# Interim Project Report — Dynamic Delta Hedging

**Course:** ISyE 6767 Systems Computational Finance  
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## 1) Problem & Models

**Goal.** Replicate (hedge) a short European call option using **daily delta rebalancing** and assess performance:

1. **Model world (Task 1):** GBM price dynamics consistent with Black–Scholes(B–S).
2. **Market world (Task 2):** Historical GOOG prices, option quotes, and daily risk-free rates.

### Black–Scholes pricing (call)

* **Price:**  
  C = S \* N(d1) − K \* exp (−r \* τ) \* N(d2)
* **Definitions:**  
  d1 = [ ln (S / K) + (r + 0.5 \* σ²) \* τ] / [ σ \* sqrt(τ)]  
  d2 = d1 − σ \* sqrt(τ)
* **Delta (call):**  
  Δ = N(d1)

where:  
S = stock price, K = strike, r = risk-free rate, τ = time to maturity (in years),  
σ = volatility, N (·) = standard normal CDF.

### Daily hedging recursion

* Bank at start:  
  B0 = V0 − Δ0 \* S0
* For day i ≥ 1:  
  Bi = Δ(i−1) \* Si + B(i−1) \* exp (r(i−1) \* Δt) − Δi \* Si
* Cumulative hedging error:  
  HEi = Δ(i−1) \* Si + B(i−1) \* exp (r(i−1) \* Δt) − Vi

### P&L series (short call at t0)

* Unhedged:  
  PNLi = V0 − Vi
* With hedge:  
  PNL\_with\_hedge\_i = HEi

### Implied volatility (Task 2)

For each trading day, solve for σ such that the Black–Scholes price equals the market mid:  
C\_BS (S, K, r, τ, σ) = V\_mkt.  
Use **bisection** on σ ∈ [1e−6, 5.0] with tolerance 1e−7.

## 2) Implementation Structure (C++17, g++-15)

* **CSV I/O**: tolerant reader/writer (quoted fields, flexible headers).
* **Date utilities**: weekday counting (Mon–Fri) for business days; intersection across files.
* **Normal utils**: standard normal PDF/CDF (uses erfc internally).
* **Black–Scholes**: callPrice, deltaCall, d1, vegaBS.
* **Implied Volatility**: impliedVolBisection (robust, intrinsic-value check).
* **Task 1 Simulator**: GBM Euler step, B–S price/delta along path, daily hedge, record terminal HE\_T.
* **Task 2 Hedger**: aligns S, r, and option mid for (K=500, expiry above), computes daily IV, delta, PNL, and HE.
* **CLI**: simulate, hedge-real, and --self-test.

## 3) Unit Tests

1. Black–Scholes price regression vs a known reference (tolerance ~0.3).
2. Implied-vol round-trip: price → IV → price (abs error < 1e−6).
3. Delta monotonicity: Δ increases with S (other parameters fixed).

## 4) Results

### 4.1 Task 1 — Model World

**Parameters:** S0 = 100, T = 0.4, μ = 0.05, σ = 0.24, r = 0.025, N = 100.  
**Paths:** 1000 simulated; plot any 100.

**Figure 1. Simulated GBM stock price paths (100 shown)**  
A graph of different colored lines

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**Figure 2. Distribution of terminal hedging errors HE\_T (1000 paths)**  
**A graph of a distribution of hedging error

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**Summary:**  
• count: 1000  
• mean: ~0 (expected under B–S with daily re-hedge)  
• std: ~0.56 (discretization + path variance)  
• distribution roughly centered at zero

**Interpretation.** In a B–S-consistent world, delta hedging is approximately unbiased; daily (discrete) re-hedging introduces variance in HE\_T.

### 4.2 Task 2 — Market World (GOOG)

**Window:** 2011-07-05 to 2011-07-29 (weekdays in common across data).  
**Contract:** Call, K = 500, expiry = 2011-09-17.  
**Daily steps:** compute τ from business-days-to-expiry/252; solve σ\_imp; compute Δ; apply hedge recursion.

**Output:** strict out/result.csv with columns  
date, S, V, iv, delta, PNL, PNL\_with\_hedge.

**Observed behavior (typical):**  
• PNL\_with\_hedge has smaller magnitude than unhedged PNL → hedge reduces exposure.  
• IV varies through time → volatility misspecification contributes to tracking error.  
• Residual deviations from perfect replication arise from discrete hedging, time-varying vol, and microstructure.

## 5) Discussion & Conclusions

* Task 1 (model world): daily delta hedging → near-zero mean hedging error; dispersion due to discrete rebalancing and volatility level.
* Task 2 (market world): hedging generally reduces risk versus an unhedged short call, but cannot eliminate P&L variance (discretization, changing IV, non-B–S effects).
* Practical takeaway: daily delta hedging is effective first-order risk control; consider higher re-hedge frequency, transaction-cost-aware rules, and extended models (local/stochastic vol), or gamma-vega hedges for deeper neutrality.

## 6) Reproducibility

**Build**  
make build

**Task 1**  
make run-task1  
python plot\_task1.py

**Task 2**  
make run-task2

**Files referenced**

* out/plots/plots\_task1\_paths.png
* out/plots/plots\_task1\_he\_hist.png
* out/result.csv

## 6) Unit test

**Build**  
make test

**Tests Implemented**

// Test implied volatility: compute IV from a known call price and compare

{

double S=100, K=100, r=0.02, tau=0.5, sigma=0.2;

double C = callPrice(S,K,r,tau,sigma);

double iv = impliedVolBisection(S,K,r,tau,C);

EXPECT\_NEAR(iv, sigma, 1e-3);

}

// Test delta: check monotonicity across strikes

{

double r=0.02, tau=0.5, sigma=0.2;

double d\_lo = deltaCall(90,100,r,tau,sigma);

double d\_hi = deltaCall(110,100,r,tau,sigma);

EXPECT\_TRUE(d\_hi > d\_lo);

}

std::cout<<"[ALL UNIT TESTS DONE]\n";

**Test Output:**

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