The Adversarial Networks Nonsense

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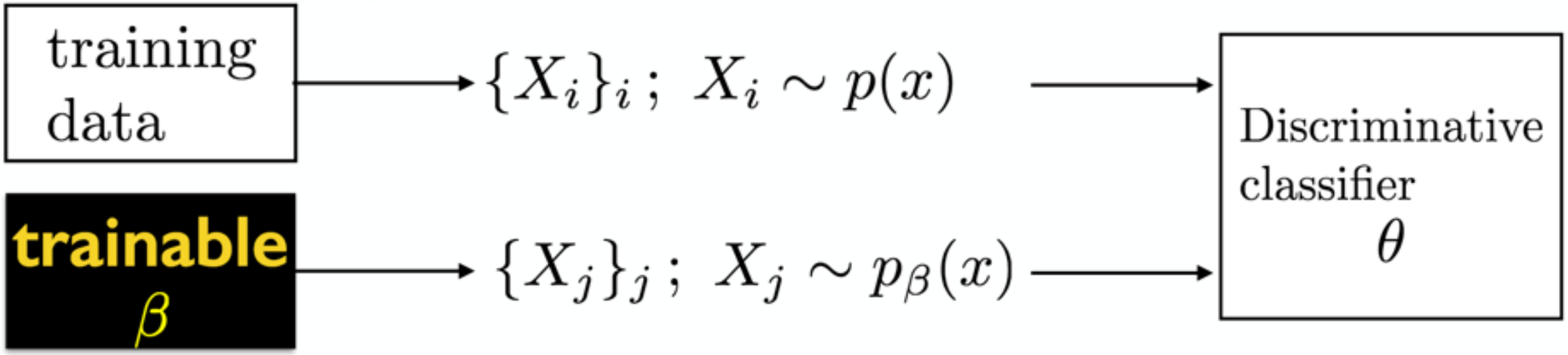
Recap of Lecture 16/17

Generative Adversarial Networks

[Goodfellow et al., '14]

- Suppose we have a *trainable* black box generator:

- Given observed data $\{X_i\}_i$; $X_i \sim p(x)$, how to force our generator to produce samples from $p(x)$?

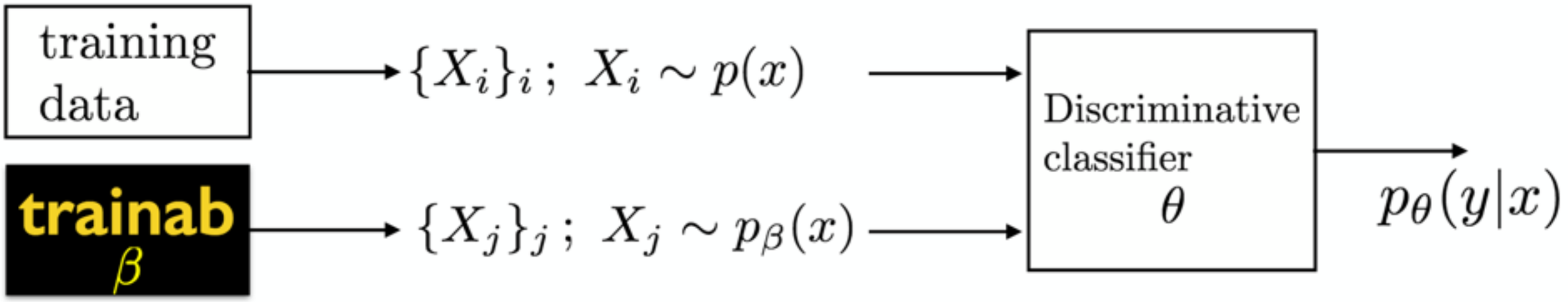


- The generator should make the classification task *as hard as possible for any discriminator.*

Generative Adversarial Networks

[Goodfellow et al., '14]

- Train generator and discriminator in a minimax setting:



$y = 1$: "real" samples
$y = 0$: "fake" samples

$$\min_{\beta} \max_{\theta} \left(\mathbb{E}_{x \sim p_{data}} \log p_\theta(y = 1|x) + \mathbb{E}_{x \sim p_\beta} \log p_\theta(y = 0|x) \right) \quad .$$

Generative Adversarial Networks

- Q: Do we have consistency? (in the limit of infinite capacity)

Given current p_β and p_{data}, the optimum discriminator is given by

$$D(x) = p(y = 1|x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_\beta(x)} \ .$$

For each x,

$$p_{data}(x) \log D(x) + p_\beta(x) \log(1 - D(x)) = (p_{data}(x) + p_\beta(x)) \left(\alpha \log \gamma + (1 - \alpha \log(1 - \gamma)\right) \ ,$$

$$\alpha = \frac{p_{data}(x)}{p_{data}(x) + p_\beta(x)} \ , \ \gamma = D(x) \ .$$

But

$$\alpha \log \gamma + (1 - \alpha) \log(1 - \gamma) = -H(\bar{\alpha}) - D_{KL}(\bar{\alpha}||p(y|x)) \leq -H(\bar{\alpha})$$

Generative Adversarial Networks

- It results that

$$\min -H(\bar{\alpha}) \text{ is attained when } \alpha = 1/2, \text{ thus}$$

$$p_\beta(x) = p_{data}(x)$$

- In practice, however, we parametrize both generator and discriminator using neural networks.

- Optimize the cost using gradient descent.

Generative Adversarial Training

$$F(\beta, \theta) = \left(\mathbb{E}_{x \sim p_{data}} \log p_\theta(y = 1|x) + \mathbb{E}_{x \sim p_\beta} \log p_\theta(y = 0|x) \right) .$$

$$\min_\beta \max_\theta F(\beta, \theta)$$

- Challenge: it is unfeasible to optimize fully in the inner discriminator loop:

$$\theta^*(\beta) = \arg\max_\theta F(\beta, \theta) . \qquad G(\beta) := F(\beta, \theta^*(\beta))$$

- Indeed, $$\frac{\partial G(\beta)}{\partial \beta} = 0 \quad w.h.p.$$

- Numerical approach: alternate *k* steps of discriminator update with 1 step of generator update.

LAPGAN

[Denton, Chintala et al.'15]

- Initial GAN models were hard to scale to large input domains.
- Laplacian Pyramid of Adversarial Networks significantly improved quality by generating independently at each scale.
- Laplacian Pyramids are invertible linear multi-scale decompositions:

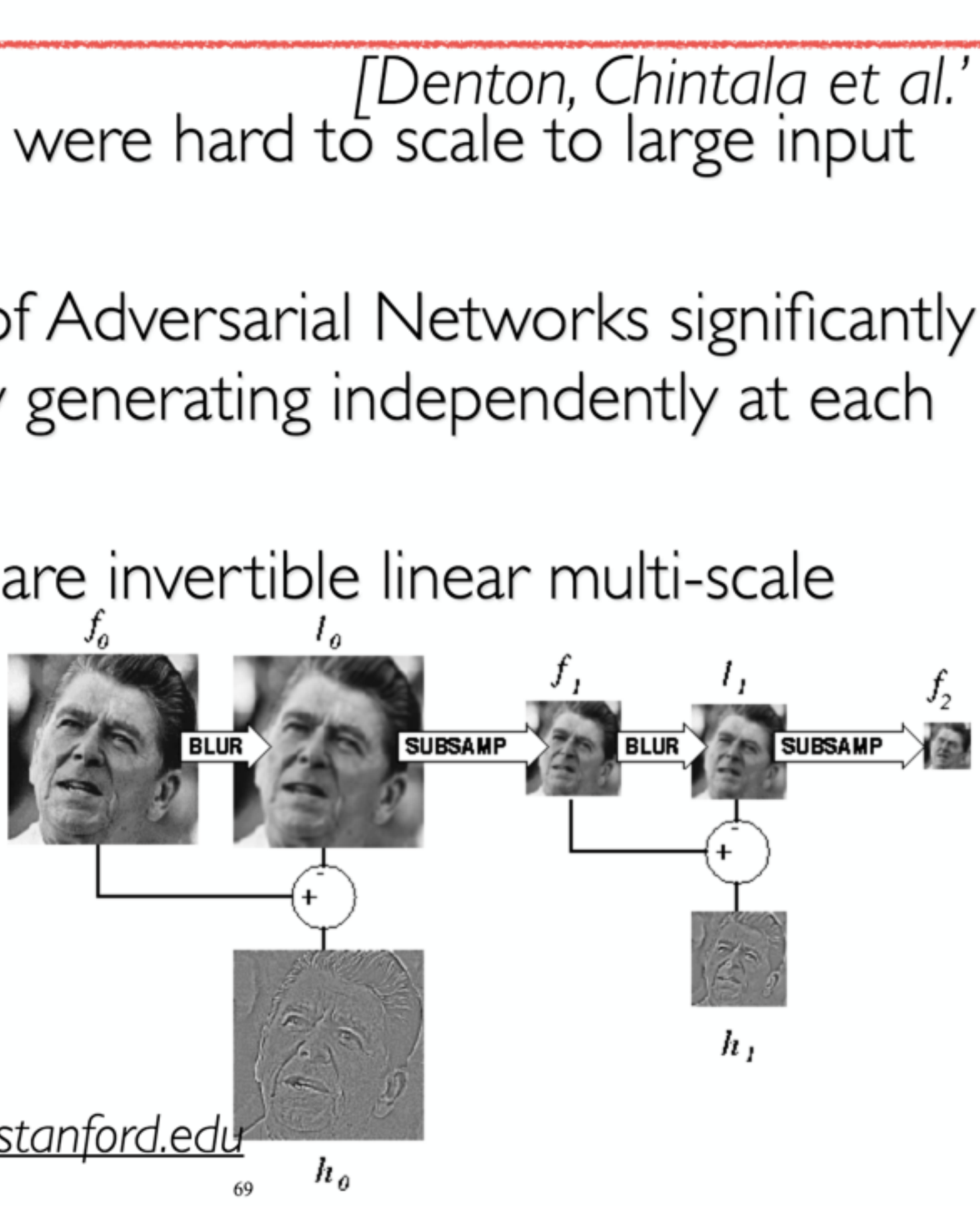
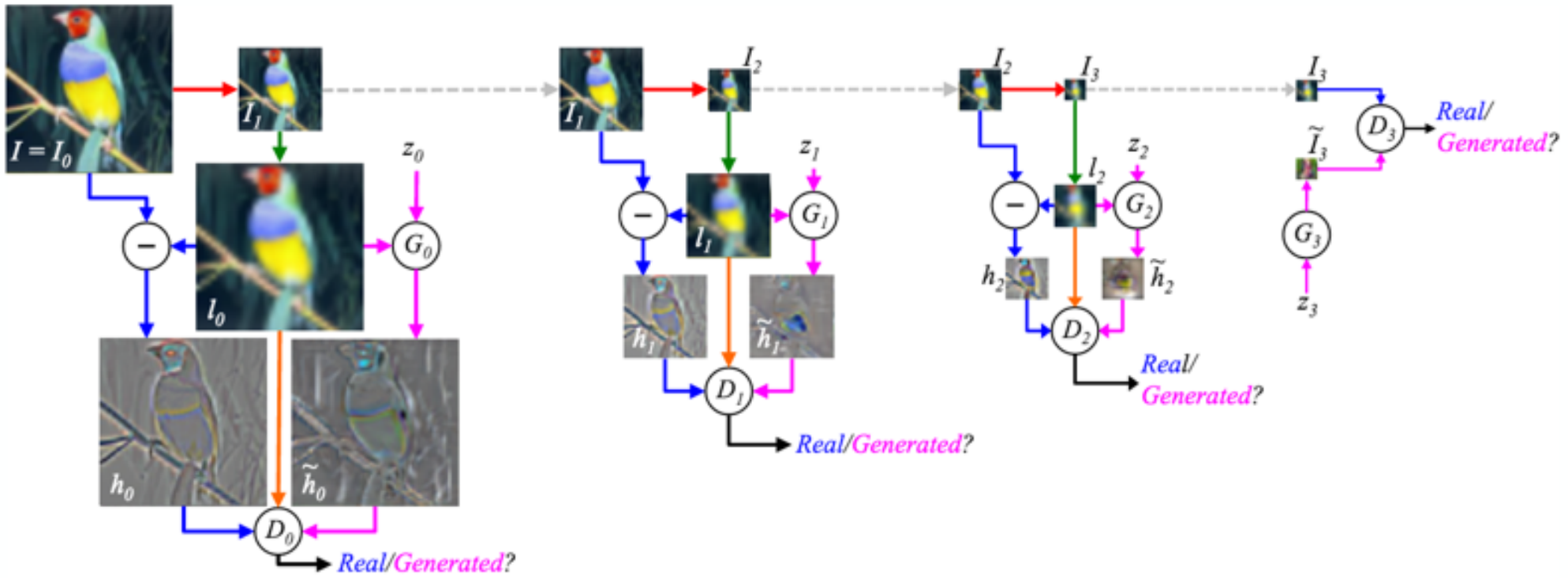


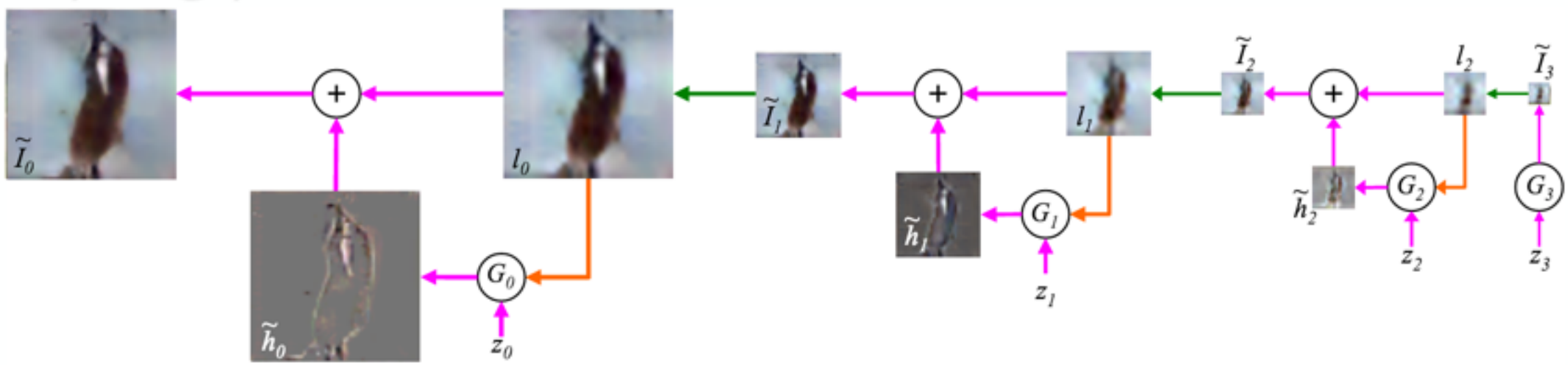
figure source: http://sepwww.stanford.edu

LAPGAN

- Training procedure:

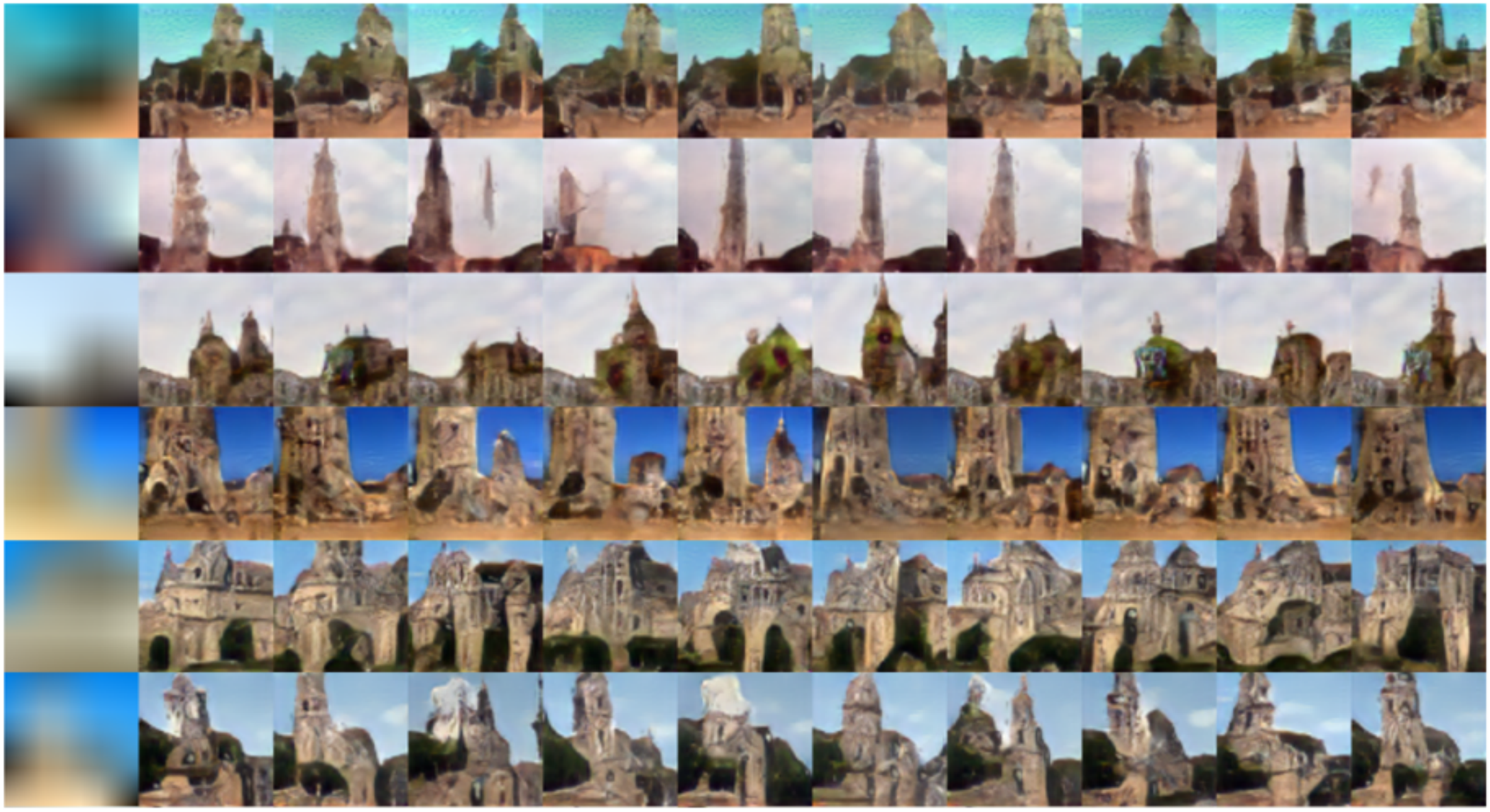


- Sampling procedure:



LAPGAN

- Samples generated from the model:



LAPGAN

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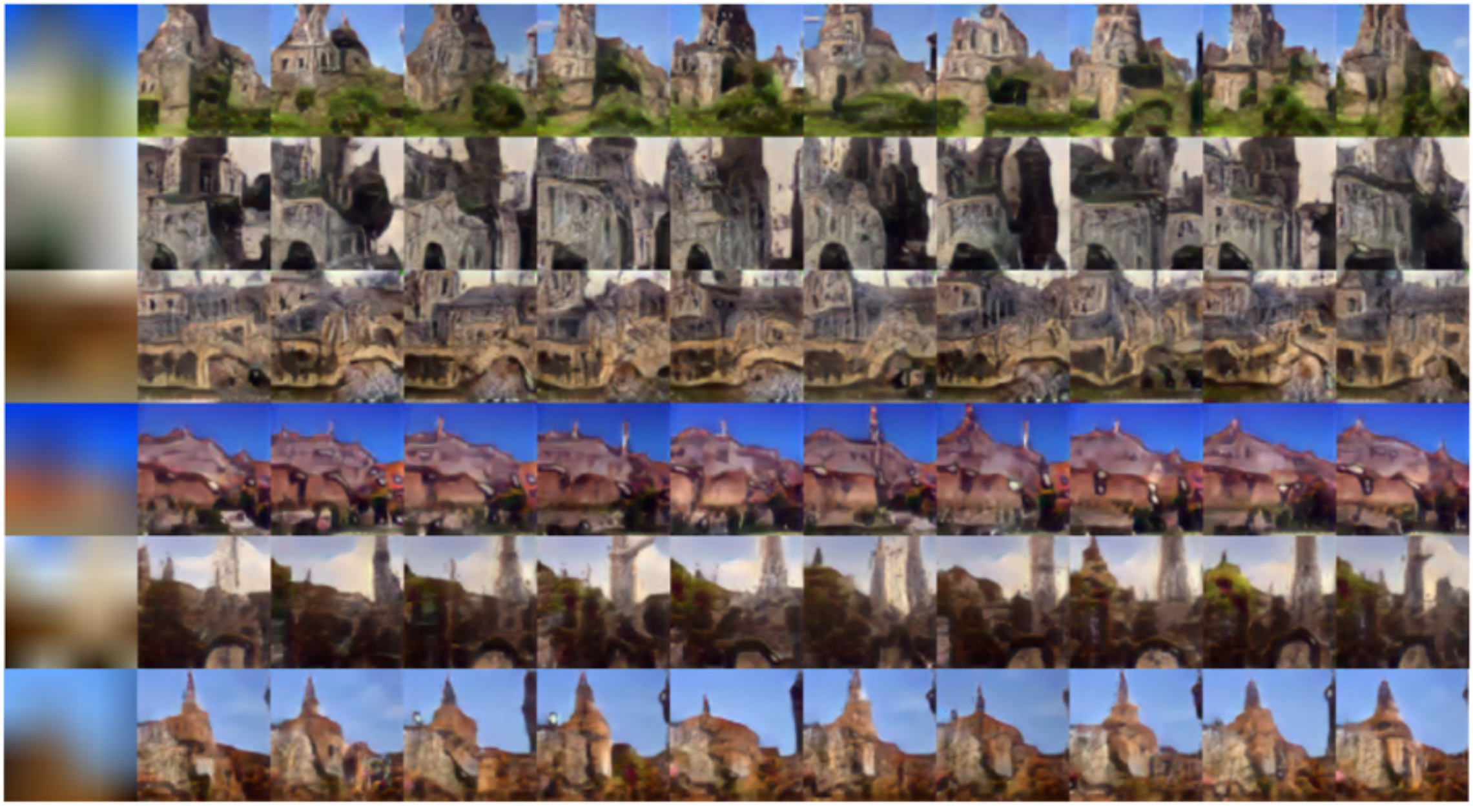
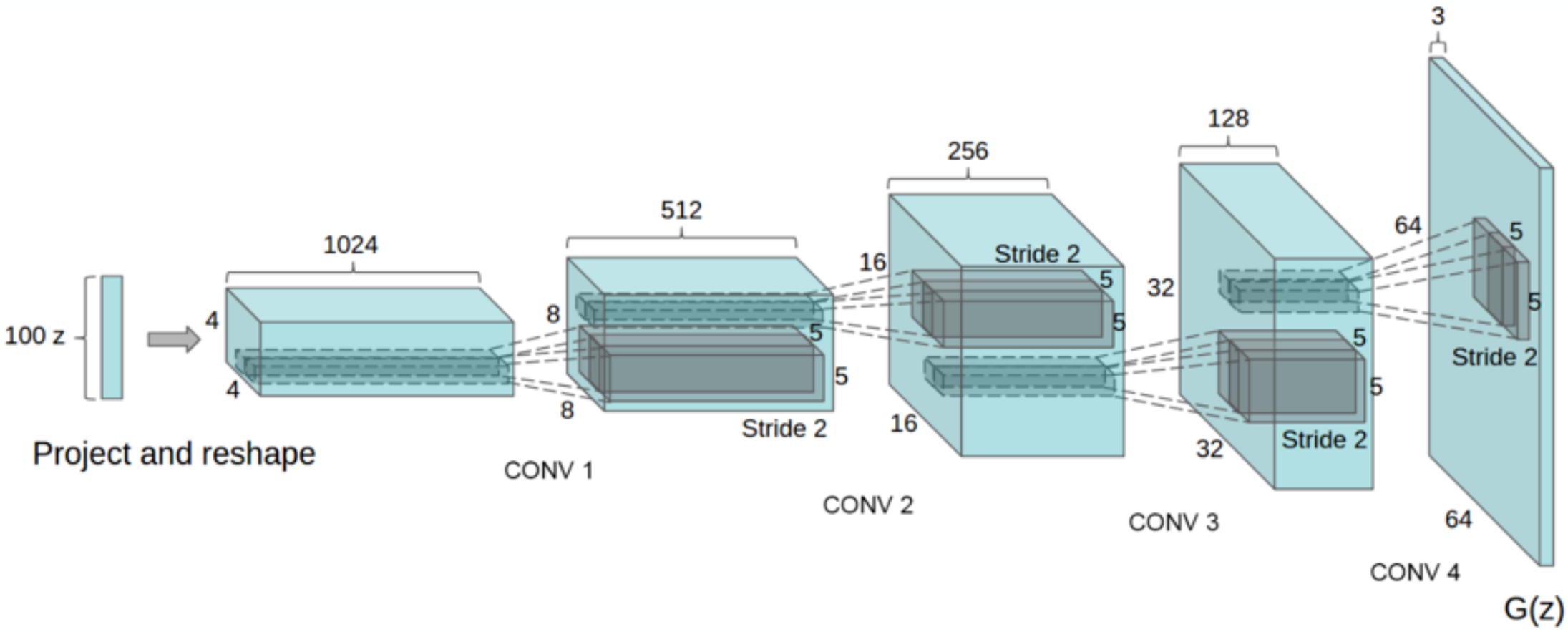


DC-GAN

[Radford et al.'16]

- Improved multi-scale architecture and Batch-Normalization:



DC-GAN

[Radford et al.'16]

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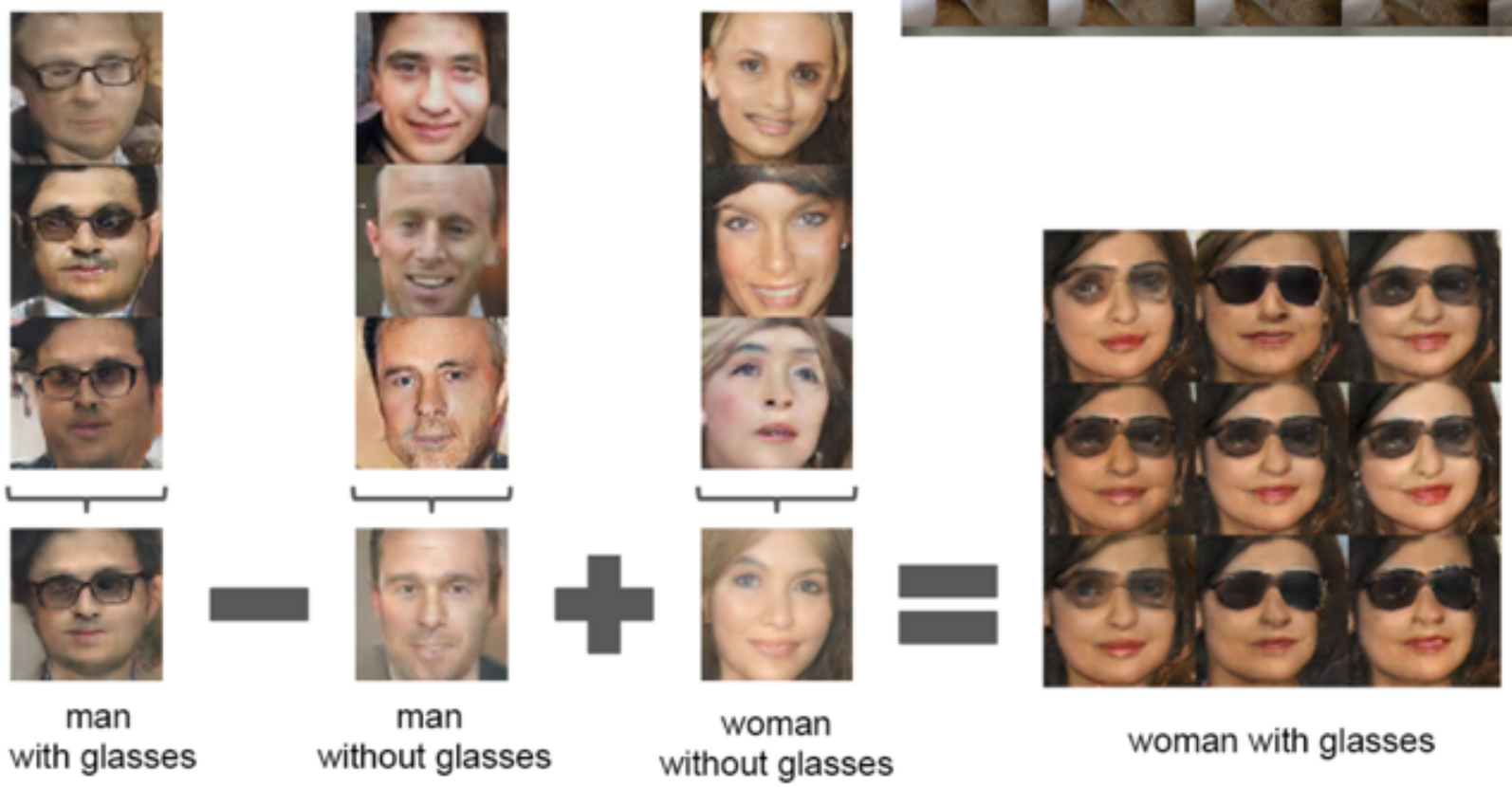


Generative Adversarial Networks

- GRAN [Generative Recurrent Adversarial Nets, Im et al.'16]

- Video Prediction [Mathieu et al.'16

- CNN Reconstruction [Brox et al.'16]

- A very *hot* topic within the Deep Learning community

Generative Adversarial Networks

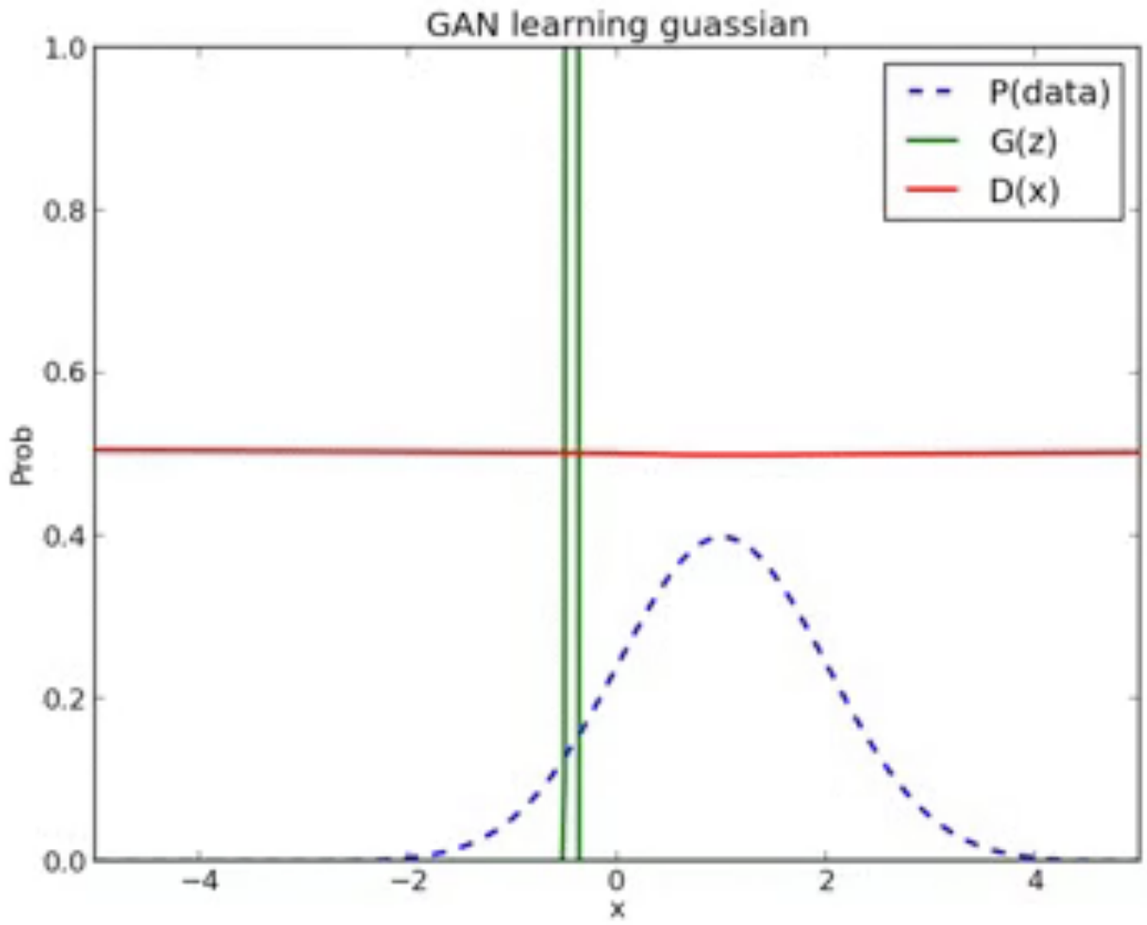
- Some open research directions:

1. **Optimization**:
   1. How to ensure a correct algorithm?
   2. Existence of a Lyapunov function?
2. **Statistics**:
   1. How to determine the discriminator power (eg VC-dimension) to obtain consistent estimators?
   2. Control of overfitting to the training distribution?
3. **Applications**:
   - Language Modeling
   - Reinforcement Learning
   - Algorithmic Tasks
   - Importance Sampling

GANs and their Properties

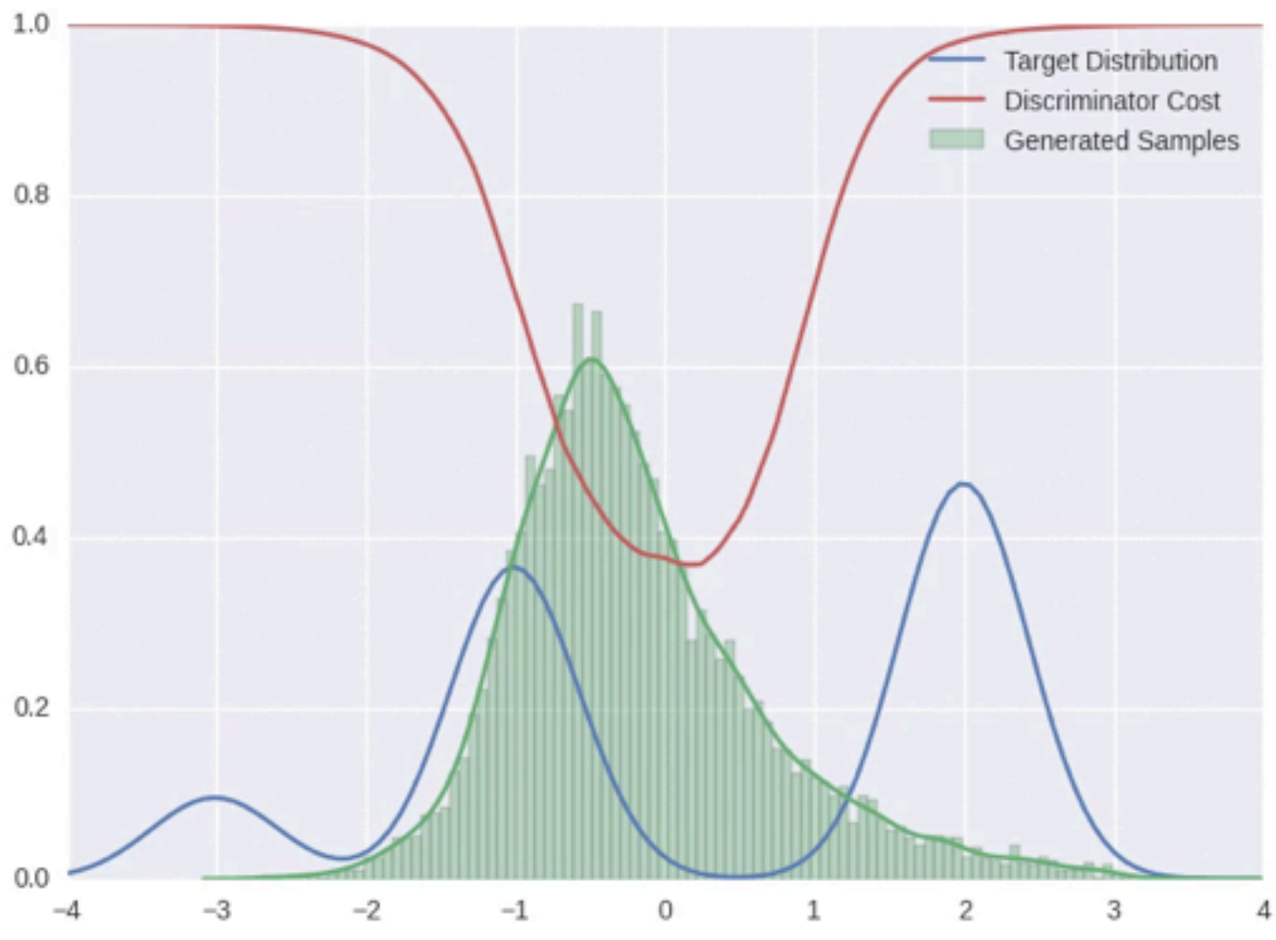
What do GANs Optimize?
Open problem: structured latent distributions

Some visualizations



https://twitter.com/AlecRad/status/619605092522676225

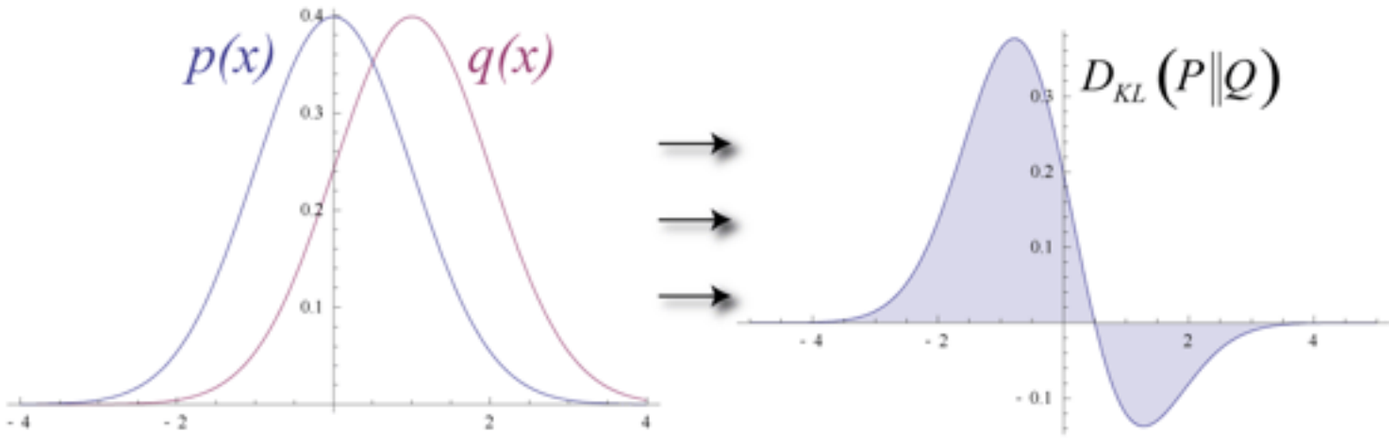
Some visualizations



https://twitter.com/Luke_Metz/status/711744051918282752

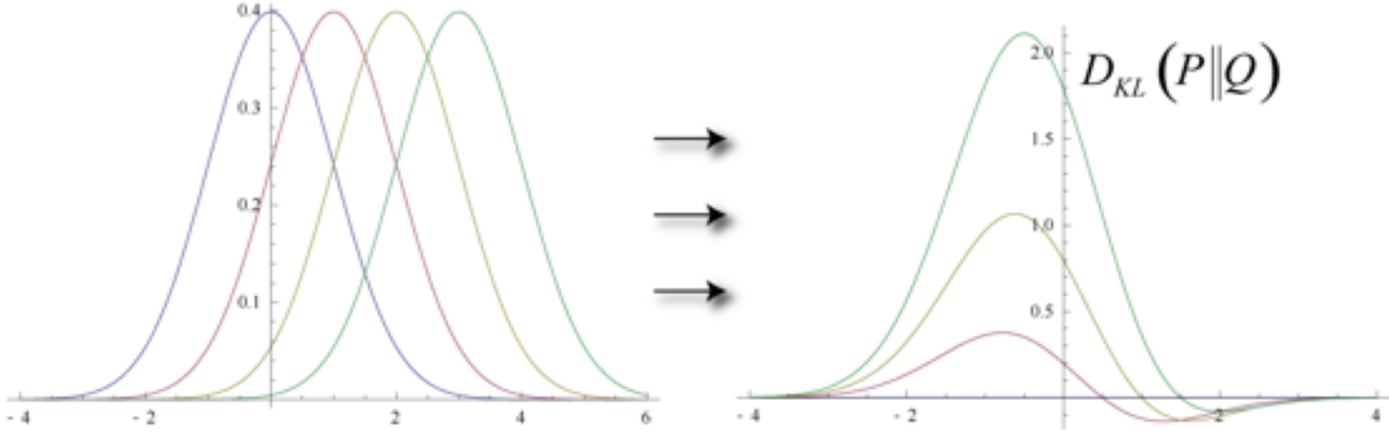
KL-Divergence

$$D_{\mathrm{KL}}(P\|Q) = \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} \mathrm{d}x,$$



Original Gaussian PDF's

KL Area to be Integrated



https://en.wikipedia.org/wiki/Kullback-Leibler_divergence

KL-Divergence: Issues

Whiteboard session

KL-Divergence: Issues

- Autoencoders optimizing KL(P/Q)
  - Blurry

JS-Divergence

$$\mathrm{JSD}(P \parallel Q) = \frac{1}{2} D(P \parallel M) + \frac{1}{2} D(Q \parallel M)$$

$$M = \frac{1}{2}(P + Q)$$

$D(P \parallel Q)$ = KL-Divergence = $D_{\mathrm{KL}}(P \| Q) = \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} \, \mathrm{d}x,$

JS-Divergence

Whiteboard session

JS-Divergence

Theorem 1. *The global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. At that point, $C(G)$ achieves the value $-\log 4$.*

Proof. For $p_g = p_{\text{data}}$, $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, (consider Eq. 2). Hence, by inspecting Eq. 4 at $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, we find $C(G) = \log \frac{1}{2} + \log \frac{1}{2} = -\log 4$. To see that this is the best possible value of $C(G)$, reached only for $p_g = p_{\text{data}}$, observe that

$$\mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}} [-\log 2] + \mathbb{E}_{\boldsymbol{x} \sim p_g} [-\log 2] = -\log 4$$

and that by subtracting this expression from $C(G) = V(D^*_G, G)$, we obtain:

$$C(G) = -\log(4) + KL\left(p_{\text{data}} \middle\| \frac{p_{\text{data}} + p_g}{2}\right) + KL\left(p_g \middle\| \frac{p_{\text{data}} + p_g}{2}\right) \tag{5}$$

where KL is the Kullback–Leibler divergence. We recognize in the previous expression the Jensen–Shannon divergence between the model's distribution and the data generating process:

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right) \tag{6}$$

Since the Jensen–Shannon divergence between two distributions is always non-negative and zero only when they are equal, we have shown that $C^* = -\log(4)$ is the global minimum of $C(G)$ and that the only solution is $p_g = p_{\text{data}}$, i.e., the generative model perfectly replicating the data generating process. □

"Generative Adversarial Nets" - Goodfellow et. al.

λ-Divergence

Symmetrised divergence [edit]

Kullback and Leibler themselves actually defined the divergence as:

$$D_{\mathrm{KL}}(P\|Q) + D_{\mathrm{KL}}(Q\|P)$$

which is symmetric and nonnegative. This quantity has sometimes been used for feature selection in classification problems, where *P* and *Q* are the conditional pdfs of a feature under two different classes.

An alternative is given via the λ divergence,

$$D_\lambda(P\|Q) = \lambda D_{\mathrm{KL}}(P\|\lambda P + (1-\lambda)Q) + (1-\lambda) D_{\mathrm{KL}}(Q\|\lambda P + (1-\lambda)Q),$$

which can be interpreted as the expected information gain about *X* from discovering which probability distribution *X* is drawn from, *P* or *Q*, if they currently have probabilities λ and (1 – λ) respectively.

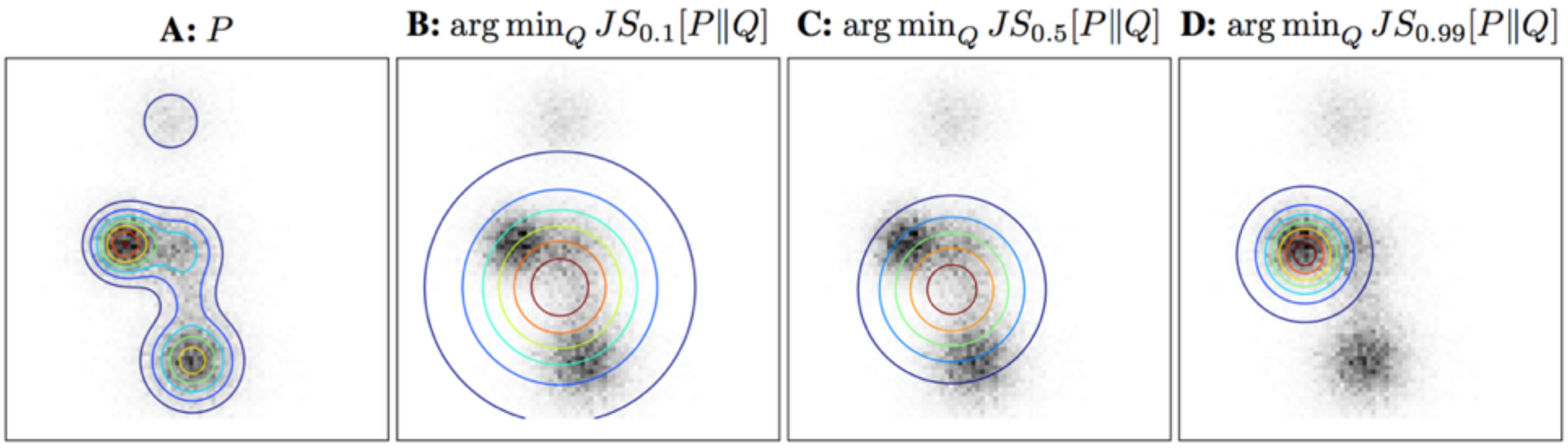
The value λ = 0.5 gives the Jensen–Shannon divergence, defined by

$$D_{\mathrm{JS}} = \tfrac{1}{2} D_{\mathrm{KL}}\left(P\|M\right) + \tfrac{1}{2} D_{\mathrm{KL}}\left(Q\|M\right)$$

where *M* is the average of the two distributions,

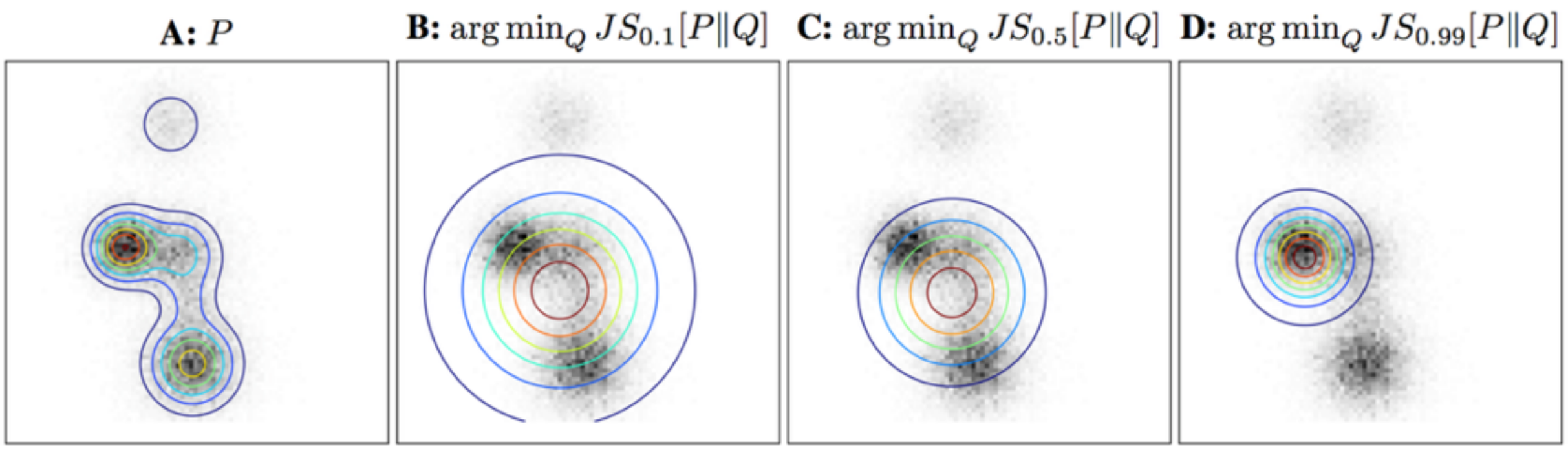
$$M = \tfrac{1}{2}(P+Q).$$

λ-Divergence



http://www.inference.vc/how-to-train-your-generative-models-why-generative-adversarial-networks-work-so-well-2/

Discussion / Questions



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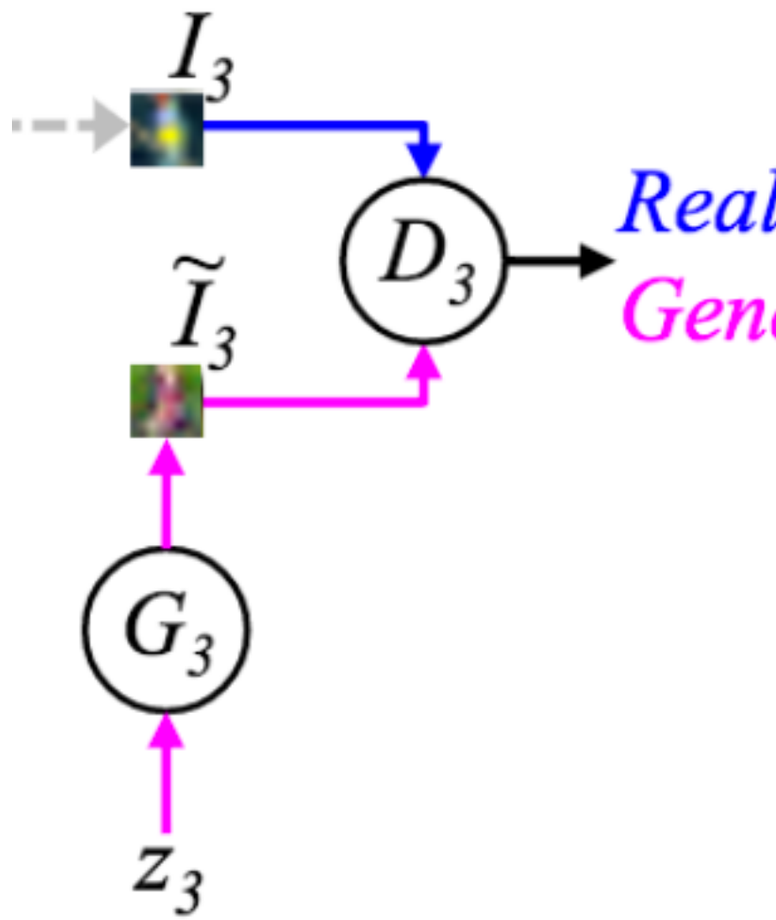
Structured Latent Distributions



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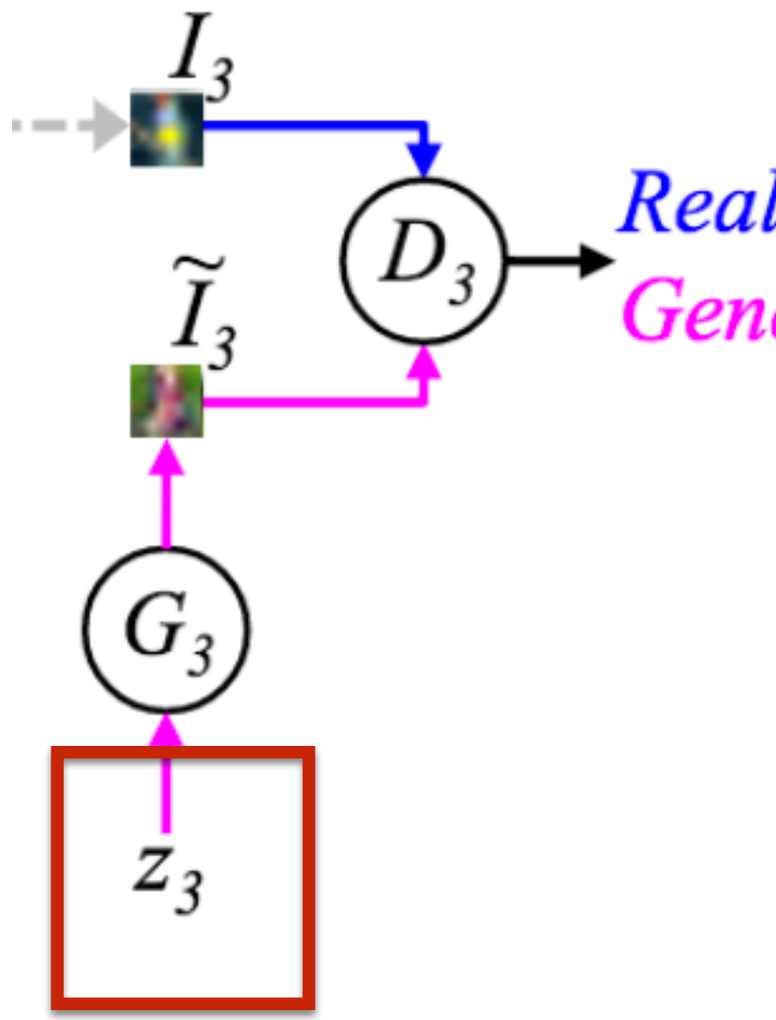

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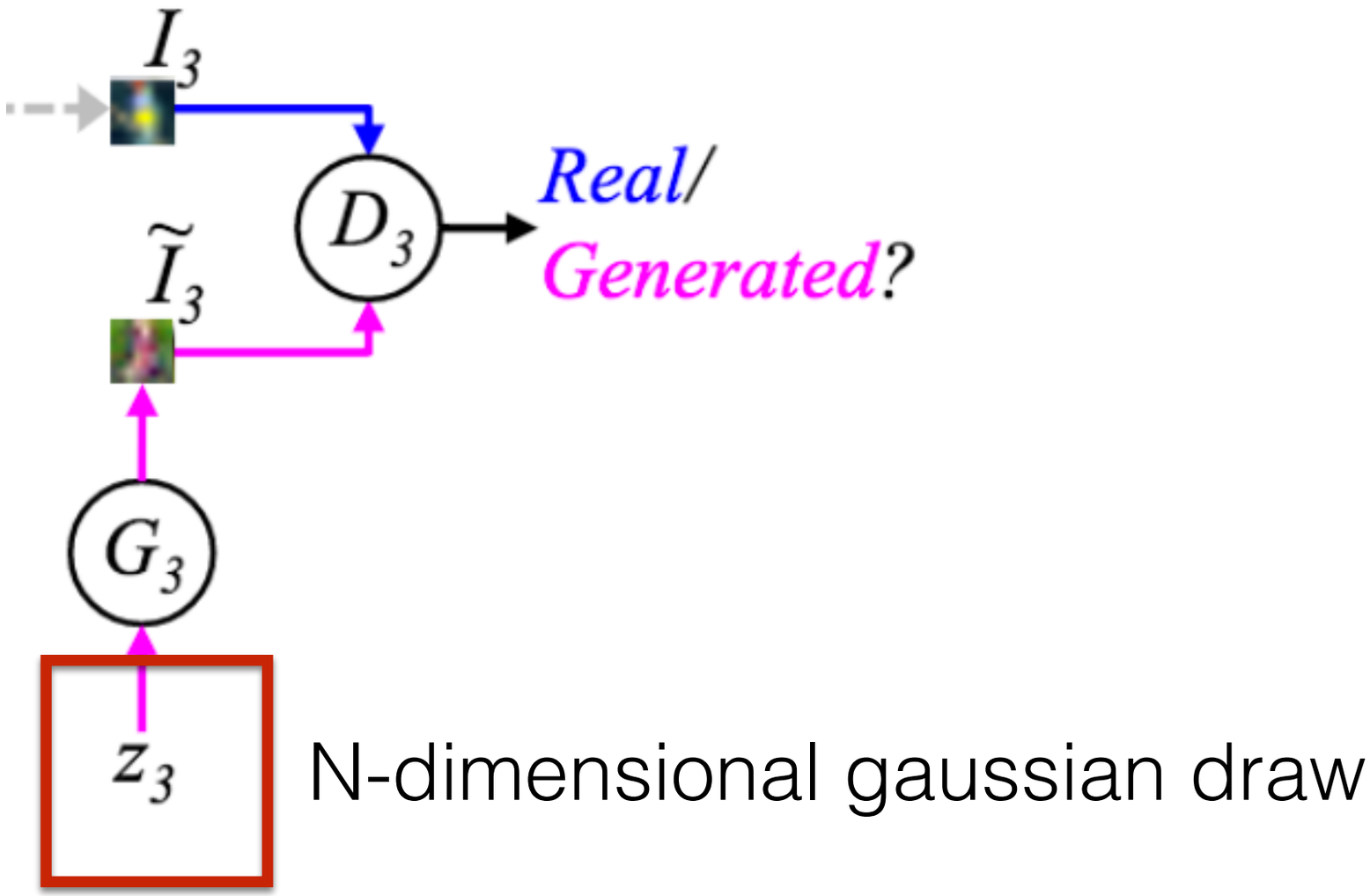

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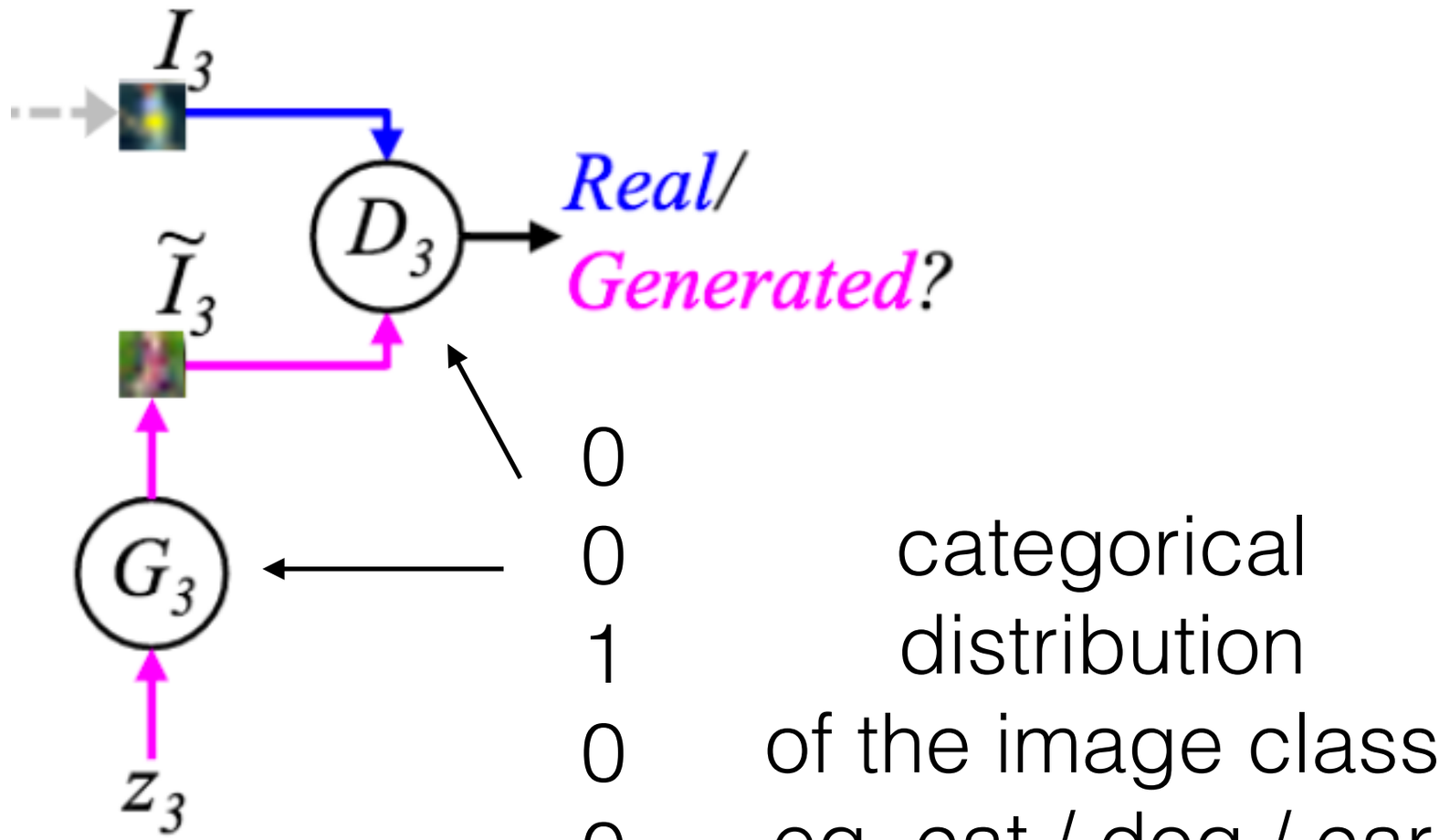
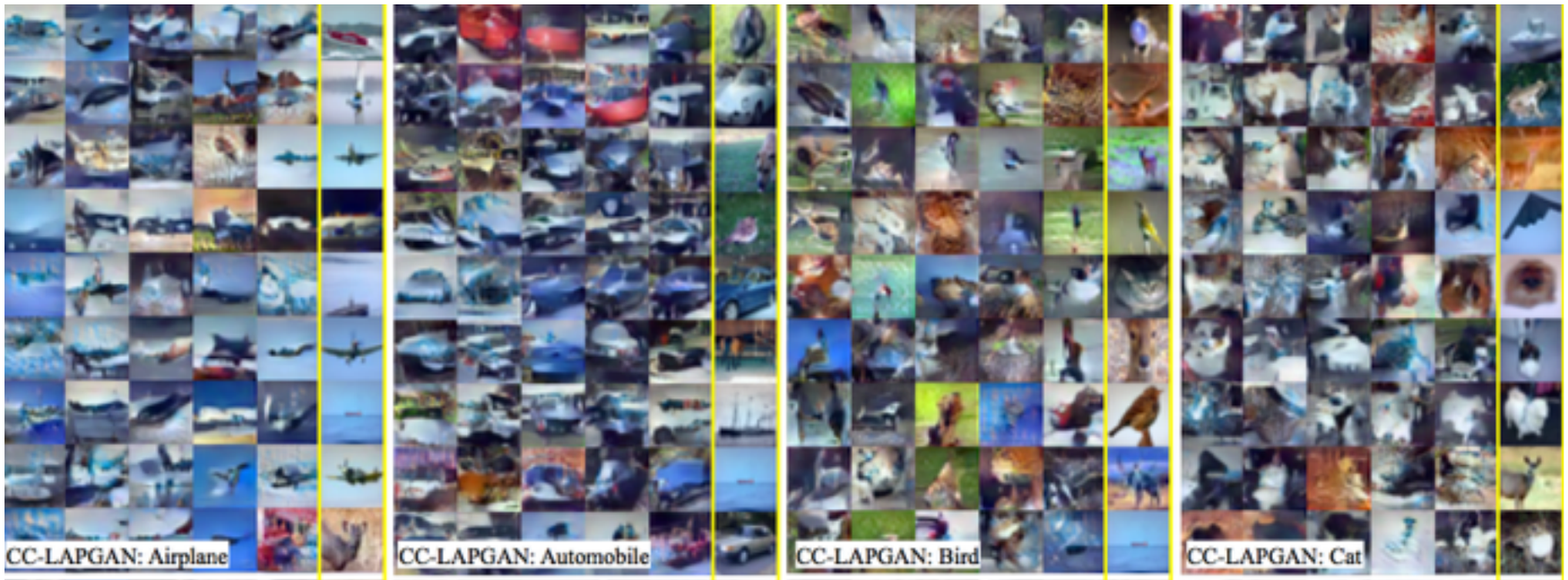
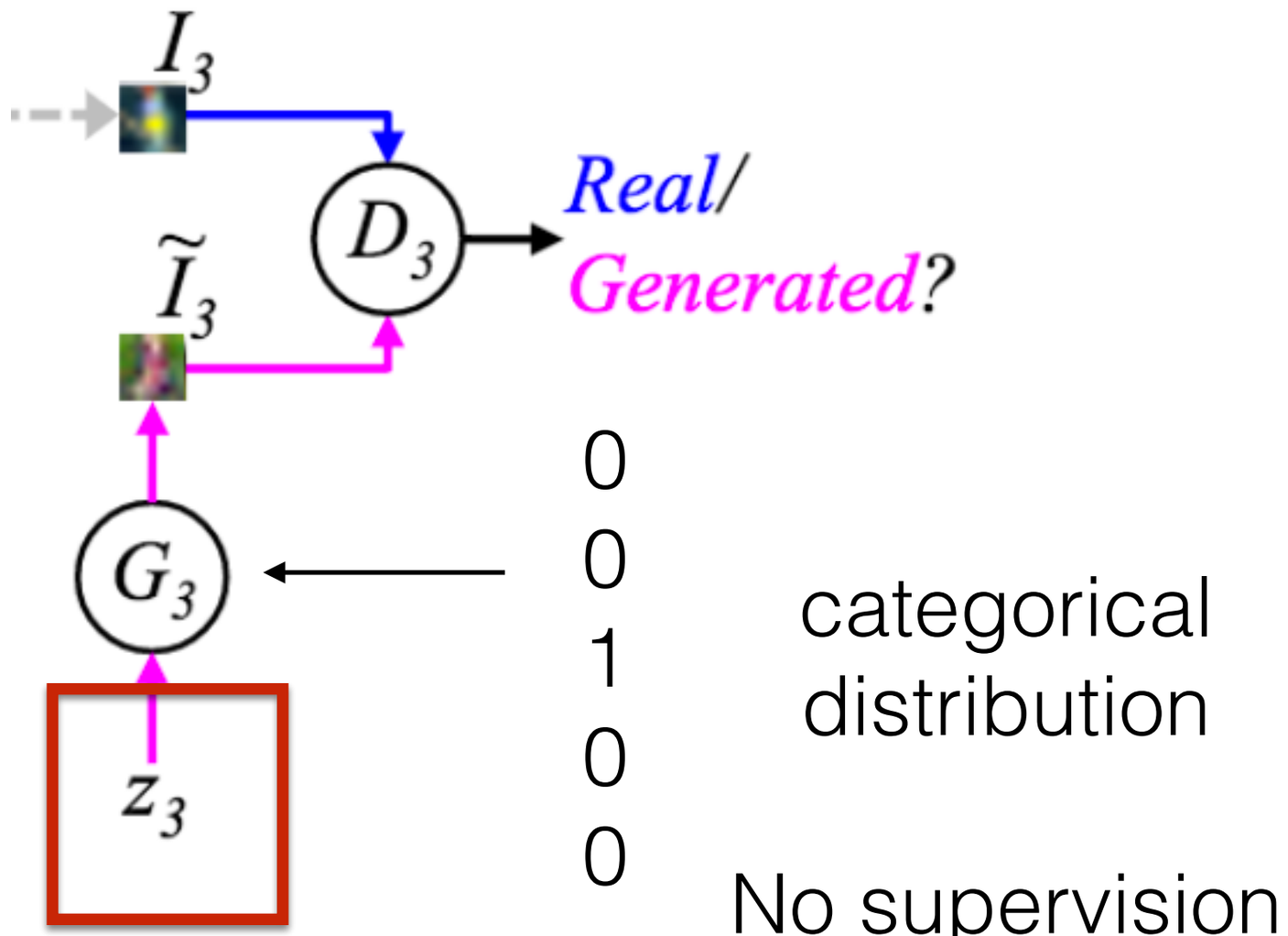


categorical
distribution
of the image class
eg. cat / dog / car


CC-LAPGAN: Airplane
CC-LAPGAN: Automobile
CC-LAPGAN: Bird
CC-LAPGAN: Cat


CC-LAPGAN: Deer
CC-LAPGAN: Dog
CC-LAPGAN: Frog
CC-LAPGAN: Horse

Structured Latent Distributions

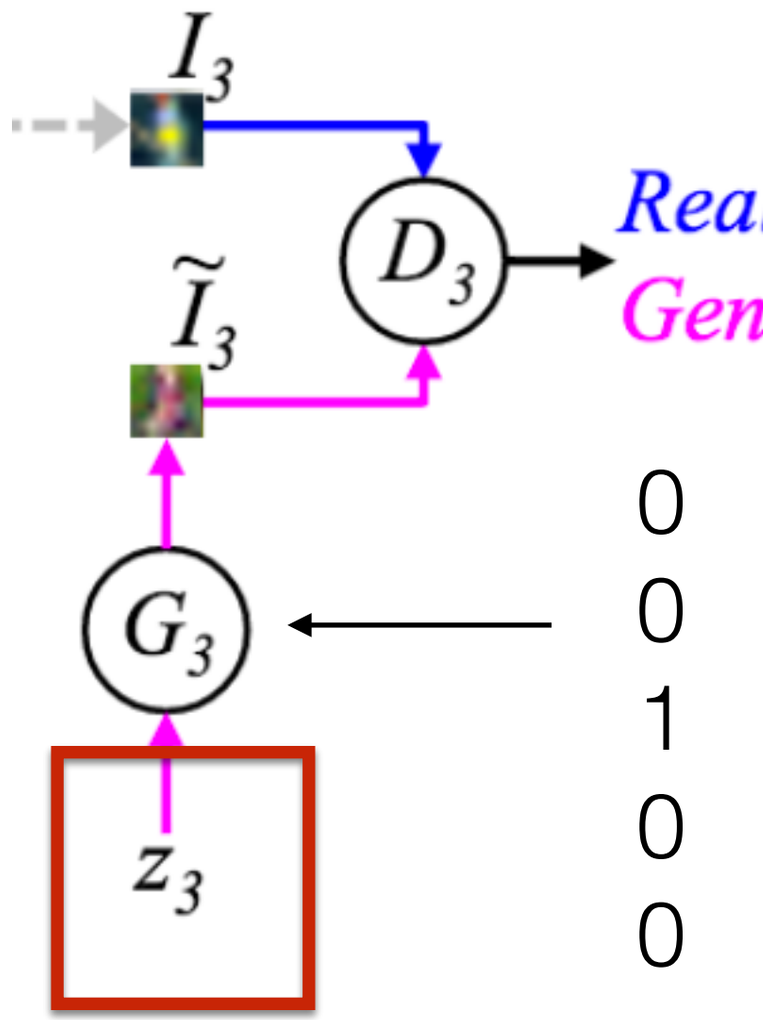


categorical distribution

No supervision

Will it work?

Discussion / Questions



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