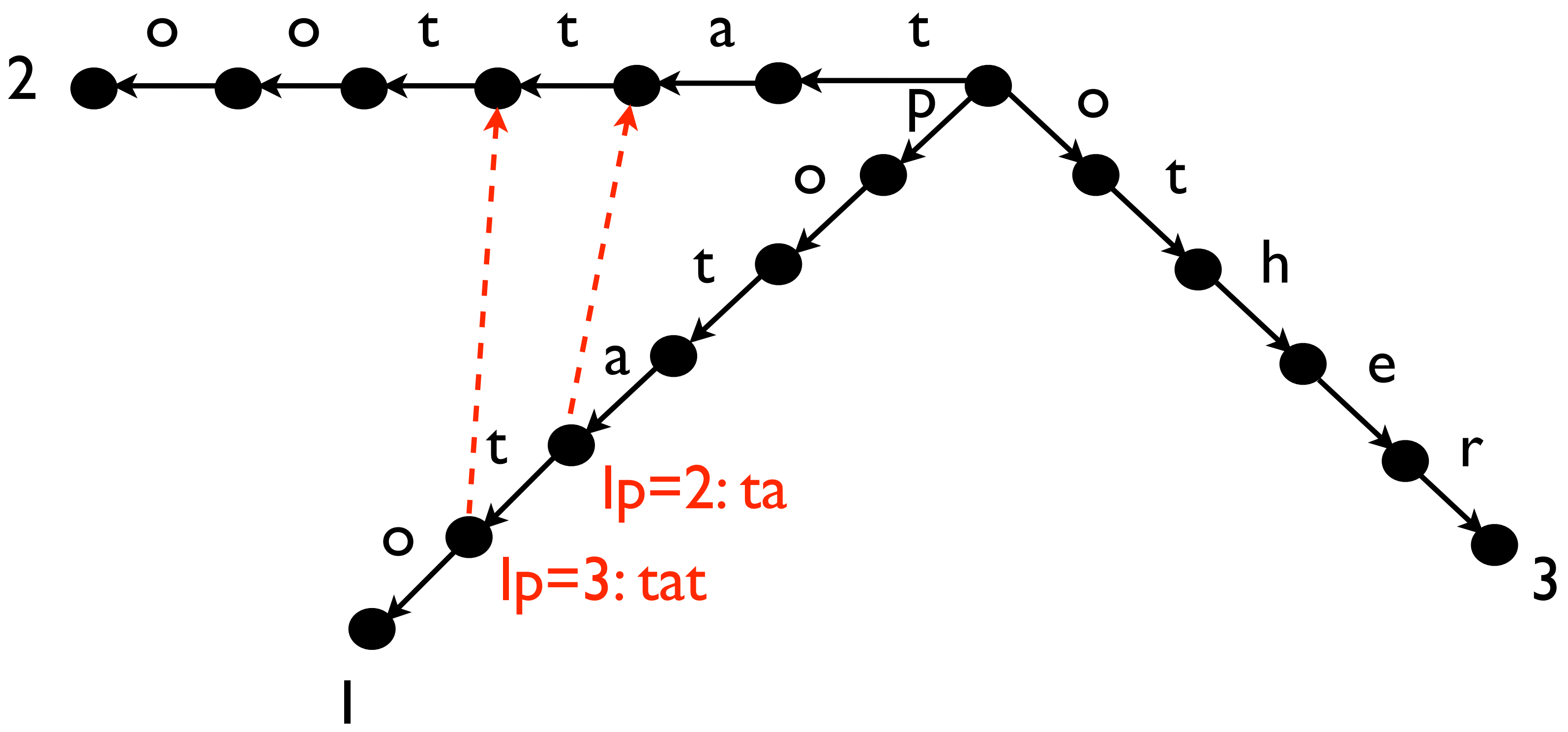

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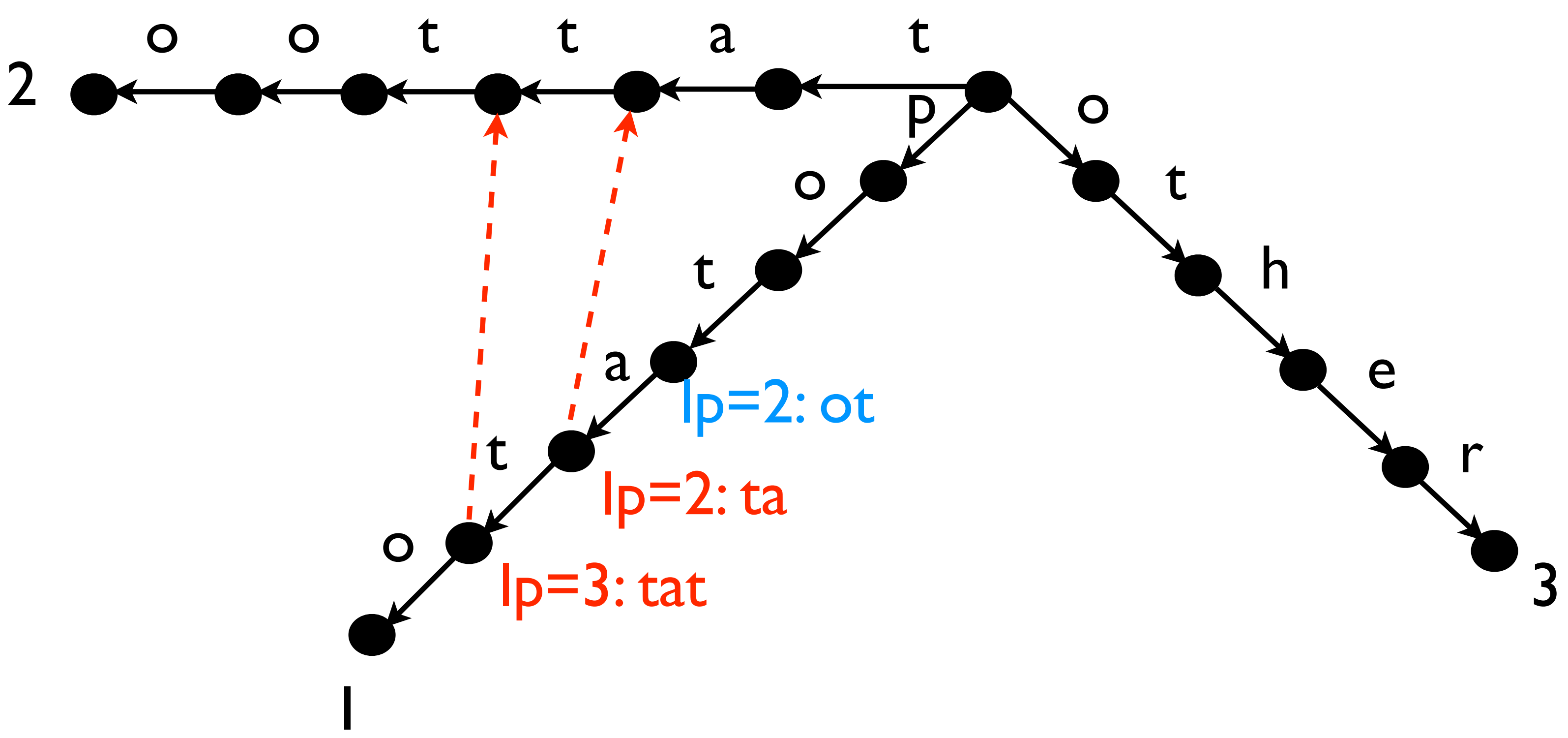

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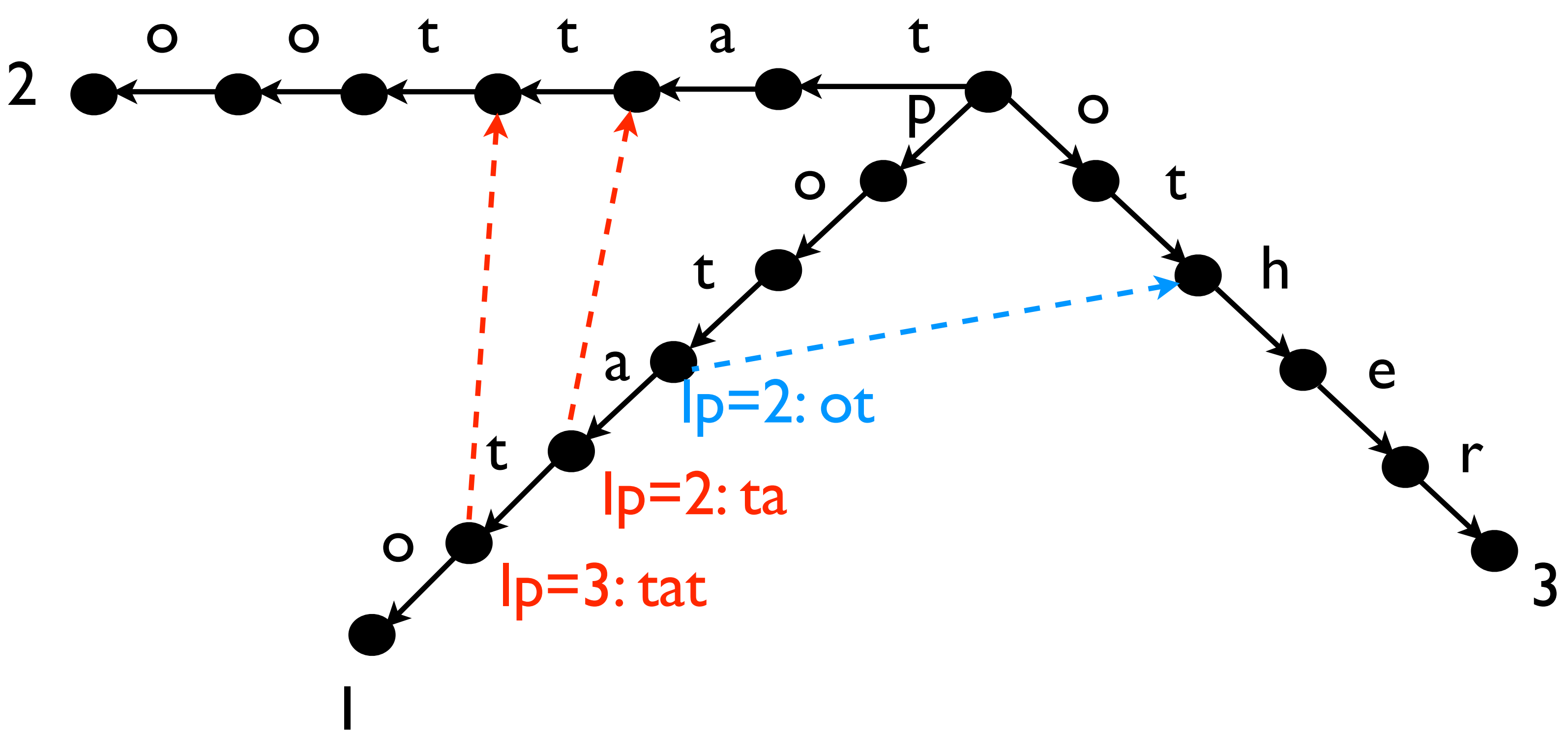

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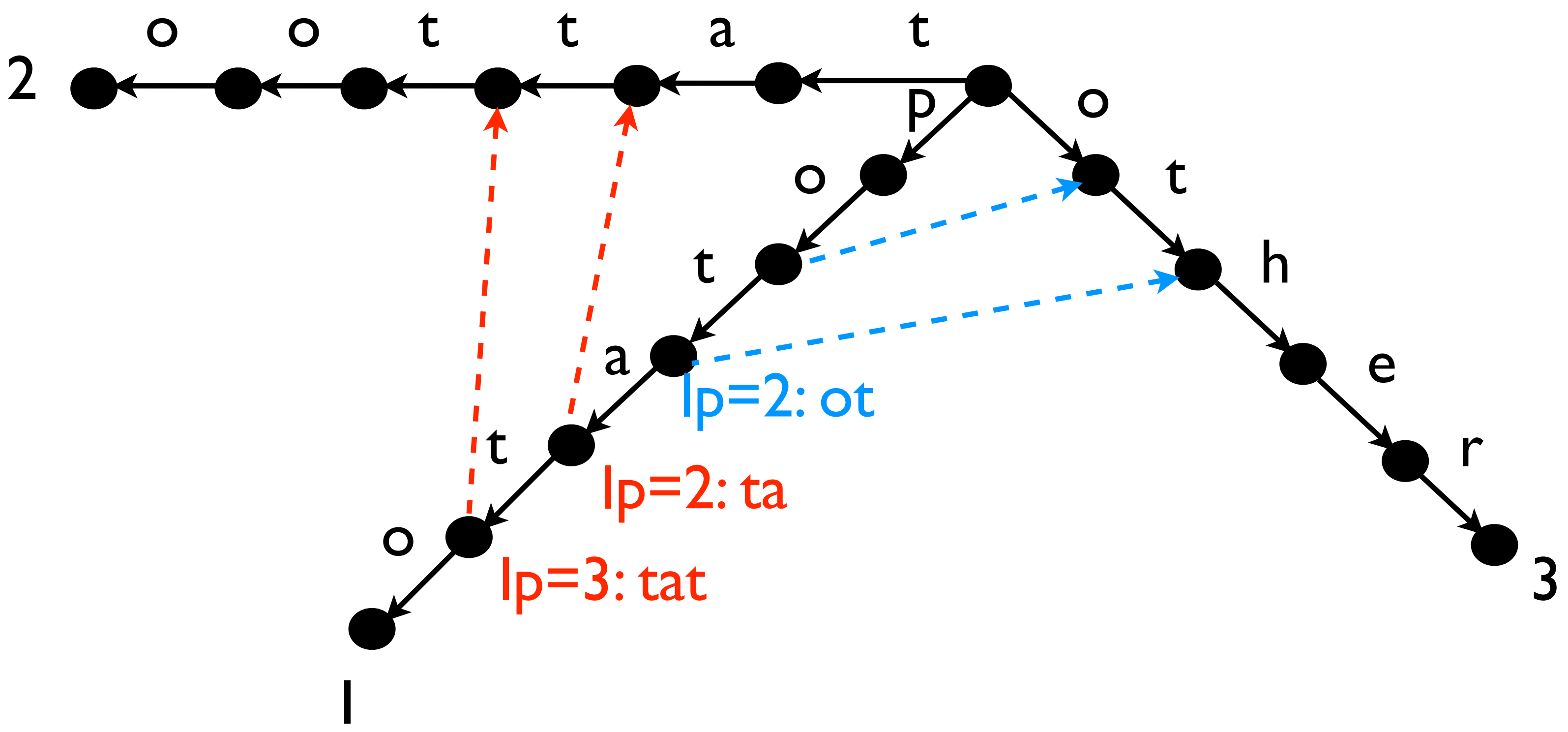

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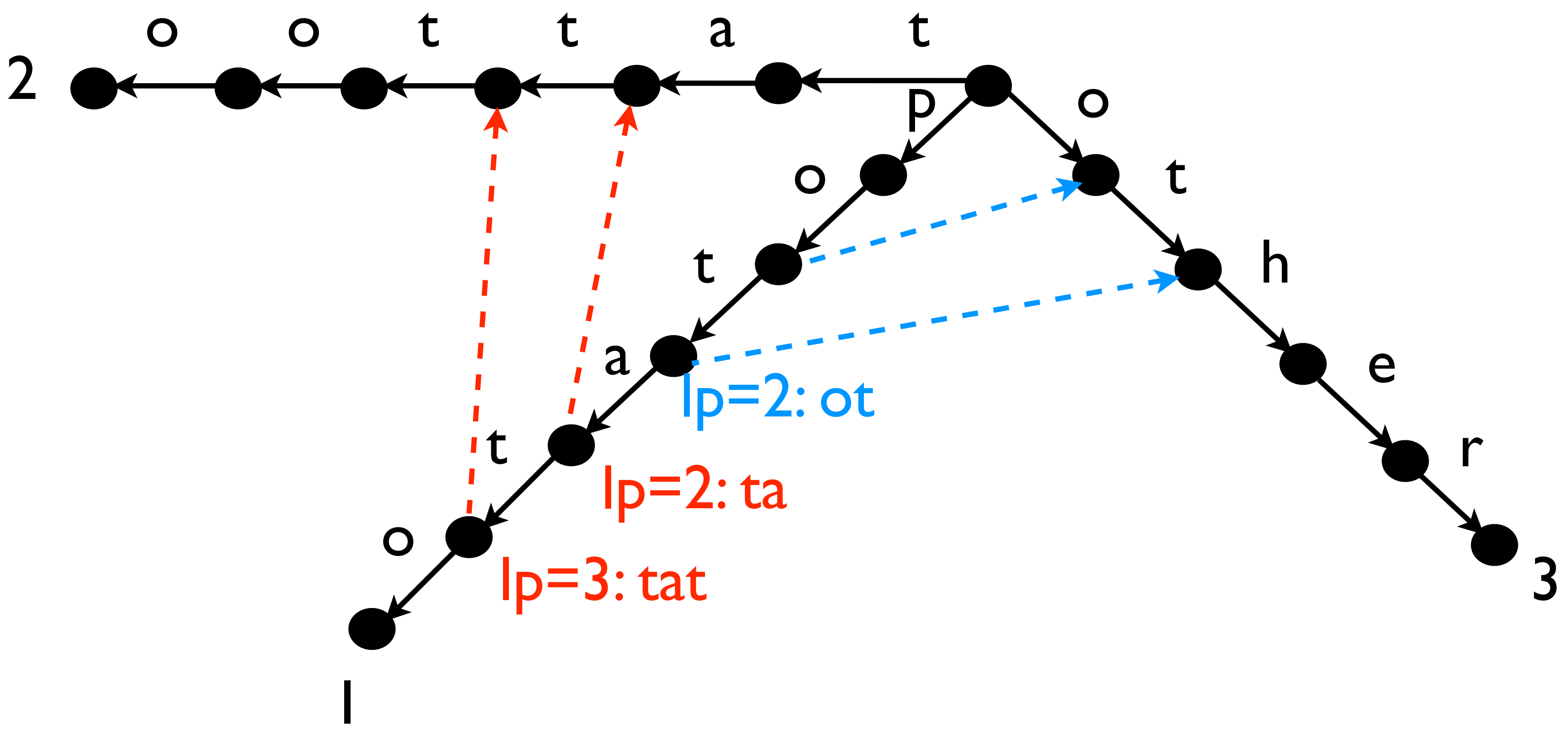

- We can compute failure links in linear time

The Aho-Corasick algorithm

```
l := 1;
c := 1;
w := root;
repeat
    while there is an edge (w,w') labeled T(c)
        if w' is numbered by pattern i or there is
        a directed path of failure links from w' to a node numbered with i
            then report that Pi occurs in T ending at position c;
        w := w' and c := c+1
    w := nw and l = c - lp(w)
until c > n
```

Inexact matching

- So far: *exact* matching problem
    - Inexact matching: approximation of pattern in text
    - From substring to subsequence matching
- The **edit distance** between two strings
    - Transformation: insertion, deletion, substitution of material

R	I	M	D	M	D	M	M	I
v		i	n	t	n	e	r	
w	r	i		t		e	r	s

- A string over the alphabet I, D, R, M, that describes a transformation of one string to another is called an *edit transcript* of the two strings

Edit distance

- Edit distance

  The **edit distance** between two strings is defined as the minimum number of edit operations - insert, delete, substitute - needed to transform the first string into the second. (Matches are not counted.)

- The edit distance problem

  The edit distance problem is to compute the edit distance between two given strings, along with an optimal edit transcript that describes the transformation.

Dynamic programming

- For strings S1 and S2, D(i,j) is the edit distance between S1[1..i] and S2[1..j]. Let n=|S1| and m=|S2|.
- Dynamic programming:
  - Recurrence relation: recursive relationship between i and j in D(i,j)
  - Tabular computation: memoization technique for computing D(i,j)
  - Traceback: computing the optimal edit transcript from the table

Recurrence relation

- Recursive relationship
  - Relate value of $D(i,j)$ for i and j positive, and values of D with index pairs smaller than i, j.
  - Base conditions: $D(i,0) = i$ and $D(0,j) = j$
- Recurrence relation for $D(i,j)$ for $i,j > 0$
  - $D(i,j) = \min[D(i-1,j)+1, D(i,j-1)+1, D(i-1,j-1)+t(i,j)]$
  - where $t(i,j)$ is 1 if $S1(i) \neq S2(j)$ and 0 if $S1(i)=S2(j)$
- Complexity issue
  - The number of recursive calls grows exponentially with n and m
  - But, there are only $(n+1) * (m+1)$ combinations of i and j, hence only $(n+1) * (m+1)$ *distinct* recursive calls

Bottom-up tabular computation

- (n+1) * (m+1) table
- Base: compute D(i,j) for the smallest possible values of i and j
- Induction: compute D(i,j) for increasing values of i and j, one row at the time

D(i,j)			w	r	i	t	e	r	s
		0	1	2	3	4	5	6	7
	0	0	1	2	3	4	5	6	7
v	1	1	1	2	3	*			
i	2	2							
n	3	3							
t	4	4							
n	5	5							
e	6	6							
r	7	7							

Base: D(i,0) = i, D(0,j) = j

Step: D(i,j) = min[D(i-1,j)+1, D(i,j-1)+1, D(i-1,j-1)+t(i,j)], t(i,j) is 1 if S1(i) ≠ S2(j) and 0 if S1(i)=S2(j)

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e	6	6							
r	7	7							

D(1,1) = min[D(0,1)+1, D(1,0)+1, D(0,0)+t(1,1)]
= min[2,2,0+1] = 1

Base: D(i,0) = i, D(0,j) = j

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D(1,2) = min[D(0,2)+1, D(1,1)+1, D(1,1)+t(1,2)]
= min[3,2,1+1] = 2

Base: D(i,0) = i, D(0,j) = j

Step: D(i,j) = min[D(i-1,j)+1, D(i,j-1)+1, D(i-1,j-1)+t(i,j)], t(i,j) is 1 if S1(i) ≠ S2(j) and 0 if S1(i)=S2(j)

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= min[3,2,1+1] = 2

D(1,3) = min[D(0,3)+1, D(1,2)+1, D(0,2)+t(1,3)]
= min[4,3,2+1] = 3

Base: D(i,0) = i, D(0,j) = j

Step: D(i,j) = min[D(i-1,j)+1, D(i,j-1)+1, D(i-1,j-1)+t(i,j)], t(i,j) is 1 if S1(i) ≠ S2(j) and 0 if S1(i)=S2(j)

Traceback

- Pointer-based approach:
  - When computing (i,j), set a pointer to the cell yielding the minimum
  - If (i,j) = D(i,j-1)+1 set a pointer from (i,j) to (i,j-1): ←
  - If (i,j) = D(i-1,j)+1 set a pointer from (i,j) to (i-1,j): ↑
  - If (i,j) = D(i-1,j-1)+t(i,j) set a pointer from (i,j) to (i-1,j-1): ↖
  - There may be several pointers if several predecessors yield the same minimum value
- To retrieve the optimal edit transcripts
  - Trace back the path(s) from (n,m) to (0,0)
  - A horizontal edge (←) represents an *insertion*
  - A vertical edge (↑) represents a *deletion*
  - A diagonal edge (↖) represents a *match* if S1(i)=S2(j), and a *substitution* if S1(i)≠S2(j)

Time-complexity

- Filling the table costs *O(nm)* time
  - To fill one cell takes a constant number of cell examinations, arithmetic operations, and comparisons
  - The table consists of n by m cells, hence O(nm) time
- Retrieving the optimal path(s) costs *O(n+m)* time

Conclusions

- Exact matching problem
  - Naive method compares character by character, single shift of P against T
  - Optimization through smarter shifting; base information for smarter shifting is provided by Z-boxes, computable in linear time
  - Boyer-Moore algorithm, Knuth-Morris-Pratt algorithm
  - We can generalize the single pattern problem to sets of patterns, using the Aho-Corasick algorithm (based on K-M-P) on keyword trees
- Inexact matching problem
  - Looking for subsequences rather than substrings
  - Dynamic programming approach to establishing edit distance between two strings, specified as an edit transcript