

ITC TUT 5 Soumen 133

classmate

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Q) show how the parity matrix for a (7,4) hamming code is generated.

code structure is

$D_7 \ D_6 \ D_5 \ P_4 \ D_3 \ P_2 \ P_1$

Hence $P_1, P_2, P_3 = 3$ is parity bits.
 $D_7, D_6, D_5, D_3 = 4$ data bits

To find parity bits

$$\begin{aligned} P_1 &= D_3 \oplus D_5 \oplus D_7 \\ P_2 &= D_3 \oplus D_6 \oplus D_7 \\ P_3 &= D_5 \oplus D_6 \oplus D_7 \end{aligned}$$

\therefore The parity matrix for hamming (7,4) is

	P_1	P_2	P_3
D_3	1	1	0
D_5	1	0	1
D_6	0	1	1
D_7	1	1	1

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2] Consider a (6,3) linear block code where generator matrix is

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 1 \\ 0 & 1 & 0 & | & 1 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & 1 \end{bmatrix}$$

- Find all code vectors
- Find the minimum hamming d's (d_{min})
- Check if the received vector = [11011] contains error using the syndrome method.

$$\rightarrow [c] = [D][g]$$

$$[c] = [d_1 d_2 d_3] \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 1 \\ 0 & 1 & 0 & | & 1 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & 1 \end{bmatrix}$$

$$[c] = [d_1, d_2, d_3, (d_1+d_2), (d_2+d_3), (d_1+d_3)]$$

The code vectors are.

code name	message vector	code vector
c_1	000	000000
c_2	001	001011
c_3	010	010110
c_4	011	011101
c_5	100	100101
c_6	101	101110
c_7	110	110011
c_8	111	111000

→ b) Find the min ham

code vector	weight
000000	0
001011	3
010110	3
011101	4
100101	3
101110	4
110011	4
111000	3

Here the min weight is 3

$$\therefore D_{\min} = 3$$

→ c) Received vector $r = [110111]$

To detect the error.

$$[S] = R H^T$$

$$[H] = [P^T; I_{n-k}]$$

$$= [P^T; I_3]$$

$$[H] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$R = [110111]$$

$$\therefore [S] = R H^T$$

$$= [11011] \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore [s] = [1, 0, 0]$$

$\therefore s \neq 0 \therefore$ There is an error.

\therefore 100 matches with the 4th row.

\therefore The 4th bit of k_1 has error and the corrected vector is

$$u = 110011$$