UIN-722007512
Name-Soumyadipta Sengupta

Report

1-D convection –Diffusion equation

$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} = 0$……(1)

$C(x,0) = 0$……(2)

$C(0,t) = 1$……(3)

$\frac{\partial C}{\partial x} = \frac{\overline{C_{\iota+\frac{1}{2}}}}{\Delta x} - \frac{\overline{C_{\iota-\frac{1}{2}}}}{\Delta x}$……(4)

$\lambda = \frac{dt}{dx}$

$\overline{C_{\iota+\frac{1}{2}}}, \overline{C_{\iota-\frac{1}{2}}}$ *are the time averaged concentrations at* $i+\frac{1}{2}$ *and* $i-\frac{1}{2}$ *respectively.*

1) Upstream weighting

$$\overline{C_{\iota+\frac{1}{2}}} = C_i^{n-1}$$

$$\overline{C_{\iota-\frac{1}{2}}} = C_{i-1}^{n-1}$$

$$C_i^n = (1-\lambda) * C_i^{n-1} + (\lambda) * C_{i-1}^{n-1}$$

$$C_1^n = (1-\lambda) * C_1^{n-1} + (\lambda) * 1, \qquad since\ C_0^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

2) Midpoint Weighting

$$\overline{C_{\iota+\frac{1}{2}}} = C_i^{n-1} + (\frac{C_{i+1}^{n-1} - C_i^{n-1}}{2})(1-\lambda)$$

$$\overline{C_{\iota-\frac{1}{2}}} = C_{i-1}^{n-1} + (\frac{C_i^{n-1} - C_{i-1}^{n-1}}{2})(1-\lambda)$$

$$C_i^n = (1-\lambda^2) * C_i^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * C_{i+1}^{n-1} + \left(\lambda * \frac{1+\lambda}{2}\right) * C_{i-1}^{n-1}$$

$$C_1^n = (1-\lambda^2) * C_1^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * C_2^{n-1} + \left(\lambda * \frac{1+\lambda}{2}\right) * 1,$$

$$since\ C_0^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

3) 2-Point Upstream weighting

$$\overline{C_{\iota+\frac{1}{2}}} = C_i^{n-1} + (\frac{C_i^{n-1} - C_{i-1}^{n-1}}{2})(1-\lambda)$$

$$\overline{C_{\iota-\frac{1}{2}}} = C_{i-1}^{n-1} + (\frac{C_{i-1}^{n-1} - C_{i-2}^{n-1}}{2})(1-\lambda)$$

UIN-722007512
Name-Soumyadipta Sengupta

$$C_i^n = (1-\lambda) * C_i^{n-1} + \lambda * C_{i-1}^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * \left(C_i^{n-1} - 2 * C_{i-1}^{n-1} + C_{i-2}^{n-1}\right)$$

$$C_1^n = (1-\lambda) * C_1^{n-1} + \lambda * 1 - \left(\lambda * \frac{1-\lambda}{2}\right) * (C_1^{n-1} - 2 * 1 + 1)$$

$$since\ C_0^{n-1} = 1\ \&\ C_{-1}^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

$$C_2^n = (1-\lambda) * C_2^{n-1} + \lambda * C_1^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * (C_2^{n-1} - 2 * C_1^{n-1} + 1)$$

$$since\ C_0^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

4) Leonard scheme

$$\overline{C_{i+\frac{1}{2}}} = \left(1 + \frac{(1-\lambda)(2\lambda-1)}{6}\right) * C_i^{n-1} + \frac{(1-\lambda)(2-\lambda)}{6} * C_{i+1}^{n-1} - \frac{(1-\lambda)(\lambda+1)}{6} * C_{i-1}^{n}$$
$$\overline{C_{i-\frac{1}{2}}} = \left(1 + \frac{(1-\lambda)(2\lambda-1)}{6}\right) * C_{i-1}^{n-1} + \frac{(1-\lambda)(2-\lambda)}{6} * C_i^{n-1} - \frac{(1-\lambda)(\lambda+1)}{6} * C_{i-2}^{n}$$

$$C_i^n = \left(1 - \lambda + \frac{\lambda*(1-\lambda)^2}{2}\right) * C_i^{n-1} + \left(1 + \lambda * \frac{1-\lambda}{2}\right) * \lambda * C_{i-1}^{n-1} + \lambda * (\lambda - 2) * \frac{(1-\lambda)}{6}) * C_{i+1}^{n-1} + (\lambda * (\lambda - 1) * \frac{(1+\lambda)}{6}) * C_{i-2}^{n-1}$$

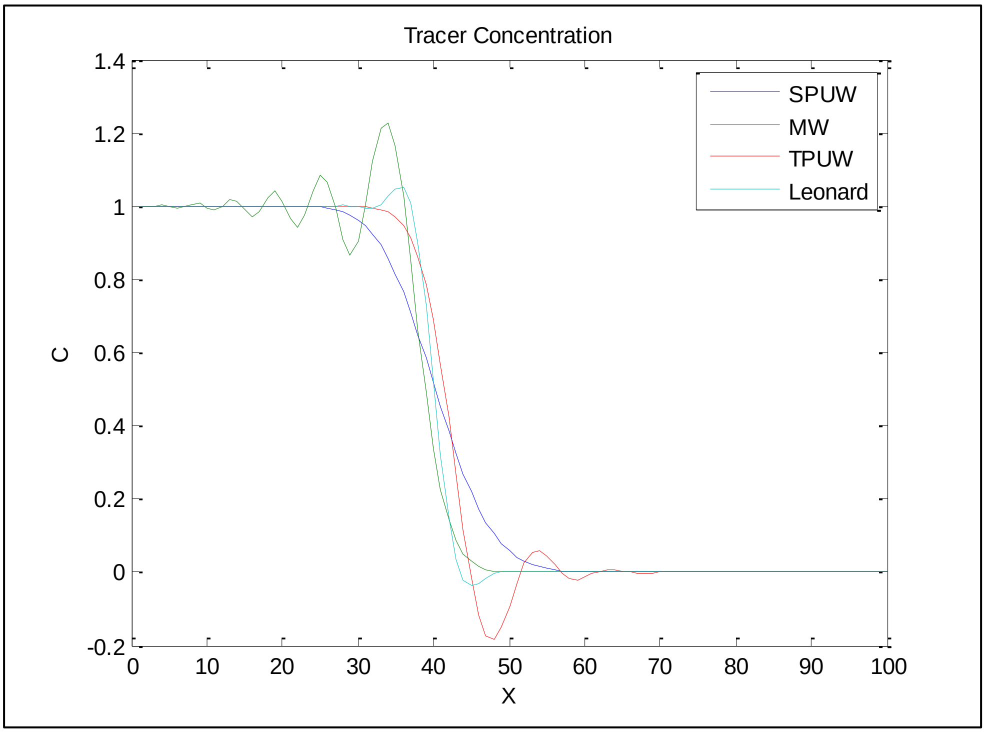
$$C_1^n = \left(1 - \lambda + \frac{\lambda*(1-\lambda)^2}{2}\right) * C_1^{n-1} + \left(1 + \lambda * \frac{1-\lambda}{2}\right) * \lambda * 1 + \lambda * (\lambda - 2) * \frac{(1-\lambda)}{6}) * C_2^{n-1} + (\lambda * (\lambda - 1) * \frac{(1+\lambda)}{6}) * 1$$

$$since\ C_0^{n-1} = 1\ \&\ C_{-1}^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

$$C_2^n = \left(1 - \lambda + \frac{\lambda*(1-\lambda)^2}{2}\right) * C_2^{n-1} + \left(1 + \lambda * \frac{1-\lambda}{2}\right) * \lambda * C_1^{n-1} + \lambda * (\lambda - 2) * \frac{(1-\lambda)}{6}) * C_3^{n-1} + (\lambda * (\lambda - 1) * \frac{(1+\lambda)}{6}) * 1$$

$$since\ C_0^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

UIN-722007512
Name-Soumyadipta Sengupta



We can see oscillations before the front in the MW scheme. There are oscillations after the front in the TPUW scheme. The Leonard scheme has oscillations both after and before the front but is more suppressed as compared to both the MW and TPUW schemes.

Tracer concentration with TVD

$$\overline{C_{i+\frac{1}{2}}} = C_i^n + \phi(r_i) * \frac{(C_{i+1}^n - C_i^n)}{2} * (1-\lambda)$$

$$\overline{C_{i-\frac{1}{2}}} = C_{i-1}^n + \phi(r_{i-1}) * \frac{(C_i^n - C_{i-1}^n)}{2} * (1-\lambda)$$

$$\phi(r_i) = \frac{(C_i^n - C_{i-1}^n)}{(C_{i+1}^n - C_i^n)}$$

$$\phi(r_{i-1}) = \frac{(C_{i-1}^n - C_{i-2}^n)}{(C_i^n - C_{i-1}^n)}$$

$$C_i^n = C_i^{n-1} + \lambda * \left(\left(\phi(r_i) + \phi(r_{i-1})\right) * \frac{1-\lambda}{2} - 1\right) * C_i^{n-1} - \lambda * \left(\frac{1-\lambda}{2}\right) * \phi(r_i) * C_{i+1}^{n-1} - (\lambda - \phi(r_{i-1}) * \lambda * \left(\frac{1-\lambda}{2}\right)) * C_{i-1}^{n-1}$$

The boundary conditions for i=1 are imposed in the same way as before, i.e. $C_0^{n-1} = 1$

The following conditions apply for the flux limiters

1) Upstream weighting

$$\phi(r) = 0$$

2) Mid –Point

UIN-722007512
Name-Soumyadipta Sengupta

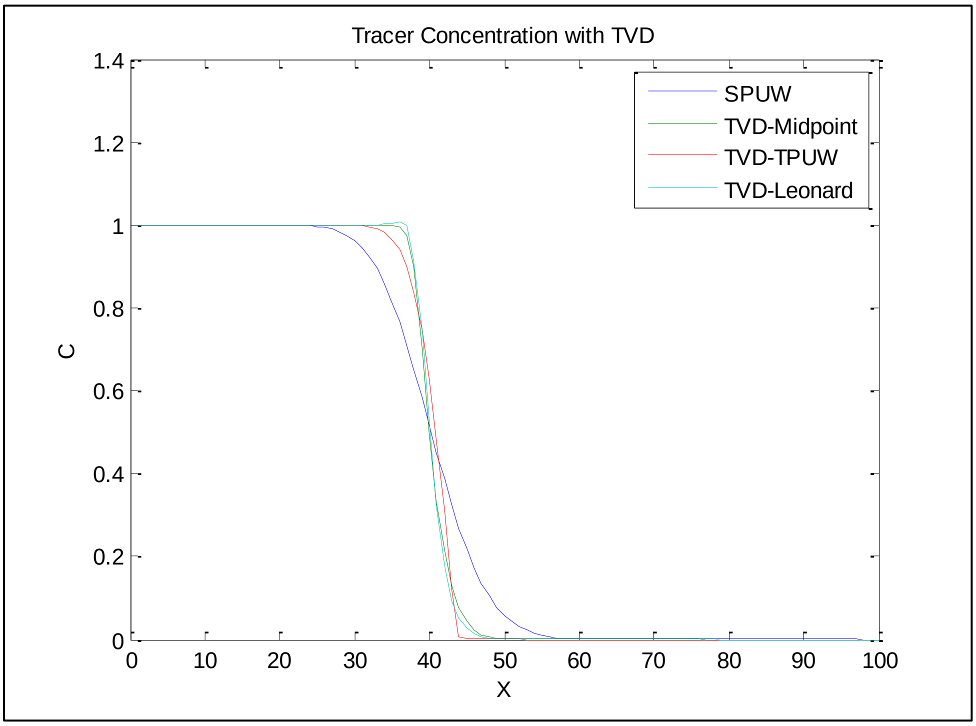
$\phi(r) = \max(0, \min(2r, 1))$ & $\phi(r) = 0$ for $r \leq 0$

3) 2-Point Upstream

$\phi(r) = \max(0, \min(r, 2))$ & $\phi(r) = 0$ for $r \leq 0$

4) Leonard Scheme

$\frac{\phi(r)}{r} \geq 0$ & $\frac{\phi(r)}{r} \leq 2$ & $\phi(r) \leq 2$ for $r > 0$ & $\phi(r) = 0$ for $r \leq 0$



Buckley –Leverett equation

$\frac{\partial S_w}{\partial t} + \frac{\partial f_w}{\partial \tau} = 0$ .... (1)

$S_w(\tau, 0) = S_{wc}$ .... (2)

$f_w(\tau, 0) = 0$ .... (3)

$f_w(0, t) = 1$ .... (4)

$\frac{\partial f_w}{\partial \tau} = \frac{\overline{f_{w_{i+\frac{1}{2}}}}}{\Delta\tau} - \frac{\overline{f_{w_{i-\frac{1}{2}}}}}{\Delta\tau}$ .... (5)

$\overline{f_{w_{i+\frac{1}{2}}}}, \overline{f_{w_{i-\frac{1}{2}}}}$ $are\ the\ time\ averaged\ fractional\ flows\ at\ i + \frac{1}{2} and\ i - \frac{1}{2} respectively.$

UIN-722007512
Name-Soumyadipta Sengupta

$S = \frac{S_{w_i}^{n} - S_{wc}}{1 - S_{wc} - S_{or}}$

$f_{w_i}^{n} = \frac{M * S^2}{M * S^2 + (1-S)^2}$

$S_{wc} = 0.2$

$S_{or} = 0.2$

1) Upstream weighting

$$\overline{f_{w_{i+\frac{1}{2}}}} = f_{w_i}^{n-1}$$

$$\overline{f_{w_{i-\frac{1}{2}}}} = f_{w_{i-1}}^{n-1}$$

$$S_{w_i}^{n} = S_{w_i}^{n-1} + (-\lambda) * f_{w_i}^{n-1} + (\lambda) * f_{w_{i-1}}^{n-1}$$
$$for\ i = 1$$
$$S_{w_1}^{n} = S_{w_1}^{n-1} + (-\lambda) * f_{w_1}^{n-1} + (\lambda) * 1,$$
$$since\ f_{w_0}^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

2) Midpoint Weighting

$$\overline{f_{w_{i+\frac{1}{2}}}} = f_{w_i}^{n-1} + \left(\frac{f_{w_{i+1}}^{n-1} - f_{w_i}^{n-1}}{2}\right)(1-\lambda)$$

$$\overline{f_{w_{i-\frac{1}{2}}}} = f_{w_{i-1}}^{n-1} + \left(\frac{f_{w_i}^{n-1} - f_{w_{i-1}}^{n-1}}{2}\right)(1-\lambda)$$

$$S_{w_i}^{n} = S_{w_i}^{n-1} + (-\lambda^2) * f_{w_i}^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * f_{w_{i+1}}^{n-1} + \left(\lambda * \frac{1+\lambda}{2}\right) * f_{w_{i-1}}^{n-1}$$

$$S_{w_1}^{n} = S_{w_1}^{n-1} + (-\lambda^2) * f_{w_1}^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * f_{w_2}^{n-1} + \left(\lambda * \frac{1+\lambda}{2}\right) * 1,$$

$$since\ f_{w_0}^{n-1} = 1\ from\ the\ given\ boundary\ conditions$$

3) 2-Point Upstream weighting

$$\overline{f_{w_{i+\frac{1}{2}}}} = f_{w_i}^{n-1} + \left(\frac{f_{w_i}^{n-1} - f_{w_{i-1}}^{n-1}}{2}\right)(1-\lambda)$$
$$\overline{f_{w_{i-\frac{1}{2}}}} = f_{w_{i-1}}^{n-1} + \left(\frac{f_{w_{i-1}}^{n-1} - f_{w_{i-2}}^{n-1}}{2}\right)(1-\lambda)$$

$$S_{w_i}^{n} = S_{w_i}^{n-1} + (-\lambda) * f_{w_i}^{n-1} + \lambda * f_{w_{i-1}}^{n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * \left(f_{w_i}^{n-1} - 2 * f_{w_{i-1}}^{n-1} + f_{w_{i-2}}^{n-1}\right)$$

$$S_{w_1}^{n} = S_{w_1}^{n-1} + (-\lambda) * f_{w_1}^{n-1} + \lambda * 1 - \left(\lambda * \frac{1-\lambda}{2}\right) * \left(f_{w_1}^{n-1} - 2 * 1 + 1\right)$$

UIN-722007512
Name-Soumyadipta Sengupta

$$since\ f_{w_0}^{\ n-1} = 1\ \&\ f_{w_{-1}}^{\ n-1} = 1\ from\ the\ given\ boundary\ conditions$$

$$S_{w_2}^{\ n} = S_{w_2}^{\ n-1} + (-\lambda) * f_{w_2}^{\ n-1} + \lambda * f_{w_1}^{\ n-1} - \left(\lambda * \frac{1-\lambda}{2}\right) * \left(f_{w_2}^{\ n-1} - 2 * f_{w_1}^{\ n-1} + 1\right)$$

$$since\ f_{w_0}^{\ n-1} = 1\ from\ the\ given\ boundary\ conditions$$

4) Leonard scheme

$$\overline{f_{w_{i+\frac{1}{2}}}} = \left(1 + \frac{(1-\lambda)(2\lambda-1)}{6}\right) * f_{w_i}^{\ n-1} + \frac{(1-\lambda)(2-\lambda)}{6} * f_{w_{i+1}}^{\ n-1} - \frac{(1-\lambda)(\lambda+1)}{6} * f_{w_{i-1}}^{\ n}$$
$$\overline{f_{w_{i-\frac{1}{2}}}} = \left(1 + \frac{(1-\lambda)(2\lambda-1)}{6}\right) * f_{w_{i-1}}^{\ n-1} + \frac{(1-\lambda)(2-\lambda)}{6} * f_{w_i}^{\ n-1} - \frac{(1-\lambda)(\lambda+1)}{6} * f_{w_{i-2}}^{\ n}$$

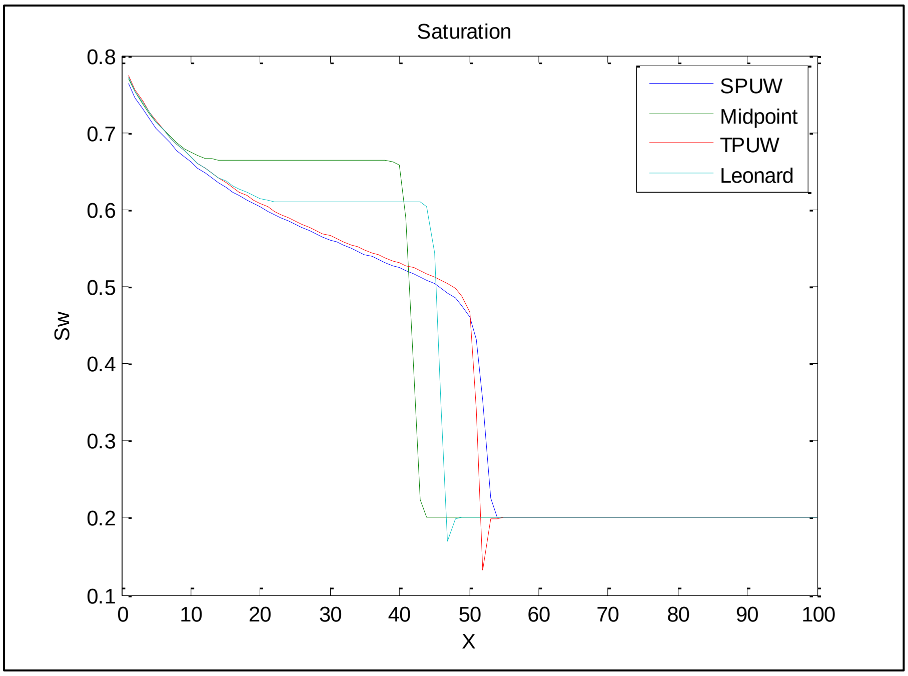
$$S_{w_i}^{\ n} = S_{w_i}^{\ n-1} + \left(-\lambda + \frac{\lambda*(1-\lambda)^2}{2}\right) * f_{w_i}^{\ n-1} + \left(1 + \lambda * \frac{1-\lambda}{2}\right) * \lambda * f_{w_{i-1}}^{\ n-1} + \lambda * (\lambda - 2) * \frac{(1-\lambda)}{6}) * f_{w_{i+1}}^{\ n-1} + (\lambda * (\lambda - 1) * \frac{(1+\lambda)}{6}) * f_{w_{i-2}}^{\ n-1}$$

$$S_{w_1}^{\ n} = S_{w_1}^{\ n-1} + \left(-\lambda + \frac{\lambda*(1-\lambda)^2}{2}\right) * f_{w_1}^{\ n-1} + \left(1 + \lambda * \frac{1-\lambda}{2}\right) * \lambda * 1 + \lambda * (\lambda - 2) * \frac{(1-\lambda)}{6}) * f_{w_2}^{\ n-1} + (\lambda * (\lambda - 1) * \frac{(1+\lambda)}{6}) * 1$$

$$since\ f_{w_0}^{\ n-1} = 1\ \&\ f_{w_{-1}}^{\ n-1} = 1\ from\ the\ given\ boundary\ conditions$$

$$S_{w_2}^{\ n} = S_{w_2}^{\ n-1} + \left(-\lambda + \frac{\lambda*(1-\lambda)^2}{2}\right) * f_{w_2}^{\ n-1} + \left(1 + \lambda * \frac{1-\lambda}{2}\right) * \lambda * f_{w_1}^{\ n-1} + \lambda * (\lambda - 2) * \frac{(1-\lambda)}{6}) * f_{w_3}^{\ n-1} + \quad (\lambda * (\lambda - 1) * \frac{(1+\lambda)}{6}) * 1$$

$$since\ f_{w_0}^{\ n-1} = 1\ from\ the\ given\ boundary\ conditions$$



We can see the mass balance errors in the MW and Leonard scheme. We can also see that there are oscillations occurring in the TPUW and Leonard schemes just after the front.

TVD schemes

$$\overline{f_{w_{i+\frac{1}{2}}}} = f_{w_i}^{\ n} + \phi(r_i) * \frac{\left(f_{w_{i+1}}^{\ n} - f_{w_i}^{\ n}\right)}{2} * (1-\lambda)$$

$$\overline{f_{w_{i-\frac{1}{2}}}} = f_{w_{i-1}}^{\ n} + \phi(r_{i-1}) * \frac{\left(f_{w_i}^{\ n} - f_{w_{i-1}}^{\ n}\right)}{2} * (1-\lambda)$$

$$\phi(r_i) = \frac{\left(f_{w_i}^{\ n} - f_{w_{i-1}}^{\ n}\right)}{\left(f_{w_{i+1}}^{\ n} - f_{w_i}^{\ n}\right)}$$

$$\phi(r_{i-1}) = \frac{\left(f_{w_{i-1}}^{\ n} - f_{w_{i-2}}^{\ n}\right)}{\left(f_{w_i}^{\ n} - f_{w_{i-1}}^{\ n}\right)}$$

$$S_{w_i}^{\ n} = S_{w_i}^{\ n-1} + \lambda * \left(\left(\phi(r_i) + \phi(r_{i-1})\right) * \frac{1-\lambda}{2} - 1\right) * f_{w_i}^{\ n-1} - \lambda * \left(\frac{1-\lambda}{2}\right) * \phi(r_i) * f_{w_{i+1}}^{\ n-1} - \left(\lambda - \phi(r_{i-1}) * \lambda * \left(\frac{1-\lambda}{2}\right)\right) * f_{w_{i-1}}^{\ n-1}$$

$$S = \frac{S_{w_i}^{\ n} - S_{wc}}{1 - S_{wc} - S_{or}}$$

UIN-722007512
Name-Soumyadipta Sengupta

$$f_{w_i}^{\,n} = \frac{M*S^2}{M*S^2+(1-S)^2}$$

The boundary conditions for i=1 are imposed in the same way as before, i.e. $f_{w_0}^{\,n-1} = 1$

The following conditions apply for the flux limiters

5) Upstream weighting

$$\phi(r) = 0$$

6) Mid –Point

$\phi(r) = \max(0, \min(2r, 1))$ & $\phi(r) = 0\ for\ r \le 0$

7) 2-Point Upstream

$\phi(r) = \max(0, \min(r, 2))$ & $\phi(r) = 0\ for\ r \le 0$

8) Leonard Scheme

$\frac{\phi(r)}{r} \ge 0$ & $\frac{\phi(r)}{r} \le 2$ & $\phi(r) \le 2\ for\ r > 0$ & $\phi(r) = 0\ for\ r \le 0$