Math 1300 Fall 2013
Quiz 2
Friday September 13 2013
No Work = No Credit

Name: ______________________ **Student Number:** ______________________

Signature: ____

Instructor: ______________________ **Section:** ______________________

Instructions: Answer all questions and show all of your work.

Problem	Points	Student's Score
1	2	
2	3	
3	5	
Total:	10	

Potentially Helpful Formulae:

$F = (1+i)^n P$

$P = \dfrac{F}{(1+i)^n}$

$\text{APY} = (1+i)^m - 1$

$F = \dfrac{(1+i)^n - 1}{i} \cdot R$

$PV = \dfrac{1-(1+i)^{-n}}{i} \cdot R$

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1. (2 points) State whether the system of equations represented by the following augmented matrix has one solution, many solutions or no solutions. If the system has one or many solutions, provide those solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x =$ $\qquad$ $y =$ $\qquad$ $z =$

> **Solution:**
> There are infinitely many solutions to this system, parametrized by the variable z:
>
> $x = -z$ $\qquad$ $y = 2 + 2z$ $\qquad$ $z = z$
>
> That is, for each value of z, we get a new solution (x, y, x), where $x = -z$ and $y = 2 + 2z$.

Name: ______________________ **Student Number:** ______________________

2. (3 points) Use the Gauss-Jordan elimination method to find all solutions for the following system of equations.

$$\begin{cases} x & & & = & -2 \\ & y & +3z & = & 5 \\ x & +y & -3z & = & 4 \end{cases}$$

Solution:

Putting these equations into augmented matrix form, we get:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 3 & 5 \\ 1 & 1 & -3 & 4 \end{array}\right]$$

Row reducing, we get:

$$\xrightarrow{R_3 := R_3 - \frac{1}{1} \times R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & -3 & 6 \end{array}\right]$$

$$\xrightarrow{R_3 := R_3 - \frac{1}{1} \times R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & -6 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 := \frac{1}{-6} \times R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & \frac{-1}{6} \end{array}\right]$$

$$\xrightarrow{R_2 := R_2 - \frac{3}{1} \times R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{11}{2} \\ 0 & 0 & 1 & \frac{-1}{6} \end{array}\right]$$

There is a unique solution, which is $(x, y, z) = (-2, \frac{11}{2}, -\frac{1}{6})$.

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3. Consider a \$200,000, 30-year mortgage with an interest rate of 4.80% compounded monthly.

   (a) (1 point) Calculate the monthly payment for this mortgage.

   > **Solution:** For mortgages, the formula $PV = \frac{1-(1+i)^{-n}}{i} \cdot R$ is applicable.
   > We have:
   >
   > 1. $PV = \$200,000$
   > 2. $n = 12 \times 30 = 360$
   > 3. $i = \frac{0.048}{12}$
   >
   > Plugging these into the formula and solving for R gives that $R = \$1049.33$.

   (b) (2 points) Calculate the amount of interest and the amount applied to principal for the first month's payment.

   > **Solution:**
   > Since the balance for the first month is $\$200,000$, and the interest rate charged for the first month is $\frac{0.048}{12} = .004$, the interest charged for the first month is $\$200,000 \times 0.004 = \$800$.
   >
   > The remainder of the monthly payment, $\$1049.33 - \$800 = \$249.33$, is used to reduce the balance.

   (c) (1 point) Calculate the unpaid balance on the mortgage after 25 years.

   > **Solution:**
   > The unpaid balance at any point in time is always exactly the present value of the remaining payments. Since there are 5 years left in the mortgage, there are $12 \times 5 = 60$ payments left. The present value of 60 payments of \$1049.33 each at a rate of 0.004 per interest period is given by using the formula $PV = \frac{1-(1+i)^{-n}}{i} \cdot R$. Solving for PV gives that the unpaid balance after 25 years is $\$55,875.67$.

   (d) (1 point) Calculate the amount of interest paid over the 30 year life of the mortgage.

Name: ____________________ **Student Number:** ____________________

Solution:
The total amount paid was $\$1049.33 \times 360 = \$377,759.06$. Of this, the principal paid was $\$200,000$. The remainder, or $\$177,759.06$, was interest.