Soumen Samanta
1601042013 3 B1

TUT1

Define self information

It is defined as the amount of information that gives knowledge about outcome of a certain event, adds to someone's overall knowledge. The amount of self information is is expressed in the unit of information : a bit and is represented by symbol I.

The smaller is the probablity of an event, the larger is the self information associated with receiving the information that the event indeed occured.

The self information $I(w_n)$ associated with the outcome w_n is

$$I(w_n) = \log_2\left(\frac{1}{P_x(w_n)}\right) = -\log_2(P_x(w_n))$$

16010420133

Problems

1. The binary symbol '0' & '1' are transmitted with probablities 1/4 & 3/4 respectively. find the corresponding self information.

Soln:- Self information in a '0' $= I_0 = \log_2 \frac{1}{P_0} = \log_2 \frac{1}{1/4} = 2$ bits

Self information in a '1' $= I_1 = \log_2 \frac{1}{P_1} = \log_2 \frac{1}{4/3} = 0.415$ bits

2. Prove that the Independent symbols are transmitted, the total self information must be equal to the sum of individual self-information

Soln:- To prove this statement, let us take 2 independent symbol I_1 and I_2 are transmitted with probablities P_1 and P_2 respectively

$I_1 = \log_2 \left(\frac{1}{P_1}\right)$ & $I_2 = \log_2 \left(\frac{1}{P_2}\right)$

Total information is addition of 2 information then the total information obtained when both statements are made in additive ie $I_1 + I_2$

$$I = \log \frac{1}{P(P_1 \text{ and } P_2)} = \log \frac{1}{P(P_1 \cap P_2)} = \log \frac{1}{P(P_1) \cdot P(P_2)}$$

$$= \log \frac{1}{P_1 \cdot P_2}$$

$$\therefore I = \log \left(\frac{1}{P_1}\right) + \log \left(\frac{1}{P_2}\right)$$

$$\boxed{I = I_1 + I_2}$$

16010420133

3. A source ~~port~~ puts out one of five possible messages during each message interval. The probability of these messages are

$P_1 = 1/2$, $P_2 = 1/4$, $P_3 = 1/8$, $P_4 = 1/16$, $P_5 = 1/16$

Soln:- Self information of each message.

$I(m_1) = \log_2 \frac{1}{p_1} = \log_2 \frac{1}{1/2} = \log_2 2 = 1$ bit

$I(m_2) = \log_2 \frac{1}{P_2} = \log_2 \frac{1}{1/4} = \log_2 4 = 2$ bit

$I(m_3) = \log_2 \frac{1}{P_3} = \log_2 \frac{1}{1/8} = \log_2 8 = 3$ bit.

$I(m_4) = \log_2 \frac{1}{P_4} = \log_2 \frac{1}{1/16} = \log_2 16 = 4$ bit

$I(m_5) = \log_2 \frac{1}{P_5} = \log_2 \frac{1}{1/16} = \log_2 16 = 4$ bit.

4) Consider a source $S = \{S_1, S_2, S_3\}$ with $P = \{1/2, 1/4, 1/4\}$ find self-information of each message

→ To find self-information $I_k = \log_2 1/P_k$

$S_1 \rightarrow I_1 = \log_2 \frac{1}{1/2} = \log_2 2 = 1$ bit

$S_2 \rightarrow I_2 = \log_2 \frac{1}{1/4} = \log_2 4 = 2$ bit

$S_3 \rightarrow I_3 = \log_2 \frac{1}{1/4} = \log_2 4 = 2$ bit.

5] A card is selected at random from a deck of playing cards. if you have been told that it is red in colour how much information you have received. How much more info you need to completely specify the card.

→ Total no of cards = 52

Total no of cards in red = 26

Probablity of getting a red card $= \frac{26}{52} = \frac{1}{2}$

∴ Amount of Information contained $I = \log_2 \frac{1}{P}$

$= \log_2 \frac{1}{1/2}$

$I = \log_2 2 = 1$ bit.

For 1 unique card $= 1/52$

$I = \log_2 52 = 5.7$ bits

More needed ⇒ $5.7 - 1$ bit $= 4.7$ bits.

b) Given 4 symbols s_1, s_2, s_3, s_4, with probability p_1, p_2, p_3 and p_4, use the different sets of values given below to show that entropy highest when all symbols have same probability.

a) $P = \{0.1, 0.2, 0.3, 0.4\}$
b) $P = \{0.2, 0.3, 0.3, 0.2\}$
c) $P = \{0.25, 0.25, 0.25, 0.25\}$

Entropy is calculated using formula.

$$H = p_1 * \log_2\left(\frac{1}{p_1}\right) + p_2 * \log_2\left(\frac{1}{p_2}\right) + p_3 * \log_2\left(\frac{1}{p_3}\right)$$
$$p_4 * \log_2\left(\frac{1}{p_4}\right)$$

i) $P = \{0.1, 0.2, 0.3, 0.4\}$

$\therefore P(A_1) = 0.1 \quad P(A_2) = 0.2$
$P(A_3) = 0.3 \quad P(A_4) = 0.4$

From above formula.

$$H(A) = 0.1 \log_2 \frac{1}{0.1} + 0.2 \log_2 \frac{1}{0.2} + 0.3 \log_2 \frac{1}{0.3}$$

$$= 0.1(3.32) + 0.2(2.32) + 0.3(1.74) + 0.4(1.32)$$

$$= 0.332 + 0.464 + 0.522 + 0.528$$

classmate
Date
Page

$$\boxed{H(A) = 1.846 \text{ bits/symbol}} \longrightarrow (i)$$

(ii) $P = \{0.2, 0.3, 0.3, 0.2\}$

$P(A_1) = 0.2$ $P(A_2) = 0.3$
$P(A_3) = 0.3$ $P(A_4) = 0.2$

From the above formula,

$$H(A) = 0.2 \log_2 \frac{1}{0.2} + 0.3 \log_2 \frac{1}{0.3} + 0.3 \log_2 \frac{1}{0.3} + 0.2 \log_2 \frac{1}{0.2}$$

$$= 0.464 + 0.522 + 0.522 + 0.464$$

$$\boxed{H(A) = 1.972 \text{ bits/symbol}} \longrightarrow (ii)$$

(iii) $P = \{0.25, 0.25, 0.25, 0.25\}$

$P(A_1) = P(A_2) = P(A_3) = P(A_4) = 0.25$

From the above formula.

$$H(A) = 0.25 \log_2 \frac{1}{0.25} \times 4 = \log_2 \frac{1}{0.245}$$

$$\boxed{H(A) = 2 \text{ bits/symbol}} \longrightarrow (iii)$$

From (i), (ii), (iii) we observe that $H(A)$ of (iii) is highest when all events have equal probablity.

(7) A pair of dice are tossed simultaneously. The outcome of first dice is recorded as x_1 and that of second dice as x_2. Three events are defined as follows

$A = \{(x_1, x_2)$ such that $(x_1 + x_2)$ is divisible exactly by $4\}$

$B = \{(x_1, x_2)$ such that $6 \leq (x_1 + x_2) \leq 8\}$

$C = \{(x_1, x_2)$ such that $x_1 x_2$ is divisible exactly by $3\}$

which event conveys max info?

→ When a pair of dice are tossed simultaneously the sample space S consists of 36 combinat of (x_1, x_2) given by

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

The event A contains the pairs given by

$A = \{(1,3), (2,2), (2,6), (3,1), (3,5), (4,4), (5,3), (6,2), (6,6)\}$

$\therefore P(A) = \frac{9}{36} = 0.25$

The event B contains:

$B = \{(1,5), (1,6), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (6,1), (6,2)\}$

$P(B) = \frac{16}{36} = 0.444$

The event - C contains

$C = \{(1,3), (1,6), (2,3), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,3), (4,6), (5,3), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$P(C) = \frac{20}{36} = 0.556$

$\therefore$ Self Information of $A = I_A = \log \frac{1}{0.25} = 2$ bits.

Self Information of $B = I_B = \log \frac{1}{0.444} = 1.17$ bits

Self Information of $C = I_C = \log \frac{1}{0.555} = 0.848$ bits