

**Question:** Find the values of  $x, y, z$  that maximize  $3x + 5y + z - x^2 - y^2 - z^2$ , subject to the constraint  $6 - x - y - z = 0$ .

First, write the objective function as  $F(x, y, z) = 3x + 5y + z - x^2 - y^2 - z^2$  and the constraint as  $G(x, y, z) = 6 - x - y - z$ .

Then form the Lagrangian  $L(x, y, z, \lambda) = F(x, y, z) + \lambda G(x, y, z) = 3x + 5y + z - x^2 - y^2 - z^2 + \lambda(6 - x - y - z)$ .

Next, set  $L_x = 0, L_y = 0, L_z = 0, L_\lambda = 0$  and solve this simultaneous system for  $x, y, z, \lambda$ .

By eliminating  $\lambda$  in the equations  $L_x = 0, L_y = 0$ , we get that  $3 - 2x = 5 - 2y \implies y = 1 + x$ .

By eliminating  $\lambda$  in the equations  $L_x = 0, L_z = 0$ , we get that  $3 - 2x = 1 - 2z \implies z = x - 1$ .

Substituting these into the equation  $L_\lambda = 0$ , we get  $6 - x - (1 + x) - (x - 1) = 0 \implies 6 - 3x = 0 \implies x = 2$ .

So, we get  $x = 2, y = 3, z = 1$ .