IMPES eqn —

$$\cancel{-\nabla\left[\frac{k k_{rw}}{B_w \mu_w}\left(\frac{\partial P_w}{\quad}\right.\right.}$$

$$-\nabla\left[\frac{k k_{rw}}{\mu_w B_w}\nabla\left(P_w - \gamma_w Z\right)\right] = \frac{\partial}{\partial t}\left(\frac{S_w \phi}{B_w}\right) + q_w$$

$$-\nabla\left[\frac{k k_{ro}}{\mu_o B_o}\nabla\left(P_o - \gamma_o Z\right)\right] = \frac{\partial}{\partial t}\left(\frac{S_o \phi}{B_o}\right) + q_o$$

$$P_{cow} = P_o - P_w$$

$$S_o + S_w = 1$$

Discretizing above eqns & eliminating $\left(S_{w_n}^{n+1} - S_{w_n}^{n}\right)$ we get,

$$\sum\left(B_{on}^{n+1} T_{ol}^{n+1} + T_{wl}^{n+1} B_{wn}^{n+1}\right) P_{ol}^{n+1}$$

$$- \left\{\sum\left(B_{on}^{n+1} T_{ol}^{n+1} + T_{wl}^{n+1} B_{wn}^{n+1}\right) + B_{on}^{n+1} C_{opn} + B_{wn}^{n+1} C_{wpn}\right\} P_{on}^{n+1}$$

$$= \sum\left(B_{on}^{n+1} T_{ol}^{n+1} \gamma_{ol}^{n} + B_{wn}^{n+1} T_{wl}^{n+1} \gamma_{wl}^{n}\right) \times \left(Z_l - Z_n\right)$$

$$+ \sum B_{wn}^{n+1} T_w^{n+1}\left(P_{cowl}^{n+1} - P_{cown}^{n+1}\right)$$

$$+\sum \left(B_{on}^{n+1} C_{opn} + B_{wn}^{n+1} C_{wpn} \right) \times P_{on}^{n}$$

$$- \left(B_{on}^{n+1} Q_o + B_{wn}^{n+1} Q_w \right)$$

And,

$$S_{wn}^{n+1} = S_{wn}^{n} + \frac{1}{C_{wwn}} \left\{ \sum T_w^{n+1} \times \left[\left(P_{ol}^{n+1} - P_{on}^{n+1} \right) - \left(P_{cowl}^{n+1} - P_{cown}^{n+1} \right) - \gamma_w^n \left(Z_l - Z_n \right) \right] + Q_w - C_{wpn} \left(P_{on}^{n+1} - P_{on}^{n} \right) \right\}$$

where, $C_{opn} = \frac{V_{bn}}{\alpha_c \Delta t} \left\{ \left(1 - S_{wn}^{n} \right) \times \left[\phi_n^{n+1} \left(\frac{1}{B_{on}} \right)' + \frac{1}{B_{on}^{n}} \left(\phi_n \right)' \right] \right\}$

$$C_{own} = \frac{V_{bn}}{\alpha_c \Delta t} \left[- \left(\frac{\phi}{B_o} \right)_n^{n+1} \right]$$

$$C_{wpn} = \frac{V_b^n}{\alpha_c \Delta t}\left\{ S_{wn}^n \left[\phi_n^{n+1} \left(\frac{1}{B_{wn}}\right)' + \frac{1}{B_{wn}^n} (\phi_n)' \right] \right\}$$

$$C_{wwn} = \frac{V_{bn}}{\alpha_c \Delta t}\left[\left(\frac{\phi}{B_w}\right)_n^{n+1} \right]$$

$$\left(\frac{1}{B_{On}}\right)' = \left(\frac{1}{B_{On}^{n+1}} - \frac{1}{B_{On}^n}\right) \Big/ \left(P_{On}^{n+1} - P_{On}^n\right)$$

$$\left(\frac{1}{B_{wn}}\right)' = \left(\frac{1}{B_{wn}^{n+1}} - \frac{1}{B_{wn}^n}\right) \Big/ \left(P_{On}^{n+1} - P_{On}^n\right)$$

$$(\phi_n)' = \frac{\phi_n^{n+1} - \phi_n^n}{P_{On}^{n+1} - P_{On}^n}$$

Here, subscript 'n' means block & 'l' means neighbouring blocks.

T_x [stb/(day-psi)] (Transmissibility)

$$= \beta_c \frac{K_x A_x k_r}{\mu B \Delta x}$$

β_c = conversion factor = 0.001127

A_x = area = ft^2

μ = cp

K_x = md

Δx = ft

Q = stb

α_c = 5.615 (conversion factor)

Z = ft

V_b = volume of block = ft^3

γ = fluid gravity (psi/ft)

T is calculated at block faces & is the harmonic mean.

Fluid properties (μ, B, ρ) are considered average

Relative permeability is found by upwinding;

for bhp control,

$$Q = J\left(P_{on}^{n+1} - P_{wf}\right)$$

$$J = \frac{2\pi k_H k_r h}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) + S\right]}$$

k_H = geometric mean

r_e = from Pearson's coefficient

In case, well passes through multiple blocks, P_{wf} is assumed same in all blocks & individual Q is found iteratively.

Using matrix notation, the same can be written as —

$$\left[B_o \cdot T_o + B_w \cdot T_w - B_{o1} \cdot C_1 - B_{w1} \cdot C_2\right] \times p^{n+1}$$

$$= \left[B_o \cdot T_o \cdot Y_o + B_w \cdot T_w \cdot Y_w\right] \times Z$$

$$+ B_w \cdot T_w \cdot P_{cow}$$

$$+ \left[B_{o1} \cdot C_1 + B_{w1} \cdot C_2\right] \times p^n$$

$$- [B_{o1} \cdot Q_o + B_{w1} \cdot Q_w - B_{w1} \cdot J_{w1} \cdot P_{cow}]$$

$$- [B_{o1} \cdot J_{o1} + B_{w1} \cdot J_{w1}] \times p^{n+1}$$

$$\longrightarrow (i)$$

$$S_w^{n+1} = S_w^n + C_3^{-1} \times [T_w \times (p^{n+1} - P_{cow}) - T_w \cdot \gamma_w \times Z + Q_w - C_2 \times (p^{n+1} - p^n)]$$

where,

$$Q_w = Q_w + J_w (P_o - P_{cow} - P_{wf})$$

Boundary condn,

Dirichilet - $P_B = \frac{P_o + P_1}{2}$

$$\therefore P_o = 2P_B - P_1$$

pressure of ~~block~~ neighbour of 1st block

Neumann,

$$q = \frac{2\pi k k_r A}{\mu B}\left(\frac{p_0 - p_1}{\Delta x}\right)$$

for $q = 0$, $p_0 = p_1$

$$p^{n+1} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \end{bmatrix} (n \times 1)$$

$$p^n = (n \times 1)$$

$$B_o = \begin{bmatrix} B_2 & -B_2 & B_2 & \cdots & \\ 0 & B_2 & B_2 & -B_2 & \cdots \\ & \cdots & & & \end{bmatrix}$$

$(n \times n)$ sparse

$$B_w = (n \times n)$$

$$T_o = \begin{bmatrix} T_1 & -(T_2 + T_3 + T_4) & T_3 + \cdots \\ & \cdots & \end{bmatrix}$$

$(n \times n)$ sparse

$T_w = (n \times n)$ sparse

$B_{o1} = (n \times n)$ identity

$$\begin{bmatrix} B_1 & 0 & 0 & \cdots \\ 0 & B_2 & 0 & \cdots \\ & \cdots & & \end{bmatrix}$$

$B_{w1} = (n \times n)$ identity

$$C_1 = C_{opn} = \begin{bmatrix} C_{11} & 0 & 0 & \cdots \\ 0 & C_{12} & 0 & 0 \\ & & \vdots & \end{bmatrix}$$

$(n \times n)$ identity

$C_2 = C_{wpn} = (n \times n)$ identity

$$Y_0 = \begin{bmatrix} Y_{01} - x Y_{02} & Y_{03} & 0 & 0 \\ 0 & Y_{02} & \cdots & \end{bmatrix}$$

$(n \times n)$ sparse

$Y_w = (n \times n)$ sparse

$$Z = (n \times 1) = \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \end{bmatrix}$$

$P_{cow} = (n \times 1)$

$$Q_0 = (n \times 1) = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ -J_{pwf} \\ \vdots \end{bmatrix}$$

$Q_w = (n \times 1)$

$$J_{01} = \begin{bmatrix} 0 & J_{12} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & J_{13} & 0 & \cdots \end{bmatrix}$$

$(n \times n)$ sparse

$J_{w1} = (n \times n)$ sparse

Only for the block where bhp control

$$S_w^{n+1} = \begin{bmatrix} S_{w1} \\ S_{w2} \\ \vdots \end{bmatrix} (n \times 1)$$

$$S_w^n = (n \times 1)$$

$$C_3 = \begin{bmatrix} C_{ww1} & 0 & 0 & \cdots \\ 0 & C_{ww2} & 0 & 0 \\ & \cdots & & \end{bmatrix}$$

$(n \times n)$ identity

$B_o, B_w, T_o, T_w, Y_o, Y_w$ ~~B~~

— for 1D tridiagonal sparse matrix

for 2D pentagonal sparse matrix

for 3D heptagonal sparse matrix