Math 1700 Summer 2013
Quiz 7
Thursday June 27 2013
No Work = No Credit

Name: ______________________ **Student Number:** ______________________

1. (5 points) Determine whether the integral $\int_1^\infty \frac{\ln(x)}{x} dx$ is convergent or divergent. If it is convergent, evaluate it.

Solution:

Try $u = \ln(x)$; then $dx = xdu$, and:

$$= \int_{x=1}^{\infty} udu$$

$$= \left[\frac{u^2}{2}\right]_{x=1}^{\infty}$$

$$= \left[\frac{(\ln(x))^2}{2}\right]_{x=1}^{\infty}$$

$$= \lim_{t\to\infty} [\frac{(\ln(t))^2}{2} - \frac{(\ln(1))^2}{2}]$$

$$= \infty$$

So the integral diverges.

...show all work...show all work...show all work...show all work...show all work...show all work...

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2. (5 points) Find the exact length of the curve

$$y = \ln(\sec(x)),$$

where $0 \leq x \leq \frac{\pi}{4}$.

Solution:

$$y = \ln(\sec(x))$$

$$\frac{d}{dx}[y] = \frac{d}{dx}[\ln(\sec(x))]$$

$$\frac{d}{dx}[y] = \frac{\sec(x)\tan(x)}{\sec(x)}$$

$$\frac{d}{dx}[y] = \tan(x)$$

$$(\frac{d}{dx}[y])^2 = \tan^2(x)$$

$$(\frac{d}{dx}[y])^2 + 1 = \tan^2(x) + 1$$

$$(\frac{d}{dx}[y])^2 + 1 = \sec^2(x)$$

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$$\sqrt{(\frac{d}{dx}[y])^2 + 1} = \sec(x)$$

So, the arc length is:

$$\int_0^{\pi/4} \sec(x)dx$$

$$= [\ln|\sec(x) + \tan(x)|]_0^{\pi/4}$$

$$= \left[\ln|\sqrt{2} + 1| - \ln|1|\right]$$

$$= \ln|\sqrt{2} + 1|$$