

**This note is intended to clarify the relationship between elasticity of demand and revenue.**

The fact that I presented in class was:

1. If  $0 < E(p) < 1$ , then  $\frac{dR}{dp} > 0$  (the change in revenue is in the same direction as a (small) change in price).
2. If  $E(p) > 1$ , then  $\frac{dR}{dp} < 0$  (the change in revenue is in the opposite direction as a (small) change in price).

In this note, I will compute some examples to illustrate these two cases.

The examples also illustrate the general idea as to why the fact is true, and can be generalized to give a proof of the fact.

*Example 1.*  $0 < E(p) < 1$ :

Suppose that  $E(p) = \frac{1}{3}$ . (Remember that  $E(p) = -\frac{\% \Delta \text{ in } q}{\% \Delta \text{ in } p}$ ).

This means that if the price is raised by 3%, the quantity demanded will fall by 1%.

Let's look at the effect this has on revenue.

Remember that *revenue* = price \* quantity demanded =  $p * q$ .

If price rises by 3%, then the new price is  $1.03p$  (103% of the old price).

If the quantity demanded falls by 1%, then the new quantity demanded is  $0.99q$  (99% of the old quantity).

So, this means that the new revenue is new price \* new quantity demanded =  $1.03p * .99q = 1.0197 * pq$ . Therefore, the revenue has increased.

*Example 2.*  $E(p) > 1$ :

Suppose that  $E(p) = 3$ . (Remember that  $E(p) = -\frac{\% \Delta \text{ in } q}{\% \Delta \text{ in } p}$ ).

This means that if the price is raised by 1%, the quantity demanded will fall by 3%.

Let's look at the effect this has on revenue.

Remember that *revenue* = price \* quantity demanded =  $p * q$ .

If price rises by 1%, then the new price is  $1.01p$  (101% of the old price).

If the quantity demanded falls by 3%, then the new quantity demanded is  $0.97q$  (97% of the old quantity).

So, this means that the new revenue is new price \* new quantity demanded =  $1.01p * .97q = 0.9797 * pq$ . Therefore, the revenue has decreased.