GRAVITATIONAL WAVES

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INTRODUCTION

Gravitational waves are propagating fluctuations of gravitational fields; ”ripples” in spacetime, generated mainly by moving massive bodies. These distortions of spacetime travel with the speed of light.

Every body in the path of such a wave feels a tidal gravitational force that acts perpendicular to the waves direction of propagation; these forces change the distance between points, and the size of the changes is proportional to the distance between the points

Introduction-general relativity

Spacetime-interval $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ where $g_{\mu\nu}$ has the information about degree of curvature(flatness) of the spacetime. Riemann

Einstein's Field Equations: $$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Where $$T_{\mu\nu} = (p+\rho) u_\mu u_\nu - p g_{\mu\nu}$$

T is the stress energy tensor, $R_{\mu\nu}$ is the Ricci Tensor, R is the Ricci scalar and G is the

Christoffel symbols: $$\Gamma^{\beta}_{\mu\nu} = \frac{1}{2} g^{\beta\alpha} \left(\frac{\partial g_{\alpha\mu}}{\partial x^\nu} + \frac{\partial g_{\alpha\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\alpha} \right)$$

Bianchi's identity: $$\nabla_\mu G^{\mu\nu} = 0$$

equation of conservation of energy and momentum $$\nabla_\mu T^{\mu\nu} = 0$$

Geodesic Deviation:

$$\frac{D^2\xi^\mu}{D\tau^2} = R^\mu{}_{\alpha\beta\gamma}\, v^\alpha\, v^\beta\, \xi^\gamma$$

Riemann tensors

$$R^\mu{}_{\alpha\beta\gamma} = \frac{\partial\Gamma^\mu_{\alpha\gamma}}{\partial x^\beta} - \frac{\partial\Gamma^\mu_{\alpha\beta}}{\partial x^\gamma} + \Gamma^\mu_{\sigma\beta}\Gamma^\sigma_{\gamma\alpha} - \Gamma^\mu_{\sigma\gamma}\Gamma^\sigma_{\beta\alpha}.$$

Ricci scalar

$$R_{\alpha\beta} = R^\mu{}_{\alpha\beta\mu}, \quad R = R_\mu{}^\mu = g^{\mu\nu} R_{\mu\nu}.$$

Equation of motion:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

Linearized theory(Weak Field)

$$g^{\mu\nu} = h^{\mu\nu} + \eta^{\mu\nu}$$ Where $|h^{\mu\nu}| << 1$ (linear terms)

where $h^{\mu\nu}$ is a tensor describing the variations induced in the spacetime metric and describes the propagation of ripples in spacetime curvature, i.e., the gravitational waves.

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h$$ where $h = g_{\mu\nu}h^{\mu\nu}$

WAVE EQUATION $$\Box\bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4}T^{\mu\nu}.$$

Which reduces in vacuum to $$\partial_\lambda\partial^\lambda\bar{h}^{\mu\nu} = 0$$

HILBERT'S GAUGE CONDITION: $$\partial_\mu\bar{h}^{\mu\nu} = 0$$

Generalized solution (Plane Wave)

$$h^{\mu\nu} = A^{\mu\nu} e^{ik_\alpha x^\alpha}$$

Where $A^{\mu\nu}$ is a constant symmetric tensor called Polarization Tensor.

$$k^\mu k_\mu = c^2 k_0^2 - k_x^2 - k_y^2 - k_z^2 = 0$$

that kµ is a lightlike or null vector, i.e., the wave propagates on the light-cone. Wave travels at ‘c’.

The gauge condition implies that the amplitude and wave vector are orthogonal i.e.

$$A^{\mu\nu} k_\nu = 0$$

Transverse Traceless(TT) Gauge

1. Traceless Amplitude: $A^{\mu}_{\mu} = \eta^{\mu\nu} A_{\mu\nu} = 0$
2. Transverse: in this gauge the metric perturbation is entirely transverse to the direction of propagation of the gravitational wave. For a wave travelling in z direction; we have $A_{\alpha z} = 0$ for all
3. The above conditions imply the TT gauge, in which $\bar{h}_{\alpha\beta} = h_{\alpha\beta}$
4. The TT gauge leaves only two independent wave amplitudes out of the original 10. Consider a wave moving in z direction $k_z = k; k_x = k_y = 0$. Aμv has only two independent components means that a gravitational wave is completely described by two dimensionless amplitudes h_+ and. $h_\times$

$$A^{\mu\nu} = h_+ \epsilon_+^{\mu\nu} + h_\times \epsilon_\times^{\mu\nu}$$

Where $\epsilon_+^{\mu\nu}$ and are $\epsilon_\times^{\mu\nu}$ the so-called unit polarization tensor.

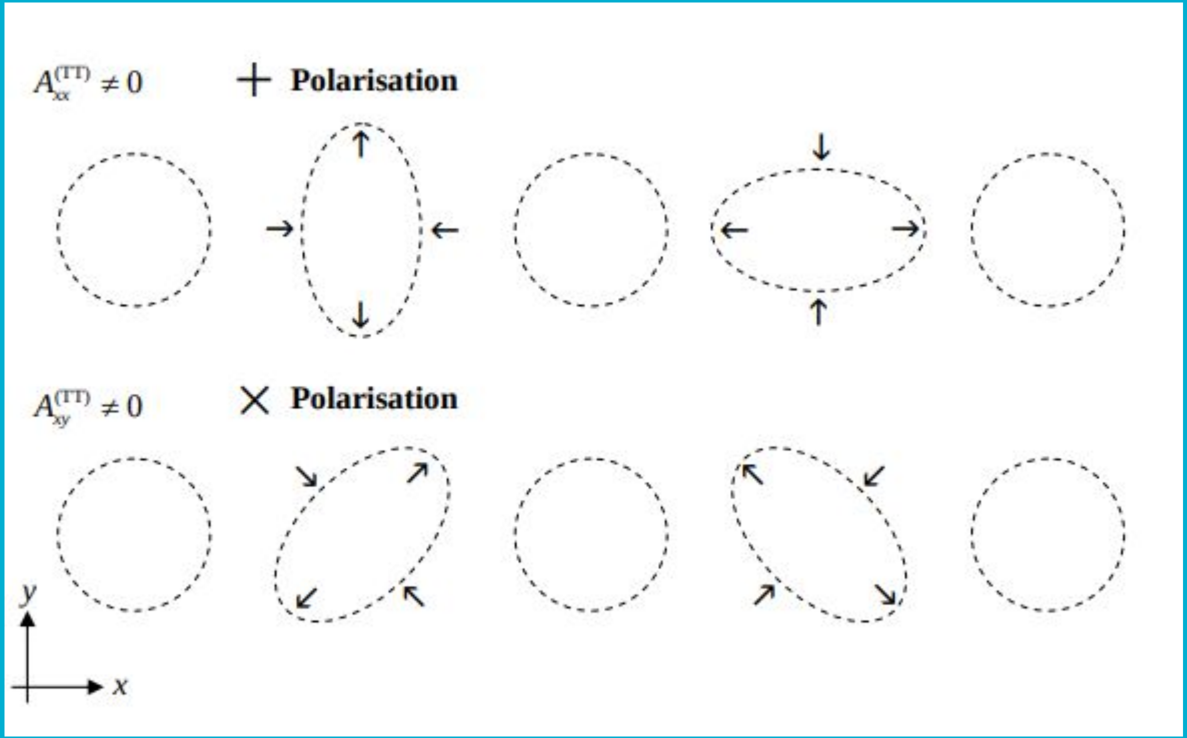
Consider the geodesic deviation of two particles – one at the origin and the other initially at coordinates x = cos θ , y = sin θ and z = 0 , i.e. in the x-y plane – as a gravitational wave propagates in the z-direction. ξ^{α} is the vector which connects the two particles.

$$\xi^x = \epsilon\cos\theta + \frac{1}{2}\epsilon\cos\theta\, A^{(\mathrm{TT})}_{xx}\cos\omega t + \frac{1}{2}\epsilon\sin\theta\, A^{(\mathrm{TT})}_{xy}\cos\omega t$$

$$\xi^y = \epsilon\sin\theta + \frac{1}{2}\epsilon\cos\theta\, A^{(\mathrm{TT})}_{xy}\cos\omega t - \frac{1}{2}\epsilon\sin\theta\, A^{(\mathrm{TT})}_{xx}\cos\omega t$$

Vary θ between 0 and 2π, so that an initially circular ring of test particles in the x-y plane, initially equidistant from the origin is formed.

Here, $A^{xx} = h_{+}$ $\quad$ $A^{xy} = h_{\times}$



The ring of test particles is shown for five different phases in the oscillation of the gravitational wave: π/2, π, 3π/2, 2π and 5π/2 .

a gravitational wave is invariant under a rotation of 180° about its direction of propagation (in this case, the z-axis)

EM Wave-invariant under rotation of 360°; neutrino invariant under 720°

In the quantum mechanical description of massless particles, the wavefunction-invariant under rotations of 360°/s, where s is the spin of the particle. The photon is a spin-1 particle and the graviton is a spin-2 particle.

Energy flux carried by GW

- Gravitational waves carry energy and cause a deformation of spacetime
- stress-energy carried by gravitational waves cannot be localized within a wavelength; extends upto several wavelengths

$$t_{\mu\nu}^{GW} = \frac{1}{32\pi}\left\langle \left(\partial_\mu h_{ij}^{TT}\right)\left(\partial_\nu h_{ij}^{TT}\right)\right\rangle.$$

- For a plane wave propagating in the z direction,we have

$$t_{00}^{GW} = \frac{t_{zz}^{GW}}{c^2} = -\frac{t_{0z}^{GW}}{c} = \frac{1}{32\pi}\frac{c^2}{G}\omega^2\left(h_+^2 + h_\times^2\right),$$

Where t_{00}^{GW} is the energy density, t_{zz}^{GW} is the momentum flux and t_{0z}^{GW} the energy flow along the z direction per unit area and unit time.

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- The energy flux has all the properties one would anticipate by analogy with electromagnetic wave:
- it is conserved (the amplitude dies out as 1/r, the flux as 1/r^2),
- it can be absorbed by detectors, and
- it can generate curvature like any other energy source in Einstein’s formulation of relativity.

Cosmic Gravitational wave background

- The gravitational wave background (also GWB and stochastic background) is a random gravitational wave signal produced by many weak, independent, and unresolved sources
- The emission of gravitational waves from astrophysical sources can create a stochastic background of gravitational waves.
- The gravitational radiation background anticipated by theorists was produced at Planck times, i.e.,10^{-32} sec or earlier after the Big Bang. Such gravitational waves have travelled almost unimpeded through the universe since they were generated.

THANK YOU

Another way of understanding the effects of gravitational waves is to study the tidal force field lines. In the TT gauge the equation of the geodesic deviation (2.16) takes the simple form

$$\frac{d^2\xi^k}{dt^2} \approx \frac{1}{2}\frac{d^2h_{jk}^{TT}}{dt^2}\xi^j \tag{2.21}$$

and the corresponding tidal force is

$$f^k \approx \frac{m}{2}\frac{d^2h_{jk}^{TT}}{dt^2}\xi^j \tag{2.22}$$

Properties of GW

- Travel almost unimpeded-change in amplitude only due to redshifts (cosmological or doppler)
- Strong GW, are emitted from regions of spacetime where gravity is very strong (black holes) and the velocities of the bulk motions of matter are near to ‘c’. EM waves won’t have much significance; but GW can be used.

in the Newtonian limit

$$R^{TT}_{j0k0} \approx \frac{\partial^2 \Phi}{\partial x^j \partial x^k},$$

$$\epsilon_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \epsilon_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R_{\kappa\lambda\mu\nu} = \frac{1}{2}\left(\partial_{\nu\kappa} h_{\lambda\mu} + \partial_{\lambda\mu} h_{\kappa\nu} - \partial_{\kappa\mu} h_{\lambda\nu} - \partial_{\lambda\nu} h_{\kappa\mu}\right),$$

which is considerably simplified by choosing the TT gauge:

$$R^{TT}_{j0k0} = -\frac{1}{2}\frac{\partial^2}{\partial t^2} h^{TT}_{jk}, \quad j, k = 1, 2, 3.$$

$$A_{\mu\nu}^{(\mathrm{TT})} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx}^{(\mathrm{TT})} & A_{xy}^{(\mathrm{TT})} & 0 \\ 0 & A_{xy}^{(\mathrm{TT})} & -A_{xx}^{(\mathrm{TT})} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{h}^{\mu\nu} = \frac{4G}{c^4} \int \frac{[T^{\mu\nu}]}{|\mathbf{r} - \mathbf{r}'|} d^3 r',$$

consider two freely falling particles hit by a gravitational wave travelling along the z-direction, with the (+) polarization present

$$h^{\mu\nu} = h_+ \epsilon^{\mu\nu} \cos[\omega(t - z)].$$

$$\frac{\xi^x}{\xi^x_0} = 1 - \frac{1}{2} h_+ \cos[\omega(t - z)] \quad \text{or} \quad \delta\xi^x = \xi^x - \xi^x_0 = -\frac{1}{2} h_+ \cos[\omega(t - z)]\xi^x_0$$

implies that the relative distance x between the two particles will oscillate

with frequency !.

$$\frac{\xi^y}{\xi^y_0} = 1 + \frac{1}{2} h_+ \cos[\omega(t - z)] \quad \text{or} \quad \delta\xi^y = \xi^y - \xi^y_0 = \frac{1}{2} h_+ \cos[\omega(t - z)]\xi^y_0$$