Math 1400 Fall 2011
Quiz 11
December 7, 2011
No Work = No Credit

Name: ______________________ **Student Number:** ______________________

1. (5 points) Find the area of the region between the curves $y = 2x^2$ and $y = 8$ from $x = -2$ to $x = 2$.

Solution: To solve this question, we will use the following fact:

Fact: Suppose that a(n upper) curve $y = f(x)$ lies above a (lower) curve $y = g(x)$ over the interval $[a, b]$. Then the area between the two curves over the interval $[a, b]$ is given by:

$$\int_a^b [f(x) - g(x)]dx$$

Notice that the function $y = 2x^2$ always lies below the curve $y = 8$ when $-2 \leq x \leq 2$ (we sketched this in class).

So, we will take for our upper curve the curve $y = 8$ and for our lower curve, the curve $y = 2x^2$.

Fact 1 says that the area of the region is:

$$\int_{-2}^{2} 8 - 2x^2 dx$$

To solve this, we'll use the following steps:

1. Find the bounds of integration. In this case, we have $a = -2, b = 2$.
2. Find an antiderivative.

$$\int 8 - 2x^2 \, dx$$

$$\int 8 \, dx - \int 2x^2 \, dx$$

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$$8x - \frac{2}{3}x^3 + C$$

where C is an arbitrary constant.

We can set $C = 0$, and take

$$F(x) = 8x - \frac{2}{3}x^3$$

for our antiderivative.

3. Compute $F(b) - F(a)$.

   We get:

$$F(b) - F(a) = F(2) - F(-2)$$

$$= [8(2) - \frac{2}{3}(2)^3] - [8(-2) - \frac{2}{3}(-2)^3]$$

$$= [16 - \frac{2}{3}(8)] - [-16 - \frac{2}{3}(-8)]$$

$$= [16 - \frac{2}{3}(8)] - [-16 + \frac{2}{3}(8)]$$

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$$= [16 - \frac{16}{3}] - [-16 + \frac{16}{3}]$$

$$= 16 - \frac{16}{3} + 16 - \frac{16}{3}$$

$$= \frac{2}{3}(16) + \frac{2}{3}(16)$$

$$= (2)\frac{2}{3}(16)$$

$$= \frac{64}{3}$$

So, the area of the region is $\frac{64}{3}\ units^2$.