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1) A binary source is emitting an independent sequence of 0's & 1's with probablities P and 1-P respectively plot the entropy of source versus P.

→ The entropy of the binary source is given by

$$H(s) = \sum_{i=1}^{2} p_i \log \frac{1}{p_i}$$

$$\therefore H(s) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2}$$

$$\therefore H(s) = P \log \frac{1}{P} + (1-P) \log \frac{1}{(1-P)}$$

$P = 0.1$, $H(s) = 0.1 \log \frac{1}{0.1} + 0.\cancel{8}9 \log \frac{1}{0.\cancel{8}9} = 0.469$ bits/symbol

$P = 0.2$, $H(s) = 0.2 \log \frac{1}{0.2} + 0.8 \log \frac{1}{0.8} = 0.722$ bits/symbol

$P = 0.3$, $H(s) = 0.3 \log \frac{1}{0.3} + 0.7 \log \frac{1}{0.7} = 0.881$ bits/symbol

$P = 0.4$, $H(s) = 0.4 \log \frac{1}{0.4} + 0.6 \log \frac{1}{0.6} = 0.971$ bits/symbol

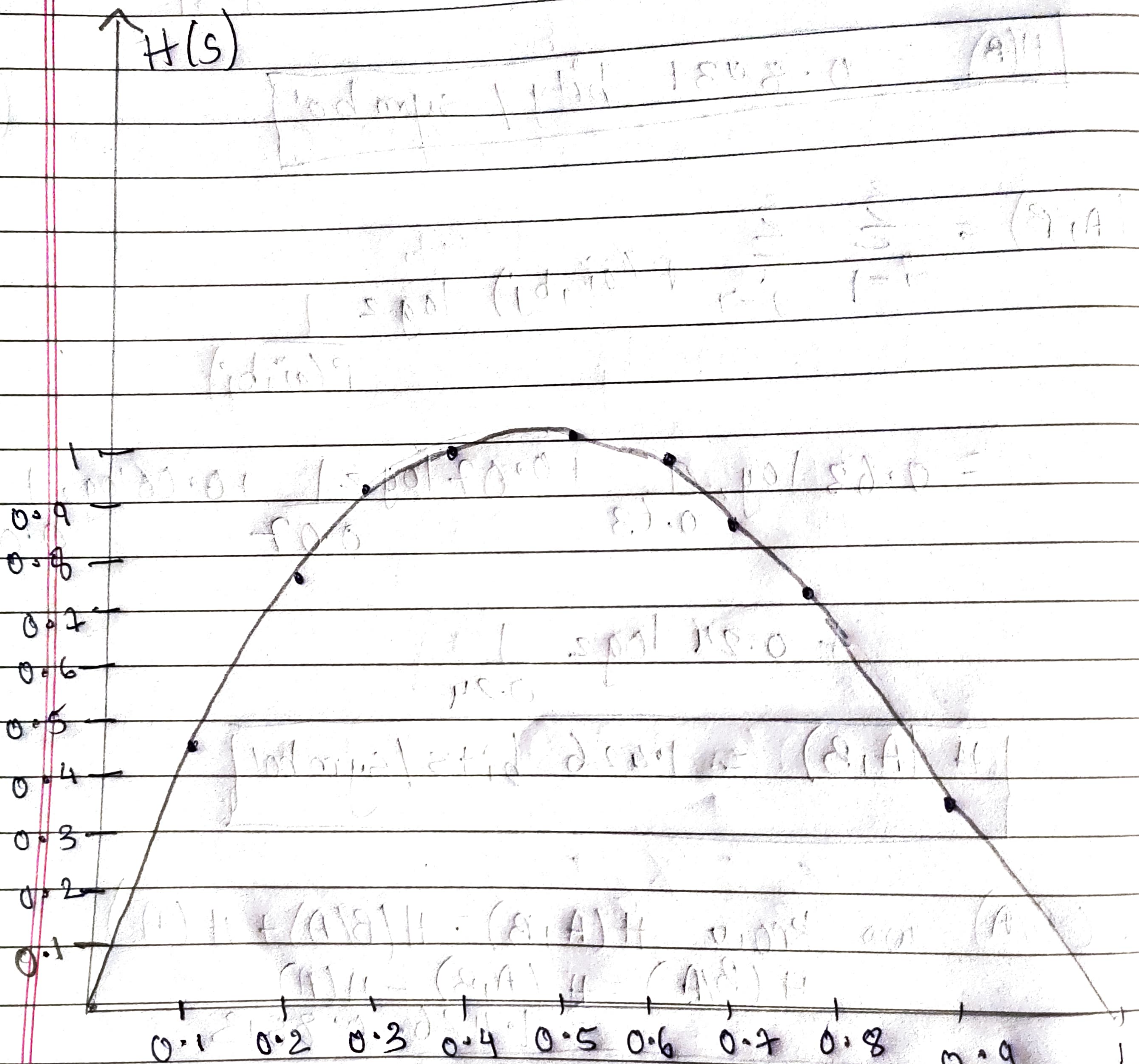
$P = 0.5$, $H(s) = 0.5 \log \frac{1}{0.5} + 0.5 \log \frac{1}{0.5} = 1$ bit/symbol

$P = 0.6 \quad H(S) = 0.6 \log \frac{1}{0.6} + 0.4 \log \frac{1}{0.4} = 0.971$ bit/symb

$P = 0.7 \quad H(S) = 0.7 \log \frac{1}{0.7} + 0.3 \log \frac{1}{0.3} = 0.881$ bit/sy

$P = 0.8 \quad H(S) = 0.8 \log \frac{1}{0.8} + 0.2 \log \frac{1}{0.2} = 0.722$ bit/s

$P = 1, \ H(S) = \log 1 + \log 1 = 0$



H(S)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

2) An LCD Panel has a resolution of 720 X 480 Pixel. It has 16 brightness levels and refreshed 30 times a second find information rate.

→ $H(s) = \sum_{i=1}^{16} P_i * \log_2\left(\frac{1}{P_i}\right)$

each brightness level = one symbol ∴ 16 symbol with equal probability

$P_1 = P_2 = P_3 = P_{16} = \frac{1}{16}$

$\therefore H(s) = 16 * \left\{\frac{1}{16} * \log_2\left(\frac{1}{1/16}\right)\right\}$

$= 4$ bits / symbol.

Data rate $= 720 \times 480 \times 30 = 10{,}368{,}000$ Pixel/sec

Information rate $= H(s) \times n$
$= 4 \times 10368000$

Information rate $= 4.147 \times 10^7$ bit/sec

3] In a communication system, a transmitter has input symbols $A = \{a_1, a_2, a_3\}$ and receiver also has 3 o/p symbols $B = \{b_1, b_2, b_3\}$ the matrix shows JPM with some marginal possibilities

$a_i \backslash b_j$	b_1	b_2	b_3
a_1	$1/12$	*	$5/36$
a_2	$5/36$	$1/9$	$5/36$
a_3	*	$1/6$	*
$P(b_j)$	$1/3$	$\frac{14}{36}$	*

i) find the missing probablities (*) in table.

(ii) Find $P(b_3/a_1)$ and $P(a_1/b_3)$

(iii) Are the event a_1 and b_1 statically independent? why?

Sol^n:- we know $\sum_{i=1}^{r} (P a_i) = 1$ for i/p symbols.

Similarly $\sum_{j=1}^{s} P(b_j) = 1$ for o/p symbols

i) given $s = 3$

$$\therefore \sum_{j=1}^{3} P(b_j) = P(b_1) + P(b_2) + P(b_3) = 1$$

$$\therefore \frac{1}{3} + \frac{14}{36} + P(b_3) = 1$$

$$\therefore P(b_3) = 5/18$$

From the first property of JPM we know, ie by adding the element of JPM column wise

$$\sum_{i=1}^{\alpha} P(a_i, b_j) = P(b_j)$$

$$P(b_1) = P(a_1, b_1) + P(a_2, b_1) + P(a_3, b_1) \rightarrow 1^{st} \text{ column}$$

$$1/3 = 1/12 + 5/36 + P(a_3, b_1)$$

$$P(a_3, b_1) = 1/9$$

lly $$P(b_2) = P(a_1, b_2) + P(a_2, b_2) + P(a_3, b_2) \rightarrow 2^{nd} \text{ column}$$

$$\frac{14}{36} = P(a_1, b_2) + \frac{1}{9} + 1/6$$

$$\therefore \boxed{P(a_1, b_2) = 1/9}$$

llly $$P(b_3) = P(a_1, b_3) + P(a_2, b_3) + P(a_3, b_3) \rightarrow 3^{rd} \text{ column}$$

$$5/18 = 5/36 + 5/36 + P(a_3, b_3)$$

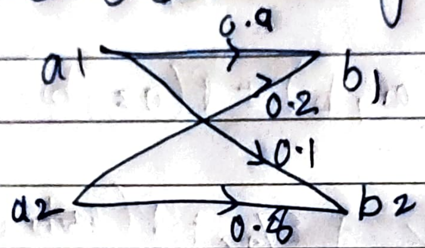
$$\boxed{\therefore P(a_3, b_3) = 0}$$

∴

$a_i \backslash b_i$	b_1	b_2	b_3
a_1	$1/12$	$1/9$	$5/36$
a_2	$5/36$	$1/9$	$5/36$
a_3	$1/9$	$1/6$	0

p( )			
	$1/3$	$14/36$	$5/18$

4 Q] A binary channel is described by the following diagram



and given $p(a_1) = 0.7$
$p(a_2) = 0.3$

find the entropies $H(A)$, $H(B)$, $H(A,B)$, $H(A/B)$, $H(B/A)$

Soln:-

$$P(B/A) = \begin{array}{c c} & \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \begin{array}{c} \times 0.7 \\ \times 0.3 \end{array} \end{array}$$

∵ (Og) x'ying the first row of the channel matrix with $P(a_1)$, by using theorem of total probability

$$P(A,B) = \begin{bmatrix} 0.63 & 0.07 \\ 0.06 & 0.24 \end{bmatrix}$$

i) $H(A) = \sum_{i=1}^{2} P(a_i) \log_2 \frac{1}{P(a_i)} = 0.7 \log_2 \frac{1}{0.7} + 0.3 \log_2$

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$$\boxed{H(A) = 0.8813 \text{ bits/symbol}}$$

(ii) $H(B) = \sum_{j=1}^{2} P(b_j) \log_2 \frac{1}{P(b_j)}$

$P(b_1) = 0.63 + 0.06 = 0.69 \quad P(b_2) = 0.07 + 0.24$
$= 0.31$

$\therefore H(B) = 0.69 \log_2 \frac{1}{0.69} + 0.31 \log_2 \frac{1}{0.31}$

$$\boxed{H(B) = 0.8931 \text{ bity / symbol}}$$

(iii) $H(A,B) = \sum_{i=1}^{2} \sum_{j=1}^{2} P(a_i, b_j) \log_2 \frac{1}{P(a_i, b_j)}$

$= 0.63 \log_2 \frac{1}{0.63} + 0.07 \log_2 \frac{1}{0.07} + 0.06 \log_2 \frac{1}{0.06}$

$+ 0.24 \log_2 \frac{1}{0.24}$

$$\boxed{H(A,B) = 1.426 \text{ bits/symbol}}$$

(v) $H(B/A)$ we know $H(A,B) = H(B/A) + H(A)$

$H(B/A) = H(A,B) - H(A)$
$= 1.426 - 0.8813$

$$\boxed{H(B/A) = 0.5417 \text{ bits / symbol}}$$

v) $H(A/B)$ $\quad H(A,B) = H(A/B) + H(B)$

|||^{by} $\quad H(A/B) = H(A,B) - H(B)$

$= 1.426 - 0.893$

$\boxed{H(A/B) = 0.533 \text{ bits / symbol}}$