Section 1.8, Question 13: A toy rocket fired straight up into the air has height $s(t) = 160t - 16t^2$ feet after t seconds.

1. What is the rocket's initial velocity?
2. What is the velocity after 2 seconds?
3. What is the acceleration when $t = 3$?
4. At what time will the rocket hit the ground?
5. At what velocity will the rocket be traveling just as it smashes into the ground?

Answer: Remember that the rate of change of a function $f(x)$ when $x = a$ is $f'(a)$.

Also, velocity is defined to be the rate of change of displacement. Acceleration is defined to be the rate of change of velocity.

In this question, we will interpret displacement as height from the ground. So, our displacement function is $s(t) = 160t - 16t^2$.

So, the velocity function is given by

$$v(t) = \frac{d}{dt}[160t - 16t^2]$$

$$= \frac{d}{dt}[160t] - \frac{d}{dt}[16t^2]$$

$$= \frac{d}{dt}[160t] - 16\frac{d}{dt}[t^2]$$

$$= 160 - 16(2t)$$

$$= 160 - 32t.$$

Now we can answer parts **1** and **2**.

The initial velocity is the velocity when the rocket leaves the ground. This happens when $t = 0$. So to find the initial velocity, we compute:

$$v(0) = (160 - 32t)|_{t=0} = (160 - 32(0)) = 160ft/s.$$

The velocity after two seconds is the velocity when $t = 2$. We compute:

$$v(2) = (160 - 32t)|_{t=2} = (160 - 32(2)) = 160 - 64 = 96ft/s.$$

To answer part **3** we need to find the acceleration function. Since acceleration is the rate of change of velocity, we can find this by taking the derivative:

$$a(t) = \frac{d}{dt}[v(t)]$$

$$= \frac{d}{dt}[160 - 32t]$$

$$= -32.$$

So the acceleration is equal to $-32ft/s^2$. It is actually independent of t, which means that it is the same, no matter what the value of t is.

So, we get that when $t = 3$, acceleration is $-32ft/s^2$.

Now we'll solve part **4**.

We need to figure out what it means for the rocket to hit the ground using our equations. When the rocket hits the ground, we know that the height of the rocket is 0. So we can try and set $s(t) = 0$ and see if we can solve for t.

So,

$$s(t) = 0$$

$$160t - 16t^2 = 0$$

$$t(160 - 16t) = 0$$

so, the rocket has height 0 when either $t = 0$ or $160 - 16t = 0 \implies t = 10$.

When $t = 0$, the rocket was being fired off the ground. So this must mean that the rocket returned to the ground when $t = 10$.

To answer part **5**, we need to find the velocity when the rocket hits the ground. But this happened when $t = 10$. So we can answer the question by finding

$$v(10) = (160 - 32t)|_{t=10} = (160 - 32(10)) = 160 - 320 = -160ft/s.$$

Note in this question that the answers to parts **3** and **5** were negative. Velocity and acceleration are *oriented* quantities. Their positive direction is upwards, moving upwards away from the ground. We can interpret the answer to part **5** as meaning that the rocket was traveling at $160ft/s$ in the negative or downward direction. Similarly, we can interpret the acceleration being $-32ft/s^2$ as meaning that the rocket was being accelerated in the negative or downward direction (by gravity).

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Section 1.8, Question 19: If $f(100) = 5000$ and $f'(100) = 10$, estimate each of the following:

1. $f(101)$
2. $f(100.5)$
3. $f(99)$
4. $f(98)$
5. $f(99.75)$

Answer: To solve this question, we are going to use the formula:

$$f(a+h) - f(a) \cong f'(a)h$$

.

The formula says that if we want to approximate the change in the value of $f(x)$ between $x = a$ and $x = a + h$, we can use the estimate $f'(a)h$.

Note that we can rewrite the formula as follows:

$$f(a+h) \cong f(a) + f'(a)h.$$

This gives us a way of approximating $f(a+h)$ if we know the values of $f(a), f'(a)$ and h.

Let's use this idea to try and solve this problem.

To solve part 1, let's set $a = 100, h = 1$. Then the formula says that

$$f(100+1) \cong f(100) + f'(100)1$$

or,

$$f(101) \cong 5000 + 10 = 5010.$$

So our estimate for the value of $f(101)$ is 5010.

Similarly, for part 2, we can set $a = 100, h = 0.5$. Then the formula says that

$$f(100+0.5) \cong f(100) + f'(100)0.5$$

or,

$$f(100.5) \cong 5000 + 10(0.5) = 5005.$$

So our estimate for the value of $f(100.5)$ is 5005.

Part 3 involves a little more care. This time, we are trying to estimate $f(99)$, and $99 < 100$. We can write

$$99 = 100 - 1 = 100 + (-1).$$

Using this method of writing 99 as $100+(-1)$, we can use the formula by setting $a = 100$ and

$$h = -1.$$

The formula,

$$f(a+h) \cong f(a) + f'(a)h$$

then gives:

$$f(99) = f(100 + (-1)) \cong f(100) + f'(100)(-1) = 5000 + 10(-1) = 4990.$$

The important point in this part of the question is that we can set h to have a negative value! This allows us to approximate values of $f(x)$ where x is less than a.

Similarly, to solve part 4, we can set $a = 100$ and

$$h = -2$$

The formula,

$$f(a+h) \cong f(a) + f'(a)h$$

then gives:

$$f(98) = f(100 + (-2)) \cong f(100) + f'(100)(-2) = 5000 + 10(-2) = 4980.$$

You should try solving part 5 by yourself!

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