Section 6.1, Question 41: Let $f(x) = 3 + x^2 + \tan(\frac{\pi x}{2})$ and let $a = 3$. Find $(f^{-1})'(a))$.

Solution:

We will solve this question using the same steps as in recitation: by finding successively the quantities:

- a
- $f^{-1}(a)$
- $f'(x)$
- $f'(f^{-1}(a))$
- $\frac{1}{f'(f^{-1}(a))}$

The question gives us that $a = 3$.

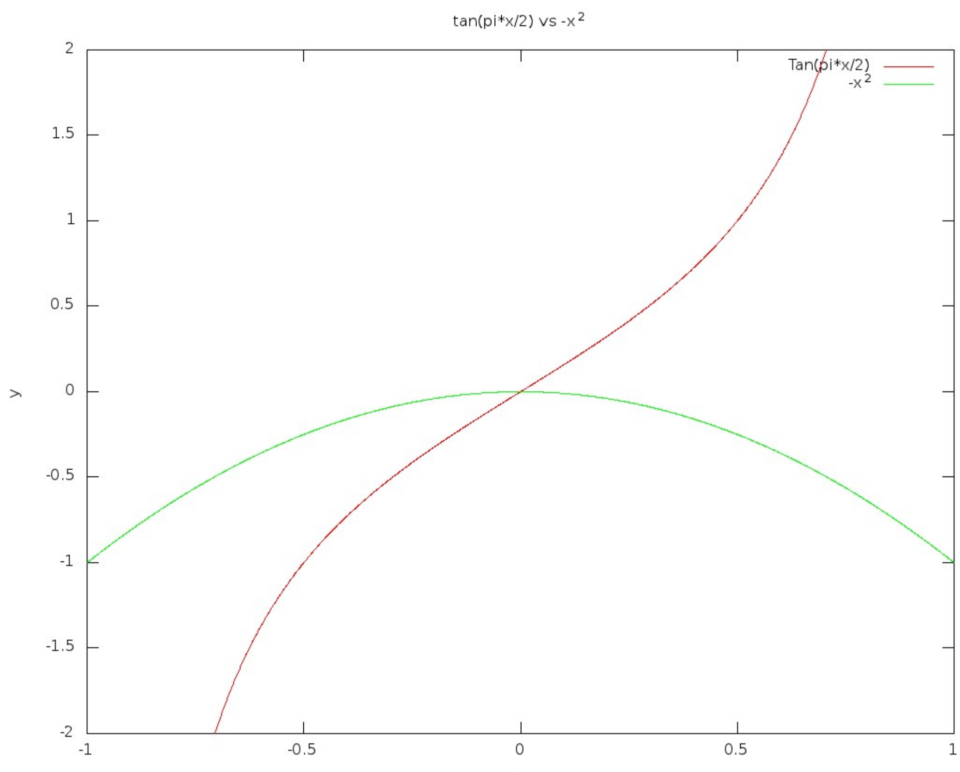
Next, to find $f^{-1}(3)$, we set $f(x) = 3$ and solve for x.

$f(x) = 3$

$\implies 3 + x^2 + \tan(\frac{\pi x}{2}) = 3$

$\implies x^2 + \tan(\frac{\pi x}{2}) = 0$

$\implies \tan(\frac{\pi x}{2}) = -x^2$



Above is a graph of the functions $y = -x^2$ (in green) and of $y = \tan(\frac{\pi x}{2})$ (in red). The graphs cross when $x = 0$, and only when $x = 0$. This tells us that the left hand side and the right hand side in the above equation are equal if, and only if, $x = 0$.

So, we see that $f^{-1}(3) = 0$.

Next, we have $f(x) = 3 + x^2 + \tan(\frac{\pi x}{2})$

$\implies f'(x) = 2x + \sec^2(\frac{\pi x}{2})\frac{\pi}{2}$

$\implies f'(f^{-1}(3)) = f'(0) = \frac{\pi}{2}$

Finally, this shows us that $(f^{-1})'(a)) = \frac{1}{f'(f^{-1}(a))} = \frac{2}{\pi}$.

$\square$