Experiment No. : 3

Title: Virtual lab on Kruskal's algorithm to construct Minimum spanning trees

Batch: B1 **Roll No.:** **16010420133** **Experiment No.: 3**

Aim: Explore the virtual lab on Kruskal's algorithm to construct minimum spanning trees.

Resource Needed:
https://ds2-iiith.vlabs.ac.in/exp/min-spanning-trees/index.html

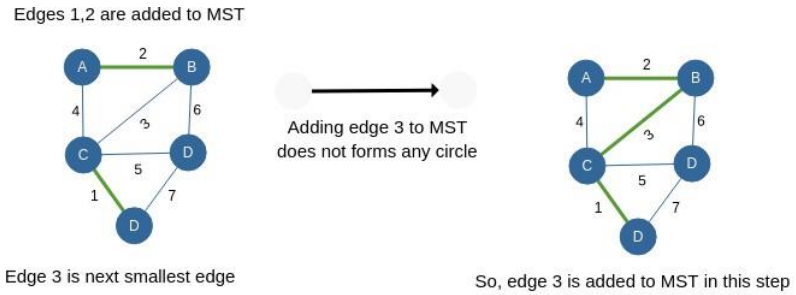
Kruskal Algorithm

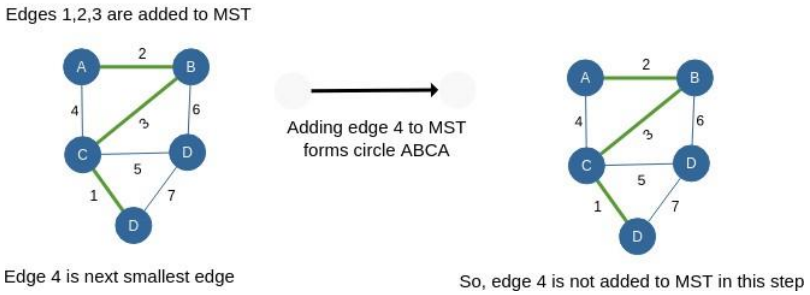
Introduction :

In Kruskal's Algorithm, we take the fastest possible approach to create MST.

- We first sort the edges of graph in increasing order.
- Check the edges in sorted order if they should be in MST or not.
- Whenever we see the newly taken edge making circle (checked by Union-find) (refer picture below), we do not keep it in MST or else we will proceed further.
- We keep performing the above steps over the array again and again till all the edges are checked.

When should we add edge to mst?





Algorithm of Kruskal Algorithm:

```
MST-KRUSKAL(G, w)
1  A = ∅
2  for each vertex v ∈ G.V
3       MAKE-SET(v)
4  sort the edges of G.E into nondecreasing order by weight w
5  for each edge (u, v) ∈ G.E, taken in nondecreasing order by weight
6       if FIND-SET(u) ≠ FIND-SET(v)
7            A = A ∪ {(u, v)}
8            UNION(u, v)
9  return A
```

Explanation and Working of Kruskal Algorithm:

Let's have a final look at the consolidated algorithm to find MST of given graph:

- **STEP 1:** Sort the given edges.
- **STEP 2:** Check each edge in sorted order if it forms a cycle with already selected edges. Ifnot Add it to list of MST. If it does move to next edge.
- **STEP 3:** Run steps 1 and 2 till v-1 edges are selected (v= no. of vertices).

Time complexity and derivation of Kruskal Algorithm:

Running Time of Kruskal's

Lets assume that we are finding MST of a N vertices graph using Kruskal's.

- To check edges we need to sort the given edges based on weights of edges. The best way to sort has a order of O(N log(N)).
- To Check one edge if it needs to be in MST or not, we apply Union-find to check if it forms a circle with edges present and add to MST exactly once and apply Union-Find algorithm of order log(E).
- Since we perform atmost N checks for a graph total complexity is O(Nlog(E)) for the checkings.
- So, total complexity is O(Nlog(E)+Nlog(N))

Best and Worst Cases for Kruskal's

For regular Kruskal's, time complexity will be O(Nlog(E)+Nlog(N)) in all cases. For Kruskal's:

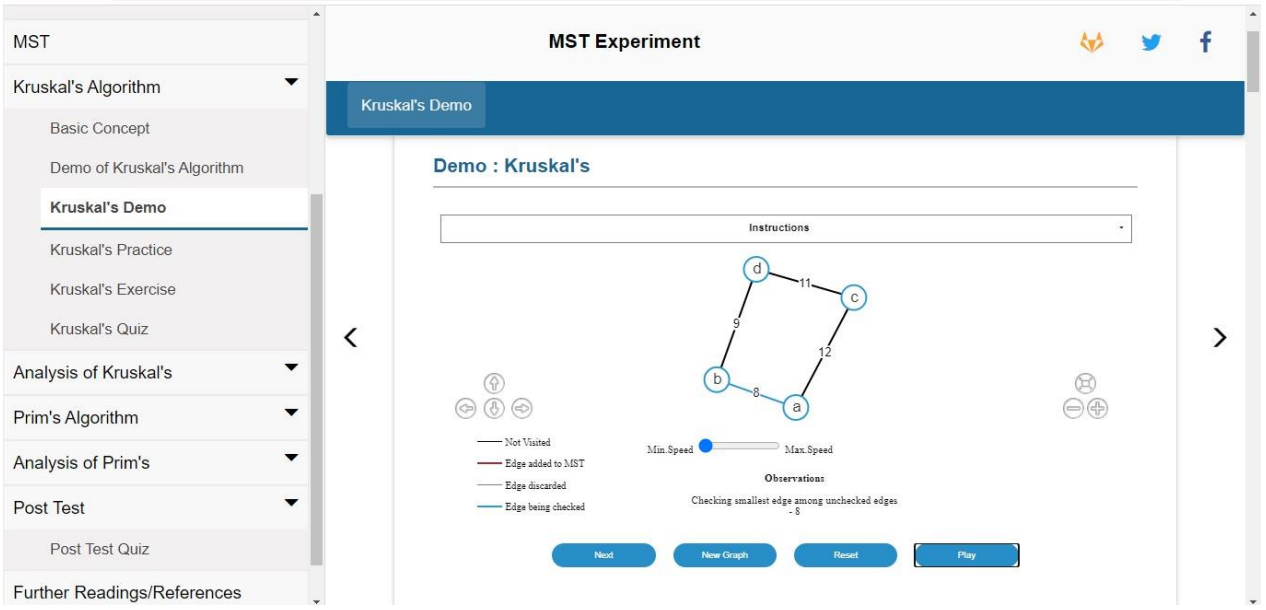
- In best case scenario, we have N no cycles and we have to run N-1 iterations to determine MST.
- Time complexity will be O((N-1)*log(E)+Elog(E)) in this case.
- In worst case we will have to check all E edges. Time complexity in such case would be O(Elog(E)+Nlog(N))

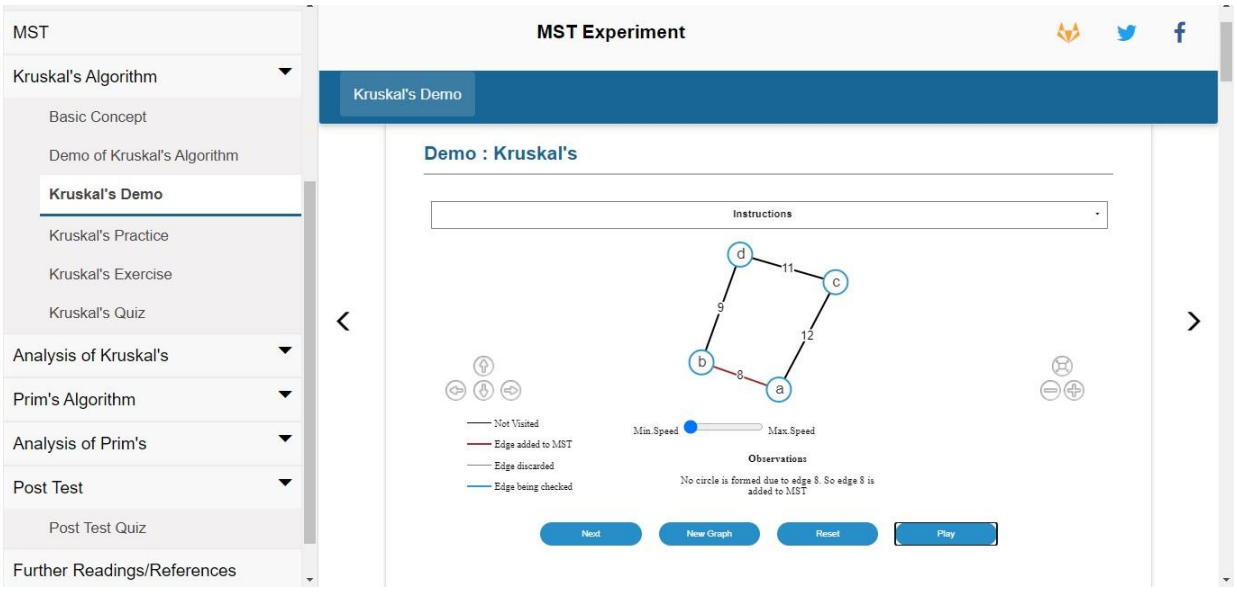
Space Complexity of Kruskal's

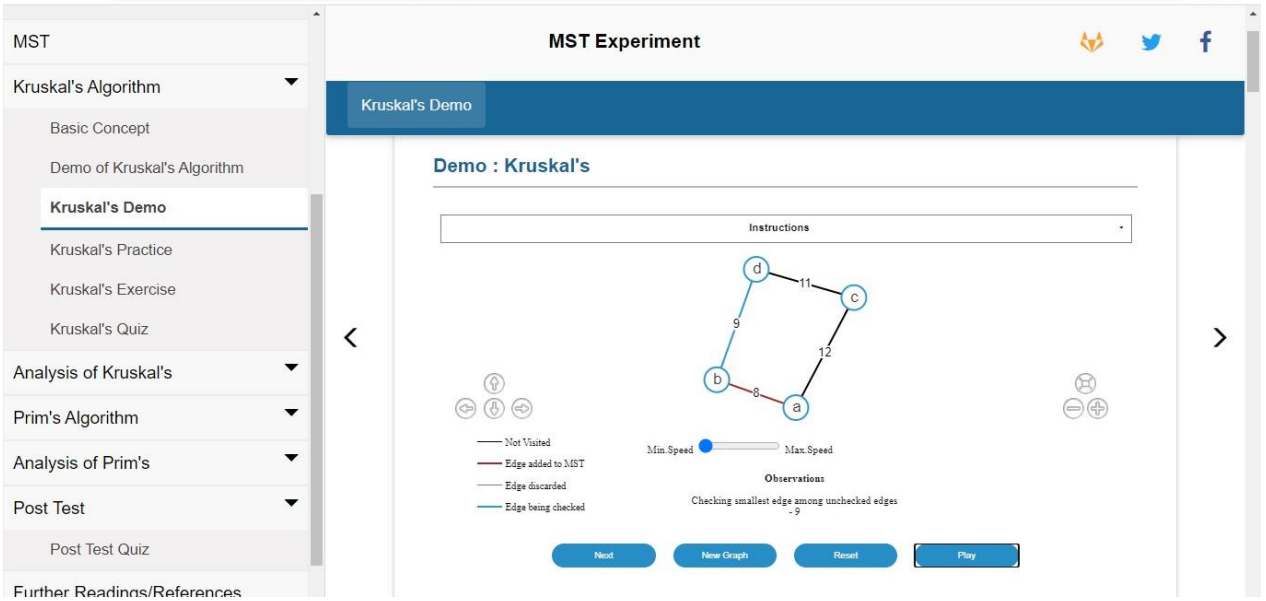
While sorting, we need an extra array to store sorted array of edges (Space comlexity of O(E)), Another array for Union-Find of size O(E). So, total Space Complexity would be O(log(E)).

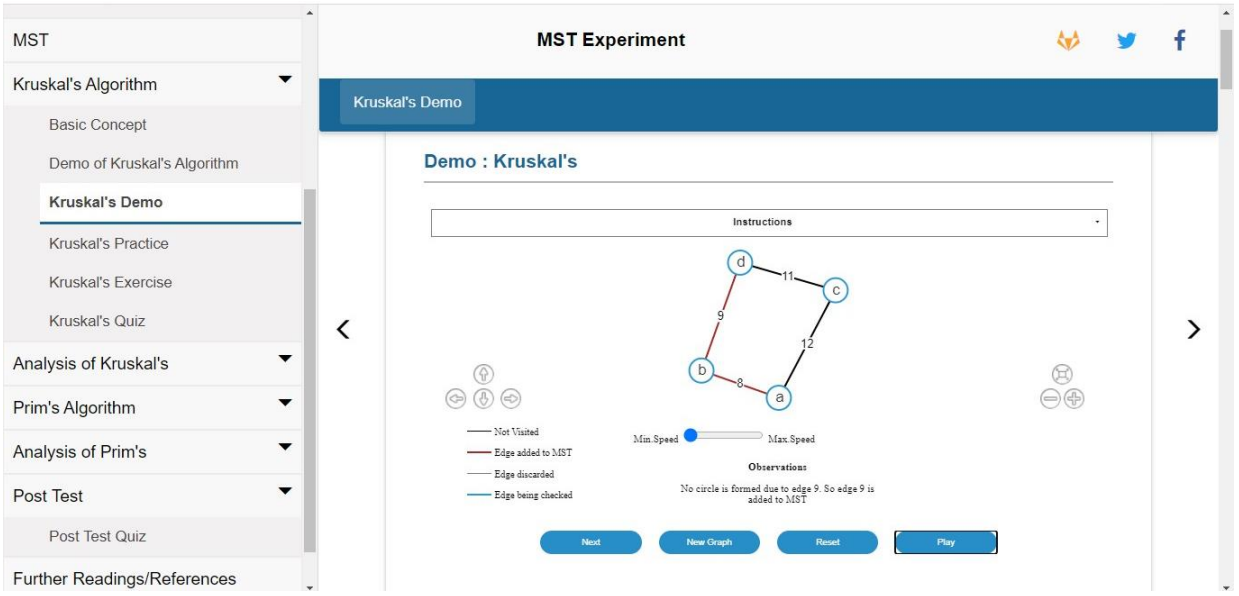
Observations from Simulation:

Kruskal's Demo:






MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Post Test Quiz
Further Readings/References
MST Experiment
Kruskal's Demo
Demo : Kruskal's
Instructions
d
c
b
a
11
9
12
8
Not Visited
Edge added to MST
Edge discarded
Edge being checked
Min.Speed
Max.Speed
Observations
Checking smallest edge among unchecked edges - 9
Next
New Graph
Reset
Play


MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
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Analysis of Prim's
Post Test
Post Test Quiz
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Kruskal's Demo
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c
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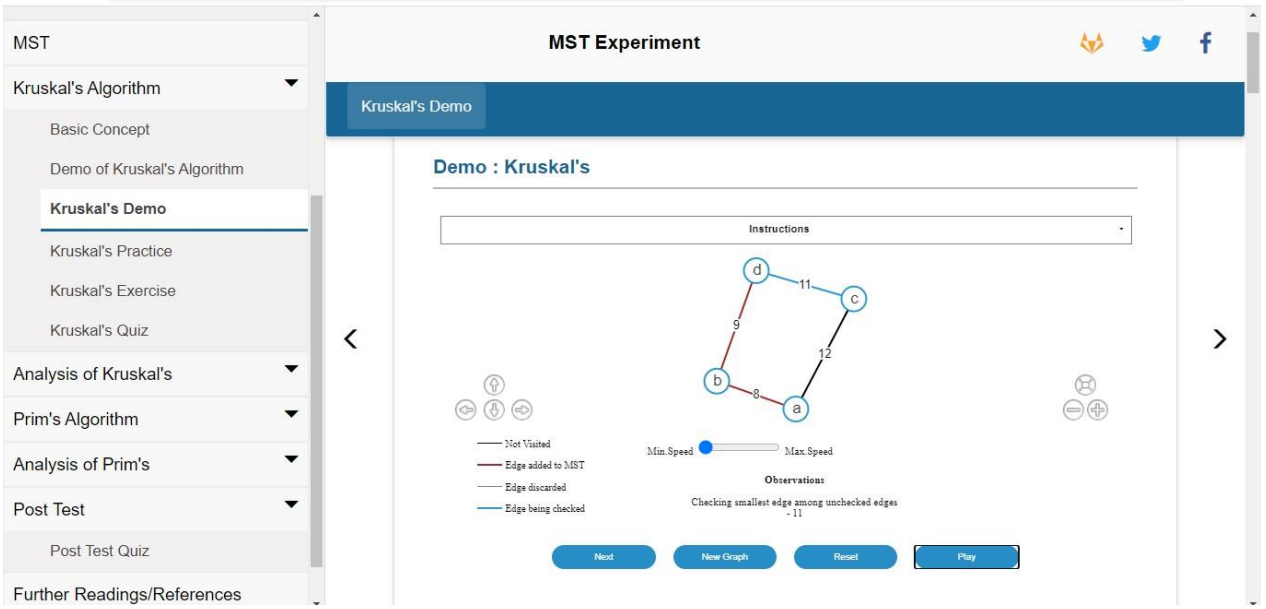
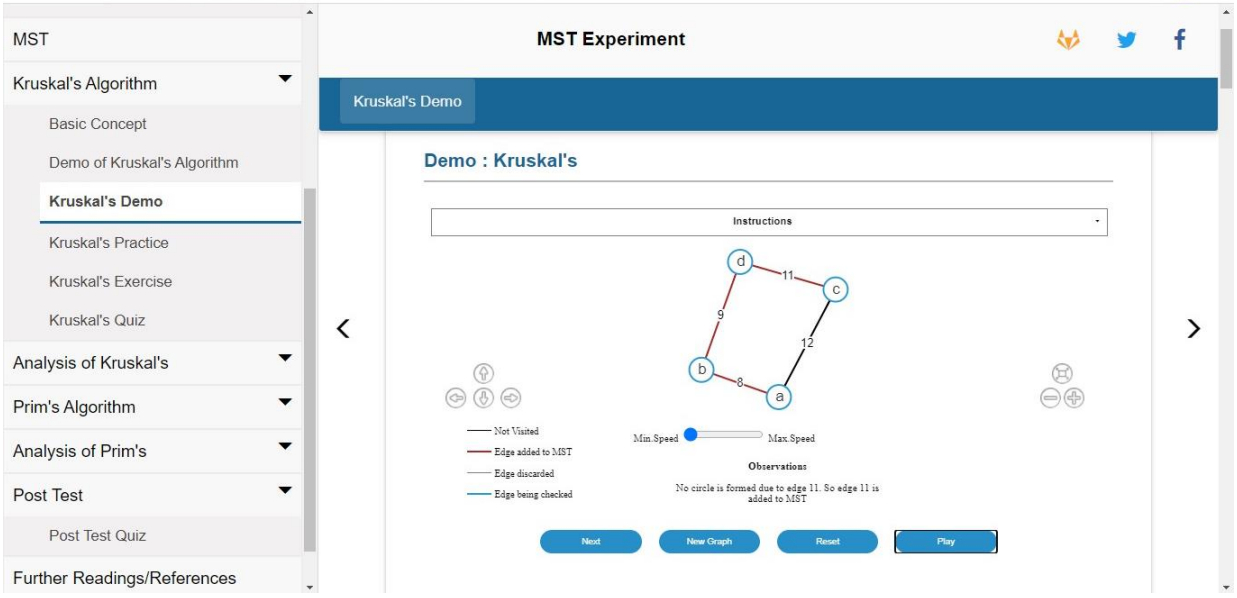
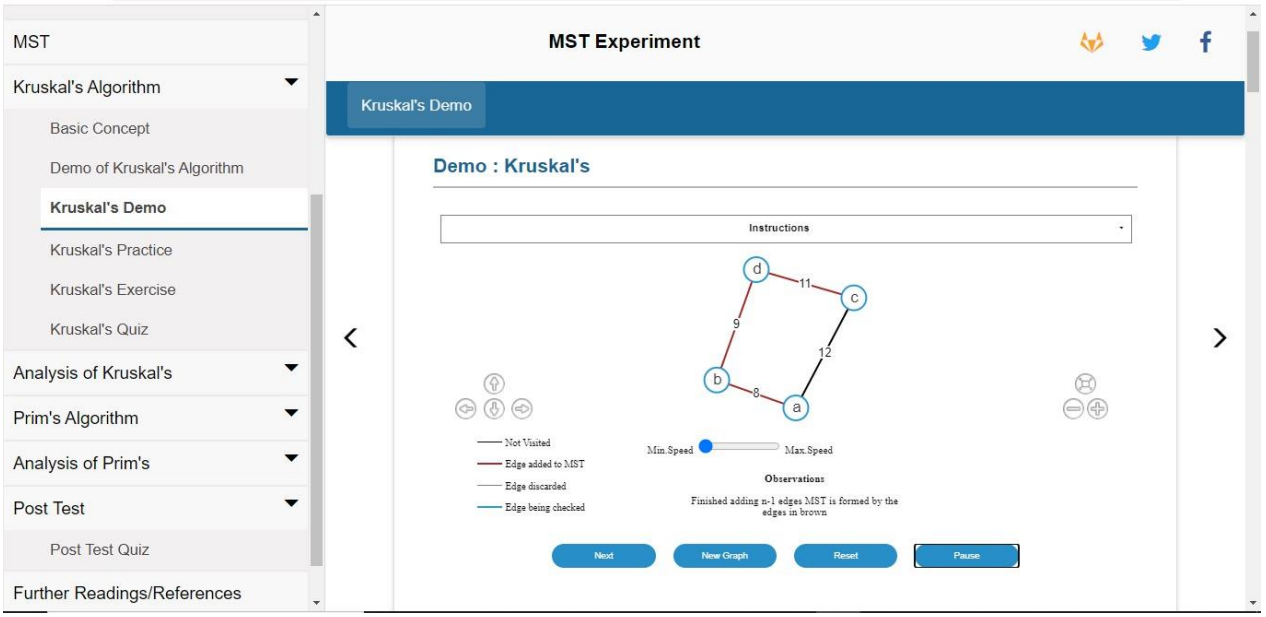
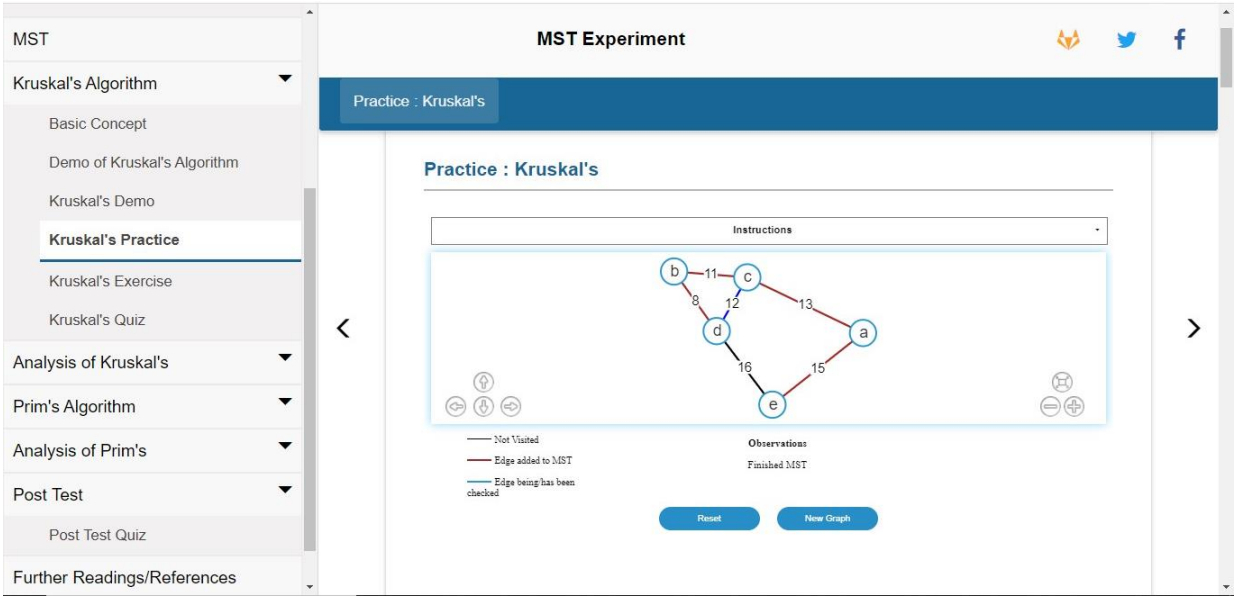
a
11
9
12
8
Not Visited
Edge added to MST
Edge discarded
Edge being checked
Min.Speed
Max.Speed
Observations
No circle is formed due to edge 9. So edge 9 is added to MST
Next
New Graph
Reset
Play


MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Post Test Quiz
Further Readings/References
MST Experiment
Kruskal's Demo
Demo : Kruskal's
Instructions
d
c
b
a
11
9
12
8
Not Visited
Edge added to MST
Edge discarded
Edge being checked
Min.Speed
Max.Speed
Observations
Checking smallest edge among unchecked edges - 11
Next
New Graph
Reset
Play

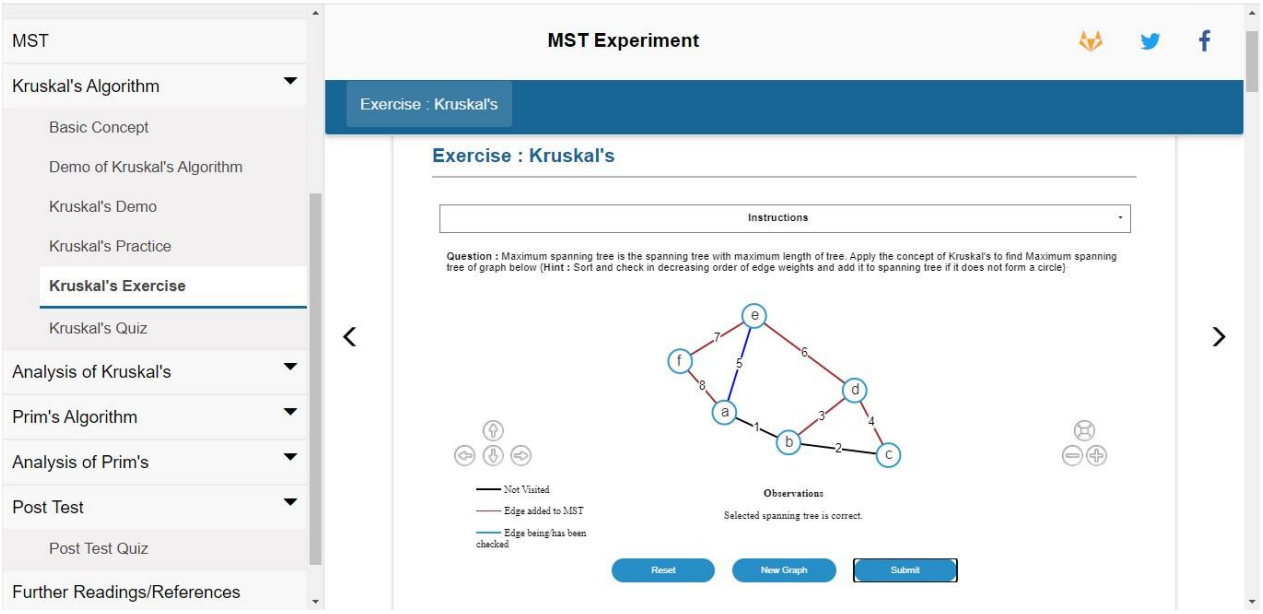

MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Post Test Quiz
Further Readings/References
MST Experiment
Kruskal's Demo
Demo : Kruskal's
Instructions
d
c
b
a
11
9
12
8
Not Visited
Edge added to MST
Edge discarded
Edge being checked
Min.Speed
Max.Speed
Observations
No circle is formed due to edge 11. So edge 11 is added to MST
Next
New Graph
Reset
Play



Kruskal's Practice:

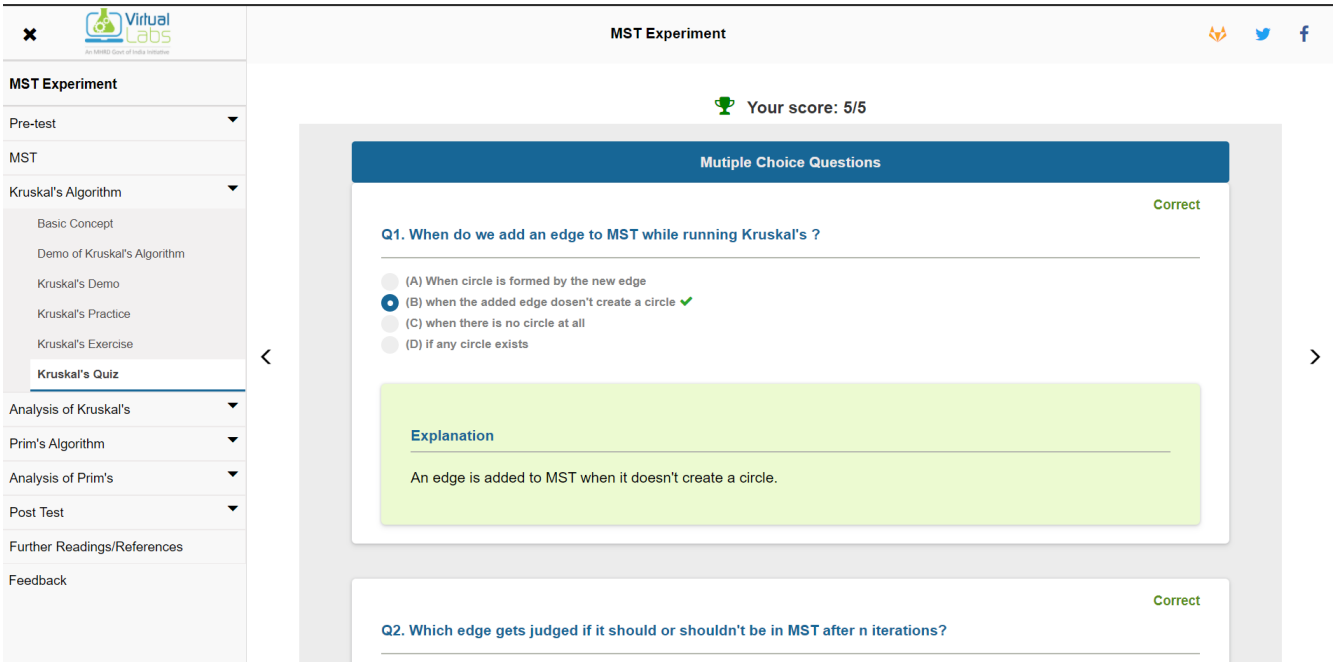


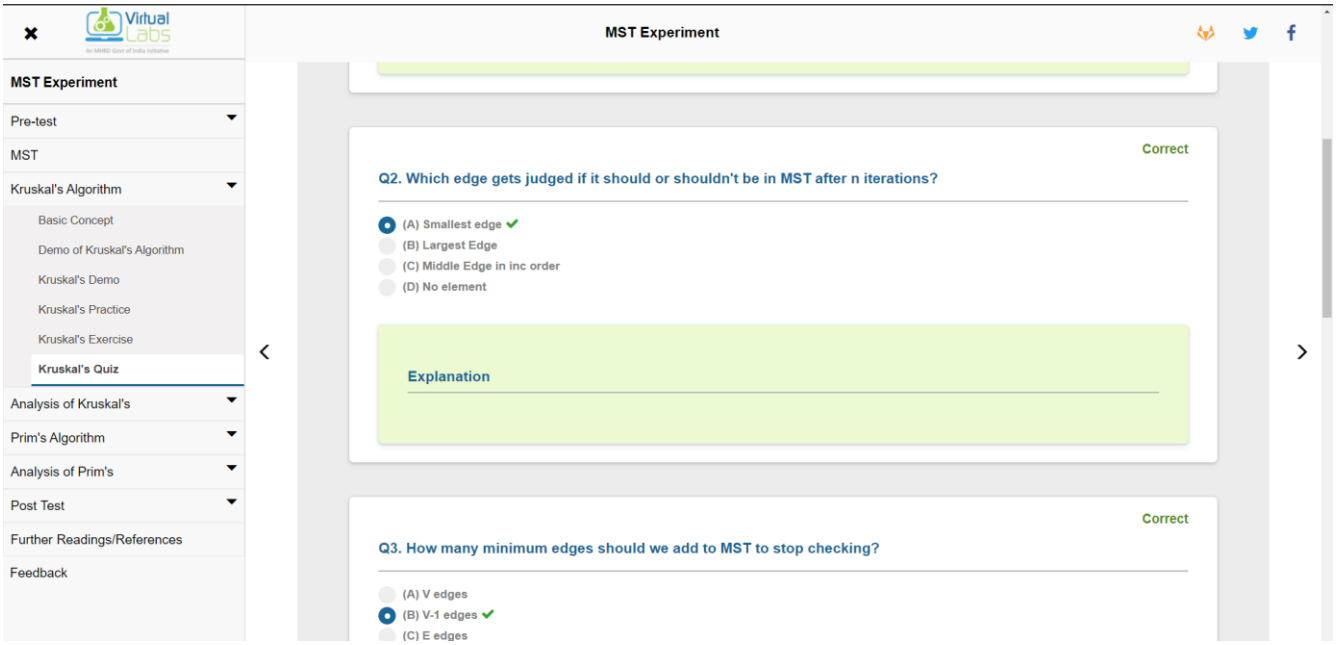
Kruskal's Exercise:

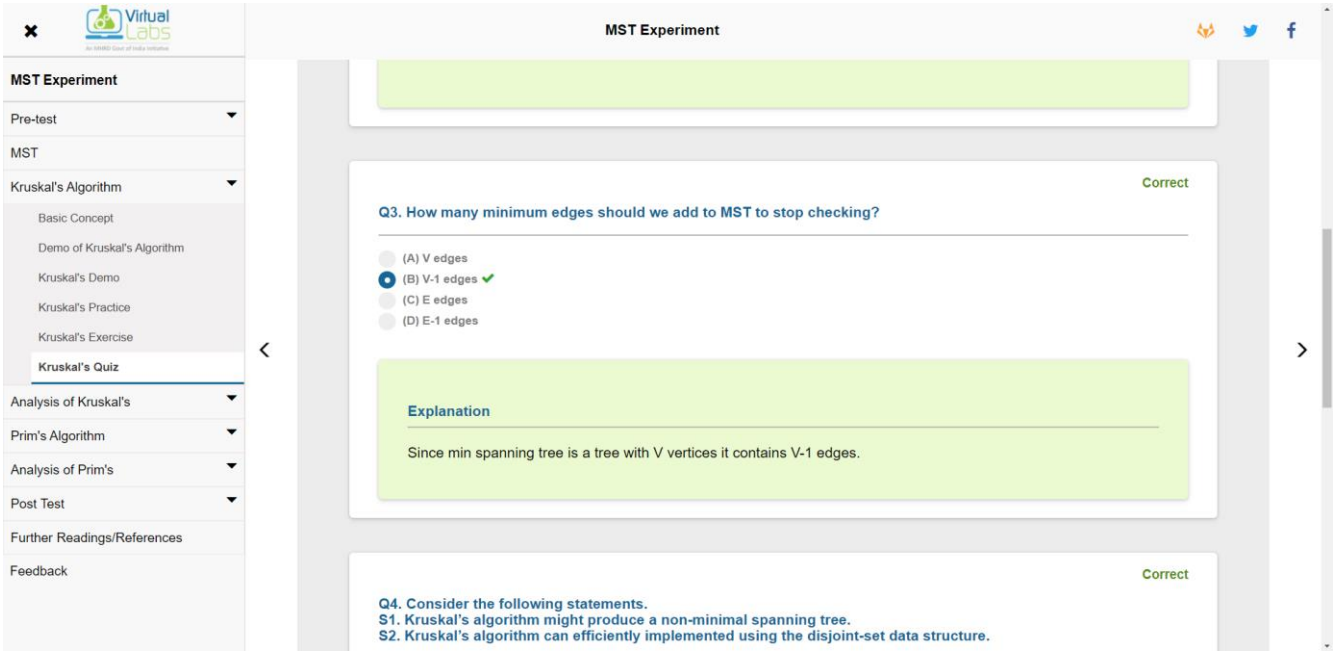


Self evaluation: Solve both Kruskal Quiz and Analysis quiz and Display the result of your first attempt.

Kruskal's Quiz:




MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Further Readings/References
Feedback
Correct
Q2. Which edge gets judged if it should or shouldn't be in MST after n iterations?
(A) Smallest edge
(B) Largest Edge
(C) Middle Edge in inc order
(D) No element
Explanation
Correct
Q3. How many minimum edges should we add to MST to stop checking?
(A) V edges
(B) V-1 edges
(C) E edges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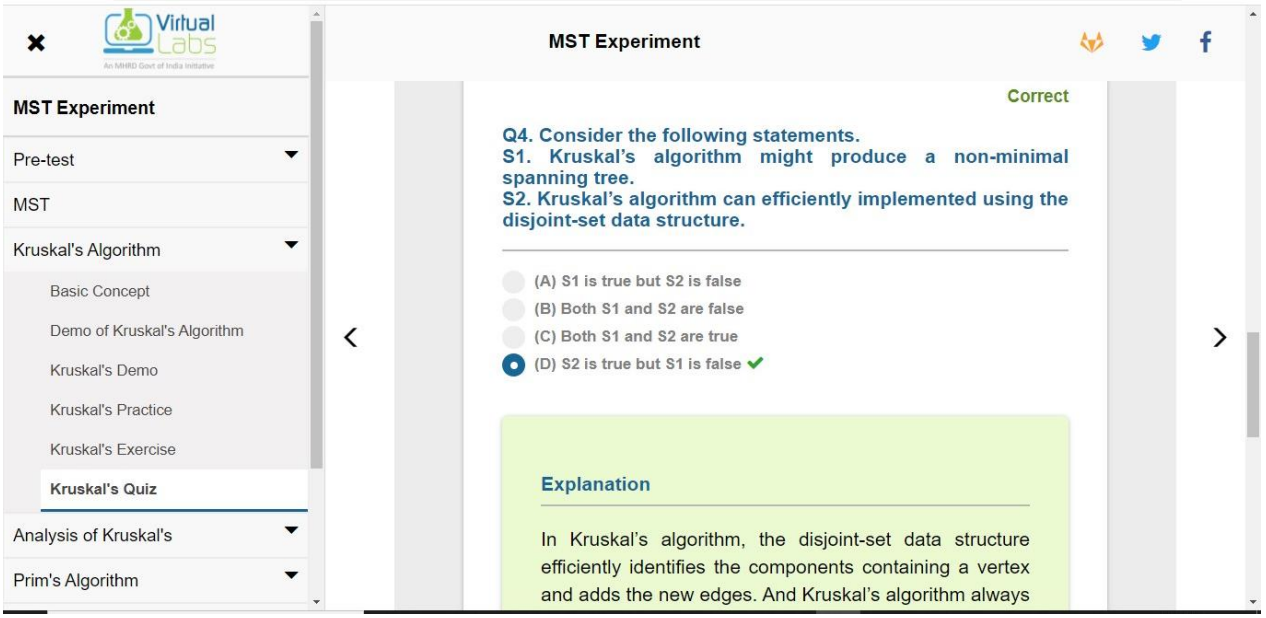
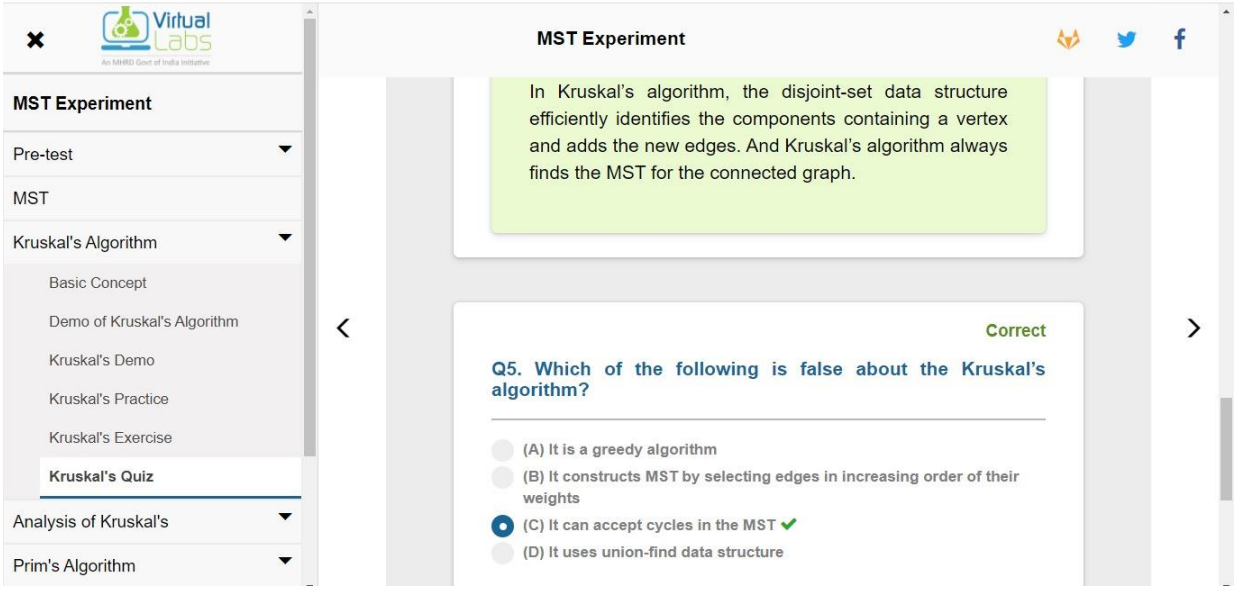
Pre-test
MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Further Readings/References
Feedback
Correct
Q3. How many minimum edges should we add to MST to stop checking?
(A) V edges
(B) V-1 edges
(C) E edges
(D) E-1 edges
Explanation
Since min spanning tree is a tree with V vertices it contains V-1 edges.
Correct
Q4. Consider the following statements.
S1. Kruskal's algorithm might produce a non-minimal spanning tree.
S2. Kruskal's algorithm can efficiently implemented using the disjoint-set data structure.


MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
Analysis of Kruskal's
Prim's Algorithm
Correct
Q4. Consider the following statements.
S1. Kruskal's algorithm might produce a non-minimal spanning tree.
S2. Kruskal's algorithm can efficiently implemented using the disjoint-set data structure.
(A) S1 is true but S2 is false
(B) Both S1 and S2 are false
(C) Both S1 and S2 are true
(D) S2 is true but S1 is false
Explanation
In Kruskal's algorithm, the disjoint-set data structure efficiently identifies the components containing a vertex and adds the new edges. And Kruskal's algorithm always


MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Basic Concept
Demo of Kruskal's Algorithm
Kruskal's Demo
Kruskal's Practice
Kruskal's Exercise
Kruskal's Quiz
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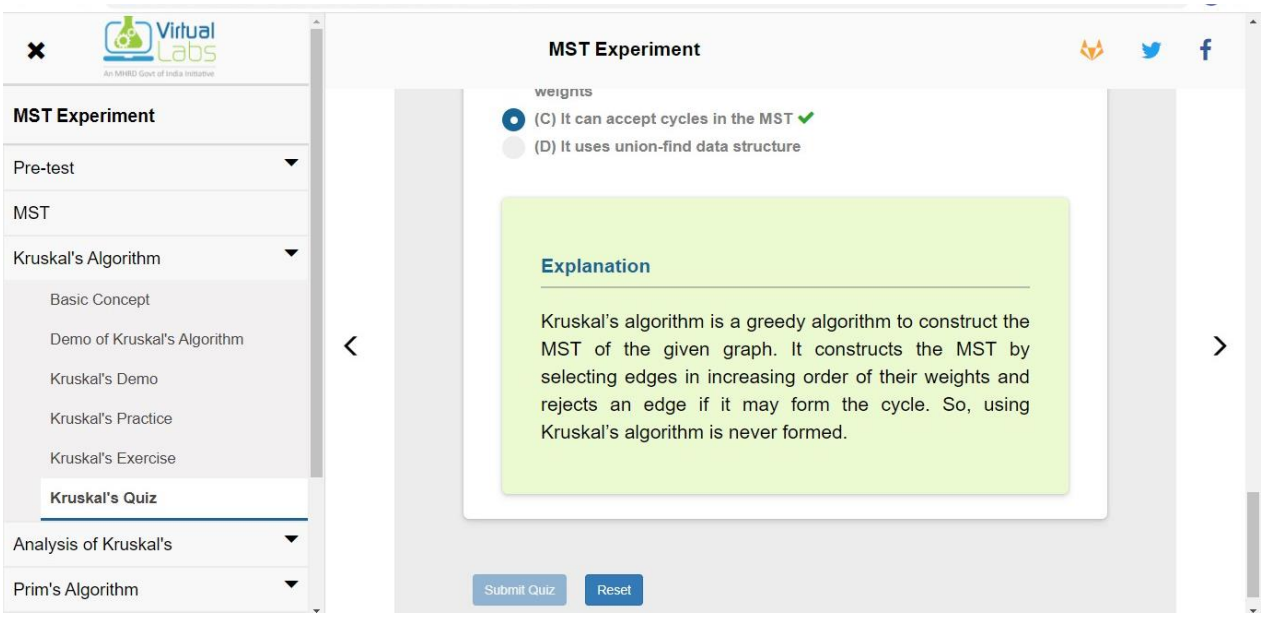
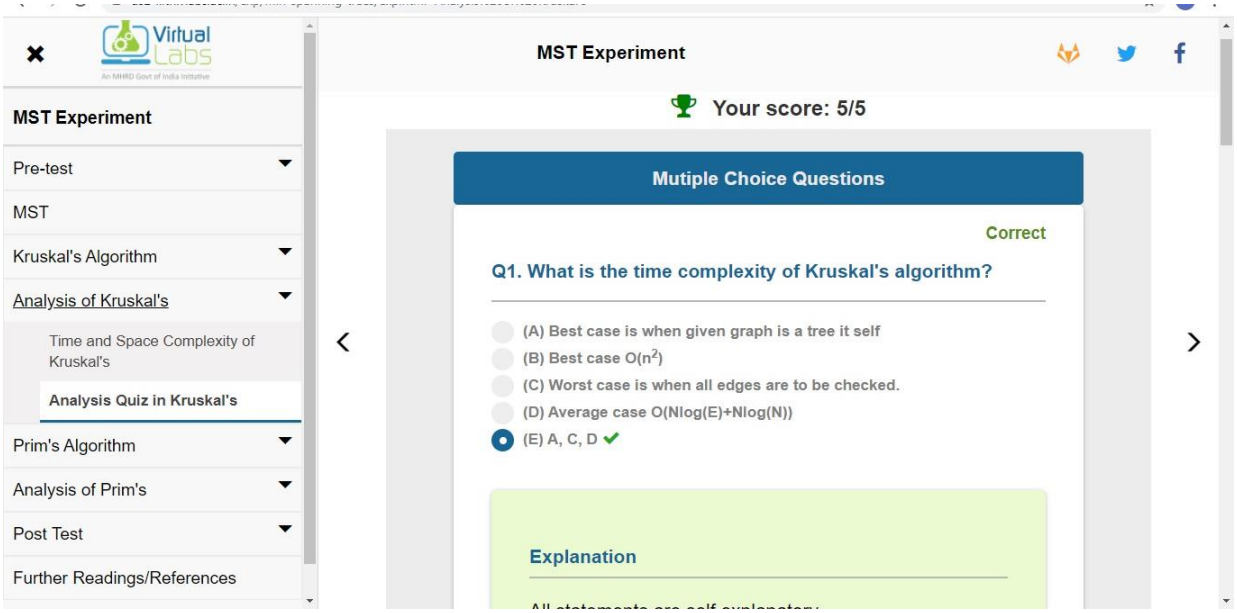
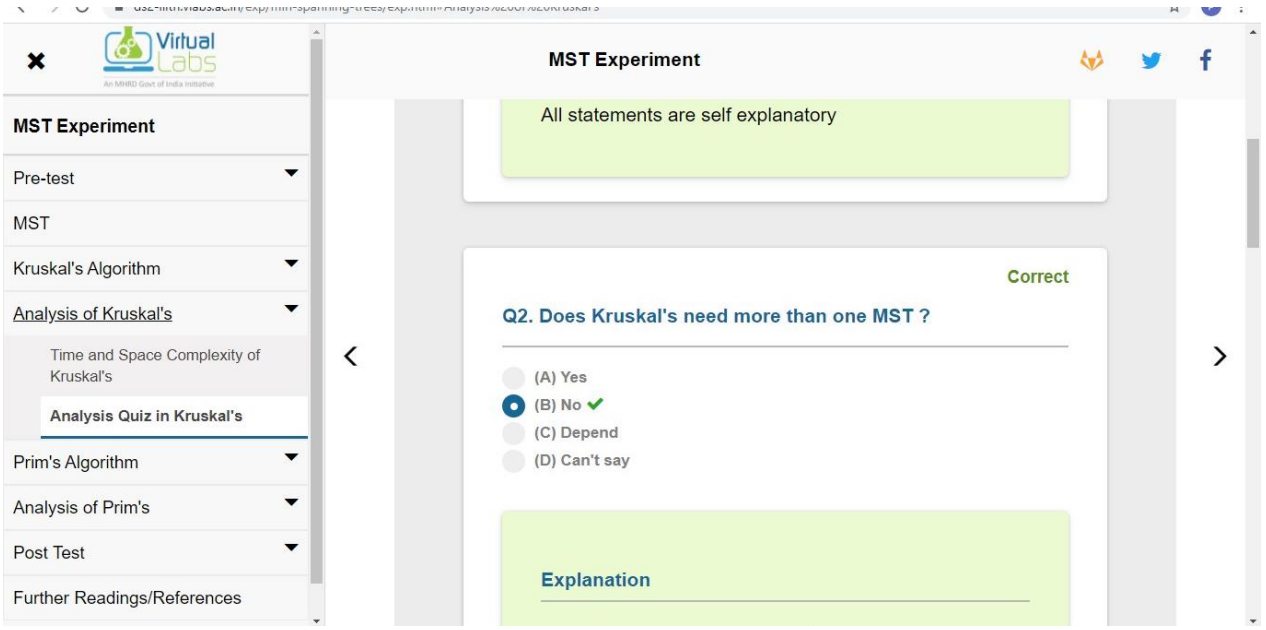
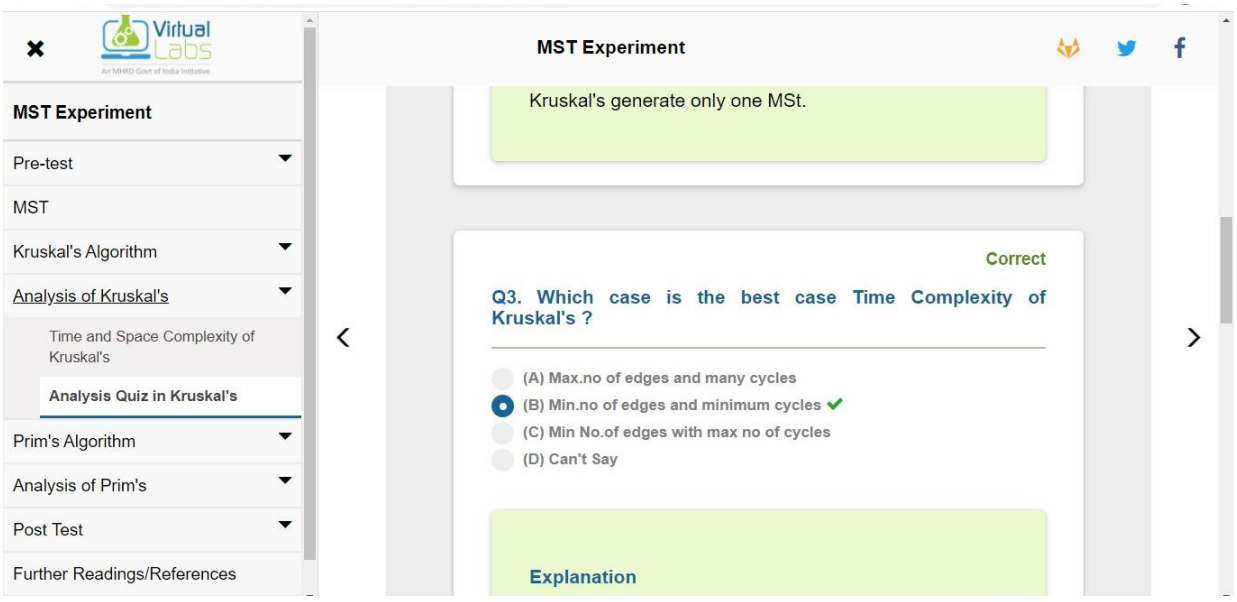
Prim's Algorithm
In Kruskal's algorithm, the disjoint-set data structure efficiently identifies the components containing a vertex and adds the new edges. And Kruskal's algorithm always finds the MST for the connected graph.
Correct
Q5. Which of the following is false about the Kruskal's algorithm?
(A) It is a greedy algorithm
(B) It constructs MST by selecting edges in increasing order of their weights
(C) It can accept cycles in the MST
(D) It uses union-find data structure

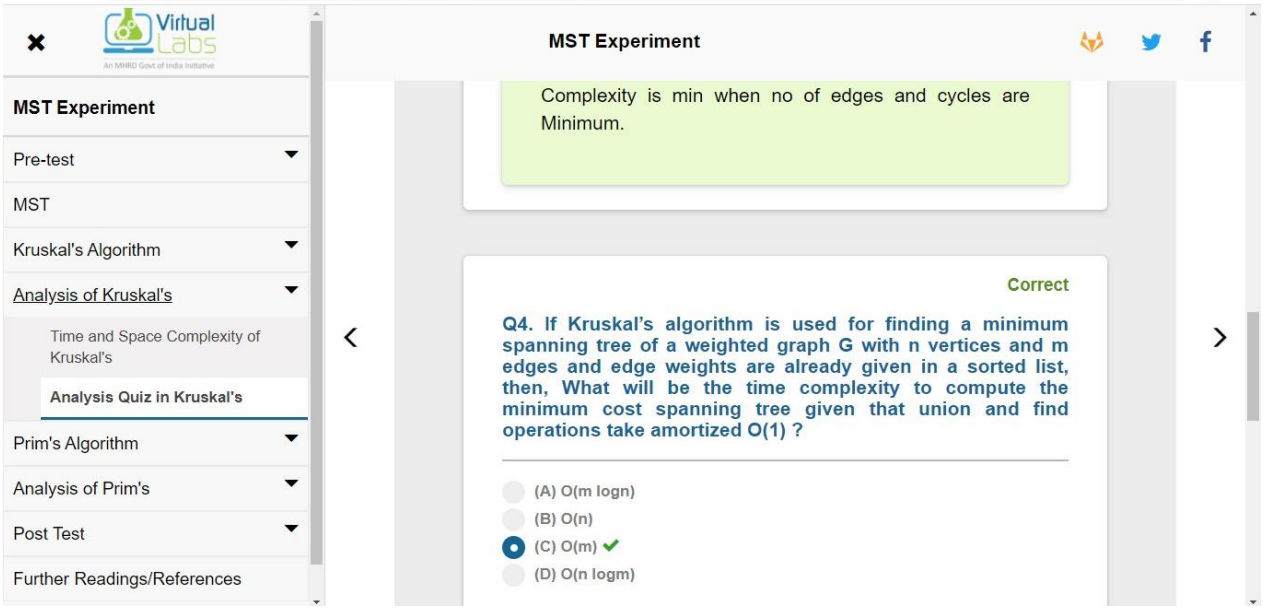


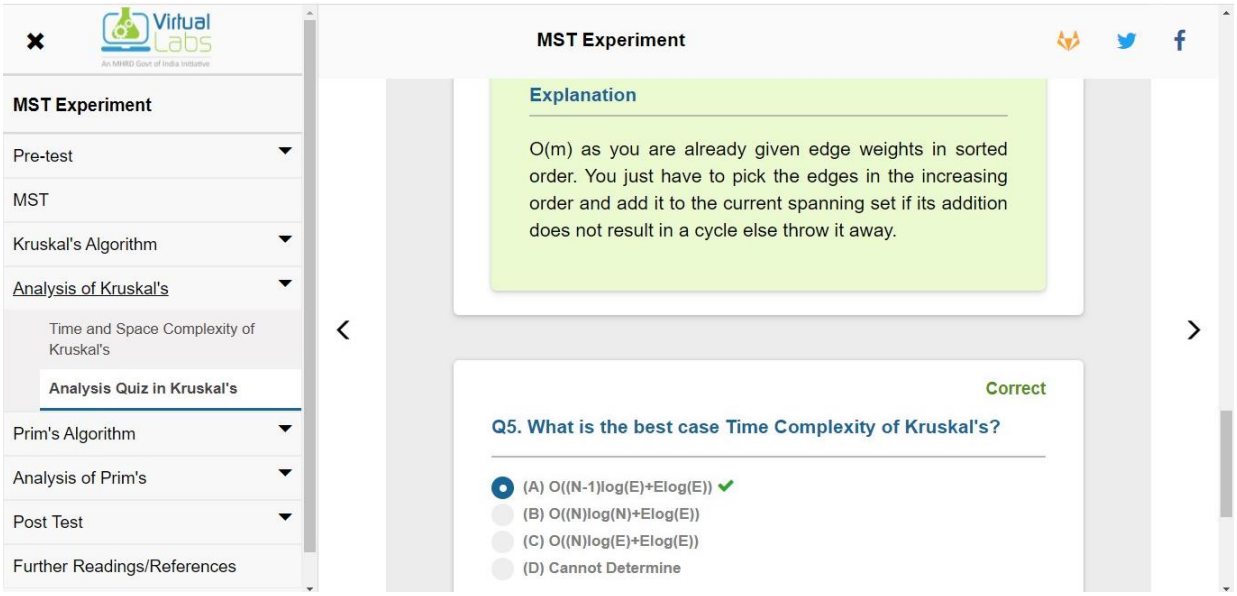
Analysis Quiz in Kruskal's:

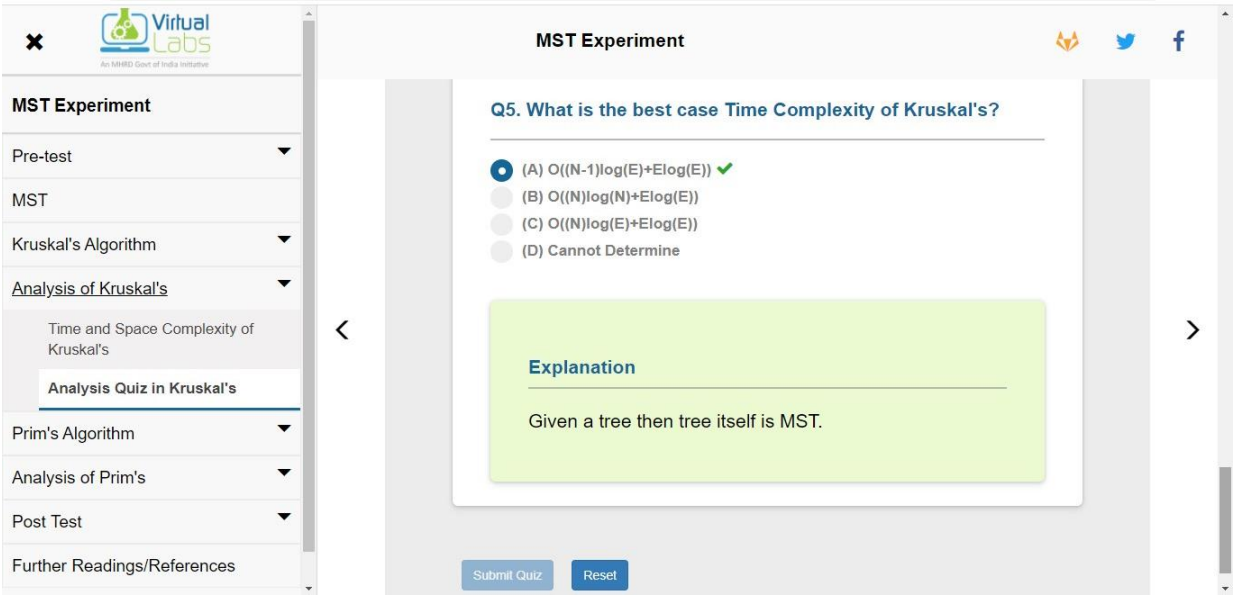



MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Analysis of Kruskal's
Time and Space Complexity of Kruskal's
Analysis Quiz in Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Further Readings/References
All statements are self explanatory
Correct
Q2. Does Kruskal's need more than one MST ?
(A) Yes
(B) No
(C) Depend
(D) Can't say
Explanation


MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Analysis of Kruskal's
Time and Space Complexity of Kruskal's
Analysis Quiz in Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Further Readings/References
Kruskal's generate only one MSt.
Correct
Q3. Which case is the best case Time Complexity of Kruskal's ?
(A) Max.no of edges and many cycles
(B) Min.no of edges and minimum cycles
(C) Min No.of edges with max no of cycles
(D) Can't Say
Explanation


MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Analysis of Kruskal's
Time and Space Complexity of Kruskal's
Analysis Quiz in Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Further Readings/References
Complexity is min when no of edges and cycles are Minimum.
Correct
Q4. If Kruskal's algorithm is used for finding a minimum spanning tree of a weighted graph G with n vertices and m edges and edge weights are already given in a sorted list, then, What will be the time complexity to compute the minimum cost spanning tree given that union and find operations take amortized O(1) ?
(A) O(m logn)
(B) O(n)
(C) O(m)
(D) O(n logm)


MST Experiment
MST Experiment
Pre-test
MST
Kruskal's Algorithm
Analysis of Kruskal's
Time and Space Complexity of Kruskal's
Analysis Quiz in Kruskal's
Prim's Algorithm
Analysis of Prim's
Post Test
Further Readings/References
Explanation
O(m) as you are already given edge weights in sorted order. You just have to pick the edges in the increasing order and add it to the current spanning set if its addition does not result in a cycle else throw it away.
Correct
Q5. What is the best case Time Complexity of Kruskal's?
(A) O((N-1)log(E)+Elog(E))
(B) O((N)log(N)+Elog(E))
(C) O((N)log(E)+Elog(E))
(D) Cannot Determine



Self-evaluation: Solve both Kruskal Quiz and Analysis quiz and Display the result of your first attempt.

Conclusion: (Based on the observations):
Performed the experiment using virtual lab to construct minimum spanning trees using Kruskal's algorithm.

Outcome: CO 3: Implement Greedy and Dynamic Programming algorithms.

References:

1. Richard E. Neapolitan, " Foundation of Algorithms ", 5th Edition 2016, Jones & Bartlett Students Edition
2. Harsh Bhasin , " Algorithms : Design & Analysis", 1st Edition 2013, Oxford Higher education, India
3. T.H. Coreman ,C.E. Leiserson,R.L. Rivest, and C. Stein, " Introduction to algorithms", 3rd Edition 2009, Prentice Hall India Publication
4. Jon Kleinberg, Eva Tardos, " Algorithm Design", 10th Edition 2013, Pearson India Education Services Pvt. Ltd.