Section 2.7, Question 13: An artist is planning to sell signed prints of her latest work. If 50 prints are offered for sale, she can charge \$400 each. However, if she makes more than 50 prints, she must lower the price of all the prints by \$5 for each print in excess of the 50. How many prints should the artist make to maximize her revenue?

Answer:

Our first step is to define the objective for this problem.

Our objective is to maximize revenue:

Obj.: Max. Revenue

Now, we need a function defining revenue.

We'll let the variable x denote the number of prints sold.

We'll first construct a sales/demand function for the business.

The question says that if upto 50 prints are sold, then the price is \$400 per print. If *more than* 50 prints are sold, then the price per print is reduced by \$5 for each print sold in excess of 50.

For examples, if 51 prints are sold, then the price per print will be $\$400 - \$5 = \$395$. If 52 prints are sold, then the price per print will be $\$400 - \$10 = \$390$.

We can summarize this information in the following demand function:

$$p(x) = \left\{ \begin{array}{ll} 400 & x \leq 50 \\ 400 - 5(x - 50) & x > 50 \end{array} \right\}.$$

Now that we have the demand function, which gives us a formula for price in terms of x, we can construct the revenue function.

Remember that revenue = price * sales. So, we get that revenue is $p(x) * x$. This gives a formula for revenue:

$$R(x) = \left\{ \begin{array}{ll} 400x & x \leq 50 \\ (400 - 5(x - 50))x & x > 50 \end{array} \right\}.$$

or,

$$R(x) = \left\{ \begin{array}{ll} 400x & x \leq 50 \\ 400x - 5x^2 + 250x & x > 50 \end{array} \right\}.$$

or,

$$R(x) = \left\{ \begin{array}{ll} 400x & x \leq 50 \\ 650x - 5x^2 & x > 50 \end{array} \right\}.$$

So now we have a revenue function.

There are no relevant constraints in this problem. Since the objective function is already a function of one variable, we do not need to simplify it any further.

Our objective is:

Max.

$$R(x) = \left\{ \begin{array}{ll} 400x & x \leq 50 \\ 650x - 5x^2 & x > 50 \end{array} \right\}.$$

To do this, we'll take the first derivative:

$$R'(x) = \left\{ \begin{array}{ll} 400 & x \leq 50 \\ 650 - 10x & x > 50 \end{array} \right\}.$$

and set $R'(x) = 0$.

Now, $400 \neq 0$, so there cannot be any maxima when $x \leq 50$.

We look at the second part of the derivative:

$650 - 10x = 0 \implies x = 65$.

So there is a possible maximum where $x = 65$. To verify this, we'll take the second derivative:

$$R''(x) = \left\{ \begin{array}{ll} 0 & x \leq 50 \\ -10 & x > 50 \end{array} \right\}.$$

and note that at $x = 65$, $R''(65) = -10 < 0$. So $R(x)$ is concave down when $x = 65$, which means that $x = 65$ is a max.

So, we get that the artist can maximize revenue by selling 65 prints.

$\square$

Section 2.7, Question 15: In the planning of a sidewalk cafe, it is estimated that for 12 tables the daily profit will be \$10 per table. Because of overcrowding, for each additional table the profit per table will be reduced by \$.50. How many tables should be provided to maximize the profit from the cafe?

Answer: Our first step is to define the objective for this problem.

Our objective is to maximize profit:

Obj.: Max. Profit

Now, we need a function defining profit.

We'll let the variable x denote the number of tables provided.

The question says that if upto 12 tables are provided, then the profit is \$10 per table. If *more than* 12 tables are provided, then the profit per table is reduced by \$.50 for each table provided in excess of 12.

For examples, if 11 tables are provided, then the profit per table will be $\$12 - \$.50 = \$11.50$. If 12 tables are provided, then the profit per table will be $\$12 - \$1 = \$11$.

We can summarize this information in the following profit function:

$$\Pi(x) = \left\{ \begin{array}{ll} 10x & x \leq 12 \\ (10 - 0.5(x - 12))x & x > 12 \end{array} \right\}.$$

or,

$$\Pi(x) = \left\{ \begin{array}{ll} 10x & x \leq 12 \\ 10x - 0.5x^2 + 6x & x > 12 \end{array} \right\}.$$

or,

$$\Pi(x) = \left\{ \begin{array}{ll} 10x & x \leq 12 \\ 16x - 0.5x^2 & x > 12 \end{array} \right\}.$$

There are no relevant constraints in this problem. Since the objective function is already a function of one variable, we do not need to simplify it any further.

Our objective is:

Max.

$$\Pi(x) = \left\{ \begin{array}{ll} 10x & x \leq 12 \\ 16x - 0.5x^2 & x > 12 \end{array} \right\}.$$

To do this, we'll take the first derivative:

$$\Pi'(x) = \left\{ \begin{array}{ll} 10 & x \leq 12 \\ 16 - x & x > 12 \end{array} \right\}.$$

and set $\Pi'(x) = 0$.

Now, $10 \neq 0$, so there cannot be any maxima when $x \leq 12$.

We look at the second part of the derivative:

$16 - x = 0 \implies x = 16$.

So there is a possible maximum where $x = 16$. To verify this, we'll take the second derivative:

$$\Pi''(x) = \left\{ \begin{array}{ll} 0 & x \leq 12 \\ -1 & x > 12 \end{array} \right\}.$$

and note that at $x = 16$, $\Pi''(16) = -1 < 0$. So $\Pi(x)$ is concave down when $x = 16$, which means that $x = 16$ is a max.

So, we get that the profit can be maximized by providing 16 tables.

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