CS-215: Experiment 5B

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1. Differential System Response

Aim

Compute the complete system response, steady state and transient response to the following linear differential system:

$$\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 3\frac{\mathrm{d}x}{\mathrm{d}t} - x$$

Theoretical Background

There are several ways to solve a linear differential system. Two of those are –

1. Complete Solution = Homogenous Solution (CF) + Particular Integral (PI)
2. Complete Response = Natural Response (Zero Input) + Forced Response (Zero State)

As expected, both methods will give same results for the same system.

The first method is mostly used in a mathematical context to solve differential equations, the CF can be viewed as the transient response and PI as the permanent / steady state.

A more intuitive approach for physical systems is the second option. Note that the Natural Response is really the response to a unit impulse input, while Forced Response is response with all initial conditions as zero. [1]

Methodology

- The Zero Input (ZIR) and Zero State (ZSR) response will be computed using DSOLVE function individually.
- The values of the 2 responses will be generated and then plotted.
- The transient response can be found by solving $\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 0$.
- The steady response is the complete response minus transient.

[1] https://electronics.stackexchange.com/q/93064

Code

```
clear all; clc

syms t y(t) x(t) ZIR(t) ZSR(t) c(t) tr(t) ss(t);

x(t) = 0;
ZIR(t) = dsolve(diff(y) + 5 * y == 3 * diff(x) - x, 'y(0) = 3');

x(t) = sin(t) * heaviside(t);
ZSR(t) = dsolve(diff(y) + 5 * y == 3 * diff(x) - x, 'y(0) = 0');

range = [0: .2: 18];
ZIR_ = double(ZIR(range));
ZSR_ = double(ZSR(range));

subplot(2, 1, 1);
plot(range, ZIR_);
% title('Zero Input Response');
hold on;
plot(range, ZSR_, 'r');
hold off;

legend('Zero Input', 'Zero State');
axis([-.5, 18.5, -1, 3.5]);
pbaspect([2, 1, 1]);
set(gca, ...
    'Box'       ,'off'          , ...
    'TickDir'   ,'out'          , ...
    'YGrid'     ,'on'           , ...
    'XTick'     ,[-30: 2: 30]   , ...
    'YTick'     ,[-30: 1: 30]   , ...
    'FontSize'  ,10             );

subplot(2, 1, 2);
plot(range, ZIR_ + ZSR_);
legend('Complete Response');

axis([-.5, 18.5, -1, 3.5]);
pbaspect([2, 1, 1]);
set(gca, ...
    'Box'       ,'off'          , ...
    'TickDir'   ,'out'          , ...
    'YGrid'     ,'on'           , ...
    'XTick'     ,[-30: 2: 30]   , ...
    'YTick'     ,[-30: 1: 30]   , ...
    'FontSize'  ,10             );
```

```

print(gcf, 'complete_response.eps', '-depsc');

tr(t) = dsolve(diff(y) + 5 * y == 0, 'y(0) = 3 - 8/13');
tr_ = double(tr(range));

ss(t) = ZIR + ZSR - tr;
ss_ = double(ss(range));

subplot(2, 1, 1);
plot(range, tr_);
hold on;
plot(range, ss_, 'r');
hold off;

legend('Transient Response', 'Steady State Response');
axis([-.5, 18.5, -1, 3.5]);
pbaspect([2, 1, 1]);
set(gca, ...
    'Box'       ,'off'          , ...
    'TickDir'   ,'out'          , ...
    'YGrid'     ,'on'           , ...
    'XTick'     ,[-30: 2: 30]   , ...
    'YTick'     ,[-30: 1: 30]   , ...
    'FontSize'  ,10             );

subplot(2, 1, 2);
plot(range, ss_ + tr_);
legend('Complete Solution');

axis([-.5, 18.5, -1, 3.5]);
pbaspect([2, 1, 1]);
set(gca, ...
    'Box'       ,'off'          , ...
    'TickDir'   ,'out'          , ...
    'YGrid'     ,'on'           , ...
    'XTick'     ,[-30: 2: 30]   , ...
    'YTick'     ,[-30: 1: 30]   , ...
    'FontSize'  ,10             );

print(gcf, 'transient_steady_response.eps', '-depsc');
```

Input Description

The system is:

$$\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 3\frac{\mathrm{d}x}{\mathrm{d}t} - x$$

Initial Conditions:

$$y(0^-) = 3$$

$$x(t) = \sin t \cdot u(t)$$

The time interval is taken to be $[0, 18]$.

Result

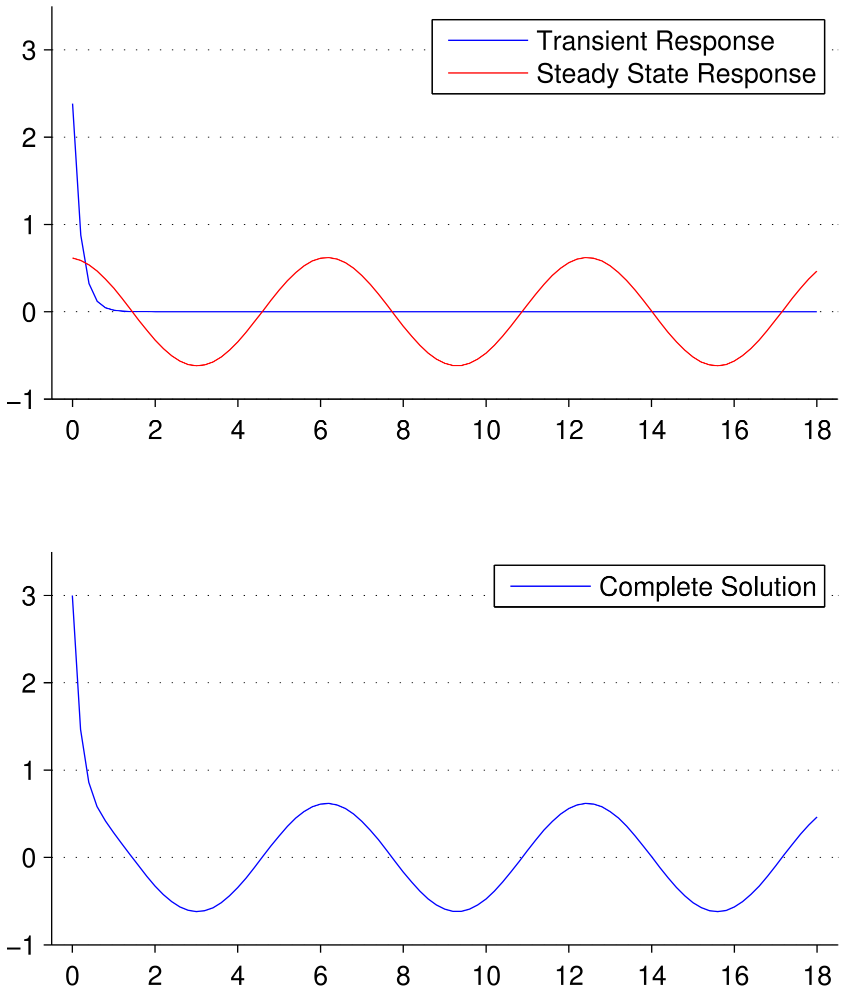


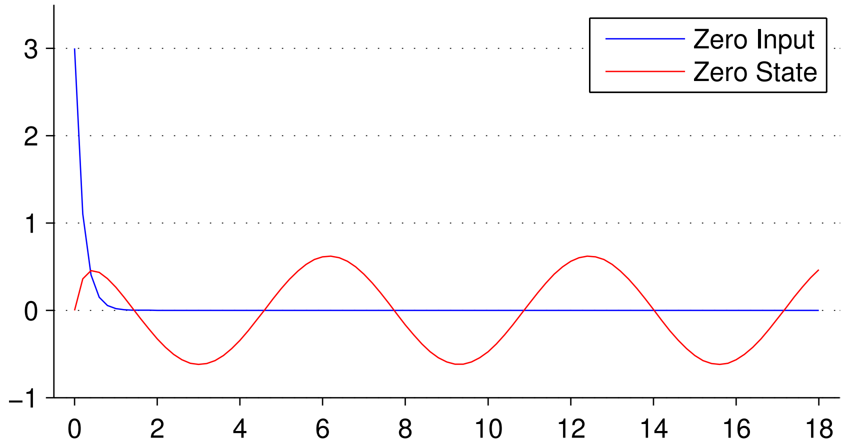
Figure 1.1: *Method 1: Complete Solution*

The Transient Response is

$$\frac{31}{13} \cdot e^{-5t} \cdot u(t)$$

Steady State Response is

$$\frac{8}{13} \cdot e^{-5t} - \frac{e^{-5t}}{13} \cdot (8 \cdot u(t) - e^{5t} \cdot (8cost - sint) \cdot u(t))$$



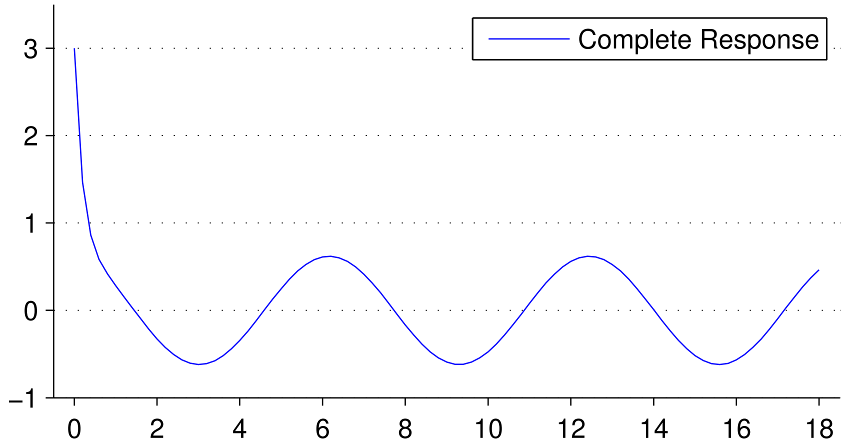


Figure 1.2: *Method 2: Complete Response*

The Zero Input Response is

$$3 \cdot e^{-5t} \cdot u(t)$$

Zero State Response is

$$-\frac{e^{-5t}}{13} \cdot (8 \cdot u(t) - e^{5t} \cdot (8cost - sint) \cdot u(t))$$

Conclusion

The complete response and transient + steady response are indeed identical.

As the time progresses, the transient part becomes zero and steady state takes over. Note, the staedy state is independent of initial system state, but dependent on the input signal.