# Advanced Binary Search 2
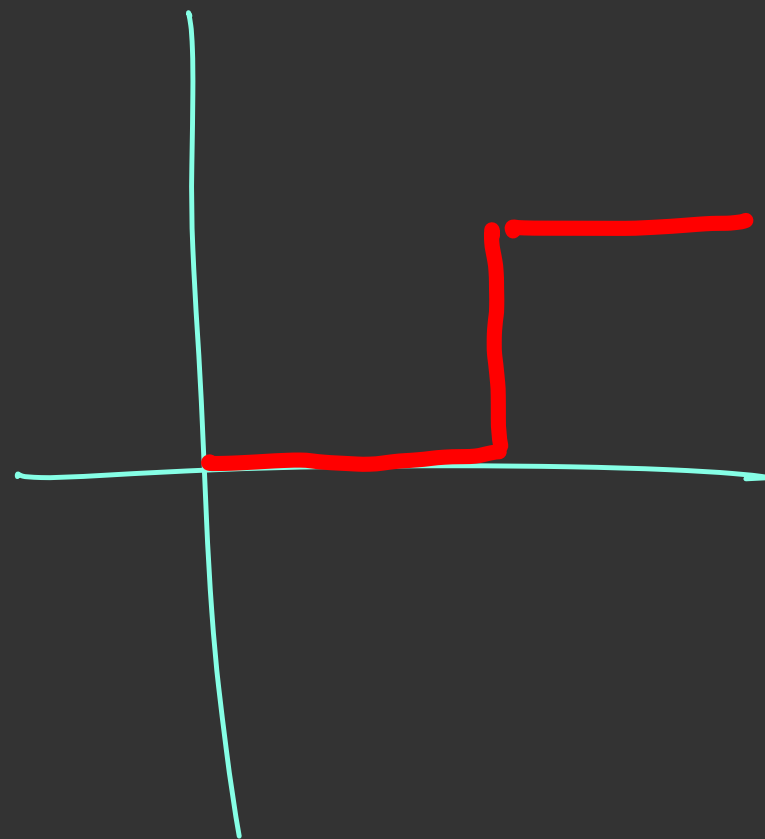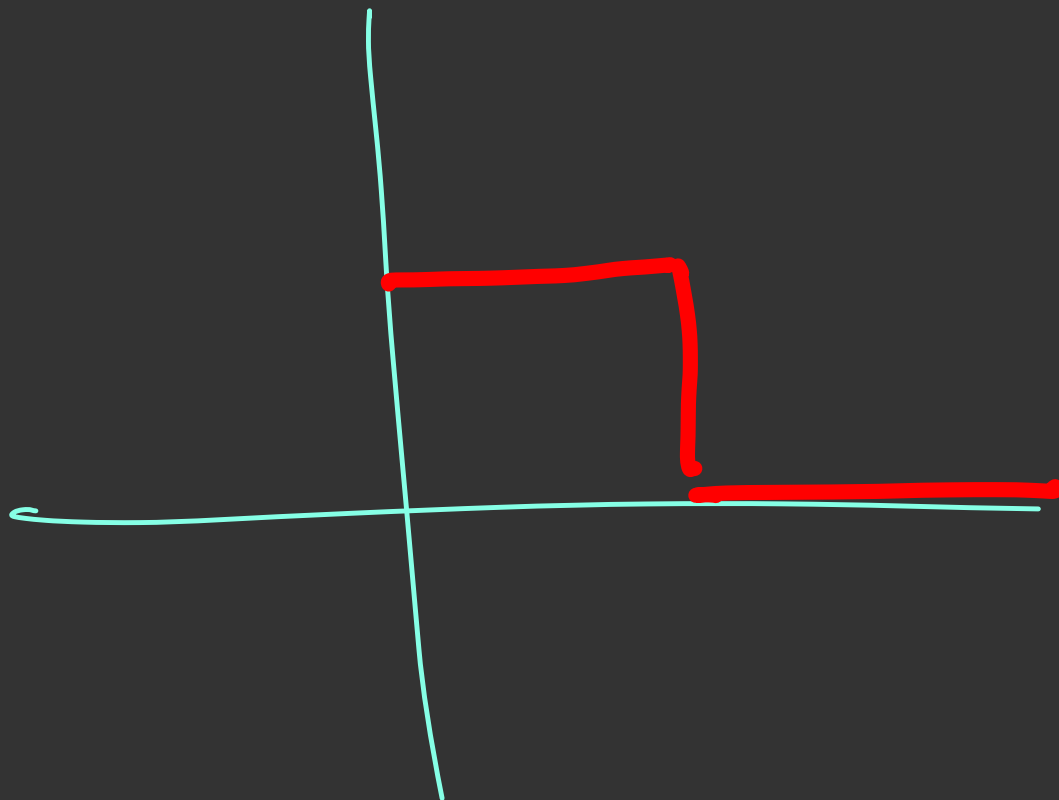
- Priyansh Agarwal
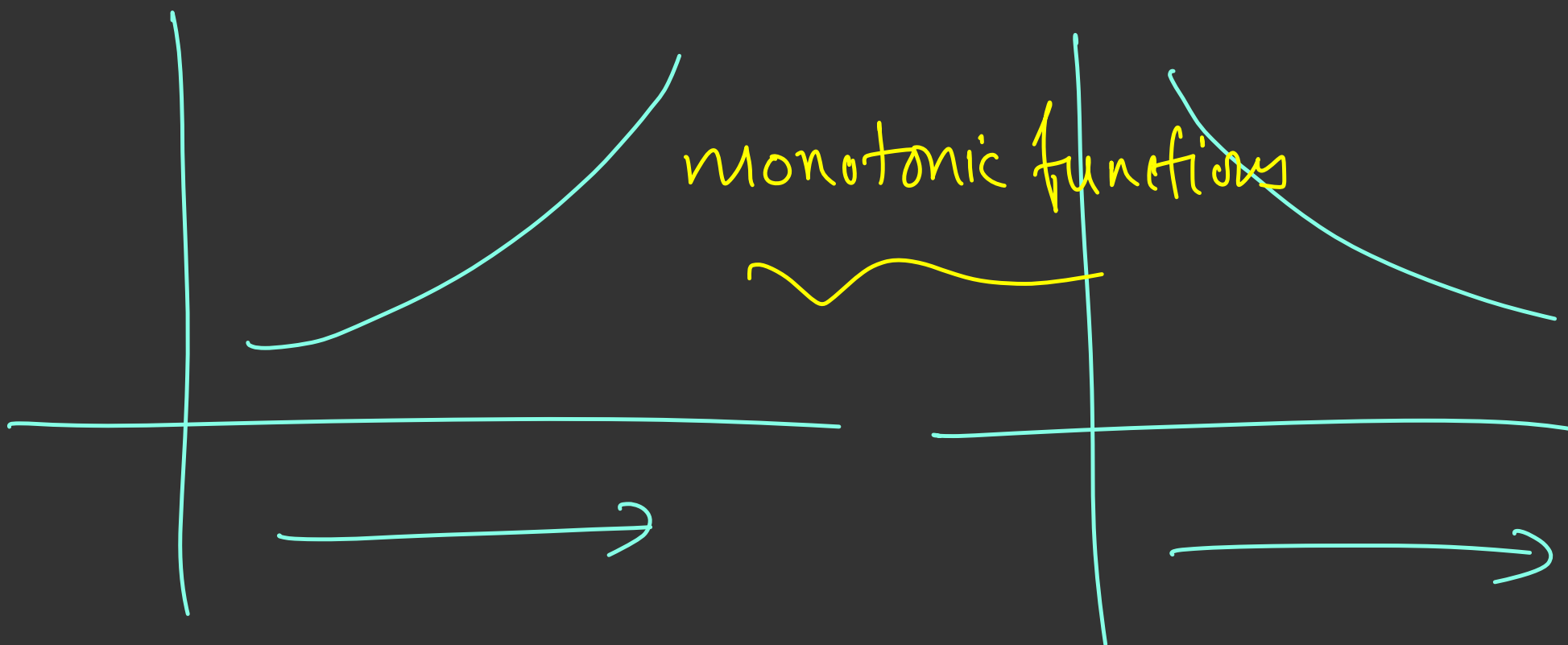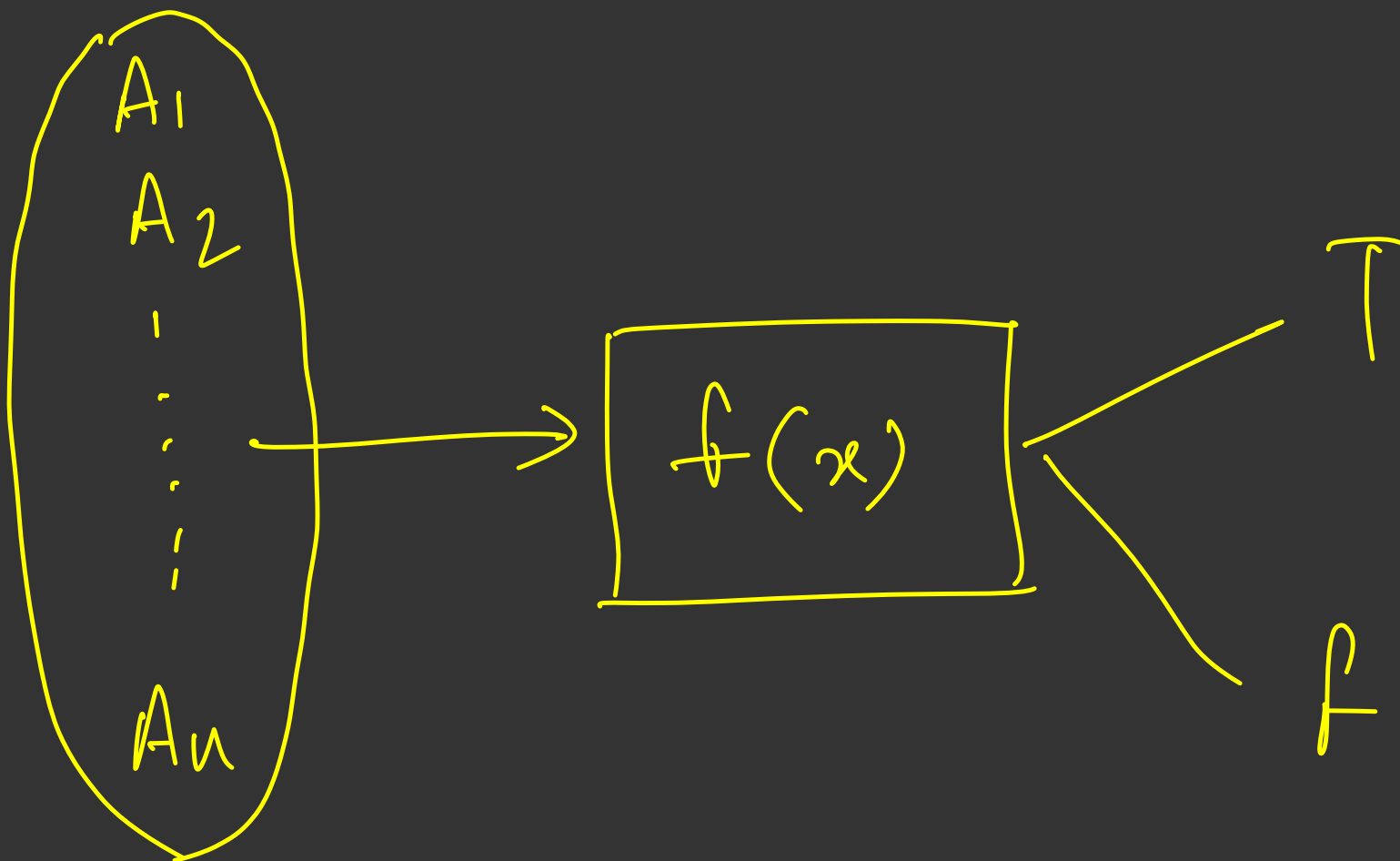
monotonic functions

$f(x) \longleftarrow$ T $\qquad$ f

(Indicate function)

monotonic predicate functions

$$A_1$$
$$A_2$$
$$\vdots$$
$$A_n$$

$$f(x)$$

$$T$$

$$f$$

# Sqrt(X)

Given a number X ($1 <= X <= 10^{18}$), find the biggest number Y such that $Y^2 <= X$

Eg:

X = 10000, Y = 100

X = 1024, Y = 32

X = 26, Y = 5

Sqrt(x)

$(X)$ $[\; 1 \leq X \leq 10^{18} \;]$

$Y$ such $\underline{Y^2 \leq X}$

$X = \underline{100}$

$Y = 1 , \; Y = 2 , \; Y = 3 , \; Y = 4$

$$y = 1$$

$$2$$

$$\boxed{3}$$

$$\vdots$$

$$9$$

$$10$$

$$11$$

$$\boxed{12}$$

$$\vdots$$

$$100$$

$$x = 100$$

$$\rightarrow f(3) \qquad \boxed{3^2 \leq 100} \ \checkmark$$

$$\rightarrow f(y) \Big< \begin{array}{l} T \ \text{if} \ y^2 \leq x \\[2ex] f \quad o/w \end{array}$$

$$f(12) \longrightarrow 12^2 \leq 100$$
$$\underline{\hspace{3cm}}$$
$$x$$

$$\underline{\underline{f(y)}} \longleftarrow y$$



TTTTTT(T)   monotonic predicate
            function

FFFFF

Biggest $y$ such that

$$y^2 \leq x$$
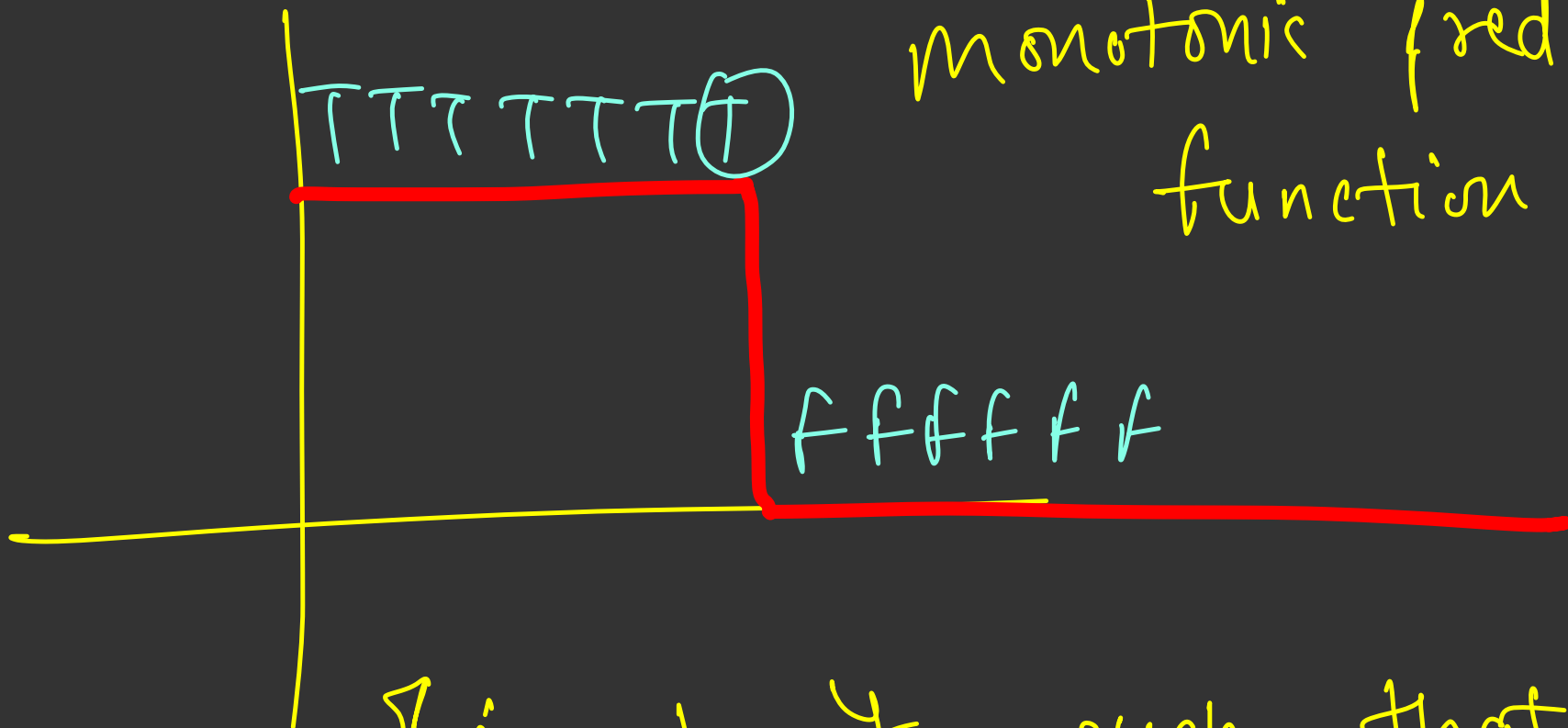
```
long long x;          (in >> x      ans = 0

left = 1  ;    right = 10^9 ;

while ( left ≤ right ) {
    mid = (left + right)/2
    if ( mid x mid ≤ x ) {
        ans = mid
        left = mid +1
    }
    else {
```

mid ≤ x
  ‾mid

right = mid -1

cout << cons;

if $x = 10^{18}$

mid $= \dfrac{10^{18}}{2}$

$= 5 \times 10^{17}$

mid x mid

① $( \text{mid} \times \text{mid} ) \leq x$
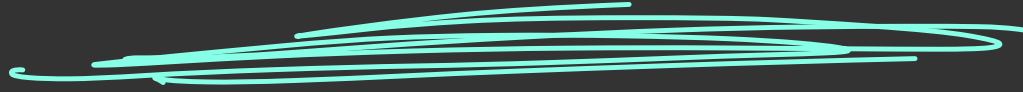
$\text{mid} \longrightarrow 10^{17}$

② $\boxed{\text{mid} \leq \dfrac{x}{\text{mid}}}$

$x \longrightarrow 10^{18}$

$(1 \text{ to } 10^{18})$

$10^9$

# Packing Rectangles

$w = 1$ , $h = 2$

$1$

$n = 5$

$0$

$x$

$1$

$2$

$2$

$3$

$3$

$8$

$4$
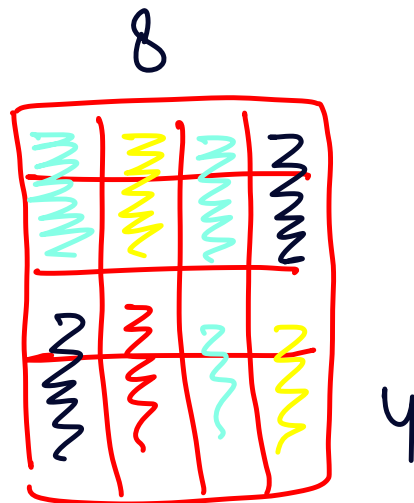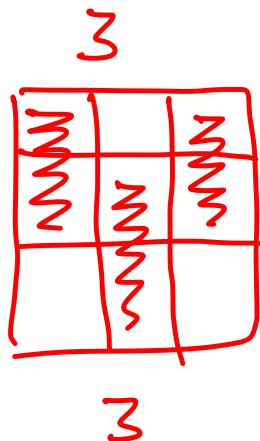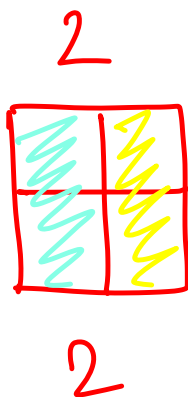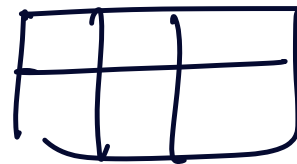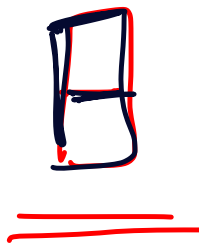
If I can pack 8 rectangles into a square of side 4

how many rect can I pack in a sq. of side 5

① $\geq 8$

② $< 8$

$$\frac{w, h}{n}$$

I

smallest $x$
such that

$f(x) \geq n$
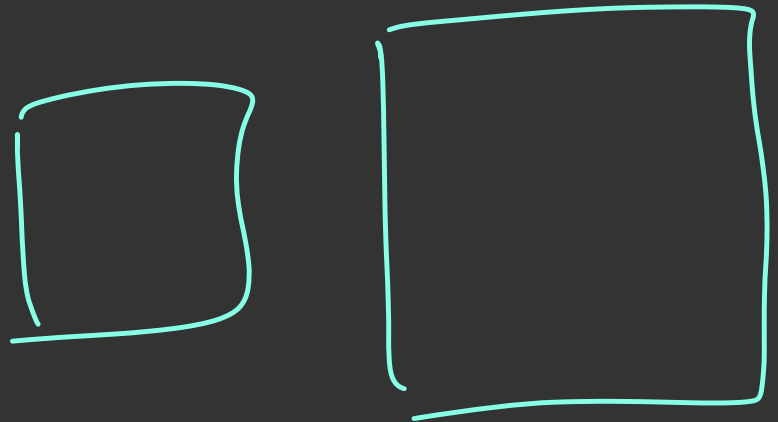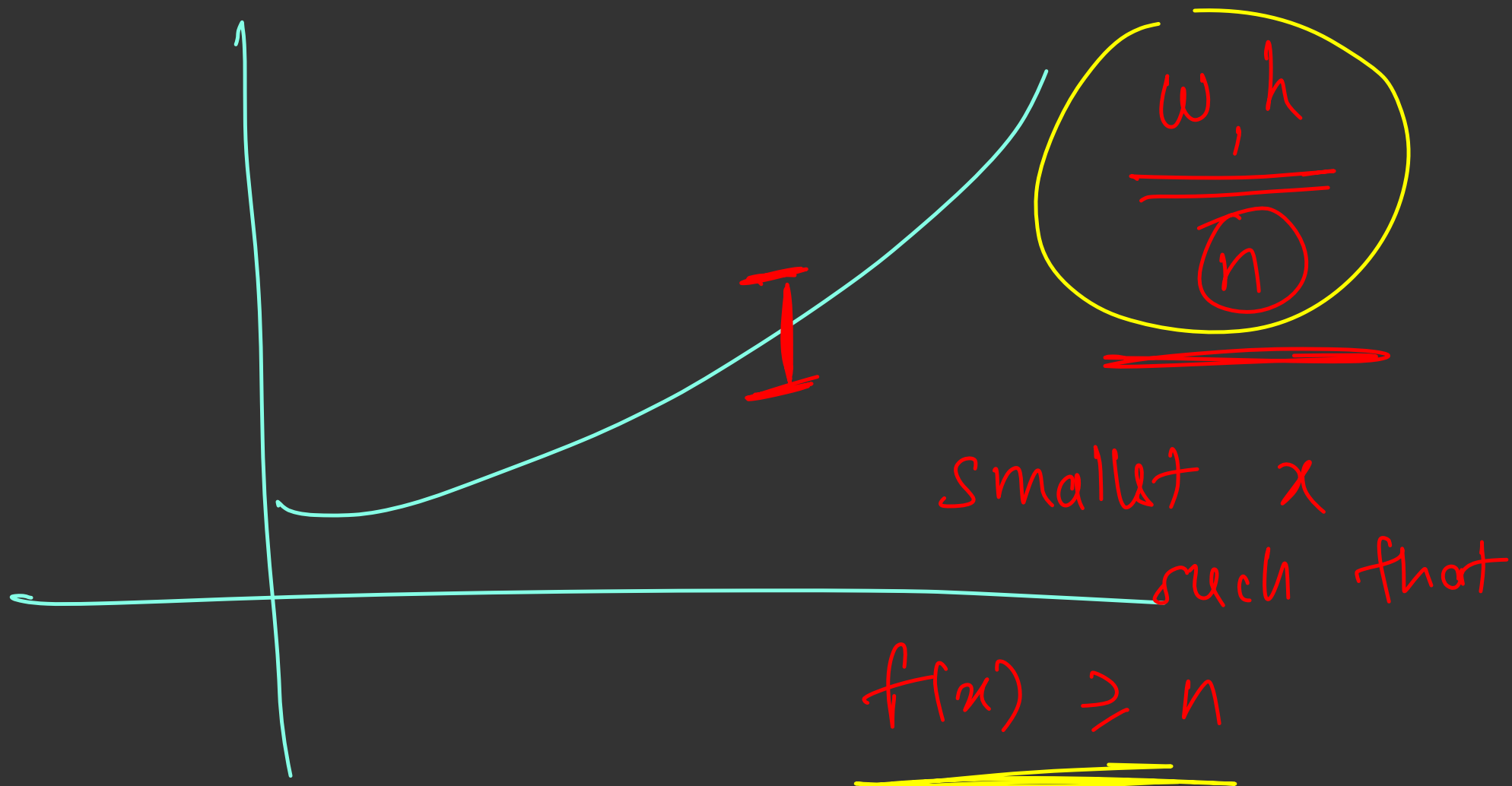
$f(x)$ ⟶ no. of rectangles I can
each in a sq. of side $x$

if $\qquad f(x) \geq n \longrightarrow T$

$$\longrightarrow < n \longrightarrow f$$

$$T\ T\ T\ T\ f\ f\ f\ f \qquad ①$$

$$f\ f\ f\ f\ T\ T\ T\ T\ T \qquad ②$$

$$f(q) \longrightarrow T$$
$$f(10) \longrightarrow T$$

we can binary search on the
length of the square

how to check what is $f(x)$

$f(x) =$ no. of rectangles we
can pack in a square of side $x$

w     w     w

h

h

h

h

w

$x$

$h, w$

$x$

$x = 5$

$w = 2$

$\lfloor 5 \rfloor_2$

$x/w$

$\lfloor x/w \rfloor \times \lfloor x/h \rfloor$

$$\underline{\underline{f(x)}} \longrightarrow \left\lfloor \frac{x}{w} \right\rfloor \times \left\lfloor \frac{x}{h} \right\rfloor \quad \text{①}$$

$$\left\lfloor \frac{x^2}{wh} \right\rfloor \quad \text{②}$$

$x = 5$

$w = 3$
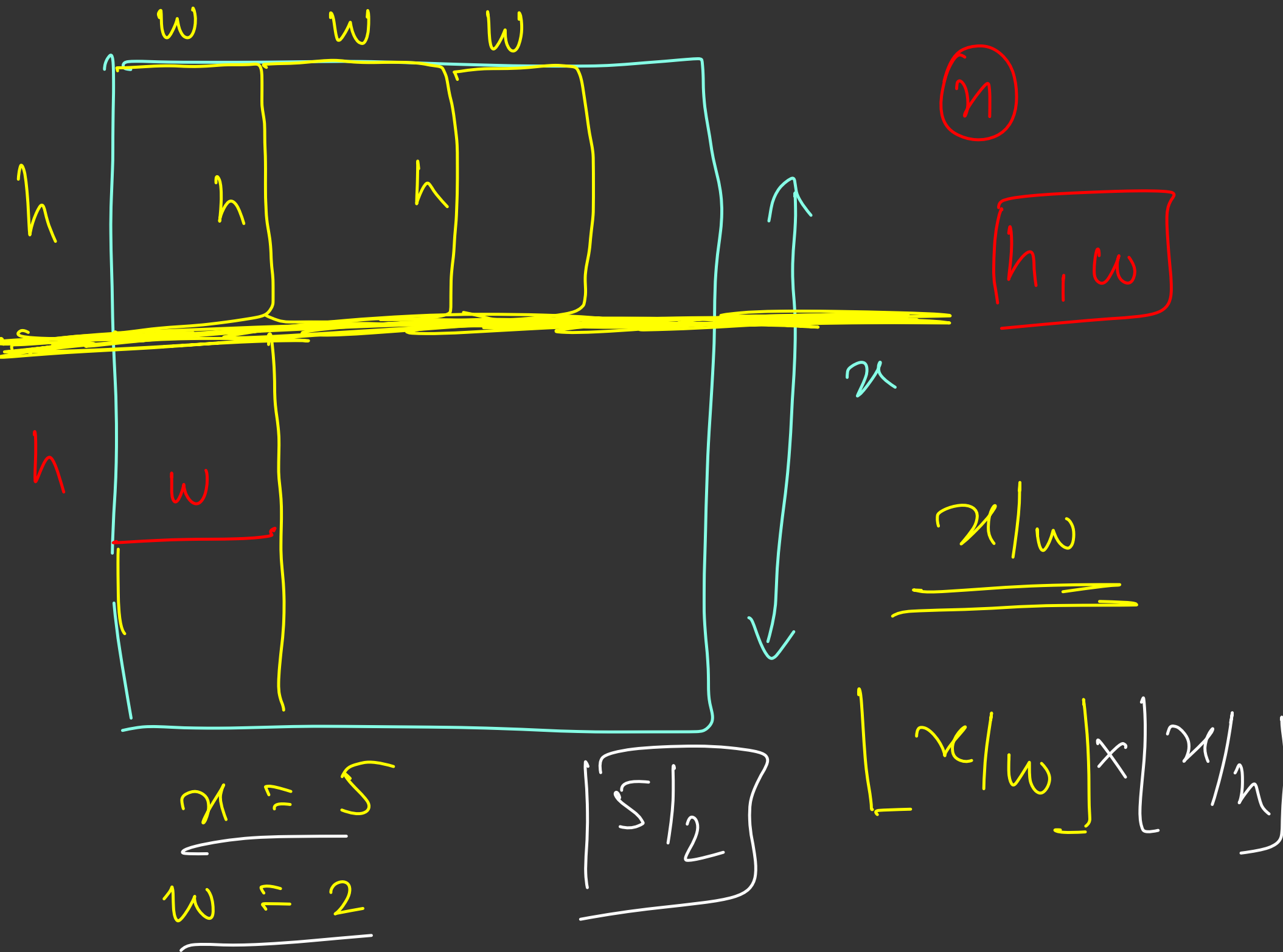
$h = 2$

$$\left\lfloor \frac{5}{3} \right\rfloor \times \left\lfloor \frac{5}{2} \right\rfloor = 1 \times 2 = 2$$

$$f(x) = \left\lfloor \frac{x}{w} \right\rfloor \times \left\lfloor \frac{x}{h} \right\rfloor$$

$$= \underline{a \times b}$$

$$a = \left\lfloor \frac{x}{w} \right\rfloor \qquad b = \left\lfloor \frac{x}{h} \right\rfloor$$

$$\underline{a \times b \geq n}$$

$\longleftrightarrow$

$(1)$

$$\frac{10^9 \text{ rectangle of } \begin{array}{c} 10^9 \text{ w} \\ 10^9 \text{ h} \end{array}}{\text{length of square} = 10^{18}}$$

$(1) \longleftrightarrow (10^{18})$
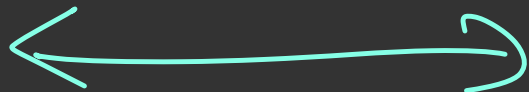
$f(x) = \dfrac{x}{w} \times \dfrac{x}{h}$

$= \dfrac{10^{18}}{10^9} \times 10^{18} / \dfrac{9}{10}$

$$= 10^9 \times 10^9$$

$$= 10^{18}$$

$$f(n) = \left\lfloor \frac{n}{w} \right\rfloor \times \left\lfloor \frac{n}{n} \right\rfloor$$

$$\underline{10^9}$$

$$= \left\lfloor \frac{10^{14}}{10^9} \right\rfloor \times \left\lfloor \frac{10^{14}}{10^9} \right\rfloor = 10^5 \times 10^5$$

$$= 10^{10}$$

f f f f f f T T T T T T T T

$$\log \frac{60}{(\text{search space})}$$

$$\frac{10^{18}}{} \qquad \uparrow 10^{14} \qquad \frac{50}{} = \boxed{10^8}$$

1 to $10^8$

$$\boxed{1 \qquad \text{to} \qquad 10^6}$$

1 to $10^{12}$

$$x = 10^9$$

$$w = 10^9, \quad h = 10^9$$

$$n = 10^9$$

$$f(x) = \left\lfloor \frac{10^9}{10^9} \right\rfloor \times \left\lfloor \frac{10^9}{10^9} \right\rfloor = \boxed{1}$$

$$1 \to 10^{18}$$

$$f(x) = \left\lfloor \frac{x}{w} \right\rfloor \times \left\lfloor \frac{x}{h} \right\rfloor \longleftarrow$$

$$1 \to 10^9 \qquad 1 \to 10^9$$

$$x = 10^{18}, \quad w = 1, \quad h = 1$$

$$f(x) = \frac{10^{18}}{1} \times \frac{10^{18}}{1}$$

$$= 10^{36}$$

is $\boxed{f(x)} \geq n$ or not

$$a = \frac{x}{w} \qquad b = \frac{x}{h}$$

$$\boxed{a \times b} \geq u \quad or \quad not$$

$$\left( a \geq \left\lceil \frac{n}{b} \right\rceil \right.$$

$$a = 2, \quad b = 3, \quad n = 5$$

$$\underline{a \times b \geq n} \qquad \text{or not}$$

$$a \geq \frac{n}{b} \qquad\qquad a \geq \frac{n}{b}$$

$$a \geq \left(\frac{n}{b}\right) \qquad\qquad 2 \geq \frac{5}{3}$$

$$2 \geq 1.67$$

find an integer $\geq 1.6$

②   $\lceil 1.6 \rceil$

$\left\lceil \dfrac{5}{3} \right\rceil = \text{ceil}(5/3)$

$\left\lceil \dfrac{a}{b} \right\rceil = \dfrac{a+b-1}{b}$

$a/b$

$$f(x) \geq n \longrightarrow \frac{x}{w} \times \frac{x}{n} \geq n$$

$$\longrightarrow a \times b \geq n$$

$$\longrightarrow a \geq \lceil n/b \rceil$$

$$\longrightarrow a \geq (n+b-1)/b$$

Solution Code

```cpp
void solve(){
    long long w, h, n;
    cin >> w >> h >> n;
    long long left = 1, right = 1e18;
    long long ans = 1e18;
    while(left <= right){
        long long mid = (left + right) / 2;
        long long rows = mid / h;
        long long cols = mid / w;
        if(cols > 0 && rows >= (n + cols - 1) / cols){
            ans = mid;
            right = mid - 1;
        }else{
            left = mid + 1;
        }
    }
    cout << ans << endl;
}
```

ans = mid;

60

$\log(10^{18}) \times O(1)$

$= \log(10^{18})$

# Doubt on ail & flcor

$$a \times b \geq n$$

① 

$$a \geq \text{Qil}(n/b)$$

$$a \times b \leq n$$

② 

$$a \leq \text{floor}(n/b)$$

$$a \times b \geqslant n$$

$$a > \boxed{floor\left(\frac{n}{b}\right)} \qquad \boxed{a > ceil\left(n/b\right)}$$

①                              ②
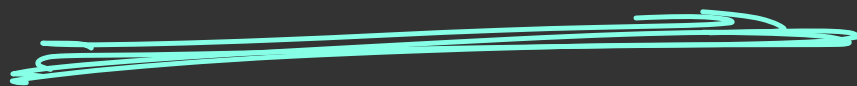
$$\frac{1.33}{2}$$               $a = 2$ ✓
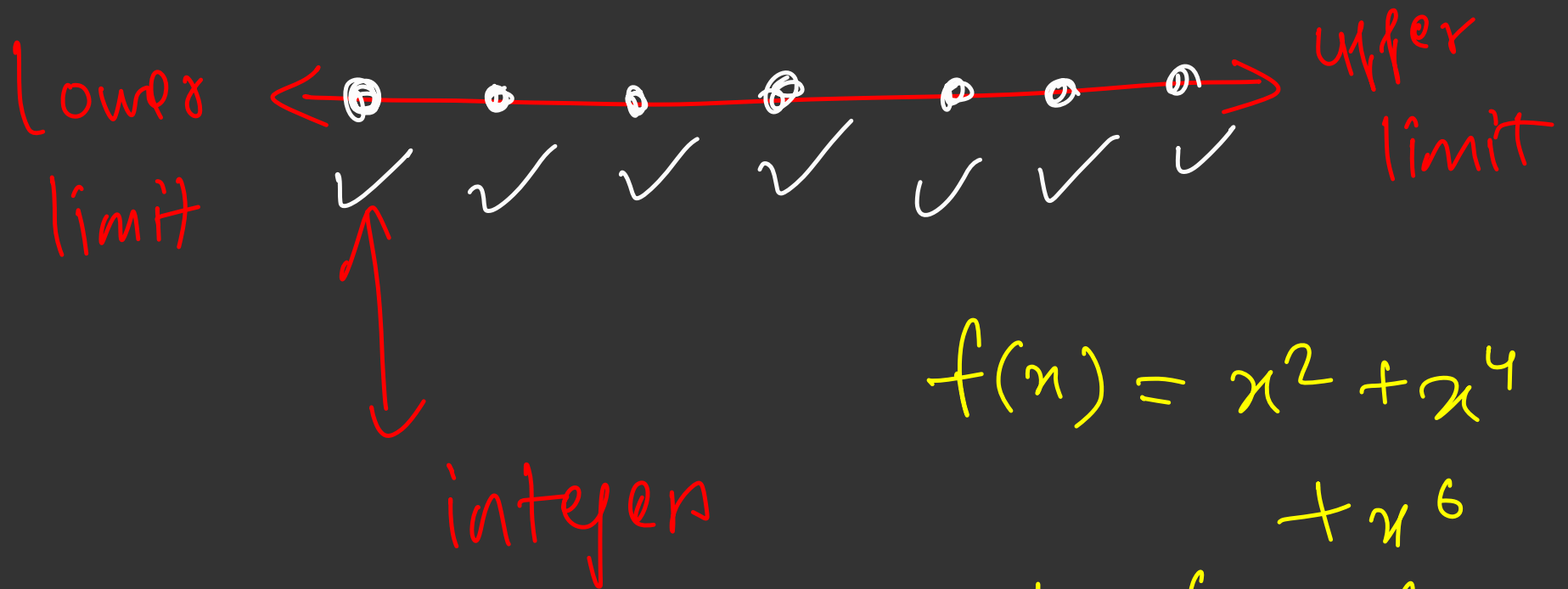
$$1 \qquad to \qquad 10^{18}$$

# Binary Search on Decimals

Find a real number between L and R following certain properties.

- How many iterations?  $($ size of search space $)$
- Precision issues?
- Ways to tackle

$10^8$

$$\left[ \text{original size} \times \frac{1}{\text{precision}} \right] \rightarrow \left( 10^8 \times \frac{1}{10^{-7}} \right)$$

lower
limit



upper
limit

integer

$$f(x) = x^2 + x^4 + x^6$$

value of $x$ for

which $f(x)$

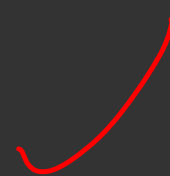$\hookrightarrow = \boxed{1029649}$ .29

1 2 3 4 5

3·01 3·02

3·015

3·0175

Codeforces $\longrightarrow$ int problem

(exact answer)

$\longrightarrow$ decimal value

1.0000000 2    actual answer

1.0000000 5 ✓  your answer

print an answer with a relative
error of upto $10^{-6}$

$$23.00000\,2000$$
$$23.00000\,2001$$
$$23.000\,001\,297$$
$$\left[\; 23.000\,001 \longleftrightarrow 23.000\,003 \;\right]$$

Answer is correct upto 8 decimal

$$\underline{10^{-6}} \quad \checkmark$$

$23.000002 \uparrow \underline{\qquad} \longrightarrow$

$23.000002 \downarrow \overline{\underline{12}} \smile$

$13$

$$A = B \qquad \text{integer} \quad \checkmark$$

$$3 = 3 \quad \longrightarrow \quad \text{Yes}$$

$$4 = 4 \quad \longrightarrow \quad \text{Yes}$$

$$4 = 5 \quad \longrightarrow \quad \text{No}$$

23. 000000 0002

23. 000 000 0001

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad ---- \quad 10^8$$

$$10^{-7}$$

$$10^7 \quad 10^7 \quad 10^7$$

1 → 2
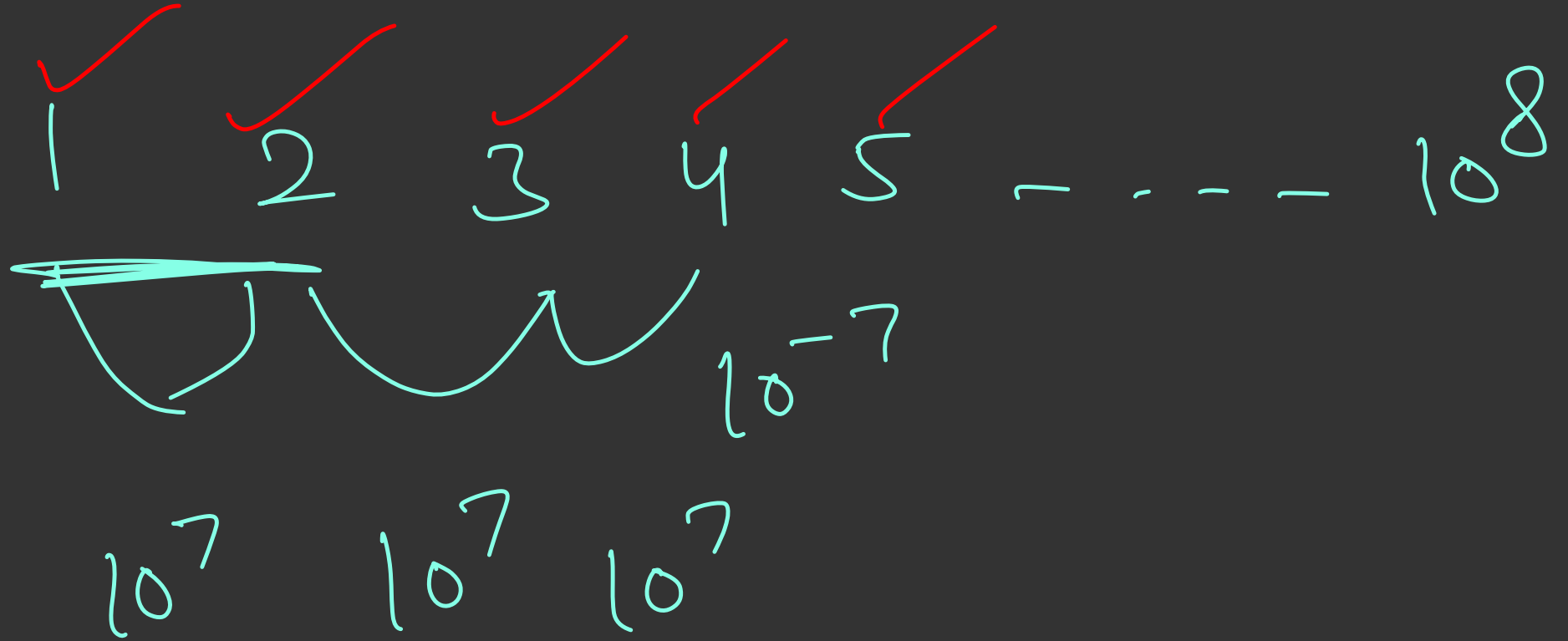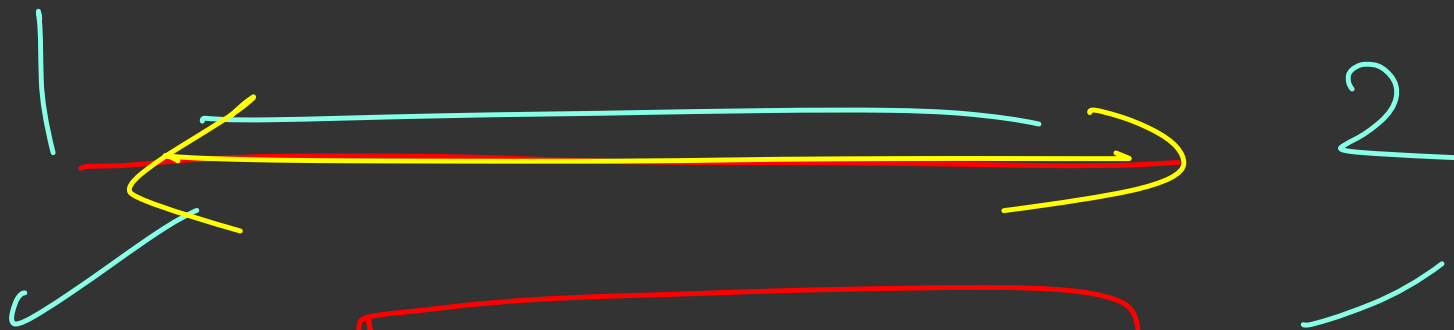
$$1.0000001$$

$$1.0000002$$

$$1.0000003$$

$$1.0000015\alpha$$

Integer $\rightarrow$ 1 $\longleftrightarrow$ $10^8$

$$10^8$$

Decimals $\rightarrow$ 1 $\longleftrightarrow$ $10^8$

$$10^7$$

$$10^8 \times 10^7 = 10^{15}$$

# Binary search on decimals

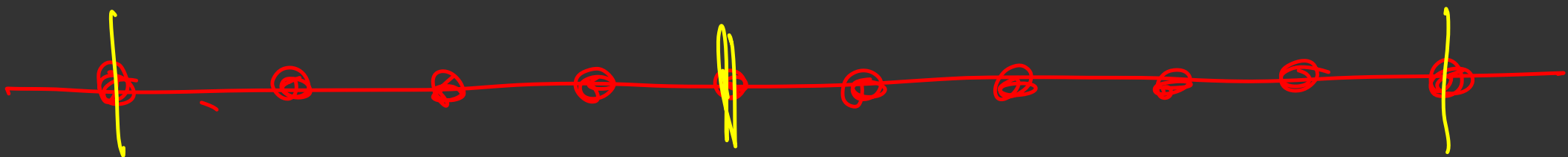Search space $\Rightarrow$ org. S.S $\times \dfrac{1}{P}$

$$org. SS = 10^8 \quad , \quad P = 10^{-7}$$

$$10^8 \times \dfrac{1}{10^{-7}} = \dfrac{10^8 \times 10^7}{10^{-7}} = 10^{15}$$

$$T.C \rightarrow \log(\text{search space})$$

$$= \log\left(\text{int. search space} \times \frac{1}{\text{precision}}\right)$$
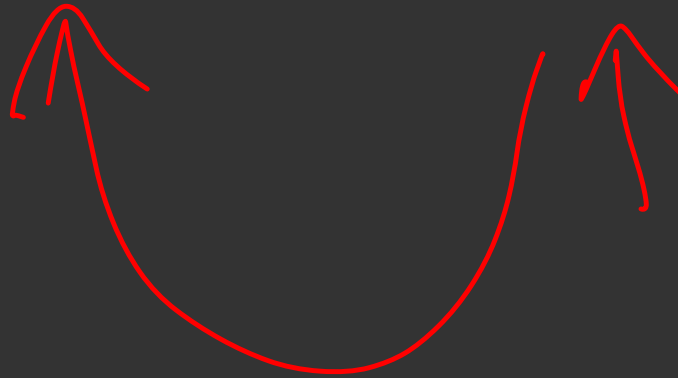
$$A_1 \quad A_2 \quad A_3 \quad --- \quad A_n$$

mid
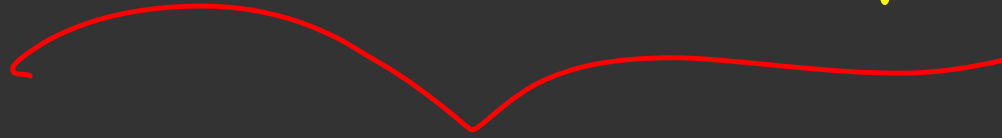
right = mid - 1

$f(mid) \rightarrow f$  right = mid - P

T T T T T T  f f f f f f f f f

1          1.0000001          1.0000002

$$right = mid - 1$$

$$\log \left( \text{int s.s.} \cdot \times \frac{1}{\beta} \right)$$

$$1 \quad 2 \quad \left[ 3 \quad 4 \right) \quad --- \quad 10^8$$

$$\log \left( \text{int ss} \right) + \log \left( \frac{1}{\beta} \right)$$

find $sqrt(x)$     find $y = sqrt(x)$

$y$ to be correct upto 7

decimal places

$1 \leq x \leq 10^{16}$