Math 1300 Fall 2013
Wednesday September 4 2013
Exercises

1. Use the Gauss-Jordan elimination method to find all solutions of the system of linear equations:

$$\left\{\begin{array}{rcrcr} 2x & + & 3y & = & 12 \\ 2x & - & 3y & = & 0 \\ 5x & - & y & = & 13 \end{array}\right\}$$

Solution:

After row operations, we get:

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

So, there is exactly *one solution*, which is: $x = 3, y = 2$. The third equation is redundant. Note that this does not contradict our fact from class (which would have said that there were infinitely many solutions), since this is not a 3×3 system.

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2. Use the Gauss-Jordan elimination method to find all solutions of the system of linear equations:

$$\left\{ \begin{array}{rcrcrcr} x & - & 3y & + & 2z & = & 10 \\ -x & + & 3y & - & z & = & -6 \\ -x & + & 3y & + & 2z & = & 6 \end{array} \right\}$$

Solution:

After row operations, we get:

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

This is a 3×3 system, so our fact from class tells us that this has infinitely many solutions.

The solution set is:

$$\{(x, y, z) \mid x = 2 + 3t, y = t, z = 4 \text{ and } t \in \mathbb{R}\}.$$

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3. Use the Gauss-Jordan elimination method to find all solutions of the system of linear equations:

$$\left\{ \begin{array}{ccccccc} x & + & 2y & + & 3z & = & 4 \\ 5x & + & 6y & + & 7z & = & 8 \\ x & + & 2y & + & 3z & = & 5 \end{array} \right\}$$

Solution:

After row operations, we get:

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

This is a 3×3 system, so our fact from class tells us that this has no solutions, since it is not possible to satisfy the equation

$$0x + 0y + 0z = 1$$

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4. An office manager placed an order for computers, printers, and scanners. Each computer cost \$1000, each printer cost \$100, and each scanner cost \$400. She ordered 15 items for $\$10,200$. Give two different combinations for the numbers of each type of item that she could have purchased.

Solution:

There are many (but not infinitely many) combinations. Here are some:

$$(\text{computer}, \text{printer}, \text{scanner}) = (1, 88, 1), (2, 78, 1), (2, 74, 2).$$

5. A quilt shop receives an order for a patchwork quilt made from square patches of three types: solid green, solid blue, and floral. The quilt is to be 8 squares by 12 squares, and there must be 15 times as many solid squares as floral squares. If the shop charges \$3 per solid square, and \$5 per floral square, and if the customer wishes to spend exactly \$300, how many of each type of square may be used in the quilt?

Solution:

After row operations, we get:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 90 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

There are many solutions to this system (caveat: there are not infinitely many as we would usually say, since x, y, z have to be whole numbers). The solution set is:

$$\{(\text{green}, \text{blue}, \text{floral}) = (n, m, 6) \mid n, m \in \mathbb{N}, n, m \geq 0, m = 96 - n\}.$$

6. You are buying some house plants out of a selection of three types, that cost \$7, \$10 and \$13. If you have budgeted exactly \$150 for house plants, and you want exactly 15 of them, what are your options?

Solution:

After row operations, we get:

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 15 \end{array}\right]$$

There are many solutions to this system (caveat: there are not infinitely many as we would usually say, since the numbers of each type that we buy, say x, y, z, have to be whole numbers). The solution set is:

$$\{(x, y, z) = (n, m, 6) \mid n, m \in \mathbb{N}, n, m \geq 0, m = 96 - n\}.$$

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7. For what value of k will the following system of linear equations have a solution?

$$\left\{ \begin{array}{ccccc} 2x & + & 6y & = & 4 \\ x & + & 7y & = & 10 \\ kx & + & 8y & = & 4 \end{array} \right\}$$

Solution: This system will have a solution when $k = 3$.