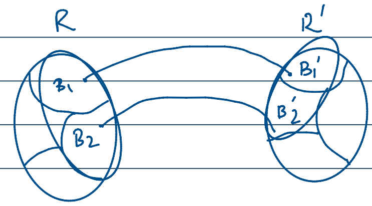
Lemma 3

If $B_{P,R_f}(A)$ has $B_1', B_2' \mid B_1' \cong B_2'$ and an edge from a bounded labelled region B_1 to B_1'

then, $\exists B_2 \mid B_1 \cong B_2$ and there is an edge from B_2 to B_2'.



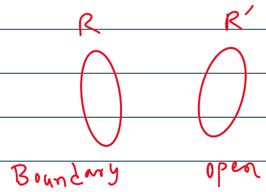
Proof: $B_1 = (R', L_1', l_1', U_1', u_1')$
$B_2 = (R', L_2', l_2', U_2', u_2')$

Let there be a region R, such that $R \rightarrow R'$ in $\mathcal{R}(A)$.

[Assumption]
There is a $B_1 = (R, L_1, l_1, U_1, u_1)$
such that $B_1 \rightarrow B_1'$ in $B_{P,R_f}(A)$.

$\exists B_2 = (R, L_2, l_2, U_2, u_2) \mid B_2 \rightarrow B_2'$ in $B_{P,R_f}(A)$.

To show that $B_1 \cong B_2$,
we look at the case when



$p(s) = a$

Transition edge 1 R' is a boundary region, R is an open region, and the clock with the k-th smallest fractional part in R is reset:

for all $0 \le i < k$, we have $a_i = a'_{i+1}$ and $b_i = b'_{i+1}$;
for all $k \le i < n$, we have $a_i = a'_i$ and $b_i = b'_i$;
if $a'_n \le a'_1 + a$ then $a_n = a'_n$, else $a_n = a'_1 + a$;
if $b'_n \ge b'_1 + a$ then $b_n = b'_n$, else $b_n = b'_1 + a$;
if $a'_n > a'_1 + a$ and $a > 0$ then $l = 1$, else $l = l'$;
if $b'_n < b'_1 + a$ and $a > 0$ then $u = 1$, else $u = u'$.

clock with k^{th} smallest frac. part is reset

prove $L_1 \cong L_2$ and either $l_1 = l_2$ or some $a_i > m$ in L_1. (Similar for upper bounds).

$L_1 = (a_0 \ldots a_n)$ $\quad$ $L_2 = (b_0 \ldots b_n)$
$L_1' = (a_0' \ldots a_n')$ $\quad$ $L_2' = (b_0' \ldots b_n')$.

1. $\forall\ 0 \le i < k,\quad a_i = a'_{i+1} \quad b_i = b'_{i+1}$
2. $\forall\ k \le i < n,\quad a_i = a_i' \quad b_i = b_i'$.

If $B_1' \cong B_2'$, $a_i' \cong b_i'\ \forall\, 0 \le i < n \Rightarrow a_i \cong b_i,\ \forall\, 0 \le i < n$

3. $a_n = \min(a_n', a_1' + a)$, 4. $b_n = \min(b_n', b_n' + a)$

We have 4 cases to consider:-

Case (i) $a_n = a_n'$ and $b_n = b_n'$.

Since, $a_n' \cong_m b_n'$, we have $a_n \cong_m b_n$

For this $l_1 = L_1'$, $l_2 = L_2'$

Since, $a_n =_m b_n$, we have $a_n =_m b_n$

For this, $l_1 = l_1'$, $l_2 = l_2'$

If $l_1' = l_2'$, then $l_1 = l_2$

Else $a_j' > m$, for some j, $a_0' = 0$ (Boundary region)

Each a_j', $1 \le j < n$, $= a_j$ or a_{j-1}, thus, some $a_k > m$

case (ii) $a_n = a_n'$, $b_n = b_1' + a$

$a_n \,\tilde{=}\, b_n'$, $a_1' \cong b_1'$, $a_n' \le a_1' + a$, $b_n' > b_1' + a$

$\Rightarrow a_n', a_1' + a, b_n', b_1' + a > m$

Thus, $a_n > m$, $b_n > m$.

As atleast one coefficient in l_1 is $\geq m$, we don't require $l_1 = l_2$.

case (iii) and (iv) follows similarly.

$a_n = a_1' + a$, $b_n = b_n'$

$a_n = a_1' + a$

b