Question: Differentiate the function $y = \cos^{-1}(\sin^{-1} t)$.

Solution:

$y = \cos^{-1}(\sin^{-1}(t))$

$\frac{d}{dx}[y] = \frac{d}{dx}[\cos^{-1}(\sin^{-1}(t))]$

$\frac{d}{dx}[y] = \frac{-1}{\sqrt{1-(\sin^{-1}(t))^2}} \frac{d}{dx}[\sin^{-1}(t)]$

$\frac{d}{dx}[y] = \frac{-1}{\sqrt{1-(\sin^{-1}(t))^2}} \frac{1}{\sqrt{1-t^2}}$

$\square$

Question: Differentiate the function $y = \tan^{-1}(\frac{1-x}{1+x})$.

Solution:

$y = \tan^{-1}(\frac{1-x}{1+x})$

$\frac{d}{dx}[y] = \frac{d}{dx}[\tan^{-1}(\frac{1-x}{1+x})]$

$\frac{d}{dx}[y] = \frac{1}{1+(\frac{1-x}{1+x})^2} \frac{d}{dx}[\frac{1-x}{1+x}]$

$\frac{d}{dx}[y] = \frac{1}{1+(\frac{1-x}{1+x})^2} \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$

$\square$

Question: Evaluate the integral: $\int \frac{1}{x\sqrt{x^2-4}} dx$.

Solution:

Try the substitution $x = 2y$. Then $x^2 = 4y^2$, $\frac{dx}{dy} = 2$, and $dx = 2dy$.

$= \int \frac{1}{2y\sqrt{4y^2-4}} 2dy$

$= \int \frac{1}{y\sqrt{4(y^2-1)}} dy$

$= \int \frac{1}{2y\sqrt{(y^2-1)}} dy$

$= \frac{1}{2} \int \frac{1}{y\sqrt{(y^2-1)}} dy$

$= \frac{1}{2} \sec^{-1}(y) + C$

$= \frac{1}{2} \sec^{-1}(\frac{x}{2}) + C$

$\square$

Question: Evaluate the integral: $\int \frac{x}{1+x^4} dx$.

Solution:

Try the substitution $u = x^2$. Then $\frac{du}{dx} = 2x$, and $dx = \frac{du}{2x}$.

$= \int \frac{x}{(1+u^2)} \frac{1}{2x} du.$

$= \int \frac{1}{(1+u^2)} \frac{1}{2} du.$

$= \frac{1}{2} \int \frac{1}{(1+u^2)} du.$

$= \frac{1}{2} \tan^{-1}(u) + C.$

$$= \frac{1}{2}\tan^{-1}(x^2) + C.$$

$\square$