Lemma 4

As the equivalence relation $\cong$ is back stable,

we can use the quotient graph $[B_{P,Rf}(A)]_{\cong_m}$

to check reachability. To indicate a suitable choice of m for solving reachability problem.

Lemma: Consider B_1, B_2 and a bounded Interval $I \subseteq \mathbb{R}$ with integer endpoints.

If $B_1 \cong B_2$ for the right-end point m of I, then $I \cap I(B_1) = \phi$

$\Leftrightarrow I \cap I(B_2) = \phi$

Proof: $B_1 = (R, L_1, l_1, U_1, u_1)$

$B_2 = (R, L_2, l_2, U_2, u_2)$

such that

$B_1 \cong B_2$

left end points of $I(B_1)$ and $I(B_2)$ are either both equal or $> m$.

To show that when left end points $\leq m$, $I(B_1)$ and $I(B_2)$ are both left-open or left closed.

If $l_1 = l_2$, then trivial,

else, $l_1 \neq l_2$. Some coefficient of L_1 and $L_2 > m$.

Since, left end points of $L_1, L_2 \leq m$.

L_1, L_2 has atleast 2 NON-ZERO coefficients.

So, both the intervals are open, irrespective of l_1, l_2.

So, both the intervals are open, irrespective of x_1, l_2.

Similarly, for right end points.

So, $I(B_1) \cap I = \phi \iff I(B_2) \cap I = \phi$.