Section 2.3, Question 13: Find all local extrema of $f(x) = 1 + 6x - x^2$ using the second derivative test.

Answer: To answer this question, we will need to use the second derivative test:

- If $f'(a) = 0$ and $f''(a) > 0$, then $f(x)$ has a local minimum at $x = a$.
- If $f'(a) = 0$ and $f''(a) < 0$, then $f(x)$ has a local maximum at $x = a$.

We will use the four steps we used in class today to solve question 9.

1. Find $f'(x)$ and set it equal to 0.

$$f(x) = 1 + 6x - x^2$$

$$f'(x) = \frac{d}{dx}[1 + 6x - x^2]$$

$$f'(x) = \frac{d}{dx}[1] + \frac{d}{dx}[6x] - \frac{d}{dx}[x^2]$$

$$f'(x) = 0 + 6 - 2x$$

$$f'(x) = 6 - 2x = 0$$

2. Solve for x.

$$6 - 2x = 0$$

$$x = 3$$

3. Find $f''(x)$.

$$f'(x) = 6 - 2x$$

$$f''(x) = \frac{d}{dx}[6 - 2x]$$

$$f''(x) = -2$$

4. Evaluate $f''(x)$ at each of the solutions.

x	3
f”(x)	-2

We can set $a = 3$ in the second derivative test. Since $f'(3) = 0$ and $f''(3) = -2 < 0$, the second derivative test tells us that $f(x)$ has a local maximum when $x = 3$.

Remember that the intuition behind this is that at $x = 3$, we know that the tangent line to $f(x)$ is horizontal, and furthermore, the graph is concave down, since $f''(3) < 0$. So this must be a local maximum.

$\square$