Math 1300 Fall 2013
Friday August 23 2013
Exercises

1. By winning a lottery, you will receive \$1000 a month for the next 5 years. If interest rates are 1.8% compounded monthly, how much is this sequence of payments worth today? Why would the lottery prefer to pay the \$60,000 in monthly installments over 5 years rather than pay it as a lump sum today? How much does it gain by doing this?

Solution: We are valuing a sequence of $n = 5 \times 12 = 60$ payments of $R = \$1000$ each, when we have access to an account paying .018 per year, or $i = .018/12 = .0015$ per interest period (which is a month).

Using the formula

$$PV = \frac{R(1-(1+i)^{-n})}{i},$$

we get that the present value is $\$57,338.12$.

The lottery benefits by paying the $\$60,000$ in monthly installments because it needs less money today to pay this amount over time, as compared to making one lump sum payment of $\$60,000$ today. In fact, it saves $\$60,000 - \$57,338.12 = \$2,661.88$ today by doing so.

2. Your company is investing in a sinking fund to replace a warehouse 5 years from now. The warehouse costs \$8 million to replace today, but this estimate will increase by 5% every year. The sinking fund earns 3.6% interest compounded monthly. How much should you pay into the fund every month in order to be able to afford the replacement?

Solution:

To afford the replacement today, we would need \$8 million. However, since the estimate increases by 5% year on year, the cost will be $(1.05) \times \$8$ million next year, $(1.05)^2 \times \$8$ million two years from now, and so on, giving us a replacement cost of $(1.05)^5 \times \$8$ million $= \$10,210,252.50$ five years from now.

To be able to have this much in its account, your company is making regular monthly deposits into an increasing annuity with the goal of having exactly $\$10,210,252.50$ in five years (or $n = 5 \times 12 = 60$ interest periods). The account pays .036 per year, but $.036/12 = .003$ per interest period, which is a month.

Rearranging the formula

$$F = \frac{R((1+i)^n - 1)}{i}$$

for an increasing annuity, we get that

$$R = \frac{Fi}{((1+i)^n - 1)}$$

, and plugging in to the formula, we get that your company should be depositing $R = \$155,569.16$ every month into the increasing annuity. (We have rounded *up* to the next cent to make sure there is no shortfall.)