Section 1.2, Question 25: Find the point on the graph of $y = x^2$ where the curve has slope $\frac{7}{2}$.

Answer: Let's recall the following from class today:

Fact 1. *The slope of the graph of* $y = x^2$ *at the point* (a, a^2) *is* $2a$.

The question asks us to find the point (let's call it P) on the graph where the slope is equal to $\frac{7}{2}$. Let's use a to denote the x-coordinate of P. So, the point P will have the coordinates (a, a^2). We can find P if we can solve for the number a.

The fact tells us that at a general point (a, a^2) on the curve $y = x^2$, the slope of the curve is always $2a$.

So, we have the following: at the point P, slope$= 2a$ (by the fact) and slope$= \frac{7}{2}$ (by the information in the question).

Putting these together, we get that $2a = \frac{7}{2}$. Now we can solve for a:

$$2a = \frac{7}{2} \implies a = \frac{7}{4}.$$

So, we get that the point P has coordinates

$$(\frac{7}{4}, (\frac{7}{4})^2) = (\frac{7}{4}, \frac{49}{16}).$$

□

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Section 1.2, Question 33: You are given that the tangent line to the graph of $f(x) = x^2$ at a point $(a, f(a))$ has equation $y = 2x - 1$ (see the graph on page 72). Find $a, f(a)$ and the slope of the parabola at $(a, f(a))$.

Answer: Recall again the following:

Fact 2. *The slope of the graph of $y = x^2$ at the point (a, a^2) is $2a$.*

We are trying to find the point (again, let's call it P) on the graph where the tangent line has the equation $y = 2x - 1$. Let's again denote the coordinates of P by (a, a^2). As in the previous question, we will be able to solve the problem if we can solve for the number a.

To solve this problem, remember that the slope of the curve at a point P is equal to the slope of the tangent line to the curve at that point.

The question tells us that at P, the tangent line has the equation $y = 2x - 1$.

This means that the tangent line at P has slope 2!

So, the curve has slope 2 at the point P.

Now, the fact tells us that at the point $P(a, a^2)$, the slope of the curve is equal to $2a$.

Putting these together, we get that

$$2a = 2 \implies a = 1.$$

So, we get that the point P has coordinates

$$(1, f(1)) = (1, 1^2) = (1, 1).$$

□