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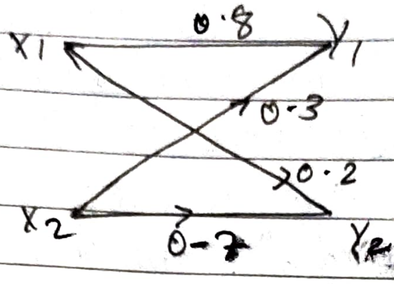
Tutorial 3

Q1] Find the mutual information for the channel shown below

$P(x_1) = 0.6$, & $P(x_2) = 0.4$



→ Solⁿ:-

Mutual Information (I (x, y))

Conditional Probability matrix is

$$P(Y/X) = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.8 & 0.2 \\ x_2 & 0.3 & 0.7 \end{array} \begin{array}{l} \times 0.6 \\ \times 0.4 \end{array}$$

Joint Prob matrix

$$P(x,y) = \begin{bmatrix} 0.48 & 0.12 \\ 0.12 & 0.28 \end{bmatrix}$$

For $H(Y) = \sum_{j=1}^{2} P(y_j) \log_2 \frac{1}{P(y_j)}$

$P(y_1) = 0.48 + 0.12 = 0.60$

$P(y_2) = 0.12 + 0.28 = 0.40$

$H(X) = 0.6 \log_2 \left(\frac{1}{0.6}\right) + 0.4 \log_2 \left(\frac{1}{0.4}\right)$

$= 0.442179 + 0.528771$

$= 0.97095$

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$H(Y/X) = H(X,Y) - H(X)$
$H(X/Y) = H(Y,X) - H(Y)$

$$H(X,Y) = \sum_{i=1}^{2} \sum_{i=1}^{2} P(x_i, y_i) \log \frac{1}{P(x_i, y_i)}$$

$$= 0.48 \log_2 \frac{1}{0.48} + 0.12 \log_2 \frac{1}{0.12}$$

$$+ 0.12 \log_2 \frac{1}{0.12} + 0.28 \log_2 \frac{1}{0.28}$$

$$= 0.5082 + 0.73413 + 0.51422 = 1.75662$$

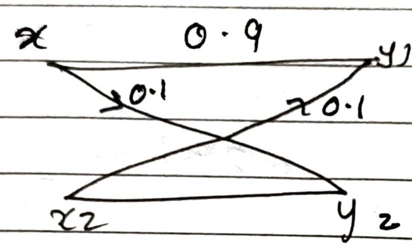
$$H(X) = 0.6 \log_2 \frac{1}{0.6} + 0.4 \log_2 \frac{1}{0.4} = 0.9705$$

$$H(Y/X) = H(X,Y) - H(X) = 1.75622 - 0.97095$$
$$= 0.7857$$

$$I(X,Y) = H(Y) - H(Y/X) = 0.97095 - 0.28507$$
$$= \underline{0.1853 \text{ bits}}$$

Q2] The BSC below is transmitted symbol 0 and 1 at the rate 100 bits calculate rate of transmission

$P(x1) = P(x2) = 0.5$



given $P = 0.1$ $\bar{P} = 0.9$

$$P(Y/X) = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.9 & 0.1 \\ x_2 & 0.1 & 0.9 \end{array}$$

Since it is a binary symmetrical channel.

$H(X) = P \log_2 \left(\frac{1}{P}\right)$

we know $I_t = H \cdot r_s$ $r_s = 100$ bits

Rate of transmission

$H(X) = 0.5 \log_2 \left(\frac{1}{0.5}\right) + 0.5 \log_2 \left(\frac{1}{0.5}\right)$

$= 1 \times \log_2(2)$

$= 1$

$I_t = 100$ bits

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Joint prob matrix

$$P(Y/X) = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \end{array} \begin{array}{c} \\ \times 0.5 \\ \times 0.5 \end{array}$$

JPM is

$$P(X, Y) = \begin{bmatrix} 0.45 & 0.05 \\ 0.05 & 0.45 \end{bmatrix}$$

$$H(Y) = \sum_{j=1}^{2} P(y_i) \log_2 \left(\frac{1}{P(y_i)} \right)$$

$$= 0.95 \log_2 \left(\frac{1}{0.95} \right) + 0.5 \log_2 \left(\frac{1}{0.5} \right)$$

$$= \underline{\underline{1 \text{ bits}}}$$

TUT-3

③ Define kraft's inequality Theorem. For the two codes shown below check whether kraft's inequality is satisfied.

symbol	code 1	code 2
a	00	0
b	01	10
c	10	110
d	11	1110

Sol^n:- Kraft's inequality is a necessary and sufficient condition to check for the existance of an instantaneous code (uniquely decodable code)

$$\sum_{k=1}^{4} r^{-l_k} \leq 1$$

For encoding 1 :- $\sum_{k=1}^{4} 2^{-l_k} \Rightarrow \sum_{k=1}^{4} \frac{1}{2^{l_k}} = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

$= 1$ KI satisfied

For code 2 :- $\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$

$= \frac{15}{16} = 0.93$

∴ For code 1, 4, 2 Kraft's inequality is satisfied since the value ≤ 1

$$\boxed{\text{Formula } \sum_{r=1}^{n} r^{-l_n} \geq 1}$$

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(4) Construct the shannon -fano encoding for the word 'MALYALAM'

→ "MALAYALAM"

frequency of M $= \frac{2}{9} = 0.22$

frequency of A $= \frac{4}{9} = 0.44$

frequency of L $= \frac{2}{9} = 0.22$

frequency of Y $= \frac{1}{9} = 0.11$

X	P(X)
M	0.22
A	0.44
L	0.22
Y	0.11

shannon - fano encoding:

X	P (x)	steps	code	c length
A	0.44	0	0	1
M	0.22	110	10	2
L	0.22	110	110	3
Y	0.11	111	111	3

encoded msg will be 100 11001110110010