Section 1.3, Question 23: Find the derivative of $f(x) = \sqrt{x}$ at $x = \frac{1}{16}$.

Answer: The question essentially is asking us to find the slope of the graph of $f(x) = \sqrt{x}$ when $x = \frac{1}{16}$. (Another way of putting this is to find the slope of the graph at the point $(\frac{1}{16}, \frac{1}{4})$.)

Notice that we can use the power rule to find the derivative by writing $f(x) = \sqrt{x} = x^{\frac{1}{2}}$. Now the power rule applies (we take $r = \frac{1}{2}$), and we can compute the derivative, which is

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

.

Now to find the derivative at $x = \frac{1}{16}$, we compute

$$f'(\frac{1}{16}) = \frac{1}{2 * \sqrt{\frac{1}{16}}} = \frac{1}{2 * \frac{1}{4}} = \frac{4}{2} = 2$$

.

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Section 1.3, Question 43: The line $y = 2x + b$ is tangent to the graph of $y = \sqrt{x}$ at the point $P(a, \sqrt{a})$. Find P and determine b.

Answer: The question tells us that at the point $P(a, \sqrt{a})$, the line $y = 2x + b$ is *tangent* to the graph $y = \sqrt{x}$.

By the definition of the derivative, the slope of the graph $y = \sqrt{x}$ must be equal to the slope of the line $y = 2x + b$, which is 2.

So, now we know that the slope of the graph at $P(a, \sqrt{a})$ is equal to 2.

On the other hand, the power rule gives us a formula for the derivative of $f(x) = \sqrt{x}$: it is $f'(x) = \frac{1}{2*\sqrt{x}}$. So, at the point $(a, \sqrt{a})$, the slope of the curve is $f'(a) = \frac{1}{2*\sqrt{a}}$.

Putting these together means that

$$\frac{1}{2 * \sqrt{a}} = 2.$$

Then we get $1 = 4 * \sqrt{a} \implies \sqrt{a} = \frac{1}{4} \implies a = \frac{1}{16}$.

So, we get that the point is $P(\frac{1}{16}, \frac{1}{4})$!

Now we have to find the value of b. Since we have now found that the line $y = 2x + b$ is tangent to $y = \sqrt{x}$ at $(\frac{1}{16}, \frac{1}{4})$, we know that the point $(\frac{1}{16}, \frac{1}{4})$ lies on the tangent line $y = 2x + b$.

But this means that if we plug in $x = \frac{1}{16}$ and $y = \frac{1}{4}$ into $y = 2x + b$, we must get

$$\frac{1}{4} = 2 * \frac{1}{16} + b$$

. This means that we can solve for b as follows: $\frac{1}{4} = 2 * \frac{1}{16} + b \implies \frac{1}{4} = \frac{1}{8} + b$

$$\implies b = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}.$$

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