CS-215: Experiment 9B

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1. Frequency Modulation and Demodulation

Aim

To compute the Frequency Modulation of given message signal and compare the reconstructed signal and original signal.

Theoretical Background

The Hilbert Transform is defined as

$$H\{x(t)\} = \hat{x}(t) = \frac{1}{\pi}\int_{\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau = x(t) * \frac{1}{\pi t}$$

The analytic equation can be written as

$$a(t) = x(t) + j\hat{x}(t)$$

So, the instantaneous phase is

$$\phi(t) = \arctan\frac{\hat{x}(t)}{x(t)}$$

and frequency is

$$f(t) = \frac{1}{2\pi}\cdot\frac{d\phi(t)}{dt}$$

Thus, this hilbert transform can be used to demodulate an FM signal, where the message signal is in the argument of the carrier as its frequency.

Methodology

- The message, its integral and the modulated signal is generated.
- Then, the frequency spectrum of message and modulated carrier is generated.
- The Hilbert transform of modulated carrier is computed using the **`hilbert`** function which returns the complex analytical function.
- The argument of the Hilbert transform is differentiated and appropriately scaled, thus resulting in the original signal.

Code

```
clearvars
clc
%%
fs = 1000;
dt = 1 / fs;
t = (-0.2:dt:0.2)';
t0 = 0.1;
%%
% The Original Message Signal

m = sinc(100*t);
m(t < -t0) = 0; m(t > t0) = 0;

mPlt = gca;
plot(t, m, 'k'); grid on
mPlt.PlotBoxAspectRatio = [2 1 1];
mPlt.YLim = [-.4 1.2];
title('Message Signal', ...
    'FontWeight','normal','FontSize',14);
xlabel('{\itt} (seconds)');

print(gcf, 'msg.eps', '-depsc');
%%
% Integrated Message Signal

mCum = cumtrapz(t, m);

mCumPlt = gca;
plot(t, mCum, 'k'); grid on
mCumPlt.PlotBoxAspectRatio = [2 1 1];
xlabel('{\itt} (seconds)');

print(gcf, 'msg_int.eps', '-depsc');
%%
% The Modulated Carrier

fc = 250;
Kf = 100;
s = cos(2*pi*fc*t + 2*pi*Kf*mCum);

sPlt = gca;
plot(t, s, 'k'); grid on
sPlt.PlotBoxAspectRatio = [2 1 1];
sPlt.YLim = [-1.5 1.5];
```

```
title('Modulated Carrier', ...
    'FontWeight','normal','FontSize',14);
xlabel('{\itt} (seconds)');

print(gcf, 'mod.eps', '-depsc');
%%
% The Frequency Spectrum of Message Signal

mft = fft(m);
N = length(m);
mfreq = (-N/2:N/2 - 1)' * (fs/N);

mftPlt = gca;
plot(mfreq, abs(fftshift(mft)), 'k'); grid on
mftPlt.PlotBoxAspectRatio = [2 1 1];
mfreqRange = mfreq(end) / 2;
mftPlt.XLim = [-mfreqRange mfreqRange];
title('Spectra of Message Signal', ...
    'FontWeight','normal','FontSize',14);
xlabel('{\itf} (Hz)');

print(gcf, 'msg_ft.eps', '-depsc');
%%
% The Frequency Spectrum of Modulated Signal

sft = fft(s);
N = length(s);
sfreq = (-N/2:N/2 - 1)' * (fs/N);

sftPlt = gca;
plot(sfreq, abs(fftshift(sft)), 'k'); grid on
sftPlt.PlotBoxAspectRatio = [2 1 1];
sftPlt.XLim = [0 sfreq(end)];
title('Spectra of Modulated Signal', ...
    'FontWeight','normal','FontSize',14);
xlabel('{\itf} (Hz)');

print(gcf, 'mod_ft.eps', '-depsc');
%%
% Demodulating the FM Signal

sH = hilbert(s) .* exp(-1i * 2*pi*fc * t);
mR = (1/(2*pi*Kf)) * [0; diff(unwrap(angle(sH))) * fs];

mRPlt = gca;
plot(t, mR, 'r', t, m, 'k'); grid on

```

```
mRPlt.PlotBoxAspectRatio = [2 1 1];
mRPlt.YLim = [-.4 1.2];
legend({'Demodulated Signal','Original Signal'})
xlabel('{\itt} (seconds)');

print(gcf, 'demod.eps', '-depsc');
%%
% The Error Signal

errPlt = gca;
plot(t, abs(mR - m), 'k'); grid on
errPlt.PlotBoxAspectRatio = [2.5 1 1];
errPlt.YLim = [-.05 .2];
title('Absolute Error in Demodulation', ...
    'FontWeight','normal','FontSize',14);
xlabel('{\itt} (seconds)');

print(gcf, 'demod_err.eps', '-depsc');
```

Input Description

The message signal is

$$m(t) = \begin{cases} \text{sinc}(100t) & |t| \leq t_0 \\ 0 & else \end{cases}$$

where the normalized sinc function is $\frac{\sin(\pi t)}{\pi t}$.

The carrier signal is $\cos(2\pi f_c t)$.

Carrier freq: $f_c = 250$ Hz, $t_0 = 0.1$ sec, Freq sensitivity: $k_f = 100$.

Sampling frequency, $f_s = 1000$ Hz.

Result

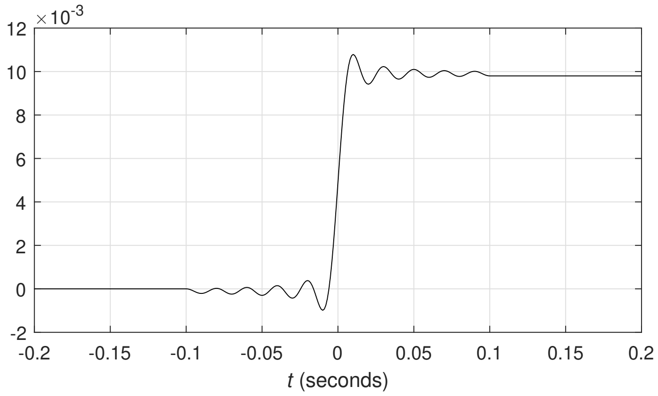

Figure 1.1: *Integral of the Message Signal*

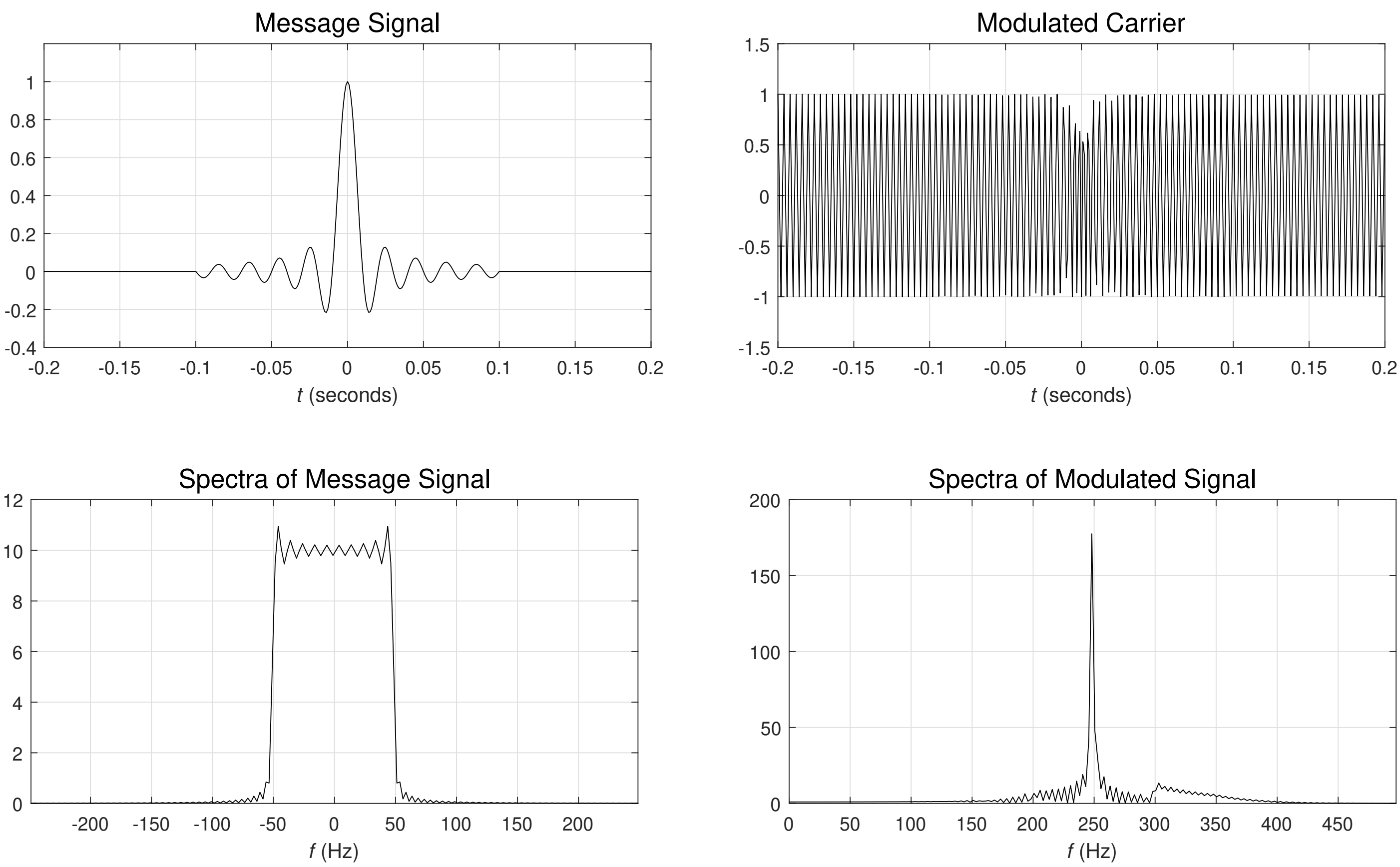

Figure 1.2: *Message and Modulated Signal*

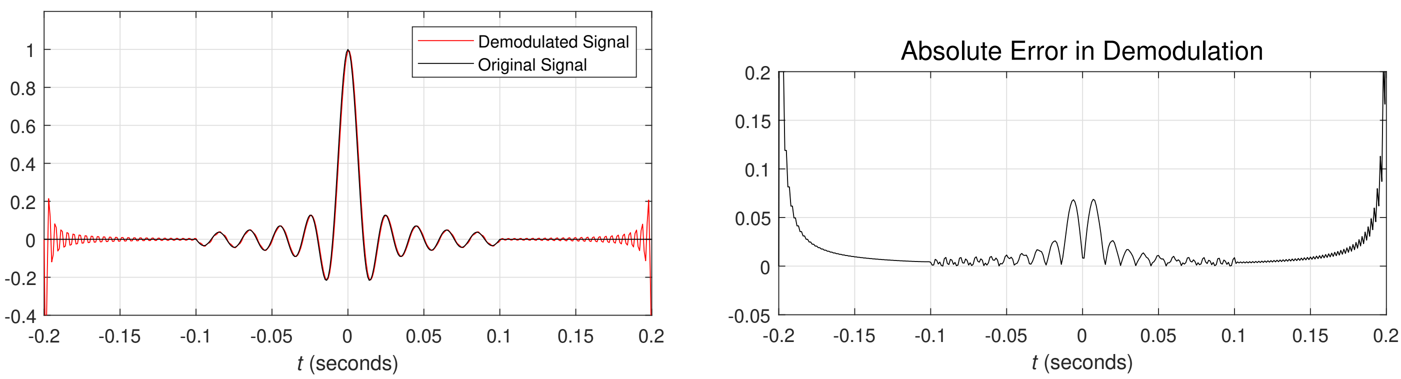

Figure 1.3: *Demoldulated Signal and error*

Conclusion and Discussion

The demodulated signal is thus, a close approximation of the original signal.

Note that there is significance divergence at the ends of the time range in demodulated signal. This is similar to Gibb's Phenomenon and can be minimized by using a higher sampling rate.