

Dynamic Array Based Stack : Worst Case Time Analysis

Let's assume we have $N=2^k$ elements and capacity = 2^k .

1) We perform a push operation and capacity is doubled.

$$\text{Capacity} = 2^{(k+1)}$$

$$\text{Elements} = 2^k + 1$$

$$\text{Time taken } t_1 = 2^{(k+1)} + 2^k + 1$$

2) We perform $2^{(k-1)}$ pop operations.

$$\text{Capacity} = 2^{(k+1)}$$

$$\text{Elements} = 2^{(k-1)} + 1$$

$$\text{Time taken } t_2 = 2^{(k-1)}$$

3) We perform another pop and capacity is halved.

$$\text{Capacity} = 2^k$$

$$\text{Elements} = 2^{(k-1)}$$

$$\text{Time taken } t_3 = 1 + 2^k + 2^{(k-1)}$$

$$\text{Total operations performed } n = 1 + 2^{(k-1)} + 1 = 2^{(k-1)} + 2$$

$$\text{Total time} = t_1 + t_2 + t_3$$

$$= 10 \cdot n - 18$$

$$\text{Amortised Time taken} = O((10 \cdot n - 18)/n) = O(10) = O(1)$$

NOTE :

1) Cost for creating new array of size $n = n$

2) Cost for simple push/pop operation = 1 (without changing capacity)

3) Cost of copying n elements into a new array = n