Instruction for supervision: Experiment ROE

Tasks

Task 1: Analyse the characteristics of x-rays of copper, molybdenum, tungsten or iron with either a KBr or a LiF analysing crystal. Identify the atomic transitions, which create the measured lines in the x-ray spectrum, and calculate the corresponding energies.
Task 2: Determine the Planck's constant from the dependence of the short-wave limit of the X-ray photons in the continuous spectrum on the acceleration voltage of the electrons.

Pre-considerations

- date, name, name of co-worker and supervisor, title of experiment, room number and place
- goal of the experiment, possible concretions, physical background, measurement principle, experimental set-up
- Basic formulas →

  For task 1:

  Bragg's deflection equation is

  $$2d \sin\theta = n\lambda \tag{1}$$

  where d is the distance between the lattice planes, θ is the glance angle at the crystal. n and λ being the order of diffraction, and the wavelength, respectively. The energy corresponds to the atomic transition can be obtained by

  $$E = h \cdot c/\lambda, \tag{2}$$

  where h is the Plank's constant, and λ can be used from Eq. (1).

For task 2:

The maximum photon energy can be equated to the anode voltage times the electronic charge i. e. $U_\mathrm{A} e$, with the corresponding wavelength being the shortest one i. e. $\lambda = \lambda_\mathrm{min}$. λ_min can be calculated using Eq. (1) with $\theta = \theta_\mathrm{min}$ being the threshold angle (minimum angle) and diffraction order n being 1. Therefore using the conditions of the short-wave limit, Eq. (2) reduces to

$$\lambda_\mathrm{min} = \frac{hc}{eU_\mathrm{A}}. \tag{3}$$

Planck's constant h is determined through the slope of the Duane-Hunt line (λ_min vs $1/U_\mathrm{A}$), m, by the following relation

$$h = \frac{e}{c} m. \tag{4}$$

Measurement

Task 1: analysis of atomic transition

The following parameters are kept fixed throughout this task.

Anode voltage	$U_\mathrm{A} = 35$ kV
Emission current	$I_\mathrm{em} = 1.0$ mA
Detector Coupling	1:2
Collimator width	$d_\mathrm{c} = 2$ mm
Integration time	$\tau = 6$ s
Increment in angle	$\delta\theta = 0.1°$

The starting and the stopping angle follows from the following table for different pairs of the anode materials and the crystals.

anode material	crystal	initial angle θ_i	final angle θ_f
Cu	LiF	4°	55°
Cu	KBr	3°	75°
Mo	LiF	3°	65°
W	LiF	4°	80°

TABLE 1: Setting of the starting and the stopping angle in the goniometer.

Experimental data

The following experimental data are obtained for various pairs of anode materials and the analysing crystals. θ_{min}, θ_{max} are the entry and exit angles corresponding to the peak, and θ is the peak angle.

$\theta_{\mathrm{min}}/°$	$\theta_{\mathrm{max}}/°$	$\theta/°$	n
19.9	20.7	20.3	1
22.0	23.1	22.6	1
43.6	44.0	43.8	2
49.7	50.4	50	2

TABLE 2: Measured data for Cu anode and LiF analyser.

$\theta_{\mathrm{min}}/°$	$\theta_{\mathrm{max}}/°$	$\theta/°$	n
12.1	12.9	12.3	1
13.1	14.4	13.6	1
24.7	25.4	25.1	2
27.4	28.6	28	2
39	39.5	39.3	3
44.2	44.9	44.6	3
57.6	58.1	57.7	4
68.8	69.5	69.1	4

TABLE 3: Measured data for Cu anode and KBr analyser.

$\theta_{\mathrm{min}}/°$	$\theta_{\mathrm{max}}/°$	$\theta/°$	n
8.6	9.4	9.1	1
9.8	10.8	10.3	1
18.1	18.6	18.3	2
20.3	21.1	20.7	2
27.8	28.5	28.1	3
31.7	32.3	31.9	3
38.5	39	38.8	4
44.4	45	44.7	4

TABLE 4: Measured data for Mo anode and LiF analyser.

$\theta_{\min}/°$	$\theta_{\max}/°$	$\theta/°$	n
17.3	18.2	18.0	1
18.3	19.3	18.5	1
21.2	21.8	21.5	1
37.8	38.4	38.1	2
39.1	39.8	39.5	2
46.7	47.4	47.1	2
67.6	68.2	67.9	3
72.3	72.9	72.6	3

TABLE 5: Measured data for W anode and LiF analyser.

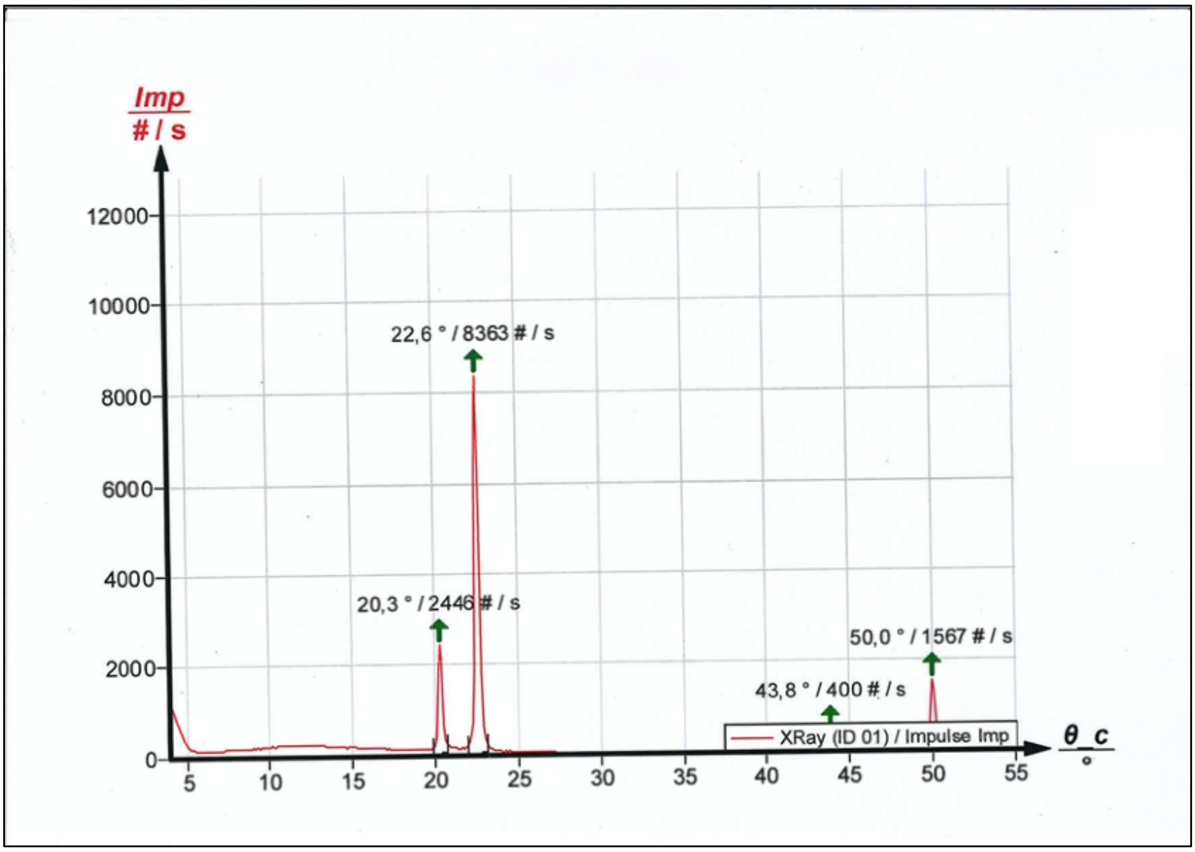

FIGURE 1: A sample of spectra of the glance angle θ, for Cu anode and LiF analyser.

Task 2: determination of the Planck's constant

The following parameters have been taken into consideration for this task.

Anode voltage $U_A = 15 - 31$ kV

Emission current $I_{em} = 1.0$ mA

Detector Coupling 1:2

Collimator width $d_c = 2$ mm

Integration time $\tau = 2$ s

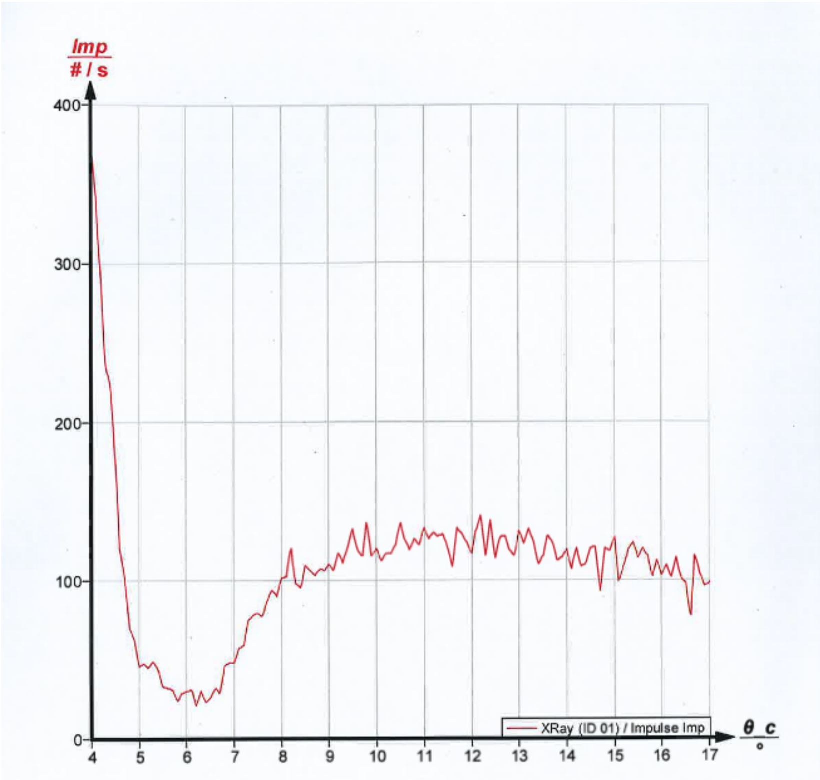

FIGURE 2: A sample of spectra of the glance angle θ, for Cu anode, LiF analyser, and anode voltage U_A being fixed at 27 kV. Here, It can be seen that the θ_{min} is obtained at 6.3°.

Increment in angle	$\delta\theta = 0.1°$
Initial angle	$\theta_{\text{i}} = 4°$
Final angle	$\theta_{\text{f}} = 17°$
Analyser	LiF
Anode	Cu

U_{A}/kV	$\theta_{\text{min}}/°$
15	12.4
17	10.4
19	9.35
21	8.5
23	7.7
25	7.2
27	6.3
29	6.0
31	5.6

TABLE 6: Threshold angles obtained for different anode voltages.

Evaluation

Task 1

The excitation energy E is calculated using the following equation

$$E = \frac{nhc}{2d\sin\theta}, \tag{5}$$

obtained from Eqs. (1)-(2).

The systematic uncertainty ΔE and the statistical uncertainty σ_E are calculated using the following formulas. ΔE is obtained by [1]

$$\Delta E = \sqrt{\left(E\frac{\Delta d}{d}\right)^2 + \left(\frac{E\Delta\theta}{\tan\theta}\right)^2}. \tag{6}$$

Here, systematic uncertainty of the lattice constant is given by the last digit, and we set $\Delta d = 0.001/\sqrt{3}$ Å. The systematic uncertainty of the angle $\Delta\theta$ is composed of the full width at the half maximum FWHM of the peaks, as given by

$$\Delta\theta = \frac{\theta_{\text{max}} - \theta_{\text{min}}}{2}.$$

The corresponding values of θ, θ_{min} and θ_{max} are provided in Tables 2-5. σ_E is obtained by

$$\sigma_E = \frac{E}{\tan\theta}\,\sigma_\theta. \tag{7}$$

So the statistical uncertainty is calculated from the reading uncertainty of the angular measurement, which is here given by $\sigma_\theta = 0.05°$.

E/eV	ΔE/eV	σ_E/eV
8872	167.48	20.86
8009	184.70	16.74
8894	32.47	8.06
8036	41.25	5.86

TABLE 7: Calculation of excitation energies for Cu anode and LiF analyser.

[1] Δ and σ always stands for the systematic and statistical uncertainty, respectively.

E/eV	ΔE/eV	σ_E/eV
8844	283.17	35.28
8013	375.75	28.81
8883	115.85	16.49
8026	158.07	13.13
8924	47.59	9.48
8050	49.88	7.10
8917	24.64	4.90
8068	18.87	2.68

Table 8: Calculation of excitation energies for Cu anode and KBr analyser.

E/eV	ΔE/eV	σ_E/eV
19462	848.26	105.70
17215	826.64	82.41
19606	258.72	51.57
17416	321.79	40.09
19605	224.35	31.94
17475	147.08	24.42
19649	106.78	21.26
17504	92.74	15.39

Table 9: Calculation of excitation energies for Mo anode and LiF analyser.

E/eV	ΔE/eV	σ_E/eV
9960	240.76	26.66
9700	252.99	25.22
8398	111.65	18.54
9976	66.67	11.06
9677	71.76	10.21
8403	47.75	6.79
9966	21.37	3.52
9676	16.11	2.63

Table 10: Calculation of excitation energies for W anode and LiF analyser.

Task 2

Using the Bragg formula with $n = 1$, the minimum wavelength λ_{min} is obtained as

$$\lambda_{\mathrm{min}} = 2d \sin\theta_{\mathrm{min}}. \tag{8}$$

λ_{min} is plotted as a function of $1/U_{\mathrm{A}}$ in Fig. 3, with the values are calculated and shown in Table 11.

U_{A}/kV	$1/U_{\mathrm{A}}$/ kV^{-1}	λ_{min}/pm	$\sigma_{\lambda_{\mathrm{min}}}$/pm
15	0.0667	82.4	0.6866
17	0.0588	72.71	0.6914
19	0.0526	65.44	0.6936
21	0.0476	59.53	0.6952
23	0.0434	53.96	0.6966
25	0.0400	50.48	0.6974
27	0.0370	44.20	0.6986
29	0.0345	42.10	0.6990
31	0.0323	39.30	0.6996

TABLE 11: Calculated minimum wavelength and the inverse anode voltage

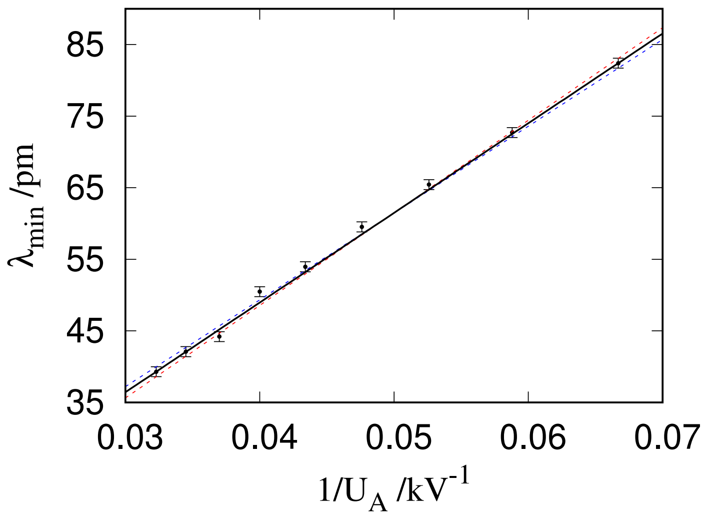

FIGURE 3: Duane Hunt plot: minimum wavelength as a function of the reciprocal acceleration voltage. The black solid line represents the regression line. The dashed red and blue lines represents the lines with maximum and minimum slope in the linear regression method. The error bars are fixed by the statistical uncertainty $\sigma_{\lambda_{\mathrm{min}}}$.

The systematic uncertainty of the slope of Duane-Hunt line is given by

$$\Delta m = \frac{m_{\mathrm{max}} - m_{\mathrm{min}}}{2}. \tag{9}$$

Here m_{min} and m_{max} can be calculated as respectively the slope of the minimum line and the maximum line of the linear regression method corresponding to the Duane-Hunt data, on knowing the extension of the error bars are given by the statistical uncertainty of the λ_{min}. This uncertainty is estimated as the following

$$\sigma_{\lambda_{\text{min}}} = 2d \cos \theta_{\text{min}} \sigma_\theta,$$

σ_θ being the reading error of the angle, which is here $0.1°$.

σ_m, the statistical uncertainty can be obtained by simply fitting the Duane-Hunt data, with the slope being one fitting parameter. The uncertainty for the slope in the fits determines the σ_m.

The final value of the Planck's constant is given by

$$h = \frac{e}{c}(m \pm \Delta m \pm \sigma_m),$$

here m is the value of the slope of the corresponding regression line. This last equation here follows from Eq. (4). For the values as obtained in Fig. (3), h is given by

$$\begin{aligned} h &= \frac{1.602}{2.9979}(1252.906 \pm 40.10 \pm 23.83) \times 10^{-36}\text{Js} \\ &= (6.69 \pm 0.21 \pm 0.13) \times 10^{-34}\text{Js} \end{aligned}$$

Discussion

- The average excitation energies are calculated as

$$\overline{E} = \frac{\sum_i^N E_i}{N}, \tag{10}$$

and the corresponding uncertainties are calculated using the Gaussian error propagation

$$\delta\overline{E} = \sqrt{\frac{1}{N}\sum_{i}^{N}(\delta E_i)^2} \tag{11}$$

	Experiment $\rightarrow$ $(\overline{E} \pm \Delta\overline{E} \pm \sigma_{\overline{E}})$/keV	Literature
$(E_\alpha^{\mathrm{Cu}})_{\mathrm{KBr}}$	8.03 $\pm$ 0.205 $\pm$ 0.016	8.04 keV
$(E_\alpha^{\mathrm{Cu}})_{\mathrm{LiF}}$	8.02 $\pm$0.133 $\pm$ 0.012	
$(E_\beta^{\mathrm{Cu}})_{\mathrm{KBr}}$	8.89 $\pm$ 0.155 $\pm$ 0.020	8.90 keV
$(E_\beta^{\mathrm{Cu}})_{\mathrm{LiF}}$	8.88 $\pm$ 0.120 $\pm$0.015	
$(E_\alpha^{\mathrm{Mo}})_{\mathrm{LiF}}$	17.40 $\pm$ 0.451 $\pm$ 0.048	17.37 keV
$(E_\beta^{\mathrm{Mo}})_{\mathrm{LiF}}$	19.58 $\pm$ 0.460 $\pm$ 0.061	19.59 keV
$(E_\alpha^{\mathrm{W}})_{\mathrm{LiF}}$	8.40 $\pm$ 0.085 $\pm$ 0.013	8.39 keV
$(E_{\beta 1}^{\mathrm{W}})_{\mathrm{LiF}}$	9.68 $\pm$0.152 $\pm$ 0.015	9.67 keV
$(E_{\beta 2}^{\mathrm{W}})_{\mathrm{LiF}}$	9.97 $\pm$ 0.144 $\pm$ 0.016	9.96 keV

TABLE 12: Obtained excitation energy and the corresponding literature value

- The Planck's constant is calculated as

$$h = (6.69 \pm 0.21 \pm 0.13) \times 10^{-34}\mathrm{Js}$$

. The actual literature value of h is 6.6256×10^{-34}Js.

- Questions:
  1. At which wavelengths and in which energy range are x-rays located?
  2. Which different x-ray spectra have to be distinguished?
  3. Why is there a short-wave limit in the brems spectrum?
  4. How is the characteristic x-ray spectrum created?
  5. How to improve the uncertainties and/or the experiment?