Assignment no 11

Title: Approximation Algorithm

Suppose you are given a set of **n** points in a 2-dimensional plane. Your task is to find the smallest possible circle that encloses all **n** points.

Hint:

- An unique circle can be identified by 3 2-dimentional points (high school geometry)
- You can use concept of high-school geometry to check whether a point lies inside or outside or on the circle
- Enclosing points mean you can allow points be inside or on the circle.

Tasks:

1. Generate **n** random 2D points using rand function in some predefined range
2. Devise an algorithm to find the optimal solution
3. Devise an algorithm that can find an approximate solution to the problem in O(n) time *(see below)*
4. Analyze the complexity of both the methods.

Solution using an Approximation Algorithm:

One approach to solve this problem is to use the Welzl's algorithm, also known as the randomized incremental algorithm. This algorithm builds the minimum enclosing circle incrementally, starting with the minimum enclosing circle of the first two points, and adding the remaining points one by one.

The key idea of the algorithm is to maintain a set of points that lie on the boundary of the current minimum enclosing circle, and a set of candidate points that could be added to the boundary. At each step, the algorithm selects a random candidate point, and checks if it lies inside the current minimum enclosing circle. If it does not, the algorithm updates the boundary with the new point and recursively applies the algorithm to the remaining candidate points. If all candidate points have been checked and none of them lie outside the current minimum enclosing circle, the algorithm returns the current minimum enclosing circle.

While this algorithm does not guarantee the optimal solution, it can provide a good approximation to the optimal solution in a reasonable amount of time. In fact, it has been shown that the algorithm can achieve a solution that is within a factor of 2 of the optimal solution.