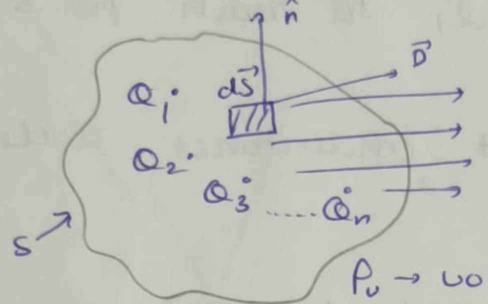
ELECTROMAGNETIC WAVES

☆ Maxwell's Equation-

1. Gauss law for Electrostatics-

The net electric flux passes through a closed circuit is equal to the net charged enclosed by closed circuit.



$\rho_v \rightarrow$ volume charge density

$$\oint\!\!\oint_S \vec{D} \cdot d\vec{S} = Q_1 + Q_2 + Q_3 + \dots + Q_n$$

$$= \iiint_V \rho_v \, dv$$

$\boxed{D = \varepsilon E}$

Medium independent · medium dependent

$$\boxed{\oint\!\!\oint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho_v \, dv}$$

↳ Integral form

Applying, the Divergence theorem,

$$\iiint_V \nabla \cdot \vec{D} \, dv = \iiint_V \rho_v \, dv$$

$\boxed{\nabla \cdot \vec{D} = \rho_v}$ Or $\boxed{\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}}$

↳ Differential form (point form)

↳ when medium is homogeneous.

* $\nabla \xrightarrow{\text{divergence}} (+)$ You are moving towards source

* $\nabla \rightarrow (-)$ You are reaching sink

* Integral form is applied when medium is highly discontinuous

* If your region / medium is homogeneous then we can use point form or differential form.

2. Gauss Law for Magnetostatics -

The magnetic flux passes through a closed circuit is 0. Because magnetic monopole does not exist. And hence magnetic charge (Net) is zero.

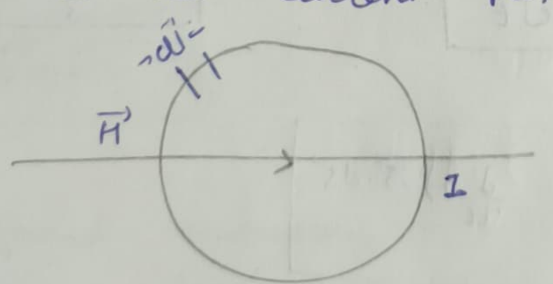
$$\boxed{\oiint \vec{B}\cdot d\vec{S} = 0}$$

Applying, divergence theorem,

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

3. Amperes Circuit Law -

The total magnetomotive force around the closed loop is equal to the net current passes through the loop.



$$\boxed{\oint_l \vec{H}\cdot d\vec{l} = I_{enc}}$$

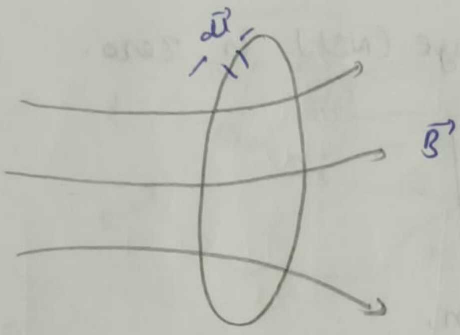
* If I is not uniform, $\boxed{\oint_l \vec{H}\cdot d\vec{l} = \iint_S \vec{J}\, d\vec{S}}$

Applying, Stoke's theorem,

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$$

4. Faraday's Law of Electromagnetic Induction-

The net EMF is equals to the rate of change of flux link with that loop.



$Emf = \vec{E} \cdot \vec{dl}$

$$\cancel{\oint_l \vec{E} \cdot \vec{dl} = \iint_S \frac{d\Phi}{dt} \cdot \vec{ds}} \quad \phi \rightarrow \text{flux}$$

$$\boxed{\oint_l \vec{E} \cdot \vec{dl} = -\frac{d\phi}{dt}} \qquad \boxed{\Phi = \iint_S \vec{B} \cdot \vec{ds}}$$

$$\boxed{\oint_l \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \iint \vec{B} \cdot \vec{ds}}$$

* Generator action → Area changes
* Transformer action → $\vec{B}$ changes

$$\boxed{\oint_l \vec{E}\cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}}$$

↳ Integral form of faraday's law of EMI

Applying Stokes Theorem,

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

↳ point form of faradays' law of EMI

* If there is time-dependent magnetic field then there must be electric field which is circulating in nature.

* <u>Maxwell Combine these four Equation-</u>

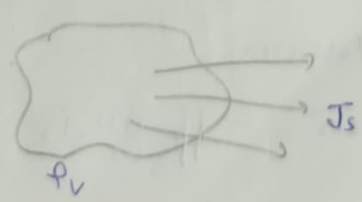
$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho_v}$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\boxed{\vec{\nabla}\times\vec{E} = \frac{\partial \vec{B}}{\partial t}}$$

$$\boxed{\vec{\nabla}\times\vec{H} = J}$$

Maxwell found inconsistency in 4th law, i.e. according to 4th law, $\vec{H}$ is only due to conductor. But there is one more factor on which $\vec{H}$ depends



$$I = \oint\!\!\oint_S \vec{J} \cdot d\vec{s}$$

$$-\frac{d}{dt} \iiint_V \rho_v \, dv = \oint\!\!\oint_S J \cdot d\vec{s}$$

$$\oint\!\!\oint_S \vec{J} \cdot d\vec{s} = -\iiint_V \frac{\partial \rho_v}{\partial t} \, dv$$

Appy, Divergence theorem,

$$\vec{\nabla} \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}$$

$$\boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0} \longrightarrow \text{Continuity Equation.}$$

Taking divergence of 4th law,

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = 0}$$ According to Ampere.

↳ It contradicts the Maxwell assumption.

Now, using Gauss law,

$$\oint\!\!\oint_S \vec{J} \cdot d\vec{s} = -\iiint_V \frac{\partial \rho_v}{\partial t} \, dv$$

$$\oint\!\!\oint_S \vec{J} \cdot d\vec{s} = -\iiint_V \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D}) \, dv$$

Taking, Divergence theorem.

$$\oint\!\!\oint_S \vec{J} \cdot d\vec{s} = -\oint\!\!\oint_S \frac{\partial D}{\partial t} \, ds$$

$$\boxed{\oiint_S \left(J + \frac{\partial \vec{D}}{\partial t}\right) d\vec{s} = 0}$$

↳ very important phenomenon for wireless communication

Here, $\frac{\partial \vec{D}}{\partial t}$ → Displacement current Density.
(This current flows ~~[illegible]~~ throw dielectric)

Now, The Modified Fourth Equation is-

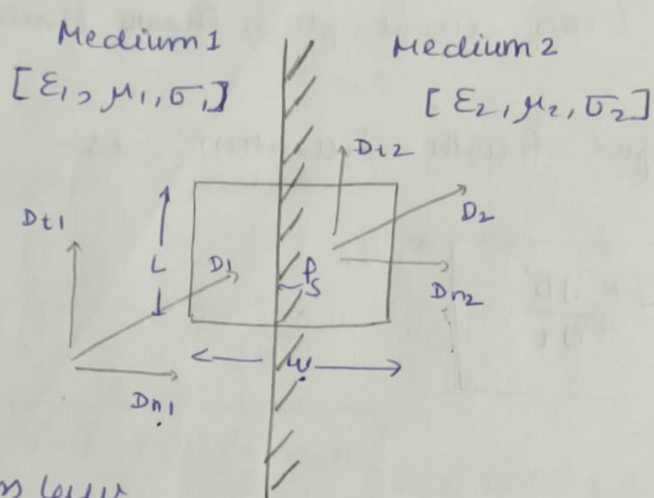
$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \quad *$$

Integral Form	Point form
(1) $\oiint_S \vec{D} \cdot d\vec{s} = \iiint_v \rho_v \, dv$	$\vec{\nabla} \cdot \vec{D} = \rho_v$
(2) $\oiint_S \vec{B} \cdot d\vec{s} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
(3) $\oint_l \vec{E} \cdot d\vec{l} = -\oiint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$	$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$
(4) $\oint_l \vec{H} \cdot d\vec{l} = \oiint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{s}$	$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

☆ Applications of Maxwell's Equations -

1. Boundary Conditions-

* Boundary conditions are applied when medium is changing abruptly. Applied in optical fibre, co-axial fibre, etc.



Applying, Gauss law,

When, $w \to 0$, Net Flux, $\Rightarrow \vec{D_{n_2}} - \vec{D_{n_1}}$

$$\boxed{\vec{D_{n_2}} - \vec{D_{n_1}} = \rho_s}$$ → when surface charges present

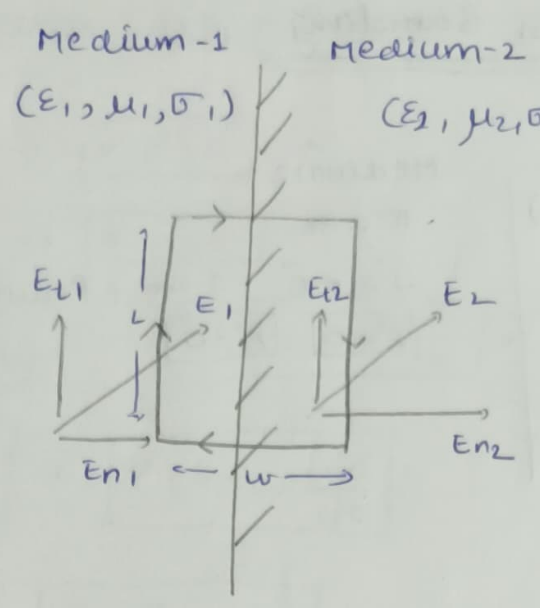
~~otherwise,~~ Otherwise,

$$\boxed{\vec{D_{n_2}} = \vec{D_{n_1}}}$$ — ①

* Normal component of electric flux density or electric displacement is cont. across the boundary if no surface charges are present (i.e. $\rho_s = 0$) otherwise it equals to the difference of two D components.

In case of Magnetic field, $\boxed{\vec{B_{n_1}} = \vec{B_{n_2}}}$ always ②

↳ it always be cont. across the boundary.

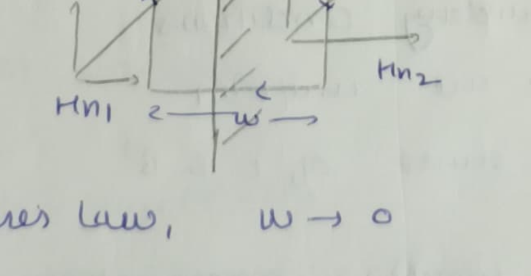


Applying the faraday's law, Assuming $w \to 0$

$$E_{t_1}L - E_{t_2}L = 0$$ ↳ As flux $\to 0$ as $w \to 0$

$$\boxed{E_{t_1} = E_{t_2}} \quad \text{— (3)}$$

* Tangential component of $\vec{E}$ always be continuous over the boundary.



Applying, Amperes Law, $w \to 0$

$$H_{t_1}L - H_{t_2}L = J(L \times W) + \frac{\partial D}{\partial t}(L \times W)$$

$$\boxed{H_{t_1} - H_{t_2} = 0} \quad \text{— (4)}$$

If surface charges present, $\boxed{H_{t_2} - H_{t_1} = J_S}$ → surface charge density.

$$\boxed{\hat{n} \times (\vec{H_2} - \vec{H_1}) = J_S}$$

2. Dielectric - Conductor Boundary - (seen in Co-axial cable)

Medium 1 $(\varepsilon_1, \mu_1, \sigma_1)$

Medium 2

$\sigma = \infty$

$\vec{J} = \sigma \vec{E}$ $\quad J \to \infty$ (Practically not Possible)

$\boxed{\vec{E} = 0}$, $\boxed{\vec{H} = 0}$

$D_{n1} = \rho_s$ (if surface charge)

$B_{n1} = 0$

$E_{t1} = 0$

$H_{t1} = J_s$

→ $\boxed{D_{n1} = \rho_s}$

→ $\boxed{B_{n1} = 0}$

→ $\boxed{E_{t1} = 0}$ ** (imp. in terms of waveguides)

→ $\boxed{H_{t1} = J_s}$

* $E_{t1} = 0$ prevents the radiation to flow out of waveguides at high freq. Hence, at high freq. waveguides are used instead co-axial cable.

* Electric & Magnetic field in the medium which is unbound, isotropic ~~to~~, homogeneous & source free -

isotropic → ε, μ are not vector quantity (Not a direction dependent)

Unbound → Not boundary conditions.

Homogeneous → ε, μ are uniform

source free → No source of $\vec{E}$ & $\vec{B}$

* generally used in wireless communication.

From Maxwell's Equations -

1. $\boxed{\nabla \cdot \vec{D} = \rho}$

2. $\boxed{\vec{\nabla} \cdot \vec{B} = 0}$

3. $\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}}$

4. $\boxed{\vec{\nabla} \times \vec{H} = J + \frac{\partial D}{\partial t}}$

Modified equations for free space - ($\rho_v = 0$, $J = 0$)

① $\boxed{\vec{\nabla} \cdot \vec{D} = 0}$ → $\boxed{\vec{\nabla} \cdot \vec{E} = 0}$ *

② $\boxed{\vec{\nabla} \cdot \vec{B} = 0}$ → $\boxed{\vec{\nabla} \cdot \vec{H} = 0}$ *

③ $\boxed{\vec{\nabla} \times \vec{E} = -\partial B/\partial t}$ → $\boxed{\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}}$ *

④ $\boxed{\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}}$ → $\boxed{\vec{\nabla} \times \vec{B} = \mu \frac{\partial \vec{D}}{\partial t}}$

↳ $\boxed{\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}}$ *

As $e^{j\omega t}$, $\frac{\partial}{\partial t} \to j\omega$

$\frac{\partial^2}{\partial t^2} \to -\omega^2$

Coupled equation
$$\boxed{\vec{\nabla} \times \vec{E} = -j\omega\mu \vec{H}}$$
$$\boxed{\vec{\nabla} \times \vec{H} = j\omega\varepsilon \vec{E}}$$

Here, $j\omega\mu$ → characteristic quantity
$j\omega\varepsilon$ - of medium

Compare it with, $\frac{\partial V}{\partial x} = -\underbrace{(R + j\omega L)}_{}I$
↳ characteristic quantity of transmission line

Taking curl on both sides.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -j\omega\mu (\vec{\nabla} \times \vec{H})$$
$$\nabla(\nabla \cdot E)^{0} - \nabla^2 E = -j\omega\mu (j\omega\varepsilon \vec{E})$$
$$\boxed{\nabla^2 \vec{E} = -\omega^2 \mu\varepsilon \vec{E}}$$

Similarly, $\boxed{\nabla^2 \vec{H} = -\omega^2 \mu\varepsilon \vec{H}}$ } → wave equation.

Case 1- Electric field is uniform throughout the 3-D space

$$\nabla^2 \vec{E} = 0 = -\omega^2 \mu \varepsilon \vec{E}$$

$$\boxed{\vec{E} = 0}$$

Hence, $\boxed{\vec{B} = 0}$

Case 2- Electric field is uniform in one plane.

Sub-case 1. we assume ~~that~~ that $\vec{E}$ is uniform in the plane which is $\perp$ to the field containing vector.

$$\boxed{\vec{E} = E_0(x)\,\hat{x}}$$

$$\vec{\nabla} \times \vec{E} = -j\omega\mu \vec{H} = 0$$

$$\boxed{\vec{H} = 0}$$

Sub-case 2- We assume that $\vec{E}$ is uniform in the plane which contains the field vector.

$$\boxed{\vec{E} = E_0(z)\,\hat{x}} \text{ or } \boxed{E_0(y)\,\hat{x}}$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial E_0(z)}{\partial z}\,\hat{y} = -j\omega\mu \{ H_x \hat{x} + H_y \hat{y} + H_z \hat{z} \}$$

$$\frac{\partial E_0(z)}{\partial z} = -j\omega\mu H_y$$

$$\boxed{H_x = 0} \qquad \boxed{H_y = -\frac{1}{j\omega\mu}\frac{\partial E_0(z)}{\partial z}}$$

$$\boxed{H_z = 0}$$

$$\nabla^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E}$$

$$\nabla^2 (E_0(z)\hat{x}) = -\omega^2 \mu \varepsilon \vec{E}$$

$$\boxed{\frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \varepsilon \vec{E}}$$

$$\boxed{\frac{\partial^2 E}{\partial z^2} = +\gamma^2 \vec{E}}$$

$\gamma \rightarrow$ propogation constant.

we know, $\gamma = \alpha + j\beta$

$$j\omega\sqrt{\mu\varepsilon} = \alpha + j\beta$$

$$\boxed{\alpha = 0} \quad \boxed{\beta = \omega\sqrt{\mu\varepsilon}}$$

It is lossless medium as $\boxed{\alpha = 0}$.

Now, $\frac{\partial^2 E}{\partial z^2} = -\beta^2 \vec{E} = 0$

$$\boxed{\frac{\partial^2 E}{\partial z^2} + \beta^2 \vec{E} = 0}$$

$$\boxed{E_x = E' e^{-j\beta z} + \bar{E} e^{j\beta z}}$$

$$\frac{\partial E_x}{\partial z} = -j\beta E^+ e^{-j\beta z} + j\beta \bar{E} e^{j\beta z}$$

$$\boxed{H_y = \frac{\beta}{\mu\omega}\left[E^+ e^{-j\beta z} - E^- e^{j\beta z}\right]}$$

$$H_{y,\,\text{forward}} = \frac{\beta E^+ e^{-j\beta z}}{\mu\omega}$$

Intrinsic impedance of medium (η)

$$\frac{E^+}{H_y} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} \Rightarrow \boxed{\frac{E}{H_y} = \sqrt{\frac{\mu}{\varepsilon}}}$$

$\rightarrow$ equivalent to ~~imdance~~ impedance.

For free space, $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-2}}}$

$$\boxed{\eta_0 = 120\pi \text{ or } 377\Omega}$$

Now, for $\boxed{\vec{E} = E(z)\,\hat{y}}$

wave eqn, $\nabla^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E}$

$$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H}$$

$$-\frac{\partial E}{\partial z}\hat{x} = -j\omega\mu\vec{H}$$

$$\frac{\partial E}{\partial z}\hat{x} = j\omega\mu\,[H_x \hat{x} + H_y \hat{y} + H_z \hat{z}]$$

$$\boxed{H_y = 0} \qquad \boxed{H_z = 0}$$

$$\boxed{H_x = \frac{1}{j\omega\mu}\frac{\partial E}{\partial z}}$$

$$\nabla^2 E = -\omega^2 \mu \varepsilon \vec{E}$$

$$\boxed{E_y = E_y^+ e^{-j\beta z} + E_y^- e^{j\beta z}}$$

$$H_x = \frac{1}{j\omega\mu}\left[-j\beta E_y^+ e^{-j\beta z} + j\beta E_y^- e^{j\beta z}\right]$$

$$H_x = \frac{-\beta}{\omega\mu}\left[E_y^+ e^{-j\beta z} - E_y^- e^{j\beta z}\right]$$

$$\boxed{\frac{E_y^+}{H_x} = -\frac{\omega\mu}{\beta} = -\sqrt{\frac{\mu}{\varepsilon}}}$$ → intrinsic impedance of F.W.

$$\boxed{\frac{E_y^-}{H_x} = \frac{\omega\mu}{\beta} = +\sqrt{\frac{\mu}{\varepsilon}}}$$ → Intrinsic impedance of E.W.

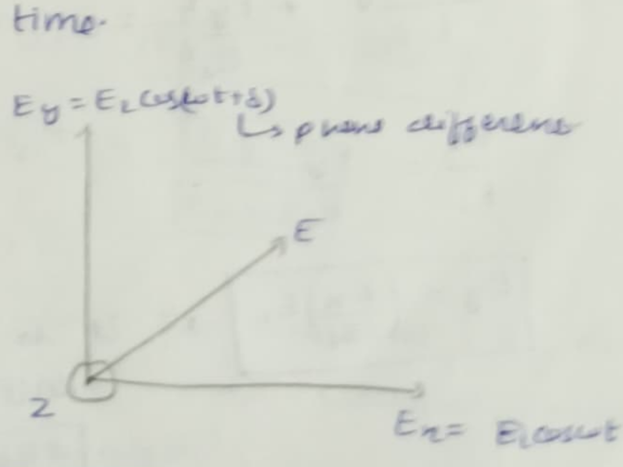
* When E_x^+ and H_y then η is +ve wave propagation is along +z-direction. Same for E_x^- and $H_y \to -\eta \to$ -z-direction.

* When E_y^+ and H_x then η is -ve. wave propagation is along -z-direction. Same for E_y^- and $H_y \to \eta \to$ +z-direction

* These Types of waves are called Transverse Electromagnetic waves. when $\vec{E}$ & $\vec{H}$ ⊥ to the direction of propagation of wave

☆ Polarisation-

* It is defined as the orientation of $\vec{E}$ as a function of time.



$$\boxed{\cos\omega t = \frac{E_x}{E_1}}$$

$$\boxed{\sin\omega t = \sqrt{1 - \frac{E_x^2}{E_1^2}}}$$

$$E_y = E_2 \cos(\omega t + \delta)$$

$$\frac{E_y}{E_2} = \cos\omega t \cos\delta - \sin\omega t \sin\delta$$

$$\frac{E_y}{E_2} = \frac{E_x}{E_1}\cos\delta - \sqrt{1 - \frac{E_x^2}{E_1^2}}\,\sin\delta$$

$$\left(\frac{E_x}{E_1}\cos\delta - \frac{E_y}{E_2}\right)^2 = \left(1 - \frac{E_x^2}{E_1^2}\right)\sin^2\delta$$

$$\frac{E_x^2}{E_1^2}\cos^2\delta + \frac{E_y^2}{E_2^2} - \frac{2E_xE_y}{E_1E_2}\cos\delta = \sin^2\delta - \frac{E_x^2\sin^2\delta}{E_1^2}$$

$$\boxed{\frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_y^2}{E_2^2} = \sin^2\delta}$$

* This equation is generalize equation of ellipse.

Case I. when $\delta = 0$ and $E_1 \neq E_2$

$$\frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1E_2} + \frac{E_y^2}{E_2^2} = 0$$

$$\frac{E_x}{E_1} = \frac{E_y}{E_2}$$

$\boxed{E_y = \left(\frac{E_x}{E_1}\right)E_2}$ → It is like straight line eqn

$\boxed{\text{slope} = \frac{E_2}{E_1}}$

* This type of polarisation is called linear polarisation

1. If $E_1 = 0$, slope $\to \infty$. This type is called vertical linear polarisation

2. If $E_2 = 0$, Slope $\to 0$. This type is ~~called~~ called horizontal linear polarisation

Case II -

$\delta = \pm 90°$, $E_1 = E_2 = E$

$$\boxed{E_x^2 + E_y^2 = E^2}$$

Orthogonality in time

$E_x \perp E_y$

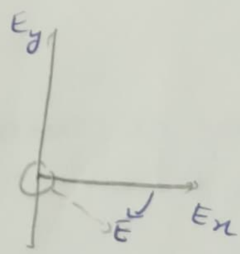
↳ orthogonality in space

* This is called circular polarisation.
* All satellite communication antennas are circularly polarised.
* This ~~make~~ polarisation make antenna orientation independent.

1. If $\delta = +\pi/2$, $\boxed{E_y = -E_2 \sin\omega t}$

If $t = 0$, $\boxed{E_y = 0}$

$\boxed{E_x = E_1}$



* On increasing ωt E_x decreases, E_y increases.
* The resultant $\vec{E}$ move clockwise, called Left Handed circular Polarisation.

2. If $\delta = -\pi/2$, $E_y = E_2 \sin\omega t$

* Resultant $\vec{E}$ move in anti- clockwise.
* This type of polarisation is called right-handed circular polarisation.

Note:-

These clockwise & anticlock as Left handed & right handed depends upon observer.

Case III - $E_1 \neq E_2$ and $\delta \neq 90° \neq 0$.

$$\boxed{\frac{E_x^2}{E_1^2} - \frac{2E_xE_y\cos\delta}{E_1E_2} + \frac{E_y^2}{E_2^2} = \sin^2\delta}$$

* It is called Elliptical Polarisation.

Case IV - $E_1 \neq E_2$ and $\delta = 90$

$$\boxed{\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1}$$

↳ special case of elliptical polarisation.

☆ Wave Propagation in Medium having finite Conductivity –

(Lossy Medium)

$$\nabla \times \vec{H} = J_c + \frac{\partial \vec{D}}{\partial t}$$ → due Bound charges (J_D)

$$\boxed{\nabla \times \vec{H} = J_c + j\omega \vec{D}}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + j\omega \varepsilon \vec{E}$$

$$\boxed{\nabla \times \vec{H} = (\sigma + j\omega\varepsilon)\vec{E}}$$ → Medium having finite conductivity

For source free, $\boxed{\nabla \times \vec{H} = j\omega\varepsilon \vec{E}}$

$$\nabla \times \vec{H} = j\omega\left(\frac{\sigma}{j\omega} + \varepsilon\right)\vec{E}$$

$$\nabla \times \vec{H} = j\omega\varepsilon_0\left[\varepsilon_r + \frac{\sigma}{j\omega\varepsilon_0}\right]\vec{E}$$

$$\nabla \times \vec{H} = j\omega\varepsilon_0\left[\varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0}\right]\vec{E}$$

$$\boxed{\nabla \times \vec{H} = j\omega\varepsilon_0 \varepsilon_r' \vec{E}}$$

$$\boxed{\varepsilon_r' = \varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0}}$$

→ impurity or losses associated with dielectric medium

* If J_c dominates over J_D then the medium is called as conductor and if J_D dominates over J_c then it is called as dielectric medium.

$\boxed{J_c >>> J_D}$ → Conductor

$\boxed{J_D >>> J_c}$ → Dielectric

$$\sigma E <<< \omega \varepsilon E$$

$\boxed{\sigma <<< \omega \varepsilon}$ → dielectric

$\boxed{\sigma >>> \omega \varepsilon}$ → Conductor

* ω → ~~Transition~~ Transition freq. The frequency above with medium become dielectric.

$$\sigma = 2\pi f_T \varepsilon$$

$\boxed{f_T = \frac{\sigma}{2\pi\varepsilon}}$ → Transition freq

Operating $\boxed{f > f_T}$ → Behaves as Dielectric

Operating $\boxed{f < f_T}$ → Behaves as Conductor.

* For Silver wire $\boxed{f_T \cong 10^{21}\ Hz}$

Now,

$$\nabla^2 E = -\omega^2 \mu \varepsilon_0 \left\{ \varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right\} \vec{E}$$

$$\nabla^2 E = \gamma^2 \vec{E} \qquad \gamma^2 = -\omega^2 \mu \varepsilon_0 \left\{ \varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right\}$$

$$\boxed{\gamma = j\omega\sqrt{\mu\varepsilon_0} \left\{ \varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0} \right\}^{1/2}}$$

$$\boxed{\gamma = \sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon)}}$$

from Transmission line, $\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

So,

$$\boxed{\eta = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon}}}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu_0\varepsilon_0\varepsilon_r}{2}}\left\{\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_0^2\varepsilon_r^2}} - 1\right\}^{1/2}}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu_0\varepsilon_0\varepsilon_r}{2}}\left\{\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_0^2\varepsilon_r^2}} + 1\right\}^{1/2}}$$

For very Good Dielectric, (Just like ideal dielectric)

$$\sigma <<< \omega\varepsilon$$

$$\boxed{\gamma = j\omega\sqrt{\mu_0\varepsilon}} \qquad \boxed{\eta = \sqrt{\frac{\mu_0}{\varepsilon}}}$$

$$\boxed{\alpha = 0} \qquad \boxed{\beta = \omega\sqrt{\mu_0\varepsilon}}$$

For very Good Conductor, (Just like ideal Conductor)

$$\sigma >>> \omega\varepsilon$$

$$\gamma = \sqrt{j\omega\mu_0\varepsilon} \Rightarrow \boxed{\gamma = \sqrt{j\omega\mu_0\sigma}}$$

$$\cancel{\gamma = \omega\sqrt{j\mu_0\varepsilon}}$$

$$\gamma = \left(e^{j\pi/2}\right)^{1/2}\sqrt{\omega\mu_0\sigma}$$

$$\gamma = e^{j\pi/4}\sqrt{\omega\mu_0\sigma}$$

$$\gamma = \left(\frac{1+j}{\sqrt{2}}\right)\sqrt{\omega\mu_0\sigma}$$

$$\alpha = \frac{1}{\sqrt{2}}\sqrt{\omega\mu_0\sigma}, \qquad \boxed{\alpha = \sqrt{\frac{\omega\mu_0\sigma}{2}}} \qquad \boxed{\beta = \sqrt{\frac{\omega\mu_0\sigma}{2}}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu_0}{\sigma}}}$$

$$\boxed{\eta = (1+j)\sqrt{\frac{\omega\mu_0}{2\sigma}}}$$

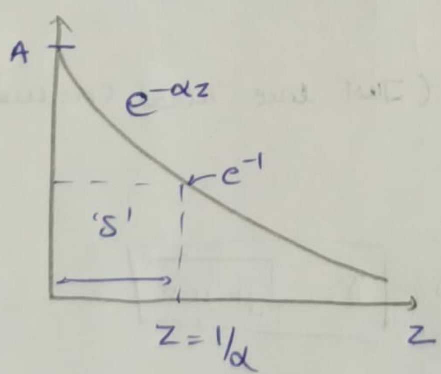
$$\boxed{\eta = \sqrt{\frac{\omega\mu_0}{\sigma}} \angle 45^\circ}$$

* For ideal conductor $\vec{E}$ leads $\vec{H}$ by amount of $\pi/4$.

~~Skin Effect~~

Skin Depth –

If we launch inside a good conductor the amplitude of E-M wave decreases exponentially.



After $Z = \frac{1}{\alpha}$, we assume that Amplitude of E-M wave is zero.

This Depth is called Skin Depth indicating by 'δ'.

$$\delta = \frac{1}{\alpha}$$

* widely used in E-M interfacing and E-M compatability.

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$$

$$\boxed{\delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma}}}$$

Loss Tangent -

* Loss Tangent is parameter which indicates how good your dielectric is, given by -

$$\boxed{\tan\delta = \frac{\sigma}{\omega \varepsilon}}$$

* If $\tan\delta = 0$, ideal dielectric.

* Generally $10^{-4} < \tan\delta < 10^{-6}$

* As $\tan\delta$ increases your material is going towards conductor