Question: Differentiate the function $g(x) = x\sin(2^x)$.

Solution:

$g'(x) = \frac{d}{dx}[x\sin(2^x)]$

$g'(x) = \frac{d}{dx}[x](\sin(2^x)) + (x)\frac{d}{dx}[\sin(2^x)]$

$g'(x) = \sin(2^x) + (x)\cos(2^x)(\frac{d}{dx}[(2^x)])$

$g'(x) = \sin(2^x) + (x)\cos(2^x)((2^x)\ln(2))$

$\square$

Question: Differentiate the function $y = \sqrt{x^x}$.

Solution:

Notice that $\sqrt{x^x} = (x^x)^{\frac{1}{2}} = x^{\frac{x}{2}}$. So:

$\frac{d}{dx}[y] = \frac{d}{dx}[x^{\frac{x}{2}}]$

$\frac{d}{dx}[y] = \frac{d}{dx}[(e^{\ln(x)})^{\frac{x}{2}}]$

$\frac{d}{dx}[y] = \frac{d}{dx}[e^{\ln(x)\frac{x}{2}}]$

$\frac{d}{dx}[y] = \frac{d}{dx}[e^{\ln(x)\frac{x}{2}}]$

$\frac{d}{dx}[y] = e^{\ln(x)\frac{x}{2}}\frac{d}{dx}[\ln(x)\frac{x}{2}]$

$\frac{d}{dx}[y] = e^{\ln(x)\frac{x}{2}}[\ln(x)\frac{d}{dx}[\frac{x}{2}] + \frac{x}{2}\frac{d}{dx}[\ln(x)]]$

$\frac{d}{dx}[y] = e^{\ln(x)\frac{x}{2}}[\ln(x)\frac{1}{2} + \frac{x}{2}(\frac{1}{x})]$

$\square$

Question: Evaluate the integral $\int \frac{2^x}{2^x+1}dx$.

Solution:

Try the substition $u = 2^x + 1$. Then $\frac{du}{dx} = 2^x\ln(2)$, and $dx = \frac{du}{2^x\ln(2)}$.

$= \int \frac{2^x}{u}\frac{1}{2^x\ln(2)}du$

$= \int \frac{1}{u}\frac{1}{\ln(2)}du$

$= \frac{1}{\ln(2)}\int \frac{1}{u}du$

$= \frac{1}{\ln(2)}\ln(u) + C$

$= \frac{1}{\ln(2)}\ln(2^x + 1) + C$

$\square$