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1. (2 points) Suppose that $f(x) = 4x^2$. Find the average rate of change of $f(x)$ over the interval from 1 to 2.

Solution:

Remember that the formula for the average rate of change of a function $f(x)$ over the interval $[a, b]$ is:

$$AROC = \frac{f(b) - f(a)}{b - a}$$

In this question, we can take $f(x) = 4x^2$, $a = 1$ and $b = 2$.

So the formula for the average rate of change gives us:

$$AROC = \frac{f(2) - f(1)}{2 - 1}$$

$$AROC = \frac{4(2^2) - 4(1^2)}{2 - 1}$$

$$AROC = \frac{4(4) - 4(1)}{2 - 1}$$

$$AROC = \frac{16 - 4}{2 - 1}$$

$$AROC = \frac{12}{1}$$

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$$AROC = 12$$

2. (3 points) If $f(100) = 5000$ and $f'(100) = 10$, estimate $f(101)$.

Solution:

To solve this question, we are going to use the formula:

$$f(a + h) - f(a) \cong f'(a)h$$

.

The formula says that if we want to approximate the change in the value of $f(x)$ between $x = a$ and $x = a + h$, we can use the estimate $f'(a)h$.

Note that we can rewrite the formula as follows:

$$f(a + h) \cong f(a) + f'(a)h.$$

This gives us a way of approximating $f(a + h)$ if we know the values of $f(a), f'(a)$ and h. Remember that we can calculate h by using $h = (a + h) - a$.

Let's use this idea to try and solve this problem.

In our question, we have that

- $a = 100$,

- $a + h = 101$,

- $h = 101 - 100 = 1$,

- $f(a) = f(100) = 5000$, and

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- $f'(a) = f'(100) = 10$.

Then the formula

$$f(a + h) \cong f(a) + f'(a)h$$

says that:

$$f(101) \cong f(100) + f'(100) * 1$$

or,

$$f(101) \cong 5000 + 10 = 5010.$$

So our estimate for the value of $f(101)$ is 5010.