Lemma 1

$A \leftarrow$ Timed automaton

$p \leftarrow$ duration measure of A.

$R_f \leftarrow$ region of A.

$\forall$ state σ of A and +ve real δ,

→ $\exists$ state $r \in R_f$ such that $(\sigma, 0) \xrightarrow{*} (r, \delta)$

iff In $\mathcal{B}_{p,R_f}(A)$, $\exists$ a path to R_f from a bounded-labelled region (R, L, l, U, u)

$$\cup$$

σ , $\delta \in I(\sigma, L, l, U, u)$

1) ⇒ 2)

Proof: Take a state σ of A and a +ve real δ,

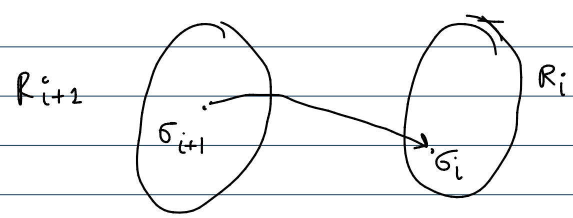
$(\sigma, 0) \xrightarrow{*} (r, \delta)$ for some $r \in R_f$.

Then: By definition of Region graph, $\mathcal{R}(A)$

$$(\sigma_n, \delta_n) \to (\sigma_{n-1}, \delta_{n-1}) \to \dots \to (\sigma_1, \delta_1) \to (\sigma_0, \delta_0) \to (r, \delta)$$

where $\sigma_n = \sigma$ and $\delta_n = 0$, $[\sigma_0] = R_f$.

$\forall\ 0 \le i < n$



Claim: $\exists\ B_0 \dots B_n$ such that

(1) $\forall\ 0 \le i \le n$, the region component of B_i is R_i.

(2) the Bound graph $\mathcal{B}_{p,R_f}(A)$ has an edge from B_0 to R_f and from B_{i+1} to B_i for all $0 \le i < n$.

(3) $\forall\ 0 \le i \le n$, $(\delta - \delta_i) \in I(\sigma_i, L_i, l_i, U_i, u_i)$ where $B_i = (R_i, L_i, l_i, U_i, u_i)$

We induct on i,

Base :- $i = 0$, (1) B_0 has the region component R_0.

(2) $B_0 \to R_f$.

(3) $\delta - \delta_0 \in I(\sigma_0, L_0, l_0, U_0, u_0)$ where $B_0(R_0, L_0, l_0, U_0, u_0)$.

Let for $i = k$, the properties hold:

For $k+1$, (1) B_k has the region component R_K.

(2) $B_{K+1} \to B_K$.

(3) $\delta - \delta_{K+1} \in I(\sigma_{K+1}, L_{K+1}, l_{K+1}, U_{K+1}, u_{K+1})$

where $B_{K+1}(R_{K+1}, L_{K+1}, l_{K+1}, U_{K+1}, u_K)$

(2) $\Rightarrow$ (1) Consider the bounded-labeled regions $B_n \dots B_0$ such that the bounds graph $B_{P,R_f}(A)$ has an edge from B_0 to R_f and from B_{i+1} to B_i $\forall\, 0 \le i < n$.

Let each B_i be $(R_i, L_i, l_i, U_i, u_i)$

Claim: $\forall\, 0 \leq i \leq n$, $\forall\, \sigma \in R_i$, and $\forall\, \delta \in I(\sigma, L_i, l_i, U_i, u_i)$,

$$\exists\, \tau \in R_f, \text{ with } (\sigma, 0) \xrightarrow{*} (\tau, \delta)$$

Again, we induct on i,

Base :- when $i = 0$,

$\forall \sigma \in R_0$, $\forall\, \delta \in I(\sigma, L_0, l_0, U_0, u_0)$, $\exists\, \tau \in R_f$ with $(\sigma, 0) \to (\tau, \delta)$. since, there is an edge from B_0 to R_f.

Let for $i = k$, the claim holds.

For $i = k+1$, $\forall \sigma \in R_{k+1}$, $\forall\, \delta \in I(\sigma, L_{k+1}, l_{k+1}, U_{k+1}, u_{k+1})$

$$\exists\, \tau \in R_f, \underbrace{(\sigma, 0) \longrightarrow (\sigma_k, \delta_k)}_{\text{from assumption}} \overbrace{\longrightarrow \dots \longrightarrow (\tau, \delta)}^{\text{By hypothesis}}$$