Comparison between Conditional and Unconditional Volatility models

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IN

FINANCIAL ENGINEERING

By

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CERTIFICATE

This is to certify that the project report entitled **"Comparison between Conditional and Unconditional Volatility models"** submitted by **Soumy Ladha** (Roll No. 13AE3FP09) to Indian Institute of Technology Kharagpur towards partial fulfilment of requirements for the award of degree of Master of Technology(Hons.) in Financial Engineering is a record of bonafide work carried out by him under my supervision and guidance during the academic session 2017-18.

Date:

Place: Kharagpur

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Abstract

The main purpose of this project is to evaluate and compare different volatility models. The evaluation is based on how well the models are predicting against implied volatility and actual volatility. The volatility models are evaluated based on daily deviations from the implied volatility and on daily changes of the modelled volatility. Statistical measurements investigated is mean squared error. The models investigated are historical volatility models, generalized autoregressive conditional heteroscedasticity (GARCH) Model, Exponentially Weighted Moving Average (EWMA) and a Parametric Model by distribution fitting.

The comparison shows that the GARCH and Distribution fitting model have a better performance than the other models. For the historical models it is shown that 50 to 75 observations are most appropriate to use to imitate the implied volatility. It has been observed that GARCH is sensitive towards irregular observation and this was major cause of varying performance of GARCH.

Contents

1 Background Study

1.1 What is Volatility?

In laymen terms, Volatility is a statistical measure of the dispersion of returns for a given security or market index. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. Commonly, the higher the volatility, the riskier the security.

1.2 Importance of Volatility

Volatility refers to the amount of uncertainty or risk about the size of changes in a security's value. A higher volatility means that a security's value can potentially be spread out over a larger range of values. This means that the price of the security can change dramatically over a short time period in either direction. A lower volatility means that a security's value does not fluctuate dramatically, but changes in value at a steady pace over a period of time. The volatility is a fundamental variable in valuations and risk calculations of derivatives.

1.3 Options

An option is a financial derivative that represents a contract sold by one party (the option writer) to another party (the option holder). The contract offers the buyer the right, but not the obligation, to buy (call) or sell (put) a security or other financial asset at an agreed-upon price (the strike price) during a certain period of time or on a specific date (exercise date).

1.3.1 Call Option

Call options give the option to buy at certain price, so the buyer would want the stock to go up. Conversely, the option writer needs to provide the underlying shares in the event that the stock's market price exceeds the strike due to the contractual obligation. An option writer who sells a call option believes that the underlying stock's price will drop relative to the option's strike price during the life of the option, as that is how he will reap maximum profit. This is exactly the opposite outlook of the option buyer. The buyer believes that the underlying stock will rise; if this happens, the buyer will be able to acquire the stock for a lower price and then sell it for a profit. However, if the underlying stock does not close above the strike price on the expiration date, the option buyer would lose the premium paid for the call option.

1.3.2 Put Option

Put options give the option to sell at a certain price, so the buyer would want the stock to go down. The opposite is true for put option writers. For example, a put option buyer is bearish on the underlying stock and believes its market price will fall below the specified strike price on or before a specified date. On the other hand, an option writer who shorts a put option believes the underlying stock's price will increase about a specified price on or before the expiration date. If the underlying stock's price closes above the specified strike price on the expiration date, the put option writer's maximum profit is achieved. Conversely, a put option holder would only benefit from a fall in the underlying stock's price below the strike price. If the underlying stock's price falls below the strike price, the put option writer is obligated to purchase shares of the underlying stock at the strike price.

1.3.3 European and American

Options are classified into two broader categories American option and European option. There is a difference between European and American options. An American option gives the holder an opportunity to exercise at any time before maturity, while a European option can be exercised only at maturity day. An American option will never be less valuable than a European option, since an American option gives the holder more opportunities than a European option. In this report, only European options will be investigated. One way of pricing a European option, is to use the well-known Black & Scholes pricing formula

The Black & Scholes pricing formula for a European call option:

$$c = S_0 \, N(d_1) - K \, e^{-rT} N(d_2)$$

$$p = K \, e^{-rT} \, N(-d_2) - S_0 \, N(-d_1)$$

$$\text{where} \quad d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

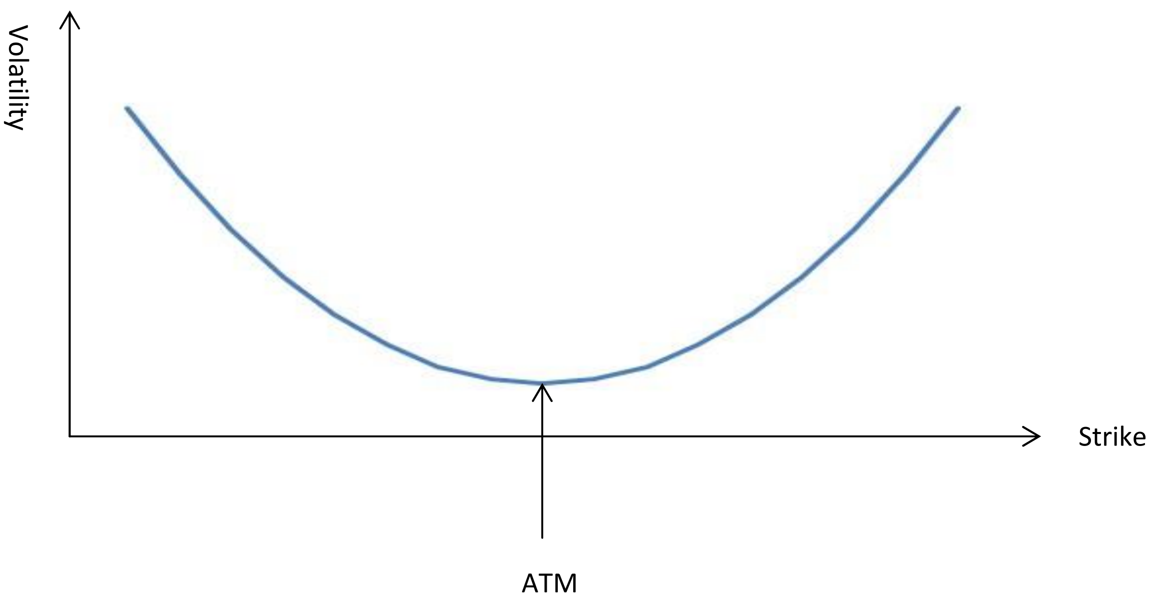
1.4 Implied Volatility (Market's Expectation)

The implied volatility is a measurement that takes the market expectations about the volatility into account. For a liquid option, it is possible to calculate the implied volatility and an interested person can get a good estimation of the market expectations of the variation in the price of the underlying asset in the future. But for an illiquid option, in the case an option is not frequently bought nor sold in a couple of days, there will be a problem to assess the current price of the option. Neither the price nor the calculation of the implied volatility will be up to date and an alternative method must be used in order to calculate the volatility. Therefore, thus this project involves two things prediction of actual volatility of returns for next few days as well as market expectations of volatility (implied volatility).

There is only one value of the volatility (σ) in the Black & Scholes formula that gives a theoretical price equal to the market price of an option. This value is called the implied volatility. When all parameters in the Black & Scholes formula are known, the contract value from quoted prices on the market included, it is possible to calculate the implied volatility. Since the value is derived from the formula it is a measurement that takes the market's expectations about the future volatility for an option into account. By using the implied volatility, the market valuation and risk calculation of the option will be more correct in a market risk perspective.

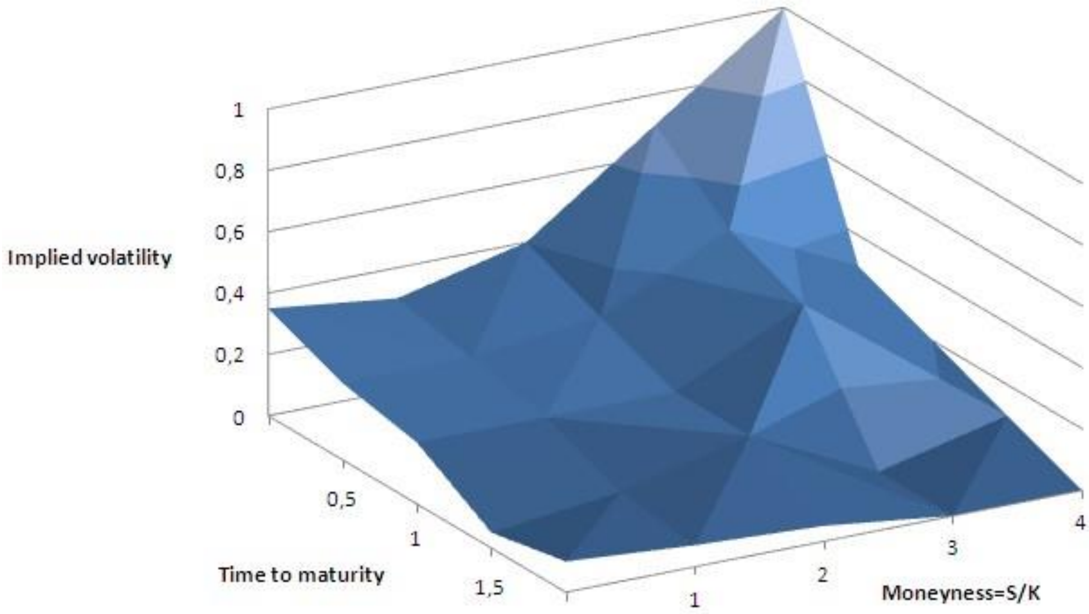
1.5 Volatility smile and skew

Options having the same underlying asset and term structure, but different strike prices are having different implied volatilities. A plot of the implied volatilities of an option as a function of theirs strike prices is known as a volatility smile.



1.6 Volatility surface

When the implied volatility is plotted in relation to both the time to maturity and the strike price the topology generated is called Volatility surface. The surface represents the value of volatility giving each traded option a theoretical volatility value equal to the market value. The surface may vary a lot over time and between different underlying assets. When the implied volatility of all known options is marked in a graph, an interpolation method is used to get the whole volatility surface. The implied volatility is calculated every day based on a weighted average of the two call options closest to the at-the-money strike. The option market is most liquid when the spot price is equal to the strike price. Therefore, the time series of the implied volatility will be calculated from at the money options in this investigation.



1.7 Volatility term structure

The term structure of implied volatility describes the relationship between time to maturity and implied volatility. Options having the same underlying asset and the same strike price may have different implied volatilities due to different term structure. The term structure is for example depending on upcoming market events. The market knows that an important report will be released at a predetermined point in time and that the result of the report will have a large impact on the company's stock price and therefore also on the volatility of the option.

1.8 Volatility Modelling and Forecasting

A volatility model should be able to forecast volatility. Virtually all the financial uses of volatility models are forecasting aspects of future returns. A volatility model is used to forecast the absolute magnitude of returns, but it may also be used to predict quantiles or, in fact, the entire density. Such forecasts are used in risk management, derivative pricing and hedging, market making, market timing, portfolio selection and many other financial activities. In each, it is the predictability of volatility that is required. A risk manager must know today the likelihood that his portfolio will decline in the future. An option trader will want to know the volatility that can be expected over the future life of the contract. To hedge this contract, he will also want to know how volatile is this forecast volatility. A portfolio manager may want to sell a stock or a portfolio before it becomes too volatile. A market maker may want to set the bid–ask spread wider when the future is believed to be more volatile.

There is now an enormous body of research on volatility models. As new approaches are proposed and tested, it is helpful to formulate the properties that these models should satisfy. At the same time, it is useful to discuss properties that standard volatility models do not appear to satisfy. The volatility models have been investigated and evaluated based on how well they are fitting the actual volatility and implied volatility. The models studied are historical volatility, GARCH volatility, EWMA and a Parametric Model. The 4 models will be evaluated both visually and with statistical measurements.

1.8.1 Historical volatility

The historical volatility σ_n is estimated from historical spot prices of the underlying asset. The calculation method of the historical volatility is described below. At first the return for each day is calculated as

$$r = (S_i - S_{i-1})/S_{i-1}$$

where S_i is the spot price of the underlying asset at day I. Then, the variance is calculated as

$$\sigma_n = \frac{1}{m-1}\sum_{i=0}^{m-1}(r_{n-i} - \bar{r})^2$$

Where m is the number of observations and $\bar{r}$ is the average return of the m observations at the time n. σ^2 given as a daily measure of the variance

1.8.2 Exponential Weighted Moving Average (EWMA)

The Exponentially Weighted Moving Average (EWMA) is a statistic for monitoring the process that averages the data in a way that gives less and less weight to data as they are further removed in time. By the choice of weighting factor, λ, the EWMA control procedure can be made sensitive to a small or gradual drift in the process. The EWMA model is a particular case of the model in follows equation

$$\sigma_t^2 = \sum_{i=1}^{m}\alpha_i R_{t-i}^2 \,,$$

Where the weights decrease exponentially as move back through time. Specifically, $\alpha_{i+1} = \lambda^*\alpha_i$ where λ is a constant between zero and one. It turns out that this weighting scheme leads to a particularly simple formula for updating volatility estimates. The formula is

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)R_{t-1}^2,$$

1.8.3 ARCH/GARCH Models

An ARCH (autoregressive conditionally heteroscedastic) model is a model for the variance of a time series. ARCH models are used to describe a changing, possibly volatile variance. Although an ARCH model could possibly be used to describe a gradually increasing variance over time, most often it is used in situations in which there may be short periods of increased variation. (Gradually increasing variance connected to a gradually increasing mean level might be better handled by transforming the variable).

ARCH models were created in the context of econometric and finance problems having to do with the amount that investments or stocks increase (or decrease) per time period, so there's a tendency to describe them as models for that type of variable. An ARCH model could be used for any series that has periods of increased or decreased variance.

ARCH models attempt to explain variance clustering in the residuals and imply linear dependence among the squared errors of the first moment model.

$$a_t = \sigma_t \varepsilon_t \text{ , and } \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2$$

where $\sigma_t^2 = E(a_t^2 \mid F_{t-1}) = Var(a_t \mid F_{t-1}) = Var(Y_t \mid F_{t-1})$.

$$a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 + e_t$$

where e_t is a white noise process. A model with σ_t^2 *autoregressive conditional heteroscedastic* (ARCH) model, or ARCH(m) model. For such models, it is required that $\alpha_0 > 0$, and $\alpha_i \geq 0$ for i > 0.

The log likelihood function of an ARCH model, with the assumption that ε_t follows a Normal distribution is

$$\ell(\underline{\alpha}) = \sum_{t=m+1}^{n} \left[-0.5\ell n(\sigma_t^2) - 0.5(a_t^2 / \sigma_t^2) \right]$$

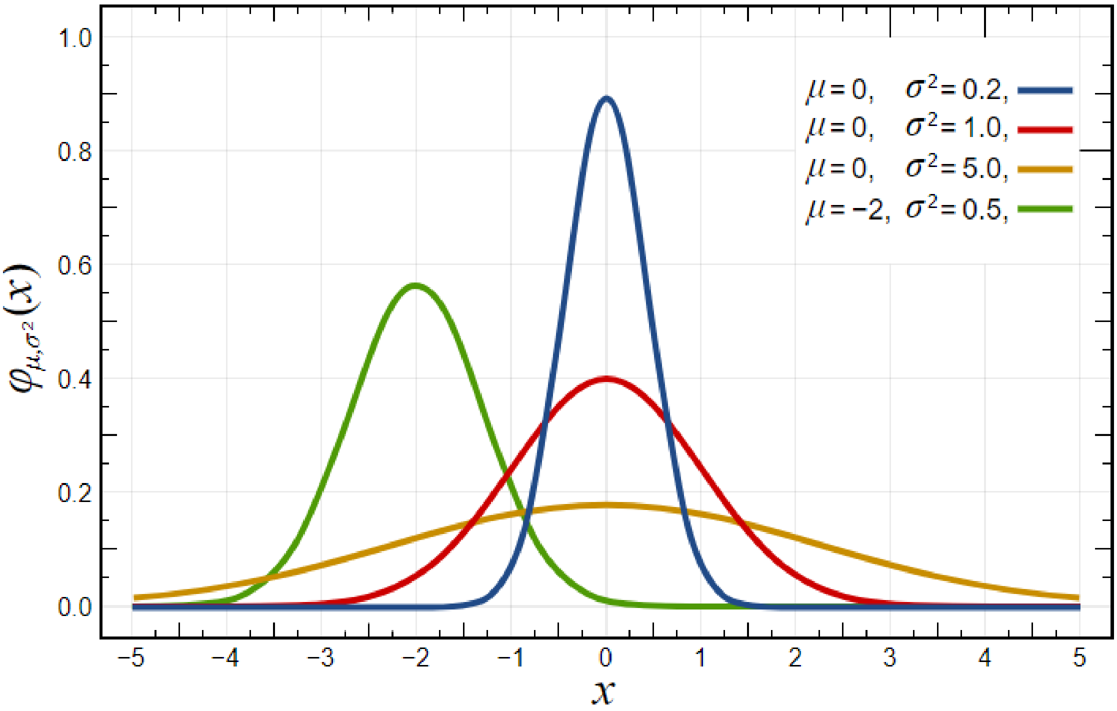
Although the ARCH model is simple, it restricts the model for the conditional variance σ_t^2 (or equivalently h_t) to follow a pure AR process and hence it may require more parameters to adequately represent the conditional variance process in comparison with other more generalized models. This model is known as a generalized ARCH model, or GARCH model. A GARCH (r, m) model can be written as

$$a_t = \sigma_t \varepsilon_t \text{ , and } \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_r \sigma_{t-r}^2$$

where $\alpha_0 > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0$, and $\sum_{i=1}^{\max(m,\, r)} (\alpha_i + \beta_i) < 1$.

The latter constraint on $\alpha_i + \beta_i$ ensures that the unconditional variance of a_t is finite, even though its conditional variance evolves over time. It is easy to see that model reduces to an ARCH(m) model if r=0. Under the Normality assumption of ε_t, the log likelihood function of $\underline{\alpha}$ for a GARCH(r, m) model is the same. This model differs to the ARCH model in that it incorporates squared conditional variance terms as additional explanatory variables. This allows the conditional variance to follow an ARMA process.

1.8.4 Distribution Modelled Volatility



The normal curve is bell-shaped and has a single peak at the exact centre of the distribution. The arithmetic mean, median, and mode of the distribution are equal and located at the peak. Half the area under the curve is above and half is below this centre point (peak). The normal probability distribution is symmetrical about its mean. It is asymptotic - the curve gets closer and closer to the x-axis but never actually touches it. A normal distribution with a mean of 0 and a standard deviation of 1 is called the standard normal distribution.

In this model an appropriate distribution is fitted for the returns, which in this case is a Normal Distribution and furthermore we assume that next day return will be sampled from this distribution. After calculating μ (return) and σ (volatility) we assume that next day return will have a mean of μ and volatility being σ.

2 Methodology and Analysis

2.1 Historical Volatility

- Calculate the average return, Calculate the deviation – Subtract the average from the actual observation.

- Square and add up all deviations. Calculate the square root of variance – this is called standard deviation.

2.2 EWMA

- Set an initial sigma which equals to zero.
- Iteratively apply the EWMA formula to generate next day volatility.
- Iterate over all values of λ, simultaneously storing all values of MS with each corresponding to different value of λ.
- Select λ with minimum MSE, set that λ as optimum.

2.3 Distribution Modelled Volatility

- Create a moving window with sufficient data at least 200 points.
- Check for normality in the data, using Pearson Chi square test.
- If data passes normality test then it can be assumed that data is sampled from the normal distribution, thus calculate its parameters μ and σ
- Generate Quartile-Quartile plot and see the fitness of Fitted distribution, furthermore extract distribution parameters as they will be used to predict expected value of next sample.

2.4 GARCH

- Create a moving windows with at least 100 data points.
- Check for stationarity preferably ADF test. Check for ARCH/GARCH effect. Using Box Test.
- If significant then model using GARCH, select best GARCH by Akaike information criterion (AIC). In this best GARCH came out to (4,3), while best ARMA was (2,1).

2.5 Implied Volatility

- Iterate over all the possible values of sigma in a given range
- Store the value which gives the theoretical value of the call price closest to actual one.
- Store the results in an array for further analysis.

3 Results

3.1 Calculation of Optimum Lambda

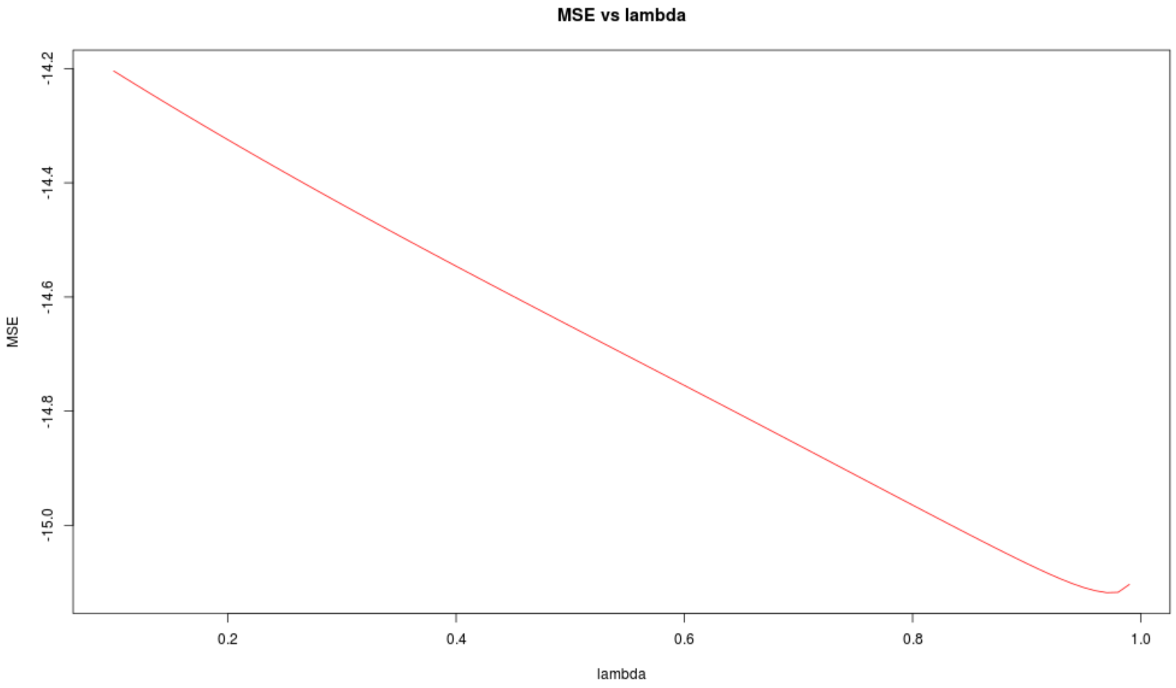


Figure 1 Calculation of optimum Lambda

Lambda Optimum = 0.96

3.2 Distribution of Returns

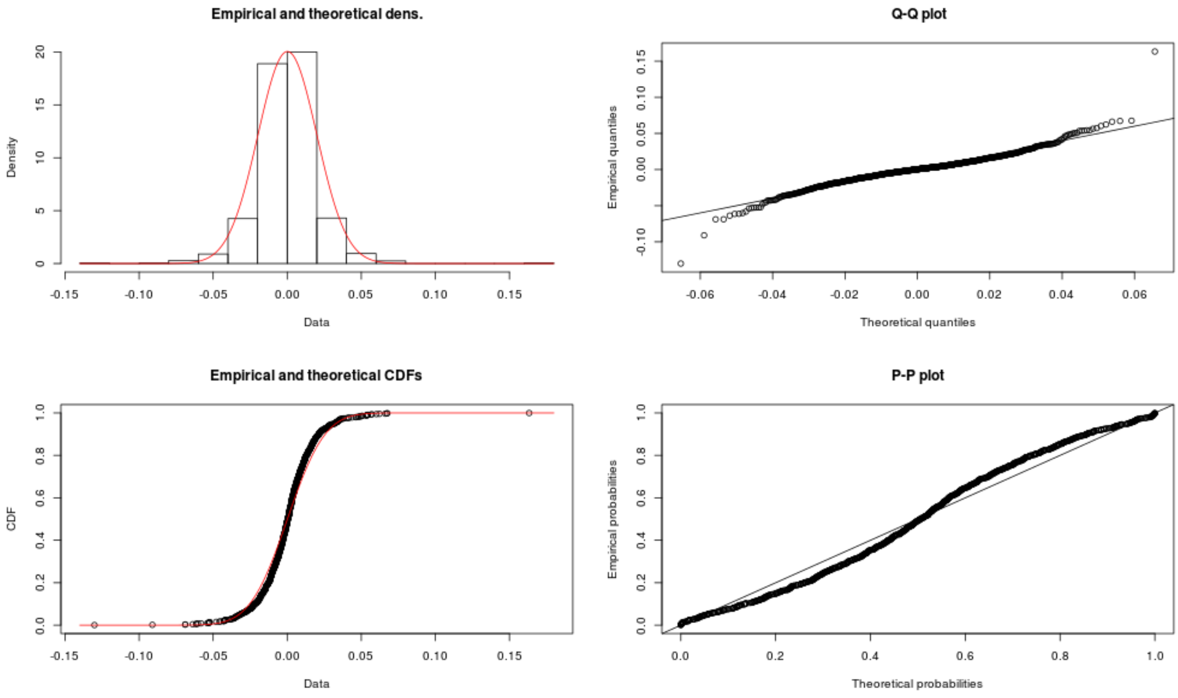


Figure 2 Distribution of Return for a random sample of size 1000

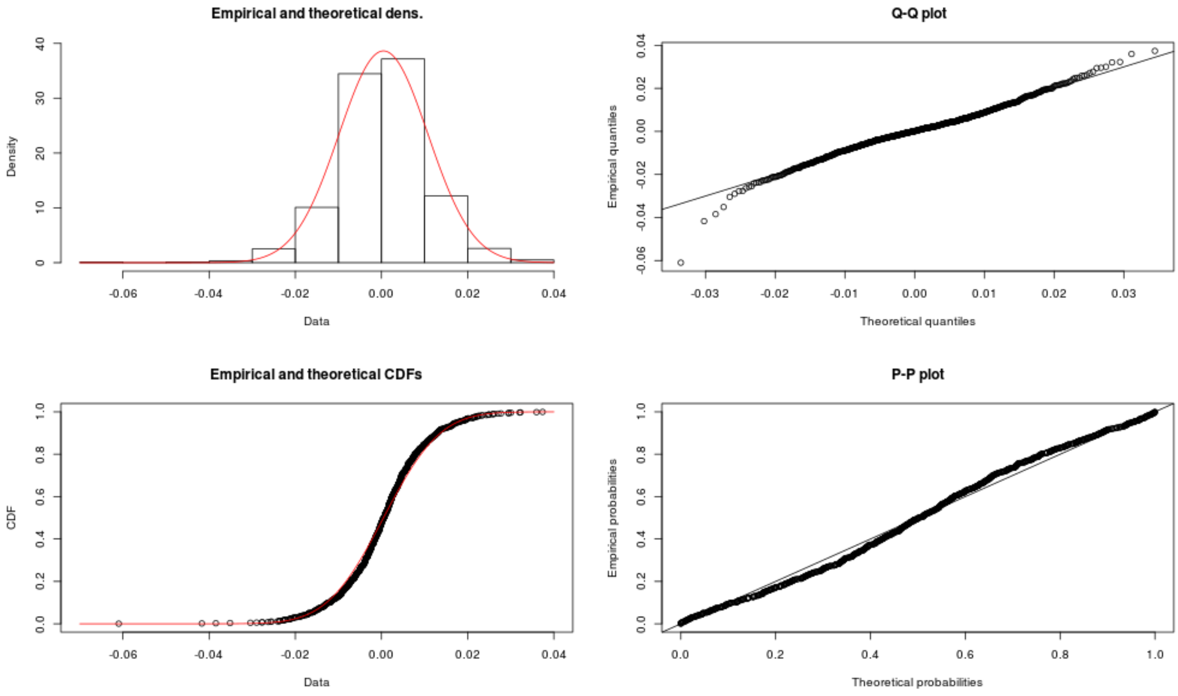


Figure 3 Distribution of returns for a random sample of size 1000

3.3 Prediction of 1 day ahead Volatility

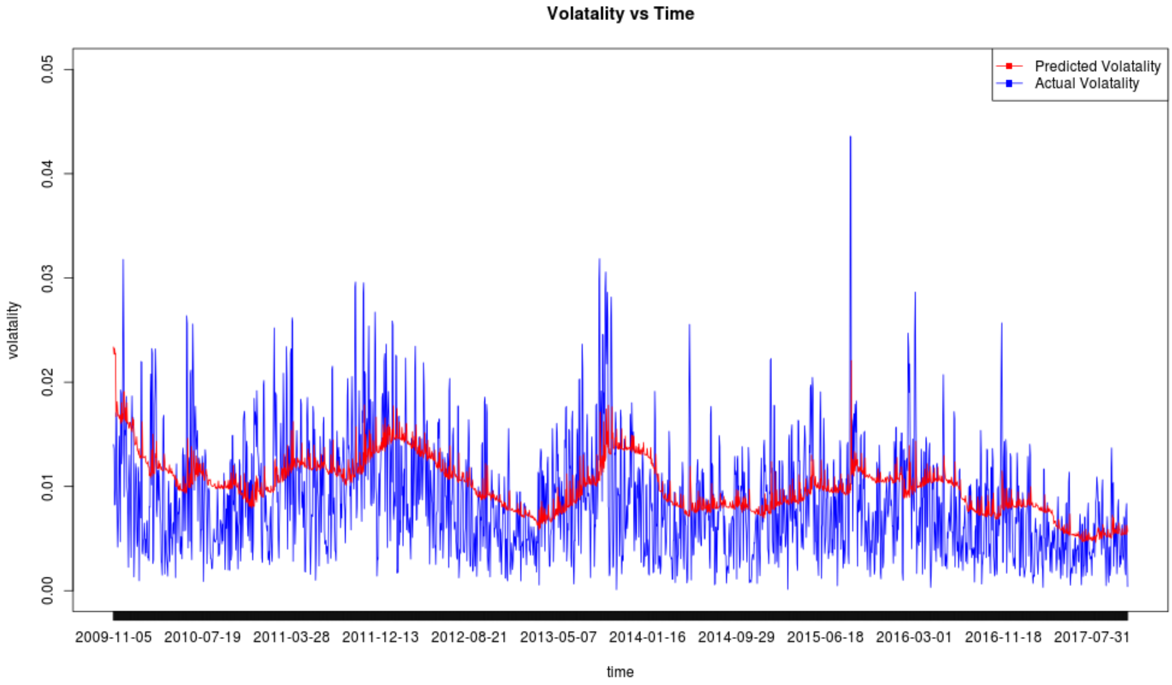


Figure 4 Volatility Modelling using Normal Distribution

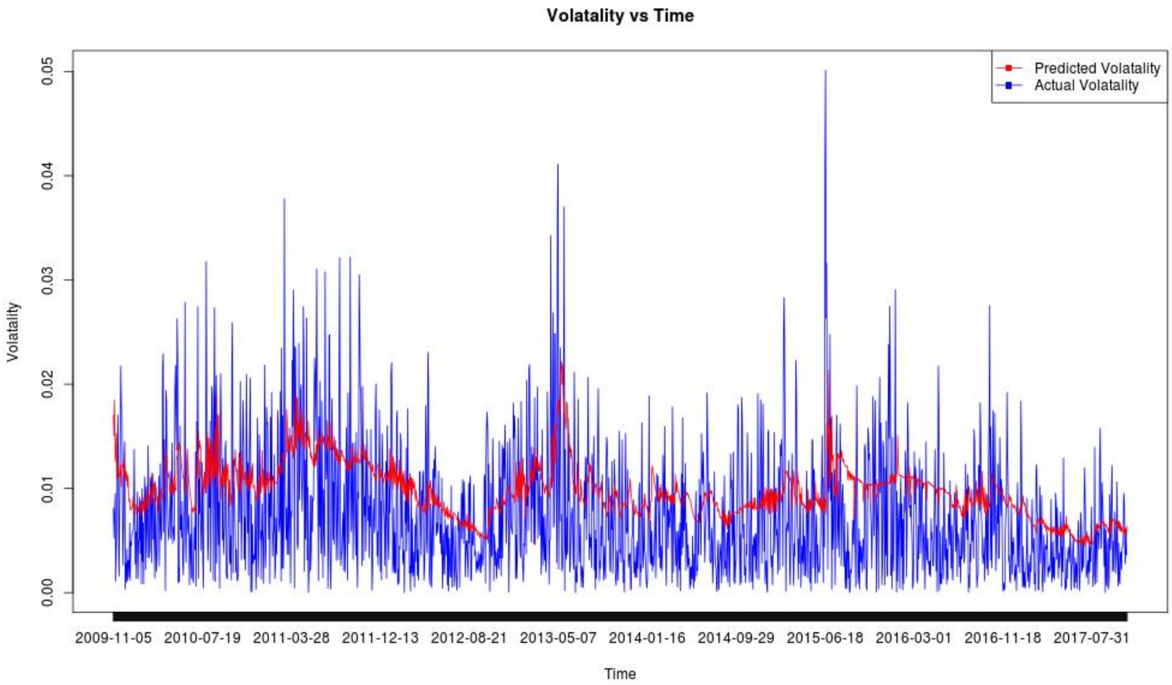


Figure 5 Volatility Modelling by GARCH

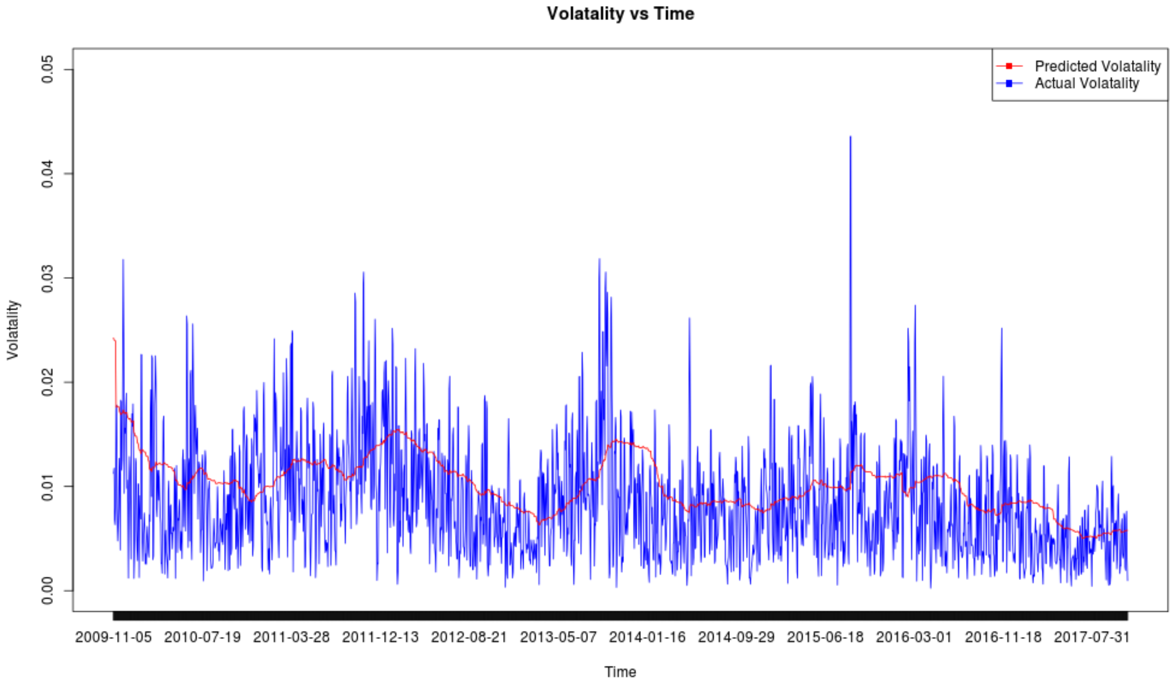


Figure 6 Volatility Modelling Historical Volatility

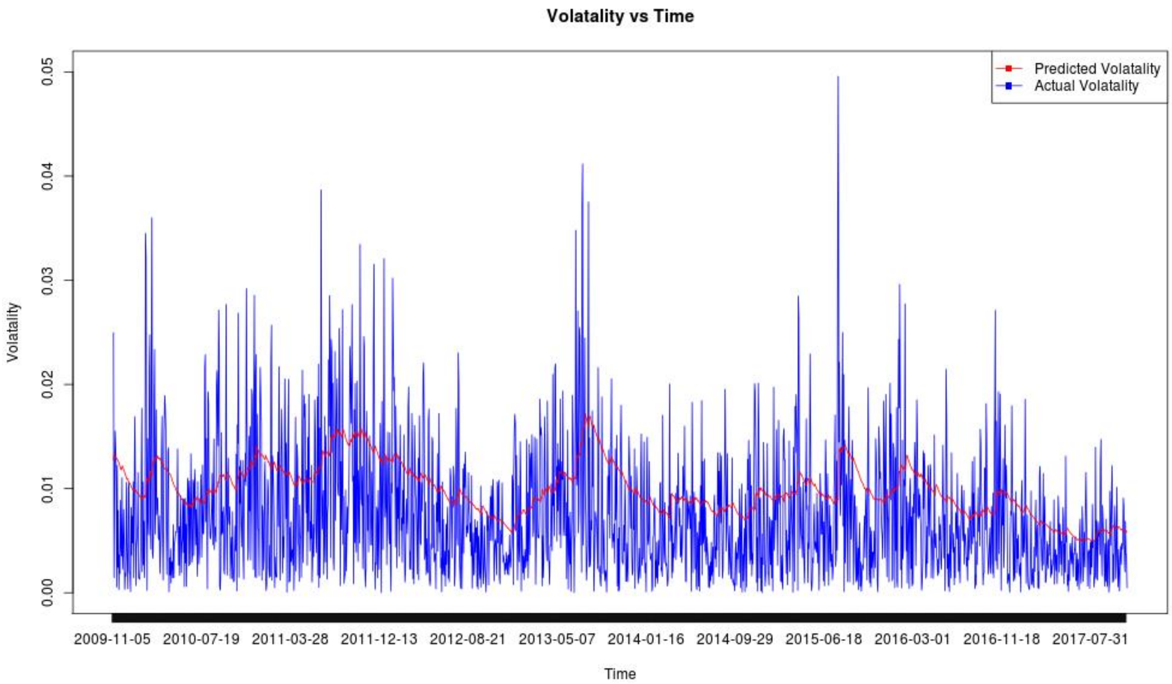


Figure 7 Volatility Modelling with EWMA

3.4 Prediction of 10 days ahead volatility

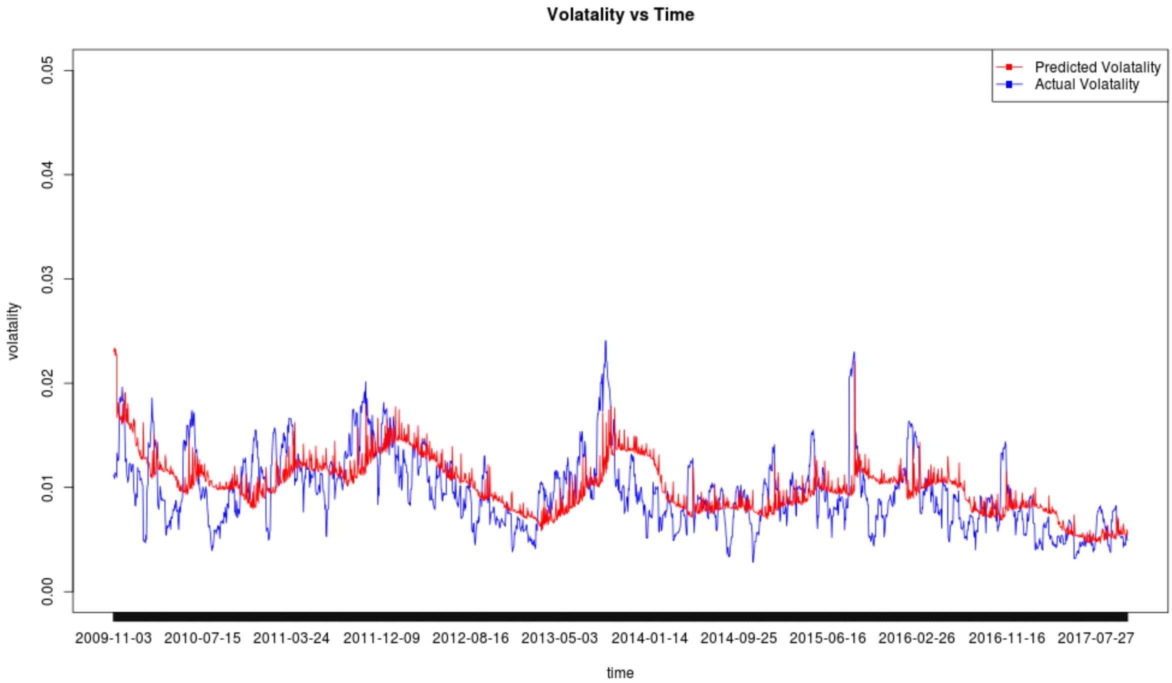


Figure 8 Volatility Modelling using Normal Distribution

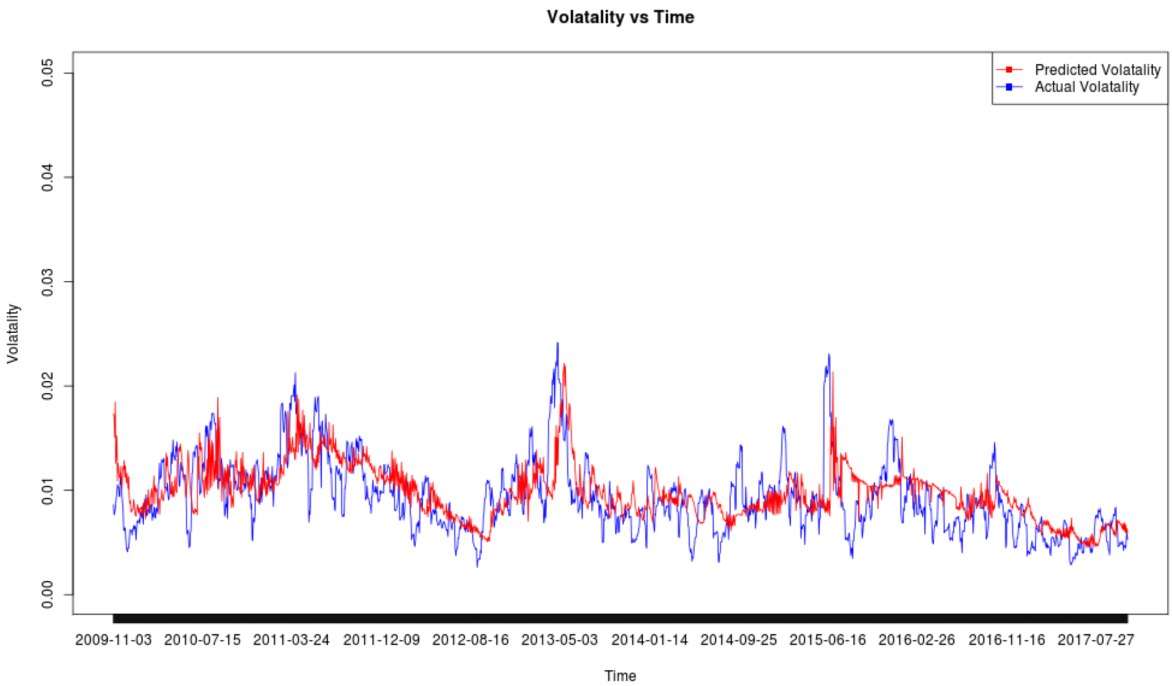


Figure 9 Volatility Modelling using GARCH

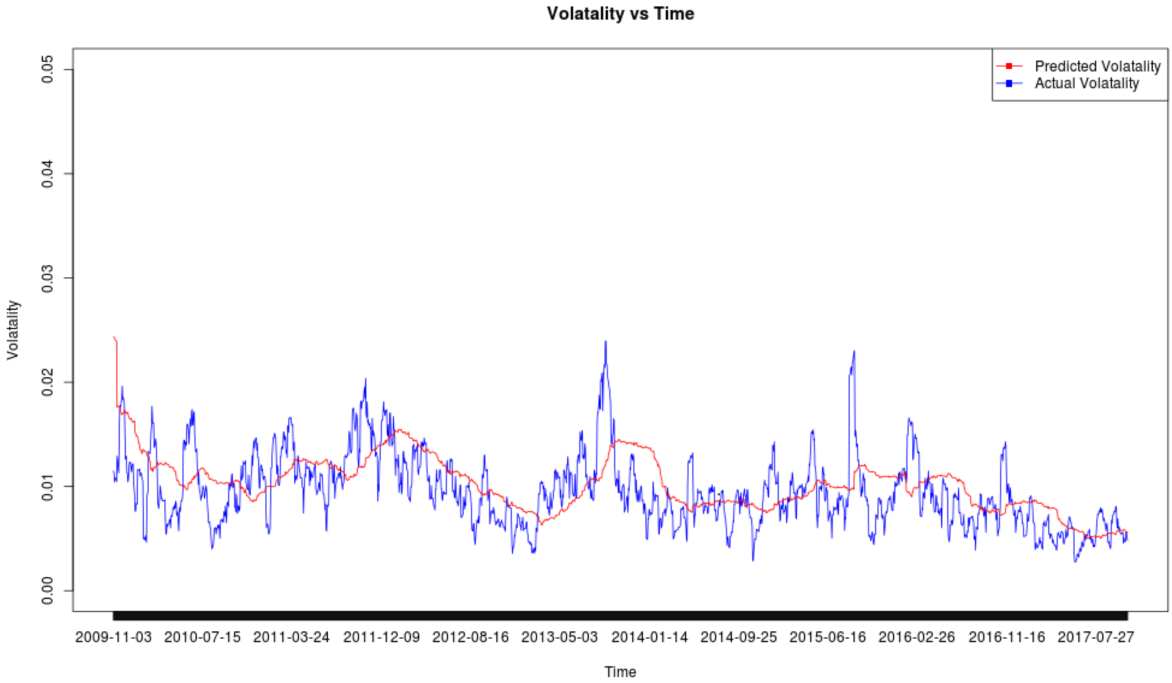


Figure 10 Historical Volatility Modelling

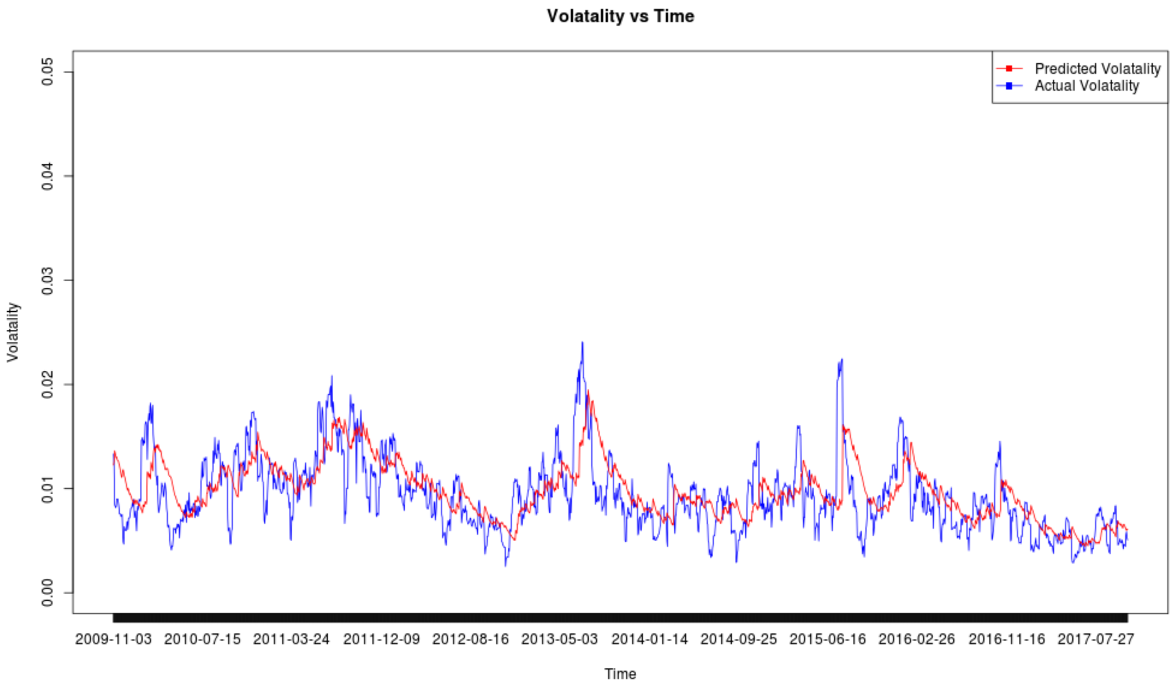


Figure 11 EWMA volatility modelling

3.5 Implied Volatility

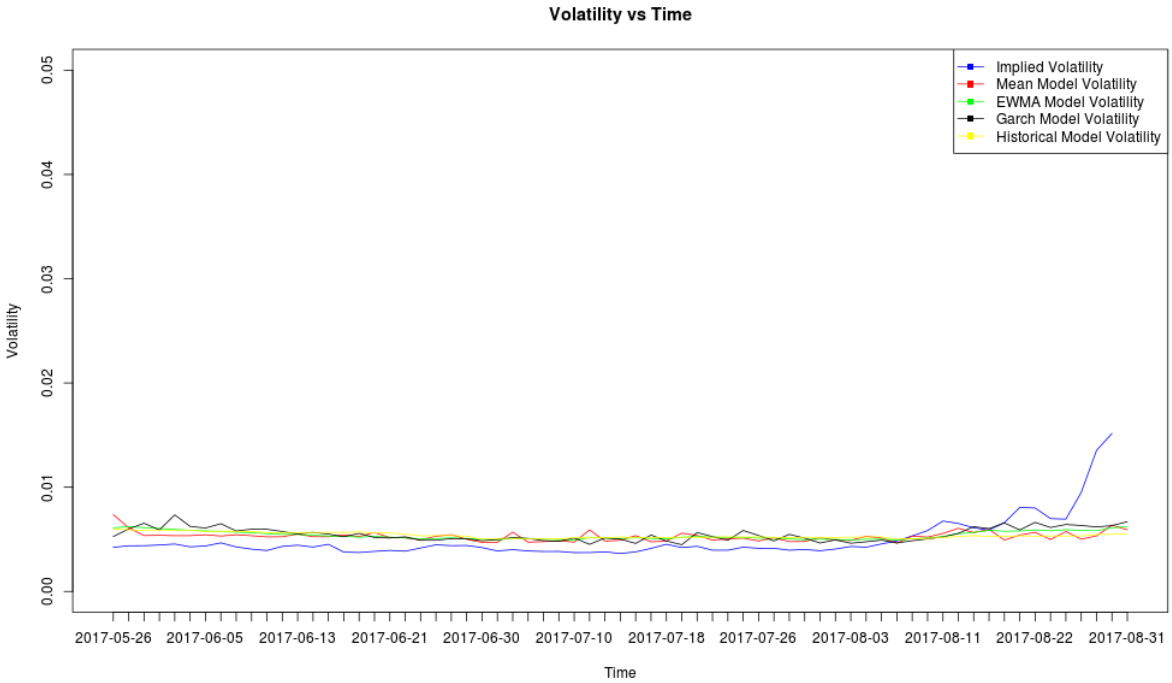


Figure 12 Implied Volatility vs Estimated Volatility

3.6 1 day ahead volatility prediction for sectoral indices

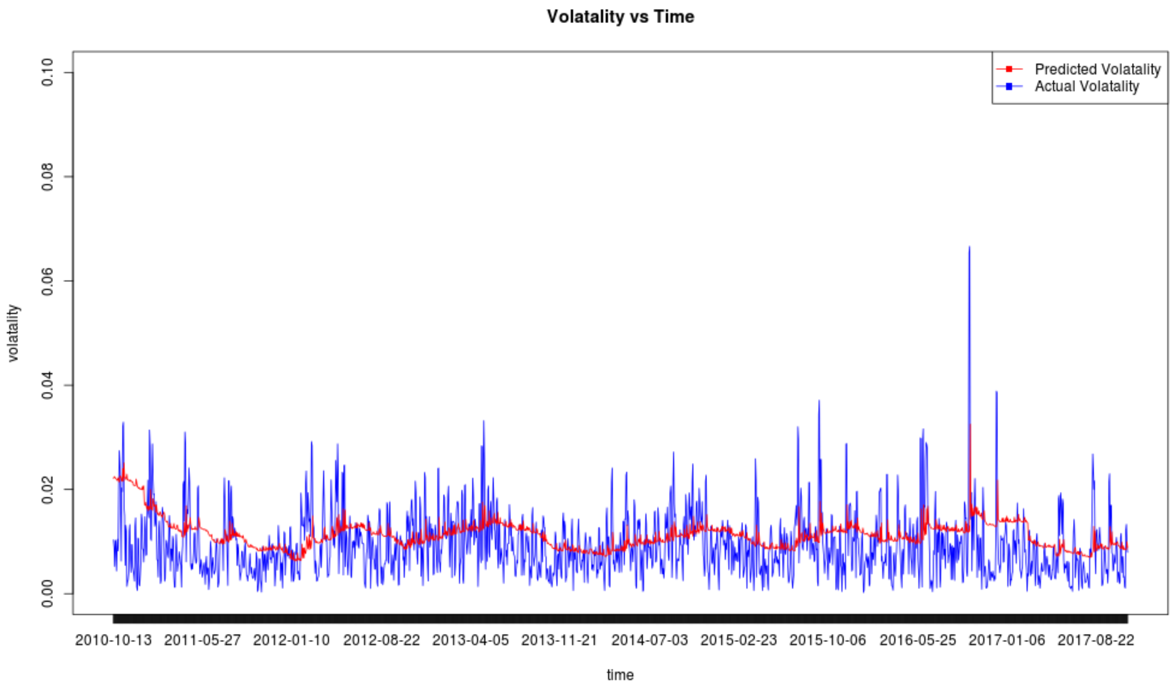


Figure 13 Volatility prediction for NIFTY 100

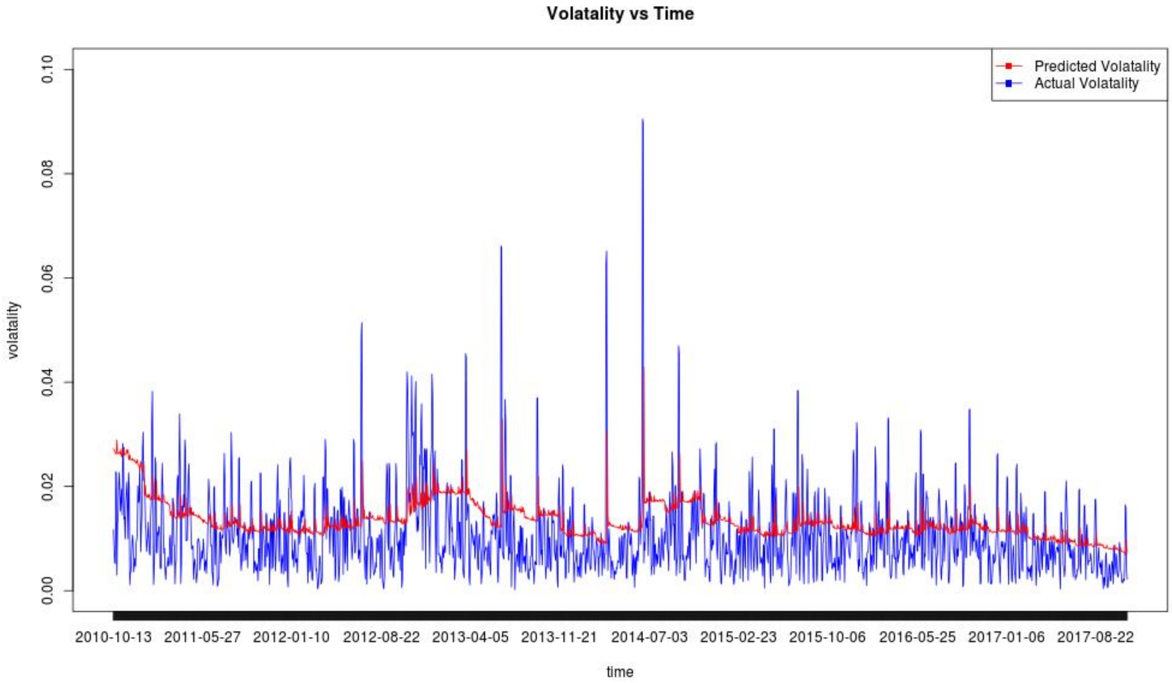


Figure 14 Volatility prediction for NIFTY IT

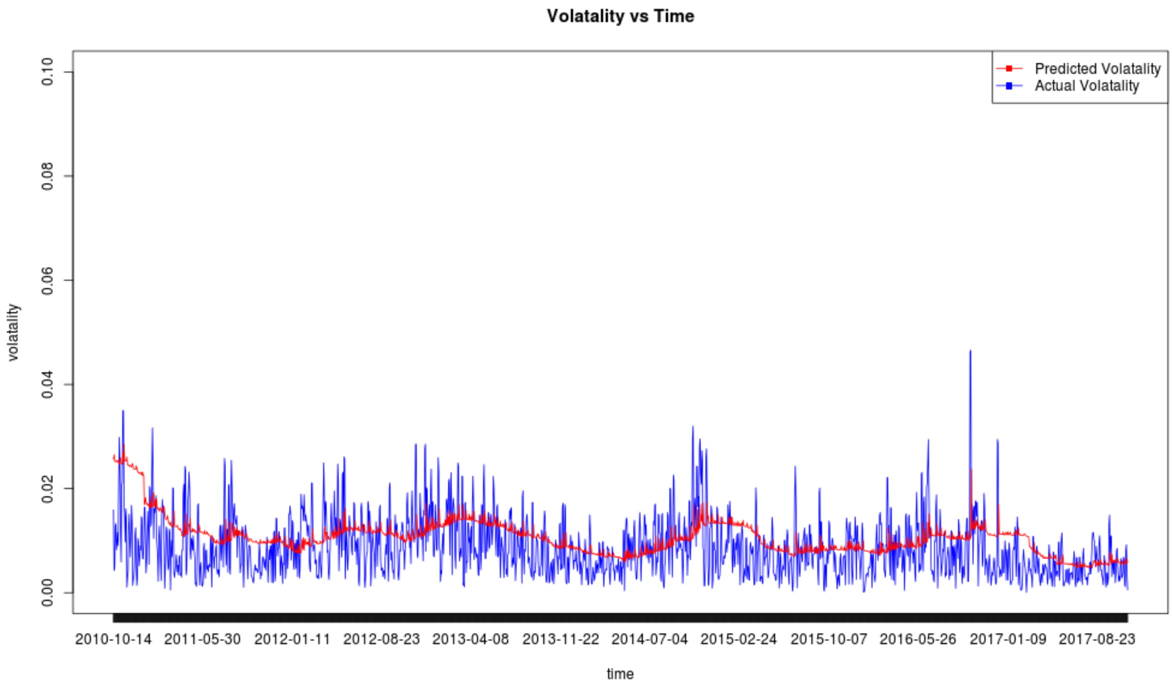


Figure 15 Volatility prediction for Mid-Cap 50

Since every model is model is based on moving windows thus the parameters are estimate continuously

3.7 Error Tables (Actual vs Predicted)

No. of days Ahead prediction	Distribution Model	EWMA	GARCH	Historical
1	0.005813	0.00680	0.006820	0.005789
10	0.003723	0.003252	0.003579	0.003776
30	0.003411	0.003303	0.003326	0.003519
60	0.003442	0.004094	0.003348	0.003581

3.8 Error Tables (Implied Vs Predicted Volatility)

	Distribution Model	EWMA	GARCH	Historical
Implied	0.001980	0.002122	0.0019078	0.002095

4 Conclusions

From the above tables one can say that parametric models tend to perform better than basic historical volatility models. GARCH has outperformed all the other models most of time for except when forecasting was being made for 1 day ahead volatility. The distribution model even though being a highly parametric model has outperformed classical models like EWMA and Historical volatility model. EWMA does not incorporate volatility clustering due to the absence of long term volatility whereas the parametric model readjusts volatility according to the previous returns. Thus eliminating the need for separate volatility model. In short one can assume that it is a nonlinear (Distribution Fitting) model built on the past values which in essence a similar approach to GARCH.

5 Future work

- In the next semester we will try to build a pricing model based on these results as well as several different hedging strategies based on volatility.
- Trading strategy based on GARCH volatility and Distribution modelled volatility.
- We will also try to build a volatility model using chi-square distribution, since returns are sampled from normal distribution and sum of square of Normal distribution is chi-square also sum of square of returns is also a proxy volatility.

6 References

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