Math 1700 Summer 2013
Quiz 5
Thursday June 20 2013
No Work = No Credit

Name: ____________________ **Student Number:** ____________________

1. (5 points) Evaluate the integral $\int \tan^4(x)\sec^6(x)dx$.

Solution:

1. $= \int \sec^4(x)\tan^4(x)\text{sec}^2(x)dx$

2. $= \int (\sec^2(x))^2\tan^4(x)\text{sec}^2(x)dx$

3. $= \int (\tan^2(x)+1)^2\tan^4(x)\text{sec}^2(x)dx$

4. $= \int (u^2+1)^2u^4du = \int (u^4+1+2u^2)u^4du = \int u^8+u^4+2u^6du$

5. $= \frac{u^9}{9}+\frac{u^5}{5}+2\frac{u^7}{7} = \frac{(\tan(x))^9}{9}+\frac{(\tan(x))^5}{5}+2\frac{(\tan(x))^7}{7}+C$

2. (5 points) Evaluate the integral $\int \frac{x}{\sqrt{x^2+2x-3}}dx$.

Solution:

First, complete the square as follows:

$x^2+2x-3 \equiv (x-c)^2+d = x^2-2cx+c^2+d$

Hence,

$-2c = 2 \implies c = -1$ and,

$c^2+d = -3 \implies 1+d = -3 \implies d = -4.$

Hence $x^2+2x-3 = (x+1)^2-4.$

$= \int \frac{x}{\sqrt{(x+1)^2-4}}dx$

This suggests the substitution $x+1 = 2\sec(\theta)$, and so we get:

$dx = 2\sec(\theta)\tan(\theta)d\theta$,
$(x+1)^2 = 4\sec^2(\theta)$,
$(x+1)^2-4 = 4\sec^2(\theta)-4 = 4(\sec^2(\theta)-1) = 4\tan^2(\theta)$, and

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$\sqrt{(x+1)^2-4} = 2\tan(\theta)$.

$= \int \frac{2\sec(\theta)-1}{2\tan(\theta)} 2\sec(\theta)\tan(\theta)d\theta$

$= \int (2\sec(\theta)-1)\sec(\theta)d\theta$

$= \int (2\sec^2(\theta)-\sec(\theta))d\theta$

$= 2\tan(\theta) - \ln|\sec(\theta)+\tan(\theta)|$

Since

$$x+1 = 2\sec(\theta),$$

we have

$$\sec(\theta) = \frac{x+1}{2}.$$

This implies that $\cos(\theta) = \frac{2}{x+1}$. Drawing a right triangle with base angle θ shows that:

$$\tan(\theta) = \frac{\sqrt{(x+1)^2-4}}{2},$$

so that

$$2\tan(\theta) = \sqrt{(x+1)^2-4}.$$

Hence we get that the integral is:

$$= \sqrt{(x+1)^2-4} - \ln\left|\frac{x+1}{2} + \frac{\sqrt{(x+1)^2-4}}{2}\right| + C$$