List of formulae needed for Chapters 4 and 5

1 Algebraic formulae

1. $b^x b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $b^{-x} = \frac{1}{b^x}$
4. $a^x b^x = (ab)^x$
5. $(b^x)^y = b^{xy}$
6. $\frac{a^x}{b^x} = (\frac{a}{b})^x$
7. $\ln(xy) = \ln(x) + \ln(y)$
8. $\ln(\frac{x}{y}) = \ln(x) - \ln(y)$
9. $\ln(x^a) = a\ln(x)$
10. $\ln(\frac{1}{x}) = -\ln(x)$
11. $\ln(e^x) = x$ for all x
12. $e^{\ln(x)} = x$ for all $x > 0$
13. $\ln(1) = 0$

2 Differentiation formulae

1. $\frac{d}{dx}[e^{g(x)}] = e^{g(x)}g'(x)$, where $g(x)$ is any function of x
2. $\frac{d}{dx}[\ln(g(x))] = \frac{1}{g(x)}g'(x)$, where $g(x)$ is any function of x

3 Modeling and Applications; Formulae

1. The exponential model is:

$$P(t) = P(0)e^{kt}$$

where $P(0)$ is the initial population/balance/sample, and k is the growth constant/ continuously compounded interest rate/ decay constant.

This exponential model gives the differential equation:

$$P'(t) = kP(t)$$

where k is the same as the growth constant/ continuously compounded interest rate/ decay constant in the model. (It is the same k as in the exponent of e in the model).

If any function satisfies the differential equation:

$$P'(t) = kP(t)$$

then it is of the form:

$$P(t) = P(0)e^{kt}$$

where $P(0)$ is the initial population/balance/sample, and k is the growth constant/ continuously compounded interest rate/ decay constant.

In this model, the number k is the same as the k in the differential equation.

2. In the case of an account that pays interest compounded continuously, the exponential model can be written as:

$$A(t) = Pe^{rt}$$

The only difference here is that we are setting $A(0) = P$. That is, we are letting P denote the initial balance. We also call P the principal.

3. The present value of an amount A payable t years in the future, from an account paying interest at rate r compounded continuously is:

$$P = Ae^{-rt}$$

4. The relative change in price of a function $f(x)$ is:

$$\frac{f'(x)}{f(x)}$$

5. The percentage change in price of a function $f(x)$ is:

$$\frac{f'(x)}{f(x)} * 100$$

6. If $q(p)$ is a demand function, then the elasticity of demand is:

$$E(p) = -\frac{q'(p)}{q(p)}p$$