Math 1300 Fall 2013
Wednesday August 28, 2013
Exercises

1. You take out a loan of $\$21,281.27$ for 7 years at 8% compounded quarterly with a payment of \$1000 per quarter year.

   (a) What is your unpaid balance after 5 years?

   Solution:
   Remember that the unpaid balance on any mortgage is the present value of the remaining payments. We have the present value formula:

   $$PV = \frac{R(1-(1+i)^{-n})}{i}.$$

   In this question, we have:

   1. Interest period is a quarter year
   2. Payment (rent) is $\$1,000$
   3. Rate per interest period $= \frac{.08}{4}$
   4. Number of interest periods remaining $= 2$ years $\times 4$ quarters $= 8$ periods.

   So the unpaid balance after 5 years (with 2 years remaining) is

   $$\frac{1000(1-(1+\frac{.08}{4})^{-(4\times 2)})}{\frac{.08}{4}} = \$7,325.48.$$

   (b) How much interest do you pay in the fifth year?

   Solution:
   The unpaid balance at the start of the fifth year (with 3 years remaining) is

   $$\frac{1000(1-(1+\frac{.08}{4})^{-(4\times 3)})}{\frac{.08}{4}} = \$10,575.23.$$

   The unpaid balance at the end of the fifth year (with 2 years remaining) is

   $$\frac{1000(1-(1+\frac{.08}{4})^{-(4\times 3)})}{\frac{.08}{4}} = \$7,325.48.$$

   The balance reduction over the fifth year is $\$3,249.86$.

   The total payments for the fifth year were $4 \times \$1000 = \$4,000$.

   Therefore the interest paid was $\$4000 - \$3249.86 = \$750.14$.

(c) How much of your first payment goes towards paying off the principal?

Solution:
The unpaid balance at the start of the loan (with 7 years remaining) is

$$\frac{1000(1-(1+\frac{.08}{4})^{-(4\times 7)})}{\frac{.08}{4}} = \$21,281.27.$$

The unpaid balance at the end of the first quarter (with 6.75 years remaining) is

$$\frac{1000(1-(1+\frac{.08}{4})^{-(4\times 6.75)})}{\frac{.08}{4}} = \$20,706.90.$$

Therefore the balance reduction over the first quarter is \$574.37.

(d) How much total interest do you pay on the loan?

Solution:
You pay a total of $4 \times \$1000 \times 7 = \$28,000$ on a principal of $\$21,281.27$. Therefore the total interest paid is $\$28,000 - \$21,281.27 = \$6,718.73$.

(e) Prepare an amortization schedule for the first 4 quarters.

Solution:

Interest Period	Unpaid Balance	Interest charged	Amount Paid	Balance Reduction
1	\$21,281.27	\$425.63	\$1000	\$574.37
2	\$20,706.90	\$414.14	\$1000	\$585.86
3	\$20,121.04	\$402.42	\$1000	\$597.58
4	\$19,523.46	\$390.47	\$1000	\$609.53