Question: Find $\lim_{x \to 2^-} e^{3/(2-x)}$.

Solution:

We'll do this step by step.

First, we'll build up the function $e^{3/(2-x)}$ as follows:

Step	Input	Function applied	Result
1:			x
2:	x	multiply by (-1)	$-x$
3:	$-x$	add 2	$2-x$
4:	$2-x$	take reciprocal	$1/(2-x)$
5:	$1/(2-x)$	multiply by 3	$3/(2-x)$
6:	$3/(2-x)$	apply exponential function	$e^{3/(2-x)}$

where in going from each step to the next, we are applying an elementary transformation to the current result.

Now we can compute limits at each step:

Step	Input	Function applied	Result	Goes to
1:			x	2^-
2:	x	multiply by (-1)	$-x$	$(-2)^+$
3:	$-x$	add 2	$2-x$	0^+
4:	$2-x$	take reciprocal	$1/(2-x)$	$+\infty$
5:	$1/(2-x)$	multiply by 3	$3/(2-x)$	$+\infty$
6:	$3/(2-x)$	apply exponential function	$e^{3/(2-x)}$	$+\infty$

So, $\lim_{x \to 2^-} e^{3/(2-x)} = +\infty$.

$\square$

Question: Differentiate the function $y = \frac{e^x}{1-e^x}$.

Solution:

$y = \frac{e^x}{1-e^x}$

$\frac{d}{dx}[y] = \frac{d}{dx}[\frac{e^x}{1-e^x}]$

$\frac{d}{dx}[y] = \frac{(1-e^x)\frac{d}{dx}[e^x]-(e^x)\frac{d}{dx}[1-e^x]}{(1-e^x)^2}$

$\frac{d}{dx}[y] = \frac{(1-e^x)(e^x)-(e^x)(-e^x)}{(1-e^x)^2}$

$\square$

Question: Differentiate the function $y = x^2 e^{-\frac{1}{x}}$.

Solution:

$y = x^2 e^{-\frac{1}{x}}$

$\frac{d}{dx}[y] = \frac{d}{dx}[x^2 e^{-\frac{1}{x}}]$

$\frac{d}{dx}[y] = \frac{d}{dx}[x^2](e^{-\frac{1}{x}}) + (x^2)\frac{d}{dx}[e^{-\frac{1}{x}}]$

$\frac{d}{dx}[y] = (2x)(e^{-\frac{1}{x}}) + (x^2)\frac{d}{dx}[e^{-\frac{1}{x}}]$

$\frac{d}{dx}[y] = (2x)(e^{-\frac{1}{x}}) + (x^2)(e^{-\frac{1}{x}})\frac{d}{dx}[-\frac{1}{x}]$

$\frac{d}{dx}[y] = (2x)(e^{-\frac{1}{x}}) + (x^2)(e^{-\frac{1}{x}})\frac{1}{x^2}$

$\square$

Question: Evaluate the integral $\int e^x(4+e^x)^5 dx$.

Solution:

Try the substition $u = 4 + e^x$. Then $\frac{du}{dx} = \frac{d}{dx}[4 + e^x] = \frac{d}{dx}[e^x] = e^x$, and $dx = \frac{du}{e^x}$.

$= \int \frac{e^x(u)^5}{e^x} du$

$= \int (u)^5 du$

$= \frac{u^6}{6} + C$

$= \frac{(4+e^x)^6}{6} + C$

$\square$