Section 1.8, Question 11: The position of a particle moving on a line is given by $s(t) = 2t^3 - 21t^2 + 60t$, where $t \geq 0$, t is measured in seconds and s is measured in feet.

1. What is velocity after 3 seconds and after 6 seconds?
2. When is the particle moving in the positive direction?
3. Find the total distance traveled by the particle during the first 7 seconds.

Answer: Remember that the rate of change of a function $f(x)$ when $x = a$ is $f'(a)$.

Velocity is defined to be the rate of change of displacement.

In this question, the displacement is given by $s(t) = 2t^3 - 21t^2 + 60t$.

So, the velocity function is given by

$$v(t) = \frac{d}{dt}[2t^3 - 21t^2 + 60t]$$

$$= \frac{d}{dt}[2t^3] - \frac{d}{dt}[21t^2] + \frac{d}{dt}[60t]$$

$$= 2\frac{d}{dt}[t^3] - 21\frac{d}{dt}[t^2] + \frac{d}{dt}[60t]$$

$$= 2 * 3t^2 - 21 * 2t + 60$$

$$= 6t^2 - 42t + 60.$$

So we can answer **part 1** by evaluating the velocity function when $t = 3$ and $t = 6$:

$$v(3) = (6t^2 - 42t + 60)|_{t=3} = 6(3)^2 - 42(3) + 60 = -12ft/s.$$

$$v(6) = (6t^2 - 42t + 60)|_{t=6} = 6(6)^2 - 42(6) + 60 = 24ft/s.$$

To answer **part 2**, we need to find out when the velocity is positive. We can do this by finding all t where $v(t) > 0$.

We'll solve this part by first setting $v(t) = 0$. This gives two roots:

$$v(t) = 6t^2 - 42t + 60 = 0$$

$$6(t^2 - 7t + 10) = 0$$

$$(t^2 - 7t + 10) = 0$$

$$(t - 5)(t - 2) = 0$$

$$t = 2, 5.$$

So the roots of $v(t) = 0$ are $t = 2, 5$. To see where $v(t) > 0$, we can divide up the domain into the intervals $[0, 2), (2, 5), (5, \infty)$ and check where $v(t) > 0$ using test points.

For $[0, 2)$,, we can use the test point $t = 0$. This gives $v(0) = (6t^2 - 42t + 60)|_{t=0} = 60ft/s$. This shows that $v(t) > 0$ on $[0, 2)$.

For $(2, 5)$, we can use a test point $t = 3$. (We've already done this in part 1!). We found that $v(3) = -12ft/s$. So, $v(t) < 0$ on $(2, 5)$.

Lastly, for $(5, \infty)$, we can use the test point $t = 6$. Again, we've already done this computation in part 1. We found that $v(6) = 24ft/s$. So, $v(t) > 0$ on $(5, \infty)$.

So, we conclude that $v(t) > 0$ on $[0, 2) \cup (5, \infty)$.

Lastly, to answer **part 3**, we need to know how far the particle has traveled over the first 7 seconds.

Note that we cannot just compute $s(7)$ to find the overall distance traveled. This is because $s(t)$ is a displacement function; it measures how far away from a reference point the particle is at time t. When $v(t) > 0$, the particle is moving *away* from the reference point, but when $v(t) < 0$, the particle is moving *back towards* the reference point.

This means that during the interval $(2, 5)$, the particle moves back towards the reference point. If we just computed the value of $s(7)$, we would get an underestimate for the distance traveled in the first 7 seconds, because we would have ignored the fact that the particle back-tracked during $(2, 5)$.

To get the actual distance the particle traveled in the first 7 seconds, we can compute the distance in the following steps:

1. find the distance traveled during $[0, 2)$
2. find the distance traveled during $(2, 5)$
3. find the distance traveled during $(5, 7)$

The distance traveled between $[0, 2)$ is

$$s(2) - s(0) = (2t^3 - 21t^2 + 60t)|_{t=2} - (2t^3 - 21t^2 + 60t)|_{t=0}$$

$$= (2(2)^3 - 21(2)^2 + 60(2)) - (2(0)^3 - 21(0)^2 + 60(0))$$

$$= (16 - 84 + 120) - (0)$$

$$= 52ft.$$

So the distance traveled during $[0, 2)$ is 52 feet in the positive direction.

The distance traveled between $(2, 5)$ is

$$s(5) - s(2) = (2t^3 - 21t^2 + 60t)|_{t=5} - (2t^3 - 21t^2 + 60t)|_{t=2}$$

$$= (2(5)^3 - 21(5)^2 + 60(5)) - (2(2)^3 - 21(2)^2 + 60(2))$$

$$= (250 - 525 + 300) - (16 - 84 + 120)$$

$$= (25) - (52) = -27ft.$$

So the distance traveled during $(2,5)$ is 27 feet in the negative direction.

The distance traveled between $(5,7)$ is

$$s(7) - s(5) = (2t^3 - 21t^2 + 60t)|_{t=7} - (2t^3 - 21t^2 + 60t)|_{t=5}$$

$$= (2(7)^3 - 21(7)^2 + 60(7)) - (2(5)^3 - 21(5)^2 + 60(5))$$

$$= (686 - 1029 + 420) - (250 - 525 + 300)$$

$$= 77 - 25 = 52 ft.$$

So the distance traveled during $(5,7)$ is 52 feet in the positive direction.

We can add up the distances to get that the total distance traveled in the first 7 seconds is

$$52 + 27 + 52 = 131 ft.$$

$\square$