

Math 1700 Summer 2013

Quiz 2

Thursday June 6 2013

No Work = No Credit

Name: _____ Student Number: _____

1. (5 points) Suppose $f(x) = 2x^3 + 3x^2 + 7x + 4$ and $a = 4$. Find $(f^{-1})'(a)$.

Solution:

Recall that:

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}.$$

We have that $a = 4$.

To find $f^{-1}(4)$, we set $f(x) = 4$, getting:

$$2x^3 + 3x^2 + 7x + 4 = 4$$

$$2x^3 + 3x^2 + 7x = 0$$

$$x(2x^2 + 3x + 7) = 0$$

We may assume that the function is one to one. Then $x = 0$ is the unique solution.

$$\text{So, } f^{-1}(4) = 0.$$

Next, $f(x) = 2x^3 + 3x^2 + 7x + 4$ implies $f'(x) = 6x^2 + 6x + 7$.

$$\text{So, } f'(0) = 7.$$

$$\text{So, } (f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(0)} = \frac{1}{7}.$$

2. (5 points) Use logarithmic differentiation to find the derivative of the function:

$$y = (x^2 + 2)^2(x^4 + 4)^4$$

.

Solution:

$$y = (x^2 + 2)^2(x^4 + 4)^4$$

...show all work...show all work...show all work...show all work...show all work...

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$$\ln(y) = \ln((x^2 + 2)^2(x^4 + 4)^4)$$

$$\ln(y) = \ln((x^2 + 2)^2) + \ln((x^4 + 4)^4)$$

$$\ln(y) = 2 \ln((x^2 + 2)) + 4 \ln((x^4 + 4))$$

$$\frac{d}{dx} [\ln(y)] = \frac{d}{dx} [2 \ln((x^2 + 2))] + \frac{d}{dx} [4 \ln((x^4 + 4))]$$

$$\frac{1}{y} \frac{d}{dx} [(y)] = 2 \frac{d}{dx} [\ln((x^2 + 2))] + 4 \frac{d}{dx} [\ln((x^4 + 4))]$$

$$\frac{1}{y} \frac{d}{dx} [(y)] = 2 \frac{1}{(x^2+2)} \frac{d}{dx} [(x^2 + 2)] + 4 \frac{1}{(x^4+4)} \frac{d}{dx} [(x^4 + 4)]$$

$$\frac{1}{y} \frac{d}{dx} [(y)] = 2 \frac{1}{(x^2+2)} (2x) + 4 \frac{1}{(x^4+4)} (4x^3)$$

$$\frac{d}{dx} [(y)] = y [2 \frac{1}{(x^2+2)} (2x) + 4 \frac{1}{(x^4+4)} (4x^3)]$$

$$\frac{d}{dx} [(y)] = (x^2 + 2)^2 (x^4 + 4)^4 [2 \frac{1}{(x^2+2)} (2x) + 4 \frac{1}{(x^4+4)} (4x^3)]$$

...show all work...show all work...show all work...show all work...show all work...show all work...