A pharmacist wants to establish an optimal inventory control policy for a new antibiotic that requires refrigeration in storage.

The pharmacist expects to sell 800 packages of this antibiotic at a steady rate during the next year. She plans to place several orders of the same size spaced equally throughout the year.

The ordering cost for each delivery is \$16, and carrying costs, based on the average number of packages in inventory, amount to \$4 per year for one package.

1. Let x be the order quantity and r the number of orders placed during the year. Find the inventory cost (ordering cost plus carrying cost) in terms of x and r.

Answer. Remember that the inventory cost is ordering cost + carrying cost. We can abbreviate this as:

$$I = O + C$$

where I stands for inventory cost, O stands for ordering cost and C stands for carrying cost.

We'll find C and O separately, and then add them together to get I.

Remember that the ordering cost is \$16 per order. If the pharmacist places r orders in the year, each will cost her \$16, for a total ordering cost of:

$$O = \$16r.$$

Now we'll compute the carrying cost.

The carrying cost is based on the average number of packages in inventory over the year.

At the beginning of an order cycle, the pharmacist will have x packages in inventory. The inventory will steadily decline (in a linear fashion) until the inventory is 0, at which point the pharmacist reorders x packages, and another cycle begins.

So the average inventory level over each of the order cycles is $\frac{x+0}{2} = \frac{x}{2}$.

Since all the order cycles are identical throughout the year, the average inventory over the year is the same as the average inventory over each order cycle. So, the average inventory over the year is also $\frac{x}{2}$, where x is the order size.

Lastly, the carrying cost is \$4 per package in the average inventory level.

So, the carrying cost for the year is:

$$C = \$4(\frac{x}{2}) = \$2x.$$

Finally, we get the inventory cost:

$$I = C + O = 16r + 2x.$$

2. Find the constraint function.

Answer. The constraint in this question arises from the fact that the pharmacist expects to sell 800 packages over the year.

Note that if the pharmacist places orders of size x, then she sells x packages per order cycle. If she places r orders in the year, then there are r order cycles, and so she sells $x * r$ packages over the year.

Combining the last two pieces of information gives that:

$$x * r = 800$$

3. Determine the economic order quantity (x) that minimizes the inventory cost, and then find the minimum inventory cost.

Answer. In this question, we are being asked to solve an optimization problem.

The optimization problem is of the minimization type, and the quantity to be minimized is I, the inventory cost.

So we can write the objective of the problem as follows:

$$minimize : I = 16r + 2x.$$

The constraint under which this minimization is to be performed is that:

$$x * r = 800.$$

We can incorporate the constraint and simplify the objective by solving for r:

$$x * r = 800 \implies r = \frac{800}{x}$$

and substituting for r in the objective:

$$minimize : I = 16(\frac{800}{x}) + 2x.$$

To minimize the objective function, our first step is to take the first derivative:

$$\frac{dI}{dx} = -16(\frac{800}{x^2}) + 2$$

and set it equal to zero:

$$\frac{dI}{dx} = -16(\frac{800}{x^2}) + 2 = 0$$

$$\implies 2 = 16(\frac{800}{x^2})$$

$$\implies x^2 = 16(\frac{800}{2})$$

$$\implies x^2 = 8(800)$$

$$\implies x^2 = 6400$$

$$\implies x = \pm 80.$$

We can discard the negative solution, because we cannot have a negative order quantity.

So, we get that $x = 80$.

Lastly, we need to verify that $x = 80$ is indeed a minimum.

We'll do this by using the second derivative test:

$$\frac{d^2I}{dx^2} = 2(16)\frac{800}{x^3}$$

When $x = 80$, we see that:

$$\frac{d^2I}{dx^2}|_{x=80} = 2(16)\frac{800}{(80)^3} > 0$$

which means that I is concave up at $x = 80$, and so $x = 80$ must be a minimum for I.

So, we conclude that I is minimized when:

1. $x = 80$ packages
2. $r = 10$ orders
3. $I = 16(10) + 2(80) = \$320$.

$\square$