

Section 1.6, Question 51: If $g(1) = 4$ and $g'(1) = 3$, find $f(1)$ and $f'(1)$, where $f(x) = 5\sqrt{g(x)}$.

Answer: We have that $f(x) = 5\sqrt{g(x)}$. So we can find $f(1)$ by setting $x = 1$ in this equation.

So,

$$f(1) = 5\sqrt{g(1)} = 5\sqrt{4} = 5 * 2 = 10.$$

Next, if $f(x) = 5\sqrt{g(x)} = 5[g(x)]^{\frac{1}{2}}$, then we can differentiate both sides to get:

$$f'(x) = \frac{d}{dx}[5[g(x)]^{\frac{1}{2}}]$$

$$f'(x) = 5 * \frac{d}{dx}[[g(x)]^{\frac{1}{2}}]$$

(constant multiple rule)

$$f'(x) = 5 * [\frac{1}{2}[g(x)]^{-\frac{1}{2}}] * \frac{d}{dx}[g(x)]$$

(general power rule)

$$f'(x) = 5 * [\frac{1}{2}[g(x)]^{-\frac{1}{2}}] * g'(x)$$

(definition of $g'(x)$)

So, we can find $f'(1)$ by setting $x = 1$ in this equation:

$$f'(1) = 5 * \frac{1}{2}[g(1)]^{-\frac{1}{2}} * g'(1) = 5 * \frac{1}{2}[4]^{-\frac{1}{2}} * 3 = 5 * \frac{1}{2} * \frac{1}{2} * 3 = \frac{15}{4}.$$

□