**Question:** Evaluate the integral $\int \sin^{-1}(x)dx$.

**Solution:**

Notice that in the problem $\int \sin^{-1}(x)dx$, there is not a natural choice of two functions $f(x)$ and $g(x)$ - it seems there is only one function there! We can be a little devious about this and write:

$\int \sin^{-1}(x)dx = \int 1 \cdot \sin^{-1}(x)dx$

In the formula $\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$,

we can then choose $f(x) = \sin^{-1}(x)$ and $g(x) = 1$. This gives $g(x) = x$ and $f'(x) = \frac{1}{\sqrt{1-x^2}}$.

Plugging into the parts formula, we get:

$I = \int 1 \cdot \sin^{-1}(x)dx = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}dx$

Let's define $J = \int \frac{x}{\sqrt{1-x^2}}dx$.

By the substitution $u(x) = \sqrt{1-x^2}$, we get that $J = -\sqrt{1-x^2}$ (**you should work this step out yourself!**).

So, plugging back in to $I$, we get that:

$I = \int 1 \cdot \sin^{-1}(x)dx = x\sin^{-1}(x) + \sqrt{1-x^2} + C$

$\square$

**Question:** Evaluate the integral $\int_0^1 xe^x dx$.

**Solution:**

Definite integrals can be solved by parts in the same way that we did indefinite integrals in class; we just modify the formula to include the limits:

$$\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b g(x)f'(x)dx.$$

For this problem, we can choose $f(x) = x$ and $g'(x) = e^x$. This gives $g(x) = e^x$ and $f'(x) = 1$.

Plugging into the parts formula, we get:

$I = \int_0^1 xe^x dx = xe^x]_0^1 - \int_0^1 e^x dx$

$= xe^x]_0^1 - e^x]_0^1$

$= [1e^1 - 0e^0] - [e^1 - e^0]$

$= e - e + 1$

$= 1$

$\square$