ENGSCI 391 Computing Assignment Part 2

Problem Set-Up (Question 1)

The problem defines the following vectors:

$\phi = [45;\ 60;\ 50;\ 50;\ 55;\ 65;\ 40;\ 30;\ 30]$

$G = [1000;\ 430;\ 400;\ 1085;\ 387;\ 640;\ 1750;\ 800;\ 885]$

$K = [500;\ 600;\ 500;\ 800;\ 600;\ 1000;\ 300;\ 200;\ 800;\ 300;\ 850]$

$d = [1952;\ 722;\ 60;\ 284;\ 855;\ 0;\ 1078;\ 225;\ 617;\ 0]$

Appendix A Shows the complete code.

Relaxed Constraints on Line and Generator Capacities (Question 2)

The optimal solution consists of all 5793 MW of supply coming from the cheapest generator which is MAN. In this situation, power does not need to be generated elsewhere as every other generator is more expensive and there are no restrictions on how power is distributed across the entire network. The total cost of this set-up is $173 790 per hour.

Relaxed Line Capacity Constraints (Question 3)

When only the line capacity constraints are relaxed, the optimal solution is produced by fully utilising each generator from the cheapest to the most expensive one, until the 5793 MW supply is met. The total cost of generation is now $233 450 per hour.

Full Problem (Question 4)

The total cost of generation when accounting for line and generator capacities is $241 150 per hour. x tells us the utilisation of generators and the duals, π, tell us the energy price at each node.

Note: Negative flows between transmission lines indicate flows against the specified direction

Generator	Utilisation (MW)
HLY	1000
E3P	52
OTA	400
MRP	1085
SFD	336
TKU	0
WTK	1270
CLY	800
MAN	850

Arc	Utilisation (MW)
A-H	-500
H-NP	-276
H-NP	139
NP-W	0
N-W	-145
W-B	-1000
B-C	278
B-D	-8
C-D	-800
T-D	233
M-T	850

City	Extra Cost ($/MWh)
A	60
H	55
NP	55
N	55
W	55
B	40
C	40
D	40
T	40
M	30

Total Cost of Generation = $241 150

Kirchhoff’s Laws (Question 5)

After adding these loop flow constraints, the **generation of each generator and energy price at each node are the same as before**. The total cost of generation is still $241 150 per hour. The flows between transmission lines have now changed:

Arc	Utilisation (MW)
A-H	-500
H-NP	-208.5
H-NP	71.5
NP-W	67.5
N-W	-212.5
W-B	-1000
B-C	292.2
B-D	-22.2
C-D	-785.8
T-D	233
M-T	850

Total Cost of Generation = $241 150

Losses in B-W HVDC Line (Question 6)

The total cost of generation is now $246 895 per hour.

Generator	Utilisation (MW)
HLY	1000
E3P	101
OTA	400
MRP	1085
SFD	387
TKU	0
WTK	1270
CLY	800
MAN	850

Arc	Utilisation (MW)
A-H	-451
H-NP	-196.75
H-NP	108.75
NP-W	130.25
N-W	-175.25
W-B	-1000
B-C	292.2
B-D	-22.2
C-D	-785.8
T-D	233
M-T	850

City	Extra Cost ($/MWh)
A	60
H	60
NP	60
N	60
W	60
B	40
C	40
D	40
T	40
M	30

Total Cost of Generation = $246 895

Appendix A: MATLAB Code

Problem Definition

```
%This script solves questions 4, 5 and 6 of the assignment. It calls
%'fullrsm' function as well as the 'build' function which is defined at the
%bottom of the file. All of the required information to answer the
%questions are stored the variable workspace.

clear; clc;

% The problem defines the following vectors
price = [45; 60; 50; 50; 55; 65; 40; 30; 30];
G = [1000; 430; 400; 1085; 387; 640; 1750; 800; 885];
K = [500; 600; 500; 800; 600; 1000; 300; 200; 800; 300; 850];
d = [1952; 722; 60; 284; 855; 0; 1078; 225; 617; 0];
```

Question 4

```
% Build the LP
[A, b, c] = build(price, G, K, d);
% Solve
[m,n] = size(A);
[result,z,x,pi]  =  fullrsm(m,n,c,A,b);

% Required information
xQ4 = x(23:31);
flowsQ4 = x(1:11) - x(12:22);
energyPriceQ4 = pi(1:10);
totalCostQ4 = z;
```

Question 5

```
% Add Kirchhoff's Law constraints to model
newRows = zeros(2,n);
newRows(1, [2 4 14 16]) = -1;
newRows(1, [3 5 13 15]) = 1;
newRows(2, [7 19]) = 1;
newRows(2, [18 8]) = -1;
newRows(2, 9) = 0.4;
newRows(2, 20) = -0.4;

A = [A; newRows];
b = [b; 0; 0];

% Solve
[m,n] = size(A);
[result,z,x,pi]  =  fullrsm(m,n,c,A,b);

% Required information
xQ5 = x(23:31);
flowsQ5 = x(1:11) - x(12:22);
```

```
energyPriceQ5 = pi(1:10);
totalCostQ5 = z;
```

Question 6

```
% Add B-W line loss constraints to model
[m,~] = size(A);
A(:,[6,17]) = 0;
newArcs = zeros(m,8);

[newArcs(5,1),newArcs(6,2),newArcs(5,3),newArcs(6,4)] = deal(-1);
[newArcs(6,1),newArcs(5,2)] = deal(0.95);
[newArcs(6,3),newArcs(5,4)] = deal(0.85);

bottom = [zeros(4,62) eye(4) eye(4)];

A = [A newArcs; bottom];
b = [b; 500; 500; 500; 500];
c = [c; zeros(8,1)];

% Solve
[m,n] = size(A);
[result,z,x,pi]  =  fullrsm(m,n,c,A,b);

%Required information
xQ6 = x(23:31);
flowsQ6 = x(1:11) - x(12:22);
flowsQ6(6) = -x(64)-x(66);
energyPriceQ6 = pi(1:10);
totalCostQ6 = z;
```

Functions

```
function [A, b, c] = build(price, G, K, d)
% This function builds the constraint matrix for the simplified electricity
% network.
% Inputs:
%   price = hourly cost of generating power at each generator
%       G = capacities of generators
%       K = capacities of network arcs
%       d = demand at network nodes (cities)
% Outputs:
%       A = constraint matrix
%       b = right-hand side
%       c = cost vector

    % Connections between cities
    arcs = [ 1 2; % A - H
             2 3; % H - NP
             2 4; % H - N
             3 5; % NP - W
             4 5; % N - W
             5 6; % W - B
             6 7; % B - C
             6 8; % B - D
```

```
              7 8; % C - D
              9 8; % T - D
             10 9  % M - T
             ];

    % Generator connections to cities
    g = [1; 1; 1; 2; 3; 4; 6; 8; 10];

    nArcs = size(arcs, 1);
    nNodes = length(d);
    nGenerators = length(G);

    A = zeros(nNodes, nArcs);
    b = d;

    % Make node-arc incidence matrix
    for i = 1:nArcs
        A(arcs(i,1),i) = -1;
        A(arcs(i,2),i) = 1;
    end

    % Add generator arcs
    genArcs = zeros(nNodes, nGenerators);
    for i = 1:nGenerators
        genArcs(g(i), i) = 1;
    end

    %Define LP
    A = [A -A genArcs];
    [m,n] = size(A);
    A = [A zeros(m,n); eye(n) eye(n)];
    b = [b; K; K; G];
    c = [zeros(22,1); price; zeros(n,1)];

end
```