Section 1.7, Question 25: Compute $\frac{d^2}{dx^2}[3x^3 - x^2 + 7x - 1]|_{x=2}$

Answer: Let's recall that

- the expression $\frac{d^2}{dx^2}[\ldots]$ asks us to find the second derivative of the expression in the brackets.
- the expression $[\ldots]|_{x=2}$ asks us to evaluate the expression in the brackets by substituting 2 for x.

We'll first compute $\frac{d^2}{dx^2}[3x^3 - x^2 + 7x - 1]$. Let's first take the first derivative:

$$\frac{d}{dx}[3x^3 - x^2 + 7x - 1]$$

$$= \frac{d}{dx}[3x^3] + \frac{d}{dx}[-x^2] + \frac{d}{dx}[7x] + \frac{d}{dx}[-1]$$

(sum rule)

$$= 3 * \frac{d}{dx}[x^3] + (-1) * \frac{d}{dx}[x^2] + \frac{d}{dx}[7x] + \frac{d}{dx}[-1]$$

(constant multiple rule)

$$= 3 * (3x^2) + (-1) * (2x) + \frac{d}{dx}[7x] + \frac{d}{dx}[-1]$$

$$= 3 * (3x^2) + (-1) * (2x) + 7 + \frac{d}{dx}[-1]$$

(since $7x$ is a linear function with slope 7)

$$= 3 * (3x^2) + (-1) * (2x) + 7 + 0$$

(since -1 is a constant function)

$$= 3 * (3x^2) + (-1) * (2x) + 7$$

So the first derivative is

$$z = 3 * (3x^2) + (-1) * (2x) + 7 = 9x^2 - 2x + 7$$

To find the second derivative, we compute $\frac{dz}{dx}$:

$$\frac{dz}{dx} = \frac{d}{dx}[9x^2 - 2x + 7]$$

$$= \frac{d}{dx}[9x^2] + \frac{d}{dx}[-2x] + \frac{d}{dx}[7]$$

(sum rule)

$$= 9 * \frac{d}{dx}[x^2] + \frac{d}{dx}[-2x] + \frac{d}{dx}[7]$$

(constant multiple rule)

$$= 9 * (2x) + \frac{d}{dx}[-2x] + \frac{d}{dx}[7]$$

$$= 9 * (2x) + (-2) + \frac{d}{dx}[7]$$

(since $-2x$ is a linear function with slope -2)

$$= 9 * (2x) + (-2) + 0$$

(since 7 is a constant function)

So,

$$\frac{d^2}{dx^2}[3x^3 - x^2 + 7x - 1] = \frac{dz}{dx} = 18x - 2.$$

To finish the question, we need to evaluate this expression by setting $x = 2$:

$$\frac{d^2}{dx^2}[3x^3 - x^2 + 7x - 1]|_{x=2} = (18x - 2)|_{x=2} = 18(2) - 2 = 36 - 2 = 34.$$

□