SPICE Simulations, Mathematical Analysis and Designing an Oscillator

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1 Designing a RC Phase Shift Oscillator

1.1 Finding the Transfer Function

Figure 2 Shows the evaluation of the Transfer function of the RC phase shift frequency selective network that I am going to use.

The transfer function comes out as

$$\frac{V_2(s)}{V_1(s)} = \frac{(RC)^3 s^3}{(RC)^3 s^3 + 6(RC)^2 s^2 + 5(RC)s + 1}$$

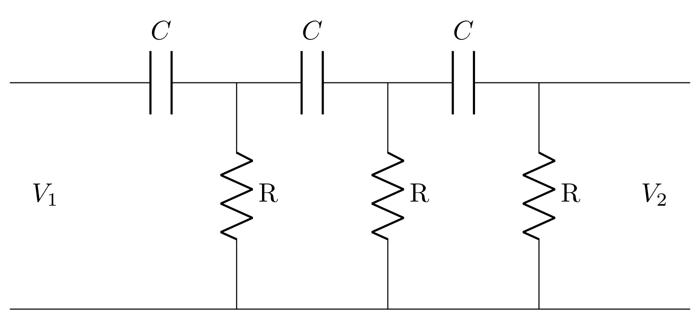
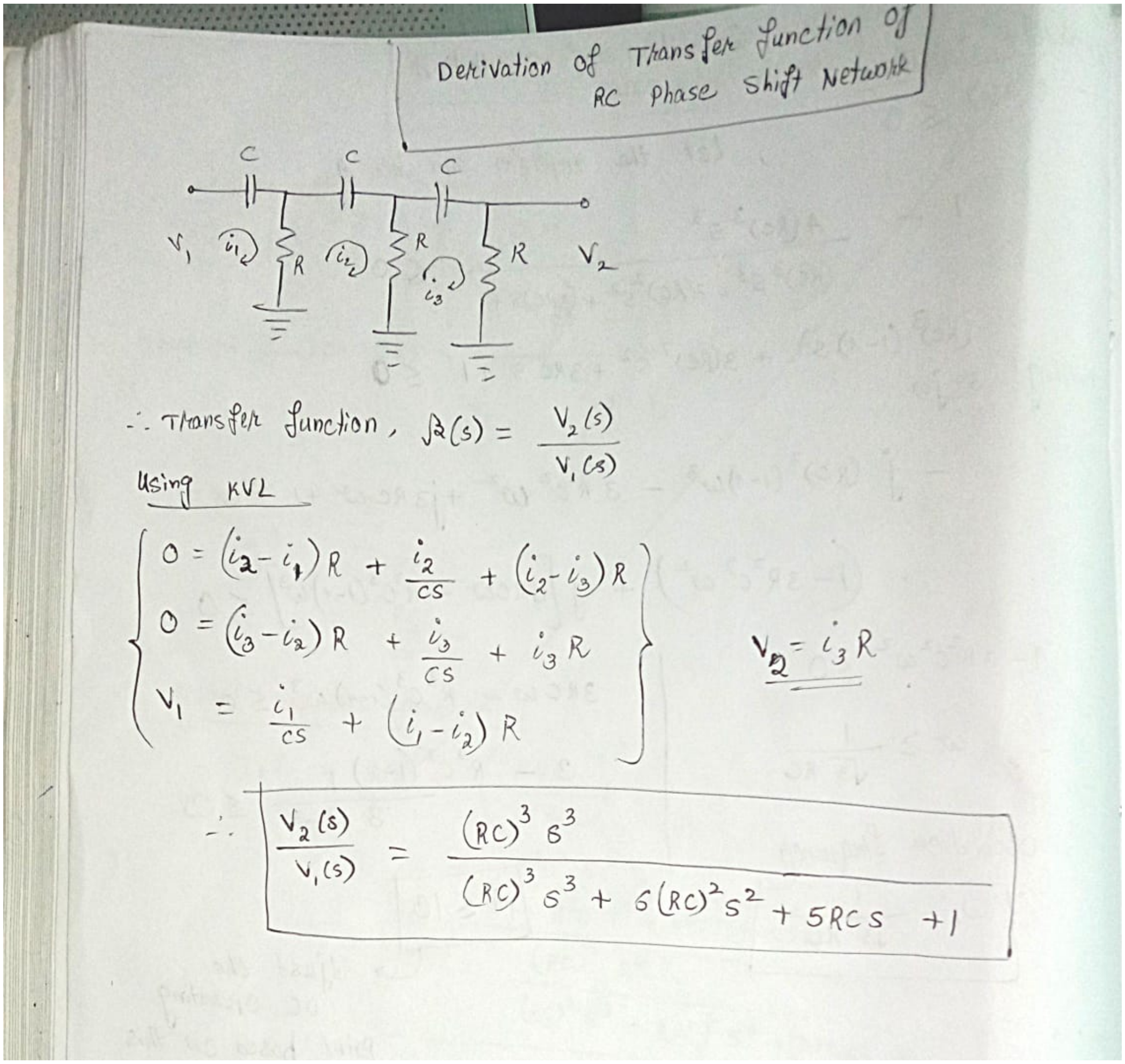


Figure 1: RC phase shift network Circuit



Derivation of Transfer function of RC Phase Shift Network

$\therefore$ Transfer function, $\Omega(s) = \dfrac{V_2(s)}{V_1(s)}$

Using KVL

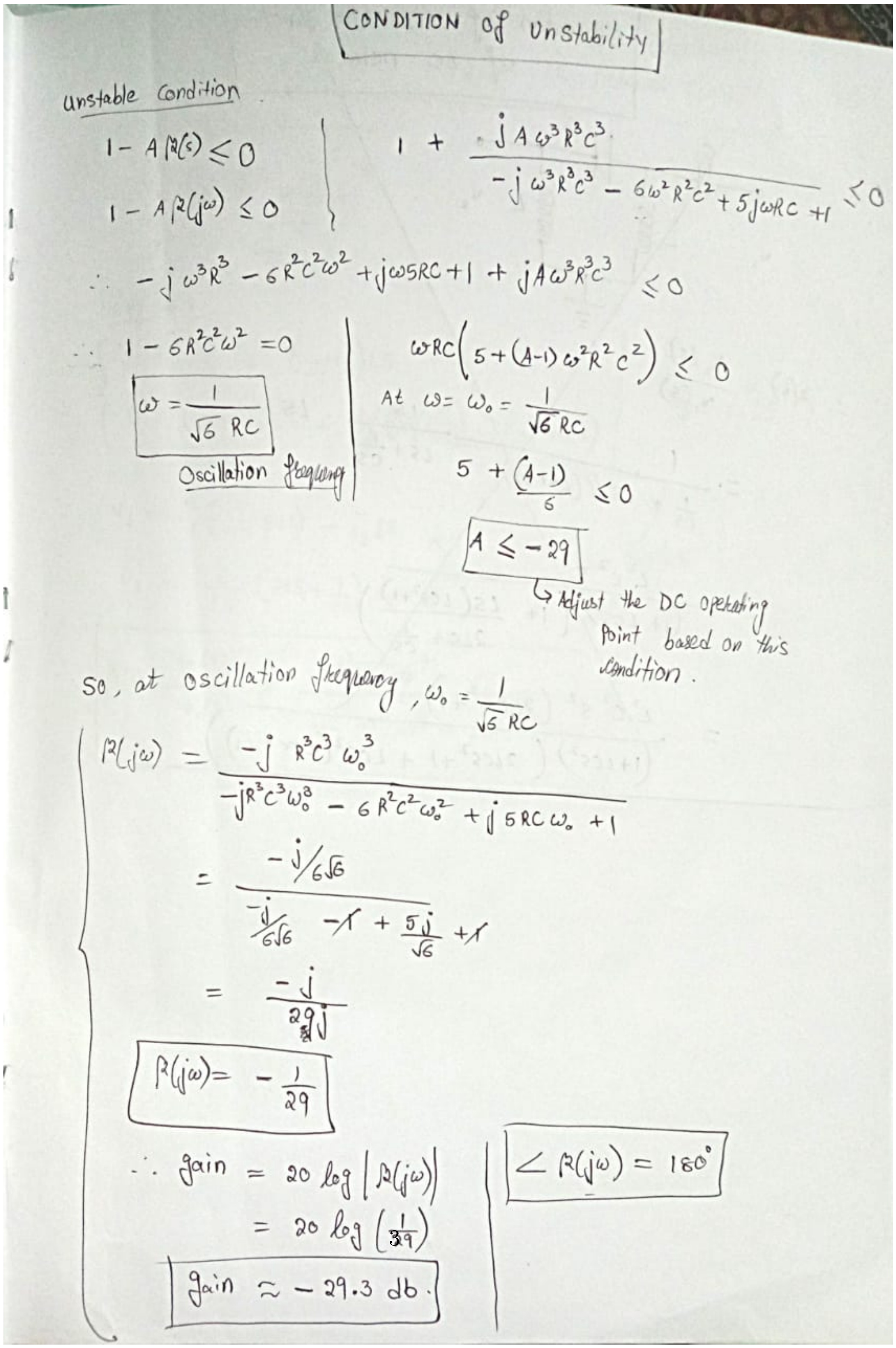
$$\begin{cases} 0 = (i_2 - i_1)R + \dfrac{i_2}{Cs} + (i_2 - i_3)R \\ 0 = (i_3 - i_2)R + \dfrac{i_3}{Cs} + i_3 R \\ V_1 = \dfrac{i_1}{Cs} + (i_1 - i_2)R \end{cases}$$

$V_2 = i_3 R$

$$\therefore \boxed{\frac{V_2(s)}{V_1(s)} = \frac{(RC)^3 s^3}{(RC)^3 s^3 + 6(RC)^2 s^2 + 5RCs + 1}}$$

Figure 2: Transfer Function

1.2 Unstability Condition



CONDITION of Unstability

unstable condition

$$1 - A\beta(s) \leq 0$$

$$1 - A\beta(j\omega) \leq 0$$

$$1 + \frac{jA\omega^3R^3C^3}{-j\omega^3R^3C^3 - 6\omega^2R^2C^2 + 5j\omega RC + 1} \leq 0$$

$$\therefore -j\omega^3R^3 - 6R^2C^2\omega^2 + j\omega 5RC + 1 + jA\omega^3R^3C^3 \leq 0$$

$$\therefore 1 - 6R^2C^2\omega^2 = 0$$

$$\boxed{\omega = \frac{1}{\sqrt{6}\,RC}}$$

Oscillation frequency

$$\omega RC\left(5 + (A-1)\omega^2R^2C^2\right) \leq 0$$

At $\omega = \omega_0 = \frac{1}{\sqrt{6}\,RC}$

$$5 + \frac{(A-1)}{6} \leq 0$$

$$\boxed{A \leq -29}$$

Adjust the DC operating point based on this condition.

So, at oscillation frequency, $\omega_0 = \frac{1}{\sqrt{6}\,RC}$

$$\beta(j\omega) = \frac{-jR^3C^3\omega_0^3}{-jR^3C^3\omega_0^3 - 6R^2C^2\omega_0^2 + j5RC\omega_0 + 1}$$

$$= \frac{-j/6\sqrt{6}}{\frac{-j}{6\sqrt{6}} \cancel{-1} + \frac{5j}{\sqrt{6}} \cancel{+1}}$$

$$= \frac{-j}{29j}$$

$$\boxed{\beta(j\omega) = -\frac{1}{29}}$$

$$\therefore \text{gain} = 20\log|\beta(j\omega)|$$

$$= 20\log\left(\frac{1}{29}\right)$$

$$\boxed{\text{gain} \approx -29.3\ \text{db.}}$$

$$\boxed{\angle\beta(j\omega) = 180^\circ}$$

Figure 3: Oscillation Condition

1.3 SPICE AC analysis of the amplifier and DC operating point

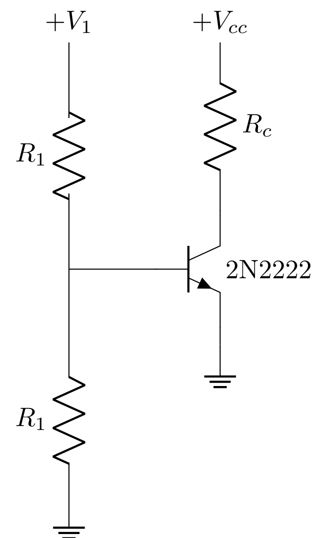


Figure 4: Amplifier circuit for DC analysis

Setting $+V_1 = 4V$ and selecting $R_1 = 1K\Omega$ and $+V_{cc} = 30V$ and changing R_c to get the optimal operating point.
For $R_c = 50\Omega$, I got $i_c = 281mA$ and $i_b = 2.21mA$. Plotting the curve along with the load line I found that it is the optimal point

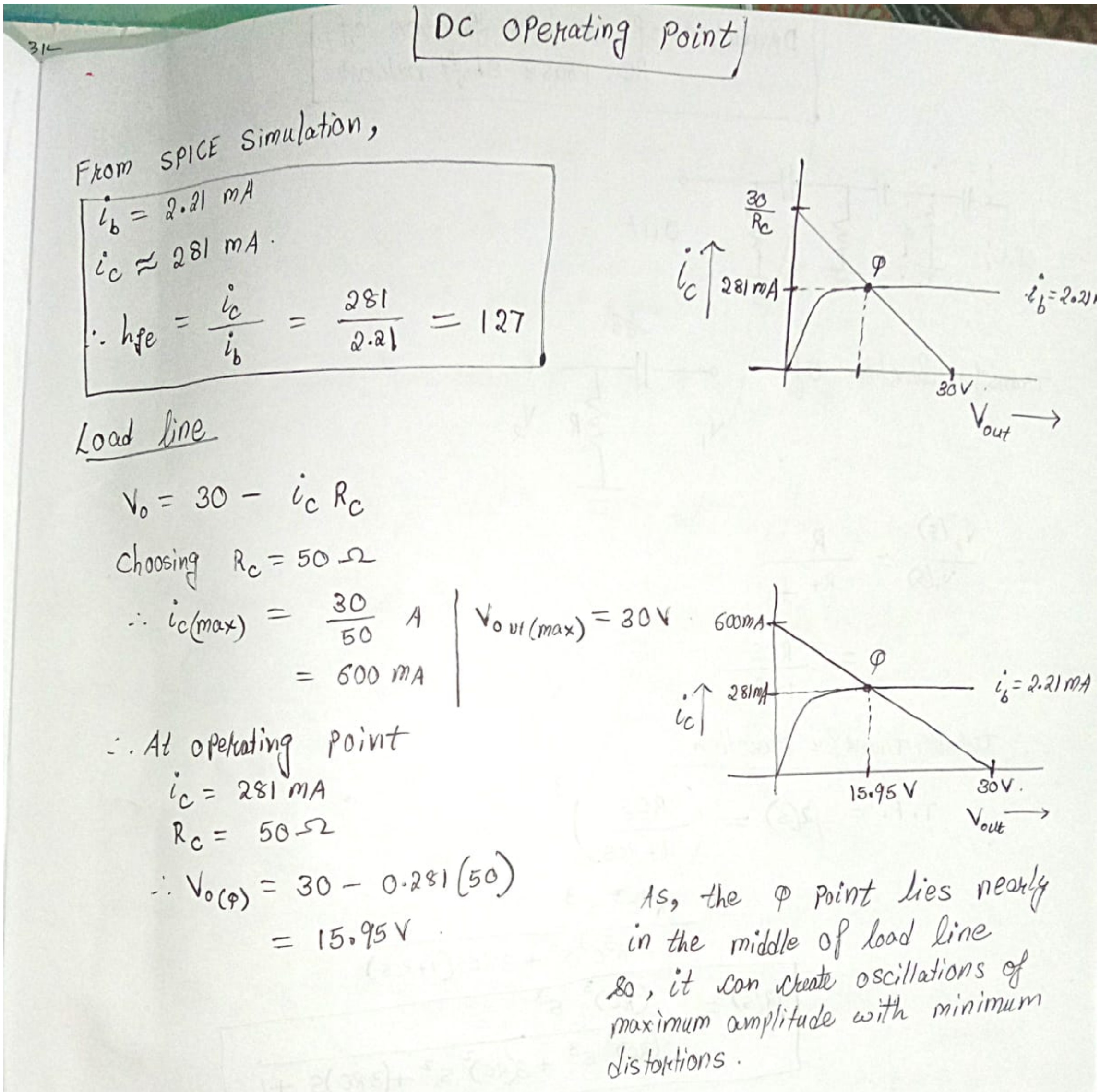


Figure 5: Choosing the right Q point based on the value of i_c from Simulation

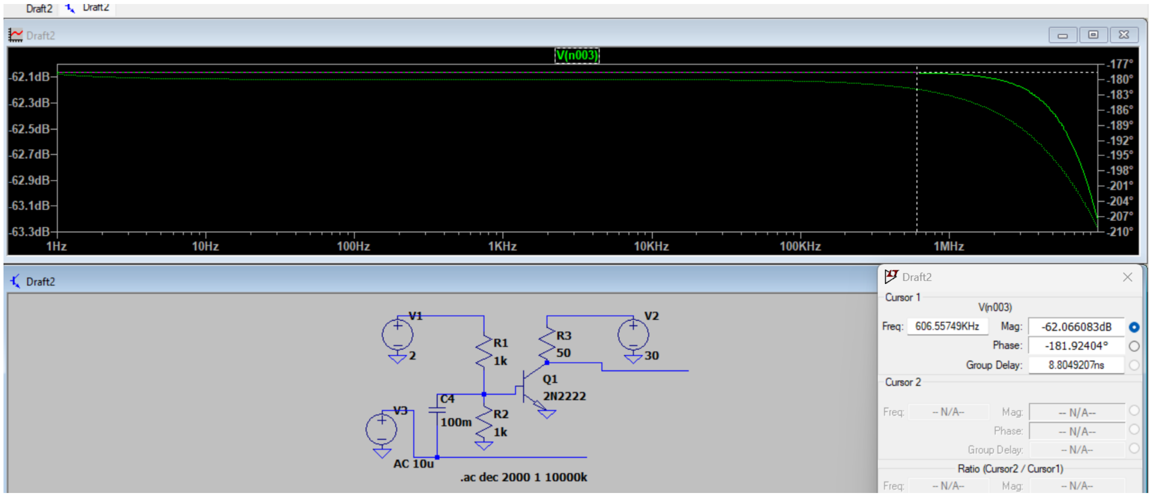


Figure 6: AC analysis of the BJT CE amplifier at the optimal DC operating point

The AC analysis of the output signal of the amplifier shows that roughly upto 606 kHz the phase and gain is constant at -180^o and -62 dB respectively.

For the AC analysis the input Voltage is given $10\mu V$.

$$V_{out} = 10^{\frac{-62dB}{20}} = 800\mu V$$

$$therefore, Amplification, A = -80$$

and it works well till 606 kHz or some MHz

1.4 Selection of desired Oscillation frequency and components

From figure 3 in derivation of condition of Oscillation, the Oscillating frequency came out to be $f_0 = \frac{1}{2\pi RC\sqrt{6}}$ So if I wish to make an oscillator oscillating at $f_0 = 700HZ$ Putting f_0 in,

$$RC = \frac{1}{2\pi f_0\sqrt{6}}$$

$$RC = 9.28 * 10^{-}5$$

Choosing $R = 93\Omega$ and $C = 1\mu F$

1.5 AC analysis of the frequency selective network

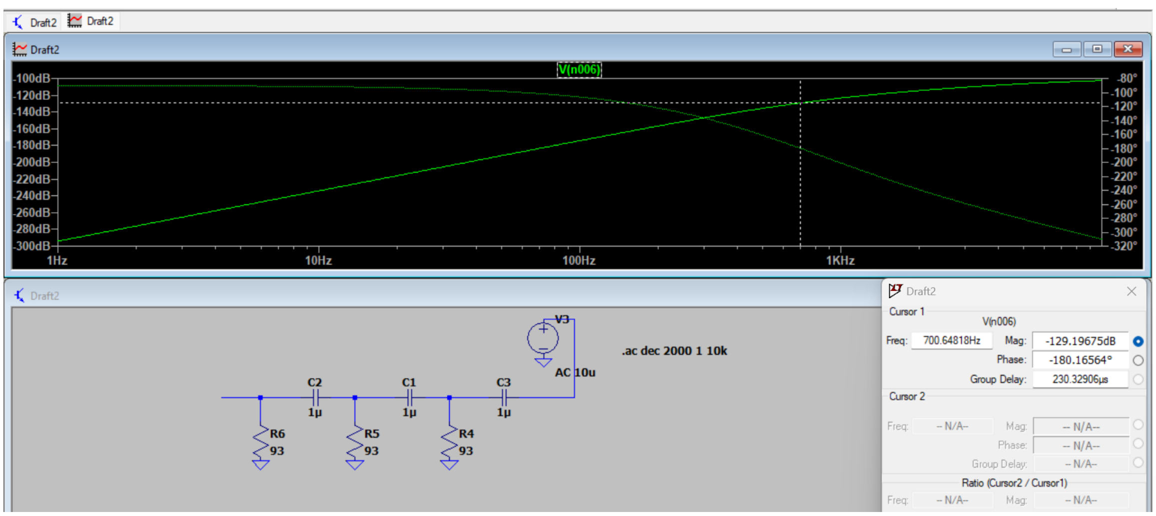


Figure 7: AC analysis of the RC frequency selective network

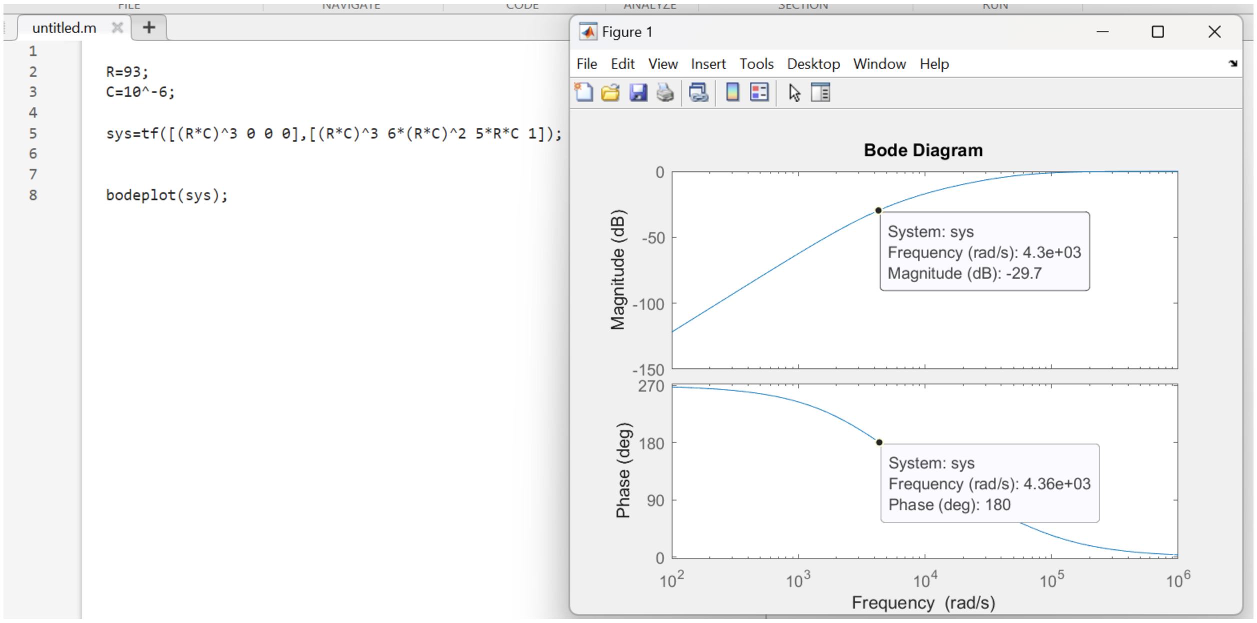


Figure 8: Bode Plot

The AC analysis shows that it gives phase shift of -180 degree at 700 hertz.

The BODE Plot plotted using the transfer function also is in accordance to the data achieved from ac analysis.

In the AC analysis the input end has -100 dB gain and output end has -129 db gain. So the Voltage gain of the network with respect to input end is about -29 dB.

1.6 Results

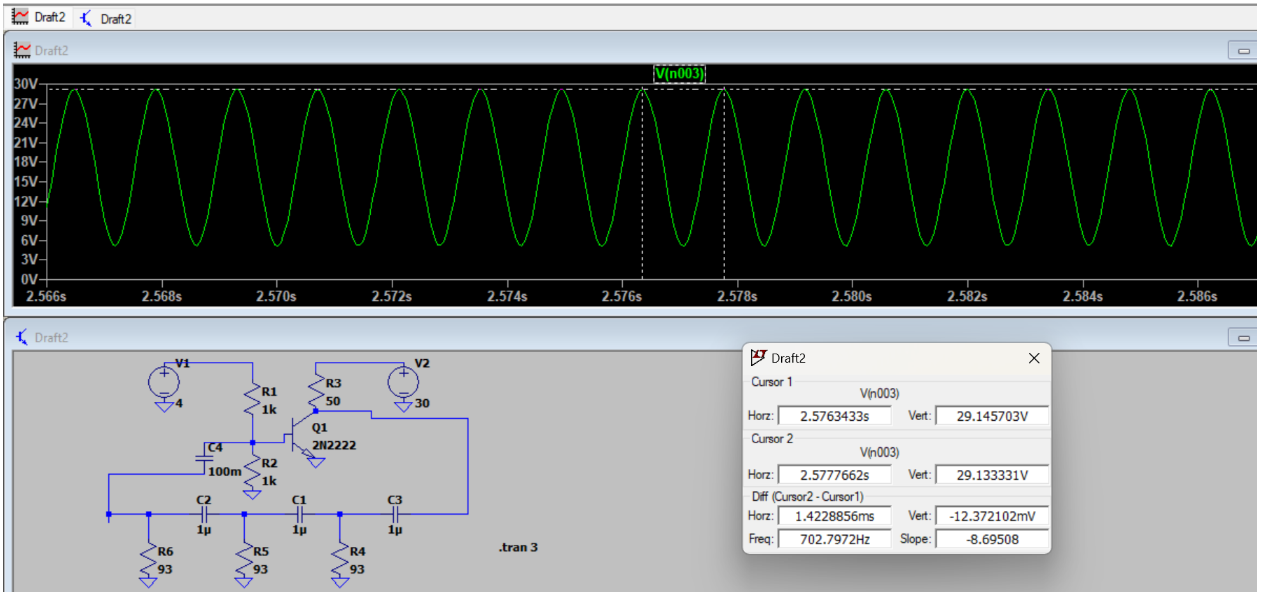


Figure 9: Result

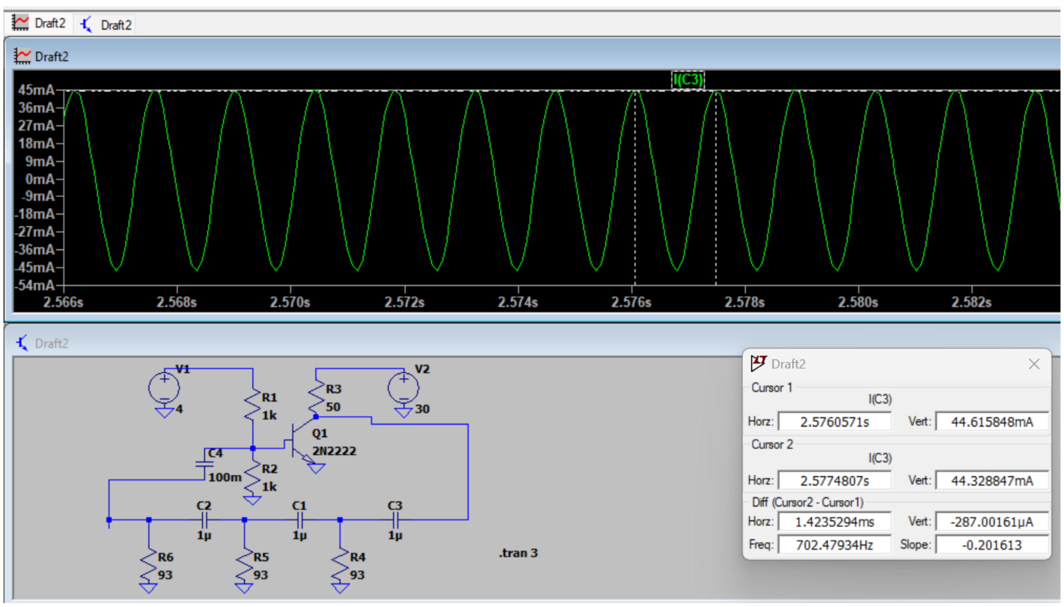


Figure 10: i_{C_3}

2 Designing LC network Oscillator

2.1 Finding the Transfer Function

Figure 11 Shows the evaluation of the Transfer function of the LC frequency selective network that I am going to use.

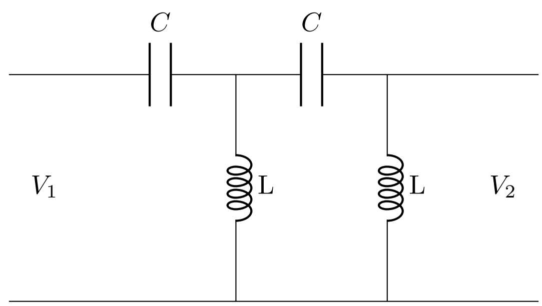
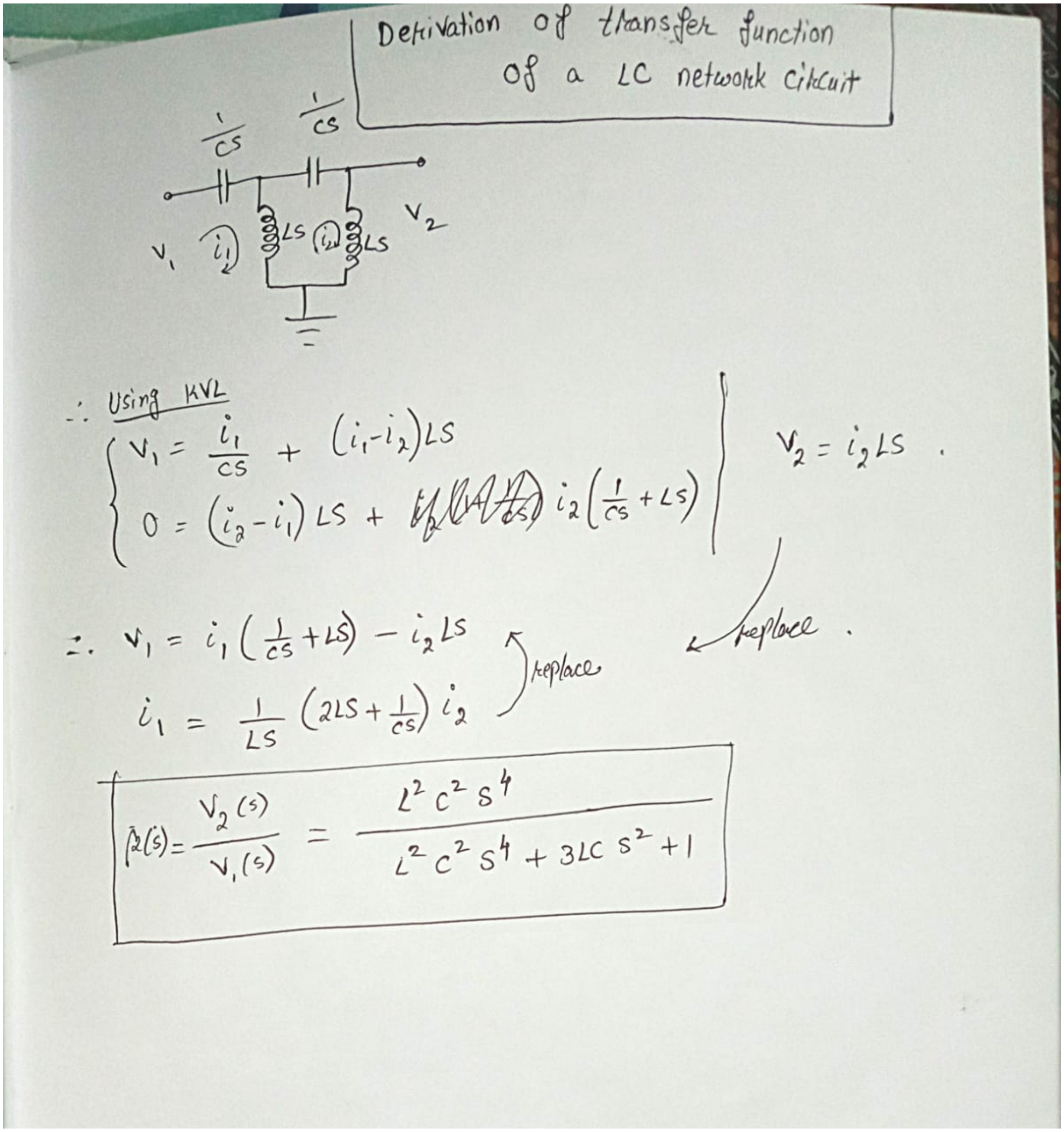


Figure 11: LC network Circuit

The transfer function comes out as

$$\frac{V_2(s)}{V_1(s)} = \frac{(LC)^2 s^4}{(LC)^2 s^4 + 3LCs^2 + 1}$$

The Oscillator is operated at the same DC point as previous. The circuit is tuned and tested for 3 frequencies i.e. 10kHz and 10 MHz



Derivation of transfer function of a LC network circuit

$\therefore$ Using KVL

$$\begin{cases} V_1 = \frac{i_1}{CS} + (i_1 - i_2)LS \\ 0 = (i_2 - i_1)LS + i_2\left(\frac{1}{CS} + LS\right) \end{cases} \qquad V_2 = i_2 LS .$$

replace .

$$\therefore V_1 = i_1\left(\frac{1}{CS} + LS\right) - i_2 LS$$

$$i_1 = \frac{1}{LS}\left(2LS + \frac{1}{CS}\right) i_2$$

replace

$$\boxed{\beta(s) = \frac{V_2(s)}{V_1(s)} = \frac{L^2C^2S^4}{L^2C^2S^4 + 3LCS^2 + 1}}$$

Figure 12: Transfer Function

2.2 10 KHZ

SPICE AC analysis and Results

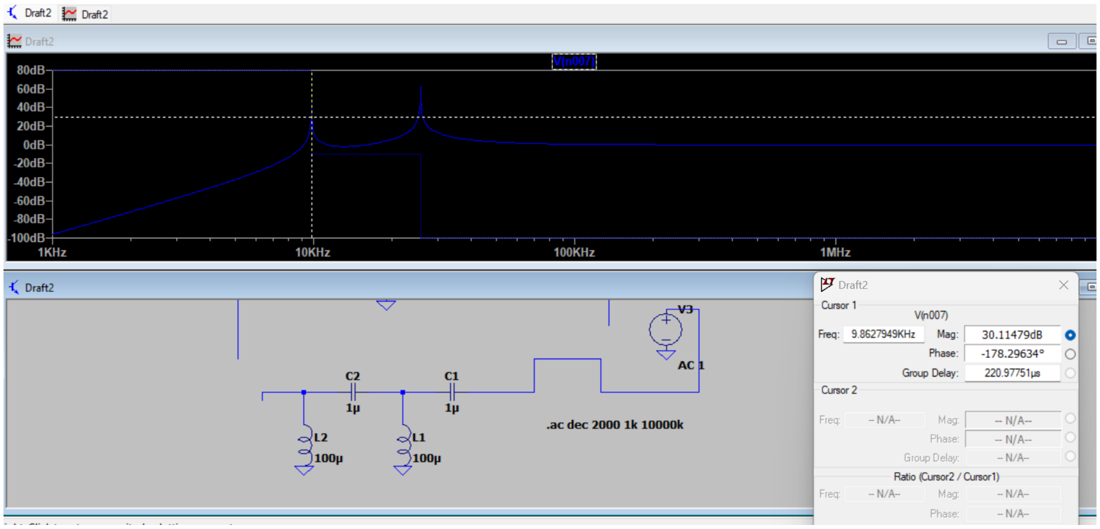


Figure 13: AC analysis of the LC frequency selective network

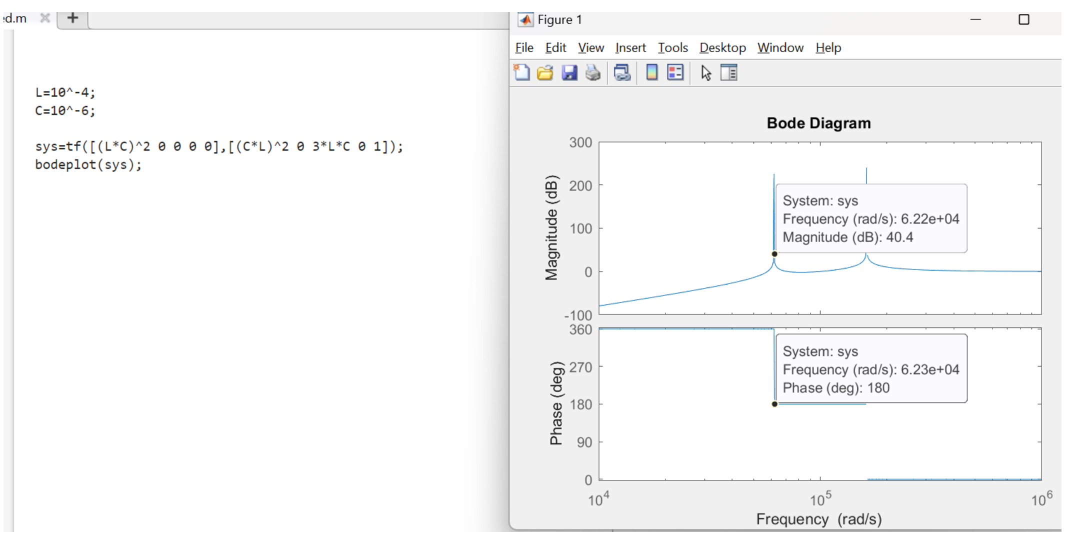


Figure 14: Bode Plot

The AC analysis shows that it gives phase shift of -180

degree at 10KHZ.
The BODE Plot plotted using the transfer function also is in accordance to the data achieved from ac analysis.

Results

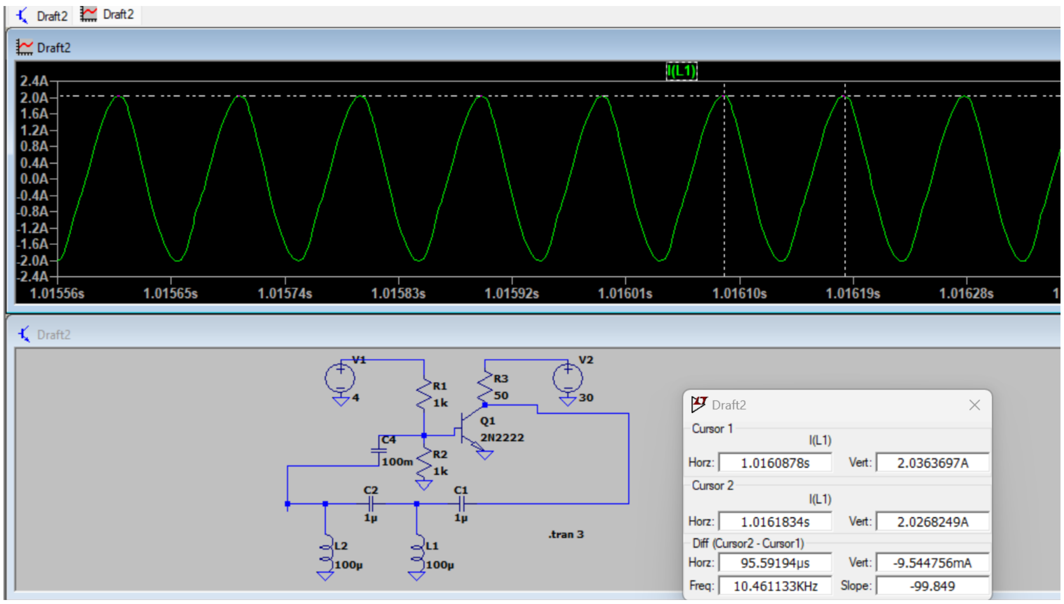


Figure 15: Result

2.3 10 MHZ

SPICE AC analysis and Results

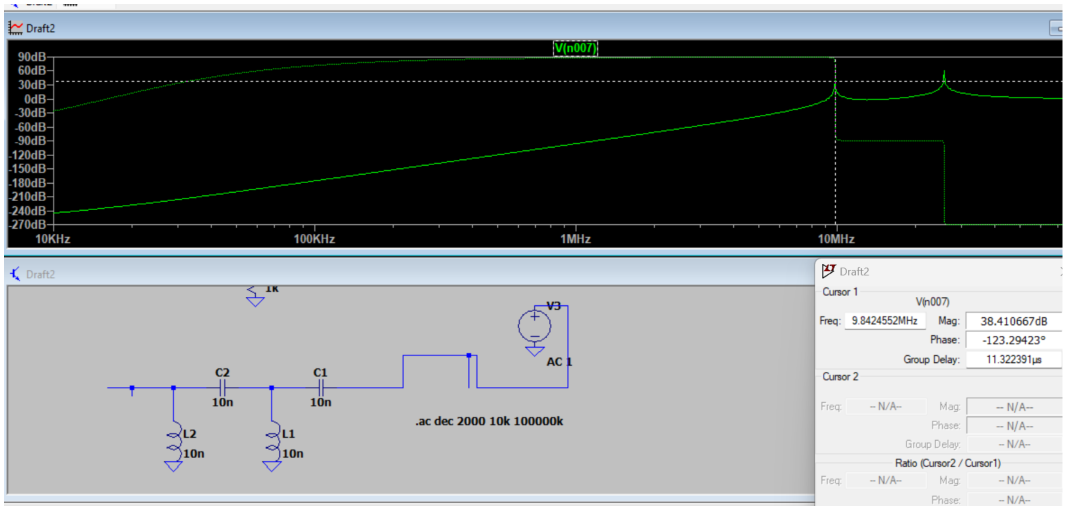


Figure 16: AC analysis of the LC frequency selective network

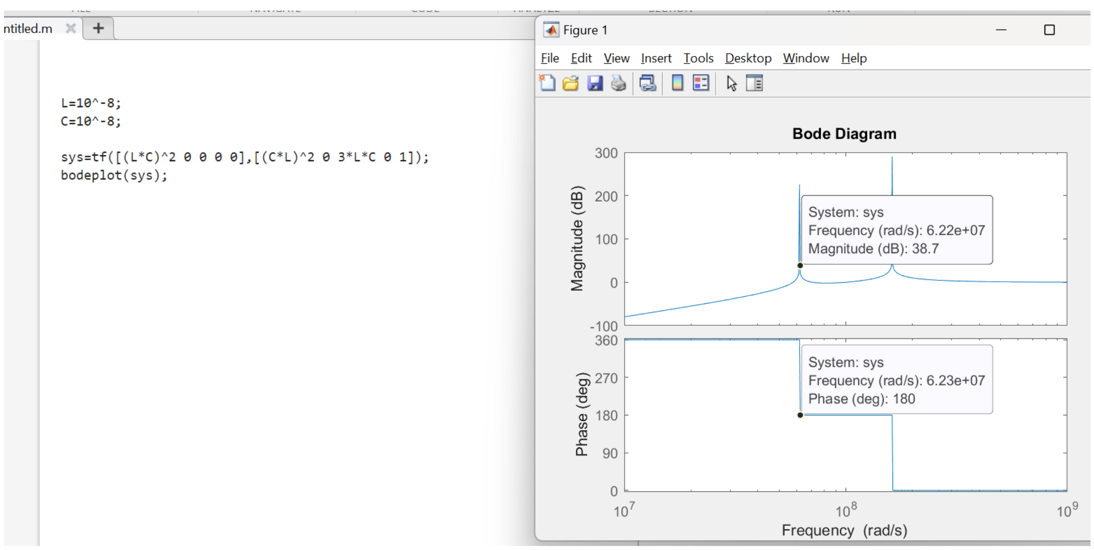


Figure 17: Bode Plot

The AC analysis shows that it gives phase shift of -180 degree at 10MHZ.
The BODE Plot plotted using the transfer function also is in accordance to the data achieved from ac analysis.

Results

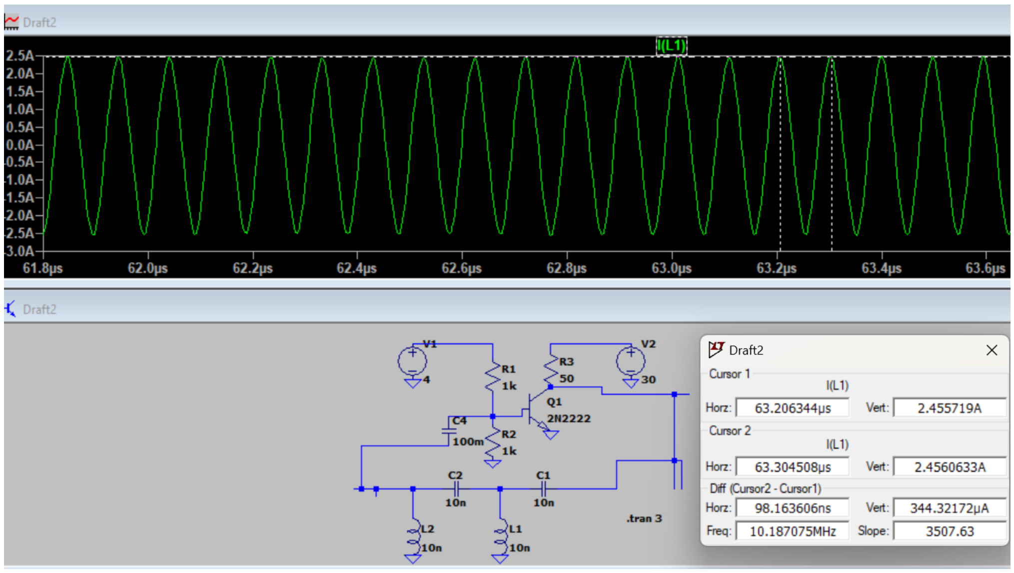


Figure 18: Result