

# Revisiting small signal stability analysis techniques in the context of stability study of a two-area multimachine power system

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**Abstract**—Regular load generation imbalances aka power system small disturbances in a network is always a matter of great concern which lead a major task of stability improvement to the technicians. Moreover when in place of single area network, there comes multi-area multi-machine power system, the overall dynamics becomes very integrated and complex to analyze. To suppress the oscillations of the system state variables i.e. to keep their deviation within some limit for a long period during small disturbances caused by load swinging or others, small signal analysis is needed first in place. Till date extensive researches has been done developing many toolboxes to analyze small signal stability like PSAT, SSST, PST, PAT, PSS/E, DigSILENT, EUROSTAG, SimPow and others. Among all PSAT is the most recommended and used software for the purpose. In this work a new algorithm has been developed to analyze the classical two-area four-machine system which is compared with PSAT's (Power system analysis toolbox) result and literature's result. It came out that there's a significant amount of inaccuracy in PSAT's outcome and inter-area oscillation mode is found to be the most critical zone for analysis rather than local modes of oscillations in power system network.

**Index Terms**—inter area oscillations, PSAT, small signal stability;

## I. INTRODUCTION

The main thing for any dynamic system to remain always in a stable operation is the synchronisation between each and every corner within the system. Power system network is no exception here. In a large scale power system there occur some regular disturbances which is not accidental like lightening effects or short circuit and also the amplitudes of the oscillations fr regular disturbances are relatively much smaller compared to those accidental disturbances. These type of regular disturbances may come from various reasons like high gain fast exciters, heavy power transfer over long distances from one plant to other, power transfer between two area over a weak path, inadequate tuning of controllers etc. Before taking steps towards diminishing the instability, the overall system should be modelled and analyzed first i.e. small signal stability analysis. But power system network is actually a non-linear network and as per mathematics is concerned for a non-linear system, state space modelling is not possible. Here, as the disturbances due to small disturbances are sufficiently small, based on the criteria the overall system is linearized around the stable point of

state variables by Taylor expansion method. Oscillations can be local or global i.e. inter area. When load profile suddenly changes followed by swinging of the generators inter area oscillations take place. If a system can be such that after oscillations takes place within a very short time it automatically decays by the action of dampers and settle down to a value that will not hamper power flow through transmission network. Such a system is small-signal stabilized. To analysis a system, first it should be mathematically modelled whereas for small signal stability modelling, synchronous machine modelling is the basic. Probably, there is more literature on synchronous machine than any other device. Most of the literature makes the thing complex and confusing. They uses many conventions and notations based on physical intuition, practical experience and years of experimentation. [1] [2] [3] [4] [5] [6] [7] In this article a straightforward methodology for multi-machine network stability analysis and the result has been compared to the literature's [8] results.

There are two types of oscillation local and global means inter- area oscialltions. Power system stabilizers(PSS) can damp the local mode oscillations whereas FACTs [9], STATCOMs can stabilize the inter area modes. However planfully placing PSS and appropriate tuning, inter area oscillation modes also can be damped. Previously Power system stabilizers (PSS) were not introduced to the system for better stability. In case of PSS, voltage profile of system loads and position of PSS in the network, are the deciding factors to reflect its ability to diminish local nd inter-area oscillations [10]. The effects of static var compensator(SVC) has been analyzed through eigenvalue analysis of a 10 machine 39 bus system and showed that SVC can stabilize the exciter modes after Hopf bifurcation which occurs at increase of load suddenly. [11] Paper [12] deals with thyristor controlled series capacitor (TCSCs) and static var compensators (SVCs) proposing a real time robust control system for monitoring stability based on local signals. Inter area oscillations has been compared with or without wind energy penetration in four machine two area network by PSAT. [13] A robust PSS design to damp inter area oscillations based on eigen value sensitivities which is performed on SSST matlab version SMAS3 [14]. Large

scale inter area network has been analyzed by PSS/E software. Here it's clear that ranging from frequent small to infrequent large scale power system oscillations, to analysis them an identical approach is not sufficient [15]. Effects of devices like PSS, AVR and governor were considered for time-domain simulation studies in DIgSILENT PowerFactory software. [16] Whereas a new method for analyzing inter area oscillations using graph laplacian presented. [17]

The objective of the paper is to review the popular techniques which is done using PSAT to analyze classical inter area network and do the same by a new algorithm in matlab environment and then comparing both with reference from literature. In section II, steps for mathematical modelling has been shown. In section III, importance of eigen value analysis and a flow chart to program any multi-machine network has been presented. In section III, results has been shown comparing the eigen values from PSAT and from the algorithm built in this article with respect to the reference.

## II. SYSTEM MODELLING

Consider a system with only one generator. For every generator in the power system, classic model is assumed. Parameters as per classic literature [8]. The dynamics of each subsystem will be expressed in a state-space model, where simplest synchronous machine modelling is where  $\delta$  stands for rotor angle in rad/s,  $\omega$  stands for speed in pu,  $T_m$  stands for mechanical torque developed by the shaft in pu and  $T_e$  stands for loading of the system in pu.

$$J \frac{d\omega}{dt} = T_m - T_e \quad (1)$$

$$\frac{d\delta}{dt} = \omega - \omega_0 \quad (2)$$

The model in matrix form will look like equation

$$\begin{bmatrix} \Delta\omega' \\ \Delta\delta' \end{bmatrix} = \begin{bmatrix} \frac{-K_D}{2H} & \frac{K_s}{2H} \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} \frac{1}{2H} \\ 0 \end{bmatrix} \Delta T_m \quad (3)$$

Added to the simplest model, effect of thyristor excitation with automatic voltage regulator (AVR) and power system stabilizer(PSS) bring four new state variables to the equation 4.

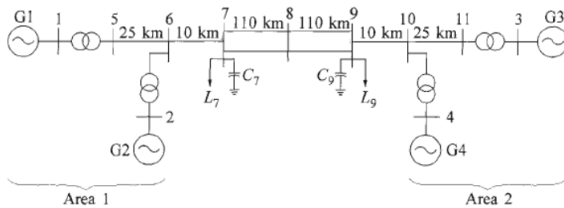


Fig. 1: classical 4 machine 11 bu system

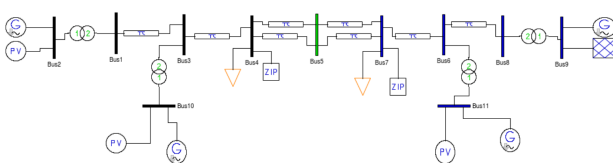


Fig. 2: PSAT simulation of the system

$$= \begin{bmatrix} \frac{-K_D}{2H} & \Delta T_e & 0 & 0 & 0 & 0 \\ \omega_s & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{T_{d0}'} & -\frac{1}{T_{q0}'} & 0 & 0 & 0 \\ 0 & \frac{1}{T_{q0}'} & -\frac{1}{T_{d0}'} & 0 & 0 & 0 \\ 0 & \frac{1}{T_{d0}'} & 0 & 0 & -\frac{1}{T_{q0}'} & 0 \\ 0 & \frac{1}{T_{q0}'} & 0 & 0 & 0 & -\frac{1}{T_{d0}'} \end{bmatrix} \begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \\ \Delta\dot{E}_q' \\ \Delta\dot{E}_d' \\ \Delta\dot{E}_q'' \\ \Delta\dot{E}_d'' \end{bmatrix} + \begin{bmatrix} \Delta\dot{T}_m \\ 0 \\ \frac{1}{T_{d0}'} \Delta E_{FD} \\ \frac{1}{T_{q0}'} \Delta E_{FD} \\ \frac{1}{T_{q0}'} \Delta E_{FD} \\ \frac{1}{T_{d0}'} \Delta E_{FD} \end{bmatrix} \quad (4)$$

Linearizing the single machine infinite bus(SMIB) against initial values of the state variables the function has been reduced to

$$\Delta\dot{x} = [A]x \quad (5)$$

In case of multimachine inter area network classical two area four machine network has been taken for analyzing. [8] fig.1.

From the network constraints using Newton-Raphson method terminal voltages is derived which is Load flow analysis. Getting the initial values, stator dynamic equations comes into the form

$$\Delta\dot{X}_i = f_i(X - i, I_{di}, I_{qi}, \theta_g, V_g) \quad (6)$$

where  $X_i$  is the state variables of the synchronous generation subsystem. Then, the stator algebraic network equations will come in the form

$$g_i(X_i, I_{di}, I_{qi}, \theta_g, V_g) = 0, \quad (7)$$

where  $i = 1, \dots, n_g$  in case of  $n_g$  generators in the system. Then, network algebraic equations will come in the form

$$h_i(X_i, I_{di}, I_{qi}, \theta_g, V_g, \theta_1, V_1) = 0, \quad (8)$$

where  $i = 1, \dots, n_g$  Load bus algebraic eqns will come into the form

$$k_i(\theta_g, V_g, \theta_1, V_1) = 0, \quad (9)$$

where  $i = n_g + 1, \dots, n$ .  $n$  is the no of total buses in the network.  $V_1$  &  $\theta_1$  is the reference of the network. Solving these equations in state space model ultimately the representation of the network will come into the form

$$\Delta\dot{X} = A_{sys}[X] + W\Delta U \quad (10)$$

Firstly, the small signal stability analysis has been carried out by Power system analysis toolbox(PSAT). fig.2.

TABLE I: Table of the results of SMIB simulation

| No | Eigen values from literature | Eigen values from the algorithm established |
|----|------------------------------|---|
| 1  | -0.171 + j6.47               | -39.3410-j0.0225                            |
| 2  | -0.171 - j6.47               | -22.2113 -j0.0021                           |
| 3  | -0.200                       | -0.2050 + j 6.3175                          |
| 4  | -2.045                       | -0.2050 -j 6.3175                           |
| 5  | -25.01                       | -0.1979 - j0.0001                           |
| 6  | -37.85                       | -1.7969 -j0.0002                            |

Then, it has been performed by an original program. A flow chart has been shown to facilitate the straight forward method to determine  $A_{sys}$  matrix of multimachine multi area network in Fig.3.

### III. IMPLICATIONS OF EIGEN VALUES AND PARTICIPATION FACTORS

Eigen values denotes the status of the system i.e. when the system is entering the oscillatory zone and to what extent. Positive eigen values means unstable mode and negative eigen values means stable mode. Participation factors states which state variable is responsible for the instability of a particular mode.

$$\psi_i[A] = \Lambda\psi_i \quad (11)$$

Participation factors shows participation of one state relative to others in the instability of a particular mode. Therefore, participation factors can tell exactly which generator is involved in this particular mode. This denotes which machine can go out of stable operation for that particular mode which will be problematic at the advent of load variation.

### IV. RESULTS

Results of SMIB simulation has been shown in Table.I. Error in eigen value positions w.r.t literature's [8] result has been calculated by the difference in real axis co-ordinates and imaginary axis co-ordinates of two most nearby positioned eigenvalue in a superimposed image of the two results. In case of SMIB there is mere 2.5 percent error.

Now the multimachine simulation result has been shown in plot in fig.4 & in table format for comparison in Table II.

Calculating and observing the participation factors of those two eigen values responsible for inter-area oscillations, it can be shown d-q axis amortisseurs flux linkages contributes towards high frequency local mode oscillations and also the change in excitation of generators from distant area largely contributes towards inter area oscillations.

### V. DISCUSSION

It can be seen, there is still a big lacuna for proposing a perfect toolbox to analyze small signal stability. More researches should be put there like [17]. Because in the coming days keeping pace with rapid depletion fossil fuels more intermittent renewable sources like solar and wind energy will penetrate the grid. In that case it will be very complex to analyze the inter area oscillations. Therefore, the popular toolboxes to analyze the eigen value of the

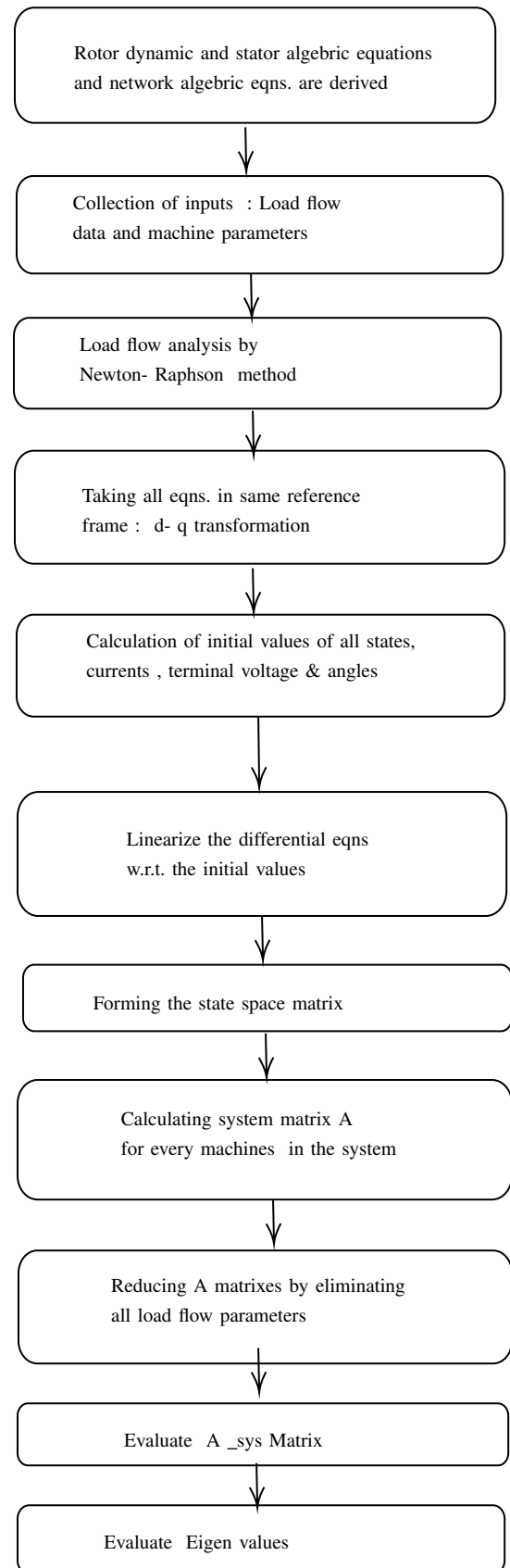


Fig. 3: Flow chart to get system characteristic matrix

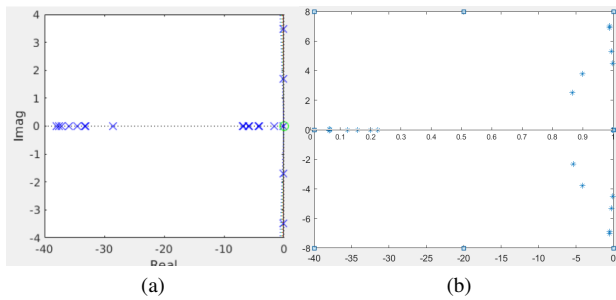


Fig. 4: Eigen value plot a. PSAT result b. newly established algorithm's output

TABLE II: Results of multi-machine simulation

| No | literature result    | PSAT result      | algorithm's result   |
|----|----------------------|------------------|----------------------|
| 1  | -0.00076<br>+j0.0022 | -28.5933 +j0     | -38.01 -j 0.038      |
| 2  | -0.00076<br>j0.0022  | -37.9218 +j0     | -38.01 +j 0.038      |
| 3  | -0.096               | -35.9073 +j0     | -37.9 -j 0.005       |
| 4  | -0.111 + j3.43       | -33.2244 +j0     | -37.9 +j0.005        |
| 5  | -0.111 - j3.43       | -37.5202 +j0     | -35.53 +j0           |
| 6  | 0.117                | -37.0992 +j0     | -34.2 +j0            |
| 7  | -0.265               | -33.1878 +j0     | -32.48 +j0           |
| 8  | -0.276               | -34.5492 + j0    | -31.5 +j0            |
| 9  | -0.492 + j 6.82      | -1.5252 +j0      | -5.35 - j2.331       |
| 10 | -0.492 - j 6.82      | -0.0332 + j3.476 | -5.5 + j2.5          |
| 11 | -0.506 +j7.02        | -0.0332 +j3.476  | -4.15 +j3.772        |
| 12 | -0.506 - j7.02       | -6.7042 +j0      | -4.15 - j3.772       |
| 13 | -3.428               | -6.9053 +j0      | -0.52 -j7.02         |
| 14 | -4.139               | -5.8999 +j0      | -0.52 +j7.02         |
| 15 | -5.287               | -4.1075 +j0      | -0.5 +j 6.9          |
| 16 | -5.303               | -0.00097 +j1.68  | -0.5 -j 6.9          |
| 17 | -31.03               | -0.00097 -j1.68  | -0.28 +j5.31         |
| 18 | -32.45               | -6.7202 + j0     | -0.265 - j5.31       |
| 19 | -34.07               | -4.2747 + j0     | -0.117 +j0           |
| 20 | -35.53               | -5.7353 + j0     | -0.111 - j4.5        |
| 21 | -37.89 +j0.142       | 0 +j0            | -0.111 + j4.5        |
| 22 | -37.89 -j0.142       | -0.00001 + j0    | -0.09 +j0            |
| 23 | -38.01 +j0.038       | 0+ j0            | -0.0009 +j<br>0.0022 |
| 24 | -38.01 -j0.038       | 0 +j0            | -0.0009 -j 0.0022    |

system and sensitivities of different state variables should be perfect with least errors possible.

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