RM 294 – Optimization I
Project 1 – Linear Programming

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Linear Problem

We have defined **10 variables corresponding to the budget spent** *(<u>in millions</u>)* on each of the 10 marketing mediums:
x_1 = amount spent in Print, x_2 = amount spent in TV, x_3 = amount spent in SEO,
x_4 = amount spent in AdWords, x_5 = amount spent in Facebook, x_6 = amount spent in LinkedIn,
x_7 = amount spent in Instagram, x_8 = amount spent in Snapchat, x_9 = amount spent in Twitter,
x_{10} = amount spent in email

We have the following budget constraints:

Constraint on the total budget:

- $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} <= 10$

Amount invested in print and TV should be no more than the amount spent on Facebook and Email:

- $x_1 + x_2 - x_5 - x_{10} <= 0$

Amount used in social media should be at least twice of SEO and AdWords:

- $x_5 + x_6 + x_7 + x_8 + x_9 - 2x_3 - 2x_4 >= 0$

For each platform, the amount invested should be no more than $3M:

- $x_1 <= 3$
- $x_2 <= 3$
- $x_3 <= 3$
- $x_4 <= 3$
- $x_5 <= 3$
- $x_6 <= 3$
- $x_7 <= 3$
- $x_8 <= 3$
- $x_9 <= 3$
- $x_{10} <= 3$

ROI Data File

Please find below the snippet from the ROI data file while provides the estimated ROIs on different marketing mediums as per the two consulting firms:

	Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
0	ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044
1	Second Firms ROI Estimate	0.049	0.023	0.024	0.039	0.044	0.046	0.026	0.019	0.037	0.026

Optimal Objective:

Using first consulting firm's ROI, maximize:

$0.031x_1 + 0.049x_2 + 0.024x_3 + 0.039x_4 + 0.016x_5 + 0.024x_6 + 0.046x_7 + 0.026x_8 + 0.033x_9 + 0.044x_{10}$

Using second consulting firm's ROI, maximize:

$0.049x_1 + 0.023x_2 + 0.024x_3 + 0.039x_4 + 0.044x_5 + 0.046x_6 + 0.026x_7 + 0.019x_8 + 0.037x_9 + 0.026x_{10}$

Optimal Budget Allocation

Part 3):

Using first consulting firm's ROI, maximize:

$0.031x_1 + 0.049x_2 + 0.024x_3 + 0.039x_4 + 0.016x_5 + 0.024x_6 + 0.046x_7 + 0.026x_8 + 0.033x_9 + 0.044x_{10}$

```
optimal_obj_1 = marketing_model1.objVal #extract optimal objective value
print("Optimal objective value:",optimal_obj_1)
```

```
Optimal objective value: 0.45600000000000007
```

```
allocation_1 = marketing_x1.x #extract optimal allocation
print("The optimal budget allocation according to the first consulting firm:", allocation_1)
```

```
The optimal budget allocation according to the first consulting firm: [0. 3. 0. 1. 0. 0. 3. 0. 0. 3.]
```

As observed from the code snippet below - if we use the ROIs estimated by the first consulting firm, the optimal budget allocation suggests that we invest **$3M each in TV, Instagram and Email, $1M in AdWords and no investment in rest of the mediums. The optimal objective value is $0.456M**

Part 4):

Using second consulting firm's ROI, maximize:

$0.049x_1 + 0.023x_2 + 0.024x_3 + 0.039x_4 + 0.044x_5 + 0.046x_6 + 0.026x_7 + 0.019x_8 + 0.037x_9 + 0.026x_{10}$

```python
optimal_obj_2 = marketing_model2.objVal # extract optimal objective value
print("Optimal objective value:",optimal_obj_2)
```

```
Optimal objective value: 0.45600000000000007
```

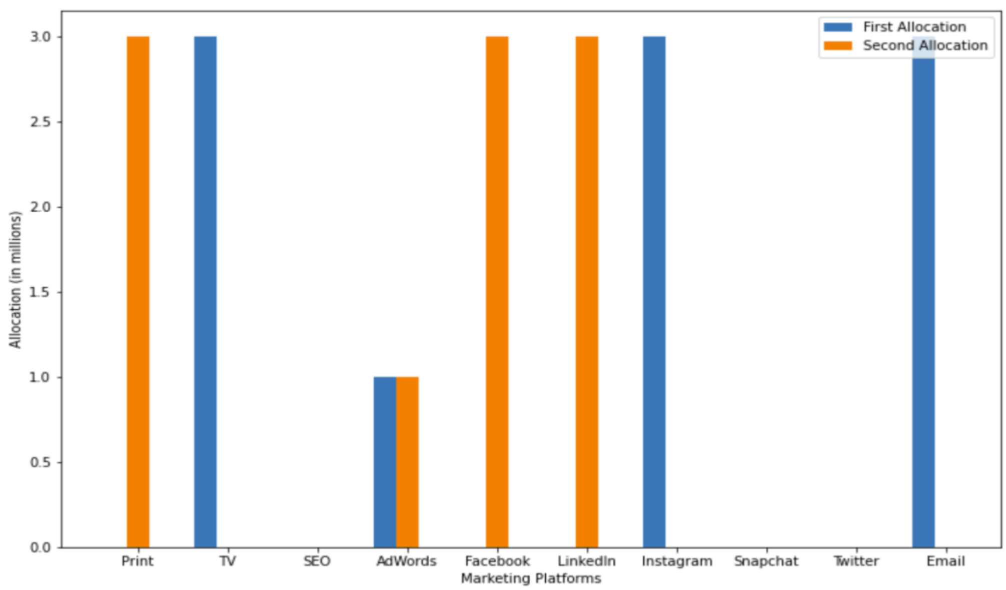
```python
allocation_2 = marketing_x2.x # extract optimal allocation
print("The optimal budget allocation according to the second consulting firm:", allocation_2)
```

```
The optimal budget allocation according to the second consulting firm: [3. 0. 0. 1. 3. 3. 0. 0. 0. 0.]
```

As observed from the code snippet below - if we use the ROIs estimated by the second consulting firm, the optimal budget allocation suggests that we invest **$3M each in Print, Facebook and LinkedIn, $1M in AdWords and no investment in rest of the mediums. The optimal objective value is $0.456M** (same as Part 3).

Comparing the two allocations

Part 5):



Although the optimal objective value remains same in utilization of ROI estimates from both firms ($0.456M), the allocations to marketing mediums are different. Comparing both, we observe:

- The first ROI data suggests that we invest $3M each in TV, Instagram and Email. However, the ROI data from the second firm suggests that we do not invest in any of these mediums and instead invest $3M each in Print, Facebook and LinkedIn.
- Both suggest that we invest $1M in AdWords
- Both suggest that we do not invest in SEO, Snapchat and Twitter

Using second allocation when first ROI data is correct:

```
print("The optimal objective decreases by",optimal_obj_1 - marketing_return1@allocation_2)
```

```
The optimal objective decreases by 0.20400000000000007
```

Assuming the first ROI data is correct, if we were to use the second allocation, the objective **decreases by $0.204 million** as compared to the optimal objective.

Using first allocation when second ROI data is correct:

```
print("The optimal objective decreases by",optimal_obj_2 - marketing_return2@allocation_1)
```

```
The optimal objective decreases by 0.19200000000000006
```

Assuming the second ROI data is correct, if we were to use the first allocation, the objective **decreases by $0.192 million** as compared to the optimal objective.

Is the third constraint useful?

```
[con.Pi for con in marketing_con1]
```

```
[0.039,
 0.010000000000000002,
 0.0,
 0.0,
 0.0,
 0.0,
 0.0,
 0.0,
 0.0,
 0.006999999999999999,
 0.0,
 0.0,
 0.015]
```

```
[con.Pi for con in marketing_con2]
```

```
[0.039,
 0.010000000000000002,
 0.0,
 0.0,
 0.0,
 0.0,
 0.0,
 0.015,
 0.006999999999999999,
 0.0,
 0.0,
 0.0,
 0.0]
```

First ROI estimates: If we relax the constraints on the amount invested in Instagram or email (<= 3M dollars constraint currently), our objective value will improve.

Second ROI estimates: If we relax the constraints on the amount invested in Facebook or LinkedIn (<= 3M dollars constraint currently), our objective value will improve.

In other words, the shadow price for these constraints is non-zero which implies these are binding constraints. Hence, the third constraint is contributing to bindings constraints which are driving the estimation of optimal value and if the constraint changes, the optimal value changes as well.

Also, it's worth noting that although the relaxation of the third constraint can increase the objective value, it's better to invest across different platforms in order to reduce risks.

Range of ROI data

Part 6):

Range of ROI values such that optimal allocation is constant:

	Min	Max
Print	-inf	0.049
TV	0.039	0.062
SEO	-inf	0.039
AdWords	0.033	0.046
Facebook	-inf	0.029
LinkedIn	-inf	0.039
Instagram	0.039	inf
Snapchat	-inf	0.039
Twitter	-inf	0.039
Email	0.029	inf

The above table gives us the range of the ROI data such that optimal allocation remains the same. Hence, using the first ROI data as our starting point, we can increase the ROI estimates by the following amounts:

	Can decrease by	Can increase by
Print	inf	0.018
TV	0.01	0.013
SEO	inf	0.015
AdWords	0.006	0.007
Facebook	inf	0.013
LinkedIn	inf	0.015
Instagram	0.007	inf
Snapchat	inf	0.013
Twitter	inf	0.006
Email	0.015	inf

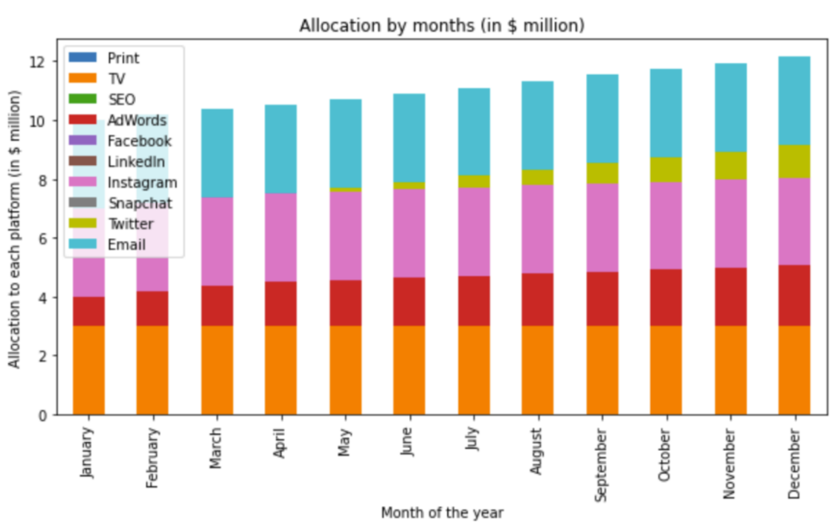
Optimal allocations for each month

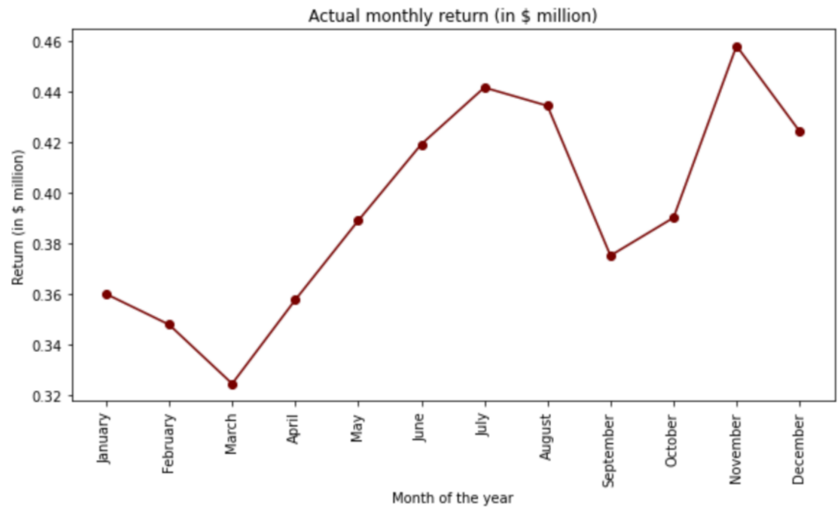
Part 7) Optimal allocations (in millions):

The table shows the optimal allocations for each month. The last column also contains the data corresponding to actual return in each month.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Actual returns
January	0.0	3.0	0.0	1.000000	0.0	0.0	3.0	0.0	0.000000	3.0	0.360000
February	0.0	3.0	0.0	1.180000	0.0	0.0	3.0	0.0	0.000000	3.0	0.347840
March	0.0	3.0	0.0	1.353920	0.0	0.0	3.0	0.0	0.000000	3.0	0.324449
April	0.0	3.0	0.0	1.505381	0.0	0.0	3.0	0.0	0.010763	3.0	0.357624
May	0.0	3.0	0.0	1.564986	0.0	0.0	3.0	0.0	0.129971	3.0	0.389058
June	0.0	3.0	0.0	1.629829	0.0	0.0	3.0	0.0	0.259657	3.0	0.419281
July	0.0	3.0	0.0	1.699709	0.0	0.0	3.0	0.0	0.399418	3.0	0.441764
August	0.0	3.0	0.0	1.773336	0.0	0.0	3.0	0.0	0.546672	3.0	0.434534
September	0.0	3.0	0.0	1.845758	0.0	0.0	3.0	0.0	0.691517	3.0	0.375183
October	0.0	3.0	0.0	1.908289	0.0	0.0	3.0	0.0	0.816578	3.0	0.390196
November	0.0	3.0	0.0	1.973321	0.0	0.0	3.0	0.0	0.946643	3.0	0.458252
December	0.0	3.0	0.0	2.049697	0.0	0.0	3.0	0.0	1.099393	3.0	0.424417

Let's see some graphs from the above table:





Stable budget

Part 8):

To determine if the budget is stable or not, we compared the investments for each platform for 2 successive months starting from January (month 0) to November (month 11) (i.e., at the end, we compare November with December) and assigned a value of 1 to the stable matrix's [month][platform] if the absolute value of the difference in investments is less than or equal to 1 million.

We then loop over the stable matrix to check if there are any zeros (i.e., the solution is unstable)

```
stable = np.zeros((11,10))

for j in range(10):    #This is the range to loop over all the 10 platforms of investment; j has values 0,1,2,3,4,5,6,7,8,9
    for i in range(11):  #This is the range to loop over all the 11 months of the year (Jan-Nov) and compare the investments
        if abs(monthly_alloc.iloc[i][j] - monthly_alloc.iloc[i+1][j]) <= 1:
            stable[i][j] = 1

for i in range(11):
    if np.product(stable[i]) == 0:
        print("Solution is unstable")
        break

    elif i==10:
        print("Solution is stable")
```

```
Solution is stable
```

As observed from the output above, all 0s in the Stable matrix converted to 1s, hence the budget is **stable**.