For the following questions, refer to the graph given in figure 23 on page 149 of our textbook. **IT IS VERY IMPORTANT TO REMEMBER THAT THIS IS THE GRAPH OF $f'(x)$, the *derivative* of the function $f(x)$. It is NOT the graph of $f(x)$ itself!**

Section 2.2, Question 25: Explain why $f(x)$ must be increasing at $x = 6$.

Answer: At $x = 6$, the graph of $f'(x)$ passes through the point $(6, 2)$.

This means that $f'(6) = 2$. In particular, $f'(6) = 2 > 0$.

Remember that the first derivative rule tells us that if $f'(x) > 0$ at $x = a$, then $f(x)$ is increasing at $x = a$.

In our case, we have $a = 6$, and $f'(x) > 0$ at $x = 6$.

So, the first derivative rule tells us that $f(x)$ must be increasing at $x = 6$.

□

Section 2.2, Question 27: Explain why $f(x)$ has a relative maximum at $x = 3$.

Answer: Remember that a point is a relative maximum for $f(x)$ if at that point, the function changes from being increasing to being decreasing.

The first derivative rule tells us that if $f'(x) > 0$ at $x = a$, then $f(x)$ is increasing at $x = a$, and if $f'(x) < 0$ at $x = b$, then $f(x)$ is decreasing at $x = b$.

Just to the left of $x = 3$, the graph of $f'(x)$ lies above the x-axis. This means that just to the left of $x = 3$, $f'(x) > 0$. By the first derivative rule, $f(x)$ is increasing just to the left of $x = 3$.

Just to the right of $x = 3$, the graph of $f'(x)$ lies below the x-axis. This means that just to the right of $x = 3$, $f'(x) < 0$. By the first derivative rule, $f(x)$ is decreasing just to the right of $x = 3$.

So, at $x = 3$, the function $f(x)$ changes from being increasing to being decreasing.

This means that $x = 3$ must be a relative maximum.

□

Section 2.2, Question 29: Explain why $f(x)$ must be concave up at $x = 0$.

Answer: Usually, to show that the function $f(x)$ is concave up at $x = 0$, we would try to show that near $x = 0$, the tangent lines to $f(x)$ lie below the graph of $f(x)$. However, the question does not give us the graph of $f(x)$, so we cannot do this!

We'll try a different approach to solving this problem. The second derivative rule will come in handy here. Remember that it says:

(Second derivative rule): If $f''(x) > 0$ at $x = a$, then $f(x)$ is concave up at $x = a$. If $f''(x) < 0$ at $x = a$, then $f(x)$ is concave down at $x = a$.

So, we can verify that $f(x)$ is concave up at $x = 0$ if we can show that $f''(x) > 0$ at $x = 0$.

Let's look at the graph of $f'(x)$. At $x = 0$, the slope of the graph is positive. This means that the derivative of $f'(x)$ is positive at $x = 0$.

Since the derivative of $f'(x)$ is positive at $x = 0$, we get that $f''(x) > 0$ at $x = 0$.

So, the second derivative rule tells us that $f(x)$ must be concave up at $x = 0$.

□

Section 2.2, Question 31: Explain why $f(x)$ has an inflection point at $x = 1$.

Answer: Remember that a point is an inflection point for $f(x)$ if at that point, the function changes from being concave up to being concave down, or vice versa.

The second derivative rule tells us that if $f''(x) > 0$ at $x = a$, then $f(x)$ is concave up at $x = a$, and if $f''(x) < 0$ at $x = b$, then $f(x)$ is concave down at $x = b$.

Just to the left of $x = 1$, the graph of $f'(x)$ has positive slope. This means that just to the left of $x = 1$, the derivative of $f(x)$ is positive, or $f'(x) > 0$. By the second derivative rule, $f(x)$ is concave up just to the left of $x = 1$.

Just to the right of $x = 1$, the graph of $f'(x)$ has negative slope. This means that just to the right of $x = 1$, the derivative of $f(x)$ is negative, or $f'(x) < 0$. By the second derivative rule, $f(x)$ is concave down just to the left of $x = 1$.

So, at $x = 1$, the function $f(x)$ changes from being concave up to being concave down.

This means that $x = 1$ must be an inflection point.

□

Hint for Webwork Set 3, Question 5A: Remember the first derivative rule: $f(x)$ is increasing wherever $f'(x) > 0$. Remember also that the graph given to you is the graph of $f'(x)$, not $f(x)$! So all you need to do is to find where $f'(x) > 0$!

Hint for Webwork Set 3, Question 5B: The second derivative rule tells us that $f(x)$ is concave down wherever $f''(x) < 0$. So all we need to do is to find out where $f''(x) < 0$. However, we are given the graph of $f'(x)$, not $f''(x)$! So we need to do a little more work.

Remember that $f''(x)$ is the first derivative of $f'(x)$. So $f''(x)$ measures the slope of the graph of $f'(x)$.

So, $f''(x) < 0$ exactly when $f'(x)$ is downwards sloping.

So, to answer this question, all we need to do is to find all the points where the graph of $f'(x)$ is downwards sloping!

Hint for Webwork Set 3, Question 5C: Remember that a local/relative minimum occurs wherever $f(x)$ changes from being decreasing to increasing. The first derivative rule tells us that $f(x)$ is decreasing if $f'(x) < 0$, and $f(x)$ is increasing if $f'(x) > 0$.

So, $f(x)$ will have a local minimum wherever $f'(x)$ changes from being negative to positive.

This means that all we have to do is look at the graph of $f'(x)$ and find all the points where it changes from being negative (below the x-axis to positive (above the x-axis)!

Hint for Webwork Set 3, Question 5D: Remember that an inflection point occurs wherever $f(x)$ changes from being concave up to concave down, or vice versa. The first derivative rule tells us that $f(x)$ is concave up if $f''(x) > 0$, and $f(x)$ is concave down if $f''(x) < 0$.

So, $f(x)$ will have an inflection point wherever $f''(x)$ changes from being negative to positive, or from being positive to negative.

Remember that $f''(x)$ measure the slope of the graph of $f'(x)$, since it is the derivative of $f'(x)$.

This means that all we have to do is look at the graph of $f'(x)$ and find all the points where it changes from being upwards sloping to downwards sloping, or changes from being downwards sloping to upwards sloping!