Section 6.5, Question 13: Find the consumer surplus for the demand curve $p = \frac{500}{x+10} - 3$ at the sales level $x = 40$.

Hints: We'll use the following fact to solve this question:

Fact 1. *The consumers' surplus in a market with demand function $p = f(x)$ and sales level A is:*

$$CS = \int_0^A [f(x) - B]dx$$

where the market price is $B = f(A)$.

We'll use this fact with:

1. Demand function $p = \frac{500}{x+10} - 3$
2. Sales level $A = 40$
3. Market price $B = f(40) = \frac{500}{(40)+10} - 3 = \frac{500}{50} - 3 = 10 - 3 = 7$

By the fact, the consumers' surplus is:

$$CS = \int_0^{40} [\frac{500}{x+10} - 3 - 7]dx$$

$$CS = \int_0^{40} [\frac{500}{x+10} - 10]dx$$

We'll solve this definite integral using the following three steps:

1. Find the bounds of integration. In this question, we have $a = 0, b = 40$.
2. Find an antiderivative. In this question, we'll need the following additional fact:

   Fact 2. *If m and c are any constants, then:*

   $$\int \frac{1}{mx+c}\,dx = \frac{\ln(mx+c)}{m} + C$$

   where C is an arbitrary constant.

So, we can find the family of antiderivatives:

$$\int \frac{500}{x+10} - 10dx$$

$$\int \frac{500}{x+10}dx - \int 10dx$$

$$500\int \frac{1}{x+10}dx - 10\int 1dx$$

$$500\text{ln}(x+10) - 10x + C$$

So, we can set $C = 0$ and take $F(x) = 500\text{ln}(x+10) - 10x$ as our antiderivative.

3. Compute $F(b) - F(a)$.

   We get:

$$F(b) - F(a)$$

$$F(40) - F(0)$$

$$[500\text{ln}((40)+10) - 10(40)] - [500\text{ln}((0)+10) - 10(0)]$$

$$[500\text{ln}(50) - 400] - [500\text{ln}(10)]$$

$$500\text{ln}(50) - 400 - 500\text{ln}(10)$$

$$500(\text{ln}(50) - \text{ln}(10)) - 400$$

$$500(\text{ln}(5)) - 400 \cong 404.71896$$

So, the consumers' surplus is \$404.72.

$\square$