Notes on limits: In recitation on Thursday, we will cover some examples of limits. Below I will state some facts about limits that we will use.

1. If $f(x)$ is a continuous function and $g(x)$ is any function, then $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$.

2. Let us denote the expression:

   (a) "$\lim_{x\to a} g(x)$" by ∞ if $\lim_{x\to a} g(x) = \infty$
   (b) "$\lim_{x\to a} g(x)$" by $-\infty$ if $\lim_{x\to a} g(x) = -\infty$

   Then when A is a constant finite number, we have:

   (a) $A + \infty = \infty + A = \infty$
   (b) $A - \infty = -\infty + A = -\infty$
   (c) $-A + \infty = \infty - A = \infty$
   (d) $-A - \infty = -\infty - A = -\infty$
   (e) $\infty + \infty = \infty$
   (f) $-\infty - \infty = -\infty$
   (g) $A * \infty = \infty * A = \infty$ if $A > 0$, and $A * \infty = \infty * A = -\infty$ if $A < 0$
   (h) $\infty * \infty = \infty$
   (i) $\frac{A}{\infty} = 0$
   (j) $\frac{\infty}{A} = \infty$ if $A > 0$, and $\frac{\infty}{A} = -\infty$ if $A < 0$

   There are two special cases that we will study in this course, where simple rules like those above no longer work.

   Case: $\infty - \infty$:

   Here are three examples showing that $\infty - \infty$ may equal any of $\infty, -\infty$, or a finite constant number A.

   (a) $\lim_{x\to\infty}[x^2 - x] = \lim_{x\to\infty}[x(x-1)] = \lim_{x\to\infty}[x] * \lim_{x\to\infty}[(x-1)] = \infty * \infty = \infty$
   (b) $\lim_{x\to\infty}[x - (x - A)] = \lim_{x\to\infty}[A] = A$
   (c) $\lim_{x\to\infty}[x^2 - x^3] = \lim_{x\to\infty}[x^2(1-x)] = \lim_{x\to\infty}[x^2] \lim_{x\to\infty}[(1-x)] = \infty * (-\infty) = -\infty$

   Case: ∞/∞:

   Here are four examples showing that ∞/∞ may equal any of $\infty, -\infty, 0$, or a nonzero finite constant number A.

   (a) $\lim_{x\to\infty}[x^2/x] = \lim_{x\to\infty}[x] = \infty$
   (b) $\lim_{x\to\infty}[Ax/x] = \lim_{x\to\infty}[A] = A$
   (c) $\lim_{x\to\infty}[x^2/x^3] = \lim_{x\to\infty}[1/x] = 0$
   (d) $\lim_{x\to\infty}[-x^2/x] = \lim_{x\to\infty}[-x] = -\infty$