Section 1.7, Question 31: A company finds that the revenue R generated by spending x dollars on advertising is given by $R = 1000 + 80x - 0.02x^2$, for $0 \le x \le 2000$. Find $\frac{dR}{dx}|_{x=1500}$.

Answer: To solve this problem, we'll

- find the derivative of R by computing $\frac{dR}{dx}$.
- evaluate our answer by substituting 1500 for x.

We have that $R = 1000 + 80x - 0.02x^2$ when $0 \le x \le 2000$. For now, we can ignore the restriction on the domain ($0 \le x \le 2000$). Let's first take the first derivative:

$$\frac{dR}{dx} = \frac{d}{dx}[1000 + 80x - 0.02x^2]$$

$$\frac{dR}{dx} = \frac{d}{dx}[1000] + \frac{d}{dx}[80x] + \frac{d}{dx}[-0.02x^2]$$

(sum rule)

$$\frac{dR}{dx} = \frac{d}{dx}[1000] + \frac{d}{dx}[80x] + (-0.02) * \frac{d}{dx}[x^2]$$

(constant multiple rule)

$$\frac{dR}{dx} = \frac{d}{dx}[1000] + \frac{d}{dx}[80x] + (-0.02) * (2x)$$

$$\frac{dR}{dx} = \frac{d}{dx}[1000] + 80 + (-0.02) * (2x)$$

(since $80x$ is a linear function with slope 80)

$$\frac{dR}{dx} = 0 + 80 + (-0.02) * (2x)$$

(since 1000 is a constant function)

$$\frac{dR}{dx} = 80 + (-0.02) * (2x)$$

So the first derivative is

$$\frac{dR}{dx} = 80 + (-0.02) * (2x) = 80 - 0.04x.$$

To finish the question, we need to evaluate this expression by setting $x = 1500$:

$$\frac{dR}{dx}|_{x=1500} = (80 - 0.04x)|_{x=1500} = 80 - 0.04(1500) = 80 - 60 = 20.$$

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Note that this solution is valid since we are evaluating the derivative inside the interval ($0 \leq x \leq 2000$) on which $R(x)$ is defined. We could not evaluate the derivative outside this interval (for example, at $x = -100$ or $x = 5000$) since $R(x)$ is not defined there!

$\square$