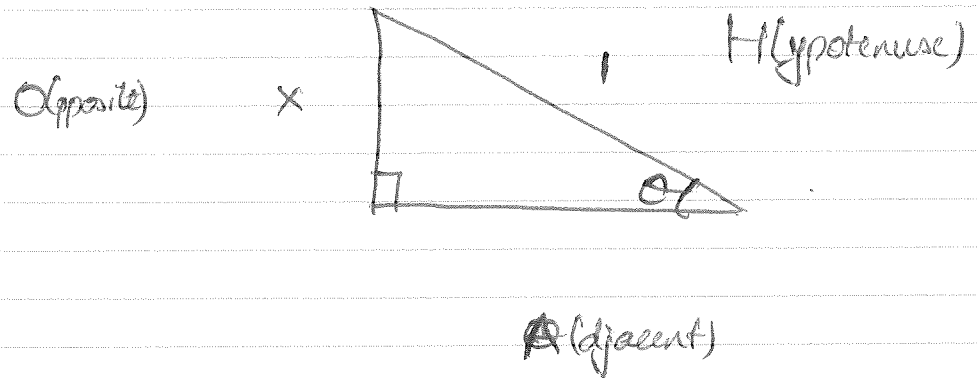
Q) Prove that $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$.

A) We will solve this by introducing a new variable, θ, and setting $\theta = \sin^{-1}(x)$.

Then $\sin(\theta) = \sin(\sin^{-1}(x)) = x$.

Since $\sin(\theta) = x$, we can draw the following triangle relating θ and x.



Since $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{1}$. Note that $\cos(\theta) = \frac{A}{H} = \frac{A}{1} = A$.

If we ~~try~~ compute the adjacent side, we get:

$$H^2 = O^2 + A^2$$

$$\Rightarrow A^2 = H^2 - O^2$$

$$\Rightarrow A = \sqrt{H^2 - O^2}$$

$$\Rightarrow A = \sqrt{1^2 - x^2} = \sqrt{1-x^2}.$$

So, we get that

$$\cos(\theta) = \sqrt{1-x^2}.$$

Or, since $\theta = \sin^{-1}(x)$,

$$\cos(\theta) = \cos(\sin^{-1}(x)) = \sqrt{1-x^2}.$$

□