Question: Expand $\ln(\frac{x^3}{y^5z^7})$.

Solution:

$\ln(\frac{x^3}{y^5z^7})$

$= \ln(x^3) - \ln(y^5z^7)$

$= \ln(x^3) - [\ln(y^5z^7)]$

$= \ln(x^3) - [\ln(y^5) + \ln(z^7)]$

$= \ln(x^3) - \ln(y^5) - \ln(z^7)$

$= 3\ln(x) - 5\ln(y) - 7\ln(z)$

□

Question: Express as a single logarithm: $\ln 3 + \frac{1}{3}\ln 8$.

Solution:

$\ln 3 + \frac{1}{3}\ln 8$

$= \ln 3 + \ln(8^{\frac{1}{3}})$

$= \ln(3(8^{\frac{1}{3}}))$

$= \ln(3(\sqrt[3]{8}))$

$= \ln(3(2))$

$= \ln(6)$

□

Question: Find the limit: $\lim_{x\to\infty}[\ln(10+x) - \ln(4+x)]$.

Solution:

$\lim_{x\to\infty}[\ln(10+x) - \ln(4+x)]$

Notice that just evaluating each limit and subtracting gives $\infty - \infty$, which is undefined. We'll try and work around this as follows:

$= \lim_{x\to\infty}[\ln(\frac{10+x}{4+x})]$

Since $\ln(x)$ is continuous for $x > 0$, the limit and the ln commute:

$= [\ln(\lim_{x\to\infty}\frac{10+x}{4+x})]$

Multiplying both numerator and denominator by $\frac{1}{x}$:

$= [\ln(\lim_{x\to\infty}\frac{\frac{10}{x}+1}{\frac{4}{x}+1})]$

$= [\ln(\frac{\lim_{x\to\infty}\frac{10}{x}+1}{\lim_{x\to\infty}\frac{4}{x}+1})]$

$= [\ln(\frac{1}{1})]$

$= [\ln(1)]$

$= 0$

□

Question: Differentiate: $f(x) = \ln(\sin^2 x)$.

Solution:

$f'(x) = \frac{d}{dx}[\ln(\sin^2 x)]$

$f'(x) = \frac{1}{\sin^2 x}\frac{d}{dx}[\sin^2 x]$

$f'(x) = \frac{1}{\sin^2 x}2\sin x\frac{d}{dx}[\sin x]$

$f'(x) = \frac{1}{\sin^2 x}2\sin x\cos x$

$\square$

Question: Use logarithmic differentiation to differentiate: $y = \frac{(x+1)^4(x-5)^3}{(x-3)^8}$.

Solution:

$y = \frac{(x+1)^4(x-5)^3}{(x-3)^8}$

$\ln(y) = \ln(\frac{(x+1)^4(x-5)^3}{(x-3)^8})$

$\ln(y) = \ln((x+1)^4(x-5)^3) - \ln((x-3)^8)$

$\ln(y) = \ln((x+1)^4) + \ln((x-5)^3) - \ln((x-3)^8)$

$\ln(y) = 4\ln((x+1)) + 3\ln((x-5)) - 8\ln((x-3))$

$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[4\ln((x+1)) + 3\ln((x-5)) - 8\ln((x-3))]$

$\frac{1}{y}\frac{d}{dx}[y] = \frac{d}{dx}[4\ln((x+1))] + \frac{d}{dx}[3\ln((x-5))] - \frac{d}{dx}[8\ln((x-3))]$

$\frac{1}{y}\frac{d}{dx}[y] = 4\frac{d}{dx}[\ln((x+1))] + 3\frac{d}{dx}[\ln((x-5))] - 8\frac{d}{dx}[\ln((x-3))]$

$\frac{1}{y}\frac{d}{dx}[y] = 4\frac{1}{(x+1)}\frac{d}{dx}[x+1] + 3\frac{1}{x-5}\frac{d}{dx}[x-5] - 8\frac{1}{x-3}\frac{d}{dx}[x-3]$

$\frac{1}{y}\frac{d}{dx}[y] = 4\frac{1}{(x+1)} + 3\frac{1}{(x-5)} - 8\frac{1}{(x-3)}$

$\frac{d}{dx}[y] = y[4\frac{1}{(x+1)} + 3\frac{1}{(x-5)} - 8\frac{1}{(x-3)}]$

$\frac{d}{dx}[y] = \frac{(x+1)^4(x-5)^3}{(x-3)^8}[4\frac{1}{(x+1)} + 3\frac{1}{(x-5)} - 8\frac{1}{(x-3)}]$

$\square$

Question: Evaluate the integral: $\int_0^3 \frac{1}{5x+1}dx$.

Solution:

$\int_0^3 \frac{1}{5x+1}dx$

Try a substitution $u = 5x+1$; then $\frac{du}{dx} = 5$, and $dx = \frac{du}{5}$:

$= \int_{x=0}^{x=3} \frac{1}{5u}du$

$= \frac{1}{5}\int_{x=0}^{x=3} \frac{1}{u}du$

$= \frac{1}{5}[\ln|u|]_{x=0}^{x=3}$

$= \frac{1}{5}[\ln|5x+1|]_{x=0}^{x=3}$

$= \frac{1}{5}\{\ln|16| - \ln|1|\}$

$= \frac{1}{5}\ln|16|$

$$= \frac{1}{5} \ln 16$$

$\square$