

Question: Find the Taylor series for $f(x) = e^{2x}$ centered at $c = 3$.

Solution:

Remember that the formula for the Taylor series is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)(x-c)^n}{n!}.$$

We'll fill in the following table:

n	$f^{(n)}(x)$	$f^{(n)}(3)$
0	e^{2x}	e^6
1	$2e^{2x}$	$2e^6$
2	2^2e^{2x}	$4e^6$
3	2^3e^{2x}	$8e^6$
4	2^4e^{2x}	$16e^6$
5	2^5e^{2x}	$32e^6$
\vdots	\vdots	\vdots
n	$2^n e^{2x}$	$2^n e^6$
\vdots	\vdots	\vdots

So the Taylor expansion for $f(x) = e^{2x}$ centered at $c = 3$ is:

$$f(x) = \sum_{n=0}^{\infty} \frac{2^n e^6 (x-3)^n}{n!}.$$

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Question: Find the Maclaurin series for $f(x) = 2^x$.

Solution:

Remember that the formula for the Maclaurin series is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}.$$

We'll fill in the following table:

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	2^x	1
1	$2^x \ln(2)$	$\ln(2)$
2	$2^x (\ln(2))^2$	$(\ln(2))^2$
3	$2^x (\ln(2))^3$	$(\ln(2))^3$
4	$2^x (\ln(2))^4$	$(\ln(2))^4$
5	$2^x (\ln(2))^5$	$(\ln(2))^5$
\vdots	\vdots	\vdots
n	$2^x (\ln(2))^n$	$(\ln(2))^n$
\vdots	\vdots	\vdots

So the Taylor expansion for $f(x) = 2^x$ is:

$$f(x) = \sum_{n=0}^{\infty} \frac{(\ln(2))^n x^n}{n!}.$$

□